§1. Números complejos

§1.1. Forma rectangular

Definición 1.1 Sea $z \in \mathbb{C}$ un número complejo. Entonces la **forma rectangular** de z está dada por $a + bi$, tal que $i^2 = -1$, con $a, b \in \mathbb{R}$. Así, $\mathrm{Re}(z) = a$ y $\mathrm{Im}(z) = b$.

Nota 1.1 $\mathrm{Im}(z)$ **no** incluye el valor de i.

$z = \underbrace{a}_{\text{Real}} + \underbrace{bi}_{\text{Imaginaria}}$ $\quad i^2 = -1$ $\quad i = \sqrt{-1}$

$\mathrm{Re}(z) = a$ $\quad \mathrm{Im}(z) = b$

$z = 4 + 3i$ $\quad \mathrm{Img}(z) = 3$

$\mathrm{Re}(z) = 4$

Resta o Suma

Parte de aritmetica

§1.2. Operaciones

- Sumar y restar
- Multiplicar
- Conjugado
- Racionalizar
- Dividir
- Factorizar

Si $z = 2 - i$ y $w = 3 + 2i$

Calcular $z - w$

$(2 - i) - (3 + 2i)$

$2 - i - 3 - 2i$

$-1 - 3i$

multiplicacion

$(a + bi)(c + di)$

$ac + adi + bci + bdi^2$ $\quad i^2 = -1$

$ac + adi + bci - bd$

$(ac - bd) + (ad + bc)i$

$(2-i)(3+2i)$

$i^2 = -1$

$6 + 4i - 3i - 2i^2 \longrightarrow (-2 \cdot -1)$

$6 + 4i - 3i + 2$ $\qquad 2$

$8 + i$ //

Conjugado:

Cambia el signo del imagino NO del real

$z = a + bi \quad , \quad \bar{z} = a - bi$

$\overline{2-i} = 2+i \qquad \overline{z+w} = \bar{z} + \bar{w}$

$\bar{\bar{z}} = z \qquad \overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}} \qquad \overline{z \cdot w} = \bar{z} \cdot \bar{w}$

Division siempre racionalizar

$\frac{2-i}{3+2i} \cdot \frac{3-2i}{3-2i}$

Siempre el reciproco del de abajo

$\frac{(2-i)(3-2i)}{(3+2i)(3-2i)}$ $\qquad 9 - 4i^2$

$(a-b)(a+b)$

$\frac{6 - 4i - 3i + 2i^2}{9 - 4i^2} \qquad i^2 = -1$

$\frac{6 - 7i - 2}{9 + 4} = \frac{4 - 7i}{13} = \frac{4}{13} - \frac{7i}{13}$

$Re(z) = \frac{4}{13} \qquad Img(z) \; \frac{-7}{13}$

1) Si z es cualquier número complejo, compruebe que $\frac{\overline{i+\bar{z}}}{i-z} = -1$.

$$\frac{-i+z}{i-z}$$

$$\frac{-\cancel{(i-z)}}{\cancel{(i-z)}} = -1$$

2) Determine todos los números complejos x, expresados en su forma rectangular, que satisfacen:

$$(ix^2 + x)\cdot\left(\frac{i \cdot x}{1-4i} - 1\right) = 0$$

Rectangular

Polar

$ix^2+x=0$

$x(ix+1)=0$

$x=0 \quad ix+1=0$

$ix=-1$

$x=\frac{-1}{i}$

$x=\frac{-1}{i}\cdot\frac{-i}{-i}$

$i^2=-1$

$x=\frac{i}{-i^2}$

$-(-1)$

1

$x=\frac{i}{1}$

$x=i$

$x=0 \wedge x=i$

$\frac{i \cdot x}{1-4i}-1=0$

$\frac{ix}{1-4i}=1$

$ix=1-4i$

$x=\frac{1-4i}{i}$

$x=\frac{1-4i}{i}\cdot\frac{-i}{-i}$

$x=\frac{(1-4i)\cdot -i}{-i^2}$

$x=\frac{-i+4i^2}{1}$

$x=-i-4$

$x=-4-i$

$a \pm bi$

$x=-4-i$

$\boxed{\text{Rta } S=\{0, i, -4-i\}}$

5) Determine la forma rectangular de $z = \frac{-3+4i}{2-i} - i$. $a+bi$

$$\frac{-3+4i}{2-i} \cdot \frac{2+i}{2+i} - i$$

$$\frac{(-3+4i)(2+i)}{(2-i)(2+i)} - i \qquad \frac{4-i^2}{(a-b)(a+b)}$$

$$\frac{-6-3i+8i+4i^2}{4-i^2} - i$$

$$\frac{-6+5i-4}{4+1} - i$$

$$\frac{-10+5i}{5} - i$$

$$\frac{\cancel{5}(-2+i)}{\cancel{5}} - i$$

$$-2+\cancel{i}-\cancel{i}$$

$$-2+0i$$

$Re(z) = -2$

$Im(z) = 0$

3) Si se tiene que $w = \dfrac{2-5ai}{1+2i}$, determine todos los valores para el número real a, de tal forma que $\operatorname{Im}(w) \neq 0$.

$$\frac{2-5ai}{1+2i} \cdot \frac{1-2i}{1-2i}$$

$$\frac{(2-5ai)(1-2i)}{(1+2i)(1-2i)}$$

$$(a-b)(a+b)$$
$$1-4i^2$$

$$\frac{2-4i-5ai-10ai^2}{1-4i^2}$$

$$\frac{2-4i-5ai+10a}{5}$$

$$\frac{(2+10a)+(-4-5a)i}{5}$$

$$\underbrace{\frac{2+10a}{5}}_{Re} + \underbrace{\frac{-4-5a}{5}}_{Im} i$$

$$\operatorname{Im}(z) = \frac{-4-5a}{5} \qquad S = \mathbb{R} - \left\{\frac{-4}{5}\right\}$$

$$\frac{-4-5a}{5} \neq 0$$

$$-4-5a \neq 0$$
$$-5a \neq 4$$
$$a \neq \frac{-4}{5}$$

4) Encuentre $x, y \in \mathbb{R}$ tales que $\dfrac{43+yi}{x-5i} = 4+3i.$

$a+bi = c+di \quad a=c \quad bi = di$

$b = d$

$\dfrac{43+yi}{x-5i} = 4+3i$

$43+yi = (4+3i)(x-5i)$

$43+yi = 4x-20i+3xi-15i^2$

$43+yi = 4x-20i+3xi+15$

$\underbrace{43}_{R}+\underbrace{yi}_{I} = \underbrace{(4x+15)}_{R} + \underbrace{(-20+3x)}_{I}i$

$43 = 4x+15$

$4x = 28$

$x = 7$

$yi = (-20+3x)i$

$y = -20+3x$

$y = -20+3(7)$

$y = 1$

R/ $x = 7 \wedge y = 1$

$a = 2 - ix \quad b = 3 - iy$, hallar $x, y \in \mathbb{R}$ tales que $a \cdot b = 8 + 4i$

$$(2 - ix)(3 - iy) = 8 + 4i$$
$$6 - 2iy - 3ix + xyi^2 = 8 + 4i$$
$$6 - 2iy - 3ix - xy = 8 + 4i$$
$$(6 - xy) + (-2y - 3x)i = 8 + 4i$$

$$6 - xy = 8$$
$$-xy = 2$$
$$xy = -2$$
$$y = \frac{-2}{x}$$

$$-2y - 3x = 4$$
$$-2\left(\frac{-2}{x}\right) - 3x = 4$$
$$\frac{4}{x} - 3x \cdot \frac{x}{x} = 4$$
$$\frac{4 - 3x^2}{x} = 4$$
$$4 - 3x^2 = 4x$$
$$-3x^2 - 4x + 4 = 0$$

$$\begin{array}{ccc} -3x & & 2 \\ x & & 2 \end{array} \quad \begin{array}{l} = 2x \\ = \underline{-6x} \\ \ \ -4x \end{array}$$

$$(-3x + 2)(x + 2) = 0$$

$$-3x + 2 = 0 \qquad x + 2 = 0$$
$$-3x = -2 \qquad \boxed{x = -2}$$
$$\boxed{x = \frac{2}{3}}$$

$$y = \frac{-2}{x}$$

$$x = -2 \rightarrow y = \frac{-2}{-2} = \boxed{1}$$
$$x = \frac{2}{3} \rightarrow y = \frac{-2}{\frac{2}{3}} = \boxed{-3}$$

$$\boxed{\begin{array}{l} R/\ x = \frac{2}{3} \wedge y = -3 \\ \quad\ x = -2 \wedge y = -1 \end{array}}$$