Potencias de i

n

$i^0 = 1$ parte entera

$i^1 = i$ $\qquad n - 4 \cdot \frac{n}{4}$

$i^2 = -1$

$i^3 = -i$

$i^{65} \longrightarrow \frac{65}{4} = 16$

$65 - 16 \cdot 4 = 1$

$i^1 = i$

$i^{30} \longrightarrow \frac{30}{4} = 7,5$

$30 - 7 \cdot 4 = 2$

$i^{30} = i^2 = -1$

$i^{19} \longrightarrow \frac{19}{4} = 4,75$

$19 - 4 \cdot 4 = 3 \longrightarrow i^3 = -i$

Logaritmo principal

$$\ln^2\left(\frac{i^{2024}}{1+i}\right) \qquad \ln^2(x) = [\ln(x)]^2$$

$$\left[\ln\left(\frac{i^{2024}}{1+i}\right)\right]^2$$

$$i^{2024} \to \frac{2024}{4} = 506$$

$$2024 - 4 \cdot 506 = 0$$

$$\to \quad i^0 = 1$$

$$\left[\ln\left(\frac{1}{1+i}\right)\right]^2$$

$1+0i$

$r = \sqrt{1^2+0^2} = 1$

$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$1 \cdot cis(\theta)$

$1 \cdot e^{i0}$

$1+i$

$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$\sqrt{2}\ cis\left(\frac{\pi}{4}\right)$

$\sqrt{2} \cdot e^{i\frac{\pi}{4}}$

$$\left[\ln\left(\frac{1 \cdot e^{i0}}{\sqrt{2} \cdot e^{i\frac{\pi}{4}}}\right)\right]^2$$

$$\left[\ln\left(\frac{1}{\sqrt{2}} \cdot e^{i\left(0-\frac{\pi}{4}\right)}\right)\right]^2$$

$$\left[\ln\left(\frac{1}{\sqrt{2}} \cdot e^{-i\frac{\pi}{4}}\right)\right]^2$$

$$\left[\ln\left(\frac{1}{\sqrt{2}} \cdot e^{-i\frac{\pi}{4}}\right)\right]^2$$

$$\left\{\ln\left(\frac{1}{\sqrt{2}}\right) + \ln\left(e^{-i\frac{\pi}{4}}\right)\right\}^2$$

$$\left[\ln\left(\frac{1}{\sqrt{2}}\right) - \frac{i\pi}{4}\right]^2 \qquad (a-b)^2$$

$$a^2 - 2ab + b^2$$

$$\ln^2\left(\frac{1}{\sqrt{2}}\right) - 2\ln\left(\frac{1}{\sqrt{2}}\right) \cdot \frac{i\pi}{4} + \frac{i^2\pi^2}{16}$$

$$\ln^2\left(\frac{1}{\sqrt{2}}\right) - 2\ln\left(\frac{1}{\sqrt{2}}\right) \cdot \frac{i\pi}{4} - \frac{\pi^2}{16}$$

$$\left\{\ln^2\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi^2}{16}\right\} + \left[2\ln\left(\frac{1}{\sqrt{2}}\right)\frac{\pi}{4}\right)i$$

$(-1+i)^i \to z = (-1+i)^i$

$z = (-1+i)^i$

$\ln(z) = \ln(-1+i)^i$

$= i\ln(-1+i)$

$\ln(x)^y$

$y\ln(x)$

$\to i\left[\ln\left(\sqrt{2}\cdot e^{i\frac{3\pi}{4}}\right)\right]$

$-1+i$

$r = \sqrt{(-1)^2+1^2} = \sqrt{2}$

$i\left[\ln(\sqrt{2}) + \cancel{\ln}\left(\cancel{e}^{i\frac{3\pi}{4}}\right)\right.$

$\theta = \arctan\left(\frac{1}{-1}\right) + \pi = \frac{3\pi}{4}$

$i\left[\ln(\sqrt{2}) + \frac{i3\pi}{4}\right)$

$z = \sqrt{2}\cdot e^{i\frac{3\pi}{4}}$

$i\ln(\sqrt{2}) + i^2\frac{3\pi}{4}$

$\ln(a\cdot b)$

$\ln(a) + \ln(b)$

$i\ln(\sqrt{2}) - \frac{3\pi}{4}$

Magnitudes $\quad r = |z|$

$|w| = |z \cdot x|$ ← magnitud del producto de 2 complejos

$|w| = |z \cdot x| = |z| \cdot |w|$

$|\overline{w}| = |w|$

$\overline{r\,\text{cis}(\theta)} = r\,\text{cis}(-\theta)$

3) Determine la forma polar de todos los números complejos que cumplen, simultáneamente, las condiciones que se muestran en cada caso:

a) $\begin{cases} |w| = |\overline{(3-i)\cdot \overline{2i-1}}| \\ \text{Arg}(w) = \text{Arg}\left[\dfrac{(-1+i)^5}{\sqrt{3}-i}\right] \end{cases}$ $\quad r = |z|$ $\quad \arg(w) = \theta$

$|w| = |\overline{(3-i)\overline{(-1+2i)}}|$

$= |(3-i)(-1+2i)|$

$|(3-i)| \cdot |(-1+2i)|$

$3-i$ $\quad r = \sqrt{3^2+(-1)^2}$ $\quad \sqrt{10}$

$-1+2i$ $\quad r = \sqrt{(-1)^2+2^2}$ $\quad \sqrt{5}$

R/ $\quad w = \sqrt{10}\cdot\sqrt{5}$

$\boxed{5\sqrt{2}}$

$$\operatorname{Arg}(w) = \operatorname{Arg}\left(\frac{(-1+i)^5}{\sqrt{3}-i}\right)$$

$-1+i$

$$\theta = \arctan\left(\frac{1}{-1}\right) + \pi = \frac{-3\pi}{4}$$

$\sqrt{3}-i$

$$\theta = \arctan\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$$

$$\operatorname{Arg}\left(\frac{\operatorname{Cis}\left(\frac{-3\pi}{4}\right)^5}{\operatorname{Cis}\left(\frac{-\pi}{6}\right)}\right)$$

$$\operatorname{Cis}\left(\frac{\frac{-15\pi}{4}}{\frac{-\pi}{6}}\right)$$

$$\operatorname{Cis}\left(\frac{-15\pi}{4} - -\frac{\pi}{6}\right)$$

$$\operatorname{Cis}\left(\frac{47\pi}{12}\right)$$

AGREGAR $+2\pi k$ A LA Respuesta en $\operatorname{Arg}(w)$

R/ $\underbrace{5\sqrt{2}}_{|w|}\ \underbrace{\operatorname{Cis}\left(\frac{47\pi}{12} + 2\pi k\right)}_{\operatorname{Arg}(w)}$

b) $\begin{cases} |z-3i| = 4 \\ \operatorname{Arg}(2-2z) = \pi/2 \end{cases}$

Siempre que le den z

$z = a+bi$

$|z-3i| = 4$

$|a+bi-3i| = 4$

$|a+(b-3)i| = 4$

$\sqrt{a^2+(b-3)^2} = 4$

$a^2+(b-3)^2 = 16 \Leftarrow 4^2$ la vuelvo

$\operatorname{Arg}(2-2z) = \frac{\pi}{2}$

Cuadrante	a	bi
1) $0 < \theta < \frac{\pi}{2}$ $0 < \theta < 90$	> 0	> 0
2) $\frac{\pi}{2} < \theta < \pi$ $90 < \theta < 180$	< 0	> 0
3) $\pi < \theta < \frac{3\pi}{2}$ $180 < \theta < 270$	< 0	< 0
4) $\frac{3\pi}{2} < \theta < 2\pi$ $270 < \theta < 360$	> 0	< 0

radianes $\cdot \frac{180}{\pi}$

Casos especiales

$\frac{\pi}{2}$	$a = 0$	$b > 0$
$-\frac{\pi}{2}$	$a = 0$	$b < 0$
$\pm\pi$	$a < 0$	$b = 0$

$z = a + bi$

$$\operatorname{Arg}(2 - 2z) = \frac{\pi}{2}$$

$$\operatorname{Arg}(2 - 2(a+bi)) = \frac{\pi}{2}$$

$$\operatorname{Arg}(2 - 2a - 2bi) = \frac{\pi}{2} \qquad \frac{\pi}{2} \quad a = 0 \quad b > 0$$

$$\underbrace{2 - 2a}_{a} + \underbrace{(-2b)i}_{b} = \frac{\pi}{2}$$

$$2 - 2a = 0 \qquad -2b > 0$$

$$-2a = -2 \qquad \boxed{b < 0}$$

$$\boxed{a = 1}$$

$$a^2 + (b-3)^2 = 16$$

$$1^2 + b^2 - 6b + 9 = 16$$

$$1 + b^2 - 6b - 7 = 0$$

$$b^2 - 6b - 6 = 0$$

Calcu

$$b = 3 - \sqrt{15} \quad , \quad b = 3 + \sqrt{15}$$

$$\boxed{1 + (3 - \sqrt{15})i}$$

d) $\begin{cases} |w-3i+3| = \operatorname{Im}(\overline{7-5i}) \\ \operatorname{Arg}(w+3i) = 3\pi/4 \end{cases}$

En cualquier caso que el angulo NO sea $\pm\pi$, $\pm\frac{\pi}{2}$, usar $\frac{b}{a} = \tan(\theta)$

$w = ai + b$

$\operatorname{Arg}(w+3i) = \frac{3\pi}{4}$

$$a + bi + 3i = \frac{3\pi}{4}$$

$$\underbrace{a}_{a} + \underbrace{(b+3)i}_{b} = \frac{3\pi}{4} \quad \leftarrow \quad \frac{b}{a} = \tan(\theta)$$

$$\frac{b+3}{a} = \tan\left(\frac{3\pi}{4}\right) \leftarrow \text{calcu}$$

$$\frac{b+3}{a} = -1$$

$$b+3 = -a$$

$$\boxed{a = -b-3}$$

$$|w - 3i + 3| = \operatorname{Im}(\overline{7-5i})$$

$$|a + bi - 3i + 3| = 5 \qquad \overline{7-5i}$$

$$|(a+3) + (b-3)i| = 5 \qquad 7+5i$$

$$\operatorname{Im}(7+5i) = 5$$

$$(a+3)^2 + (b-3)^2 = 5^2$$

$$\boxed{a = -b-3}$$

$$(-b \cancel{-3+3})^2 + (b-3)^2 = 25$$

$$b^2 + b^2 - 6b + 9 = 25$$

$$2b^2 - 6b - 16 = 0$$

$$b_1 = \frac{3+\sqrt{41}}{2} \qquad , \quad b_2 = \frac{3-\sqrt{41}}{2}$$

$$4{,}7 \qquad\qquad -1{,}7$$

$$\boxed{a = -b-3}$$

$$a_1 \; - \frac{3+\sqrt{41}}{2} - 3 \approx -1{,}30$$

$$a_2 \; - \frac{3-\sqrt{41}}{2} - 3 = -7{,}70$$

$$\operatorname{Arg}(w+3i) = \frac{3\pi}{4}$$

$$z_1 = -1{,}30 - 1{,}70i \qquad z_2 = -7{,}70 + 4{,}70i$$