1) $\sum_{n=2}^{\infty} n \cdot e^{-n}$

$f(x) = x \cdot e^{-x}$

$f'(x) = e^{-x} + x \cdot e^{-x} \cdot -1$

$e^{-x} - xe^{-x}$

$e^{-x}(1-x) = 0$

$\cancel{e^{-x} = 0}$ $\quad x = 1$

	$-\infty$	1	$+\infty$
$1-x$	+	o	-
$f'(x)$	+		-
$f(x)$	$\nearrow$		$\searrow$

Decrece

$$\int_{2}^{+\infty} n \cdot e^{-x} \, dx \Rightarrow \int x \cdot e^{-x} \, dx \quad u = x \quad dv = e^{-x}$$

$$du = dx \quad v = -e^{-x}$$

$$-xe^{-x} - \int -e^{-x} \, dx$$

$$-xe^{-x} - e^{-x}$$

$$-xe^{-x} \Big|_{2}^{+\infty}$$

$$\lim_{x \to +\infty} \left(-xe^{-x} - e^{-x}\right) - \left(-2e^{-2} - e^{-2}\right)$$

$$\lim_{x \to +\infty} \frac{-x}{e^{x}}, \frac{\infty}{\infty}, \text{l'hôpital} - \lim_{x \to +\infty} e^{-x}$$

$$\lim_{x \to +\infty} \cancel{\frac{-1}{e^{x}}} - \lim_{x \to +\infty} \cancel{e^{-x}}$$

$$0 - 0 + 2e^{-2} + e^{-2}$$

$$3e^{-2} \text{ //}$$

$\boxed{\text{Converge}}$

3) $\sum_{n=0}^{\infty} n \cdot e^{1-n^2}$

$f(x) = x \cdot e^{1-x^2}$

$f'(x) = e^{1-x^2} + x \cdot e^{1-x^2} \cdot -2x$

$= e^{1-x^2} - 2x^2 \cdot e^{1-x^2}$

$= e^{1-x^2}(1-2x^2)$

$\cancel{e^{1-x^2} = 0} \quad 1-2x^2 = 0$

	$-\infty$	$\sqrt{\frac{1}{2}}$	$+\infty$
e^{1-x^2}	+		+
$1-2x^2$	+	o	-

$x = \cancel{-}\sqrt{\frac{1}{2}} \quad x = \sqrt{\frac{1}{2}}$

$f'(x)$	$+$	$-$
$f(x)$	$\nearrow$	$\searrow$

Decrece

$$\int_0^{+\infty} n \cdot e^{1-n^2} \longrightarrow \int x \cdot e^{1-x^2}$$

$u = 1 - x^2$

$du = -2x\,dx$

$\frac{-1}{2} du = x\,dx$

$$\frac{-1}{2}\int e^u\,du$$

$$\frac{-1}{2} e^{1-x^2}$$

$$\lim_{x \to +\infty} \cancel{\frac{-1}{2} e^{1-x^2}} - \frac{-1}{2} e^{1-0^2}$$

$$\frac{e}{2}$$

$\boxed{\text{Converge}}$

4) $\sum_{n=1}^{\infty} n^2 \cdot e^{-n^3}$

$f(x) = x^2 \cdot e^{-x^3}$

$f'(x) = 2x \cdot e^{-x^3} + x^2 \cdot e^{-x^3} \cdot -3x^2$

$= 2x \cdot e^{-x^3} - 3x^4 \cdot e^{-x^3}$

$x \cdot e^{-x^3} (2 - 3x^3) = 0$

$x = 0 \quad \cancel{e^{-x^3} = 0} \quad 2 - 3x^3 = 0$

$x = \sqrt[3]{\frac{2}{3}}$

	$-\infty$ to 0	0 to $\sqrt[3]{\frac{2}{3}}$	$\sqrt[3]{\frac{2}{3}}$ to $+\infty$
x	$-$	$+$	$+$
$2-3x^3$	$+$	$+$	$-$
e^{-x^3}	$+$	$+$	$+$
$f'(x)$	$-$	$+$	$-$
$f(x)$	$\searrow$	$\nearrow$	$\searrow$

Decrece

$$\int_1^{+\infty} n^2 \cdot e^{-x^3} \longrightarrow \int x^2 \cdot e^{-x^3}$$

$u = -x^3$

$du = -3x^2\,dx$

$\frac{-1}{3} du = x^2\,dx$

$$\frac{-1}{3}\int e^u$$

$$\frac{-1}{3} e^{-x^3}$$

$$\frac{-1}{3} e^{-x^3} \Big|_1^{+\infty}$$

$$\lim_{x \to +\infty} \frac{-1}{3} \cdot e^{-x^3} - -\frac{1}{3} \cdot e^{-1}$$

$$\frac{1}{3e}$$

$\boxed{\text{Converge}}$

5) $\sum^{\infty} \frac{n}{\ldots}$

$$n=1 \quad 1+n^2$$

$$f(x) = \frac{x}{1+x^2}$$

$$f'(x) = \frac{1+x^2 - x\cdot 2x}{(1+x^2)^2}$$

$$= \frac{1+x^2-2x^2}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2} = 0 \qquad f\nexists \; x=-1$$

$$1-x^2=0$$

$$x=1$$

	$-\infty$	-1	1	$+\infty$
$(1+x^2)^2$	+	+	+	
$1-x^2$	+	+	0	−
$f'(x)$	+	+	−	
$f(x)$	↗	↗	↘	

Decrece

$$\int_1^{+\infty} \frac{n}{1+n^2} \longrightarrow \int \frac{x}{1+x^2} \qquad u = 1+x^2$$

$$du = 2x\,dx$$

$$\frac{1}{2}du = x\,dx$$

$$\frac{1}{2}\int \frac{1}{u}$$

$$\frac{1}{2}\ln|u|$$

$$\frac{1}{2}\ln|1+x^2|$$

$$\frac{1}{2}\ln|1+x^2| \Big|_1^{+\infty}$$

$$\lim_{x\to+\infty} \frac{1}{2}\ln|1+x^2| - \frac{1}{2}\ln|1+1^2|$$

$$+\infty$$

$\boxed{\text{Diverge}}$

6) $$\sum_{n=1}^{\infty} \frac{2n+3}{(n^2+3n)^2}$$

$$f(x) = \frac{2x+3}{(x^2+3x)^2}$$

$$f'(x) = \frac{2(x^2+3x)^2 - (2x+3)\cdot 2(x^2+3x)\cdot(2x+3)}{(x^2+3x)^4}$$

$$= \frac{\cancel{(x^2+3x)}\left(2(x^2+3x) - 2(2x+3)^2\right)}{\cancel{(x^2+3x)^4}}$$

$$= \frac{2x^2+6x-8x^2+24x+18}{(x^2+3x)^3}$$

	$-\infty$	0	-3	$+\infty$
-6	−	−	−	
$2x^2$ …		+	+	

$$= \frac{-6x^2+18x+18}{(x^2+3x)^3}$$

$$= \frac{-6(x^2+3x+3)}{(x^2+3x)^3} = 0 \qquad f \nexists \quad x^2+3x=0$$

$$x(x+3)=0$$

$$x=0 \quad x=-3$$

$$-6(x^2+3x+3)=0$$

$$\underbrace{x^2+3x+3}_{>0} = 0$$

x^2+3x+3	+	+	+
x	+	+	+
$x+3$	−	−	+
$f'(x)$	+	+	−
$f(x)$	↗	↗	↘

Decreciente

$$\int_1^{+\infty} \frac{2n+3}{(n^2+3n)^3} \longrightarrow \int \frac{2x+3}{(x^2+3x)^2} \qquad u = x^2+3x \quad du = 2x+3\,dx$$

$$\int \frac{1}{u^2}$$

$$\frac{-1}{u}$$

$$\frac{-1}{x^2+3x}$$

$$\left.\frac{-1}{x^2+3x}\right|_1^{+\infty}$$

$$\lim_{x\to\infty} \underbrace{\frac{-1}{\cancel{x^2+3x}}}_{=0} - \frac{-1}{1^2+3(1)}$$

$$\frac{1}{4}$$

$\boxed{\text{Converge}}$

7) $$\sum_{n=3}^{\infty} \frac{1}{n\ln^2(n)}$$

$$f(x) = \frac{1}{x\ln^2(x)}$$

$$f'(x) = -\frac{1}{(x\ln^2(x))^2} \cdot \ln^2(x) + \cancel{x}\, 2\ln(x) \cdot \frac{1}{\cancel{x}}$$

$$= -\frac{\ln^2(x)+2\ln(x)}{x^2\ln^4(x)}, \text{ Decreciente}$$

$$\int_3^{+\infty} \frac{1}{n\ln^2(n)}\,dx \longrightarrow \int \frac{1}{x\ln^2(x)}\,dx \qquad u = \ln(x) \quad du = \frac{1}{x}$$

$$\int \frac{1}{u^2}\,du$$

$$= \frac{-1}{u}$$

$$= \frac{-1}{\ldots}$$

$$\left.\frac{-1}{\ln(x)}\right|_3^{+\infty}$$

$$\lim_{x\to+\infty} \underbrace{\frac{-1}{\cancel{\ln(x)}}}_{=0} - \frac{-1}{\ln(3)}$$

$\frac{1}{\ln(3)}$ $\ln(x)$

$\therefore$ Converge

8) $$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$$

$$f(x) = \frac{\ln(x)}{x^2}$$

$$f'(x) = \frac{\frac{1}{\cancel{x}} \cdot \cancel{x^2} - \ln(x) \cdot 2x}{x^4}$$

$$= \frac{x - \ln(x) \cdot 2x}{x^4}$$

$$= \frac{\cancel{x}(1 - 2\ln(x))}{\cancel{x^4}}$$

$$\frac{1 - 2\ln(x)}{x^3},$$

$f \nexists\ x = 0$

$1 - 2\ln(x) = 0$

$2\ln(x) = 1$

$\ln(x) = \frac{1}{2}$

$x = e^{\frac{1}{2}}$

$x = \sqrt{e}$

	$-\infty$ … 0	0 … $\sqrt{e}$	$\sqrt{e}$ … $+\infty$
$1-2\ln(x)$	$+$	$+$	0 $-$
x^3	$-$	0 $+$	$+$
$f'(x)$	$-$	$+$	$-$
$f(x)$	↘	↗	↘

Decrece

$$\int_1^{+\infty} \frac{\ln(n)}{n^2} \longrightarrow \int \frac{\ln(x)}{x^2}$$

$u = \ln(x)$ $dv = \frac{1}{x^2}$

$du = \frac{1}{x}dx$ $v = \frac{-1}{x}$

$$\left.\frac{-\ln(x)}{x} + \frac{1}{x^2}\right|_1^{+\infty}$$

$$\frac{-\ln(x)}{x} - \int \frac{-1}{x} \cdot \frac{1}{x} du$$

$$\lim_{x \to +\infty} \frac{-\ln(x)}{x} , \frac{+\infty}{+\infty} , \text{l'hôpital} \quad \lim_{x \to +\infty} \underbrace{\frac{1}{\cancel{x^2}}}_{=0} \quad \frac{-\ln(x)}{x} + \frac{1}{x^2}$$

$$\lim_{x \to +\infty} -\frac{1}{\cancel{x}} = 0$$

$$= \frac{-\ln(1)}{1} + \frac{1}{1^2}$$

$= 1$

$\boxed{\text{Converge}}$