Magnitudes de números complejos

$|w| = |z \cdot x|$ Si nos pidieran la magnitud del producto de dos números complejos z y x

Pasándolos a polar

$|r_z \operatorname{cis}(\theta_z) \cdot r_x \cdot \operatorname{cis}(\theta_x)|$

$= |r_z \cdot r_x \cdot \underbrace{\operatorname{cis}(\theta_z + \theta_x)}_{=1}|$

$= r_z \cdot r_x$

$\boxed{\therefore |w| = |z \cdot x| = |z| \cdot |x|}$

$|\overline{w}|$ Si nos pidieran la magnitud del conjugado de un número complejo, pero ojo

$|\overline{w}| = |w|$, es la misma magnitud del original

$\overline{r \operatorname{cis}(\theta)}$ Si nos pidieran el conjugado en forma polar

$\overline{r \operatorname{cis}(\theta)} = r \operatorname{cis}\underbrace{(-\theta)}_{\text{cambiar signo del ángulo}}$

$r(\cos(-\theta) + i \operatorname{sen}(-\theta))$

$= r(\cos(\theta) - i \operatorname{sen}(\theta))$

3) Determine la forma polar de todos los números complejos que cumplen, simultáneamente, las condiciones que se muestran en cada caso:

a) $\begin{cases} |w| = |(3-i)\cdot\overline{2i-1}| \\ \operatorname{Arg}(w) = \operatorname{Arg}\left[\dfrac{(-1+i)^5}{\sqrt{3}-i}\right] \end{cases}$

$|w| = |(3-i) \cdot (2i-1)|$ ⇒ Para calcular

$|w| = |(3-i)\cdot(2i-1)|$ → Para calcular

$= |3-i|\cdot|2i-1|$ magnitud, se quitan

conjugados, no afecta

$r=\sqrt{3^2+(-1)^2} \quad r=\sqrt{(-1)^2+(2)^2}$

$=\sqrt{10}\cdot\sqrt{5}$

$$\boxed{|w| = 5\sqrt{2}}$$

(θ)

$$\mathrm{Arg}(w) = \mathrm{Arg}\left(\frac{(-1+i)^5}{\sqrt{3}-i}\right) \longrightarrow$$

Interesa solo el angulo

A polar

$z=-1+i$

$a=-1$

$b=1$

$r=\sqrt{(-1)^2+1^2}=\sqrt{2}$

$\theta = \arctan\left(\frac{1}{-1}\right)+\pi$ $a<0 \wedge b\geq 0$

$=\frac{3\pi}{4}$

$$\boxed{z=\sqrt{2}\cdot\mathrm{cis}\left(\frac{3\pi}{4}\right)}$$

$$= \frac{\left[\sqrt{2}\cdot\mathrm{cis}\left(\frac{3\pi}{4}\right)\right]^5}{2\cdot\mathrm{cis}\left(\frac{-\pi}{6}\right)}$$

$$= \frac{4\sqrt{2}\quad\mathrm{cis}\left(\frac{15\pi}{4}\right)}{2\quad\mathrm{cis}\left(\frac{-\pi}{6}\right)}$$

$z=\sqrt{3}-i$

$a=\sqrt{3}$

$b=-1$

$$= \frac{4\sqrt{2}}{2}\,\mathrm{cis}\left(\frac{15\pi}{4}-\frac{\pi}{6}\right)$$

$r=\sqrt{(\sqrt{3})^2+(-1)^2}=2$

$$\frac{4\sqrt{2}}{2}\,\mathrm{cis}\left(\frac{47\pi}{12}\right)$$

$\theta=\arctan\left(\frac{-1}{\sqrt{3}}\right)$

$=\frac{-\pi}{6}$

$$\boxed{z=2\cdot\mathrm{cis}\left(\frac{-\pi}{6}\right)}$$

$$\mathrm{Arg}(w) = \mathrm{cis}\left(\frac{47\pi}{12}\right)$$

$$\boxed{R/\ w=\underbrace{5\sqrt{2}}_{|w|}\quad\underbrace{\mathrm{cis}\left(\frac{47\pi}{12}+2\pi k\right)}_{\mathrm{Arg}(w)},\ k\in\mathbb{Z}}$$

Agregar $2\pi k$ a la respuesta

b) $$\begin{cases} |z-3i| & = 4 \\ \mathrm{Arg}(2-2z) = \pi/2 \end{cases}$$

Tener cuidado con z, escribirlo $z=a+bi$

$z=a+bi$

$|z-3i| = 4$

$|a+bi-3i| = 4$ Agrupar real ∧ img

$|a+i(b-3)| = 4$

$\sqrt{a^2+(b-3)^2} = 4$ $|z| = \sqrt{a^2+b^2}$

$a^2+(b-3)^2 = 16$

Can't be solved yet

Continuamos...

$\text{Arg}(2-2z) = \dfrac{\pi}{2}$

Para que el argumento (θ) de un z sea $\frac{\pi}{2}, -\frac{\pi}{2}, \pi, -\pi$ se hacen 2 ecuaciones

Parte real = 0

Parte img $i<0 \vee i>0$

Cuando el argumento es negativo

Cuando el argumento es positivo

$= \text{Arg}(2-2(a+bi)) = \dfrac{\pi}{2}$

$\text{Arg}(2-2a-2bi) = \dfrac{\pi}{2}$

$2-2a+i(-2b)$

$2-2a=0$ se iguala = 0

$\boxed{a=1}$

$-2b>0$

$\boxed{b<0}$

Volvemos

$a^2+(b-3)^2 = 16$

$1^2+b^2-6b+9 = 16$

$1+b^2-6b-7=0$

$b^2-6b-6=0$

$b=3-\sqrt{15}$ $b=3+\sqrt{15}$ Calcu

$$\boxed{R/\ z = \underbrace{1}_{a=1} + i\underbrace{(3-\sqrt{15})}_{b=3-\sqrt{15}}}$$

c) $\begin{cases} |w+1-i| = 5 \\ \text{Arg}(w-2) = -\pi/2 \end{cases}$

$w = a+bi$

$|a+bi+1-i| = 5$

$|(a+1)+i(b-1)| = 5$

$|(a+1)+i(b-1)|=5$

$\sqrt{(a+1)^2+(b-1)^2}=5$

$(a+1)^2+(b-1)^2=25$

Can't be solved yet

$\operatorname{Arg}(w-2)=-\frac{\pi}{2}$

$\operatorname{Arg}(a+bi-2)=-\frac{\pi}{2}$

$a-2+bi$ arg < 0

$a-2=0 \qquad b<0$

$a=2$

$(a+1)^2+(b-1)^2=25$

$(2+1)^2+b^2-2b+1=25$

$9+b^2-2b-24=0$

$b^2-2b-15=0$

$b=5 \quad b=-3$ Calcu

$$\boxed{R/\ \underbrace{2}_{a=2}-\underbrace{3i}_{b=-3}}$$

d) $\begin{cases} |w-3i+3| = \operatorname{Im}(\overline{7-5i}) \\ \operatorname{Arg}(w+3i)=3\pi/4 \end{cases}$

$z=x+yi \rightarrow \theta=\arctan\left(\frac{y}{z}\right)$ y si ya conocemos

$\tan(\theta)=\frac{y}{x}$ $\frac{y}{z}$, en cualquier caso

$\frac{y}{x}=\tan(\theta)$ que no se arg $=\pm\pi$ v $\pm\frac{\pi}{2}$

usar $\frac{y}{x}=\tan(\theta)$

$w=ai+b$

$\operatorname{Arg}(w+3i)$ donde $z=\underbrace{x}_{\text{Real}}+\underbrace{yi}_{\text{Imaginaria}}$

$=\operatorname{Arg}(a+bi+3i)$

$=\operatorname{Arg}(\underbrace{a}_{x}+\underbrace{(b+3)i}_{y})$

$$\boxed{\text{De rad a deg} \quad \text{rad}\cdot\frac{180}{\pi}}$$

Cuadrante a bi

$$\frac{b+3i}{a} = \tan\left(\frac{3\pi}{4}\right) \qquad \frac{3\pi}{4}\cdot\frac{180}{\pi}$$

$$= 135^{\circ}$$

$$\boxed{\begin{matrix} b+3>0 \\ a<0 \end{matrix}}$$

$$\boxed{\begin{matrix} 2do \quad 90<135<180 \\ a<0\ ,\ b>0 \end{matrix}}$$

$$\frac{b+3i}{a} = \tan\left(\frac{3\pi}{4}\right) = -1$$

(calculando $\tan\left(\frac{3\pi}{4}\right)$)

$$\frac{b+3}{a} = -1$$

$$\boxed{b+3 = -a}$$

$b = -a-3 \;\}\, 1$

$a = -b-3 \;\}\, 2$

Ahora magnitud

$$|w - 3i + 3| = \operatorname{Im}(\overline{7-5i})$$

$$\overline{7-5i}$$

$$= 7+5i$$

$$\operatorname{Im}(7+5i) = \boxed{5}$$

$$|w-3i+3| = 5$$

$$w = a+bi$$

$$|a+bi-3i+3| = 5$$

$$|(a+3)+(b-3)i| = 5$$

$$\sqrt{(a+3)^2+(b-3)^2} = 5$$

$$(a+3)^2+(b-3)^2 = 25$$

$$(-b-3+3)^2+(b-3)^2 = 25$$

$$(-b)^2+(b-3)^2 = 25$$

$$b^2+b^2-6b+9 = 25$$

$$2b^2-6b-16 = 0$$

$$\boxed{\begin{matrix} b = -a-3 \\ a = -b-3 \end{matrix}}$$

Se puede usar cualquiera

$$b_1 = \frac{3+\sqrt{41}}{2} \qquad b_2 = \frac{3-\sqrt{41}}{2}$$ (calcu

$$\boxed{b_1 \approx 4{,}7} \qquad \boxed{b_2 \approx -1{,}7}$$

1 $\quad 0<\theta<\frac{\pi}{2} \quad >0 \quad >0$

$0<a<90$

2 $\quad \frac{\pi}{2}<\theta<\pi \quad <0 \quad >0$

$90<a<180$

3 $\quad \pi<\theta<\frac{3\pi}{2} \quad <0 \quad <0$

$180<a<270$

4 $\quad \frac{3\pi}{2}<\theta<2\pi \quad >0 \quad <0$

$270<a<360$

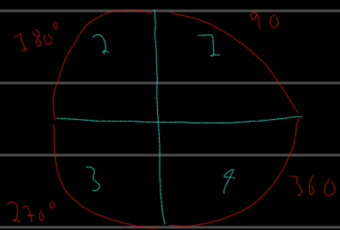
Si un angulo da <0 se hace $360+\theta$ hasta que de >0

Ej: $360+(-45) = 315 \rightarrow 270<315<360$ 4to cuadrante

:

$-370 \rightarrow 360+(-370) = -10$

Again $360+(-10) = 350 \rightarrow$ cuarto cuadrante



Ya usamos b, buscamos a

$a = -b - 3$

$b = -1,70 \quad a_1 = -1,30$

$b = 4,70 \quad a_2 = -7,70$

Entonces

$\text{Arg}(w+3i) = \frac{3\pi}{4} \approx 2,36$

$\boxed{\begin{matrix} b+3>0 \\ a<0 \end{matrix}}$

Pasamos a polar

$z_1 = -1,30 - 1,70i \qquad z_2 = -7,70 + 4,70i$

$z_1 = -1,30 + (-1,70+3)i \qquad z_2 = -7,70 + (4,70+3)i$

$\boxed{z_1 = -1,30 + 1,3i} \qquad \boxed{z_2 = -7,70 + 7,70}$

$\text{Pol}(-1,30; 1,3) \qquad \text{Pol}(-7,70; 7,70)$

$\boxed{\theta = 2,35} \qquad \boxed{\theta = 2,35}$

Se busca el angulo

$\frac{3\pi}{4} \approx 2,36$

$$\boxed{\begin{matrix} R/ \ -1,30 - 1,70i \\ -7,70 + 4,70i \end{matrix}}$$

e) $\begin{cases} |z-2| = 5 \\ \text{Arg}(z-1) = 3\pi/4 \end{cases}$

$z = a + bi$

$|z-2| = 5$

$|a + bi - 2 = 5$

$|(a-2) + (b)i| = 5$

$\sqrt{(a-2)^2 + b^2} = 5$

$(a-2)^2 + b^2 = 25$

Can't be solve yet

$$\text{Arg}(z-1) = \frac{3\pi}{4}$$

$$\text{Arg}(a+bi-1) = \frac{3\pi}{4} \longrightarrow \frac{3\pi}{4} \cdot \frac{180}{\pi} = 135^\circ$$

$$\frac{b}{a-1} = \tan\left(\frac{3\pi}{4}\right)$$

$90 < 135 < 180$

2do cuadrante

$a < 0 \wedge b > 0$

$$\frac{b}{a-1} = -1$$

$b = 1 - a$

$a = 1 - b$ ← Despejando a

Continuamos

$(a-2)^2 + b^2 = 25$

$\boxed{\begin{array}{l} b = 1-a \\ a = 1-b \end{array}}$

$(1-b-2)^2 + b^2 = 25$

$(-b-1)^2 + b^2 = 25$

usar cualquiera

$b^2 + 2b + 1 + b^2 - 25 = 0$

Preferible b

$2b^2 + 2b - 24 = 0$

$\boxed{b = 3}$ $\boxed{\cancel{b = -4}}$

$\boxed{\begin{array}{l} \frac{3\pi}{4} \to 2do \\ a < 0 \\ b > 0 \end{array}}$

$b = 3$

$b > 3$ ✓

$a = 1 - b \to a = 1 - 3$

$a < -2$ ✓

$a = -2$

$\boxed{R/\ -2+3i}$

Comprobar

$\text{Arg}(z-1)$

$\text{Arg}(-2+3i-1)$

$\to -3+3i$

Convertir a polar

$z = -3+3i$

$a = -3$

$b = 3$

Pol $(-3;\ 3)$

$\theta = 2{,}35 \qquad \frac{3\pi}{4} \approx 2{,}35$

$\therefore$ Es correcto

$\boxed{R/\ \ -2+3i}$

f) $\begin{cases} |w+2i| &= |w| \\ \operatorname{Arg}(\overline{w}+2) &= -\pi/4 \end{cases}$

$w = a+bi$

$|w+2i| = |w|$

$|a+bi+2i| = |a+bi|$

$|a+(2+b)i| = |a+(bi)|$

$\sqrt{a^2+(2+b)^2} = \sqrt{a^2+b^2}$

$a^2+(2+b)^2 = a^2+b^2$

$a^2+4+4b+b^2 = a^2+b^2$

$\cancel{a^2}+4+4b+\cancel{b^2}-\cancel{a^2}-\cancel{b^2} = 0$

$4+4b = 0$

$4b = -4$

$\boxed{b = -1}$

Ahora Arg

$w = a+bi$

$\operatorname{Arg}(\bar{w}+2) = \frac{-\pi}{4} \longrightarrow \frac{-\pi}{4} \cdot \frac{180}{\pi} = -45$

$a-bi+2 = \frac{-\pi}{4}$

$360+(-45) = 315$

$(a+2)-(b)i = \frac{-\pi}{4}$

$270 < 315 < 360$

$\frac{-b}{a+2} = \tan\left(\frac{-\pi}{4}\right)$

4to cuadrante

$\boxed{a>0 \ \wedge \ b<0}$

$\frac{-b}{a+2} = -1$

$-b<0$

$b>0$

Despejar Aquí

$-b = -a-2$

$\boxed{b = a+2}$

$\boxed{a+2>0}$

$a = b - 2$ Despejando

$a = -1 - 3$

$\boxed{a = -3}$

$b = a + 2$

$b = -3 + 2$

$\boxed{b = -1}$

$\boxed{R/\ -3 - i}$

Comprobacion

$\text{Arg}(\bar{w} + 2) = \frac{-\pi}{4} \approx -0{,}78$

$w = -3 - i \rightarrow \bar{w} = -3 + i$

$\text{Arg}(-3 + i + 2)$

$-1 + i$

$\text{Pol}(-1; 1)$

$= 2{,}35$

$\therefore$ $\boxed{R/\ \nexists}$

Por que no da