1. Considere los números complejos $z = (p+2) + (2-q)i$ y $w = (q-4p)i + 6q$. Determine los valores de $p, q \in \mathbb{R}$ para que $z = w$

R/ $p = \frac{-4}{11}$ y $q = \frac{3}{11}$

$$z = w$$

$$(p+2) + (2-q)i = (q-4p)i + 6q$$

$$p+2 = 6q$$
$$p = 6q - 2$$
$$p = 6\left(\frac{3}{11}\right) - 2$$
$$p = \frac{-4}{11}$$

$$2-q = q-4p$$
$$2-q = q-4(6q-2)$$
$$2-q = q-24q+8$$
$$2-q-q+24q-8=0$$
$$22q-6=0$$
$$22q=6$$
$$q=\frac{3}{11}$$

R/ $\boxed{p = \frac{-4}{11} \quad q = \frac{3}{11}}$

2. Encuentre los valores de $a, b \in \mathbb{R}$ si $(3-a) + (b+1)i = (4a-b)i - 3$

$$(3-a) + (b+1)i = (4a-b)i - 3$$

$$3-a = -3$$
$$a = 6$$

$$b+1 = 4a-b$$
$$2b+1 = 4a$$
$$2b+1 = 24$$
$$2b = 23$$
$$b = \frac{23}{2}$$

R/ $\boxed{a = 6 \quad b = \frac{23}{2}}$

Encuentre los valores de $x, y \in \mathbb{R}$ si $(2x - y)i - (x + y) = 6 - 9i$

$(2x - y)i - (x+y) = 6 - 9i$

$2x - y = -9 \qquad -x - y = 6$

$-y = -9 - 2x \qquad -x - 2x - 9 = 6$

$y = 2x + 9 \qquad -3x = 15$

$y = -10 + 9 \qquad x = -5$

$y = -1$

$\boxed{R/ \quad x = -5 \quad \wedge \quad y = -1}$

4. Encuentre los valores de $a, b \in \mathbb{R}$ tales que $2a + 1 - bi = b - (a + 2)i$

$2a + 1 = b \qquad -b = -a - 2$

$b = 3 \qquad -2a - 1 = -a - 2$

$-a = -1$

$a = 1$

$\boxed{R/ \quad a = 1 \quad \wedge \quad b = 3}$

5. Determine el valor de $x, y \in \mathbb{R}$ que satisfacen la ecuación dada por el criterio $(3 - 4i)^2 - 2(x - yi) = x + i$. R/ $x =$

$9 - 24i - 16 - 2x + 2yi = x + i$

$-7 - 2x + (-24 + 2y)i = x + i$

$-7 - 2x = x \qquad -24 + 2y = 1$

$-7 - 3x = 0 \qquad 2y = 25$

$-3x = 7 \qquad y = \frac{25}{2}$

$x = -\frac{7}{3}$

$\boxed{R/ \quad x = -\frac{7}{3} \quad \wedge \quad y = \frac{25}{3}}$

Encuentre dos números reales x, y que cumplan $(1-i)x+2yi=4+2i$

$x - ix + 2yi = 4 + 2i$

$x + (-x + 2y)i = 4 + 2i$

$x = 4 \qquad -x + 2y = 2$

$-4 + 2y = 2$

$2y = 6$

$y = 3$

$\boxed{R \quad x = 4 \wedge y = 3}$

7. Encuentre los valores de $a, b \in \mathbb{R}$ tales que $5a - 7i + 3bi = 8b - i - 1$

$5a = 8b - 1 \qquad -7 + 3b = -1$

$5a = 15 \qquad 3b = 6$

$a = 3 \qquad b = 2$

$\boxed{R/ \quad a = 3 \ , \ b = 2}$

8. Encuentre los valores de $a, b \in \mathbb{R}$ tales que $bi - 10 = 2b + 3a - 5i$

$-10 = 2b + 3a \qquad b = -5$

$\cancel{-10} = \cancel{-10} + 3a$

$a = 0$

$\boxed{R \quad a = 0 \ , \ b = -5}$

9. Encuentre los valores de $a, b \in \mathbb{R}$ tal que $3a + 2 - 5bi = 10ai + 1 - 2b$

$3a + 2 = 1 - 2b$ $\quad\quad$ $-5b = 10a$

$3a + 2 = 1 + 4a$ $\quad\quad$ $b = -2a$

$1 = a$ $\quad\quad$ $b = -2$

$a = 1$

$\boxed{R/ \quad a = 1 \quad \wedge \quad b = -2}$

10. Encuentre los números reales x, y tales que $\dfrac{43 + yi}{x - 5i} = 4 + 3i$

$43 + yi = (x - 5i)(4 + 3i)$

$43 + yi = 4x + 3xi - 20i - 15i^2$

$43 + yi = 4x + 15 + (3x - 20)i$

$43 = 4x + 15$ $\quad\quad$ $y = 3x - 20$

$4x = 28$ $\quad\quad$ $y = 1$

$x = 7$

R/ $x = 7 \wedge y = 1$

12. Sea $x = 4 - 3i$, determine un número $y \in \mathbb{C}$ tal que $\overline{x} \cdot \overline{y} = 2i - 1$

$$x = 4 - 3i$$
$$\overline{x} = 4 + 3i$$

$$(4 + 3i)\,\overline{y} = 2i - 1$$
$$\overline{y} = \frac{2i - 1}{4 + 3i}$$

$$\overline{y} = \frac{(2i - 1)(4 - 3i)}{16 - 9i^2}$$

$$\overline{y} = \frac{8i + 6 - 4 + 3i}{25}$$

$$\overline{y} = \frac{11i + 2}{25}$$

$$\overline{y} = \frac{2}{25} + \frac{11}{25}i$$

$$\boxed{y = \frac{2}{25} - \frac{11i}{25}}$$

13. Dado $z = 3 - 4i$, encuentre un $w \in \mathbb{C}$ tal que $\overline{z} \cdot \overline{w} = 2i - 1$.

$z = 3 - 4i$

$\overline{z} = 3 + 4i$

$(3 + 4i) \cdot \overline{w} = 2i - 1$

$\overline{w} = \frac{2i - 1}{3 + 4i} \cdot \frac{3 - 4i}{3 - 4i}$

$\overline{w} = \frac{(2i - 1)(3 - 4i)}{9 - 16i^2}$

$\overline{w} = \frac{6i + 8 - 3 + 4i}{25}$

$\overline{w} = \frac{10i + 5}{25}$

$\overline{w} = \frac{5(2i + 1)}{\cancel{25}}$

$\overline{w} = \frac{1}{5} + \frac{2}{5}i$

$$\boxed{w = \frac{1}{5} - \frac{2}{5}i}$$

14. Sea $w = (x - i)(x + 3 - 4i)$. Halle los valores reales de x para los cuales w es imaginario puro. R/ $x = -4 \vee x = 1$

$(x-i)(x+3-4i)$

$x^2+3x-4xi-xi+3i-4$

$x^2+3x-4=0$ $\quad$ $-5xi+5i=0$

$x \quad 4 = 4x$ $\quad$ $5i(-x+1)=0$

$x \quad -1 = \underline{-x}$ $\quad$ $x=1$

$3x$

$(x+4)(x-1)=0$

$x=-4$ $\quad$ $x=1$

$\boxed{R/ \quad x=-4 \quad \wedge \quad x=1}$

15. Sean $z = -3 + ix^2y$ y $w = x^2 + y + 4i$. Halle los valores de x y y para que z y w sean números complejos conjugados. R/ $x = \pm 1$, $y = -4$

$-3+ix^2y = x^2+y+4i$

$-3=x^2+y$ $\quad$ $ix^2y=4i$

$y=-x^2-3$ $\quad$ $x^2y=4$

$x^2(-x^2-3)=4$

$-x^4-3x^2=4$

$-x^4-3x^2-4=0$

1. Resuelva en $\mathbb{C}$ las siguientes ecuaciones:

a) $9x^2 + 4 = 0$

$$9x^2 = -4$$
$$x^2 = \frac{-4}{9}$$
$$x = \sqrt{\frac{-4}{9}}$$
$$x_1 = -\frac{2}{3}i \qquad x_2 = \frac{2}{3}i$$

2. Encuentre el o los números $z \in \mathbb{C}$ que satisfacen simultáneamente las siguientes condiciones: R/ $w = 1 + (3 - \sqrt{15})i$

$$\begin{cases} |z - 3i| = 4 \\ \arg(2 - 2z) = \frac{\pi}{2} \end{cases}$$

$\frac{\pi}{2}$, $a = 0 \quad b > 0$

$$|a + bi - 3i| = 4$$
$$|a + (b-3)i| = 4$$
$$a^2 + (b-3)^2 = 16$$

$$\arg(2 - 2(a+bi)) = \frac{\pi}{2}$$
$$2 - 2a - 2bi = \frac{\pi}{2}$$

$$2 - 2a = 0 \qquad -2b > 0$$
$$2(1 - a) = 0 \qquad b < 0$$
$$1 - a = 0$$
$$a = 1$$

$$a^2 + (b-3)^2 = 16$$
$$1 + b^2 - 6b + 9 = 16$$
$$b^2 - 6b + 10 = 16$$
$$b^2 - 6b - 6 = 0$$
$$b = 3 + \sqrt{15}, \quad b = 3 - \sqrt{15}$$

$\boxed{R/ \; 1 + (3 - \sqrt{15})i}$

3. Encuentre el o los números $z \in \mathbb{C}$ que satisfacen simultáneamente las siguientes condiciones:

$$\begin{cases} |z-2i| = 2 \\ \arg(z+\sqrt{3}) = \frac{\pi}{2} \end{cases}$$

$|a+bi-2i| = 2$

$a^2 + (b-2)^2 = 4$

$3 + b^2 - 4b + \cancel{4} = \cancel{4}$

$b^2 - 4b + 3 = 0$

$b \quad -3 = -3b$

$b \quad -1 = -b$

$-4b$

$(b-3)(b-1)$

$b=3 \quad b=1$

$\arg(a+bi+\sqrt{3}) = \frac{\pi}{2}$

$a+\sqrt{3} = 0 \quad b>0$

$a = -\sqrt{3}$

R/ $\boxed{-\sqrt{3}+3i, \; -\sqrt{3}+i}$

4. Encuentre el o los números $w \in \mathbb{C}$ que satisfacen simultáneamente las siguientes condiciones: R/ $w = -3 - \frac{5}{2}i$

$$\begin{cases} |w-3| = \frac{13}{2} \\ \arg(w+3) = \frac{-\pi}{2} \end{cases}$$

$a+bi-3 = \frac{13}{2}$

$(a-3)^2 + b^2 = \frac{169}{4}$

$\arg(a+bi+3) = -\frac{\pi}{2} \quad a=0 \quad b<0$

$a+3=0$

$a=-3$

$(-3-3)^2 + b^2$

$36 + b^2 = \frac{169}{4}$

$b^2 = \frac{25}{4}$

$b = \pm \frac{5}{2}$

R/ $\boxed{-3 - \frac{5}{2}}$

12. Encuentre el o los números $z \in \mathbb{C}$ que satisfacen simultáneamente las siguientes condiciones:

R/ $z = -2 - i$, $z = 1 - 4i$

$$\begin{cases} |z+2i| = \sqrt{5} \\ \arg(\overline{z}+3) = \frac{\pi}{4} \end{cases}$$

$|a+bi+2i| = \sqrt{5}$

$a^2 + (b+2)^2 = 5$

$\arg(a - bi + 3) = \frac{\pi}{4}$

$\frac{-b}{a+3} = \tan\left(\frac{\pi}{4}\right)$

$\cancel{\frac{\pi}{4}} \cdot \frac{180}{\cancel{\pi}} = 45$

$0 < 45 < 90$

I cuadrante

$a > 0 \quad b > 0$

$-b = a + 3$

$b = -a - 3$

$a^2 + (-a-3+2)^2 = 5$

$a^2 + (-a-1)^2 = 5$

$a^2 + a^2 + 2a + 1 = 5$

$2a^2 + 2a - 4 = 0$

$2a \quad -2 = -2a$

$a \quad 2 = \frac{4a}{2a}$

$(2a-2)(a+2) = 0$

$a = 1 \qquad a = -2$

$b = -a - 3 \qquad b = -a - 3$

$b = -1 - 3 \qquad b = 2 - 3$

$b = -4 \qquad b = -1$

$\boxed{R/ \ 1 - 4i, \ -2 - i}$

14. Encuentre el o los números $z \in \mathbb{C}$ que satisfacen simultáneamente las siguientes condiciones: R/ $z = -3\sqrt{2}i$

$$\begin{cases} |z - \overline{z}| = 6\sqrt{2} \\ \arg(\overline{z} - 3\sqrt{2}) = \frac{3\pi}{4} \end{cases}$$

$|a + \cancel{bi} + a - \cancel{bi}| = 6\sqrt{2}$

$(2a)^2 = (6\sqrt{2})^2$

$4a^2 = 72$

$a^2 = 18$

$a = 3\sqrt{2}, -3\sqrt{2}$

$\arg(a - bi - 3\sqrt{2}) = \frac{3\pi}{4}$ $\quad \frac{3\pi}{4} \cdot \frac{180}{\pi} = 135$

$\frac{-b}{a - 3\sqrt{2}} = \tan\left(\frac{3\pi}{4}\right)$ $\quad 90 < 135 < 180$

II cuadrante

$a < 0 \quad b > 0$

$-b = 3\sqrt{2} - a$

$b = a - 3\sqrt{2}$

$a = 3\sqrt{2} \to b = 3\sqrt{2} - 3\sqrt{2} \to 0$

$\boxed{R/\ z = 3\sqrt{2}}$

2) Resuelva en $\mathbb{C}$ las siguientes ecuaciones:

a) $x^3 - 8i = 0$

$x^3 = 8i$ $\quad z = 0 + 8i$

$x = \sqrt[3]{8i}$ $\quad a = 0$, $b = 8$

$= \sqrt[3]{8} \cdot \operatorname{cis}\left(\frac{\frac{\pi}{2} + 2\pi k}{3}\right)$ $\quad r = \sqrt{0 + 8^2} = 8$

$\theta = \quad a = 0 \wedge b > 0 \to \frac{\pi}{2}$

$= 2 \cdot \operatorname{cis}\left(\frac{\frac{\pi}{2} + 2\pi k}{3}\right)$

$$\boxed{\begin{array}{l} k = 0 \to 2 \cdot \operatorname{cis}\left(\frac{\pi}{6}\right) \\ k = 1 \to 2 \cdot \operatorname{cis}\left(\frac{5\pi}{6}\right) \\ k = 2 \to 2 \cdot \operatorname{cis}\left(\frac{3\pi}{2}\right) \end{array}}$$

b) $z^3 + 2 = -2i$

$z^3 = -2-2i$ $\qquad$ $z = -2-2i$

$z = \sqrt[3]{-2-2i}$ $\qquad$ $a = -2$, $b = -2$

$\operatorname{cis}\left(\frac{-\frac{3\pi}{4}+2\pi k}{3}\right)$ $\qquad$ $r = \sqrt{(-2)^2+(-2)^2} = 2\sqrt{2}$

$\theta = \arctan\left(\frac{-2}{-2}\right) - \pi = -\frac{3\pi}{4}$

$z = 2\operatorname{cis}\left(-\frac{3\pi}{4}\right)$

$$\boxed{\begin{aligned} k=0 &\rightarrow \sqrt[3]{2\sqrt{2}}\cdot\operatorname{cis}\left(-\frac{\pi}{4}\right)\\ k=1 &\rightarrow \sqrt[3]{2\sqrt{2}}\cdot\operatorname{cis}\left(\frac{5\pi}{12}\right)\\ k=2 &\rightarrow \sqrt[3]{2\sqrt{2}}\cdot\operatorname{cis}\left(\frac{13\pi}{12}\right)\end{aligned}}$$

c) $z^3 = e^{\ln 5 + i\frac{\pi}{4}}$

$e^{\ln(5)}\cdot\operatorname{cis}\left(\frac{\pi}{4}\right)$

$5\cdot\operatorname{cis}\left(\frac{\pi}{4}\right)$

$z = \sqrt[3]{5}\cdot\operatorname{cis}\left(\frac{\frac{\pi}{4}+2\pi k}{3}\right)$

$$\boxed{\begin{aligned} k=0 &\rightarrow \sqrt[3]{5}\cdot\operatorname{cis}\left(\frac{\pi}{12}\right)\\ k=1 &\rightarrow \sqrt[3]{5}\cdot\operatorname{cis}\left(\frac{3\pi}{4}\right)\\ k=2 &\rightarrow \sqrt[3]{5}\cdot\operatorname{cis}\left(\frac{17\pi}{12}\right)\end{aligned}}$$

d) $x^6 + 1 = 0$

$$z = -1 + 0i$$

$$r = \sqrt{(-1)^2 + 0^2} = 1$$

$$\theta = \arctan\left(\frac{0}{-1}\right) + \pi = \pi$$

$$z = \text{cis}(\pi)$$

$$x^6 = -1$$

$$x = \sqrt[6]{-1}$$

$$x = \text{cis}\left(\frac{\pi + 2\pi k}{6}\right)$$

$$\boxed{\begin{array}{ll} k = 0 \to \text{cis}\left(\frac{\pi}{6}\right) & k = 3 \to \text{cis}\left(\frac{7\pi}{6}\right) \\ k = 1 \to \text{cis}\left(\frac{\pi}{2}\right) & k = 4 \to \text{cis}\left(\frac{3\pi}{2}\right) \\ k = 2 \to \text{cis}\left(\frac{5\pi}{6}\right) & k = 5 \to \text{cis}\left(\frac{11\pi}{2}\right) \end{array}}$$

Ejemplo 2.1 Considere las matrices $A_{2\times 3}$, $B_{2\times 3}$, $C_{2\times 2}$, con $C^2 + BA^T$ invertible. Despeje X si

$$X^T AB^T = 2C - \left(C^2 X\right)^T$$

$$C^2 + BA^T \to \left(C^2 + BA^T\right)^{-1} \to \frac{1}{C^2 + BA^T}$$

$$X^T \cdot AB^T = 2C - (C^2 X)^T$$

$$(X^T \cdot AB^T)^T = (2C - (C^2 X)^T)^T$$

$$BA^T \cdot X = 2C^T - C^2 X$$

$$BA^T X + C^2 X = 2C^T$$

$$X(BA^T + C^2) = 2C^T$$

$$X(C^2 + BA^T) = 2C^T$$

$$X = \frac{2C^T}{C^2 + BA^T}$$

$$\boxed{X = 2C^T \cdot (C^2 + BA^T)^{-1}}$$

Ejemplo 2.2 Considere $XA + B^T B = 2X$. Despeje X.

$$XA + B^T B = 2X$$
$$XA - 2X = -B^T B$$
$$X(A - 2I) = -B^T B$$
$$X = -B^T B(A-2I)^{-1}$$

Ejemplo 2.3 Si se sabe que A y $(2\mathbb{I} - A^T)$ son matrices invertibles, despeje X:

$$2(XA)^T = B + A^T A X^T$$

$A \to A^{-1}$ $\quad (2I - A^T) \to (2I - A^T)^{-1}$

$$2A^T X^T - A^T \cdot A X^T = B$$
$$(2A^T - A^T A)X^T = B$$
$$A^T(2I - A)X^T = B$$
$$X^T = B(A^T(2I-A))^{-1}$$
$$X = B^T[(2I - A^T)\cdot A)]^{-1}$$

Ejemplo 2.4 Si $A - 2\mathbb{I}$ es invertible, determine X tal que $(XA - B)^T - C = 2X^T$.

$$A - 2I = (A-2I)^{-1}$$

$$A^{-T} = (A^{-1})^T = (A^T)^{-1}$$
$$(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$$

$$(XA - B)^T - C = 2X^T$$
$$(XA)^T - B^T - C = 2X^T$$
$$A^T X^T - 2X^T = B^T + C$$
$$X^T(A^T - 2I) = B^T + C$$
$$X^T = (B^T + C)(A^T - 2I)^{-1}$$
$$X = (B + C^T)(A - 2I)^{-1}$$

Si es invertible A^{-1} entonces A^T también lo es $(A^T)^{-1}$

$$I^T = I$$

6. [3 pts] Considere matrices A, B y C de orden $n \times n$ y $A^T - 2B$ invertible, tales que

$$(AX^T)^T = C^T + 2(XB)$$

Si se sabe que C es una matriz simétrica, use las operaciones con matrices y sus propiedades para demostrar que

$$X = C(A^T - 2B)^{-1}$$

$(A^T - 2B)^{-1}$ $\quad C^T = C$

$$(AX^T)^T = C^T + 2XB$$
$$XA^T - 2XB = C^T$$
$$X(A^T - 2B) = C^T$$
$$X = C \cdot (A^T - 2B)^{-1} \quad //$$

#6 Sean A, B, C y X matrices, tales que $(XA - B)^T - C = 2X^T$. Si se sabe que $A - 2I$ es una matriz que posee inversa, utilice propiedades de las matrices y álgebra matricial para obtener la matriz X en términos de las demás matrices. **(4 puntos)**

$(A - 2I)^{-1}$

$$A^T X^T - B^T - C = 2X^T$$
$$(A^T - 2I)X^T = B^T + C$$
$$X^T = (B^T + C)(A^T - 2I)^{-1}$$
$$X = (B + C^T)(A - 2I)^{-1}$$

3. Considere las matrices $Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 2 & 0 \end{pmatrix}$, $P = \begin{pmatrix} \cancel{x} & 3 & 0 \\ -1 & 0 & 2 \end{pmatrix}$, con $\cancel{x} \in \mathbb{R}$, determine $(PQ)^{-1}$

$$PQ = \begin{pmatrix} x & 3 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 2 & 0 \end{pmatrix}$$

$$PQ = \begin{pmatrix} x+0+0 & 0-3+0 \\ -1+0+4 & 0+0+0 \end{pmatrix}$$

$$PQ = \begin{pmatrix} x & -3 \\ 3 & 0 \end{pmatrix}$$

$$(PQ)^{-1} = \left(\begin{array}{cc|cc} x & -3 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{array}\right)$$

$$F1 \leftrightarrow F2 \left(\begin{array}{cc|cc} 3 & 0 & 0 & 1 \\ x & -3 & 1 & 0 \end{array}\right)$$

$$\begin{array}{r} \frac{1}{3} \cdot \widetilde{F1} \\ -x \cdot F1 + \widetilde{F2} \end{array} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{3} \\ 0 & -3 & 1 & -\frac{3}{x} \end{array}\right)$$

$$\begin{array}{r} \\ \frac{-1}{3} \cdot \widetilde{F2} \end{array} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{-1}{3} & \frac{x}{9} \end{array}\right)$$

$$(PQ)^{-1} = \begin{pmatrix} 0 & \frac{1}{3} \\ \frac{-1}{3} & \frac{x}{9} \end{pmatrix}$$

6. Considere las matrices:

$$C = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix},\ B = \begin{pmatrix} -3 & 0 \\ 1 & -1 \end{pmatrix}$$

Calcule $B + B^T(C + 2I_2)^{-1}$

$$B^T = \begin{pmatrix} -3 & 1 \\ 0 & -1 \end{pmatrix} \qquad 2 \cdot I_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$C + 2I_2 = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$C + 2I_2 = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$$

$$(C + 2I_2)^{-1} = \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right)$$

$$-F1 + \widetilde{F2} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{array}\right)$$

$$\begin{array}{l} -2 \cdot F2 + \widetilde{F1} \\ -1 \cdot \widetilde{F2} \end{array} \left(\begin{array}{cc|cc} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{array}\right)$$

$$(C + 2I_2)^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$$

$$B^T (C + 2I_2)^{-1} = \begin{pmatrix} -3 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 + 1 & -6 - 1 \\ 0 - 1 & 0 + 1 \end{pmatrix}$$

$$B^T (C + 2I_2)^{-1} = \begin{pmatrix} 4 & -7 \\ -1 & 1 \end{pmatrix}$$

$$B + B^T (C + 2 I_2)^{-1} = \begin{pmatrix} -3 & 0 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 4 & -7 \\ -1 & 1 \end{pmatrix}$$

R/ $\begin{pmatrix} 1 & -7 \\ 0 & 0 \end{pmatrix}$

7. Considere las matrices:

$$C = \begin{pmatrix} 2 & -3 & -1 \\ 2 & 2 & -1 \\ 1 & -2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & -3 \\ 2 & -1 & 2 \end{pmatrix}$$

Calcule $(C - I_3)^{-1} + 3B^TB$

$$C - I_3 = \begin{pmatrix} 2 & -3 & -1 \\ 2 & 2 & -1 \\ 1 & -2 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C - I_3 = \begin{pmatrix} 1 & -3 & -1 \\ 2 & 1 & -1 \\ 1 & -2 & -1 \end{pmatrix}$$

$$(C - I_3)^{-1} = \left(\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} \\ -2 \cdot F1 + \widetilde{F2} \\ -F1 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & 0 & 0 \\ 0 & 7 & 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} 3 \cdot F2 + \widetilde{F1} \\ \frac{1}{7} \cdot \widetilde{F2} \\ -F2 + F3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-4}{7} & \frac{1}{7} & \frac{3}{7} & 0 \\ 0 & 1 & \frac{1}{7} & \frac{-2}{7} & \frac{1}{7} & 0 \\ 0 & 0 & -\frac{1}{7} & \frac{-5}{7} & \frac{-1}{7} & 1 \end{array}\right)$$

$$\begin{array}{r} \frac{4}{7} \cdot F3 + \widetilde{F1} \\ -\frac{1}{7} \cdot F3 + \widetilde{F2} \\ -7 \cdot \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & -4 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 5 & 1 & -7 \end{array}\right)$$

$$(C - I_3)^{-1} = \begin{pmatrix} 3 & 1 & -4 \\ -1 & 0 & 1 \\ 5 & 1 & -7 \end{pmatrix}$$

$$3 \cdot B^T = 3 \begin{pmatrix} 2 & 2 \\ 1 & -1 \\ -3 & 2 \end{pmatrix}$$

$$3 \cdot B^T = \begin{pmatrix} 6 & 6 \\ 3 & -3 \\ -9 & 6 \end{pmatrix}$$

$$3 \cdot B^T \cdot B = \begin{pmatrix} 6 & 6 \\ 3 & -3 \\ -9 & 6 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & -3 \\ 2 & -1 & 2 \end{pmatrix}$$

$$3 \cdot B^T \cdot B = \begin{matrix} 12+12 & 6-6 & -18+12 \\ 6-6 & 3+3 & -9-6 \\ -18+12 & -9-6 & 27+12 \end{matrix}$$

$$3 \cdot B^T \cdot B = \begin{pmatrix} 24 & 0 & -6 \\ 0 & 6 & -15 \\ -6 & -15 & 39 \end{pmatrix}$$

$$((C - I_3)^{-1} + 3B^T \cdot B = \begin{pmatrix} 3 & 1 & -4 \\ -1 & 0 & 1 \\ 5 & 1 & -7 \end{pmatrix} \begin{pmatrix} 24 & 0 & -6 \\ 0 & 6 & -15 \\ -6 & -15 & 39 \end{pmatrix}$$

$$R(\quad \begin{pmatrix} 27 & 1 & -10 \\ -1 & 6 & -14 \\ -1 & -14 & 32 \end{pmatrix}$$

8. Considere las matrices: R/

$$A = \begin{pmatrix} 2 & 5 & 0 \\ 1 & 3 & 0 \\ 2 & 0 & -1 \end{pmatrix}, B = \begin{pmatrix} -2 & 3 & 0 \\ 3 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 0 & -2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

Calcule $A^{-1} - 2B^TC$

$$A^{-1} = \left(\begin{array}{ccc|ccc} 2 & 5 & 0 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 & 1 & 0 \\ 2 & 0 & -1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} \frac{1}{2} \cdot \widetilde{F1} \\ -F1 + \widetilde{F2} \\ -2 \cdot F1 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & \frac{5}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{-1}{2} & 1 & 0 \\ 0 & -5 & -1 & -1 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} \frac{-5}{2} \cdot F2 + \widetilde{F3} \\ 2 \cdot \widetilde{F2} \\ 5 \cdot F2 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -5 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & -6 & 10 & 1 \end{array}\right)$$

$$\begin{array}{r} \\ \\ -1 \cdot \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -5 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & 6 & -10 & -1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 3 & -5 & 0 \\ -1 & 2 & 0 \\ 6 & -10 & -1 \end{pmatrix}$$

$$2B^T = 2 \begin{pmatrix} -2 & 3 \\ 3 & 0 \\ 0 & 1 \end{pmatrix}$$

$$2B^T = \begin{pmatrix} -2 & 6 \\ 6 & 0 \\ 0 & 2 \end{pmatrix}$$

$$2B^T \cdot C = \begin{pmatrix} -2 & 6 \\ 6 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & -2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$2B^T \cdot C = \begin{pmatrix} 0+6 & 4+0 & -2+6 \\ 0+0 & -12+0 & 6+0 \\ 0+2 & 0+0 & 0+2 \end{pmatrix}$$

$$2B^T \cdot C = \begin{pmatrix} 6 & 4 & 4 \\ 0 & -12 & 6 \\ 2 & 0 & 2 \end{pmatrix}$$

$$A^{-1} - 2B^T C = \begin{pmatrix} 3 & -5 & 0 \\ -1 & 2 & 0 \\ 6 & -10 & -1 \end{pmatrix} \begin{pmatrix} 6 & 4 & 4 \\ 0 & -12 & 6 \\ 2 & 0 & 2 \end{pmatrix}$$

$$A^{-1} - 2B^T C = \begin{pmatrix} -3 & -9 & -4 \\ -1 & 14 & -6 \\ 4 & -10 & -3 \end{pmatrix}$$

9. Considere las matrices:

$$C = \begin{pmatrix} -1 & 2 & 0 \\ 1 & -1 & 2 \\ 1 & 2 & -1 \end{pmatrix}, B = \begin{pmatrix} 2 & -3 & 0 \\ 1 & 0 & -1 \\ -2 & 1 & 2 \end{pmatrix}$$

Calcule $B^T \cdot (C + 2I_3)^{-1}$

$$C + 2I_3 = \begin{pmatrix} -1 & 2 & 0 \\ 1 & -1 & 2 \\ 1 & 2 & -1 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$C + 2I_3 = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

$$(C + 2I_3)^{-1} = \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} \\ -F1 + \widetilde{F2} \\ -F1 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} -2 \cdot F2 + \widetilde{F1} \\ -1 \cdot \widetilde{F2} \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & -1 & 2 & 0 \\ 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} -4 \cdot F3 + \widetilde{F1} \\ 2 \cdot F3 + \widetilde{F1} \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & -4 \\ 0 & 1 & 0 & -1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$B^T \cdot (C + 2I_3)^{-1} = \begin{pmatrix} 2 & 1 & -2 \\ -3 & 0 & 1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 & -4 \\ -1 & -1 & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

R/ $$\begin{pmatrix} 6 - 1 + 2 & 4 - 1 + 0 & -8 + 2 - 2 \\ -9 + 0 - 1 & -6 + 0 + 0 & 12 + 0 + 1 \\ 0 + 1 - 2 & 0 + 1 + 0 & 0 - 2 + 2 \end{pmatrix} = \begin{pmatrix} 7 & 3 & -8 \\ -10 & -6 & 13 \\ -1 & 1 & 0 \end{pmatrix}$$

10. Dadas las matrices:

$$C = \begin{pmatrix} 5 & 7 & -2 \\ -2 & -3 & 0 \\ 3 & 4 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & -1 & -2 \end{pmatrix}.$$

Calcule $I_3 - C^{-1} + 3B^TB$

$$C^{-1} = \left(\begin{array}{ccc|ccc} 5 & 7 & -2 & 1 & 0 & 0 \\ -2 & -3 & 0 & 0 & 1 & 0 \\ 3 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} \frac{1}{5}\cdot \widetilde{F1} \\ 2\cdot F1 + \widetilde{F2} \\ -3\cdot F1 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & \frac{7}{5} & \frac{-2}{5} & \frac{1}{5} & 0 & 0 \\ 0 & \frac{-1}{5} & \frac{-4}{5} & \frac{2}{5} & 1 & 0 \\ 0 & \frac{-1}{5} & \frac{11}{5} & \frac{-3}{5} & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} -\frac{7}{5}\cdot F2 + \widetilde{F1} \\ -5\cdot \widetilde{F2} \\ \frac{1}{5}\cdot F2 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & -6 & 3 & 7 & 0 \\ 0 & 1 & 4 & -2 & -5 & 0 \\ 0 & 0 & 3 & -1 & -1 & 1 \end{array}\right)$$

$$\begin{array}{r} 6\cdot F3 + \widetilde{F1} \\ -4\cdot F3 + \widetilde{F2} \\ \frac{1}{3}\cdot \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 5 & 2 \\ 0 & 1 & 0 & \frac{-2}{3} & -\frac{11}{3} & \frac{-4}{3} \\ 0 & 0 & 1 & \frac{-1}{3} & \frac{-1}{3} & \frac{1}{3} \end{array}\right)$$

$$C^{-1} = \begin{pmatrix} 1 & 5 & 2 \\ \frac{-2}{3} & \frac{-11}{3} & \frac{-4}{3} \\ -\frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{pmatrix}$$

$$I_3 - C^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 5 & 2 \\ \frac{-2}{3} & \frac{-11}{5} & \frac{-4}{3} \\ \frac{-1}{3} & \frac{-1}{3} & \frac{1}{3} \end{pmatrix}$$

$$I_3 - C^{-1} = \begin{pmatrix} 0 & -5 & -2 \\ \frac{2}{3} & -\frac{6}{5} & \frac{4}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \end{pmatrix}$$

10. Dadas las matrices:

$$C = \begin{pmatrix} 5 & 7 & -2 \\ -2 & -3 & 0 \\ 3 & 4 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & -1 & -2 \end{pmatrix}$$

Calcule $I_3 - C^{-1} + 3B^T B$

$$3B^T = 3\begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 6 & -3 \\ 12 & -6 \end{pmatrix}$$

$$3B^T \cdot B = \begin{pmatrix} 3 & 6 \\ 6 & -3 \\ 12 & -6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 4 \\ 2 & -1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3+12 & 6-6 & 12-12 \\ 6-6 & 6+3 & 12+6 \\ 12-12 & 24+6 & 48+12 \end{pmatrix}$$

$$= \begin{pmatrix} 15 & 0 & 0 \\ 0 & 9 & 18 \\ 0 & 30 & 60 \end{pmatrix}$$

$$I_3 - C^{-1} + 3B^T B = \begin{pmatrix} 0 & -5 & -2 \\ \frac{2}{3} & -\frac{6}{5} & \frac{4}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \end{pmatrix} + \begin{pmatrix} 15 & 0 & 0 \\ 0 & 9 & 18 \\ 0 & 30 & 60 \end{pmatrix}$$

$$\begin{pmatrix} 15 & -5 & -2 \\ \frac{2}{3} & \frac{39}{5} & \frac{58}{3} \\ \frac{1}{3} & \frac{91}{3} & \frac{179}{3} \end{pmatrix}$$

11. Considere las matrices:

$$B = \begin{pmatrix} -1 & -2 & 2 \\ 0 & 2 & 2 \\ 1 & 1 & -2 \end{pmatrix}, H = \begin{pmatrix} 3 & -1 & 0 \\ 1 & 4 & 2 \end{pmatrix}$$

Calcule $B^{-1} + 2I_3 - H^T H$

$$B^{-1} = \left(\begin{array}{ccc|ccc} -1 & -2 & 2 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} -1 \cdot \widetilde{F1} \\ \\ -1 \cdot F1 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & -2 & -1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} -2 \cdot F2 + \widetilde{F1} \\ \frac{1}{2} \cdot \widetilde{F2} \\ F2 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & -4 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 1 & \frac{1}{2} & 1 \end{array}\right)$$

$$\begin{array}{l} 4 \cdot F3 + \widetilde{F1} \\ -F3 + \widetilde{F2} \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & \frac{1}{2} & 1 \end{array}\right)$$

$$B^{-1} = \begin{pmatrix} 3 & 1 & 4 \\ -1 & 0 & -1 \\ 1 & \frac{1}{2} & 1 \end{pmatrix}$$

$$2I_3 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$H^T H = \begin{pmatrix} 3 & 1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 & 0 \\ 1 & 4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 1 & 2 \\ 1 & 17 & 8 \\ 2 & 8 & 4 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 3 & 1 & 4 \\ -1 & 0 & -1 \\ 1 & \frac{1}{2} & 1 \end{pmatrix} \quad 2I_3 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$H^T \cdot H = \begin{pmatrix} 10 & 1 & 2 \\ 1 & 17 & 8 \\ 2 & 8 & 4 \end{pmatrix}$$

$$B^{-1} + 2I_3 = \begin{pmatrix} 3 & 1 & 4 \\ -1 & 0 & -1 \\ 1 & \frac{1}{2} & 1 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$B^{-1} + 2I_3 = \begin{pmatrix} 5 & 1 & 4 \\ -1 & 2 & -1 \\ 1 & \frac{1}{2} & 3 \end{pmatrix}$$

$$B^{-1} + 2I_3 - H^T H = \begin{pmatrix} 5 & 1 & 4 \\ -1 & 2 & -1 \\ 1 & \frac{1}{2} & 3 \end{pmatrix} - \begin{pmatrix} 10 & 1 & 2 \\ 1 & 17 & 8 \\ 2 & 8 & 4 \end{pmatrix}$$

$$B^{-1} + 2I_3 - H^T H = \begin{pmatrix} -5 & 0 & 2 \\ 0 & -15 & -9 \\ -1 & -\frac{15}{2} & -1 \end{pmatrix}$$

12. Dadas las matrices:

$$A=\begin{pmatrix}1 & -1 & 2\\ -2 & -8 & 0\\ 1 & -2 & -2\end{pmatrix},\ B=\begin{pmatrix}-3 & -1 & 2\end{pmatrix},\ C=\begin{pmatrix}1 & 0 & 2\\ 2 & 3 & -2\end{pmatrix}.$$

Calcule $C(A+3I_3)^{-1}\cdot B^T$

$$A+3I_3=\begin{pmatrix}1 & -1 & 2\\ -2 & -8 & 0\\ 1 & -2 & -2\end{pmatrix}+\begin{pmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{pmatrix}$$

$$A+3I_3=\begin{pmatrix}4 & -1 & 2\\ -2 & -5 & 0\\ 1 & -2 & 1\end{pmatrix}$$

$$(A+3I_3)^{-1}=\left(\begin{array}{ccc|ccc}4 & -1 & 2 & 1 & 0 & 0\\ -2 & -5 & 0 & 0 & 1 & 0\\ 1 & -2 & 1 & 0 & 0 & 1\end{array}\right)$$

$$\begin{array}{r}\frac{1}{4}\cdot \widetilde{F1}\\ 2\cdot F1+\widetilde{F2}\\ -F1+\widetilde{F3}\end{array}\left(\begin{array}{ccc|ccc}1 & \frac{-1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0\\ 0 & \frac{-11}{2} & 1 & \frac{1}{2} & 1 & 0\\ 0 & \frac{-7}{4} & \frac{1}{2} & \frac{-1}{4} & 0 & 1\end{array}\right)$$

$$\begin{array}{r}\frac{1}{4}\cdot F2+\widetilde{F1}\\ -\frac{2}{11}\cdot \widetilde{F2}\\ \frac{7}{4}\cdot F2+\widetilde{F3}\end{array}\left(\begin{array}{ccc|ccc}1 & 0 & \frac{5}{11} & \frac{5}{22} & \frac{-1}{22} & 0\\ 0 & 1 & \frac{-2}{11} & \frac{-1}{11} & \frac{-2}{11} & 0\\ 0 & 0 & \frac{2}{11} & \frac{-9}{22} & \frac{-7}{22} & 1\end{array}\right)$$

$$\begin{array}{r}-\frac{5}{11}\cdot F3+\widetilde{F1}\\ \frac{2}{11}\cdot F3+\widetilde{F2}\\ \frac{11}{2}\cdot \widetilde{F3}\end{array}\left(\begin{array}{ccc|ccc}1 & 0 & 0 & \frac{5}{4} & \frac{3}{4} & \frac{-5}{2}\\ 0 & 1 & 0 & \frac{-1}{2} & \frac{-1}{2} & 1\\ 0 & 0 & 1 & \frac{-9}{4} & \frac{-7}{4} & \frac{11}{2}\end{array}\right)$$

$$(A+3I_3)^{-1}=\begin{pmatrix}\frac{5}{4} & \frac{3}{4} & \frac{-5}{2}\\ \frac{-1}{2} & \frac{-1}{2} & 1\\ \frac{-9}{4} & \frac{-7}{4} & \frac{11}{2}\end{pmatrix}$$

Despejar x con algebra matricial

2. Considere las matrices $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 & 0 \\ 1 & 4 & 2 \end{pmatrix}$

Si se sabe que la matriz X de 2×2 satisface la ecuación $XA = 3X - 2BB^T$:

$$XA = 3X - 2BB^T$$
$$XA - 3X = -2BB^T$$
$$X(A - 3I) = -2BB^T$$
$$X = -2BB^T(A - 3I)^{-1}$$

3. Considere las matrices $A = \begin{pmatrix} 3 & 4 & 0 \\ 0 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$ y $C = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ despejar D

Si se sabe que $D + AB^T = C$

$$D = C - AB^T$$

30. Considere las matrices cuadradas del mismo orden A, B y X, tales que A y $(2I - A^T)$ son invertibles y se tiene que $2(XA)^T = B + A^T AX^T$ despejar x

$$2A^T X^T = B + A^T A X^T$$
$$2A^T X^T - A^T \cdot A X^T = B$$
$$(2A^T - A^T A) X^T = B$$
$$X^T = [(2A^T - A^T A)^{-1} B]$$
$$X = [(2A^T - A^T A)^{-1} B]^T$$

Ejemplo 2.1 Considere las matrices $A_{2\times 3}$, $B_{2\times 3}$, $C_{2\times 2}$, con $C^2 + BA^T$ invertible. Despeje X si

$$X^T AB^T = 2C - (C^2X)^T$$

$$X^T AB^T = 2C - X^T (C^2)^T$$

$$X^T AB^T + X^T (C^2)^T = 2C$$

$$X^T (AB^T + (C^2)^T) = 2C$$

$$X = [2C(AB^T + (C^2)^T]^{-1}$$

$$X = [2C(AB^T + (C^2)^T]^{-T}$$

Ejemplo 2.2 Considere $XA + B^TB = 2X$. Despeje X.

$$XA + B^TB = 2X$$

$$XA - 2X = -B^TB$$

$$X(AI - 2) = -B^TB$$

$$X = -B^TB(AI - 2)^{-1}$$

Ejemplo 2.3 Si se sabe que A y $(2\mathbb{I} - A^T)$ son matrices invertibles, despeje X:

$$2(XA)^T = B + A^TAX^T$$

$$2A^TX^T = B + A^TAX^T$$

$$2A^TX^T - A^TAX^T = B$$

$$(2A^T - A^TA)X^T = B$$

$$A^T(2I - A)X^T = B$$

$$X^T = B(A^T(2I - A))^{-1}$$

$$X = B(A^T(2I - A))^{-T}$$

Ejemplo 2.5 Determine $A \cdot B^T - 5C$, dadas las matrices

$$A = \begin{pmatrix} 2 & -4 & 0 \\ -3 & -2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -4 & 3 & 1 \end{pmatrix} \quad \text{y} \quad C = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$B^T = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} \qquad 5C = \begin{pmatrix} -15 \\ 10 \end{pmatrix}$$

$$AB^T = \begin{pmatrix} 2 & -4 & 0 \\ -3 & -2 & 1 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$$

$$A \cdot B^T = \begin{pmatrix} -8 & -12 & 0 \\ 12 & -6 & 1 \end{pmatrix} \qquad R| \ A \cdot B^T - 5C = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

$$A \cdot B^T = \begin{pmatrix} -20 \\ 7 \end{pmatrix}$$

Ejemplo 2.9 Determine la matriz A si:

$$(\mathbb{I}_2 - 2A)^{-1} = \begin{pmatrix} -1 & 2 \\ 4 & 5 \end{pmatrix}$$

$$(I_2 - 2A)^{-1} = M$$

$$I_2 - 2A = M^{-1}$$

$$\left(\begin{array}{cc|cc} -1 & 2 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right) \qquad M^{-1} = \begin{pmatrix} \frac{-5}{13} & \frac{2}{13} \\ \frac{4}{13} & \frac{1}{13} \end{pmatrix}$$

$$\begin{array}{l} -1 \cdot F1 \\ -4 \cdot F1 + F2 \end{array} \left(\begin{array}{cc|cc} 1 & -2 & -1 & 0 \\ 0 & 13 & 4 & 1 \end{array}\right) \qquad I_2 - 2A = \begin{pmatrix} \frac{-5}{13} & \frac{2}{13} \\ \frac{4}{13} & \frac{1}{13} \end{pmatrix}$$

$$\begin{array}{l} 2 \cdot F2 + F1 \\ \frac{1}{13} \cdot F2 \end{array} \left(\begin{array}{cc|cc} 0 & 0 & \frac{-5}{13} & \frac{2}{13} \\ 0 & 1 & \frac{4}{13} & \frac{1}{13} \end{array}\right) \qquad I_2 - \begin{pmatrix} \frac{-5}{13} & \frac{2}{13} \\ \frac{4}{13} & \frac{1}{13} \end{pmatrix} = 2A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{-5}{13} & \frac{2}{13} \\ \frac{4}{13} & \frac{1}{13} \end{pmatrix} = \begin{pmatrix} \frac{18}{13} & -\frac{2}{13} \\ -\frac{4}{13} & \frac{12}{13} \end{pmatrix}$$

$$2A = \begin{pmatrix} \frac{18}{13} & -\frac{2}{13} \\ -\frac{4}{13} & \frac{12}{13} \end{pmatrix}$$

R/

$$A = \frac{1}{2} \begin{pmatrix} \frac{18}{13} & -\frac{2}{13} \\ -\frac{4}{13} & \frac{12}{13} \end{pmatrix} = \begin{pmatrix} \frac{9}{13} & \frac{-1}{13} \\ \frac{-2}{13} & \frac{6}{13} \end{pmatrix}$$

Ejemplo 2.10 Determine $A^T(A-3i\mathbb{I})$, donde

$$A = \begin{pmatrix} -i & 0 & i \\ 0 & 2i & 3 \\ i & 4 & 0 \end{pmatrix}.$$

Lo hice sin la i del $3i\mathbb{I}$, solo $3\mathbb{I}$

$$A - 3I = \begin{pmatrix} -i & 0 & i \\ 0 & 2i & 3 \\ i & 4 & 0 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -i-3 & 0 & i \\ 0 & 2i-3 & 3 \\ i & 4 & -3 \end{pmatrix}$$

$$A^T = \begin{pmatrix} -i & 0 & i \\ 0 & 2i & 4 \\ i & 3 & 0 \end{pmatrix}$$

$$A^T(A-3I) = \begin{pmatrix} -i & 0 & i \\ 0 & 2i & 4 \\ i & 3 & 0 \end{pmatrix} \begin{pmatrix} -i-3 & 0 & i \\ 0 & 2i-3 & 3 \\ i & 4 & -3 \end{pmatrix}$$

$$\begin{pmatrix} i^2+3i+0+i^2 & 0+0+4i & -i^2+0-3i \\ 0+0+4i & 0+4i^2-6i+16 & 0+6i-12 \\ -i^2-3i+0+0 & 0+6i-9+0 & i^2+9+0 \end{pmatrix}$$

R/ $$\begin{pmatrix} -2+3i & 4i & 1-3i \\ 4i & 12-6i & -12+6i \\ 1-3i & -9+6i & 8 \end{pmatrix}$$

Ejemplo 2.11 Sean $a \in \mathbb{R}$ y las matrices

$$A = \begin{pmatrix} 3 & 0 & a \\ 0 & 3 & -1 \\ 0 & 1 & 2 \end{pmatrix} \quad \text{y} \quad B = \begin{pmatrix} a & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & a \end{pmatrix}.$$

Determine la matriz X si

$$AX^T + A = (2B)^T + 2X^T.$$

$$AX^T + A = 2B^T + 2X^T$$

$$AX^T - 2X^T = 2B^T - A$$

$$(A - 2I)X^T = 2B^T - A$$

$$X^T = (2B^T - A)(A - 2I)^{-1}$$

$$X = [(2B^T - A)(A - 2I)^{-1}]^T$$

Ejemplo 2.12 Considere

$$(3A + XB)^T = -(2X)^T + B$$

a) Determine la matriz X, asumiendo que $B + 2\mathbb{I}$ es invertible

$$3A^T + (XB)^T = -2X^T + B$$

$$3A^T + B^T X^T = -2X^T + B$$

$$B^T X^T + 2X^T = B - 3A^T$$

$$(B^T + 2I)X^T = B - 3A^T$$

$$X^T = (B - 3A^T)(B^T + 2I)^{-1}$$

$$X = \{(B - 3A^T)(B^T + 2I)^{-1}\}^T$$

Ejemplo 2.14 Considere el **sistema homogéneo**:

$$\begin{cases} -x + 2y + z = 0 \\ 3x - y + 2z = 0 \\ y + pz = 0 \end{cases}$$

a) Determine el o los valores de p tal que el sistema tenga solución única.

b) Determine el o los valores de p tal que el sistema tenga infinitas soluciones.

a)
$$\left(\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 3 & -1 & 2 & 0 \\ 0 & 1 & p & 0 \end{array}\right)$$

$$\begin{array}{r} -\widetilde{F1} \\ -3 \cdot F1 + \widetilde{F2} \\ \end{array} \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 1 & p & 0 \end{array}\right)$$

$$\begin{array}{r} 2 \cdot F2 + \widetilde{F1} \\ \frac{1}{5} \cdot \widetilde{F2} \\ -F2 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1+p & 0 \end{array}\right)$$

$x + z = 0$

$y + z = 0$

$(-1 + p) z = 0$

$-1 + p = 0$

$p = 1$ $\qquad\qquad$ $p \neq 1$

Infinitas sols $\qquad$ solucion unica

Ejemplo 2.16 Considere:

$$A = \begin{pmatrix} 2 & 4 & 1 \\ -1 & 1 & -1 \\ 1 & 4 & 0 \end{pmatrix}, \; C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 5 \end{pmatrix} \;\text{ y }\; D = \begin{pmatrix} 1 & 1 \\ 3 & 0 \\ 1 & 3 \end{pmatrix}$$

a) Calcule A^{-1} (**¡¡innecesario!!**)

b) Determine X si $A \cdot (X^T - 2C) = D$

a) $A^{-1} = \left(\begin{array}{ccc|ccc} 2 & 4 & 1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{array}\right)$

$$\begin{array}{l} \frac{1}{2} \cdot \widetilde{F1} \\ F1 + \widetilde{F2} \\ -F1 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 3 & \frac{-1}{2} & \frac{1}{2} & 1 & 0 \\ 0 & 2 & \frac{-1}{2} & \frac{-1}{2} & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} -2F2 + \widetilde{F1} \\ \frac{1}{3} \cdot \widetilde{F2} \\ -2 \cdot F2 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{6} & \frac{1}{6} & \frac{-2}{3} & 0 \\ 0 & 1 & \frac{-1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{-1}{6} & \frac{-5}{6} & \frac{-2}{3} & 1 \end{array}\right)$$

$$\begin{array}{l} -\frac{5}{6} \cdot F3 + \widetilde{F1} \\ \frac{1}{6} \cdot F3 + \widetilde{F2} \\ -6 \cdot \widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -4 & 5 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 5 & 4 & -6 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -4 & -4 & 5 \\ 1 & 1 & -1 \\ 5 & 4 & -6 \end{pmatrix}$$

b) $A \cdot (X^T - 2C) = D$

$X^T - 2C = A^{-1} D$

$X^T = A^{-1} D + 2C$

$X = D^T \cdot A^{-T} + 2C^T$

Ejemplo 2.17

$$\begin{cases} x + 2y + 3z = 6 \\ -2x + y - z = -2 \\ -4x + 7y + 3z = 6 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ -2 & 1 & -1 & -2 \\ -4 & 7 & 3 & 6 \end{array}\right)$$

$$\begin{array}{r} \\ 2 \cdot F1 + \widetilde{F2} \\ 4 \cdot F1 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 5 & 5 & 10 \\ 0 & 15 & 15 & 30 \end{array}\right)$$

$$\begin{array}{r} -2 \cdot F2 + \widetilde{F1} \\ \frac{1}{5} \cdot \widetilde{F2} \\ -15 \cdot F2 + \widetilde{F3} \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x + z = 2$$
$$y + z = 2$$
$$x = 2 - z$$
$$y = 2 - z$$

R/ $S = \left\{ \begin{pmatrix} 2 - z \\ 2 - z \\ z \end{pmatrix}, z \in \mathbb{R} \right\}$

Practica de examen

1. **[3 puntos]** Factorice en $\mathbb{C}$ el polinomio $K(p) = p^4 - 5p^3 + 3p^2 + 19p - 30$ si se sabe que $p = 2 + i$ es un cero de K.

1	-5	3	19	-30	
	$2+i$	$-7-i$	$-7-6i$	30	$2+i$
1	$-3+i$	$-4-i$	$12-6i$	0	$(x-(2+i))$

$$x^3 + (-3+i)x^2 + (-4-i)x + (12-6i)$$

1	$-3+i$	$-4-i$	$12-6i$	
	$2-i$	$-2+i$	$-12+6i$	$2-i$
1	-1	-6	0	$(x-(2-i))$

$$x^2 - x - 6$$

$x \quad -3 = -3x$

$x \quad 2 = 2x$

$-x$

R/ $(x-2-i)(x-2+i)(x-3)(x+2)$

2. **[4 puntos]** Determine $z \in \mathbb{C}$ que satisface simultáneamente las siguientes condiciones:

$$\begin{cases} |\overline{z} - i| = \sqrt{29} \\ \arg(z - 6i) = \dfrac{3\pi}{4} \end{cases}$$

$|a - bi - i| = \sqrt{29}$

$|a + (-b-1)i| = \sqrt{29}$

$a^2 + (-b-1)^2 = 29$ Aun no

$\arg(a + bi - 6i) = \dfrac{3\pi}{4}$

$\arg(a + (b-6)i) = \dfrac{3\pi}{4}$

$\dfrac{b-6}{a} = \tan\left(\dfrac{3\pi}{4}\right)$

$\dfrac{3\pi}{4} \cdot \dfrac{180}{\pi} = 135$

$\dfrac{b-6}{a} = -1$

$90 < 135 < 180$

II Cuadrante

$a < 0 \quad b > 0$

seis

$b - 6 = -a$

$b = 6 - a$

$b = 6 - a$

Volviendo

$a^2 + (-b-1)^2 = 29$

$a^2 + (-(6-a) - 1)^2 = 29$

$a^2 + (-6 + a - 1)^2 = 29$

$a^2 + (a-7)^2 = 29$

$a^2 + a^2 - 14a + 49 = 29$

$2a^2 - 14a + 20 = 0$

$2a \quad -10 = -10a$

$a \quad -2 = -4a$

$-14a$

$(2a - 10)(a - 2)$

$a = 5 \qquad a = 2$

$a = 5 \qquad a = 2$

$b = 6 - 5 \qquad b = 6 - 2$

$b = 1 \qquad b = 4$

$\boxed{\text{R/ } 5 + i \ , \ 2 + 4i}$

$n = 3$

3. [**4 puntos**] Determine las tres raíces cúbicas de $z = -8 + 8i\sqrt{3}$ y exprese su resultado en forma polar.

$z = -8 + 8\sqrt{3}\, i$

$a = -8 \qquad b = 8\sqrt{3}$

$a < 0$

$r = \sqrt{(-8)^2 + (8\sqrt{3})^2} = 16$

$\theta = \arctan\left(\dfrac{8\sqrt{3}}{-8}\right) + \pi = \dfrac{2\pi}{3}$

$z = 16 \operatorname{cis}\left(\dfrac{2\pi}{3}\right)$

$n = 0 \quad \sqrt[3]{16} \operatorname{cis}\left(\dfrac{\frac{2\pi}{3} + 2\pi \cdot 0}{3}\right) = \sqrt[3]{16} \cdot \operatorname{cis}\left(\dfrac{2\pi}{9}\right)$

$n = 1 \quad \sqrt[3]{16} \operatorname{cis}\left(\dfrac{\frac{2\pi}{3} + 2\pi \cdot 1}{3}\right) = \sqrt[3]{16} \cdot \operatorname{cis}\left(\dfrac{8\pi}{9}\right)$

$n = 1 \quad \sqrt[3]{16} \operatorname{cis}\left(\dfrac{\frac{2\pi}{3} + 2\pi \cdot 1}{3}\right) = \sqrt[3]{16} \cdot \operatorname{cis}\left(\dfrac{14\pi}{9}\right)$

4. **[5 puntos]** Calcule y exprese el número $z = (\sqrt{3} - i)^{2i} \cdot (-\sqrt{2} + \sqrt{2}i)^6$ en forma polar.

$z = (\sqrt{3} - i)^{2i}$

$\ln(z) = 2i \ln(\sqrt{3} - i)$

$= 2i \cdot \ln(2 \cdot e^{-\frac{\pi}{6} i})$

$2i \{ \ln(2) + \cancel{\ln(e^{-\frac{\pi}{6} i})}$

$2i \{ \ln(2) + -\frac{\pi}{6})$

$2\ln(2) i + \frac{\pi}{3}$

$\boxed{e^{\frac{\pi}{3}} \cdot cis(2\ln(2))}$

$z = \sqrt{3} - i$

$a = \sqrt{3} \quad b = -1$

$r = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$

$\theta = \arctan\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

$z = 2 \cdot cis\left(\frac{-\pi}{6}\right)$

$z = 2 \cdot e^{-\frac{\pi}{6} i}$

Ahora el otro

$z = (-\sqrt{2} + \sqrt{2} i)^6$

$a = -\sqrt{2} \quad b = \sqrt{2}$

$r = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2} = 2$

$\theta = \arctan\left(\frac{\sqrt{2}}{-\sqrt{2}}\right) + \pi = \frac{3\pi}{4}$

$z = 2\, cis\left(\frac{3\pi}{4}\right)$

$= \left(2\, cis\left(\frac{3\pi}{4}\right)\right)^6$

$= \boxed{64\, cis\left(\frac{9\pi}{2}\right)}$

$\left(e^{\frac{\pi}{3}} \cdot cis(2\ln(2))\right)\left(64\, cis\left(\frac{9\pi}{2}\right)\right)$

$\boxed{64 \cdot e^{\frac{\pi}{3}} \cdot cis\left(2\ln(2) + \frac{9\pi}{2}\right)}$

5. **[3 puntos]** Determine los valores de a y d tales que $AA^T = B$, sabiendo que

$$A = \begin{pmatrix} a & 3 \\ -1 & d \end{pmatrix} \text{ y } B = \begin{pmatrix} 13 & -7 \\ -7 & 10 \end{pmatrix}$$

$$AA^T = \begin{pmatrix} a & 3 \\ -1 & d \end{pmatrix}\begin{pmatrix} a & -1 \\ 3 & d \end{pmatrix}$$

$$= \begin{pmatrix} a^2+9 & -a+3d \\ -a+3d & 1+d^2 \end{pmatrix} = \begin{pmatrix} 13 & -7 \\ -7 & 10 \end{pmatrix}$$

$$a^2+9 = 13 \qquad 1+d^2 = 10$$
$$a^2 = 4 \qquad d^2 = 9$$
$$a = \pm 2 \qquad d = \pm 3$$

$a = -2 \quad d = -3$	$a = 2 \quad d = 2$
$-a+3d = -7$	$-a+3d = -7$
$2 + -9 = -7$	$-2+9 = -7$
$-7 = -7$ ✓	$7 = -7$ ✗

$a = 2 \quad d = -3$	$a = -3 \quad d = 2$
$-a+3d = -7$	$-a+3d = -7$
$-2 + -9 = -7$	$-2 + -9 = -7$
$-11 = -7$ ✗	$-9 = -7$ ✗

$$\boxed{a = -2 \qquad d = -2}$$

6. **[5 puntos]** Sean A, B y C matrices cuadradas de orden tres invertibles. Si se sabe que $C = AB^{-1}$, donde $B^{-1} = \begin{pmatrix}1 & -1 & 1\\ 1 & 0 & -1\\ 0 & 0 & 1\end{pmatrix}$ y $A = \begin{pmatrix}2 & 3 & 0\\ 1 & 1 & -1\\ 0 & 1 & -3\end{pmatrix}$, determine $(C^{-1})^T$

$$A \cdot B^{-1} = \begin{pmatrix}2 & 3 & 0\\ 1 & 1 & -1\\ 0 & 1 & -3\end{pmatrix}\begin{pmatrix}1 & -1 & 1\\ 1 & 0 & -1\\ 0 & 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}2+3+0 & -2+0+0 & 2-3-0\\ 1+1+0 & -1+0+0 & 1-1-1\\ 0+1+0 & 0+0+0 & 0-1-3\end{pmatrix}$$

$$C = \begin{pmatrix}5 & -2 & -1\\ 2 & -1 & -1\\ 1 & 0 & -4\end{pmatrix}$$

$$C^{-1} = \left(\begin{array}{ccc|ccc}5 & -2 & -1 & 1 & 0 & 0\\ 2 & -1 & -1 & 0 & 1 & 0\\ 1 & 0 & -4 & 0 & 0 & 1\end{array}\right)$$

$$\begin{array}{l}\frac{1}{5}\cdot \widetilde{F1}\\ -2\cdot F1+\widetilde{F2}\\ -F1+\widetilde{F2}\end{array} \quad \left(\begin{array}{ccc|ccc}1 & \frac{-2}{5} & \frac{-1}{5} & \frac{1}{5} & 0 & 0\\ 0 & \frac{-1}{5} & \frac{-3}{5} & \frac{-2}{5} & 1 & 0\\ 0 & \frac{2}{5} & \frac{-19}{5} & \frac{-1}{5} & 0 & 1\end{array}\right)$$

$$\begin{array}{l}\frac{2}{5}\cdot F2+\widetilde{F1}\\ -5\cdot \widetilde{F2}\\ -\frac{2}{5}\cdot \widetilde{F2}+\widetilde{F3}\end{array} \quad \left(\begin{array}{ccc|ccc}1 & 0 & 1 & 1 & -2 & 0\\ 0 & 1 & 3 & 2 & -5 & 0\\ 0 & 0 & -5 & -1 & 2 & 1\end{array}\right)$$

$$\begin{array}{l}-F3+\widetilde{F1}\\ -3\cdot F3+\widetilde{F2}\\ -\frac{1}{5}\cdot \widetilde{F3}\end{array} \quad \left(\begin{array}{ccc|ccc}1 & 0 & 0 & \frac{4}{5} & \frac{-8}{5} & \frac{1}{5}\\ 0 & 1 & 0 & \frac{7}{5} & \frac{-19}{5} & \frac{3}{5}\\ 0 & 0 & 1 & \frac{1}{5} & \frac{-2}{5} & \frac{-1}{5}\end{array}\right)$$

$$C^{-1} = \begin{pmatrix} \frac{4}{5} & \frac{-8}{5} & \frac{1}{5} \\ \frac{7}{5} & \frac{-19}{5} & \frac{3}{5} \\ \frac{1}{5} & \frac{-2}{5} & \frac{-1}{5} \end{pmatrix}$$

$$(C^{-1})^T = \begin{pmatrix} \frac{4}{5} & \frac{7}{5} & \frac{1}{5} \\ \frac{-8}{5} & \frac{-19}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{3}{5} & \frac{-1}{5} \end{pmatrix}$$

7. **[5 puntos]** Utilizando el método de Gauss-Jordan, determine el conjunto solución de:

$$\begin{cases} x + 2y + z - w = 2 \\ x - y + z + 3w = 2 \\ 2x + y + 2z + 2w = 4 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 1 & -1 & 1 & 3 & 2 \\ 2 & 1 & 2 & 2 & 4 \end{array}\right)$$

$$\begin{array}{c} \\ -F1+\widetilde{F2} \\ -2\cdot F1+\widetilde{F3} \end{array} \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & -3 & 0 & 4 & 0 \\ 0 & -3 & 0 & 4 & 0 \end{array}\right)$$

$$\begin{array}{c} -2\cdot F2+\widetilde{F1} \\ -\frac{1}{3}\cdot \widetilde{F2} \\ 3\cdot F2+\widetilde{F3} \end{array} \left(\begin{array}{cccc|c} 1 & 0 & 1 & \frac{5}{3} & 2 \\ 0 & 1 & 0 & \frac{-4}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$x + z + \frac{5}{3}w = 2 \quad \rightarrow \quad x = 2 - z - \frac{5}{3}w$$

$$y - \frac{4}{3}w = 0 \qquad y = \frac{4}{3}w$$

$$S = \left\{ \left(2 - z - \frac{5}{3}w,\ \frac{4}{3}w,\ z,\ w \right),\ z, w \in \mathbb{R} \right\}$$