1. [3 pts] Calcule $\overline{(i+3)} \cdot \frac{5i}{2-i}$.

$$3-i \cdot \frac{5i}{2-i} \cdot \frac{2+i}{2+i}$$

$$3-i \quad \frac{(5i)(2+i)}{(2-i)(2+i)} \rightarrow \begin{array}{c} a^2-b^2 \\ (a-b)(a+b) \\ 4^2-i^2 \end{array}$$

$$3-i \quad \frac{10i+5i^2}{4-i^2}$$

$$3-i \quad \frac{10i-5}{4--1}$$

$$3-i \quad \frac{10i-5}{5}$$

$$3-i \quad \frac{\cancel{5}(2i-1)}{\cancel{5}}$$

$$(3-i)(2i-1)$$

$$6i-3-2i^2+i$$

$$7i-3--2$$

$$\boxed{7i-1}$$

2. Sean $x = 3\,\text{cis}(110°)$, $y = 2\,\text{cis}(85°)$, $z = xy$.

(a) **[2 pts]** Calcule z en forma rectangular.

$$3\,\text{cis}(110°) \cdot 2\,\text{cis}(85°)$$

$$3 \cdot 2\,\text{cis}(110° + 85°)$$

$$6\,\text{cis}(195°)$$

$$6\left[\cos(195°) + i\,\text{sen}(195°)\right]$$

$$\boxed{\frac{-3\sqrt{6}+3\sqrt{2}}{2} - \frac{3\sqrt{6}-3\sqrt{2}}{2}\,i}$$

(b) [1 pto] Encuentre el argumento principal de z.

$$z = \underbrace{\frac{-3\sqrt{6}+3\sqrt{2}}{2}}_{a} - \underbrace{\frac{3\sqrt{6}-3\sqrt{2}}{2}}_{b}\,i$$

$$\theta = \arctan\left(\frac{-\dfrac{3\sqrt{6}+3\sqrt{2}}{2}}{\dfrac{-3\sqrt{6}-3\sqrt{2}}{2}}\right) = \boxed{15}$$

3. **[3 pts]** Considere el polinomio $P(x) = x^5 - 3x^4 + 4x^3 - 4x^2 + 3x - 1$. Si se sabe que $P(i) = 0$, factorice completamente en $\mathbb{C}$ el polinomio $P(x)$.

$P(i) = 0 \rightarrow x - i = 0 \quad x = i$ $\qquad 2 - i$

$2 + i$

$x^5 - 3x^4 + 4x^3 - 4x^2 + 3x - 1$

$$\begin{array}{cccccc|c} 1 & -3 & 4 & -4 & 3 & -1 & \\ & i & -3i-1 & 3+3i & -3-i & 1 & i \\ \hline 1 & -3+i & -3i+3 & -1+3i & -i & 0 & (x-i) \end{array}$$

$x^4 + x^3(-3+i) + x^2(-3i+3) + x(-1+3i) + (-i)$

$$\begin{array}{ccccc|c} 1 & -3+i & 3-3i & -1+3i & -i & \\ & -i & 3i & -3i & i & -i \\ \hline 1 & -3 & 3 & -1 & 0 & (x-(-i)) \end{array}$$

$(x+i)$

$x^3 \quad -3x^2 \quad +3x \quad -1$

$x = 1$

$(x-1) = 0$

$(x-1)^3 = 0$

$$\boxed{(x-i)(x+i)(x-1)^3}$$

4. [4 pts] Escriba en la forma $a+ib$ el número complejo $(2+i)^{1-i}$.

$$z=(2+i)^{1-i}$$

$$\ln(z)=\ln(2+i)^{1-i}$$

$$r=\sqrt{2^2+1^2}=\sqrt{5}$$

$$=(1-i)\ln(2+i)$$

$$\theta=\tan^{-1}\left(\frac{1}{2}\right)$$

$$=(1-i)\left[\ln(\sqrt{5}\cdot e)^{i\arctan\left(\frac{1}{2}\right)}\right] \qquad z=\sqrt{5}\cdot e^{i\arctan\left(\frac{1}{2}\right)}$$

$$=(1-i)\left[\ln(\sqrt{5})+\cancel{\ln}(\cancel{e})^{i\arctan\left(\frac{1}{2}\right)}\right]$$

$$=(1-i)\left[\ln(\sqrt{5})+i\arctan\left(\frac{1}{2}\right)\right]$$

$$\ln(\sqrt{5})+i\arctan\left(\frac{1}{2}\right)-\ln(\sqrt{5})i-i^2\arctan\left(\frac{1}{2}\right)$$

$$\ln(\sqrt{5})+i\arctan\left(\frac{1}{2}\right)-\ln(\sqrt{5})i+\arctan\left(\frac{1}{2}\right)$$

$$\left[\ln(\sqrt{5})+\arctan\left(\frac{1}{2}\right)\right]+\left[-\ln\sqrt{5})+\arctan\left(\frac{1}{2}\right)\right]i$$

$$e^{\ln(\sqrt{5})+\arctan\left(\frac{1}{2}\right)}\ \mathrm{cis}\left(-\ln\sqrt{5})+\arctan\left(\frac{1}{2}\right)\right)$$

$$e^{\ln(\sqrt{5})+\arctan\left(\frac{1}{2}\right)}\left[\cos\left(-\ln\sqrt{5})+\arctan\left(\frac{1}{2}\right)\right)+\operatorname{sen}\left(-\ln\sqrt{5})+\arctan\left(\frac{1}{2}\right)\right)i\right]$$

$$\boxed{3{,}350259315-1{,}189150222i}$$

5) [3 pts] Sea a una constante real no nula y sean

$$A=\begin{pmatrix}1 & -1\\ \frac{a+3}{2} & -2\end{pmatrix},\quad B=\begin{pmatrix}-1 & 0 & a\\ 2 & -1 & 0\end{pmatrix}\ \text{y}\ C=\begin{pmatrix}0 & 3\\ -2 & 0\\ 0 & -1\end{pmatrix}$$

Compruebe que $A+\frac{1}{2}(BC)^T=I_2$.

$$I_2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$B\cdot C=\begin{pmatrix}-1 & 0 & a\\ 2 & -1 & 0\end{pmatrix}\cdot\begin{pmatrix}0 & 3\\ -2 & 0\\ 0 & -1\end{pmatrix}$$

$$BC=\begin{pmatrix}0+0+0 & -3+0-a\\ 0+2+0 & 6+0+0\end{pmatrix}$$

$$BC=\begin{pmatrix}0 & -3-a\\ 2 & 6\end{pmatrix}$$

$$(BC)^T=\begin{pmatrix}0 & 2\\ -3-a & 6\end{pmatrix}$$

$$\frac{1}{2}\cdot(BC)^T=\begin{pmatrix}0 & 1\\ \frac{-3-a}{2} & 3\end{pmatrix}$$

$$A+\frac{1}{2}\cdot(B\cdot C)^T=\begin{pmatrix}1 & -1\\ \frac{a+3}{2} & -2\end{pmatrix}+\begin{pmatrix}0 & 1\\ \frac{-3-a}{2} & 3\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

6. **[3 pts]** Considere matrices A, B y C de orden $n \times n$ y $A^T - 2B$ invertible, tales que

$$(AX^T)^T = C^T + 2(XB)$$

Si se sabe que C es una matriz simétrica, use las operaciones con matrices y sus propiedades para demostrar que

$$X = C(A^T - 2B)^{-1}$$

$A^T - 2B = (A^T - 2B)^{-1}$ $\quad C^T = C$

$$(AX^T)^T = C^T + 2(XB)$$

$$(X^T)^T \cdot A^T = C^T + 2XB$$

$$X \cdot A^T = C^T + 2XB$$

$$XA^T - 2XB = C^T$$

$$X(A^T - 2B) = C^T$$

$$X(A^T - 2B) = C$$

$$\boxed{X = C(A^T - 2B)^{-1}}$$

7. [5 pts] Use el método de Gauss-Jordan para determinar el conjunto solución del sistema:

$$\begin{cases} x+y+z=6 \\ 2x-y+3z=14 \\ 2x+2y+2z=12 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ 2 & 2 & 2 & 12 \end{array}\right)$$

$$\begin{array}{c} \\ \\ -2\cdot F1+\widetilde{F2} \\ -2\cdot F1+\widetilde{F3} \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \end{array} \quad \begin{array}{c} \\ 1 \\ 0 \\ 0 \end{array}$$

$$\begin{array}{c} \\ -F2+\widetilde{F1} \\ -\frac{1}{3}\cdot\widetilde{F2} \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left(\begin{array}{ccc|c} 1 & 0 & \frac{4}{3} & \frac{20}{3} \\ 0 & 1 & \frac{-1}{3} & \frac{-2}{3} \\ 0 & 0 & 0 & 0 \end{array}\right) \end{array} \quad \begin{array}{c} \\ 0 \\ 1 \\ 0 \end{array}$$

$$\begin{cases} x+\frac{4}{3}z=\frac{20}{3} \Longrightarrow x=\frac{20}{3}-\frac{4}{3}z \\ y-\frac{1}{3}z=\frac{-2}{3} \Longrightarrow y=\frac{-2}{3}+\frac{1}{3}z \end{cases}$$

$$S=\left\{\left(x=\frac{20}{3}-\frac{4}{3}z,\ y=\frac{-2}{3}+\frac{1}{3}z,\ z\right),\ z\in\mathbb{R}\right\}$$

8. [4 pts] Considere la matriz $A = \begin{pmatrix} -7 & 5 & 4 \\ 2 & -1 & -1 \\ 4 & -3 & -2 \end{pmatrix}$ y determine A^{-1}.

$$\left(\begin{array}{ccc|ccc} -7 & 5 & 4 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 4 & -3 & -2 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} -\frac{1}{7}\cdot \widetilde{F1} \\ -2\cdot F1+\widetilde{F2} \\ -4\cdot F1+\widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & \frac{-5}{7} & \frac{-4}{7} & \frac{-1}{7} & 0 & 0 \\ 0 & \frac{3}{7} & \frac{1}{7} & \frac{2}{7} & 1 & 0 \\ 0 & \frac{-1}{7} & \frac{2}{7} & \frac{4}{7} & 0 & 1 \end{array}\right) \quad \begin{array}{c} 1 \\ 0 \\ 0 \end{array}$$

$$\begin{array}{r} \frac{5}{7}\cdot F2+\widetilde{F1} \\ \frac{7}{3}\cdot \widetilde{F2} \\ \frac{1}{7}\cdot F2+\widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{3} & \frac{1}{3} & \frac{5}{3} & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{7}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 1 \end{array}\right) \quad \begin{array}{c} 0 \\ 1 \\ 0 \end{array}$$

$$\begin{array}{r} \frac{1}{3}\cdot F3+\widetilde{F1} \\ -\frac{1}{3}\cdot F3+\widetilde{F1} \\ 3\cdot \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 2 & 1 & 3 \end{array}\right) \quad \begin{array}{c} 0 \\ 0 \\ 1 \end{array}$$

$$\boxed{A^{-1} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & -1 \\ 2 & 1 & 3 \end{pmatrix}}$$