- **Ejercicios combinados:**

  Determine si las siguientes series convergen o divergen y calcule su suma si son convergentes.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \cdot (n^2-n) \cdot 3^n + 5^n}{5^n \cdot (n^2-n)}$

$$\sum_{n=2}^{\infty} \frac{(-1)^n \cdot \cancel{(n^2-n)} \cdot 3^n}{5^n \cdot \cancel{(n^2-n)}} + \sum_{n=2}^{\infty} \frac{\cancel{5^n}}{\cancel{5^n}(n^2-n)}$$

$$\sum_{n=2}^{\infty} \left(\frac{-3}{5}\right)^n + \sum_{n=2}^{\infty} \frac{1}{n(n-1)}$$

$|r| = \frac{3}{5} < 1$

$$\frac{\left(\frac{-3}{5}\right)^2}{1 - -\frac{3}{5}} +$$

$$= \frac{9}{40}$$

$$\frac{1}{n(n-1)} = \frac{A}{n-1} + \frac{B}{n}$$

$$1 = A(n) + B(n-1)$$

$n = 1 \to 1 = A \to A = 1$

$n = 0 \to 1 = -B \to B = -1$

$$\sum_{2}^{\infty} \left[\frac{1}{n-1} - \frac{1}{n}\right]$$

$$1 - \lim_{n \to +\infty} \cancel{\frac{1}{n}} = 0$$

$$\frac{9}{40} + 1$$

$$\frac{49}{40}$$

$$\boxed{\text{Converge a } \frac{49}{40}}$$

2. $\displaystyle\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+1}\right) - \frac{2^{1-2n}}{3^{n-1}}$

$$\sum_{n=2}^{\infty} \left[\ln(n) - \ln(n+1)\right] - \sum_{n=2}^{\infty} \frac{2^{1-2n}}{3^{n-1}}$$

$\ln(2) - \lim \ln(n+1)$ $\quad \infty \quad 2^1 \cdot 2^{-2n}$

$$\lim_{n\to+\infty} \quad \sum_{n=2} \frac{\cdots}{3^n \cdot 3^{-1}}$$

$$+\infty \quad - 6 \sum_{n=2}^{\infty} \left(\frac{1}{12}\right)^n \qquad |r| = \frac{1}{12} < 1$$

$$6\left[\frac{\left(\frac{1}{12}\right)^2}{1-\frac{1}{12}}\right]$$

$$+\infty \quad - \quad \frac{1}{22}$$

$$\boxed{\text{Diverge}}$$

3. $$\sum_{m=2}^{\infty} \frac{1}{2^{2m+1}} - \frac{1}{m} + \frac{1}{m+1}$$

$$\sum_{n=2}^{\infty} \frac{1}{2^{2n+1}} - \sum_{n=2}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

$$\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{4}\right)^n - \frac{1}{2} - \lim_{n\to+\infty} \frac{1}{\cancel{n}+1} = 0$$

$$|r| = \frac{1}{4} < 1$$

$$\frac{1}{2}\left[\frac{\left(\frac{1}{4}\right)^2}{1-\frac{1}{4}}\right] - \frac{1}{2}$$

$$= \frac{1}{24} - \frac{1}{2}$$

$$= \frac{-11}{24}$$

$$\boxed{\text{Converge a } \frac{-11}{24}}$$

4. $$\sum_{n=2}^{\infty} \frac{1}{n(n+1)} - 5^{3-n}$$

$$\sum_{n=2}^{\infty} \frac{1}{n(n+1)} - \sum_{n=2}^{\infty} 5^3 \cdot 5^{-n}$$

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \qquad - 125 \sum_{n=2}^{\infty} \left(\frac{1}{5}\right)^n \qquad |r| = \frac{1}{5} < 1$$

$$1 = A(n+1) + B(n)$$

$$n=0 \to 1 = A \to A = 1 \qquad - 125 \left[\frac{\left(\frac{1}{5}\right)^2}{1-\frac{1}{5}}\right]$$

$$n=-1 \to 1 = -B \to B = -1$$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \qquad - \frac{25}{4}$$

$$\sum_{n=2}^{\infty} \left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$\frac{1}{2} - \lim_{n \to +\infty} \cancel{\frac{1}{n+1}} = 0$$

$$\frac{1}{2} - \frac{25}{4}$$

$$\frac{-23}{4}$$

$$\boxed{\text{Converge, Suma} = \frac{-23}{4}}$$

5. $\displaystyle\sum_{n=3}^{\infty} \cos\left(\frac{3\pi}{n+1}\right) - \cos\left(\frac{3\pi}{n+2}\right) - 2^{3-n}$

$$\sum_{n=3}^{\infty} \left[\cos\left(\frac{3\pi}{n+1}\right) - \cos\left(\frac{3\pi}{n+2}\right)\right] - \sum_{n=3}^{\infty} 2^{3-n}$$

$$\cos\left(\frac{3\pi}{3+1}\right) - \lim_{n \to +\infty} \cos\left(\cancel{\frac{3\pi}{n+2}}\right) = 0 - 8 \sum_{n=3}^{\infty} \left(\frac{1}{2}\right)^n \qquad |r| = \frac{1}{2} < 1$$

$$\cos\left(\frac{3\pi}{4}\right) - 8 \left[\frac{\left(\frac{1}{2}\right)^3}{1-\frac{1}{2}}\right]$$

$$\cos\left(\frac{3\pi}{4}\right) - 2$$

$\boxed{\text{Converge a ESU}}$

6. $\displaystyle\sum_{k=3}^{\infty} \frac{9\cdot 5^{-k}}{(-3)^{2-k}} - \frac{2}{k^2-1}$

$$\sum_{k=3}^{\infty} \frac{9\cdot 5^{-k}}{(-3)^{2-k}} - \sum_{k=3}^{\infty} \frac{2}{(k-1)(k+1)}$$

$$\sum_{k=3}^{\infty} \frac{\cancel{9}\cdot 5^{-k}}{\cancel{(-3)^2}\cdot(-3)^{-k}}$$

$$\sum_{k=3}^{\infty} \left(\frac{-3}{5}\right)^k$$

$$|r| = \frac{3}{5} < 1$$

Suma

$$\frac{\left(\frac{-3}{5}\right)^3}{1-\frac{-3}{5}}$$

$$= \frac{27}{200}$$

$$\frac{2}{(k-1)(k+1)} = \frac{A}{(k-1)} + \frac{B}{(k+1)}$$

$$2 = A(k+1) + B(k-\;)$$

$$k=1 \to 2=2A \to A=1$$

$$k=-1 \to 2=-2B \to B=-1$$

$$\frac{2}{(k-1)(k+1)} = \frac{1}{k-1} - \frac{1}{k+1}$$

$$\sum_{k=3}^{\infty}\left[\frac{1}{k-1} - \frac{1}{k}\right] + \sum_{k=3}^{\infty}\left[\frac{1}{k} - \frac{1}{k+1}\right]$$

$$\frac{1}{2} - \lim_{k\to+\infty} \cancel{\frac{1}{k}}^{=0} + \frac{1}{3} - \lim_{k\to+\infty} \cancel{\frac{1}{k+1}} = 0$$

$$\frac{1}{2} + \frac{1}{3}$$

$$\frac{5}{6}$$

$$\frac{27}{200} - \frac{5}{6}$$

$$\boxed{= \frac{-581}{600}}$$

1. Considere la serie convergente $\sum_{n=1}^{\infty} b_n$. Utilice el criterio de la divergencia para verificar

que la serie $\displaystyle\sum_{n=1}^{\infty} \frac{1+3n}{4n+b_n}$ diverge.

Si $\displaystyle\sum_{n=1}^{\infty} b_n$ converge entonces

$$\lim_{n \to +\infty} b_n = 0$$

Ahora

$$\sum_{n=1}^{\infty} \frac{1+3n}{4n+\cancel{b_n}} = 0$$

$$\sum_{n=1}^{\infty} \frac{3\cancel{n}}{4\cancel{n}} = \frac{3}{4} \neq 0$$

$$\boxed{\therefore \text{ Diverge}}$$

2. Determine todos los valores de b para que la serie $\displaystyle\sum_{n=5}^{\infty} \frac{n}{b^n} - \frac{n+1}{b^{n+1}}$ converja sea igual a $\dfrac{5}{b^5}$

$$\sum_{n=5}^{\infty} \frac{n}{b^5} - \frac{n+1}{b^{n+1}}$$

$$\frac{5}{b^5} - \lim_{n \to +\infty} \frac{n+\cancel{1}}{\cancel{b}^{n+1}} = 0$$

$$\boxed{b > 1}$$

3. Considere la siguiente serie $\displaystyle\sum_{n=3}^{\infty} \frac{(5p)^n}{2^{n+1}}$

a) Determine para qué valores de $p \in \mathbb{R}$, la serie es convergente.

$$\sum_{n=3}^{\infty} \frac{(5p)^n}{2^n \cdot 2}$$

$$\frac{1}{2} \sum_{n=3}^{\infty} \left(\frac{5p}{2}\right)^n$$ geome $|r|$ must be < 1

$$\left|\frac{5p}{2}\right| < 1$$

$$-1 < \frac{5p}{2} < 1$$

$$-2 < 5p < 2$$

$-2 \quad\quad 2$

$$\frac{-2}{5} < p < \frac{2}{5}$$

$$\boxed{\text{Converge en } x \in \left]\frac{-2}{5}, \frac{2}{5}\right[}$$

b) Para los valores de p donde la serie converge, determine el valor de la suma en términos de p.

$$\frac{1}{2} \sum_{n=3}^{\infty} \left(\frac{5p}{2}\right)^n \qquad |r| = \frac{5p}{2}$$

$$\frac{1}{2} \cdot \left[\frac{\left(\frac{5p}{2}\right)^3}{1-\frac{5p}{2}}\right]$$

$$\frac{1}{2} \left[\frac{\frac{125p^3}{\cancel{8}}}{\frac{2-5p}{\cancel{2}}}\right]$$

$$\frac{1}{2} \left[\frac{125p^3}{8-20p}\right]$$

$$\boxed{\frac{125p^3}{16-40p}}$$

4. Considere la serie $\displaystyle\sum_{n=1}^{\infty} \frac{(2p^2)^{n+1}}{(3p)^{n-1}}$, donde p es constante y $p \neq 0$

a) Determine para qué valores de $p \in \mathbb{R}$, la serie es convergente.

$$\sum_{n=1}^{\infty} \frac{(2p^2)^n \cdot (2p^2)}{(3p)^n \cdot (3p)^{-1}}$$

$$6p^3 \sum_{n=1}^{\infty} \left(\frac{2\cancel{p^2}}{3\cancel{p}}\right)^n$$

$$6p^3 \sum_{n=1}^{\infty} \left(\frac{2p}{3}\right)^n \qquad |r| = \left|\frac{2p}{3}\right| < 1$$

$$-1 < \frac{2p}{3} < 1$$

$$-3 < 2p < 3$$

$$\frac{-3}{2} < p < \frac{3}{2}$$

$$\boxed{\text{Converge para } p \in \left]\frac{-3}{2}, \frac{3}{2}\right[}$$

$b)$ Para los valores de p donde la serie converge, determine el valor en términos de p.

$$6p^3\left[\frac{\left(\frac{2p}{3}\right)^1}{1-\frac{2p}{3}}\right]$$

$$6p^3\left[\frac{\frac{2p}{\cancel{3}}}{\frac{3-2p}{\cancel{3}}}\right]$$

$$\frac{12p^4}{3-2p}$$

$$\sum_{k=2}^{\infty}\frac{3^{k+2}-2\cdot 5^{k-1}}{7^{k+1}}$$

$$\sum_{k=2}^{\infty}\frac{3^{k+2}}{7^{k+1}}-\sum_{k=2}^{\infty}\frac{2\cdot 5^{k-1}}{7^{k+1}}$$

$$\frac{9}{7}\sum_{k=2}^{\infty}\left(\frac{3}{7}\right)^k-\frac{2}{35}\sum_{k=2}^{\infty}\left(\frac{5}{7}\right)^k$$

$$\frac{9}{7}\left[\frac{\left(\frac{3}{7}\right)^2}{1-\frac{3}{7}}\right]-\frac{2}{35}\left[\frac{\left(\frac{5}{7}\right)^2}{1-\frac{5}{7}}\right]$$

$$=\boxed{\frac{61}{196}}$$

a) $$\sum_{n=1}^{\infty}\frac{2}{(2n+1)(2n+5)}$$

$$\frac{2}{2n+1}=\frac{A}{2n+1}+\frac{B}{2n+5}$$

$$2=A(2n+5)+B(2n+1)$$

$$n=\frac{-1}{2}\to 2=4A\to A=\frac{1}{2}$$

$$n=\frac{-5}{2}\to 2=-4B\to B=\frac{-1}{2}$$

$$\frac{1}{2}\left[\sum^{\infty}\left(\frac{1}{\ }-\frac{1}{\ }\right)+\sum^{\infty}\left(\frac{1}{\ }-\frac{1}{\ }\right)\right]$$

$$2\left[\sum_{n=1}\left(\frac{}{2n+1} \quad \frac{}{2n+3}\right) \quad \sum_{n=1}\left(\frac{}{2n+3} \quad \frac{}{2n+5}\right)\right]$$

$$\frac{1}{2}\cdot\left(\frac{1}{3}-\lim_{n\to+\infty}\cancel{\frac{1}{2n+3}}^{=0}\right)+\frac{1}{2}\left(\frac{1}{5}-\lim_{n\to+\infty}\cancel{\frac{1}{2n+5}}^{=0}\right)$$

$$\frac{1}{6}+\frac{1}{10}$$

$$\boxed{\frac{4}{15}}$$