Fórmulas sugerías para integración por sustitución trigonométrica

Expresión	Sustitución	Fórmula
$\sqrt{a^2-k^2x^2}$	$x=\frac{a}{k}sen\sigma$	$1-sen^2\sigma=\cos^2\sigma$
$\sqrt{a^2+k^2x^2}$	$x=\frac{a}{k}\tan\sigma$	$1+\tan^2\sigma=\sec^2\sigma$
$\sqrt{k^2x^2-a^2}$	$x=\frac{a}{k}\sec\sigma$	$\sec^2\sigma-1=\tan^2\sigma$

$$\sqrt{a^2-k^2\left(\frac{a}{k}\,sen\right)^2}$$

$$\sqrt{a^2-\cancel{k^2}\frac{a^2}{\cancel{k^2}}sen^2(x)}$$

$$\sqrt{a^2-a^2sen^2(x)}$$

$$\sqrt{a^2(1-sen^2(x)}$$ Formula reconendada

$$a\sqrt{1-sen^2(x)}\rightarrow\cos^2(x)$$ $1-sen^2(x)=\cos^2(x)$

$$a\sqrt{\cos^2(x)}$$

$$a\cos(x)$$

Nota: Existen variantes de la x

$$\sqrt{a^2-k^2x^2}$$

$$\sqrt{a^2-k^2(x-2)}\quad,\ x-2=\frac{a}{k}\,sen(x)$$

1) $$\int\frac{x}{\sqrt{9-x^2}}\,dx$$

$$\int\frac{x}{\sqrt{3^2-1^2\cdot x^2}}\,dx \rightarrow \text{forma } \sqrt{a^2-k^2x^2}$$

$$a=3\quad k=1\quad x=\frac{a}{k}\,sen(x)$$

$$\int\frac{3\,sen(x)}{\sqrt{9-(3sen(x))^2}}\,3\cos(x)\,dx$$ $x=3\,sen(x)$

$dx=3\cos(x)$

$$\int\frac{9\,sen(x)\cos(x)}{\sqrt{9-9\,sen^2(x)}}$$

$$\int\frac{\cancel{9}\,sen(x)\cos(x)}{3\sqrt{1-sen^2(x)}}$$

$$\int \frac{3\,sen(x)\cos(x)}{\sqrt{\cos^2(x)}}$$

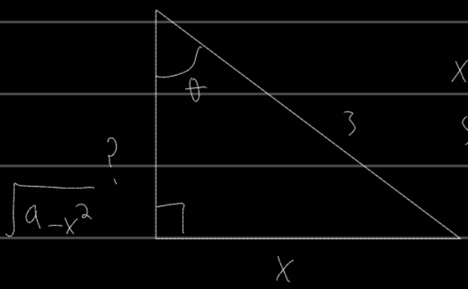
$$3\int \frac{sen(x)\cancel{\cos(x)}}{\cancel{\cos(x)}}$$

$$3\int sen(x)$$

$$-3\cos(x)+C \qquad x = 3\,sen(x)$$

Ahora se hace un triangulo to go back to x

Los lados se establecen de acuerdo al cambio de var original



$x = 3\,sen(x)$

$sen(x) = \frac{x}{3}$ $\frac{\text{opuesto}}{\text{hipotenusa}}$

Para el que falte, pitaguras

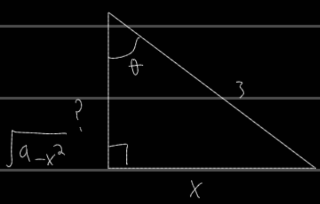
$h^2 = a^2 + b^2$

$3^2 = x^2 + ?^2$

$9 = x^2 + ?^2$

$9 - x^2 = ?^2$

$\sqrt{9-x^2} = ?$



$$\int \frac{x}{\sqrt{9-x^2}}\,dx = -3\cos(x) + C, \qquad \cos(x) = \frac{\sqrt{9-x^2}}{3}$$

$$\cancel{-3}\,\frac{\sqrt{9-x^2}}{\cancel{3}} + C$$

$$-\sqrt{9-x^2} + C$$

2) $\int \frac{1}{\sqrt{4x^2+4x-8}}\,dx \to$ Completar cuadrados $\qquad \sqrt{k^2x^2-a^2}$

$4x^2+4x-8 \qquad x = \frac{a}{k}\sec(x)$

$$\int \frac{1}{\underset{k^2}{4}\underset{x^2}{(x+\frac{1}{2})^2}-\underset{a^2}{9}}\ \frac{3}{2}\sec(x)\tan(x)\,dx$$

$4(x+\frac{1}{2})^2-9 \qquad \sec^2(x)-1=\tan^2(x)$

$x + \frac{1}{2} = \frac{3}{2}\sec(x)$

$dx = \frac{3}{2}\sec(x)\tan(x)$

$$\frac{3}{2}\int \frac{\sec(x)\tan(x)}{\sqrt{4\left(\frac{3}{2}\sec(x)\right)^2-9}}\,dx$$

$$\int \frac{\sec(x)\tan(x)}{\sqrt{\cancel{4}\,\frac{9\sec^2(x)}{\cancel{4}}-9}}\,dx$$

$$\int \frac{\sec(x)\tan(x)}{\sqrt{9\sec^2(x)-9}}$$

$$\int \frac{\sec(x)\tan(x)}{\sqrt{9(\sec^2(x)-1}}$$

$$\int \frac{\sec(x)\tan(x)}{3\sqrt{\tan^2(x)}}$$

$$\int \frac{\sec(x)\cancel{\tan(x)}}{3\cancel{\tan(x)}}$$

$$\frac{1}{3}\int \sec(x)$$

$$\ln|\sec(x)+\tan(x)|+C$$

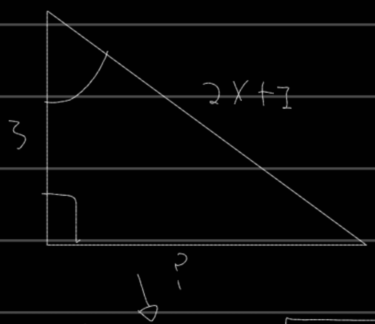
$$\ln\left|\frac{2x+1}{3}+\frac{\sqrt{(2x+1)^2-9}}{3}\right|+C$$

$x+\frac{1}{2}=\frac{3}{2}\sec(x)$

$2(x+\frac{1}{2})=3\sec(x)$

$\frac{2(x+\frac{1}{2})}{3}=\sec(x)$

$\frac{2x+1}{3}=\sec(x)$ $\quad \frac{H}{A}$



$\sqrt{9(x+\frac{1}{2})^2-9}=\sqrt{(2x+1)^2-9}$

$2x+1 \to$ Hip $\quad \sec(x)$

$3 \to$ Ady