

Ejercicio #4

Objetivo Simplex Gran M \rightarrow Dual

$$\text{Max } z = x + 3y$$

$$x + 2y \geq 7$$

$$2x - y \leq 4$$

$$4x - y \geq 1$$

$$3x + 2y \leq 20$$

$$x, y \geq 0$$

	X	Y	A1	H1	A2	H2	E1	E2	VS
Y	0	1	0	0	$-3/11$	$4/11$	0	$3/11$	7
H1	0	0	0	1	$-7/11$	$2/11$	0	$7/11$	7
X	1	0	0	0	$2/11$	$1/11$	0	$-2/11$	2
E1	0	0	-1	0	$-4/11$	$9/11$	1	$4/11$	9
Z	0	0	M	0	$M \cdot 7/11$	$13/11$	0	$7/11$	23

Primal

$$\text{max } z = x + 3y$$

Sujeto a:

$$x + 2y \geq 7 \quad (y1) \quad A1-E1$$

$$2x - y \leq 4 \quad (y2) \quad H1$$

$$4x - y \geq 1 \quad (y3) \quad A2-E2$$

$$3x + 2y \leq 20 \quad (y4) \quad H2$$

Primal con variables

$$\text{max } z = x + 3y - MA1 - MA2$$

Sujeto a:

$$x + 2y - E1 + A1 = 7$$

$$2x - y + H1 = 4$$

$$4x - y - E2 + A2 = 1$$

$$3x + 2y + H2 = 20$$

$$\text{max } z - x - 3y + MA1 + MA2 = 0$$

Dual

$$\text{min } w = x + 3y - MA1 - MA2$$

$$\text{Sujeto a: min } w = 7y1 + 4y2 + y3 + 20y4$$

Sujeto a:

$$y1 + 2y2 + 4y3 + 3y4 \geq 1 \quad (\text{Variable } x)$$

$$2y1 - y2 - y3 + 2y4 \geq 3 \quad (\text{Variable } y)$$

$$y1 \leq 0, y2 \geq 0, y3 \leq 0, y4 \geq 0$$

Tabla Inicial

	X	Y	A1	H1	A2	H2	E1	E2	VS
A1	1	2	1	0	0	0	-1	0	7
H1	2	-1	0	1	0	0	0	0	4
A2	4	-1	0	0	1	0	0	-1	1
H2	3	2	0	0	0	1	0	0	20
Z	1	3	-M	0	-M	0	0	0	0

Penalización

Nuevo $z = z - M A1 - M A2$

	X	Y	A1	H1	A2	H2	E1	E2	VS
Z	-1	-3	M	0	M	0	0	0	0
-MA1	-M	$\frac{2}{3}$	-M	0	0	0	M	M	-7M
-MA2	-4M	3	0	0	-M	0	0	M	-M
Nuevo Z	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0	M	M	-8M

x^* y y^* negativos, se itera

* Iteración #1

$VE = X$ RM: $\frac{7}{1} = 7$ $\frac{4}{2} = 2$ $\frac{1}{4} = \frac{1}{4}$ $\frac{20}{3} = \frac{20}{3}$ $Pivote = 4$

$VS = A2$

	X	Y	A1	H1	A2	H2	E1	E2	VS
A2-0X	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$
	=1	=-1/4	=0	=0	=1/4	=0	=0	=0	=1/4

$\frac{1}{4}$

$w = -5M - 1$

	A1	H1	H2	Z
X	$1 - 1 \cdot 0 = 0$	$2 - 2 \cdot 1 = 0$	$3 - 3(1) = 0$	$(-8M - 1) - (-5M - 1) = -3M$
Y	$2 - 1(-1/4) = 9/4$	$-1 - 2(-1/4) = -1/2$	$2 - 3(-1/4) = 11/4$	$(-M - 3) - (-5M - 1) = 4M - 2$
A1	$1 - 1 \cdot 0 = 1$	$0 - 2(0) = 0$	$0 - 3(0) = 0$	$0 - (-5M - 1) = 5M + 1$
H1	$0 - 1 \cdot 0 = 0$	$1 - 2(0) = 1$	$0 - 3(0) = 0$	$0 - (-5M - 1) = 5M + 1$
A2	$0 - 1(1/4) = -1/4$	$0 - 2(1/4) = -1/2$	$0 - 3(1/4) = -3/4$	$0 - (-5M - 1) = 5M + 1$
H2	$0 - 1(0) = 0$	$0 - 2(0) = 0$	$1 - 3(0) = 1$	$0 - (-5M - 1) = 5M + 1$
E1	$-1 - 1 \cdot 0 = -1$	$0 - 2(0) = 0$	$0 - 3(0) = 0$	$M - (-5M - 1) = 6M + 1$
E2	$0 - 1(-1/4) = 1/4$	$0 - 2(-1/4) = 1/2$	$0 - 3(-1/4) = 3/4$	$M - (-5M - 1) = 6M + 1$
VS	$7 - 1(1/4) = 27/4$	$4 - 2(1/4) = 7/2$	$20 - 3(1/4) = 77/4$	$(-8M) - (-5M - 1) = -3M + 1$

Tabla luego de iteración 1

	X	y	A1	H1	A2	H2	E1	E2	VS
A1	0	$9/4$	1	0	$-1/4$	0	-1	$1/4$	$27/4$
H1	0	$-1/2$	0	1	$-1/2$	0	0	$1/2$	$7/2$
X	1	$-1/4$	0	0	$0/4$	0	0	$-1/4$	$1/4$
H2	0	$11/4$	0	0	$-3/4$	1	0	$3/4$	$77/4$
Z	0	$\frac{-(9M+13)}{4}$	0	0	$\frac{(5M+1)}{4}$	0	M	$\frac{-(M+1)}{4}$	$\frac{(1-27M)}{4}$

\nearrow Negativo, a iterar

* Iteración #2

$VE = y$ $RM: \frac{(27/4)}{9/4} = 3$ $\frac{(77/4)}{11/4} = 7$ $Pivote = 9/4$
 $VS = A1$ VS

	X	y	A1	H1	A2	H2	E1	E2	VS
$A1 \rightarrow y$	$0/(9/4) = 0$	$(9/4)/(9/4) = 1$	$1/(9/4) = 4/9$	$0/(9/4) = 0$	$(-1/4)/(9/4) = -1/9$	$0/(9/4) = 0$	$-1/(9/4) = -4/9$	$(1/4)/(9/4) = 1/9$	$(27/4)/(9/4) = 3$

	H1	X	H2	Z
X	$0 + (1/2) \cdot 0 = 0$	$1 + (1/4) \cdot 0 = 1$	$0 - 11/4 \cdot 0 = 0$	$0 + \frac{(9M+13)}{4} (0) = 0$
y	$-1/2 + (1/2) \cdot 1 = 0$	$-1/4 + (1/4) \cdot 1 = 0$	$1/4 - 11/4 \cdot 1 = 0$	$\frac{-(9M+13)}{4} + \frac{9M+13}{4} \cdot 1 = 0$
A1	$0 + (1/2)(4/9) = 2/9$	$0 + (1/4)(4/9) = 1/9$	$0 - 11/4 \cdot (4/9) = -11/9$	$0 + \frac{9M+13}{4} (4/9) = M + 13/9$
H1	$1 + (1/2) \cdot 0 = 1$	$0 + 1/4 \cdot 0 = 0$	$0 - 1/4 \cdot 0 = 0$	$0 + \frac{9M+13}{4} \cdot 0 = 0$
A2	$-1/2 + 1/2 \cdot (-1/9) = -5/9$	$1/4 + 1/4 \cdot (-1/9) = 2/9$	$-3/4 - 11/4 \cdot (-1/9) = -4/9$	$\frac{(5M+1)}{4} + \frac{(9M+13)}{4} \cdot (-1/9) = \frac{5M+1}{4} - \frac{9M+13}{36}$
H2	$0 + 1/2 \cdot 0 = 0$	$0 + 1/4 \cdot 0 = 0$	$1 - 11/4 \cdot 0 = 1$	$0 + \frac{9M+13}{4} \cdot 0 = 0$
E1	$0 + 1/2 \cdot (-4/9) = -2/9$	$0 + 1/4 \cdot (-4/9) = -1/9$	$0 - 11/4 (1/9) = -11/36$	$M + \frac{9M+13}{4} \cdot (-4/9) = -13/9$
E2	$1/2 + 1/2 (1/9) = 5/9$	$-1/4 + 1/4 (1/9) = -2/9$	$3/4 - 11/4 (1/9) = 4/9$	$\frac{-(M+1)}{4} + \frac{9M+13}{4} (1/9) = 1/9$
VS	$7/2 + 1/2 (3) = 5$	$1/4 + 1/4 (3) = 1$	$77/4 - 11/4 (3) = 11$	$\frac{(1-27M)}{4} + \frac{9M+13}{4} (3) = 10$

Tabla luego de iteración 2

	X	Y	A1	H1	A2	H2	E1	E2	VS
Y	0	1	$4/9$	0	$-1/9$	0	$-4/9$	$1/9$	3
H1	0	0	$2/9$	1	$-5/9$	0	$-2/9$	$5/9$	5
X	1	0	$1/9$	0	$2/9$	0	$-1/9$	$-2/9$	1
H2	0	0	$-11/9$	0	$-4/9$	1	$11/9$	$4/9$	11
Z	0	0	$M+13/9$	0	$M-1/9$	0	$-13/9$	$1/9$	10

↑ Negativo, iterar

* Iteración #3

$VE = E1$
 $VS = H2$
 $RM: \frac{11}{(11/9)} = 9$
 $Pivote = 11/9$

	X	Y	A1	H1	A2	E1	E2	VS	H2
H2 → E1	$0/(11/9)$	$0/(11/9)$	$0/(-11/9)$	$0/(11/9)$	$(-4/9)/(11/9)$	$11/9$	$(11/9)/(11/9)$	$10/(11/9)$	$1/(11/9)$
	= 0	= 0	= 0	= 0	= $-\frac{4}{11}$	= 1	= 1	= 9	= $9/11$

	Y	H1	X	Z
X	$0 + 4/9 \cdot 0 = 0$	$0 + 2/9 \cdot 0 = 0$	$1 + 1/9 \cdot 0 = 1$	$0 + 13/9 \cdot 0 = 0$
Y	$1 + 4/9 \cdot 0 = 1$	$0 + 2/9 \cdot 0 = 0$	$0 + 1/9 \cdot 0 = 0$	$0 + 13/9 \cdot 0 = 0$
A1	$4/9 + 4/9(-1) = 0$	$2/9 + 2/9(-1) = 0$	$1/9 + 1/9(-1) = 0$	$(M+13/9) + 13/9(-1) = M$
H1	$0 + 4/9 \cdot 0 = 0$	$1 + 2/9(0) = 1$	$0 + 1/9 \cdot 0 = 0$	$0 + 13/9 \cdot 0 = 0$
A2	$-1/9 + 4/9(-4/11) = -3/11$	$-5/9 + 2/9(-4/11) = -7/11$	$2/9 + 1/9(-4/11) = 2/11$	$(M-1/9) + 13/9(-4/11) = M-7/11$
H2	$0 + 4/9(9/11) = 4/11$	$0 + 2/9(9/11) = 2/11$	$0 + 1/9(9/11) = 1/11$	$0 + 13/9(9/11) = 13/11$
E1	$-4/9 + 4/9 \cdot 1 = 0$	$-2/9 + 2/9(1) = 0$	$-1/9 + 1/9(1) = 0$	$-13/9 + 13/9 \cdot 1 = 0$
E2	$1/9 + 4/9(4/11) = 3/11$	$5/9 + 2/9(4/11) = 7/11$	$2/9 + 1/9(4/11) = 2/11$	$1/9 + 13/9(4/11) = 7/11$
VS	$3 + 4/9(9) = 7$	$5 + 2/9 \cdot 9 = 7$	$1 + 1/9(9) = 2$	$10 + 13/9 \cdot 9 = 23$

Tabla luego de la iteración 3

	X	Y	A1	M1	A2	H2	E1	E2	VS
Y	0	1	0	0	-3/11	4/11	0	3/11	7
H1	0	0	0	1	-7/11	2/11	0	7/11	7
X	1	0	0	0	2/11	1/11	0	-2/11	2
E1	0	0	-1	0	-4/11	9/11	1	4/11	9
Z	0	0	M	0	M/11	13/11	0	7/11	23

Matriz Óptima Inversa

Comprobación

"X"

Matriz Óptima Inversa	Operación	X	Desarrollo	Resultado
0 0 -3/11 4/11	*	1	$0 \cdot 1 + 0 \cdot 2 + (-3/11) \cdot 4 + 4/11 \cdot 3$	0
0 1 -7/11 2/11	*	2	$0 \cdot 1 + 1 \cdot 2 + (-7/11) \cdot 4 + 2/11 \cdot 3$	0
0 0 2/11 1/11	*	34	$0 \cdot 1 + 0 \cdot 2 + 2/11 \cdot 4 + 1/11 \cdot 3$	1
-1 0 -4/11 9/11	*	39	$-1 \cdot 1 + 0 \cdot 2 + (-4/11) \cdot 4 + 9/11 \cdot 3$	0

"Y"

Matriz Opt Inv	Operación	Y	Desarrollo	Resultado
0 0 -3/11 4/11	*	2	$0 \cdot 2 + 0 \cdot (-1) + (-3/11) \cdot (-1) + 4/11 \cdot 2$	1
0 1 -7/11 2/11	*	-1	$0 \cdot 2 + 1 \cdot (-1) + (-7/11) \cdot (-1) + 2/11 \cdot 2$	0
0 0 2/11 1/11	*	-1	$0 \cdot 2 + 0 \cdot (-1) + 2/11 \cdot (-1) + 1/11 \cdot 2$	0
-1 0 -4/11 9/11	*	2	$-1 \cdot 2 + 0 \cdot (-1) + (-4/11) \cdot (-1) + 9/11 \cdot 2$	0

"A1"

Matriz Opt Inv	Operación	A1	Desarrollo	Resultado
0 0 -3/11 4/11	*	1	$0 \cdot 1 + 0 \cdot 0 + (-3/11) \cdot 0 + 4/11 \cdot 0$	0
0 1 -7/11 2/11	*	0	$0 \cdot (1) + 1 \cdot 0 + (-7/11) \cdot 0 + 2/11 \cdot 0$	0
0 0 2/11 1/11	*	0	$0 \cdot 1 + 0 \cdot 0 + 2/11 \cdot 0 + 1/11 \cdot 0$	0
-1 0 -4/11 9/11	*	0	$-1 \cdot 0 + 0 \cdot 0 + (-4/11) \cdot 0 + 9/11 \cdot 0$	-1

"H1"

Matriz	Opt	Inv	Oper	H1	Desarrollo	Resultado
0	0	$-3/11$	$4/11$	*	$0 \cdot 0 + 0 \cdot 1 + (-3/11) \cdot 0 + 4/11 \cdot 0$	0
0	1	$-7/11$	$2/11$	*	$0 \cdot 0 + 1 \cdot 1 + (-7/11) \cdot 0 + 2/11 \cdot 0$	1
0	0	$2/11$	$1/11$	*	$0 \cdot 0 + 0 \cdot 1 + 2/11 \cdot 0 + 1/11 \cdot 0$	0
-1	0	$-4/11$	$9/11$	*	$-1 \cdot 0 + 0 \cdot 1 + (-4/11) \cdot 0 + 9/11 \cdot 0$	0

"A2"

			Oper	A2	Desarrollo	Resultado
0	0	$-3/11$	$4/11$	*	$0 \cdot 0 + 0 \cdot 0 + (-3/11) \cdot 1 + 4/11 \cdot 0$	$-3/11$
0	1	$-7/11$	$2/11$	*	$0 \cdot 0 + 1 \cdot 0 + (-7/11) \cdot 1 + 2/11 \cdot 0$	$-7/11$
0	0	$2/11$	$1/11$	*	$0 \cdot 0 + 0 \cdot 0 + 2/11 \cdot 1 + 1/11 \cdot 0$	$2/11$
-1	0	$-4/11$	$9/11$	*	$-1 \cdot 0 + 0 \cdot 0 + (-4/11) \cdot 1 + 9/11 \cdot 0$	$-4/11$

"H2"

Matriz	Opt	Inv	Oper	H2	Desarrollo	Resultado
0	0	$-3/11$	$4/11$	*	$0 \cdot 0 + 0 \cdot 0 + (-3/11) \cdot 0 + 4/11 \cdot 1$	$4/11$
0	1	$-7/11$	$2/11$	*	$0 \cdot 0 + 1 \cdot 0 + (-7/11) \cdot 0 + 2/11 \cdot 1$	$2/11$
0	0	$2/11$	$1/11$	*	$0 \cdot 0 + 0 \cdot 0 + 2/11 \cdot 0 + 1/11 \cdot 1$	$1/11$
-1	0	$-4/11$	$9/11$	*	$-1 \cdot 1 + 0 \cdot 0 + (-4/11) \cdot 0 + 9/11 \cdot 0$	$9/11$

"E1"

Matriz	Opt	Inv	Oper	E1	Desarrollo	Resultado
0	0	$-3/11$	$4/11$	*	$0 \cdot 1 + 0 + (-3/11) + 4/11 \cdot 0$	0
0	1	$-7/11$	$2/11$	*	$0 \cdot -1 + 1 \cdot 0 + (-7/11) \cdot 0 + 2/11 \cdot 0$	0
0	0	$2/11$	$1/11$	*	$0 \cdot -1 + 0 + 2/11 \cdot 0 + 1/11 \cdot 0$	0
-1	0	$-4/11$	$9/11$	*	$-1 \cdot -1 + 0 + (-4/11) \cdot 0 + 9/11 \cdot 0$	1

"E2"

Mat	Opt	Inv	Oper	E2	Desarrollo	Resultado
0	0	$-3/11$	$4/11$	*	$0 + 0 + (-3/11) \cdot (-1) + 4/11 \cdot 0$	$3/11$
0	1	$-7/11$	$2/11$	*	$0 + 1 \cdot 0 + (-7/11) \cdot (-1) + 2/11 \cdot 0$	$7/11$
0	0	$2/11$	$1/11$	*	$0 + 0 + 2/11 \cdot (-1) + 1/11 \cdot (0)$	$-2/11$
-1	0	$-4/11$	$9/11$	*	$-1 \cdot 0 + 0 + (-4/11) \cdot (-1) + 9/11 \cdot 0$	$4/11$

"VS"

Matriz Opt Inv				Op	VS	Desarrollo	Resultado
0	0	$-3/11$	$4/11$	*	7	$0 \cdot 7 + 0 \cdot 4 + (-3/11) \cdot 1 + 4/11 \cdot 20$	7
0	1	$-7/11$	$2/11$	*	4	$0 \cdot 7 + 1 \cdot 4 + (-7/11) \cdot 1 + 2/11 \cdot 20$	7
0	0	$2/11$	$1/11$	*	1	$0 \cdot 7 + 0 \cdot 4 + 2/11 \cdot 1 + 1/11 \cdot 20$	2
-1	0	$-4/11$	$9/11$	*	20	$-1 \cdot 7 + 0 \cdot 4 + (-4/11) \cdot 1 + 9/11 \cdot 20$	9

→ Comprobamos los valores de z : $y=3, x=1, H1=E1=0$

y	H1	x	E1	Matriz Opt Inv	Op	Desarrollo	Resultado
3	0	1	0	$0 \ 0 \ -3/11 \ 4/11$	*	$3 \cdot 0 + 0 \cdot 1 + 0 \cdot 1 + 0 \cdot 1$	0 $\Rightarrow y1=0$
				$0 \ 1 \ -7/11 \ 2/11$	*	$3 \cdot 0 + 0 \cdot 1 + 0 \cdot 1 + 0 \cdot 1$	0 $\Rightarrow y2=0$
				$0 \ 0 \ 2/11 \ 1/11$	*	$3 \cdot (-3/11) + 0 \cdot (-7/11) + 1 \cdot 2/11$	$-7/11 \Rightarrow y3 = -7/11$
				$-1 \ 0 \ -4/11 \ 9/11$	*	$3 \cdot 4/11 + 0 \cdot 2/11 + 1 \cdot (-4/11) + 0 \cdot 9/11$	$13/11 \Rightarrow y4 = 13/11$

Dual:

$$\begin{aligned} \min w &= 7(y1) + 4(y2) + y3 + 20(y4) \\ &= 7(0) + 4(0) + (-7/11) + 20(13/11) \\ &= 23 \end{aligned}$$

Sujeto a:

$$y1 + 2y2 + 4y3 + 3y4 \geq 1$$

$$0 + 2 \cdot (0) + 4(-7/11) + 3(13/11) = 1$$

$$1 = 1$$

$$0 = 0$$

$$2(y1) - (y2) - (y3) + 2(y4) \geq 3$$

$$2 \cdot 0 - 2 \cdot 0 - 1 \cdot (-7/11) + 2(13/11) = 3$$

$$3 = 3$$

$$0 = 0$$

Tipo de caso: Es factible porque las variables artificiales salen en la base de la solución final. No hay coeficientes negativos, por lo cual es aceptado