Convergencia de sucesiones

$\lim\limits_{n\to+\infty} a_n$, a_n una sucesión , $+\infty$ Diverge

K, Converge

→ Número

Teoremas

→ Dominancia

$$\lim_{n\to+\infty}\frac{3n^2+\cancel{6n}}{2n^3+\cancel{7n}} = \lim_{n\to+\infty}\frac{3n^2}{2n^3}$$

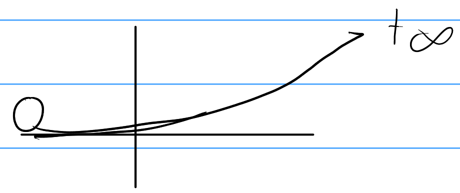
$$\frac{\pm\infty}{K}=\pm\infty \qquad \frac{K}{\pm\infty}=0$$

Se puede usar l'hôpital

I) Es $\left\{\dfrac{2^n}{n^2}\right\}$ Convergente?

$$f(x)=\frac{2^x}{x^2}$$

a^x



$$\lim_{x\to+\infty}\frac{2^x}{x^2} \qquad \frac{2^{+\infty}=+\infty}{+\infty^2=+\infty}\text{, l'hôpital}$$

$$\lim_{x\to+\infty}\frac{2^x\ln(2)}{2x}$$

$\longrightarrow a^x = a^x\cdot\ln(a)$

$\longrightarrow x^n = n\cdot x^{n-1}$

$$\lim +\infty = +\infty$$

$$\lim_{x\to+\infty} \frac{2^x \ln(2)}{2x} = \frac{+\infty}{+\infty} \circ \text{l'hôpital}$$

$$\lim f(x) = \lim f'(x)$$

$$\lim_{x\to+\infty} \frac{2^x \ln(2)\cdot\ln(2)}{2}$$

$$\frac{2^{+\infty}\cdot\ln(2)\cdot\ln(2)}{2}$$

$$\frac{+\infty}{2}$$

$$+\infty$$

$$\boxed{\therefore \text{La sucesion diverge}}$$

2) $\left\{ \dfrac{2p^3-54}{3p-p^2} \right\}$

$$\lim_{x\to+\infty} \frac{2x^3 \cancel{-54}}{\cancel{3x}-x^2}$$

$$\lim_{x\to+\infty} \frac{2\cancel{x^3}}{-\cancel{x^2}} = -2x$$

$$= -2\cdot +\infty$$

$$= +\infty$$

$\therefore$ Diverge

3) $\left\{ \dfrac{2x^2+3x}{5x^2-2x+2} \right\}$

$$\lim_{x \to +\infty} \frac{2x^2 \cancel{+3x}}{5x^2 \cancel{-2x+2}}$$

$$\lim_{x \to +\infty} \frac{2\cancel{x^2}}{5\cancel{x^2}} = \frac{2}{5} \quad \therefore \boxed{\text{Converge a } \frac{2}{5}}$$

4) $\left\{ \dfrac{\cos(\pi x)}{2^x} \right\}$

Ambito de $\underbrace{\operatorname{sen}(x) \ \text{o} \ \cos(x)}_{x}$

$$-1 \le x \le 1$$

Ambito de $\underbrace{\operatorname{sen}(x)^2 \ \text{o} \ \cos(x)^2}_{x}$

$$0 \le x \le 1$$

$$\lim_{x \to +\infty} \frac{\cos(\pi x)}{2^x}$$

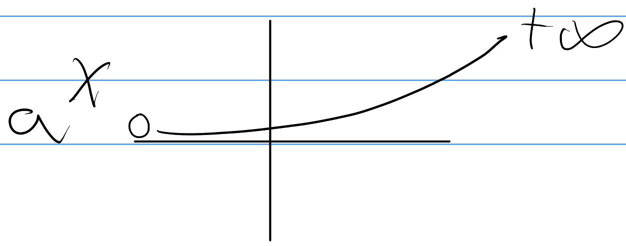
Teorema del Sandwich

$$-1 \le \cos(\pi x) \le 1$$

$$\frac{-1}{2^x} \le \frac{\cos(\pi x)}{2^x} \le \frac{1}{2^x}$$

Puede agarrar el que quiera

$$\lim_{x \to +\infty} \frac{1}{2^x} = \frac{1}{2^{+\infty}} = \frac{1}{+\infty} = 0$$ ∴ Converge



5) $\left\{ \dfrac{\cos(x)+1}{x^2} \right\}$, $-1 \le \cos \le 1$

$$-1 \le \cos(x) \le 1$$

$$-1+1 \le \cos(x)+1 \le 1+1$$

$$0 \le \cos(x)+1 \le 2$$

$$0 \le \frac{\cos(x)+1}{x^2} \le \frac{2}{x^2}$$

$$\lim_{x \to +\infty} \frac{2}{x^2} = \frac{2}{+\infty^2} = \frac{2}{+\infty} = 0$$

$\therefore$ Converge

Sucesion $1, 2, 3, 4 \ldots$

Series $1+2+3 \ldots$

$$\sum_{i=p}^{\infty} P(x) \pm Q(x) = \sum_{i=p}^{\infty} P(x) \pm \sum_{i=p} Q(x)$$

$$\sum_{i=p}^{\infty} k \cdot P(x) = k \sum_{i=p}^{\infty} P(x)$$

Serie geometrica

$$\sum_{n=p}^{\infty} r^n, \quad r \text{ es un numero}, \quad 2^n, \left(\frac{1}{2}\right)^n$$

$$1^n = 1, \quad 1^{1000} = 1$$

$$\frac{1}{2^n} = \frac{1^n}{2^n} = \left(\frac{1}{2}\right)^n, \quad r = \frac{1}{2}$$

Converge $|r| < 1$ Converge a $\dfrac{r^p}{1-r}$

Diverge $|r| \geq 1$ Diverge

Solo la geometrica y la telescopica son las que piden valores de convergencia o suma

valor de convergencia = Suma

1)

$$\sum_{n=1}^{\infty} \frac{3^{n+1}}{5\cdot 2^{-n}}$$

$$\sum_{n=1}^{\infty} \frac{3^{n}\cdot 3^{1}}{5\cdot 2^{-n}}$$

$$\sum_{r=p}^{\infty} k\cdot f(x) = k \sum_{r=p}^{\infty} f(x)$$

$$\frac{3}{5} \sum_{n=1}^{\infty} \frac{3^{n}}{2^{-n}}$$

$2^{1}=2^{1}$

$2^{-1}=\frac{1}{2}$,

$$\boxed{x^{-n} = \frac{1}{x^{n}}}$$

$2^{-n} = \frac{1}{2^{n}}$

$$\frac{3}{5} \sum_{n=1}^{\infty} \frac{\frac{3^{n}}{1}}{\frac{1}{2^{n}}}$$

$(a\cdot b)^{x} = a^{x}\cdot b^{x}$

$$\frac{3}{5} \sum_{n=1}^{\infty} \frac{3^{n}\cdot 2^{n}}{1\cdot 1}$$

$3^{n}\cdot 2^{n} = (3\cdot 2)^{n} = 6^{n}$

$$\frac{3}{5} \sum_{n=1}^{\infty} 6^{n},$$

$|r| = |6| = 6 \geq 1$

$\boxed{\therefore \text{Diverge}}$

2. $\sum_{k=3}^{\infty} \frac{4^{k+1}}{5^k}$

$$\sum_{k=3}^{\infty} \frac{4^{k+1}}{5^k}$$

$$\sum_{k=3}^{\infty} \frac{4^k \cdot 4^1}{5^k}$$

$$4 \sum_{k=3}^{\infty} \frac{4^k}{5^k} \qquad \left(\frac{4}{5}\right)^k = \frac{4^k}{5^k}$$

$$4 \cdot \sum_{k=3}^{\infty} \left(\frac{4}{5}\right)^k , \quad \left|\frac{4}{5}\right| < 0{,}8$$

$0{,}8 < 1$

Converge $\qquad \frac{r^p}{1-r}$

$$4 \cdot \left[\frac{\left(\frac{4}{5}\right)^3}{1-\frac{4}{5}}\right] = \boxed{\frac{256}{25}}$$

$\boxed{\therefore \text{ Converge y su suma es } \frac{256}{25}}$

5. $\displaystyle\sum_{k=-1}^{\infty} \frac{5-2^{k+1}}{3^{2k}}$

$(x^a)^b = x^{a \cdot b}$

$3^{2k} = (3^2)^k$
$= 9^k$

$$\sum_{k=-1}^{\infty} \frac{5-2^{k+1}}{9^k}$$

$$\sum_{k=-1}^{\infty} \frac{5}{9^k} - \sum_{k=-1}^{\infty} \frac{2^{k+1}}{9^k}$$

		Resultado
Converge	Converge	Converge
Diverge	Converge	Diverge
Converge	Diverge	Diverge
Diverge	Diverge	Diverge

$$\sum_{k=-1}^{\infty} \frac{5}{9^k} - \sum_{k=-1}^{\infty} \frac{2^{k+1}}{9^k}$$

$$\sum_{k=-1}^{\infty} \frac{5}{9^k} - \sum_{k=-1}^{\infty} \frac{2^k \cdot 2}{9^k}$$

$$5\sum_{k=-1}^{\infty} \frac{1}{9^k} - 2\sum_{k=-1}^{\infty} \frac{2^k}{9^k}$$

$$5\sum_{k=-1}^{\infty}\frac{1}{9^k}-2\sum_{k=-1}^{\infty}\frac{2^k}{9^k}$$

$$5\sum_{k=-1}^{\infty}\frac{1^k}{9^k}-2\sum_{k=-1}^{\infty}\frac{2^k}{9^k}$$

$$5\sum_{k=-1}^{\infty}\left(\frac{1}{9}\right)^k-2\sum_{k=-1}^{\infty}\left(\frac{2}{9}\right)^k$$

$r=\frac{1}{9}$, $|r|=\left|\frac{1}{9}\right|=\frac{1}{9}$ $\quad$ $r=\frac{2}{9}$, $|r|=\left|\frac{2}{9}\right|=\frac{2}{9}$

$\Rightarrow \frac{1}{9}<1$, Converge $\quad$ $\Rightarrow \frac{2}{9}<1$, Converge

$$\frac{r^p}{1-r}$$

$$5\cdot\left[\frac{\left(\frac{1}{9}\right)^{-1}}{1-\frac{1}{9}}\right]-2\cdot\left[\frac{\left(\frac{2}{9}\right)^{-1}}{1-\frac{2}{9}}\right]$$

Converge, Suma= $\boxed{\frac{2187}{56}}$

2. Determine si $\sum_{n=3}^{\infty} \frac{9 \cdot 5^{-2n}}{(-3)^{2-n}}$ es convergente o divergente. En caso de ser convergente, determine el valor de convergencia. **[5 puntos]**

$a-b$

$$\sum_{n=3}^{\infty} \frac{9 \cdot 5^{-2n}}{(-3)^{2-n}} \quad (-3)^{2-n} \quad (-3)^2 \cdot (-3)^{-n} \quad 9 \cdot (-3)^{-n}$$

$x^a \cdot x^b = x^{a+b}$

$x^a \cdot x^{-b} = x^{a-b}$

$$\sum_{n=3}^{\infty} \frac{9 \cdot 5^{-2n}}{(-3)^2 \cdot (-3)^{-n}}$$

$x^{-a} \cdot x^b = x^{-a+b}$

$x^{-a} \cdot x^{-b} = x^{-a-b}$

$$\sum_{n=3}^{\infty} \frac{\cancel{9} \cdot 5^{-2n}}{\cancel{9} \quad (-3)^{-n}}$$

$$\sum_{n=3}^{\infty} \frac{5^{-2n}}{(-3)^{-n}} \quad (5^{-2})^n, \ (5^2)^{-n}$$

$(5^x)^{-1} = 5^{-x1}$

$$\sum_{n=3}^{\infty} \frac{(5^2)^{-n}}{(-3)^{-n}}$$

$\left(\frac{x}{y}\right)^{-1} = \frac{y}{x}$

$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

$$\sum_{n=3}^{\infty} \left[\left(\frac{25}{-3}\right)^{-1}\right]^n$$

$(x^a)^b \quad (x^a)^{-b}$

$x^{a \cdot b} \quad x^{a \cdot -b} = x^{-ab}$

$x^{ab} = (x^b)^a = (x^a)^b$

$x^{-ab} = (x^{ab})^{-1}, \ (x^a)^{-b}, \ (x^b)^{-a}$

$$\sum_{n=3}^{\infty} \left[\left(\frac{25}{-3}\right)^{-1}\right]^{n} \qquad \left(\frac{x}{y}\right)^{-1} = \frac{y}{x}$$

$$\sum_{n=3}^{\infty} \left(\frac{-3}{25}\right)^{n}, \quad r = \frac{-3}{25}, \quad |r| = \left|\frac{-3}{25}\right| = \frac{3}{25}$$

$= 0,12$

$$\frac{r^{p}}{1-r}$$

Ojo!!! NO se usa valor absoluto, por que la formula NO lo trae

$0,12 < 1$ ∴ Converge

$$r = \frac{-3}{25} \qquad p = 3 \qquad \frac{r^{p}}{1-r}$$

$$\frac{\left(\frac{-3}{25}\right)^{3}}{1 - -\frac{3}{25}} = \frac{-27}{17500}$$

3. Determine si las siguientes series convergen o divergen. En caso de alguna ser convergente, determine su valor de convergencia.

a) **[5 puntos]** $\sum_{n=3}^{\infty} \frac{(-2)^{n+3}+6}{5^n}$ $\frac{-2}{-1}$. Hay que dividirlas

$$\sum_{n=3}^{\infty} \frac{(-2)^{n+3}+6}{5^n}$$

$$\sum_{n=3}^{\infty} \frac{(-2)^n \cdot (-2)^3}{5^n} + \sum_{n=3}^{\infty} \frac{6}{5^n}$$

$$(-2)^3 \sum_{n=3}^{\infty} \frac{(-2)^n}{5^n} + 6 \sum_{n=3}^{\infty} \frac{1^n}{5^n}$$

$$-8 \sum_{n=3}^{\infty} \left(\frac{-2}{5}\right)^n + 6 \sum_{n=3}^{\infty} \left(\frac{1}{5}\right)^n$$

$r = \frac{-2}{5}$ $|r| = \frac{2}{5} < 1$ ✓ $\qquad$ $r = \frac{1}{5}$ $|r| = \frac{1}{5} < 1$

$p = 3$ $\qquad$ $p = 3$

Convergen a $\frac{r^p}{1-r}$

$$-8 \cdot \left[\frac{\left(-\frac{2}{5}\right)^3}{1 - -\frac{2}{5}}\right] + 6 \cdot \left[\frac{\left(\frac{1}{5}\right)^3}{1-\frac{1}{5}}\right]$$

$$\boxed{\frac{179}{350}}$$

(8) [4 puntos] Determine si la serie dada converge o diverge. En caso de converger, calcule su suma.

$$\sum_{n=3}^{\infty} \frac{\boxed{(-3)^{n+1}} - \boxed{(-6)^{n-1}}}{8^{n+3}} \to 2 \qquad \to 1 \quad , \text{Separar}$$

$$\sum_{n=3}^{\infty} \frac{(-3)^{n+1}}{8^{n+3}} - \sum_{n=3}^{\infty} \frac{(-6)^{n-1}}{8^{n+3}}$$

$$\sum_{n=3}^{\infty} \frac{(-3)^{n} \cdot (-3)^{1}}{8^{n} \cdot 8^{3}} - \sum_{n=3}^{\infty} \frac{(-6)^{n} \cdot (-6)^{-1}}{8^{n} \cdot 8^{3}}$$

$$\frac{-3}{8^3} \sum_{n=3}^{\infty} \frac{(-3)^n}{8^n} - \frac{(-6)^{-1}}{8^3} \sum_{n=3}^{\infty} \frac{(-6)^n}{8^n}$$

$$\frac{-3}{512} \sum_{n=3}^{\infty} \cdot \left(\frac{-3}{8}\right)^n + \frac{1}{3072} \sum_{n=3}^{\infty} \left(\frac{-6}{8}\right)^n$$

$$\underbrace{r = \frac{-3}{8},\ |r| = \frac{3}{8} < 1 \checkmark \quad p = 3 \qquad\qquad r = \frac{-6}{8},\ |r| = \frac{6}{8} < 1 \checkmark \quad p = 3}$$

Convergen a $\dfrac{r^p}{1-r}$

$$\frac{-3}{512}\left[\frac{\left(\frac{-3}{8}\right)^3}{1 - -\frac{3}{8}}\right] + \frac{1}{3072} \cdot \left[\frac{\left(-\frac{6}{8}\right)^3}{1 - -\frac{6}{8}}\right]$$

$$\boxed{= 0{,}00017} \checkmark$$