1) $n = 1$ $\quad 3n + 2 = 5n$

$3 \cdot 1 + 2 = 5 \cdot 1$

$5 = 5$ ✓

2) $n = p$ $\quad 3p + 2 = 5p$, Hipotesis de inducción

3) $n = p + 1$ $\quad 3(p+1) + 2 = 5(p+1)$, H(Q1)

$\to 5p + 5$

1. $1+2+3+\ldots+n=\dfrac{n(n+1)}{2}, \forall n \geq 1$

$n=1 \qquad 1=\dfrac{1\cdot(1+1)}{2}$

$1=1$ ✓

$n=p \qquad 1+2+3+\ldots+p=\dfrac{p(p+1)}{2}$, H.I.

$n=p+1 \qquad 1+2+3+\ldots+p+(p+1)=\dfrac{(p+1)[(p+1)+1]}{2}$

Agrega este

$$=\frac{(p+1)(p+2)}{2}$$

Demostracion.

$1+2+3+\ldots+p+(p+1)$

$$\frac{p(p+1)}{2}+(p+1)\cdot\frac{2}{2}, \quad \text{H.I.} \qquad \frac{a}{b}+c\cdot\frac{b}{b}$$

$$\frac{p(p+1)}{2}+\frac{(p+1)\cdot 2}{2} \qquad \frac{a}{b}+\frac{cb}{b}$$

$$\frac{p(p+1)+(p+1)\cdot 2}{2} \qquad \frac{a+cb}{b}$$

$$\frac{p^2+p+2p+2}{2} \qquad \frac{d}{d}\cdot\frac{a}{b}+\frac{c}{d}\cdot\frac{b}{b}$$

$$\frac{da+cb}{db}$$

$$\frac{p^2+p+2p+2}{2}$$

$$\frac{p^2+3p+2}{2} \Rightarrow p^2+3p+2$$

$$\begin{array}{ccc} p & 2 & = 2p \\ p & 1 & = p \\ & & \overline{3p} \end{array}$$

$$\boxed{\frac{(p+1)(p+2)}{2}}$$

$$\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$$

$n=1$ $$r^0 = \frac{1-r^{0+1}}{1-r}$$

$$1 = \frac{1-r}{1-r} \longrightarrow 1=1 \checkmark$$

$n=p$ $$\sum_{k=0}^{p} r^k = \frac{1-r^{p+1}}{1-r} \quad , \text{H. I.}$$

$n=p+1$ $$\sum_{k=0}^{p+1} r^k = \frac{1-r^{\overset{p+1+1}{p+2}}}{1-r} \quad , \text{H. Q. D.}$$

Demostracion

$$\sum_{k=0}^{p+1} r^k$$

$$\sum_{k=0}^{p} r^k + r^{p+1}$$

$$\sum_{k=i}^{p+1} r^k$$

$$\sum_{k=i}^{p} r^k + r^{p+1}$$

$$\sum_{k=0}^{p} r^k + r^{p+1}$$

$$\frac{1-r^{p+1}}{1-r} + r^{p+1}\frac{1-r}{1-r}$$, H.I.

$$\frac{1-r^{p+1}}{1-r} + \frac{r^{p+1}(1-r)}{1-r}$$

$$\frac{1\boxed{-r^{p+1}}+\boxed{r^{p+1}}(1-r)}{1-r}$$

$$\frac{1-r^{p+1}(\overset{0}{\cancel{-1+1}}+r)}{1-r}$$, Factor Comun

$$\frac{1-r^{p+1}\cdot r^{1}}{1-r}$$ $x^a \cdot x^b = x^{a+b}$

$$\frac{1-r^{p+2}}{1-r}$$

$$S_n = \sum_{k=0}^{n} \frac{1}{2^k}, \quad S_0, S_2, S_4$$

$$S_0 = \frac{1}{2^0} = 1$$

$$S_2 = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} = \frac{7}{4}$$

$$S_4 = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} = \frac{31}{16}$$

$$S_n = \sum_{\underbrace{k=0}_{p}}^{n} \frac{1}{2^k}$$

r^p, r es un numero

$$\sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \qquad \frac{r^p}{1-r}, \quad r = \frac{1}{2}$$

$$\frac{\left(\frac{1}{2}\right)^0}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = \boxed{2}$$

$$\frac{1}{x^n} = \frac{1^n}{x^n} = \left(\frac{1}{x}\right)^n$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

5. $\sum_{k=1}^{n}[(k-1)(k-1)!+1] = n!+n-1$ Propiedades de Factorial.

$n! = n(n-1)! = n(n-1)\cdot(n-2)\cdot(n-3)!$

$(n+1)! = (n+1)\cdot(n+1-1)!$ $\quad 0! = 1$

$= (n+1)\cdot n!$

$n=1 \quad (1-1)(1-1)!+1 = 1!+1-1$

$1 = 1$ ✓

$n=p \quad \sum_{k=1}^{p}[(k-1)(k-1)!+1] = p!+p-1$, H. I.

$n=p+1 \quad \sum_{k=1}^{p+1}[(k-1)(k-1)!+1] = (p+1)!+(p+\cancel{1})\cancel{-1}$

$= (p+1)!+p$

Demostracion

$\sum_{k=1}^{p+1}[(k-1)(k-1)!+1]$

Remplazando en k el p+1

$\sum_{k=i}^{p+1} r^k \rightarrow \sum_{k=i}^{p} r^k + r^{p+1}$

$\sum_{k=1}^{p}[(k-1)(k-1)!+1] + ((p+1)-1)(p+1-1)!+1$

$$\sum_{k=1}^{p} [(k-1)(k-1)! + 1] + (p+\cancel{1-1})(p+\cancel{1-1})! + 1$$

$$\sum_{k=1}^{p} [(k-1)(k-1)! + 1] + p \cdot p! + 1$$

$$p! + p \cancel{-1} + p \cdot p! + \cancel{1}$$

$$p! + p + p \cdot p!$$

$$p! + p \cdot p! + p$$

$$p!(p+1) + p$$

$$(p+1)! + p$$

Propiedades de Factorial

$$(p+1)! = (p+1) \cdot (p+1-1)!$$

$$(p+1) \cdot p!$$

7. $\sum_{j=1}^{k}(2j-1)\cdot 3^j = 3+(k-1)\cdot 3^{k+1}$ Sea $n=k$

$n=1$ $(2\cdot 1-1)\cdot 3^1 = 3+(\cancel{1-1})\cdot 3^{1+1}$

$3 = 3$ ✓

$n=p$ $\sum_{j=1}^{p}(2j-1)\cdot 3^j = 3+(p-1)\cdot 3^{p+1}$, H.I.

$n=p+1$ $\sum_{j=1}^{p+1}(2j-1)\cdot 3^j = 3+(p+1-1)\cdot 3^{p+1+1}$

$= 3+p\cdot 3^{p+2}$, H.Q.D.

Demostración

$\sum_{j=1}^{p+1}(2j-1)\cdot 3^j$

$j=p+1$

$\sum_{j=1}^{p}(2j-1)\cdot 3^j + (2(p+1)-1)\cdot 3^{p+1}$

$\sum_{j=1}^{p}(2j-1)\cdot 3^j + (2p+2-1)\cdot 3^{p+1}$

$\sum_{j=1}^{p}(2j-1)\cdot 3^j + (2p+1)\cdot 3^{p+1}$

$$\sum_{i=1}^{p} (2i-1)\cdot 3^{i} + (2p+1)\cdot 3^{p+1}$$

$$3+(p-1)\cdot\boxed{3^{p+1}} + (2p+1)\boxed{3^{p+1}} \quad \text{, H.I.}$$

$$3+3^{p+1}\cdot((p-1)+(2p+1)) \quad \text{, F.C.}$$

$$3+3^{p+1}\cdot 3^{1}\cdot p$$

$$x^{a}+x^{b}=x^{a+b}$$

$$3+3^{p+2}\cdot p$$

$$\boxed{3+p\cdot 3^{p+2}}$$

Sucesiones

Derivables NO tienen .... o !

$f'(x) > 0 \rightarrow$ Creciente
$f'(x) < 0 \rightarrow$ Decreciente

NO Derivables tienen .... o !

$\frac{a_{n+1}}{a_n} \geq 1$, vamos a asumir que es creciente

$3 \geq 1$ ✓ $\rightarrow$ creciente
$-1 \geq 1$ ✗ $\rightarrow$ Decreciente

5. $b_n = 3n^4 - 4n^3 + 4,$

$f(x) = 3x^4 - 4x^3 + 4$

$f'(x) = 12x^3 - 12x^2 + 0 = 0$

$12x^3 - 12x^2 = 0$ $\quad$ $a \cdot b = 0$
$12x^2(12x - 1) = 0$ $\quad$ $a = 0$ $\vee$ $b = 0$

$12x^2 = 0$ $\quad$ $12x - 1 = 0$
$x^2 = 0$ $\quad$ $12x = 1$
$x = 0$ $\quad$ $x = \frac{1}{12}$

$12x^2$ $\qquad$ $12x-1$

$x = 0$ $\qquad$ $x = \frac{1}{12}$

	$-\infty$	0		$\frac{1}{12}$		$+\infty$
$12x^2$	$-$	$\bullet$	$+$		$+$	
$12x-1$	$-$		$-$	$\bullet$	$+$	
$f'(x)$	$+$		$-$		$+$	
$f(x)$	↗		↘		↗	

$\boxed{\text{Creciente}}$

x	$-$	$-$	$\bullet$	$+$	$+$
$-x$	$+$	$+$		$-$	$-$

$n \geq 3,\ n \geq 7,\ n \geq k,\ k \in \mathbb{N}$

10. $a_n = \frac{7^n}{n!}$ $\quad \frac{a_{n+1}}{a_n} \geq 1,$ vamos a asumir que es creciente

Se asume a_n creciente

$a_n = \frac{7^n}{n!} \qquad a_{n+1} = \frac{7^{n+1}}{(n+1)!}$

$$\frac{\frac{7^{n+1}}{(n+1)!}}{\frac{7^n}{n!}} \geq 1$$

Props. Fact.

$$\frac{7^{n+1} \cdot n!}{7^n \cdot (n+1)!} \geq 1$$

$(n+1)! =$
$(n+1) \cdot (n+1-1)!$
$(n+1) \cdot n!$

$$\frac{\cancel{7^n} \cdot 7^1 \cdot \cancel{n!}}{\cancel{7^n} \cdot (n+1) \cdot \cancel{n!}} \geq 1$$

$$\frac{7}{n+1} \geq 1$$

$$7 \geq n+1$$

$6 \geq n \qquad 6 \leq n$

Decreciente

$\therefore$ Creciente

$\boxed{\begin{array}{c} -x \geq 3 \\ x \leq 3 \end{array}} \qquad -\frac{1}{x} \geq 1 \quad x \leq 1$

$\frac{-1}{x}, \frac{1}{-x}, -\frac{1}{x}$