(a) [*] $1+4+7+\cdots+(3n-2)=\dfrac{3n^2-n}{2}$

$$\sum_{i=1}^{n} 3n-2 = \frac{3n^2-n}{2}$$

$n=1 \Rightarrow 1=1$ ✓

$n=p \quad \sum_{i=1}^{p} 3p-2 = \frac{3p^2-p}{2}$, Hi

HQD

$n=p+1 \quad \sum_{i=1}^{p+1} 3p+1 = \frac{3(p+1)^2-p-1}{2}$

$= 3p^2+6p+3-p-1$

$\boxed{\frac{3p^2+5p+2}{2}}$

Demo

$$\sum_{i=1}^{p+1} 3p+1$$

$$\sum_{i=1}^{p} (3p-2) + 3p+1$$

$$\frac{3p^2-p}{2} + 3p+1$$

$$\frac{3p^2-p+6p+2}{2}$$

$$\frac{3p^2+5p+2}{2}$$

(b) [*] $\dfrac{2}{3^2}+\dfrac{2^2}{3^3}+\dfrac{2^3}{3^4}+\cdots+\dfrac{2^{n-1}}{3^n}=\dfrac{2}{3}-\left(\dfrac{2}{3}\right)^n.$

$$\sum_{i=2}^{n} \frac{2^{n-1}}{3^n} = \frac{2}{3} - \left(\frac{2}{3}\right)^n$$

$n=2 \quad \frac{2^{1-1}}{3^1} + \frac{2^{2-1}}{3^2} = \frac{2}{3} - \left(\frac{2}{3}\right)^2$

$$\frac{2}{9} = \frac{2}{9}$$

$n=p \quad \sum_{i=2}^{p} \frac{2^{p-1}}{3^p} = \frac{2}{3} - \left(\frac{2}{3}\right)^p$, Hi

HQD

$k=p+1 \quad \sum_{i=2}^{p+1} \frac{2^{p}}{3^{p+1}} = \frac{2}{3} - \left(\frac{2}{3}\right)^{p+1}$

$\sum_{i=2}^{p} \frac{2^{p-1}}{3^{p}} + \frac{2^{p}}{3^{p+1}}$

$\frac{2}{3} - \left(\frac{2}{3}\right)^{p} + \frac{2^{p}}{3^{p+1}}$

$\frac{2}{3} - \left(\frac{2}{3}\right)^{p} + \left(\frac{2}{3}\right)^{p} \cdot \frac{1}{3}$

$\frac{2}{3} - \left(\frac{2}{3}\right)^{p}\left(1 - \frac{1}{3}\right)$

$\frac{2}{3} - \left(\frac{2}{3}\right)^{p+1}$

(c) [*] $3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + (n+1) \cdot 2^n = n \cdot 2^{n+1} - 4$

$\sum_{i=2}^{k} (k+1) \cdot 2^{k} = k \cdot 2^{k+1}$

$k=1 \qquad 4=4 \checkmark$

$k=p \quad \sum_{i=2}^{p} (p+1) \cdot 2^{p} = p \cdot 2^{p+1} - 4$, Hi

HQD

$k=p+1 \quad \sum_{i=2}^{p+1} (p+2) \cdot 2^{p+1} = (p+1) \cdot 2^{p+2} - 4$

Demo

$\sum_{i=2}^{p} (p+1) \cdot 2^{p} + (p+2) \cdot 2^{p+1}$

$p \cdot 2^{p+1} + (p+2) \cdot 2^{p+1} - 4$

$2^{p+1}(p+p+2) - 4$

$2^{p+1} \cdot (2p+2) - 4$

$2^{p+1} \cdot 2(p+1) - 4$

$2^{p+2} \cdot (p+1) - 4$

(d) [*] $\displaystyle\sum_{k=1}^{n} \frac{k-(k-1)^2}{k!} = 1 + \frac{n-1}{n!}$

$n=1 \qquad 1=1$

$n=p \qquad \displaystyle\sum_{k=1}^{p} \frac{p-(p-1)^2}{p!} = 1 + \frac{p-1}{p!}$, Hi

$H(Q)$

$n=p+1 \qquad \displaystyle\sum_{k=1}^{p+1} \frac{p+1-p^2}{(p+1)!} = 1 + \frac{p}{p!}$

Demo

$$\sum_{k=1}^{p+1} \frac{p+1-p^2}{(p+1)!}$$

$$\sum_{k=1}^{p} \frac{p-(p-1)^2}{p!} + \frac{p+1-p^2}{(p+1)!}$$

$$1 + \frac{p-1}{p!} + \frac{p+1-p^2}{(p+1)!}, \text{ Hi} \qquad 1 + \frac{p}{p!}$$

$$1 + \frac{p-1}{p!} + \frac{p+1-p^2}{(p+1)\cdot p!}$$

$$1 + \frac{(p-1)(p+1) + p+1 - p^2}{(p+1)\,p!}$$

$$1 + \frac{\cancel{p^2} \cancel{-1} + p \cancel{+1} - \cancel{p^2}}{(p+1)\,p!}$$

$$1 + \frac{p}{p!}$$

(e) [*] $\displaystyle\sum_{i=0}^{n}(7-11i) = \frac{(n+1)(14-11n)}{2}$, $n \geq 1$.

$h = 0 \qquad 1 = 1$ ✓

$h = p \qquad \sum_{i=0}^{p} (7 - 11i) = \frac{(p+1)(14 - 11p)}{2}$, Hi

$h = p+1 \qquad \sum_{i=0}^{p+1} (7 - 11i) = \underset{\text{HOD}}{\frac{(p+2)(3 - 11p)}{2}} \to 3p - 11p^2 + 6 - 22p$

$$\frac{-11p^2 - 19 + 6}{2}$$

Demo $\sum_{i=0}^{p} (7 - 11i) + 7 - 11(p+1)$

$7 - 11p - 11$

$-4 - 11p$

$$= \frac{(p+1)(14 - 11p)}{2} - 4 - 11p$$

$$= \frac{(p+1)(14 - 11p) - 8 - 22p}{2}$$

$$= \frac{14p - 11p^2 + 14 - 11p - 8 - 22p}{2}$$

$$\frac{-11p^2 - 19 + 6}{2}$$

(f) [*] $1 \cdot 2 + 2 \cdot 3 + \cdots + (n-1) \cdot n = \frac{(n-1)n(n+1)}{3}$, $n \geq 3$.

$h = 1 \qquad 0 = 0$

$h = p \qquad 1 \cdot 2 + 2 \cdot 3 \ldots + (p-1) \cdot p = \frac{(p-1)p(p+1)}{3}$, Hi

$h = p+1 \qquad 1 \cdot 2 + 2 \cdot 3 \ldots + p(p+1) = \frac{p(p+1)(p+2)}{3}$ HOD

Demo $1 \cdot 2 + 2 \cdot 3 \ldots + (p-1)p + p(p+1)$

$$\frac{(p-1)p(p+1)}{3} + p(p+1)$$

$$\frac{(p-1)p(p+1) + 3(p(p+1))}{3}$$

$$\frac{(p+1)((p-1)p + 3p)}{3}$$

$$\frac{(p+1)(p^2 - p + 3p)}{3}$$

$$\frac{(p+1)(p^2 + 2p)}{3}$$

$$\frac{(p+1)p(p+2)}{3}$$

(g) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$n=1 \qquad 1=1 \ \checkmark$

$n=p \qquad \sum_{i=1}^{p} p^2 = \frac{p(p+1)(2p+1)}{6}$, Hi

HQD

$n=p+1 \qquad \sum_{i=1}^{p+1} (p+1)^2 = \frac{(p+1)(p+2)(2p+3)}{6}$

Demo

$$\sum_{i=1}^{p+1} (p+1)^2$$

$$\sum_{i=1}^{p} p^2 + (p+1)^2$$

$$\frac{p(p+1)(2p+1)}{6} + (p+1)^2$$

$$\frac{(p+1)(p(2p+1)+6(p+1))}{6}$$

$$\frac{(p+1)(2p^2+p+6p+6)}{6}$$

$$\frac{(p+1)(2p^2+5p+6)}{6}$$

$$\frac{(p+1)(2p+3)(p+2)}{6}$$

(h) $\frac{2}{3}+\frac{2}{9}+\cdots+\frac{2}{3^n} = 1-\frac{1}{3^n}$

$n=1 \qquad \frac{2}{3} = \frac{2}{3} \ \checkmark$

$n=p \qquad \frac{2}{3}+\frac{2}{9}+\dots+\frac{2}{3^p} = 1-\frac{1}{3^p}$, Hi

HQD

$n=p+1 \qquad \frac{2}{3}+\frac{2}{9}+\dots+\frac{2}{3^{p+1}} = 1-\frac{1}{3^{p+1}}$

Demo

$$\frac{2}{3}+\frac{2}{9}+\cdots+\frac{2}{3^{p+1}}$$

$$\frac{2}{3}+\frac{2}{9}+\cdots+\frac{2}{3^p}+\frac{2}{3^{p+1}}$$

$$1-\frac{1}{3^p}+\frac{2}{3^{p+1}}$$

$$1-\frac{3-2}{3^{p+1}}$$

$$1-\frac{1}{3^{p+1}}$$

(i) $1^3+2^3+3^3+\cdots+n^3=\dfrac{n^2(n+1)^2}{4}$

$n=1$ $\quad 1=1$ ✓

$n=p$ $\quad p^3=\dfrac{p^2(p+1)^2}{4}$, Hi

HQD

$n=p+1$ $\quad (p+1)^3=\dfrac{(p+1)^2(p+2)^2}{4}$

Demo

$$p^3+(p+1)^3$$

$$\frac{p^2(p+1)^2}{4}+(p+1)^3$$

$$\frac{(p+1)^2(p^2+4p+4)}{4}$$

$$\frac{(p+1)^2(p+2)^2}{4}$$

(j) $\dfrac{1}{1\cdot 3}+\dfrac{1}{3\cdot 5}+\cdots+\dfrac{1}{(2n-1)(2n+1)}=\dfrac{n}{2n+1}$

$n=1$ $\quad \dfrac{1}{3}=\dfrac{1}{3}$

$n=p$ $\quad \dfrac{1}{(2p-1)(2p+1)}=\dfrac{p}{2p+1}$, Hi

HQD

$n=p+1$ $\quad \dfrac{1}{(2p+1)(2p+3)}=\dfrac{p+1}{2p+3}$

Demo

$$\frac{1}{(2p-1)(2p+1)}+\frac{1}{(2p+1)(2p+3)}$$

$$\frac{p}{2p+1}+\frac{1}{(2p+1)(2p+3)}$$

$$\frac{2p^2+5p+2}{(2p+1)(2p+3)}$$

$$\frac{(p+1)(2p+1)}{(2p+1)(2p+3)}$$

$$\frac{p+1}{2p+3}$$

(k) $\frac{1}{1\cdot 5}+\frac{1}{5\cdot 9}+\cdots+\frac{1}{(4n-3)(4n+1)}=\frac{n}{4n+1}$,

$n=1 \qquad \frac{1}{5}=\frac{1}{5}$

$n=p \qquad \frac{1}{(4p-3)(4p+1)}=\frac{p}{4p+1}$, Hi

$n=p+1 \qquad \frac{1}{(4p+1)(4p+5)}=\frac{p+1}{4p+5}$, HQD

Demo

$$\frac{p}{4p+1}+\frac{1}{(4p+1)(4p+5)}$$

$$\frac{4p^2+5p+1}{(4p+1)(4p+5)}$$

$$\frac{(4p+1)(p+1)}{(4p+1)(4p+5)}$$

$$\frac{p+1}{4p+5}$$

(l) $1\cdot 3+2\cdot 4+\cdots+n(n+2)=\frac{n(n+1)(2n+7)}{6}$

$n=1 \qquad 3=3$

$n=p \qquad p(p+2)=\frac{p(p+1)(2p+7)}{6}$, Hi

$n=p+1 \qquad (p+1)(p+3)=\frac{(p+1)(p+2)(2p+9)}{6}$, HQD

Demo

$$p(p+2)+(p+1)(p+3)$$

$$\frac{p(p+1)(2p+7)}{6}+(p+1)(p+3)$$

$$\frac{(p+1)(p(2p+7)+6(p+3))}{6}$$

$$\frac{(p+1)(2p^2+7p+6p+18)}{6}$$

$$\frac{(p+1)(p+2)(2p+9)}{6}$$

(m) $\frac{4}{5^0} + \frac{4}{5^1} + \frac{4}{5^2} + \cdots + \frac{4}{5^n} = 5 - \frac{1}{5^n}$,

$n = 1 \quad 4 + \frac{4}{5} = 5 - \frac{1}{5}$

$\frac{24}{5} = \frac{24}{5}$

$n = p \quad \frac{4}{5^p} = 5 - \frac{1}{5^p}$, HI

$n = p+1 \quad \frac{4}{5^{p+1}} = 5 - \frac{1}{5^{p+1}}$, HQD

Demo

$\frac{4}{5^p} + \frac{4}{5^{p+1}}$

$5 - \frac{1}{5^p} + \frac{4}{5^{p+1}}$, HI

$5 - \frac{1}{5^{p+1}}$

(n) $1{\cdot}3+2{\cdot}4+\cdots+(n-1)(n+1) = \frac{n(n-1)(2n+5)}{6}$,

$n = 1 \quad 0 = 0$

$n = p \quad (p-1)(p+1) = \frac{p(p-1)(2p+5)}{6}$, HI

$n = p+1 \quad p(p+2) = \frac{(p+1)\,p(2p+7)}{6}$, HQD

Demo

$(p-1)(p+1) + p(p+2)$

$\frac{p(p-1)(2p+5)}{6} + p(p+2)$

$\frac{p((p-1)(2p+5) + 6(p+2))}{6}$

$\frac{p(2p^2+5p-2p-5+6p+12)}{6}$

$\frac{p(2p^2+9p-5)}{6}$

$\frac{p(p+1)(2p+7)}{6}$

(ñ) $1 \cdot 4 + 2 \cdot 5 + \cdots + n \cdot (n+3) = \frac{n(n+1)(n+5)}{3}$

$n = 1 \quad 4 = 4$ ✓

$n = p \quad p(p+3) = \frac{p(p+1)(p+5)}{3}$

$n = p+1 \quad (p+1)(p+4) = \frac{(p+1)(p+2)(p+6)}{3}$

Demo

$p(p+5)+(p+1)(p+4)$

$\frac{p(p+1)(p+5)}{3}+(p+1)(p+4)$

$\frac{(p+1)(p(p+5)+3p+12)}{3}$

$\frac{(p+1)(p^2+5p+3p+12)}{3}$

$\frac{(p+1)(p^2+8p+12)}{3}$

$\frac{(p+1)(p+2)(p+6)}{3}$

(o) $1\cdot 5+2\cdot 6+\cdots+n(n+4)=\dfrac{n(n+1)(2n+13)}{6}$,

$n=1 \qquad 5=5 \ \checkmark$

$n=p \qquad p(p+4)=\frac{p(p+1)(2p+13)}{6}$

$n=p+1 \qquad (p+1)(p+5)=\frac{(p+1)(p+2)(2p+15)}{6}$

Demo

$p(p+4)+(p+1)(p+5)$

$\frac{p(p+1)(2p+13)}{6}+(p+1)(p+5)$

$\frac{(p+1)(p(2p+13)+p+5)}{6}$

$\frac{(p+1)(2p^2+13p+6p+30)}{6}$

$\frac{(p+1)(2p^2+19p+30)}{6}$

$\frac{(p+1)(p+2)(2p+15)}{6}$

(p) $1+2+2^2+\cdots+2^n=2^{n+1}-1,\ n\geq 1.$

$n=1 \qquad 2^0+2^1=2^{1+1}-1$

$3=3 \ \checkmark$

$n=p \qquad 2^p=2^{p+1}-1$, Hi

$n=p+1 \qquad 2^{p+1}=2^{p+2}-1$, H(p)

Demo

2^p+2^{p+1}

$2^{p+1}-1+2^{p+1}$

$2^{p+1}+2^{p+1}-1$

$2(2^{p+1})-1$

$2^{p+2}-1$

Q) $a_n = 2 a_{n-1} + 3$

$a_n = 2^{n+3} - 3 = 5$

$n = 0 \quad 2^{0+3} - 3 = 5$

$n = p \quad 2^{p+3} - 3$, Hi

$n = p+1 \quad 2^{p+4} - 3$

Demo

$2 \cdot \overbrace{a_{n-1}}^{n=p} + 3 \to$ usar def recursiva

$2(2^{p+3} - 3) + 3$, Hi

$2^{p+4} - 6 + 3$

$2^{p+4} - 3$ //

R) **Ejemplo 2.2** Considere la sucesión $\{a_n\}_{n\geq 1}$, definida por $a_n = \frac{b_{n+1}}{6 \cdot b_n}$, con $b_1 =$ 2, $b_2 = 5$, $b_{n+2} = b_n + b_{n+1}$. Calcule a_3.

$b_1 = 2$

$b_2 = 5$

$\underbrace{b_{n+2}}_{b_3} = b_n + b_{n+1}$

$b_3 = b_1 + b_2 = 2 + 5 = 7$

$b_4 = b_2 + b_3 = 5 + 7 = 12$

$a_n = \frac{b_{n+1}}{6 \cdot b_n}$

Para $n = 3$ (a_3) ocupamos b_4

$a_3 = \frac{b_3 + 1}{6 \cdot b_3} = \frac{b_4}{6 \cdot b_3}$

$a_3 = \frac{12}{6 \cdot 7} = \frac{2}{7}$