1) **(3 pts)** Analice la convergencia de la serie $\sum_{k=5}^{\infty} \frac{(-1)^k 3^{2k+1}}{8^k}$. En caso de ser convergente, determine su valor de convergencia.

$$\sum_{h=5}^{\infty} \frac{(-1)^h \cdot 3^{2h+1}}{8^h}$$

$$\sum_{h=5}^{\infty} \frac{(-1)^h \cdot (9)^h \cdot 3}{8^h}$$

$$3 \sum_{h=5}^{\infty} \frac{(-9)^h}{8^h}$$

$$3 \sum_{h=5}^{\infty} \left(\frac{-9}{8}\right)^h$$ , Serie geometrica con $|r| = \frac{9}{8} < 1$

$\boxed{\therefore \text{Diverge}}$

2) Para cada una de las series siguientes, determine si es convergente o divergente. Debe indicar los criterios que utiliza.

a) **(4 pts)** $\sum_{k=2}^{\infty} \frac{1}{k \cdot \ln k}$

Por criterio de la integral

$$f(x) = \frac{1}{x\ln(x)}$$

$$f'(x) = \frac{-1}{(x\ln(x))^2} \cdot \ln(x) + 1$$

$$= \frac{-\ln(x)+1}{x^2 \ln^2(x)} \qquad f \nexists \; x = 0$$

$$-\ln(x) + 1 = 0$$

$$\ln(x) = 1$$

$$x = e$$

| | $-\infty$ to $0$ | $0$ to $e$ | $e$ to $\infty$ |
|---|---|---|---|
| $(x\ln(x))^2$ | + | + | + |
| $-\ln(x)+1$ | + | + | − |
| $f'(x)$ | + | + | − |
| $f(x)$ | ↗ | ↘ | ↘ |

Decrece

$\therefore f(x)$ es decreciente

$$\int_2^{+\infty} \frac{1}{x\ln(x)}\,dx \to \int \frac{1}{x\ln(x)}\,dx \qquad u = \ln(x), \quad du = \frac{1}{x}\,dx$$

$$\int \frac{1}{u}\,du$$

$$\ln|\ln(x)| \Big|_2^{+\infty}$$

$\ln|\ln(x)|$

$$\lim_{x\to+\infty} \ln|\ln(x)| - \ln|\ln(2)|$$

$+\infty$

$\boxed{\therefore \text{ Diverge}}$

b) **(3 pts)** $\displaystyle\sum_{k=1}^{\infty} \frac{3k-2}{k^3-2k^2+4}$

Por crit de cmp al límite

$$\sum_{n=1}^{\infty} \frac{3n-2}{n^3-2n^2+4} \sim \frac{3n}{n^3} \sim \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2}, \; p \text{ serie con } p>1$$

converge

$$\lim_{n\to+\infty} \frac{3n-2}{n^3-2n^2+4} \cdot \frac{n^2}{}$$

$$\lim_{n\to+\infty} \frac{3n^3-2n^2}{n^3-6n^2+12}$$

$$\lim_{n\to+\infty} \frac{3n^3}{n^3} = 3$$

Como $b_n$ converge $\wedge$ $L \neq 0$

$\boxed{\text{Converge}}$

c) **(4 pts)** $\displaystyle\sum_{k=2}^{\infty} \frac{k^5\left(k^3+2\right)^{k+1}}{(2k+1)^{3k}}$

Por crit de la raiz

$$\lim_{n\to+\infty} \sqrt[n]{\frac{n^5 (n^3+2)^{n+1}}{(2n+1)^{3n}}}$$

$$\lim_{n\to+\infty} \frac{\underbrace{\cancel{\sqrt[n]{n^5}}}_{=1} \cdot \sqrt[\cancel{n}]{(n^3+2)^{\cancel{n}}} \cdot \underbrace{\cancel{\sqrt[n]{n^3+2}}}_{=1}}{\sqrt[\cancel{n}]{[(2n+3)^3]^{\cancel{n}}}}$$

$$\lim_{n\to+\infty} \frac{n^3+2}{(2n+3)^3}$$

$$\lim_{n\to+\infty} \frac{n^3+2}{8n^3+36n^2+54n+27}$$

$$\lim_{n\to+\infty} \frac{\cancel{n^3}}{8\cancel{n^3}} = \frac{1}{8} < 1$$

$$\boxed{\therefore \text{Converge}}$$

3) **(5 pts)** Determine el interior del intervalo de convergencia (sin analizar extremos) de:

$$\sum_{k=1}^{\infty} \frac{(k+3)! \cdot 3^k \cdot (2x-1)^k}{5 \cdot 7 \cdot 9 \cdots (2k+3)}$$

Por crit de razon

$$\lim_{n\to+\infty} \frac{(n+4)! \cdot 3^{n+1} \cdot (2x-1)^{n+1}}{\cancel{5 \cdot 7 \cdot 9 \cdots (2n+5)}(2n+6)} \cdot \frac{\cancel{5 \cdot 7 \cdot 9 \cdots (2n+3)}}{(n+3)! \cdot 3^n \cdot (2x-1)^n}$$

$$\lim_{n\to+\infty} \frac{(n+4) \cdot \cancel{(n+3)!} \cdot \cancel{3^n} \cdot 3 \cdot \cancel{(2x-1)^n} \cdot (2x-1)}{(2n+6) \cdot \cancel{(n+3)!} \cdot \cancel{3^n} \cdot \cancel{(2x-1)^n}}$$

$$|2x-1| \lim_{n\to+\infty} \frac{3n+12}{2n+6}$$

$$|2x-1| \lim_{n\to+\infty} \frac{3\cancel{n}}{2\cancel{n}} = \frac{3}{2}$$

$$\frac{3}{2}|2x-1| < 1$$

$$|2x-1| < \frac{2}{3}$$

$$-\frac{2}{3} < 2x-1 < \frac{2}{3}$$

$$\frac{1}{3} < 2x < \frac{5}{3}$$

$$\frac{1}{6} < x < \frac{5}{6}$$

$$\boxed{\text{Intervalo: } \left]\frac{1}{6}, \frac{5}{6}\right[}$$

4) **(4 pts)** Si se sabe que $\int_0^1 \frac{\ln(1+x)}{x}\,dx = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$, aproxime el valor de dicha integral, con un error menor que 0.001.

| $N$ | $N+1$ | $a_{n+1}\left(\frac{1}{(n+1)^2}\right)$ |
|---|---|---|
| 31 | 32 | $\frac{1}{32^2} = 9{,}76 \cdot 10^{-4} < 1 \cdot 10^{-3}$ |

Suma

$$\sum_{x=1}^{31} \frac{(-1)^{x+1}}{x^2} = 0{,}822$$

$$\boxed{\therefore \int_0^1 \frac{\ln(1+x)}{x} \approx 0{,}822}$$