(1) [1 punto] Si se sabe que la serie $\sum_{n=2}^{\infty} b_n$ converge, entonces para la serie $\sum_{n=2}^{\infty} \frac{1}{b_n}$ se puede decir con seguridad que:

A) Depende de la expresión b_n ✓

B) Converge

C) Diverge

Depende de b_n

(2) [1 punto] Para la serie $\sum_{n=0}^{\infty} \frac{9n^3}{\sqrt[4]{3n^{12}-1}}$

A) Converge con certeza

B) No es posible determinar la convergencia.

C) Diverge con certeza

$$\sum_{n=0}^{\infty} \frac{9n^3}{\sqrt[4]{3n^{12}-1}} \sim \frac{9n^3}{\sqrt[4]{3n^{12}}} \sim \frac{9n^3}{\cancel{n^3}\sqrt[4]{3}} \sim$$

$$\sim \frac{9}{\sqrt[4]{3}} \sim \frac{9}{3^{\frac{1}{4}}} \sim 3^{\frac{7}{4}}$$

Por crit de Cmp al lim

$$a_n = \frac{9n^3}{\sqrt[4]{3n^{12}-1}} \qquad b_n = 3^{\frac{7}{4}}$$

Serie constante divergente

$$\lim_{n\to+\infty} \frac{\frac{9n^3}{\sqrt[4]{3n^{12}-1}}}{3^{\frac{7}{4}}}$$

$$\lim_{n\to+\infty} \frac{9n^3}{3^{\frac{7}{4}}\sqrt[4]{3n^{12}-1}}$$

$$\lim_{n\to+\infty} \frac{9n^3}{3^{\frac{7}{4}}\sqrt[4]{n^{12}\left(3-\frac{1}{n^{12}}\right)}}$$

$$\lim_{n\to+\infty} \frac{9\cancel{n^3}}{\cancel{n^3}\cdot 3^{\frac{7}{4}}\left(\sqrt[4]{3-\frac{1}{n^{12}}}\right)}$$

$$\lim_{n\to+\infty} \frac{9}{3^{\frac{7}{4}}\left(\sqrt[4]{3-\underbrace{\frac{1}{\cancel{n^{12}}}}_{=0}}\right)} = \frac{9}{3^{\frac{7}{4}}\cdot\sqrt[4]{3}}$$

$$\boxed{\begin{array}{l} b_n \text{ diverge } \wedge L \neq 0 \\ \therefore \displaystyle\sum_{n=0}^{\infty} \frac{9n^3}{\sqrt[4]{3n^{12}-1}} \quad \text{Diverge} \end{array}}$$

(3) [1 punto] Determine si la sucesión $\{a_n\}$ es convergente o divergente. En caso de converger, determine el valor al cual converge.

$$a_n = \frac{8n^6+7n^4}{4n^2+n^6}$$

$$\lim_{n\to+\infty} \frac{8n^6+7n^4}{4n^2+n^6}$$

$$\lim_{n\to+\infty} \frac{8\cancel{n^6}}{\cancel{n^6}} = 8$$

$$\boxed{\text{La serie converge a } 8}$$

(4) [1 punto] Determine si la sucesión $\{b_n\}$ es convergente o divergente. En caso de converger, determine el valor al cual converge.

$$b_n = \left(1+\frac{4}{n}\right)^n = e^4 \quad \text{converge}$$

$$\lim_{n\to+\infty} \left(1+\frac{4}{n}\right)^n \qquad e^{\lim_{n\to+\infty} n\ln\left(1+\frac{4}{n}\right)}$$

$$e^L$$

Calculando L

$$\lim_{n\to+\infty} n\ln\left(1+\frac{4}{n}\right)$$

$$\lim_{n\to+\infty} \frac{\ln\left(1+\frac{4}{n}\right)}{\frac{1}{n}} \quad , \frac{0}{0} , \text{ L'hôpital}$$

$$\lim_{n\to+\infty} \frac{\frac{1}{1+\frac{4}{n}} \cdot -\frac{4}{\cancel{n}}}{\frac{-1}{\cancel{n}}}$$

$$\lim_{n\to+\infty} \frac{4}{1+\frac{4}{n}} = 4$$

$$= e^4$$

$$\boxed{\text{Converge a } e^4}$$

(5) **[1 punto]** Determine si la sucesión $\{c_n\}$ es convergente o divergente. En caso de converger, determine el valor al cual converge.

$$c_n = \frac{2-1n^4}{\sqrt{3n^8-6}}$$

$$\lim_{n\to+\infty} \frac{2-n^4}{\sqrt{3n^8-6}}$$

$$\lim_{n\to+\infty} \frac{2-n^4}{\sqrt{n^8\left(3-\frac{6}{n^8}\right)}}$$

$$\lim_{n\to+\infty} \frac{\cancel{n^4}\left(\frac{2}{n^4}-1\right)}{\cancel{n^4}\cdot\sqrt{3-\frac{6}{n^8}}}$$

$$\lim_{n\to+\infty} \frac{\cancel{\frac{2}{n^4}}^{=0}-1}{\sqrt{3-\cancel{\frac{6}{n^8}}^{=0}}} = \frac{-1}{\sqrt{3}}$$

$$\boxed{\text{Converge a } \frac{-1}{\sqrt{3}}} \boxed{= \frac{-\sqrt{3}}{3}}$$

(6) **[1 punto]** Determine si la sucesión $\{c_n\}$ es convergente o divergente. En caso de converger, determine el valor al cual converge.

$$c_n = \frac{5^{n-2}+3}{5^{n-1}+9}$$

$$\lim_{n\to+\infty} \frac{5^{n-2}+3}{5^{n-1}+9} ,$$

$$\lim_{n\to+\infty} \frac{\cancel{5^{n-1}}\cdot 5^{-1}\left(1+\frac{3}{5^{n-1}}\right)}{\cancel{5^{n-1}}\left(1+\frac{9}{5^{n-1}}\right)}$$

$$\lim_{n\to+\infty} \frac{5^{-1}\left(1+\cancel{\frac{3}{5^{n-1}}}^{=0}\right)}{1+\cancel{\frac{9}{5^{n-1}}}^{=0}} = \frac{1}{5}$$

$$\boxed{\text{Converge a } \frac{1}{5}}$$

(7) [2 puntos] La sucesión $\{c_n\}$ converge a $e^{24/7}$. Determine el valor de α.

$$c_n = \left(1+\frac{4}{n}\right)^{\alpha n}$$

$$\lim_{n\to+\infty} \left(1+\frac{4}{n}\right)^{\alpha n} = e^{4\alpha}$$

$$e^{4\alpha} = e^{\frac{24}{7}}$$

$$4\alpha = \frac{24}{7}$$

$$\alpha = \frac{6}{7}$$

$$\boxed{\therefore\ \alpha = \frac{6}{7}}$$

(8) [4 puntos] Determine si la serie dada converge o diverge. En caso de converger, calcule su suma.

$$\sum_{n=3}^{\infty} \frac{(-3)^{n+1}-(-6)^{n-1}}{8^{n+3}}$$

$$\sum_{n=3}^{\infty} \frac{(-3)^{n+1}-(-6)^{n-1}}{8^{n+3}}$$

$$\sum_{n=3}^{\infty} \frac{(-3)^n\cdot(-3)}{8^n\cdot 8^3} - \sum_{n=3}^{\infty} \frac{(-6)^n\cdot(-6)^{-1}}{8^n\cdot 8^3}$$

$$\frac{-3}{512}\sum_{n=3}^{\infty}\left(\frac{-3}{8}\right)^n - \frac{-1}{3072}\sum_{n=3}^{\infty}\left(\frac{-3}{4}\right)^n$$

$$|r| = \frac{3}{8} < 1 \qquad |r| = \frac{6}{8} < 1$$

$$\frac{-3}{512}\left[\frac{\left(\frac{-3}{8}\right)^3}{1--\frac{3}{8}}\right] + \frac{1}{3072}\left[\frac{\left(\frac{-3}{4}\right)^3}{1--\frac{3}{4}}\right]$$

$$\frac{81}{360448} - \frac{9}{114688} = 0{,}00014624657568993506$$

(usar python)

(9) [4 puntos] Determine si la serie dada converge o diverge. En caso de converger, calcule su suma.

$$\sum_{n=3}^{\infty} \frac{1}{(4n+3)(4n+11)}$$

$$\sum_{n=3}^{\infty} \frac{1}{(4n+3)(4n+11)}$$

$$\frac{1}{(4n+3)(4n+11)} = \frac{A}{4n+3} + \frac{B}{4n+11}$$

$$1 = A(4n+11) + B(4n+3)$$

$$n = \frac{-3}{4} \to 1 = 8A \to A = \frac{1}{8}$$

$$n = \frac{-11}{4} \to 1 = -8B \to B = \frac{-1}{8}$$

$$\frac{1}{(4n+3)(4n+11)} = \frac{\frac{1}{8}}{4n+3} - \frac{\frac{1}{8}}{4n+11}$$

$$\frac{1}{8} \sum_{n=3}^{\infty} \frac{1}{4n+3} - \frac{1}{4n+11} \qquad \frac{1}{4(n+1)+3} = \frac{1}{4n+4+3} = \frac{1}{4n+7}$$

$$\frac{1}{8} \sum_{n=3}^{\infty} \left(\frac{1}{4n+3} - \frac{1}{4n+7} \right) + \frac{1}{8} \sum_{n=3}^{\infty} \left(\frac{1}{4n+7} - \frac{1}{4n+11} \right)$$

$$\frac{1}{8} \left(\frac{1}{4(3)+3} - \underbrace{\lim_{n \to +\infty} \frac{1}{4n+7}}_{=0} \right) + \frac{1}{8} \left(\frac{1}{4(3)+7} - \underbrace{\lim_{n \to +\infty} \frac{1}{4n+11}}_{=0} \right)$$

$$= \frac{1}{8} \left(\frac{1}{15} \right) + \frac{1}{8} \left(\frac{1}{19} \right)$$

$$= \frac{1}{120} + \frac{1}{152}$$

$$= \frac{17}{1140}$$

$$\boxed{\text{Converje a } \frac{17}{1140}}$$