

Sucesiones

Si tiene a_1, a_2, \dots

$a_{n+1} \geq a_n$, a_n es una sucesión

Asumir, que es creciente

$$\begin{aligned} a_{n+1} &\geq a_n && \checkmark \text{ Creciente} \\ a_n &\geq a_{n-1} && \times \text{ Decreciente} \end{aligned}$$

$$a_n = n! \quad a_{n+1} = (n+1)!$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{n!}$$

Derivables NO tiene a_1, a_2, \dots

$f(x) = a_n$ $f'(x) > 0$, es creciente
 $f'(x) < 0$, es decreciente

$$1. p_q = 14q - q^2, \forall q \geq 7$$

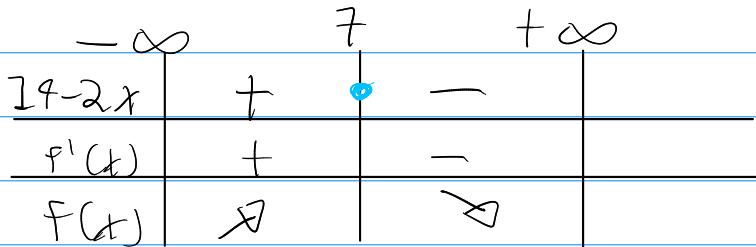
$$f(x) = 14x - x^2 \quad x \cdot k = k$$

$$f'(x) = 14 - 2x = 0$$

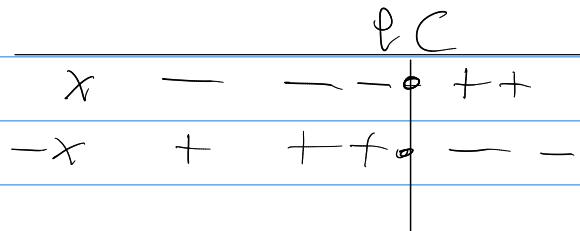
$$2(7-x) = 0$$

$$7-x = 0$$

$$x = 7$$



; Decrease



$$2. f_m = m^3 - 5m^2 - 25m, \forall m \geq 5$$

$$f(x) = x^3 - 5x^2 - 25x$$

$$f'(x) = 3x^2 - 10x - 25 = 0$$

$$\cancel{3x} - 25 = 5x$$

$$\cancel{x} - 25 = -15x$$

$$-10x$$

$$(3x+5)(x-5) = 0$$

$$3x+5=0 \quad x-5=0$$

$$x = \underline{-5} \quad x = 5$$

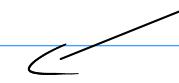
$$(3x+5)(x-5)$$

$$x = -\frac{5}{3} \quad x = 5$$

$$-\infty \quad -\frac{5}{3} \quad 5 \quad +\infty$$

$3x+5$	-	+	+
$x-5$	-	-	+
$f'(x)$	+	-	+
$f(x)$	↗	↘	↗

crece



$$9. a_m = \frac{m!}{m^2} \quad \frac{a_{n+1}}{a_n} \geq 1 \quad \begin{array}{l} \checkmark \text{ Crecce} \\ \times \text{ Decre} \end{array}$$

Se asume que a_m es creciente

$$a_{n+1} = \frac{(n+1)!}{(n+1)^2} \quad a_n = \frac{n!}{n^2}$$

$$\frac{(n+1)!}{(n+1)^2} \geq 1$$

$\frac{n!}{n^2}$

$$\frac{(n+1)! \cdot n^2}{(n+1)^2 \cdot n!} \geq 1 \quad \begin{array}{l} (p+1)! \\ (p+1) \cdot (p+1-1)! \\ (p+1) \cdot p! \\ (p+1) \cdot p \cdot (p-1)! \end{array}$$

$$\frac{(n+1) \cancel{n!} \cdot n^2}{(n+1)^2 \cdot \cancel{n!}} \geq 1$$

$$\frac{n^2}{n+1} \geq 1$$

$$n^2 \geq n+1$$

$$n^2 - n - 1 \geq 0$$

$$\frac{\sqrt{5} + 1}{2} \quad \checkmark \quad \frac{\sqrt{5} - 1}{2}$$

Crecce

$$a_n = \frac{3^n \cdot n!}{2 \cdot 4 \cdot 6 \cdots (2n)}$$

$$\frac{a_{n+1}}{a_n} \geq 1$$

$$a_{n+1} = \frac{3^{n+1} \cdot (n+1)!}{2 \cdot 4 \cdot 6 \cdot \cancel{2n} \cdot 2(n+1)} = \frac{3^{n+1} \cdot (n+1)!}{2 \cdot 4 \cdot 6 \cdot 2n \cdot (2n+2)}$$

$\cancel{2n+2}$

$$\frac{\frac{3^{n+1} \cdot (n+1)!}{2 \cdot 4 \cdot \cancel{2n} \cdot (2n+2)}}{\frac{3^n \cdot n!}{2 \cdot 4 \cdot 6 \cdots (2n)}} \geq 1$$

$$\frac{\frac{3^{n+1} \cdot (n+1)!}{3^n \cdot n! (2n+2)}}{(n+1-1)!} \geq 1$$

$3^{n+1} = 3^n \cdot 3^1$

$(n+1-1)!$

~~$\frac{3^n \cdot 3 \cdot (n+1)!}{3^n \cdot n! (2n+2)}$~~ ≥ 1

$$\frac{3(n+1)}{2n+2} \geq 1$$

$$\frac{3(n+1)}{2(n+1)} \geq 1$$

$$\frac{3}{2} \geq 1$$

[Creciente ✓]

$$a_n = \frac{n^4}{e^n}, \quad n > 0 \quad \nearrow$$

$$n < 0 \quad \searrow$$

$$f(x) = \frac{x^4 - a}{e^x - b} \quad \frac{a' \cdot b - a \cdot b'}{b^2}$$

$$\frac{4x^3 \cdot e^x - x^4 \cdot e^x}{e^{x^2}} = 0$$

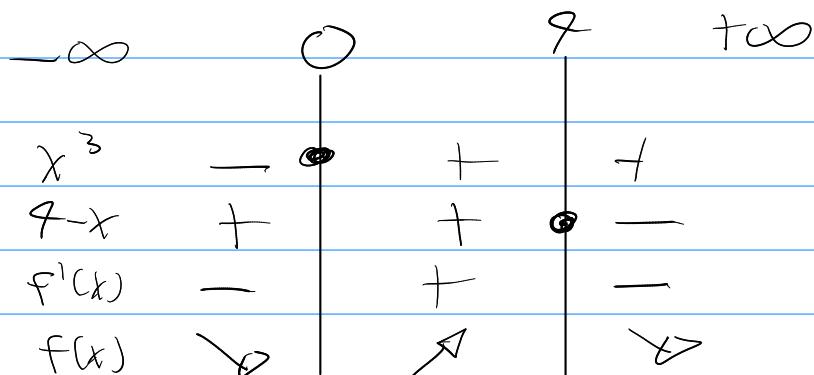
$$\frac{x^3 \cdot e^x (4-x)}{e^x} = 0$$

$$\frac{x^3 (4-x)}{e^x} = 0$$

$$x^3 (4-x) = 0$$

$$x^3 = 0 \quad 4-x = 0$$

$$x = 0 \quad x = 4$$



\therefore Decrease

Induction

$$(g) \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$n=1 \quad 1^2 = \frac{1(1+1)(2 \cdot 1 + 1)}{6}$$

$1=1 \checkmark$

$$n=p \quad p$$
$$\sum_{i=1}^p i^2 = \frac{p(p+1)(2p+1)}{6}, \text{ hi:}$$

$$p+1+p = p+2$$

$$n=p+1 \quad p+1$$
$$\sum_{i=1}^{p+1} i^2 = \frac{(p+1)(p+2)(2p+3)}{6}$$

$$\frac{2(p+1)+1}{2p+2+1} \\ 2p+3$$

$$\sum_{i=1}^{p+1} i^2$$

$$\sum_{i=1}^p i^2 + (p+1)^2$$

$$\frac{p(p+1)(2p+1)}{6} + (p+1)^2, \frac{6}{6}, \text{ hi:}$$

$$\frac{p(p+1)(2p+1)}{6} + \frac{(p+1)^2}{6}$$

$$\frac{p(p+1)(2p+1)}{6} + \frac{(p+1)^2}{6} 6$$

$$\frac{p(p+1)(2p+1)}{6} + \frac{(p+1)^2}{6} 6$$

$$\frac{(p+1)p(2p+1) + 6(p+1)}{6}$$

$$\frac{(p+1)(2p^2+p+6p+6)}{6}$$

$$\frac{(p+1)(2p^2+7p+6)}{6}$$

$$\begin{aligned} 2p^2 + 7p + 6 \\ 2p \cancel{- 3} = 3p \\ p \cancel{- 2} = 7p \end{aligned}$$

$$\boxed{\frac{(p+1)(p+2)(2p+3)}{6}}$$

$$\frac{(p+1)(p+2)(2p+3)}{6}$$

$$(j) \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

$$n=1 \quad \frac{1}{(2 \cdot 1 - 1) \cdot (2 \cdot 1 + 1)} = \frac{1}{2 \cdot 1 + 1}$$

$$\frac{1}{3} = \frac{1}{3} \quad \checkmark$$

$$n=p \quad \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2p-1)(2p+3)} = \frac{p}{2p+1}, \text{ H.I.}$$

$$n=p+1 \quad \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2p+1)(2p+3)} = \frac{p+1}{2p+3}$$

$$\begin{array}{r} 2(\cancel{p+1})-1 \\ 2p+2-1 \\ 2p+1 \end{array} \quad \begin{array}{r} 2(\cancel{p+1})+1 \\ 2p+2+1 \\ 2p+3 \end{array}$$

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2p-1)(2p+3)} + \frac{1}{(2p+1)(2p+3)}$$

$$\frac{p}{2p+1} + \frac{1}{(2p+1)(2p+3)}, \text{ H.I.}$$

$$\frac{(2p+3)}{2p+3} \cdot \frac{p}{2p+1} + \frac{1}{(2p+1)(2p+3)}$$

$$\frac{(2p+3)p}{(2p+3)(2p+1)} + \frac{1}{(2p+1)(2p+3)}$$

$$\frac{(2p+3)p + 1}{(2p+3)(2p+1) \quad (2p+3)(2p+3)}$$

$$\frac{2p^2 + 3p + 1}{(2p+1)(2p+3)}$$

$$\begin{aligned} 2p^2 + 3p + 1 \\ 2p \cancel{-} 1 = p \\ p \cancel{-} 1 = 2p \\ 3p \end{aligned}$$

$$\frac{(2p+1)(p+1)}{(2p+1)(2p+3)}$$

$p+1$
$2p+3$

$$6. \sum_{k=1}^n k \cdot k! = (n+1)! - 1$$

$$\begin{aligned} n &= 2 & 2 \cdot 2! &= (2+2)! - 2 \\ && 2 &= 2 \end{aligned}$$

$$\begin{aligned} n &= p & p \\ &\in \mathbb{N}, n! &= (p+2)! - 2, \text{ H.S.} \\ &n = 2 & (p+2+2)! - 2 \end{aligned}$$

$$\begin{aligned} n &= p+2 & p+2 \\ &\in \mathbb{N}, n! &= (p+2)! - 2, \text{ H.Q.B.} \\ &n = 2 & \end{aligned}$$

$$\begin{aligned} p+2 \\ \in \mathbb{N}, n! \\ n = 2 \end{aligned}$$

$$\begin{aligned} p \\ \in \mathbb{N}, n! + (p+2), (p+2)! \\ n = 2 \end{aligned}$$

$$\begin{aligned} (p+2)! - 2 + (p+2), (p+2)! \\ (p+2)! + (p+2), (p+2)! - 2 \end{aligned}$$

$$(p+2)! (2+p+2) - 2$$

$$\begin{aligned} (p+2)! (p+2) - 2 \\ (p+2)! - 2 \end{aligned}$$

$$(\rho+2)! (\rho+2) - 1$$

$$(\rho+2)! = (\rho+2) \cdot (\rho+2-1)! \\ (\rho+2) \cdot (\rho+1)!$$

$$(\rho+2)! = (\rho+2) \cdot (\rho+2-1) \\ (\rho+2) \cdot (\rho+1) \cdot \rho \cdot (\rho-1) \cdot (\rho-2)!$$

$$9. \sum_{j=1}^k (2j-1) \cdot 3^j = 3 + (k-1) \cdot 3^{k+1}$$

$$h=1 \quad (2 \cdot 1 - 1) \cdot 3^1 = 3 + (1-1) \cdot 3^{1+1}$$

$$3 = 3$$

$$h=p \quad p$$

$$\leq (2 \cdot 1 - 1) \cdot 3^1 = 3 + (p-1) \cdot 3^{p+1} \quad H_1$$

$$j=1$$

$$(p+1)-1 = p+1$$

$$h=p+1 \quad p+1$$

$$\leq (2 \cdot 1 - 1) \cdot 3^1 = 3 + p \cdot 3^{p+2} \quad H_1 \text{ (Q)}$$

$$j=2$$

$$(p+1)-1$$

$$p$$

$$p+1$$

$$\leq (2 \cdot 1 - 1) \cdot 3^1$$

$$j=2$$

$$\xrightarrow{\quad} 2 \overbrace{(p+1)-1}^{(2p+2-1)} \cdot 3^{p+1}$$

$$(2p+2-1) \cdot 3^{p+2}$$

$$(2p+1) \cdot 3^{p+1}$$

$$p$$

$$\leq (2 \cdot 1 - 1) \cdot 3^1 + (2p+1) \cdot 3^{p+1}$$

$$j=2$$

$$3 + (p-1) \cdot 3^{p+1} + (2p+1) \cdot 3^{p+1}$$

$$3 + 3^{p+1} (p-1 + 2p+1)$$

$$3 + 3^{p+1} \cdot 3^1 = \boxed{3 + 3^{p+2}}$$

$$10. \sum_{k=1}^n \frac{k+2}{(k^2+k) \cdot 2^k} = 1 - \frac{1}{(n+1) \cdot 2^n}$$

$$h=1 \quad \frac{1+2}{(1^2+1) \cdot 2^1} = 1 - \frac{1}{(1+1) \cdot 2^1}$$

$$\frac{3}{2} = \frac{3}{2} \quad \checkmark$$

$$h=p \quad p \\ \leq \quad \frac{k+2}{(k^2+k) \cdot 2^k} = 1 - \frac{1}{(p+1) \cdot 2^p}, \text{ Hi} \\ k=1 \quad (p+1) \cdot 2^p \quad \underbrace{p+1+2=p+3}_{p+1+2=p+3}$$

$$h=p+1 \quad \leq \quad \frac{k+2}{(k^2+k) \cdot 2^k} = 1 - \frac{1}{(p+2) \cdot 2^{p+1}}, \text{ H(1)}$$

$$p+1 \\ \leq \frac{k+2}{(k^2+k) \cdot 2^k}$$

$$p+1+2=p+3 \\ p+1 \quad + \quad \frac{p+3}{[(p+1)^2 + (p+1)] \cdot 2^{p+1}}$$

$$1 - \frac{1}{(p+1) \cdot 2^p} + \frac{p+3}{[(p+1)^2 + p+1] \cdot 2^{p+1}}$$

$$I - \frac{I}{(\rho+1) \cdot 2^P} + \frac{\rho+3}{\{(\rho+1)^2 + (\rho+1)\} \cdot 2^{\rho+1}}$$

$$I - \frac{I}{(\rho+1) \cdot 2^P} + \frac{\rho+3}{(\rho+1)(\rho+1+1) \cdot 2^{\rho+1}}$$

$$I - \frac{I}{(\rho+1) \cdot 2^P} + \frac{\rho+3}{(\rho+1) \cdot (\rho+2) \cdot 2^{\rho+1}}$$

$$I - \frac{I}{(\rho+1) \cdot 2^P}, \frac{(\rho+2) \cdot 2}{(\rho+2) \cdot 2} + \frac{\rho+3}{(\rho+1) \cdot (\rho+2) \cdot 2^{\rho+1}}$$

$$I - \frac{\rho+3}{(\rho+1) \cdot (\rho+2) \cdot 2^{\rho+1}} - \frac{I}{(\rho+1) \cdot 2^P}, \frac{(\rho+2) \cdot 2}{(\rho+2) \cdot 2} I$$

$$I - \frac{-\rho-3 + 2\rho+7}{(\rho+1) \cdot (\rho+2) \cdot 2^{\rho+1}}$$

$$I - \frac{\cancel{\rho+1}}{(\cancel{\rho+1}) \cdot (\rho+2) \cdot 2^{\rho+1}}$$

$$I - \frac{I}{(\rho+2) \cdot 2^{\rho+2}}$$

$$I - \frac{I}{(\rho+2) \cdot 2^{\rho+2}}$$

1. Demuestre utilizando el Principio de Inducción Matemática que $\sum_{k=1}^n \frac{2k-1}{2^k} = 3 - \frac{2n+3}{2^n}$, para todo entero $n \geq 1$. (4 pts)

$$n=1 \quad \frac{2 \cdot 1 - 1}{2^1} = 3 - \frac{2 \cdot 1 + 3}{2^1}$$

$$\frac{1}{2} \cdot \frac{1}{2} \checkmark$$

$$n=p \quad p \\ \leq \quad \frac{2^k - 1}{2^k} = 3 - \frac{2p+3}{2^p}, \text{ H.I.}$$

$$n=p+1 \quad p+1 \\ \leq \quad \frac{2^k - 1}{2^k} = 3 - \frac{2p+5}{2^{p+1}}. \text{ H.Q.D}$$

$$\frac{2(p+1)+3}{2^{p+2}} = 2p+5$$

$$p+1 \\ \leq \quad \frac{2^k - 1}{2^k}$$

$$p \\ \leq \quad \frac{2^k - 1}{2^k} + \frac{2p+1}{2^{p+1}}$$

$$2(p+2)-1 \\ 2p+2-1 = 2p+1$$

$$3 - \frac{2p+3}{2^p} + \frac{2p+1}{2^{p+1}}$$

$$3 - \frac{2p+3}{2^p} \cdot \frac{2}{2} + \frac{2p+1}{2^{p+1}}$$

$$3 - \frac{7p+6+2p+1}{2^{p+1}}$$

$$3 + \frac{-4p - 6 + 2p + 7}{2^{p+1}}$$

$$3 - \frac{2p + 5}{2^{p+1}}$$

$$3 + \frac{-2p - 5}{2^{p+1}}$$

$$x+2 = -(x-2)$$

$$3 + -\frac{2p + 5}{2^{p+1}}$$