2) $\sum_{n=2}^{\infty} n\cdot e^{-n}$

$\int_{2}^{+\infty} n\cdot e^{-n}dx \longrightarrow \int x\cdot e^{-x}dx$ $\quad u=x \quad dv = e^{-x}$

$du = dx \quad v = -e^{-x}$

$-xe^{-x} - \int -e^{-x}dx$

$-xe^{-x} - e^{-x} + C$

$-xe^{-x} - e^{-x}\Big|_{2}^{+\infty}$

$\lim_{x\to+\infty} -xe^{-x} - e^{-x} - (-2e^{-2} - e^{-2})$

$\lim_{x\to+\infty} \frac{-x}{e^{x}}, \frac{+\infty}{+\infty}$, l'hôpital $- \lim_{x\to+\infty} \cancel{e^{-x}} - (-2e^{-2} -- e^{-2})$

$\lim_{x\to+\infty} \cancel{\frac{-1}{e^{x}}} + 2e^{-2} + e^{-2}$

$3e^{-2}$

$\boxed{\text{Converge a } 3e^{-2}}$

3) $\sum_{n=0}^{\infty} n\cdot e^{1-n^2}$

$\int_{0}^{+\infty} n\cdot e^{1-n^2} \longrightarrow \int x\cdot e^{1-x^2}$ $\quad u = 1-x^2$

$du = -2x$

$-\frac{1}{2}du = x$

$\frac{-1}{2}\int e^{u}du$

$\frac{-1}{2}e^{u}$

$\frac{-1}{2}e^{1-x^2} + C$

$\frac{-1}{2}e^{1-x^2}\Big|_{0}^{+\infty}$

$\lim_{x\to+\infty} \cancel{\frac{-1}{2}e^{1-x^2}} -- \frac{1}{2}\cdot e^{1}$

$\frac{e}{2}$

$\boxed{\text{Converge a } \frac{e}{2}}$

4) $\sum_{n=1}^{\infty} n^{2}\cdot e^{-n^3}$

$\int_{1}^{+\infty} n^{2}\cdot e^{-n^3} \longrightarrow \int x^{2}\cdot e^{-x^3}$ $\quad u = -x^3$

$du = -3x^2$

$$\frac{-1}{3} e^{-x^3} \Big|_{1}^{+\infty} \qquad \frac{-1}{3}\int e^u \, du \qquad \frac{-1}{3} du = x^2$$

$$\lim_{x \to +\infty} \cancel{\frac{-1}{3} e^{-x^3}} - -\frac{1}{3} \cdot e^{-1} \qquad \frac{-1}{3} e^u + C$$

$$\frac{-1}{3} e^{-x^3} + C$$

$$\frac{1}{3e}$$

$$\boxed{\text{Converge a } \frac{1}{3e}}$$

5) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\int_{1}^{+\infty} \frac{1}{n^2} \longrightarrow \int \frac{1}{x^2} dx \qquad u = x \quad du = dx$$

$$\int \frac{1}{u^2} du$$

$$\frac{-1}{x} \Big|_{1}^{+\infty} \qquad \int u^{-2} du$$

$$\frac{u^{-1}}{-1} + C$$

$$\lim_{x \to +\infty} \cancel{\frac{-1}{x}} - -\frac{1}{1} \qquad -\frac{1}{u} + C$$

$$-\frac{1}{x} + C$$

$$1$$

$$\boxed{\text{Converge a } 1}$$

7. $\sum_{n=1}^{\infty} \frac{2}{1+n^2}$

$$\int_{1}^{+\infty} \longrightarrow 2\int \frac{1}{1+x^2}$$

$$2\arctan(x) \Big|_{1}^{+\infty} \qquad 2\arctan(x) + C$$

$$\lim_{x \to +\infty} 2\arctan(x) - 2\arctan(1)$$

$$2\,\frac{\pi}{2} - \frac{\pi}{2} = 0$$

$$\boxed{\text{Converge a } 0}$$

8. $\sum_{n=1}^{\infty} \frac{n}{1+n^2}$

$$\int_1^{+\infty} \frac{n}{1+n^2} \longrightarrow \int \frac{x}{1+x^2}$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x$

$$\frac{1}{2}\int \frac{1}{u}\,du$$

$$\frac{1}{2}\ln|u| + C$$

$$\frac{1}{2}\ln|1+x^2| + C$$

$$\frac{1}{2}\ln|1+x^2| \Big|_1^{+\infty}$$

$$\lim_{x\to+\infty} \frac{1}{2}\ln|1+x^2| - \frac{1}{2}\ln|1+1^2|$$

$$+\infty$$

$\therefore$ $\boxed{\text{Diverge}}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+3}{(n^2+3n)^2}$

$$\int_1^{+\infty} \frac{2n+3}{(n^2+3n)^2} \longrightarrow \int \frac{2x+3}{(x^2+3x)^2}$$

$u = x^2+3x$

$du = 2x+3$

$$\int \frac{1}{u^2}\,du$$

$$\frac{-1}{u} + C$$

$$\frac{-1}{x^2+3x} + C$$

$$\frac{-1}{x^2+3x} \Big|_1^{+\infty}$$

$$\lim_{x\to+\infty} \cancel{\frac{-1}{x^2+3x}} - -\frac{1}{1^2+3(1)}$$

$$= \frac{1}{4}$$

$\boxed{\text{Converge a } \frac{1}{4}}$

10. $\displaystyle\sum_{j=3}^{\infty} \frac{1}{j \cdot \ln^2(j)}$

$$\int_3^{+\infty} \frac{1}{n\ln^2(n)}\,dx \longrightarrow \int \frac{1}{x\ln^2(x)}\,dx$$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$$\int \frac{1}{u^2}\,dx$$

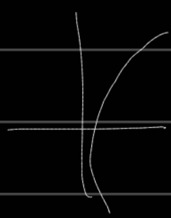
$$\frac{-1}{u} + C$$

$$\frac{-1}{\ln(x)} + C$$

$$\frac{-1}{\ln(x)} \Big|_3^{+\infty}$$

$$\lim_{x\to+\infty} \cancel{\frac{-1}{\ln(x)}} - -\frac{1}{\ln(3)}$$

$$\frac{1}{\ln(3)}$$



$\boxed{\text{Converge a } \frac{1}{\ln(3)}}$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$

$n=1$

$$\int_{1}^{+\infty} \frac{\ln(x)}{x^2}\,dx \longrightarrow \int \frac{\ln(x)}{x^2}\,dx \qquad u=\ln(x) \qquad dv=\frac{1}{x^2}$$

$$du=\frac{1}{x}\,dx \qquad v=\frac{-1}{x}$$

$$\left. -\frac{\ln(x)}{x}+\frac{1}{x} \right|_{1}^{+\infty} \qquad -\frac{\ln(x)}{x}-\int \frac{-1}{x}\cdot\frac{1}{x}\,dx$$

$$\lim_{x\to+\infty} -\frac{\ln(x)}{x}+\cancel{\lim_{x\to+\infty}\frac{1}{x}}-\left(\cancel{-\frac{\ln(1)}{1}}+\frac{1}{1}\right) \qquad -\frac{\ln(x)}{x}-\int\frac{-1}{x^2}\,dx$$

$$\lim_{x\to+\infty} -\frac{\ln(x)}{x}, \ \frac{-\infty}{+\infty}, \ \text{L'hôpital} \qquad -\frac{\ln(x)}{x}+\frac{1}{x}+C$$

$$\lim_{x\to+\infty} -\frac{1}{x}=0$$

1

$\boxed{\text{Converge a } 1}$