1) $\sum_{n=3}^{\infty} \frac{(3n+5)!}{20n-7}$

$\lim_{n \to +\infty} \dfrac{\frac{(3n+8)!}{20n+13}}{\frac{(3n+5)!}{20n-7}}$ (crit de razon

$\lim_{n \to +\infty} \frac{(3n+8)!\,(20n-7)}{(3n+5)!\,(20n+13)}$

$\lim_{n \to +\infty} \frac{(3n+8)(3n+7)(3n+6)\cancel{(3n+5)!}\,(20n-7)}{\cancel{(3n+5)!}\,(20n+13)}$

$\lim_{n \to +\infty} \frac{(3n+8)(3n+7)(3n+6)(20n-7)}{20n+13}$

$\lim_{n \to +\infty} \frac{(3n)(3n)(3n)\cancel{(20n)}}{\cancel{20n}}$ Por crit de dominancia

$\lim_{n \to +\infty} (3n)^3 = +\infty$

Diverge

2) $\sum_{n=1}^{\infty} \frac{(n+1) \cdot 2^n}{n!}$

$\lim_{n \to +\infty} \dfrac{\frac{(n+2) \cdot 2^{n+1}}{(n+1)!}}{\frac{(n+1) \cdot 2^n}{n!}}$

$$\lim_{n \to +\infty} \frac{(n+2)\cdot \cancel{2^n}\cdot 2\cdot \cancel{n!}}{(n+1)\cdot \cancel{2^n}\cdot (n+1)\cdot \cancel{n!}}$$

$$\lim_{n \to +\infty} \frac{2(n+2)}{(n+1)^2}$$

$$\lim_{n \to +\infty} \frac{2n+4}{n^2+2n+1}$$

$$\lim_{n \to +\infty} \frac{2\cancel{n}}{n^{\cancel{2}}}$$

$$\lim_{n \to +\infty} \frac{2}{\cancel{n}}^{=0} = 0$$

3) $$\sum_{n=1}^{\infty} \frac{4^{n-1} - n^4}{n! + 3^n}$$

$$\lim_{n \to +\infty} \frac{\dfrac{4^n - (n+1)^4}{(n+1)! + 3^{n+1}}}{\dfrac{4^{n-1} - n^4}{n! + 3^n}}$$ Crit de razon

$$\lim_{n \to +\infty} \frac{\left[4^n - (n+1)^4\right](n! + 3^n)}{\left[(n+1)! + 3^{n+1}\right]\cdot\left[4^{n-1} - n^4\right]}$$

$$\lim_{n \to +\infty} \frac{4^n \cdot n!}{\left[(n+1)(n)! + 3^{n+1}\right]\left[4^{n-1} - n^4\right]}$$ Por dominancia

$$\lim_{n \to +\infty} \frac{\cancel{4^n}\cdot \cancel{n!}}{(n+1)\cdot \cancel{n!}\cdot \cancel{4^n}\cdot 4^{-1}}$$ Por dominancia

$$\lim_{n\to+\infty} \frac{4}{n+1} = 0$$

$\boxed{\text{Converge}}$

9) $$\sum_{n=1}^{\infty} \frac{n^n}{3^n \cdot n!}$$

$$\lim_{n\to+\infty} \frac{\dfrac{(n+1)^{n+1}}{3^{n+1}\cdot(n+1)!}}{\dfrac{n^n}{3^n \cdot n!}}$$

$$\lim_{n\to+\infty} \frac{(n+1)^n \cdot \cancel{(n+1)} \cdot \cancel{3^n} \cdot \cancel{n!}}{\cancel{3^n} \cdot 3 \cdot \cancel{(n+1)} \cdot \cancel{n!} \cdot n^n}$$

$$\lim_{n\to+\infty} \frac{(n+1)^n}{3n^n}$$

$$\frac{1}{3}\lim_{n\to+\infty} \left(\frac{(n+1)}{n}\right)^n \qquad e^{\frac{1}{3}\lim\limits_{n\to+\infty} n\ln\left(\frac{n+1}{n}\right)}$$

$$e^1$$

$$\frac{1}{3}\lim_{n\to+\infty} n\ln\left(1+\frac{1}{n}\right)$$

$$\frac{1}{3}\lim_{n\to+\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}, \quad \frac{+\infty}{+\infty}, \text{ L'hôpital}$$

$$\frac{1}{3}\lim_{n\to+\infty} \frac{\dfrac{1}{1+\frac{1}{n}} \cdot \cancel{\dfrac{-1}{n}}}{\cancel{\dfrac{-1}{n}}}$$

$$\frac{1}{3}\lim_{n\to+\infty} \frac{1}{1+\cancel{\frac{1}{n}}=0} = \frac{1}{3}e = \frac{e}{3} < 1$$

$\boxed{\text{Converge}}$

5) $$\sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n}$$

$$\lim_{n\to+\infty} \frac{\frac{3^{n+1}\cdot(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\cdot n!}{n^n}}$$

$$\lim_{n\to+\infty} \frac{\cancel{3^n}\cdot 3\cdot \cancel{(n+1)}\cdot \cancel{n!}\cdot n^n}{(n+1)^n\cdot\cancel{(n+1)}\cdot \cancel{3^n}\cdot\cancel{n!}}$$

$$\lim_{n\to+\infty} \frac{3\cdot n^n}{(n+1)^n}$$

$$3 \lim_{n\to+\infty} \left(\frac{n}{n+1}\right)^n$$

$$\begin{array}{r|l} \cancel{n} & n+1 \\ \hline -\cancel{n}-1 & 1 \\ -1 & \end{array}$$

$$3 \lim_{n\to+\infty} \left(1-\frac{1}{n+1}\right)^n \qquad 3\, e^{\lim_{n\to+\infty} n\ln\left(1-\frac{1}{n+1}\right)}$$

$$e^L$$

$$3 \lim_{n\to+\infty} n\ln\left(1-\frac{1}{n+1}\right)$$

$$3 \lim_{n\to+\infty} \frac{\ln\left(1-\frac{1}{n+1}\right)}{\frac{1}{n}}, \ \frac{0}{0}, \ \text{l'hôpital}$$

$$3 \lim_{n\to+\infty} \frac{\frac{1}{1-\frac{1}{n+1}}\cdot - -\frac{1}{(n+1)^2}}{-\frac{1}{n^2}}$$

$$3 \lim_{n\to+\infty} \frac{\frac{1}{\cancel{n}(n+1)}}{\frac{-1}{\cancel{n^2}}}$$

$$3 \lim_{n\to+\infty} \frac{-n}{n+1}, \ \frac{0}{0} \ \text{l'hôpital}$$

$$3 \lim_{n\to+\infty} \frac{-1}{1} = -1$$

$$3 \cdot e^{-1}$$

$$\frac{3}{e} > 1$$

$$\boxed{\text{Diverge}}$$

8) $$\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{n!}$$

$$\lim_{n\to+\infty} \frac{\dfrac{\cancel{2 \cdot 4 \cdot 6 \cdots (2n)} \cdot (2n+2)}{(n+1)!}}{\dfrac{\cancel{2 \cdot 4 \cdot 6 \cdots (2n)}}{n!}}$$

$$\lim_{n\to+\infty} \frac{2\cancel{(n+1)} \cdot \cancel{n!}}{\cancel{(n+1)} \cdot \cancel{n!}} = 2 > 1 \qquad \boxed{\text{Diverge}}$$

9) $$\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n+1)}{(2n)!}$$

$$\lim_{n\to+\infty} \frac{\dfrac{\cancel{2 \cdot 4 \cdot 6 \cdots (2n+1)} \cdot (2n+3)}{(2n+2)!}}{\dfrac{\cancel{2 \cdot 4 \cdot 6 \cdots (2n+1)}}{(2n)!}}$$

$$\lim_{n\to+\infty} \frac{(2n+3)\cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}}$$

$$\lim \quad 2n+3$$

$$\lim_{n\to+\infty} \frac{\quad}{2n^2+2n+4n+2}$$

$$\lim_{n\to+\infty} \frac{2\cancel{n}}{2n^{\cancel{2}}}$$

$$\lim_{n\to+\infty} \frac{1}{n} = 0 < 1$$

$\boxed{\text{Converge}}$

10) $$\sum_{n=0}^{\infty} \frac{[1\cdot3\cdot5\cdots(2n-1)]\cdot n!}{(2n+1)!}$$

$$\lim_{n\to+\infty} \frac{\dfrac{[1\cdot3\cdot5\cdots(2n-1)(2n+1)](n+1)!}{(2n+3)!}}{\dfrac{[1\cdot3\cdot5\cdots(2n-1)]\cdot n!}{(2n+1)!}}$$

$$\lim_{n\to+\infty} \frac{\cancel{[1\cdot3\cdot5\cdots(2n-1)}(2n+1)]\cdot(n+1)\cdot\cancel{n!}\cdot\cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}\cdot\cancel{[1\cdot3\cdot5\cdots(2n-1)]}\cancel{n!}}$$

$$\lim_{n\to+\infty} \frac{(2n+1)\cancel{(n+1)}}{(2n+3)2\cancel{(n+1)}}$$

$$\lim_{n\to+\infty} \frac{2n+1}{4n+6}$$

$$\lim_{n\to+\infty} \frac{2\cancel{n}}{4\cancel{n}} = \frac{1}{2} < 1$$

$\boxed{\text{Converge}}$

11) $$\sum_{n=0}^{\infty} \frac{1\cdot4\cdot7\cdot(3n+1)}{(2n+1)!}$$

$$\lim_{n\to+\infty} \frac{\dfrac{\cancel{1\cdot 4\cdot 7\cdots(3n+1)}(3n+4)}{(2n+4)!}}{\dfrac{\cancel{1\cdot 4\cdot 7\cdots(3n+1)}}{(2n+2)!}}$$

$$\lim_{n\to+\infty} \frac{(3n+4)\cancel{(2n+2)!}}{(2n+4)(2n+3)\cancel{(2n+2)!}}$$

$$\lim_{n\to+\infty} \frac{3n+4}{4n^2+6n+8n+12}$$

$$\lim_{n\to+\infty} \frac{3\cancel{n}}{4\cancel{n}^2}$$

$$\lim_{n\to+\infty} \frac{3}{4\cancel{n}} \overset{=0}{=} 0 < 1$$

$$\boxed{\text{Converge}}$$

12) $$\sum_{n=0}^{\infty} \frac{1\cdot 4\cdot 7\cdots(3n+1)}{2^{n+1}\cdot(n+1)!}$$

$$\lim_{n\to+\infty} \frac{\dfrac{\cancel{1\cdot 4\cdot 7\cdots(3n+1)}(3n+4)}{2^{n+2}\cdot(n+2)!}}{\dfrac{\cancel{1\cdot 4\cdot 7\cdots(3n+1)}}{2^{n+1}\cdot(n+1)!}}$$

$$\lim_{n\to+\infty} \frac{(3n+4)\cdot\cancel{2^{n+1}}\cdot\cancel{(n+1)!}}{\cancel{2^{n+1}}\cdot 2\cdot(n+2)\cancel{(n+1)!}}$$

$$\lim_{n\to+\infty} \frac{3n+4}{2n+4}$$

$$\lim_{n \to +\infty} \frac{3\cancel{n}}{2\cancel{n}} = \frac{3}{2} > 1$$

$\boxed{\text{Diverge}}$