

$$\text{Magnitudes} \leftarrow \sqrt{a^2 + b^2}$$

$|w| = |z \cdot x|$ , Ambos complejos

$$|w| = |z \cdot w| = |z| \cdot |w| = \sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2}$$

$$|\bar{w}| = w \quad \text{if } a > 0 \rightarrow \theta - \frac{180}{\pi}$$

(cuadrante I)  $a > 0, b > 0 \rightarrow -x + 360$

$$\begin{cases} 0 < \theta < \frac{\pi}{2} \\ 0 < \theta < 90 \end{cases}$$

$$\begin{cases} \frac{\pi}{2} < \theta < \pi \\ 90 < \theta < 180 \end{cases}$$

$$\begin{cases} \pi < \theta < \frac{3\pi}{2} \\ 180 < \theta < 270 \end{cases}$$

$$\begin{cases} \frac{3\pi}{2} < \theta < 2\pi \\ 270 < \theta < 360 \end{cases}$$

$$\left. \begin{array}{l} \theta = \frac{\pi}{2} \quad a=0 \quad b>0 \\ \theta = -\frac{\pi}{2} \quad a=0 \quad b<0 \\ \theta = \pm \pi \quad a \neq 0 \quad b=0 \end{array} \right\}$$

En todos los casos que el  $\theta$  NO sea un caso especial

$$\operatorname{Arg}(w), \frac{b}{a} = \tan(\theta)$$

Todos los complejos que satisfacen esto

$$|z - 3i| = 4$$

$$z = a + bi \in \mathbb{C} \cup \{0\}$$

$$\operatorname{Arg}(z - 2i) = \frac{\pi}{2}$$

$$z - 2i(a + bi) = \frac{\pi}{2}$$

$$a = 0 \quad b > 0$$

$$z - 2a - 2bi = \frac{\pi}{2}$$

$$\underbrace{(z - 2a)}_a + \underbrace{(-2b)i}_b = \frac{\pi}{2}$$

$$z - 2a = 0 \quad -2b > 0$$

$$-2a = -2 \quad b < 0$$

$$\boxed{a = 1}$$

$$|z - 3i| = 4$$

$$\sqrt{a^2 + b^2}$$

$$|a + bi - 3i| = 4$$

$$|(a) + (b-3)i| = 4$$

$$a^2 + (b-3)^2 = 4^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$1^2 + b^2 - 6b + 9 = 16$$

$$b^2 - 6b + 20 - 16 = 0$$

$$b^2 - 6b - 4 = 0$$

$$\checkmark b = 3 - \sqrt{15}, b = 3 + \sqrt{15}$$

$$\boxed{1 + (3 - \sqrt{15})i}$$

$$\frac{b}{a} = \tan(\theta)$$

$$|\bar{z} + 1 - i| = 5$$

$$\operatorname{Arg}(z - (1+i)) = \frac{3\pi}{4}$$

$$\frac{3\pi}{4}, \frac{180}{\pi}$$

$$a+bi - 1-2i = \frac{3\pi}{4}$$

135

q.v (135 < 180)

$$(a-1) + (b-2)i = \frac{3\pi}{4} - 1$$

$$\frac{b-2}{a-1} = \tan\left(\frac{3\pi}{4}\right)$$

$$a < 0 \quad b > 0$$

$$\frac{b-2}{a-1} = -1$$

$$b-2 = 1-a$$

$$a = 3-b$$

$$|\bar{z} + 1 - i| = 5$$

$$|\overline{a+bi} + 1-i| = 5$$

$$|a - bi + 1 - i| = 5$$

$$|(a+1) + (-b-1)i| = 5$$

$$(-a-b)^2 = (a+b)^2$$

$$(a+1)^2 + (-b-1)^2 = 5^2$$

$$(3-b+1)^2 + b^2 + 2b + 1 - 25 = 0$$

$$a^2 + 2ab + b^2$$

$$(4-b)^2 + b^2 + 2b + 1 - 25 = 0$$

$$(4-b)^2$$

$$16 - 8b + b^2 + b^2 + 2b + 1 - 25 = 0$$

$$4^2 - 2 \cdot 4 \cdot b + b^2$$

$$2b^2 - 6b - 8 = 0$$

$$b = 4$$

$$b = -1$$

$$a < 0$$

$$b > 0$$

$$a = 3-b$$

$$3-4 = -1$$

$$M \quad \boxed{-1 + 4i}$$

$$P(x), Q(x)$$

$$\begin{array}{c} P(x) \\ \hline Q(x) \\ (x - Q(x)) \end{array}$$

$$P(x) = 3x^2 + 2x \dots$$

$$Q(x) = \underbrace{2-i}_{2\text{re}} \quad Q(x) = \underbrace{2+i}_{2\text{Im}}$$

$$P(x) = x^4 - x^3 + 8x^2 - 4x + 26 \quad Q(x) = 2i$$

$$\begin{array}{rcccc|c} 1 & -1 & 8 & -4 & 26 & 2i \\ & +2i & -4-2i & 4+8i & -26 & \\ \hline 1 & -1+2i & 4-2i & 8i & 0 & (x-2i) \end{array} \quad \begin{array}{l} 2i \cdot 8i \\ 16i^2 \\ -16 \end{array}$$

$$x^4 + (-1+2i)x^3 + (4-2i)x^2 + 8i$$

$$\begin{array}{rccccc|c} 1 & -1+2i & 4-2i & 8i & -2i & -2i+8i^2 & , \\ & -2i & 2i & -8i & -2i & -4-2i & \\ \hline 1 & -1 & 4 & 0 & (x-(-2i)) & (x+2i) & \end{array} \quad \begin{array}{l} (-1+2i) \cdot 2i \\ -2i+8i^2 \\ -4-2i \\ (4-2i) \cdot 2i \\ 8i - 8i^2 \\ 4+8i \end{array}$$

$$x^2 - x + 4$$

$$x = \left( \frac{-1 + i\sqrt{25}}{2} \right), x = \left( \frac{-1 - i\sqrt{25}}{2} \right)$$

$$(x-2i)(x+2i)\left(x - \frac{-1 + i\sqrt{25}}{2}\right)\left(x - \frac{-1 - i\sqrt{25}}{2}\right)$$

$m \neq n$      $2 \times 3$      $3 \times 2$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \leftarrow \text{Nula} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \quad A^T = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

$$A = A^T$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 7 & 8 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 7 & 8 \end{bmatrix}$$

Operacion

Suma y resta, OCUPAN SI O SI  
ser del mismo tamaño

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \pm \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix}$$

multiplicacion por escalar

$$2 \cdot \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$$

Multiplicación de matrices

Si o Si (obligatorio) el numero de columnas de  $A_1$  tiene que ser igual al numero de filas de  $A_2$

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 \cdot 9 + 0 \cdot 5 \\ -1 \cdot 9 + 1 \cdot 5 \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1 \cdot 5 + 2 \cdot 7 & 1 \cdot 6 + 2 \cdot 8 \\ 3 \cdot 5 + 4 \cdot 7 & 3 \cdot 6 + 4 \cdot 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix}, B = \begin{bmatrix} 6 & 5 & 9 \\ 3 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 \cdot 6 + 3 \cdot 3 + 4 \cdot 2 & 2 \cdot 5 + 3 \cdot 2 + 4 \cdot 1 & 2 \cdot 9 + 3 \cdot 1 + 4 \cdot 2 \\ 5 \cdot 6 + 6 \cdot 3 + 7 \cdot 2 & 5 \cdot 5 + 6 \cdot 2 + 7 \cdot 1 & 5 \cdot 9 + 6 \cdot 1 + 7 \cdot 2 \\ 8 \cdot 6 + 9 \cdot 3 + 1 \cdot 2 & 8 \cdot 5 + 9 \cdot 2 + 1 \cdot 1 & 8 \cdot 9 + 9 \cdot 1 + 1 \cdot 2 \end{bmatrix}$$

Propiedades de matrices,  $\lambda$  = es scalar

- 1)  $A_{mn} + B_{mn} = C_{mn}$  cerrada
- 2)  $(A+B)+C = A+(B+C)$  Asociativa
- 3)  $A+0 = 0+A = A$
- 4)  $A+(-A) = -A+A = 0$
- 5)  $A+B = B+A$  Comutativa
- 6)  $A \cdot 0 = 0$
- 7)  $\lambda(A+B) = \lambda A + \lambda B$
- 8)  $(\lambda I + \chi)A = \lambda A + \chi A$
- 9)  $(\lambda \cdot \chi)A = \lambda(\chi \cdot A)$
- 10)  $I A = A \cdot I = A$
- 11)  $A \cdot B \neq B \cdot A$
- 12)  $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
- 13)  $(A+B)C = AC + BC$

Teoremas

- 1)  $(AT)^T = A$
- 2)  $(A \cdot B)^T = B^T \cdot A^T$
- 3)  $(A \pm B)^T = A^T \pm B^T$
- 4)  $(A^{-1})^{-1} = A$
- 5)  $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$
- 6)  $I^T = I$  Tener en cuenta
  - $A + AB = A(I + B)$
  - $AI + AB = A(I + B)$
  - $B + AB = (I + A)B$

Sean  $A, B, C$  matrices tales que

$$(xA - B)^T - C = 2x^T, \text{ si se sabe que}$$

$A - 2I$  tiene inversa, encuentra  $X$

$$X \cdot (A - 2I) = 7 \quad (A - 2I)^{-1} = (A - 2I)^{-1} \quad X^{-1} = X \\ X^{-1} = I \quad X$$
$$X = 7 \cdot (A - 2I)^{-1}$$

$$(xA - B)^T - C = 2x^T$$

$$(xA)^T - B^T - C = 2x^T \quad (a+b)^T = a^T + b^T$$

$$A^T \cdot x^T - B^T - C = 2x^T \quad (a \cdot b)^T = b^T \cdot a^T$$

$$A^T \cdot x^T - 2x^T = B^T + C$$

$$(A^T - 2I)x^T = B^T + C$$

$$[(A^T - 2I) \cdot x^T]^T = [B^T + C]^T$$

$$(x^T)^T \cdot (A^T - 2I)^T = (B^T)^T + C^T$$

$$X \cdot [(A^T)^T - (2I)^T] = B^T + C^T \quad (A^T)^T = A$$

$$X \cdot [A - 2I] = B^T + C^T \quad I^T = I$$

$$\boxed{X = (B^T + C^T) \cdot (A - 2I)^{-1}}$$