1) $\sqrt{\frac{n-\ln(n)}{5n^3+3n^2}}$

$\sqrt{\frac{\cancel{n}}{\cancel{n}^3}}$

$\sqrt{\frac{1}{n^2}}$

aplicando cmp al lim

$\frac{1}{n} \to$ Armonica divergente

$$\frac{\sqrt{\frac{n-\ln(n)}{5n^3+3n^2}}}{\frac{1}{n}}$$

$$\frac{\sqrt{\frac{\cancel{n}\left(1-\frac{\ln(n)}{n}\right)}{\cancel{n}^3\left(5n+\frac{3}{n}\right)}}}{\frac{1}{n}}$$

$$\frac{\sqrt{\frac{1-\cancel{\frac{\ln(n)}{n}}}{n^2\left(5n+\cancel{\frac{3}{n}}\right)}}}{\frac{1}{n}}$$

$$\frac{\sqrt{\frac{1}{5n^2}}}{\frac{1}{n}}$$

$$\frac{\frac{1}{\cancel{n}\sqrt{5}}}{1}$$

$\cancel{n}$

$\frac{1}{\sqrt{5}}$ D_n diverge y $\lim_{n\to+\infty} \neq 0$

$\therefore$ Diverge

2) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[3]{2+\ln(n)}} < \frac{1}{n\sqrt[3]{\ln(n)}}$

$\sum_{n=1}^{\infty} \frac{1}{n\sqrt[3]{\ln(n)}}$

$f(x) = \frac{1}{x\sqrt[3]{\ln(x)}}$

$f'(x) = \frac{-1}{(x^3 \ln^{\frac{2}{3}}(x))^2} \cdot \ln^{\frac{2}{3}}(x) + \cancel{x}\frac{2}{3}\ln(x)\cdot\frac{1}{\cancel{x}}$

$\frac{-\ln^{\frac{2}{3}}(x) + \frac{2}{3}\ln(x)}{x^2 \ln^{\frac{2}{3}}(x)}$, Decrece

$\int_1^{+\infty} \frac{1}{x\ln^{\frac{2}{3}}(x)} \longrightarrow \int \frac{1}{x\ln^{\frac{2}{3}}(x)}\,dx$ $u=\ln(x)$ $du=\frac{1}{x}$

$\int \frac{1}{u^{\frac{2}{3}}}\,dx$

$\int u^{-\frac{1}{3}}$

$\frac{\ln^{\frac{2}{3}}(x)}{\frac{2}{3}}$

$\left.\frac{\ln(x)^{\frac{2}{3}}}{\frac{2}{3}}\right|_1^{+\infty}$

$\lim_{x\to+\infty} \frac{\ln^{\frac{2}{3}}(x)}{\frac{2}{3}} - \frac{\ln(1)^{\frac{2}{3}}}{\frac{2}{3}}$

$+\infty$, Diverge

aplicando cmp al lim

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt[3]{2+\ln(n)}} \leq \frac{1}{n\sqrt[3]{\ln(n)}} \Big\}\ \text{Diverge}$$

$$\frac{\dfrac{1}{\cancel{n}\sqrt[3]{2+\ln(n)}}}{\dfrac{1}{\cancel{n}\sqrt[3]{\ln(n)}}}$$

$$\frac{\sqrt[3]{\ln(n)}}{\sqrt[3]{2+\ln(n)}}$$

$$\frac{\sqrt[3]{\ln\cancel{(n)}}}{\cancel{\sqrt[3]{\ln(n)}}}, \quad \text{Por crit de cmp al lim}$$

$$= 1$$

$$\boxed{\begin{array}{l} \text{Como } \lim\limits_{n\to+\infty} \dfrac{a_n}{b_n} = 1 \\ \wedge\ b_n \text{ diverge} \\ \to \displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[3]{2+\ln(n)}} \end{array}}$$

3) $$\sum_{n=1}^{\infty} \frac{\ln(n+1)}{(n+1)^2}$$

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2} \leq \frac{1}{n^{\frac{3}{2}}}$$

aplicando cmp al lim

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}, \quad \text{p serie convergente}$$

$p > 1$

Por crit de cmp al lim

$$\lim_{n \to +\infty} \frac{\frac{\ln(n)}{n^2}}{\frac{1}{n^{\frac{3}{2}}}}$$

$$\lim_{n \to +\infty} \frac{\ln(n) \cdot n^{\frac{3}{2}}}{n^2}$$

$$\lim_{n \to +\infty} \frac{\ln(n)}{n^{\frac{1}{2}}} = 0$$

$$\boxed{\begin{array}{l} b_n \text{ converge y } \lim\limits_{n \to +\infty} \frac{a_n}{b_n} = 0 \\ \sum\limits_{n=1}^{\infty} \frac{\ln(n+1)}{(n+1)^2} \text{ converge} \end{array}}$$

4) $$\sum_{n=1}^{\infty} \frac{n}{n^2+4}$$

$$\frac{n}{n^2+4} \leq \frac{\cancel{n}}{\cancel{n}^2}$$

$$\frac{n}{n^2+4} \leq \frac{1}{n}$$

aplicando cmp al lim

$$\sum_{n=1}^{\infty} \frac{1}{n}$$, p serie con $p=1$ diverge

usando crit de cmp al lim

$$\lim_{n \to +\infty} \frac{\frac{n}{n^2+4}}{\frac{1}{n}}$$

$$\lim_{n\to+\infty} \frac{n^2}{n^2+4}$$

$$\lim_{n\to+\infty} \frac{n^2}{n^2} = 1$$

$$\boxed{\begin{array}{l} b_n \text{ diverge} \wedge \lim\limits_{n\to+\infty} \dfrac{a_n}{b_n} = 1 \\ \displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+4} \quad \text{Diverge} \end{array}}$$

5) $$\sum_{n=1}^{\infty} \frac{2n-1}{n^3+n^2+5}$$

$$\sim \sum_{n=1}^{\infty} \frac{2n}{n^3}$$

$$\sim 2 \sum_{n=1}^{\infty} \frac{1}{n^2}$$, aplicando cmp al lim p serie con $p > 1$ convergente

Crit de cmp al lim

$$\lim_{n\to+\infty} \frac{\frac{2n-1}{n^3+n^2+5}}{\frac{2}{n^2}}$$

$$\lim_{n\to+\infty} \frac{2n^3-n^2}{2n^3+2n^2+10}$$

$$\lim_{n\to+\infty} \frac{2n^3}{2n^3} = 1$$

$$\boxed{\begin{array}{l} b_n \text{ converge} \wedge L \neq 0 \\ \displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{n^3+n^2+5} \quad \text{Converge} \end{array}}$$

6) $$\sum_{n=4}^{\infty} \frac{4n+1}{2n^3-2n-1}$$

$$\sim \sum_{n=4}^{\infty} \frac{\cancel{4}n}{\cancel{2}n^3}$$

aplicando cmp al lim

$$\sim 2\sum_{n=4}^{\infty} \frac{1}{n^2}$$, p serie, p>1

converge

$$\lim_{n\to+\infty} \frac{\frac{4n+1}{2n^3-2n-1}}{\frac{1}{n^2}}$$

$$\lim_{n\to+\infty} \frac{4n^3+n^2}{4n^3-4n-4}$$

$$\lim_{n\to+\infty} \frac{4n^3}{4n^3} = 1$$

$$\boxed{b_n \text{ converge } \wedge \lim_{n\to+\infty} \frac{a_n}{b_n} \neq 0 \quad \therefore \sum_{n=4}^{\infty} \frac{4n+1}{2n^3-2n-1} \text{ converge}}$$

7) $$\sum_{n=1}^{\infty} \frac{1}{3^n-1}$$

Aplicando crit de cmp al lim

$$\sim \sum_{n=1}^{\infty} \frac{1}{3^n}$$, geometrica con $|r|<1$

converge

$$\lim_{n\to+\infty} \frac{\frac{1}{3^n-1}}{\frac{1}{3^n}}$$

$$\lim_{n\to+\infty} \frac{3^n}{3^n-1}$$

$$\lim_{n\to+\infty} \frac{\cancel{3^n}}{\cancel{3^n}\left(1-\frac{1}{3^n}\right)}$$

$$\lim_{n\to+\infty} \frac{1}{1-\cancel{\frac{1}{3^n}}=0} = \frac{1}{1} = 1$$

$$\boxed{\begin{array}{l} b_n \text{ conver } \wedge L \neq 0 \\ \therefore \sum_{n=1}^{\infty} \frac{1}{3^n-1} \text{ converge} \end{array}}$$

8) $\sum_{n=1}^{\infty} \frac{1}{2^n-n}$

$\sim \sum_{n=1}^{\infty} \frac{1}{2^n}$, geometrica con $|r|<1$ converge

Aplicando el criterio de comparacion al limite

$a_n = \frac{1}{2^n-n}$ $\quad b_n = \frac{1}{2^n}$

$$\lim_{n\to+\infty} \frac{\frac{1}{2^n-n}}{\frac{1}{2^n}}$$

$$\lim_{n\to+\infty} \frac{2^n}{2^n-n}$$

$$\lim_{n\to+\infty} \frac{2^n}{2^n} = 1$$

$$\boxed{\begin{array}{l} \text{Como } b_n \text{ converge } \wedge L \neq 0 \\ \sum_{n=1}^{\infty} \frac{1}{2^n-n} \text{ converge} \end{array}}$$

9) $\sum_{n=1}^{\infty} \frac{1}{5\sqrt{n}-12} \sim \frac{1}{5\sqrt{n}} \sim \frac{1}{\sqrt{n}}$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}, \text{ p serie con } p<1 \text{ diverge}$$

$$\lim_{n\to+\infty} \frac{\frac{1}{5\sqrt{n}-12}}{\frac{1}{\sqrt{n}}}$$

$$\lim_{n\to+\infty} \frac{\sqrt{n}}{5\sqrt{n}-12}$$

$$\lim_{n\to+\infty} \frac{\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}\left(5-\frac{12}{\sqrt{n}}\right)}$$

$$\lim_{n\to+\infty} \frac{1}{5-\frac{12}{\cancel{n^{\frac{1}{2}}}}\to 0} = \frac{1}{5}$$

$$\boxed{\begin{array}{l} b_n \text{ diverge} \wedge L \neq 0 \\ \therefore \displaystyle\sum_{n=1}^{\infty} \frac{1}{5\sqrt{n}-12} \text{ diverge} \end{array}}$$

10) $$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+2}} \sim \frac{1}{n\sqrt{n^2}} = \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2}, \text{ p serie, } p>1 \text{ converge}$$

$$a_n = \frac{1}{n\sqrt{n^2+2}} \qquad b_n = \frac{1}{n^2}$$

$$\lim_{n\to+\infty} \frac{\frac{1}{\cancel{n}\sqrt{n^2+2}}}{\frac{1}{\cancel{n^2}}}$$

$$\lim_{n\to+\infty} \frac{n}{\sqrt{n^2+2}}$$

$$\lim_{n\to+\infty} \frac{n}{\sqrt{n^2\left(1+\frac{2}{n^2}\right)}}$$

$$\lim_{n\to+\infty} \frac{\cancel{n}}{\cancel{n}\sqrt{1+\frac{2}{n^2}}}$$

$$\lim_{n\to+\infty} \frac{1}{\sqrt{1+\underbrace{\cancel{\frac{2}{n^2}}}_{=0}}} = 1$$

$$\boxed{\begin{array}{l} b_n \text{ converge } \wedge\ L \neq 0 \\ \therefore \displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+2}} \text{ converge} \end{array}}$$

11) $$\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}} \sim \frac{2n^2}{\sqrt{n^5}} \sim \frac{\cancel{n^2}}{\cancel{n^2}\sqrt{n}} \sim \frac{1}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$$, p serie, $p < 1$ diverge

$$a_n = \frac{2n^2+3n}{\sqrt{5+n^5}} \qquad b_n = \frac{1}{\sqrt{n}}$$

$$\lim_{n\to+\infty} \frac{\frac{2n^2+3n}{\sqrt{5+n^5}}}{\frac{1}{\sqrt{n}}}$$

$$\lim_{n\to+\infty} \frac{n^{\frac{1}{2}}(2n^2+3n)}{\sqrt{5+n^5}}$$

$$\lim_{n\to+\infty} \frac{n^{\frac{1}{2}}(2n^2+3n)}{\sqrt{n^5\left(1+\frac{5}{n^5}\right)}}$$

$$\lim_{n\to+\infty} \frac{n^{\frac{1}{2}}\, n^{2}\left(2+\frac{3}{n}\right)}{\sqrt{n^5}\cdot\sqrt{1+\frac{5}{n^5}}}$$

$$\lim_{n\to+\infty} \frac{\cancel{n^{\frac{5}{2}}}\left(2+\frac{3}{n}\right)}{\cancel{n^{\frac{5}{2}}}\cdot\sqrt{1+\frac{5}{n^5}}}$$

$$\lim_{n\to+\infty} \frac{2+\cancel{\frac{3}{n}}=0}{\sqrt{1+\cancel{\frac{5}{n^5}}=0}} = 2$$

$$\boxed{\begin{array}{l} b_n \text{ diverge } \wedge L \neq 0 \\ \therefore \displaystyle\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}} \text{ diverge} \end{array}}$$

12) $$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^3+1}} \sim \frac{\cancel{\sqrt{n}}}{n\cancel{\sqrt{n}}} \sim \frac{1}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}, \quad p \text{ serie}, \; p=1$$

diverge

$$a_n = \frac{\sqrt{n}}{\sqrt{n^3+1}} \qquad b_n = \frac{1}{n}$$

$$\lim_{n\to+\infty} \frac{\frac{\sqrt{n}}{\sqrt{n^3+1}}}{\frac{1}{n}}$$

$$\lim_{n\to+\infty} \frac{n\sqrt{n}}{\sqrt{n^3+1}}$$

$$\lim_{n\to+\infty} \frac{n\sqrt{n}}{\sqrt{n^3\left(1+\frac{1}{n^3}\right)}}$$

$$\lim_{n\to+\infty} \frac{\cancel{n\sqrt{n}}}{\cancel{n\sqrt{n}}\left(1+\frac{2}{n^3}\right)}$$

$$\lim_{n\to+\infty} \frac{1}{1+\underbrace{\cancel{\frac{1}{n^3}}}_{=0}} = 1$$

$$\boxed{\begin{array}{l} b_n \text{ diverge } \wedge L \neq 0 \\ \therefore \sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^3+2}} \end{array}}$$