(a) $2+5+8+\ldots+(3n-1)=\frac{1}{2}n(3n+1), \ \forall n \geq 1$

$n=1 \rightarrow 2=2$ ✓

$n=p \rightarrow 2+5+8 \ldots + (3p-1) = \frac{1}{2}p(3p+1)$, Hi

$n=p+1 \rightarrow 2+5+8 \ldots + (3p-1)+(3p+2) = \frac{1}{2}(p+1)(3p+4)$, HQD

Demo

$2+5+8 \ldots + (3p+2)$

$2+5+8 \ldots + (3p-1)+(3p+2)$

$\frac{1}{2}p(3p+1) + 3p+2$

$\frac{1}{2}\left(p(3p+1)+2(3p+2)\right)$

$\frac{1}{2}(3p^2+p+6p+4)$

$\frac{1}{2}(3p^2+7p+4)$

$$\begin{array}{ccl} 3p & \times & 4 = 4p \\ p & & 1 = \underline{3p} \\ & & \quad 7p \end{array}$$

$\frac{1}{2}(3p+4)(p+1)$ //

(b) $1 \cdot 1!+2 \cdot 2!+3 \cdot 3!+\ldots+n \cdot n!=(n+1)!-1, \ \forall n \geq 1$

$n=1 \rightarrow 1=1$

$n=p \rightarrow 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + p \cdot p! = (p+1)! - 1$, Hi

$n=p+1 \rightarrow 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + (p \cdot p!) + (p+1) \cdot (p+1)! = (p+2)! - 1$, HQD

Demo

$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + (p \cdot p!) + (p+1) \cdot (p+1)!$

$(p+1)! - 1 + (p+1) \cdot (p+1)!$

$(p+1)!(1 + p+1) - 1$

$(p+1)!((p+2))-1$

$(p+1)! \cdot (p+2) - 1$

$(p+2)! - 1$ //

(c) $1^2 - 2^2 + 3^2 - \ldots + (-1)^{n+1}n^2 = \dfrac{(-1)^{n+1}n(n+1)}{2}, \quad \forall n \geq 1$

$n = 1 \rightarrow 1 = 1$

$n = p \quad 1^2 - 2^2 - 3^2 \ldots + (-1)^{p+1} \cdot p^2 = \dfrac{(-1)^{p+1} \cdot p \cdot (p+1)}{2}$, Hi

$n = p+1 \quad 1^2 - 2^2 - 3^2 \ldots + (-1)^{p+1} \cdot p^2 + (-1)^{p+2} \cdot (p+1)^2 = \dfrac{(-1)^{p+2} \cdot (p+1) \cdot (p+2)}{2}$, HQD

Demo

$1^2 - 2^2 - 3^2 \ldots + (-1)^{p+1} \cdot p^2 + (-1)^{p+2} \cdot (p+1)^2$

$\dfrac{(-1)^{p+1} \cdot p \cdot (p+1)}{2} + (-1)^{p+2} \cdot (p+1)^2$, Hi

$$\frac{(-1)^{p+1} \cdot p \cdot (p+1) + 2 \cdot (-1)^{p+2} \cdot (p+1)^2}{2}$$

$$\frac{(-1)^{p+1} \cdot p \cdot (p+1) + 2 (-1)^{p+1} \cdot (-1) \cdot (p+1)^2}{2}$$

$$\frac{(-1)^{p+1} (p+1) \left[p + 2(-1)(p+1) \right]}{2}$$

$$\frac{(-1)^{p+1} (p+1) \left[p - 2p - 2 \right]}{2}$$

$$\frac{(-1)^{p+1} (p+1) \left[-p - 2 \right]}{2}$$

$$\frac{(-1)^{p+1} (p+1) (-1) (p+2)}{2}$$

$$\boxed{\frac{(-1)^{p+2} (p+1)(p+2)}{2}} \; //$$

$$2$$

(d) $\frac{1}{2^2-1}+\frac{1}{3^2-1}+\ldots+\frac{1}{(n+1)^2-1}=\frac{3}{4}-\frac{1}{2(n+1)}-\frac{1}{2(n+2)},\quad \forall n\geq 1$

$$n=1 \to \frac{1}{3}=\frac{1}{3} \checkmark$$

$$n=p \quad \frac{1}{2^2-1}+\frac{1}{3^2-1}+\ldots+\frac{1}{(p+1)^2-1}=\frac{3}{4}-\frac{1}{2(p+1)}-\frac{1}{2(p+2)}, \text{ Hi}$$

$$n=p+1 \quad \frac{1}{2^2-1}+\frac{1}{3^2-1}+\ldots+\frac{1}{(p+1)^2-1}+\frac{1}{(p+2)^2-1}=\frac{3}{4}-\frac{1}{2(p+2)}-\frac{1}{2(p+3)}, \text{ HQD}$$

Demo

$$\frac{1}{2^2-1}+\frac{1}{3^2-1}+\ldots+\frac{1}{(p+1)^2-1}+\frac{1}{(p+2)^2-1}$$

$$\frac{3}{4}-\frac{1}{2(p+1)}-\frac{1}{2(p+2)}+\frac{1}{(p+2)^2-1}, \text{ Hi}$$

$$\frac{3}{4}-\frac{1}{2(p+2)}+\frac{1}{(p+2)^2-1}-\frac{1}{2(p+1)}$$

$$\frac{3}{4}-\frac{1}{2(p+2)}+\frac{1}{((p+2)-1)((p+2)+1)}-\frac{1}{2(p+1)}$$

$$\frac{3}{4}-\frac{1}{2(p+2)}+\frac{1}{(p+1)(p+3)}-\frac{1}{2(p+1)}$$

$$\downarrow$$

$$\frac{1}{(p+1)(p+3)}=\frac{A}{(p+1)}+\frac{B}{(p+3)}$$

$$1=A(p+3)+B(p+1)$$

$$p=-1 \to 1=2A \to A=\frac{1}{2}$$

$$p=-3 \to 1=-2B \to B=\frac{-1}{2}$$

$$\frac{1}{(p+1)(p+3)}=\frac{\frac{1}{2}}{\frac{(p+1)}{1}}-\frac{\frac{1}{2}}{\frac{(p+3)}{1}}$$

$$=\frac{1}{2(p+1)}-\frac{1}{2(p+3)}$$

$$\frac{3}{4} - \frac{1}{2(p+2)} + \cancel{\frac{1}{2(p+1)}} - \frac{1}{2(p+3)} - \cancel{\frac{1}{2(p+1)}}$$

$$\boxed{\frac{3}{4} - \frac{1}{2(p+2)} - \frac{1}{2(p+3)}} \; //$$

(e) $2 + 6 + 18 + \ldots + 2 \cdot 3^{n-1} = 3^n - 1, \quad \forall n \geq 1$

$n=1 \longrightarrow 2=2$

$n=p \quad 2+6+18+\ldots+2\cdot 3^{p-1} = 3^p - 1$, H:

$n=p+1 \quad 2+6+18+\ldots 2\cdot 3^{p-1} + 2\cdot 3^p = 3^{p+1} - 1$, HQD

Demo

$2\cdot 3^{p-1} + 2\cdot 3^p$

$3^p - 1 + 2\cdot 3^p$

$3^p(1+2) - 1$

$\boxed{3^{p+1} - 1}$ //

(f) $(-1)^1 + (-1)^2 + (-1)^3 + \ldots + (-1)^n = \frac{1}{2}\left((-1)^n - 1\right), \quad \forall n \geq 1$

$n=1 \qquad -1=-1$

$n=p \quad (-1)^1 + (-1)^2 + (-1)^3 + \ldots + (-1)^p = \frac{1}{2}\left[(-1)^p - 1\right]$, H:

$n=p+1 \quad (-1)^1 + (-1)^2 + (-1)^3 + \ldots + (-1)^p + (-1)^{p+1} = \frac{1}{2}\left[(-1)^{p+1} - 1\right]$, HQD

Demo

$(-1)^1 + (-1)^2 + (-1)^3 + \ldots + (-1)^p + (-1)^{p+1}$

$\frac{1}{2}\left[(-1)^p - 1\right] + (-1)^{p+1}$, H:

$$\frac{(-1)^p}{2} - \frac{1}{2} + \frac{2(-1)^{p+1}}{2}$$

$$\frac{(-1)^p - 1 + 2(-1)^{p+1}}{2}$$

$$\frac{1}{2}\left[(-1)^p - 1 + 2(-1)^{p+1}\right]$$

$$\frac{1}{2}\left[(-1)^p + 2(-1)^{p+1} - 1\right]$$

$$2\left[\qquad \right]$$

$$\frac{1}{2}\left[(-1)^p \left(1+2(-1) \right) - 1 \right]$$

$$\frac{1}{2}\left[(-1)^p (-1) - 1 \right]$$

$$\boxed{\frac{1}{2}\left[(-1)^{p+1} - 1 \right]} \; //$$

(g) $\dfrac{1}{1\cdot2\cdot3}+\dfrac{1}{2\cdot3\cdot4}+\dfrac{1}{3\cdot4\cdot5}+\cdots+\dfrac{1}{n(n+1)(n+2)}=\dfrac{n(n+3)}{4(n+1)(n+2)}, \quad \forall n \geq 1$

$n=1 \qquad \frac{1}{6} = \frac{1}{6}$

$n=p \qquad \frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\cdots+\frac{1}{p(p+1)(p+2)} = \frac{p(p+3)}{4(p+1)(p+2)}$, HI

$n=p+1 \qquad \frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\cdots+\frac{1}{p(p+1)(p+2)}+\frac{1}{(p+1)(p+2)(p+3)} = \frac{(p+1)(p+4)}{4(p+2)(p+3)}$, HQD

Demo

$$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\cdots+\frac{1}{p(p+1)(p+2)}+\frac{1}{(p+1)(p+2)(p+3)}$$

$$\frac{p(p+3)}{4(p+1)(p+2)}+\frac{1}{(p+1)(p+2)(p+3)}$$

$$\frac{p(p+3)^2+4}{4(p+1)(p+2)(p+3)}$$

$$\frac{p(p^2+6p+9)+4}{4(p+1)(p+2)(p+3)}$$

$$\frac{p^3+6p^2+9p+4}{4(p+1)(p+2)(p+3)} \longrightarrow p^3+6p^2+9p+4$$

$$\begin{array}{rrrr|r} 1 & 6 & 9 & 4 & \\ & -1 & -5 & -4 & -1 \\ \hline 1 & 5 & 4 & & \end{array}$$

$$(p^2+5p+4)(p+1)$$

$$p \times 4 = 4p, \quad p \times 1 = p$$

$$(p+4)(p+1)(p+1)$$

$$\frac{(p+4)(p+1)\cancel{(p+1)}}{4\cancel{(p+1)}(p+2)(p+3)}$$

$$\boxed{\frac{(p+1)(p+4)}{4(p+2)(p+3)}} \; //$$

(h) $2+5+13+\cdots+(2^{n-1}+3^{n-1})=\frac{1}{2}(2^{n+1}+3^{n}-3), \quad \forall n \geq 1$

(ii) $2+5+13+\ldots+(2^{n-1}+3^{n-1})=\frac{1}{2}(2^{n+1}+3^{n}-3), \quad \forall n\geq 1$

$n=1 \qquad 2=2$

$n=p \qquad 2+5+13+\ldots+(2^{p-1}+3^{p-1})=\frac{1}{2}(2^{p+1}+3^{p}-3)$, Hi

$n=p+1 \qquad 2+5+13+\ldots+(2^{p-1}+3^{p-1})+(2^{p}+3^{p})=\frac{1}{2}(2^{p+2}+3^{p+1}-3)$, HQD

Demo

$$2+5+13+\ldots+(2^{p-1}+3^{p-1})+(2^{p}+3^{p})$$

$$\frac{1}{2}(2^{p+1}+3^{p}-3)+(2^{p}+3^{p}) \quad \text{, Hi}$$

$$\frac{2^{p+1}}{2}+\frac{3^{p}}{2}-\frac{3}{2}+\frac{2(2^{p}+3^{p})}{2}$$

$$\frac{2^{p+1}}{2}+\frac{3^{p}}{2}-\frac{3}{2}+\frac{2\cdot 2^{p}}{2}+\frac{2\cdot 3^{p}}{2}$$

$$\frac{2^{p+1}+3^{p}-3+2^{p+1}+2\cdot 3^{p}}{2}$$

$$\frac{1}{2}\left(2^{p+1}+3^{p}-3+2^{p+1}+2\cdot 3^{p}\right)$$

$$\frac{1}{2}\left(2(2^{p+1})+3^{p}(1+2)-3\right)$$

$$\boxed{\frac{1}{2}\left(2^{p+2}+3^{p+1}-3\right)} \qquad //$$

(i) $1\cdot 5+2\cdot 5^{2}+3\cdot 5^{3}+\ldots+n\cdot 5^{n}=\dfrac{5+(4n-1)5^{n+1}}{16}, \quad \forall n\geq 1$

$n=1 \qquad 5=5$

$n=p \qquad 1\cdot 5+2\cdot 5^{2}+3\cdot 5^{3}+\ldots+p\cdot 5^{p}=\dfrac{5+(4p-1)\cdot 5^{p+1}}{16}$, Hi

$n=p+1 \qquad 1\cdot 5+2\cdot 5^{2}+3\cdot 5^{3}+\ldots+(p+1)\cdot 5^{p+1}=\dfrac{5+(4p+3)\cdot 5^{p+2}}{16}$, Hi

Demo

$$1\cdot 5+2\cdot 5^{2}+3\cdot 5^{3}+\ldots+(p+1)\cdot 5^{p+1}$$

$$\frac{5+(4p-1)\cdot 5^{p+1}}{16}+(p+1)\cdot 5^{p+1}$$

$$\frac{5+(4p-1)\cdot 5^{p+1}+16(p+1)\cdot 5^{p+1}}{16}$$

$$5+5^{p+1}(4p-1+16p+16)$$

$$16$$

$$\frac{5+5^{p+1}(20p+15)}{16}$$

$$\frac{5+5^{p+1}(5(4p+3)}{16}$$

$$\frac{5+5^{p+2}(4p+3)}{16} \quad //$$

(j) $a+(a+d)+(a+2d)+\ldots+[a+(n-1)d]=\dfrac{n\,[2a+(n-1)d]}{2}, \;\; \forall n \geq 1$

$k=1 \qquad a = a$

$k=p \qquad a+(a+d)+(a+2d)+\ldots+[a+(p-1)d) = \dfrac{p(2a+(p-1)d}{2}$, Hi

$k=p+1 \qquad a+(a+d)+(a+2d)+\ldots+[a+(p-1)d)+[a+(p)d] = \dfrac{(p+1)(2a+(p)d)}{2}$, HQI

Demo

$a+(a+d)+(a+2d)+\ldots+[a+(p-1)d)+[a+(p)d]$

$$\frac{p[(2a+(p-1)d]}{2}+a+(p)d \quad , \text{Hi}$$

$$\frac{p[2a+(p-1)d]+2a+2pd}{2}$$

$$\frac{p[2a+pd-d]+2a+2pd}{2}$$

$$\frac{2ap+p^2d\cancel{-pd}+2a+\cancel{2pd}}{2}$$

$$\frac{2ap+p^2d+2a+pd}{2}$$

$$\frac{pd(p+1)+2a(p+1)}{2}$$

$$\boxed{(p+1)(2a+pd)} \quad //$$

(k) $1 \cdot 3 \cdot 5 \cdots (2n-1) = \dfrac{(2n)!}{2^n \cdot n!}, \quad \forall n \geq 1$

$k = 1 \qquad 1 = 1$

$k = p \qquad 1 \cdot 3 \cdot 5 \cdots (2p-1) = \dfrac{(2p)!}{2^p \cdot p!}$, Hi

$k = p+1 \qquad 1 \cdot 3 \cdot 5 \cdots (2p-1) \cdot (2p+1) = \dfrac{(2p+2)!}{2^{p+1} \cdot (p+1)!}$, HQD

$$= \frac{(2p+2)(2p+1)(2p)!}{2^{p+1}(p+1)!}$$

Demo

$1 \cdot 3 \cdot 5 \cdots (2p-1) + (2p+1)$

$$\frac{(2p)!}{2^p \cdot p!} \cdot (2p+1)$$

$$= \frac{\cancel{2}(p+1)(2p+1)(2p)!}{\cancel{2} \cdot 2^p (p+1)!}$$

$$\frac{\cancel{(p+1)}(2p+1)(2p)!}{2^p \cancel{(p+1)}(p)!}$$

$$\frac{(2p+1)(2p)!}{2^p \cdot p!}$$

$$\boxed{\frac{2p!}{2^p \cdot k!} \cdot (2p+1)}$$

//

(l) $\left(1-\dfrac{1}{4}\right)\left(1-\dfrac{1}{9}\right)\cdots\left(1-\dfrac{1}{n^2}\right) = \dfrac{n+1}{2n}, \quad \forall n \geq 2$

$k = 2 \qquad \frac{3}{4} = \frac{3}{4}$

$k = p \qquad \left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\cdots\left(1-\frac{1}{p^2}\right) = \frac{p+1}{2p}$, Hi

$k = p+1 \qquad \left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\cdots\left(1-\frac{1}{p^2}\right)\left(1-\frac{1}{(p+1)^2}\right) = \frac{(p+2)}{2(p+1)}$, HQD

Demo

$$\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\cdots\left(1-\frac{1}{p^2}\right) \left(1-\frac{1}{(p+1)^2}\right)$$

$$\left(\frac{p+1}{2p}\right)\left(1-\frac{1}{(p+1)^2}\right) \quad , \text{Hi}$$

$$\left(\frac{\cancel{p+1}}{2p}\right)\left(\frac{(p+1)^2-1}{\cancel{(p+1)}^2}\right)$$

$$\frac{(p+1)^2-1}{2p(p+1)}$$

$2p(p+1)$

$$\frac{((p+1)-1)((p+1)+1)}{2p(p+1)}$$

$$\frac{\cancel{p}(p+2)}{2\cancel{p}(p+1)}$$

$$\boxed{\frac{p+2}{2(p+1)}} \quad //$$

(m) $4\left(1+5+5^2+\ldots+5^n\right)+1=5^{n+1}$

$k=1 \qquad 5=5$

$k=p \quad 4(1+5+5^2+\ldots 5^p)+1=5^{p+1}$, Hi

$k=p+1 \quad 4(1+5+5^2+\ldots+5^p+5^{p+1})+1=5^{p+2}$, HQD

Demo

$4(1+5+5^2+\ldots+5^k)+4\cdot 5^{p+1}$

$5^{p+1}+4\cdot 5^{p+1}$, Hi!

$5^{p+1}(1+4)$

$5^{p+1}\cdot 5$

$\boxed{5^{p+2}}$ //

(n) $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\ldots+\frac{2n-1}{2^n}=3-\frac{2n+3}{2^n}, \ \forall n\geq 1$

$k=1 \qquad \frac{1}{2}=\frac{1}{2}$

$k=p \qquad \frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\ldots+\frac{2p-1}{2^p}=3-\frac{2p+3}{2^p}$, Hi

$k=p+1 \qquad \frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\ldots+\frac{2p-1}{2^p}+\frac{2p+1}{2^{p+1}}=3-\frac{2p+5}{2^{p+1}}$, HQD

Demo

$$\frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\ldots+\frac{2p-1}{2^p}+\frac{2p+1}{2^{p+1}}$$

$$3-\frac{2p+3}{2^p}+\frac{2p+1}{2^{p+1}}$$, Hi

$$3 + \frac{2p+1}{2^{p+1}} - \frac{2p+3}{2^{p}}$$

$$3 + \frac{2p+1-4p-6}{2^{p+1}}$$

$$3 - \frac{1}{2^{p+1}}\left(-2p-1+4p+6\right)$$

$$3 - \frac{1}{2^{p+1}}\left(2p+5\right)$$

$$\boxed{3 - \frac{2p+5}{2^{p+1}}} \quad //$$

4. Usando inducción matemática demuestre la validez de cada una de las siguientes igualdades.

(a) $\displaystyle\sum_{i=0}^{n} \frac{1}{(2i+1)(2i+3)} = \frac{n+1}{2n+3}$

$n=0 \qquad \frac{1}{3} = \frac{1}{3}$

$n=p \qquad \displaystyle\sum_{i=0}^{p} \frac{1}{(2i+1)(2i+3)} = \frac{p+1}{2p+3}$, Hi

$n=p+1 \qquad \displaystyle\sum_{i=0}^{p+1} \frac{1}{(2i+1)(2i+3)} = \frac{p+2}{2p+5}$, HQD

Demo

$$\sum_{i=0}^{p+1} \frac{1}{(2i+1)(2i+3)}$$

$$\sum_{i=0}^{p} \frac{1}{(2i+1)(2i+3)} + \frac{1}{(2p+3)(2p+5)}$$

$$\frac{p+1}{2p+3} + \frac{1}{(2p+3)(2p+5)} \text{ , Hi}$$

$$\frac{(2p+5)(p+1)+1}{(2p+3)(2p+5)}$$

$$\frac{2p^2+2p+5p+5+1}{(2p+3)(2p+5)}$$

$$\frac{2p^2+7p+6}{(2p+3)(2p+5)} \qquad \begin{array}{l} 2p \times 3 = 3p \\ p \quad\;\; 2 = \underline{4p} \\ \qquad\quad\;\; 7p \end{array}$$

$(2p+1)(2p+3)$ $\qquad$ $(2p+3)(p+2)$

$$\frac{\cancel{(2p+3)}(p+2)}{\cancel{(2p+3)}(2p+5)}$$

$$\boxed{\frac{p+2}{2p+5}} \quad //$$

(b) $\displaystyle\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

$k=1 \qquad \frac{1}{2} = \frac{1}{2}$

$k=p \qquad \displaystyle\sum_{i=1}^{p} \frac{1}{i(i+1)} = \frac{p}{p+1}$, Hi

$k=p \qquad \displaystyle\sum_{i=1}^{p+1} \frac{1}{i(i+1)} = \frac{(p+1)}{p+2}$, HQD

Demo

$$\sum_{i=1}^{p+1} \frac{1}{i(i+1)}$$

$$\sum_{i=1}^{p} \frac{1}{i(i+1)} + \frac{1}{(p+1)(p+2)}$$

$$\frac{p}{p+1} + \frac{1}{(p+1)(p+2)} \text{ , Hi}$$

$$\frac{p(p+2)+1}{(p+1)(p+2)}$$

$$\frac{p^2+2p+1}{(p+1)(p+2)}$$

$$\frac{\cancel{(p+1)}^2}{\cancel{(p+1)}(p+2)}$$

$$\boxed{\frac{p+1}{p+2}} \quad //$$

(c) $\sum_{i=1}^{k} \ln i = \ln(k!)$ sea $k = n$

$k=1 \quad 0 = 0$

$k=p \quad \sum_{i=1}^{p} \ln(i) = \ln(p!)$, Hi

$k=p+1 \quad \sum_{i=1}^{p+1} \ln(i) = \ln(p+1)!$, HQD

Demo

$\sum_{i=1}^{p+1} \ln(i)$

$\sum_{i=1}^{p} \ln(i) + \ln(p+1)$

$\ln(p!) + \ln(p+1)$

$\ln(p! \cdot (p+1))$

$\boxed{\ln(p+1)!}$ //

(d) $\sum_{i=1}^{n} \frac{1}{i(i+1)(i+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

$k=1 \quad \frac{1}{6} = \frac{1}{6}$

$k=p \quad \sum_{i=1}^{p} \frac{1}{i(i+1)(i+2)} = \frac{p(p+3)}{4(p+1)(p+2)}$, Hi

$k=p+1 \quad \sum_{i=1}^{p+1} \frac{1}{i(i+1)(i+2)} = \frac{(p+1)(p+4)}{4(p+2)(p+3)}$, HQD

Demo

$\sum_{i=1}^{p+1} \frac{1}{i(i+1)(i+2)}$

$\sum_{i=1}^{p} \frac{1}{i(i+1)(i+2)} + \frac{1}{(p+1)(p+2)(p+3)}$

$\frac{p(p+3)}{4(p+1)(p+2)} + \frac{1}{(p+1)(p+2)(p+3)}$

$\frac{p(p+3)^2 + 4}{4(p+1)(p+2)(p+3)}$

$p^3 + 6p^2 + 9p + 4$

$1 \quad 6 \quad 9 \quad 4 \;|$

$$\frac{p(p^2+6p+9)+4}{4(p+1)(p+2)(p+3)}$$

$$\begin{array}{cccc|c} & -1 & -5 & -4 & -1 \\ \hline 1 & 5 & 4 & 0 & \end{array}$$

$$(p^2+5p+4)(p+1)$$

$$(p+4)(p+1)(p+1)$$

$$\frac{p^3+6p^2+9p+4}{4(p+1)(p+2)(p+3)}$$

$$\frac{(p+4)\cancel{(p+1)}^2}{4\cancel{(p+1)}(p+2)(p+3)}$$

$$\boxed{\frac{(p+4)(p+1)}{4(p+2)(p+3)}} \quad //$$

(e) $\displaystyle\sum_{i=1}^{n} \log\left(\frac{i+1}{i}\right)^i = \log\frac{(n+1)^n}{n!}$

$n=1$ $\quad \log(2)=\log(2)$

$n=p$ $\quad \displaystyle\sum_{i=1}^{p} \log\left(\frac{i+1}{i}\right)^i = \log\left(\frac{(p+1)^p}{p!}\right)$, Hi

$n=p+1$ $\quad \displaystyle\sum_{i=1}^{p+1} \log\left(\frac{i+1}{i}\right)^i = \log\left(\frac{(p+2)^{p+1}}{(p+1)!}\right)$, HQD

Demo

$$\sum_{i=1}^{p+1} \log\left(\frac{i+1}{i}\right)^i$$

$$\sum_{i=1}^{p} \log\left(\frac{i+1}{i}\right)^i + \log\left(\frac{p+2}{(p+1)}\right)^{p+1}, \text{ Hi}$$

$$\log\left(\frac{(p+1)^p}{p!}\right) + \log\left(\frac{p+2}{(p+1)}\right)^{p+1}$$

$$\log\left(\frac{(p+1)^p}{p!} \cdot \frac{(p+2)^{p+1}}{(p+1)^{p+1}}\right)$$

$$\log\left(\cancel{(p+1)}^p \cdot (p+2)^{p+1}\right.$$

$$\log\left(\frac{[illegible]}{(p)!\,(p+1)\,\cancel{(p+1)^{p}}}\right)$$

$$\boxed{\log\left(\frac{(p+2)^{p+1}}{(p+1)!}\right)}$$

//