$P(x) = 9x^5 + 27x^4 + 28x^3 + 12x^2 + 3x + 1$

$Q(x) = \frac{i}{3}$

9	27	28	12	$\cancel{3}$	1	$\frac{i}{3}$
	$3i$	$-1+9i$	$-3+9i$	$3i-3$	-1	
9	$3i+27$	$27+9i$	$9+9i$	$3i$	0	$(x - \frac{i}{3})$

$9x^4 + (3i+27)x^3 + (27+9i)x^2 + (9+9i)x + 3i$

9	$3i+27$	$27+9i$	$9+9i$	$3i$	$-\frac{i}{3}$
	$-3i$	$-9i$	$-9i$	$-3i$	
9	27	27	9	0	$(x - \frac{-i}{3})$
					$(x+3i)$

$9x^3 + 27x^2 + 27x + 9$

9	27	27	9	
	-9	-18	-9	-1
9	18	9	0	$(x-(-1))$
				$(x+1)$

$9x^2 + 18x + 9$

9	18	9	
	-9	-9	-1
9	9	0	$(x+1)$

$9x + 9$

$$\boxed{(x - \tfrac{i}{3})(x + \tfrac{i}{3})(x+1)^2 (9x+9)}$$

$\frac{i}{3} \cdot 9$

$(3i+27)\frac{i}{3}$

$\frac{3i^2}{3} + \frac{27i}{3}$

$-\frac{i}{3} \cdot 9$

$-1 + 9i$

$-\frac{i}{3} \cdot 27$

$(27+9i)\frac{i}{3}$

$\frac{27i}{3} + \frac{9i^2}{3}$

$9i - 3$

$(9+9i)\frac{i}{3}$

$\frac{9i}{3} + \frac{9i^2}{3}$

$3i - 3$

$3i \cdot \frac{i}{3}$

$\frac{3i^2}{3} = -1$

$$\frac{(-1+i\sqrt{3})^6}{16i^{15}}$$ Exprese en forma polar

i^{15}

$15-4\cdot\text{int}\left(\frac{15}{4}\right)$

$15-4\cdot 3$

$15-12$

3

$i^3=-i$

$$\frac{\overbrace{(-1+i\sqrt{3})^6}^{z_1}}{\underbrace{-16i}_{z_2}}$$

A polar

$z_1=-1+i\sqrt{3}$

$r=\sqrt{(-1)^2+(\sqrt{3})^2}=2$

$\theta=\arctan\left(\frac{\sqrt{3}}{-1}\right)+\pi=\frac{2\pi}{3}$

$z_1=2\text{ cis}\left(\frac{2\pi}{3}\right)$

$z_2=0-16i$

$r=\sqrt{0^2+(-16)^2}=16$

$\theta=-\frac{\pi}{2}$ $\quad a=0 \wedge b<0$

$z_2=16\text{ cis}\left(-\frac{\pi}{2}\right)$

$$\frac{\left(2\text{ cis}\left(\frac{2\pi}{3}\right)\right)^6}{16\text{ cis}\left(-\frac{\pi}{2}\right)}$$

$$\frac{2^6\cdot\text{cis}\left(\frac{2\pi}{3}\cdot 6\right)}{16\text{ cis}\left(-\frac{\pi}{2}\right)}$$

$$\frac{2^6\cdot\text{cis}(4\pi)}{16\text{ cis}\left(-\frac{\pi}{2}\right)}$$

$$\frac{2^6}{16}\cdot\text{cis}\left(4\pi+\frac{-\pi}{2}\right)$$

$$4\cdot\text{cis}\left(\frac{7\pi}{2}\right)$$

Sea $z = 1 - e^{\frac{\pi}{2} i}$ $r \operatorname{cis}(\theta)$

$r e^{i\theta}$

a) Compruebe que $z = 1 - i$

$$1 - e^{\frac{\pi}{2} i}$$

$$1 - \left[\operatorname{cis}\left(\frac{\pi}{2}\right) \right]$$

$$1 - \left\{ \cancel{\cos\left(\frac{\pi}{2}\right)}^{0} + i \cancel{\operatorname{sen}\left(\frac{\pi}{2}\right)}^{1} \right\}$$

$$1 - i$$

b) Calcule $\ln(z)$

$$\ln(1 - i)$$

$$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\theta = \arctan\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$$

$$z = \sqrt{2} \cdot e^{-\frac{\pi}{4} i}$$

$$\ln\left(\sqrt{2} \cdot e^{-\frac{\pi}{4} i}\right)$$

Cuadrante	a	b	Rango	Que hago?
I	+	+	✓	NADA
II	−	+	X	Sumar π al θ
III	−	−	X	Resta π al θ
IV	+	−	✓	NADA

Casos especiales

$\theta = \frac{\pi}{2}$ si $a = 0 \wedge b > 0 \to$ Positivo

$\theta = -\frac{\pi}{2}$ si $a = 0 \wedge b < 0 \to$ Negativo

$$\ln(\sqrt{2}) + \cancel{\ln}(e)^{-\frac{\pi}{4} i}$$

$$\ln(\sqrt{2}) - \frac{\pi}{4} i$$

$$\approx \boxed{0{,}3465 - 0{,}7853 i}$$

$$|z-3-4i| = 5$$
$$\operatorname{Arg}(\bar{z}+i) = -\frac{\pi}{2}$$

Casos especiales

$\theta = \frac{\pi}{2}$	$a=0$	$b>0$
$\theta = -\frac{\pi}{2}$	$a=0$	$b<0$
$\theta = \pm\pi$	$a<0$	$b=0$

$$\operatorname{Arg}(\bar{z}+i) = -\frac{\pi}{2}$$

$$\overline{a+bi}+i = -\frac{\pi}{2}$$
$$a-bi+i = -\frac{\pi}{2}$$
$$a+(-b+1)i = -\frac{\pi}{2}$$

$$a=0 \qquad -b+1<0$$
$$-b<-1$$
$$b>1$$

$$|z-3-4i| = 5$$
$$|a+bi-3-4i| = 5$$
$$|(a-3)+(b-4)i| = 5$$
$$(a-3)^2+(b-4)^2 = 5^2$$
$$(0-3)^2+b^2-8b+16 = 25$$
$$9+b^2-8b+16-25 = 0$$
$$b^2-8b = 0$$
$$b(b-8) = 0$$
$$b = 8$$

$$\boxed{0+8i}$$

Sean A, B, C, Q matrices $\in \mathbb{R}$ |

$$AB+2C-Q=0$$

$$A=\begin{pmatrix} a & 0 & 2 \\ 3 & 1 & b \end{pmatrix}, \quad B=\begin{pmatrix} 2 & -1 & 3 \\ 0 & 3 & 1 \\ a & 2 & 2 \end{pmatrix}, \quad C=\begin{pmatrix} 3a & 1 & 0 \\ -1 & -b & -4 \end{pmatrix}$$

a) Determine el tamaño de Q

$$AB+2C-Q=0$$
$$AB+2C=Q$$

$$A=\begin{pmatrix} a & 0 & 2 \\ 3 & 1 & b \end{pmatrix}, \quad B=\begin{pmatrix} 2 & -1 & 3 \\ 0 & 3 & 1 \\ a & 2 & 2 \end{pmatrix}$$

$$AB=\begin{pmatrix} 2a+0+2a & -a+0+4 & 3a+0+4 \\ 6+0+ba & -3+3+2b & 9+1+2b \end{pmatrix}$$

$$AB=\begin{pmatrix} 4a & 4-a & 4+3a \\ 6+6a & 2b & 10+2b \end{pmatrix}$$

$$AB+2C-Q=0$$

$$C=\begin{pmatrix} 3a & 1 & 0 \\ -1 & -b & -4 \end{pmatrix}, \quad 2C=\begin{pmatrix} 6a & 2 & 0 \\ -2 & -2b & -8 \end{pmatrix}$$

$$AB + 2C = Q$$

$$\begin{pmatrix} 4a & 4-a & 4+3a \\ 6+6a & 2b & 10+2b \end{pmatrix} + \begin{pmatrix} 6a & 2 & 0 \\ -2 & -2b & -8 \end{pmatrix}$$

$$Q = \begin{pmatrix} 10a & 6-a & 4+3a \\ 4+6a & 0 & 2+2b \end{pmatrix}, \quad \boxed{2 \times 3}$$

b) Calcule $Q_{11} \wedge Q_{22}$

$10a \qquad 0$ ✓

6) $Ax + (xB)^T = I$, x es simetrica $(x^T = x)$

a) Demuestre $x = (A + B^T)^{-1}$

$A + B^T$ es invertible $(A+B^T)^{-1}$

$$Ax + (xB)^T = I$$
$$Ax + B^T \cdot x^T = I$$
$$A \cdot x + B^T \cdot x = I, \quad \text{Pues } x \text{ es simetrica}$$
$$(A + B^T)x = I$$
$$x = I\,(A+B^T)^{-1}$$
$$\boxed{x = (A+B^T)^{-1}}$$ ✓

b) Calcule x si, $x = (A + B^T)^{-1}$

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$$

$$A + B^T = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ 2 & 4 \end{pmatrix}$$

$$\begin{matrix} \frac{1}{3} \cdot \widetilde{F1} \\ -2 \cdot F1 + \widetilde{F2} \end{matrix} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 4 & -\frac{2}{3} & 1 \end{array}\right) \quad \begin{matrix} 1 \\ 0 \end{matrix}$$

$$\begin{matrix} \\ \frac{1}{4} \cdot \widetilde{F2} \end{matrix} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{4} \end{array}\right) \quad \begin{matrix} 0 \\ 1 \end{matrix}$$

$$\boxed{(A + B^T)^{-1} = \begin{pmatrix} \frac{1}{3} & 0 \\ -\frac{1}{6} & \frac{1}{4} \end{pmatrix}}$$

Use Gauss-Jordan para este sistema

$$\begin{cases} x + 2y + w = 10 \\ x + 2y + z + w = 5 \\ -2y + 2z + 2w = 4 \end{cases}$$

$$\begin{cases} x + 2y + 0z + w = 10 \\ x + 2y + z + w = 5 \\ 0x - 2y + 2z + 2w = 4 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 10 \\ 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 2 & 2 & 4 \end{array}\right)$$

$$-F1+\widetilde{F2} \quad \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 10 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & -2 & 2 & 2 & 4 \end{array}\right)$$

$$F1 \leftrightarrow F2 \quad \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 10 \\ 0 & -2 & 2 & 2 & 4 \\ 0 & 0 & 1 & 0 & -5 \end{array}\right)$$

$$\begin{array}{l} -2\cdot F2+\widetilde{F1} \\ -\frac{1}{2}\cdot \widetilde{F2} \end{array} \quad \left(\begin{array}{cccc|c} 1 & 0 & 2 & 3 & 14 \\ 0 & 1 & -1 & -1 & -2 \\ 0 & 0 & 1 & 0 & -5 \end{array}\right)$$

$$\begin{array}{l} -2\cdot F3+\widetilde{F1} \\ F3+\widetilde{F2} \end{array} \quad \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 24 \\ 0 & 1 & 0 & -1 & -7 \\ 0 & 0 & 1 & 0 & -5 \end{array}\right)$$

$$\begin{array}{cccc} x & y & z & w \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 24 \\ 0 & 1 & 0 & -1 & -7 \\ 0 & 0 & 1 & 0 & -5 \end{array}\right)$$

$$x + 3w = 24 \longrightarrow x = 24 - 3w$$

$$y - w = -7 \longrightarrow y = -7 + w$$

$$z = -5 \longrightarrow z = -5$$

$$w \in \mathbb{R}$$

$$S = \{ 24 - 3w, -7 + w, -5, w \}$$