

Magnitudes $\leftarrow \sqrt{a^2 + b^2}$

$|w| = |z \cdot x|$, Ambos complejos

$|w| = |z \cdot w| = |z| \cdot |w| = \sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2}$

$|\bar{w}| = |w| \quad \text{if } a, b \rightarrow \theta = \frac{180}{\pi}$

cuadrante a b $-\theta + 360$

1) $0 < \theta < \frac{\pi}{2}$ $a > 0$ $b > 0$

$0 < \theta < 90$

2) $\frac{\pi}{2} < \theta < \pi$ $a < 0$ $b > 0$

$90 < \theta < 180$

3) $\pi < \theta < \frac{3\pi}{2}$ $a < 0$ $b < 0$

$180 < \theta < 270$

4) $\frac{3\pi}{2} < \theta < 2\pi$ $a > 0$ $b < 0$

$270 < \theta < 360$

$\theta = \frac{\pi}{2}$ $a = 0$ $b > 0$

$\theta = -\frac{\pi}{2}$ $a = 0$ $b < 0$

$\theta = \pm \pi$ $a < 0$ $b = 0$

En Todos los casos que el θ NO sea un caso especial

$\text{Arg}(w), \frac{b}{a} = \tan(\theta)$

Todos los complejos que satisfacen esto

$$|z - 3i| = 4$$

$$\operatorname{Arg}(z - 2z) = \frac{\pi}{2}$$

$$z = a + bi \in \mathbb{C} \setminus \{0\}$$

$$z - 2z(a + bi) = \frac{\pi}{2}$$

$$a = 0 \quad b > 0$$

$$z - 2a - 2bi = \frac{\pi}{2}$$

$$\underbrace{(z - 2a)}_a + \underbrace{(-2b)i}_b = \frac{\pi}{2}$$

$$z - 2a = 0 \quad -2b > 0$$

$$-2a = -2 \quad b < 0$$

$$\boxed{a = 1}$$

$$|z - 3i| = 4$$

$$|a + bi - 3i| = 4$$

$$|(a) + (b - 3)i| = 4$$

$$a^2 + (b - 3)^2 = 4^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$1^2 + b^2 - 6b + 9 = 16$$

$$b^2 - 6b + 10 - 16 = 0$$

$$b^2 - 6b - 6 = 0$$

$$b = 3 - \sqrt{15}, \quad b = 3 + \sqrt{15}$$

$$\boxed{1 + (3 - \sqrt{15})i}$$

$$\frac{b}{a} = \tan(\theta)$$

$$|\bar{z} + 1 - i| = 5$$

$$\text{Arg}(z - (1 + 2i)) = \frac{3\pi}{4}$$

$$\frac{3\pi}{4}, \frac{180}{\pi}$$

$$a + bi - 1 - 2i = \frac{3\pi}{4}$$

$$135$$

$$90 < 135 < 270$$

$$(a-1) + (b-2)i = \frac{3\pi}{4}$$

$$\frac{b-2}{a-1} = \tan\left(\frac{3\pi}{4}\right)$$

$$a < 0 \quad b > 0$$

$$\frac{b-2}{a-1} = -1$$

$$b-2 = 1-a$$

$$a = 3-b$$

$$|\bar{z} + 1 - i| = 5$$

$$|a + bi + 1 - i| = 5$$

$$|a - bi + 1 - i| = 5$$

$$|(a+1) + (-b-1)i| = 5$$

$$(-a-b)^2 = (a+b)^2$$

$$(a+1)^2 + (-b-1)^2 = 5^2$$

$$(3-b+1)^2 + b^2 + 2b + 1 - 25 = 0$$

$$a^2 + 2ab + b^2$$

$$(4-b)^2 + b^2 + 2b + 1 - 25 = 0$$

$$(4-b)^2$$

$$16 - 8b + b^2 + b^2 + 2b + 1 - 25 = 0$$

$$4^2 - 2 \cdot 4 \cdot b + b^2$$

$$2b^2 - 6b - 8 = 0$$

$$b = 4$$

$$b = -1$$

$$a < 0$$

$$b > 0$$

$$a = 3-b$$

$$3-4 = -1$$

$$-1 + 4i$$

$$P(x), Q(x) \quad P(x) \overline{Q(x)}$$

$$P(x) = 3x^2 + 2x \dots$$

$$Q(x) = \frac{2-i}{2i0}, \quad Q(x) = \frac{2+i}{2i0}$$

$$P(x) = x^4 - x^3 + 8x^2 - 9x + 16 \quad Q(x) = 2i$$

$$\begin{array}{r|rrrrr} 1 & -1 & 8 & -9 & 16 & \\ & +2i & -4-2i & 4+8i & -16 & 2i \\ \hline 1 & -1+2i & 4-2i & 8i & 0 & (x-2i) \end{array}$$

$2i \cdot 8i$
 $16i^2$
 -16

$$x^3 + (-1+2i)x^2 + (4-2i)x + 8i$$

$$\begin{array}{r|rrrr} 1 & -1+2i & 4-2i & 8i & \\ & -2i & 2i & -8i & -2i \\ \hline 1 & -1 & 4 & 0 & (x-(-2i)) \end{array}$$

$(-1+2i) \cdot 2i$
 $-2i + 4i^2$
 $-4 - 2i$
 $-4 - 2i$

$(4-2i) \cdot 2i$
 $8i - 4i^2$
 $4 + 8i$

$-2i + 4i^2, i^2 = -1$

$$x^2 - x + 4$$

$$x = \left(\frac{1 + i\sqrt{15}}{2} \right), \quad x = \left(\frac{1 - i\sqrt{15}}{2} \right)$$

$$(x-2i)(x+2i) \left(x - \frac{1 + i\sqrt{15}}{2} \right) \left(x - \frac{1 - i\sqrt{15}}{2} \right)$$

$m \neq n$ 2×3 3×2

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \leftarrow \text{Nula} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \quad A^T = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

$$A = A^T$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 5 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

Operacion

Suma Resta, OCUPLAN si o si
ser del mismo tamaño

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \pm \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 \pm 5 & 2 \pm 6 \\ 3 \pm 7 & 4 \pm 8 \end{bmatrix}$$

Multiplacion por escalon

$$2 \cdot \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$$

Multiplicación de matrices

Si \wedge Si (obligatorio) el número de columnas de A_1 tiene que ser igual al número de filas de A_2

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 \cdot 9 + 0 \cdot 5 \\ -1 \cdot 9 + 1 \cdot 5 \end{bmatrix} = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1 \cdot 5 + 2 \cdot 7 & 1 \cdot 6 + 2 \cdot 8 \\ 3 \cdot 5 + 4 \cdot 7 & 3 \cdot 6 + 4 \cdot 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{bmatrix}, B = \begin{bmatrix} 6 & 5 & 4 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 \cdot 6 + 3 \cdot 3 + 4 \cdot 2 & 2 \cdot 5 + 3 \cdot 2 + 4 \cdot 1 & 2 \cdot 4 + 3 \cdot 1 + 4 \cdot 2 \\ 5 \cdot 6 + 6 \cdot 3 + 7 \cdot 2 & 5 \cdot 5 + 6 \cdot 2 + 7 \cdot 1 & 5 \cdot 4 + 6 \cdot 1 + 7 \cdot 2 \\ 8 \cdot 6 + 9 \cdot 3 + 1 \cdot 2 & 8 \cdot 5 + 9 \cdot 2 + 1 \cdot 1 & 8 \cdot 4 + 9 \cdot 1 + 1 \cdot 2 \end{bmatrix}$$

Propiedades de matrices, λ es escalar

- 1) $A_{mn} + B_{mn} = C_{mn}$ Cerrada
- 2) $(A+B)+C = A+(B+C)$ Asociativa
- 3) $A+0 = 0+A = A$
- 4) $A+(-A) = -A+A = 0$
- 5) $A+B = B+A$ Conmutativa
- 6) $A \cdot 0 = 0$
- 7) $\lambda(A+B) = \lambda A + \lambda B$
- 8) $(\lambda I + \mu)A = \lambda A + \mu A$
- 9) $(\lambda \cdot \mu)A = \lambda(\mu A)$
- 10) $IA = A \cdot I = A$
- 11) $A \cdot B \neq B \cdot A$
- 12) $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
- 13) $(A+B)C = AC + BC$

Teoremas

- 1) $(A^T)^T = A$
- 2) $(A \cdot B)^T = B^T \cdot A^T$
- 3) $(A \pm B)^T = A^T \pm B^T$
- 4) $(A^{-1})^{-1} = A$
- 5) $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$
- 6) $I^T = I$ Tener en cuenta
 $A + AB = A(I + B)$
 $AI + AB = A(I + B)$
 $B + AB = (I + A)B$

Sean A, B, C matrices tales que

$$(XA-B)^T - C = 2X^T, \text{ si se sabe que}$$

$A-2I$ tiene inversa, encuentre X

$$X \cdot (A-2I) = 7 \quad (A-2I) = (A-2I)^{-1} \quad \begin{array}{l} X^T = X \\ X^{-1} = \frac{1}{X} \end{array}$$

$$X = 7 \cdot (A-2I)^{-1}$$

$$(XA-B)^T - C = 2X^T$$

$$(XA)^T - B^T - C = 2X^T$$

$$(a+b)^T = a^T + b^T$$

$$A^T \cdot X^T - B^T - C = 2X^T$$

$$(a \cdot b)^T = b^T \cdot a^T$$

$$A^T \cdot X^T - 2X^T = B^T + C$$

$$(A^T - 2I)X^T = B^T + C$$

$$[(A^T - 2I), X^T]^T = [B^T + C]^T$$

$$(X^T)^T \cdot (A^T - 2I)^T = (B^T)^T + C^T$$

$$X \cdot [(A^T)^T - (2I)^T] = B + C^T$$

$$(A^T)^T = A$$

$$X \cdot [A - 2I] = B + C^T$$

$$I^T = I$$

$$X = (B + C^T) \cdot (A - 2I)^{-1}$$