Ejemplo 206

Resuelva $\mathbb{R}$ la siguiente ecuación aplicando leyes de potencia:

$$2^x\left(\frac{2^{2x}}{4^{-3+x}}\right)^{x+1} = \sqrt{4^{6-x}}$$

$$2^{1}\left(\frac{2^{2x}}{4^{-3+x}}\right)^{x+1} = \sqrt{4^{6-x}}$$

$$2\left(\frac{2^{2x}}{2^{2(-3+x)}}\right)^{x+1} = \sqrt{2^{2(6-x)}}$$

$$2^{x}\left(\frac{2^{x}}{2^{-6+2x}}\right)^{x+1} = \left[2^{2(6-x)}\right]^{\frac{1}{2}} \qquad 2\cdot\frac{1}{2} = 1$$

$$2^{x}\left(2^{2x-(-6+2x)}\right)^{x+1} = 2^{6-x}$$

$$2^{x}\left(2^{2x+6-2x}\right)^{x+1} = 2^{6-x}$$

$$2^{x}\left(2^{6}\right)^{x+1} = 2^{6-x}$$

$$2^{x}\left(2^{6x+6}\right) = 2^{6-x}$$

$$2^{x+6x+6} = 2^{6-x}$$

$$2^{7x+6} = 2^{6-x}$$

$$7x+6 = 6-x$$

$$8x = 0 \qquad S=\{0\}$$

$$x=0$$

Ejemplo 207

Resuelva $\mathbb{R}$ *la siguiente ecuación aplicando leyes de potencia:*

$$2^{2x} - 6 \cdot 2^{x} + 8 = 0$$

$$2^{2x} - 6 \cdot 2^{x} + 8 = 0$$

$$(2^{x})^{2} - 6 \cdot 2^{x} + 8 = 0$$

Sea $u^{2} = 2^{x}$

$$u^{2} - 6u + 8 = 0$$

$$\begin{array}{lll} u & -4 = & -4u \\ u & -2 = & -2u \end{array}$$

$$(u-4)(u-2) = 0$$

$u = 4 \qquad u = 2$

$2^{x} = 2^{2} \qquad 2^{x} = 2$

$x = 2 \qquad x = 1$

$$S\{1, 2\}$$

$$3^{2x+2} - 5 \cdot 3^{x+1} - 6 = 0$$

$$3^{2x} \cdot 3^{2} - 5 \cdot 3^{x} \cdot 3 - 6 = 0$$

$$9(3^{x})^{2} - 15 \cdot 3^{x} - 6 = 0$$

Sea $u = 3^{x}$

$$9u^2 - 15u - 6 = 0$$

$$\begin{array}{ll} 3u & 1 = 3u \\ 3u & -6 = -18u \\ & \overline{\quad -15} \end{array}$$

$$(3u+1)(3u-6) = 0$$

$$\boxed{u = \frac{-1}{3}} \qquad \boxed{u = 2}$$

$$3^x = \frac{-1}{3} \qquad\qquad 3^x = 2$$

$$\log_3(3^x) = \log_3\left(\frac{-1}{3}\right) \qquad x\log_3(x) = \log_3(2)$$

$$\boxed{\begin{array}{l} S = \varnothing \\ \text{Error} \end{array}} \qquad \boxed{x = \log_3(2)}$$

Ejemplo 208

Resuelva $\mathbb{R}$ *la siguiente ecuación:*

$$e^{5x-2} - 7^{1-x} = 0$$

$$e^{5x-2} - 7^{1-x} = 0$$

$$e^{5x-2} = 7^{1-x}$$

$$\log_7(e^{5x-2}) = \log_{(7)} 7^{1-x}$$

$$(5x-2)\log_7(e) = 1-x \log_7(7)$$

$$5\log_7(e)x - 2\log_7(e) = 1-x$$

Primero la constante y luego la variable

$$5\log_7(e)x + x = 1 + 2\log_7(e)$$

$$x\,(5\log_7(e)+1) = 1 + 2\log_7(e)$$

$$x = 1 + 2 \log (e)$$

$$x = \frac{1+2\log_7(e)}{5\log_7(e)+1}$$

$$5^{3x+2} - 2^{4x+1} = 0$$

$$5^{3x+2} = 2^{4x+1}$$

$$\log_5(5^{3x+2}) = \log_5(2^{4x+1})$$

$$3x+2 = 4x+1\ \log_5(2)$$

$$3x+2 = 4\log_5(2)x + \log_5(2)$$

$$3x - 4\log_5(2)x = \log_5(2)-2$$

$$x(3-4\log_5(2) = \log_5(2)-2$$

$$\boxed{x = \frac{\log_5(2)-2}{3-4\log_5(2)}}$$

Ejemplo 209

Resuelva $\mathbb{R}$ *la siguiente ecuación:*

$$3\cdot 2^{2x} - 29\cdot 2^{x} = -40$$

$$3\cdot 2^{2x} - 29\cdot 2^{x} = -40$$

$$3\cdot(2^x)^2 - 29\cdot 2^x = -40$$

$u = 2^x$

$$3u^2 - 29u + 40 = 0$$

$$x_1 = 8 \qquad x_2 = \frac{5}{3}$$

$$2^x = 8 \qquad 2^x = \frac{5}{3}$$

$$2^x = 2^3 \qquad 2^x = \frac{5}{3}$$

$$x = 3 \qquad x\log_2(2) = \log_2\left(\frac{5}{3}\right)$$

$$x = \log_2\left(\frac{5}{3}\right)$$

$$S = \left\{3, \log_2\left(\frac{5}{3}\right)\right\}$$