Induccion matematica

Notaciones: suma valor final

Formula de incremento

$$1+2+3,\ldots n = \sum_{i=1}^{n} i$$

valor inicial

$$\sum_{k=0}^{n} 2^k$$

Lo mismo con el producto

Final

$$1\cdot 2\cdot 3\cdot \ldots \cdot n \qquad \prod_{i=1}^{n} i$$

Formula de multiplicacion

valor inicial

1) Considere la serie $\sum_{k=3}^{n} [3k+\ln(k+1)]$

Forma del primero, segundo y ultimo?

a) $3(3)+\ln(3+1) = 9+\ln(4)$, $k=3$

b) $3(4)+\ln(4+1) = 12+\ln(5)$, $k=4$

Suma Parcial (2 primeros terminos)

$$\sum_{k=3}^{4} [3k+\ln(k+1)] = [9+\ln(4)] + \underbrace{12+\ln(5)}_{3(4)+\ln(4+1)}, \quad n=4$$

c) $3n+\ln(n+1)$, ultimo termino

Suma Final

$$\sum_{k=3}^{n} [3k+\ln(k+1)] = [9+\ln(4)] + [12+\ln(5)] + \ldots + [3n+\ln(n+1)]$$

Notacion factorial

$n! = n(n-1)\cdot(n-2)\ldots 3\ 2\ 1$

$n! = n(n-1)(n-2) \cdots 3.2.1$

$0! = 1$

Ej. $(2x+1)!$

$= (2x+1)(2x+1-1)(2x+1-2) \cdots 3.2.1$

$= \underbrace{(2x+1)(2x)(2x-1)}! \cdots 2.1$

Se desarrollan los terminos necesarios ($\nexists$ max y min)

Al ultimo se le pone !

Ej. $\dfrac{x(2x+1)}{(2x+1)!}$

$= \dfrac{\cancel{x(2x+1)}}{\cancel{(2x+1)}(2x)(2x-1)!}$

$\boxed{= \dfrac{1}{2(2x-1)!}}$

Props de !

$n! = n(n-1)!$

$(n+1)! = (n+1)\,n! \leftarrow (n+1-1)$

Se sacan todos los que quieran

Ej: $\displaystyle\sum_{k=0}^{n} (2^k+1)$

Queremos que empieze en $k=1$, remplazando y sumando pero con $k=1$

$\underbrace{2^0+1=2}_{k=0} + \displaystyle\sum_{k=1}^{n} (2^k+1)$

Ej $\displaystyle\sum_{i=0}^{n} i$, dejando 0 y 1 afuera

$\underbrace{0}_{i=0} + \underbrace{1}_{i=1} + \displaystyle\sum_{i=2}^{n} i$ Dejar fuera de la suma terminos iniciales

Ej

$$\sum_{i=0}^{n-1} i + n$$ dejando el ultimo afuera

Ej

$$\sum_{k=0}^{n} a_k = a_0 + \sum_{k=1}^{n} a_k$$ Primero afuera

$$= \sum_{k=0}^{n-1} a_k + a_n$$ ultimo afuera

Primero y ultimo afuera

$$= a_0 + \sum_{k=1}^{n-1} a_k + a_n$$

Mas Props de suma

k constante

(1) $$\sum_{i=0}^{n} [k \cdot a_i] = k \sum_{i=0}^{n} a_i$$

$$\sum_{i=0}^{n} (3 \cdot 2^i) = 3 \sum_{i=0}^{n} 2^i$$

$$\sum_{i=0}^{n} [x^2 (3i-1)] = x^2 \sum_{i=0}^{n} 3i-1$$

Como x^2 ≠ de la variable contadora (i), sale afuera

(2) $$\sum_{i=0}^{n} (a_i \pm b_i) = \sum_{i=0}^{n} a_i \pm \sum_{i=0}^{n} b_i$$

$$\sum_{i=2}^{n} 2^k + i^3 = \sum_{i=2}^{n} 2^k + \sum_{i=2}^{n} i^3$$

Ejemplos de induccion matematica, demostrar validez

1) $\sum^{k}$ ...

$$\underbrace{\sum_{i=1} \ln(i)}_{\text{Suma}} = \underbrace{\ln(k!)}_{\text{Formula explicita}}$$

Sea $P(k): \sum_{i=1}^{k} \ln(i) = \ln(k!)$

y $S = \{k \in \mathbb{N} \mid P(k) \text{ es verdadera}\}$

a) Probar para $k=1$

ahi acaba

$$\sum_{i=1}^{1} \ln(i) = \ln(1!)$$

$$\ln(1) = \ln(1) \checkmark$$

$\therefore$ Es verdadero

b) Probar para $k=p$

$$\sum_{i=1}^{p} \ln(i) = \ln(p!)$$, Hipotesis de induccion

HQD

c) Probar para $k=p+1$

$$\sum_{i=1}^{p+1} \ln(i) = \boxed{\ln(p+1)!}$$ Hay que llegar a esto

Demostracion

$$\sum_{i=1}^{p+1} \ln(i)$$ Transformar para usar hi

$$\sum_{i=1}^{p} \ln(i) + \ln(p+1)$$

$$\ln(p!) + \ln(p+1)$$ Hi

$$\ln[p!\,(p+1)]$$ Prop de log

$p+1 > p \longrightarrow n! = n\cdot(n-1)! \longrightarrow (n+1)! = (n+1)\cdot n!$

$$\ln(p+1)!$$

$10 \cdot 9! = 10!$

$\therefore$ Lo propuesto es cierto

2) $\underbrace{1 \cdot 3 \cdot 5 \cdots (2n-1)}_{} = \dfrac{(2n)!}{2^n \cdot n!}, \quad \forall n \geq 1$

$$2n-1 = \frac{(2n)!}{2^n \cdot n!}$$

a) Se demuestra para $n=1$

$$2n-1 = \frac{(2(1))!}{2^1 \cdot 1!}$$

$$2 \cdot 1 - 1 = \frac{(2(1))!}{2^1 \cdot 1!}$$

$$1 = 1$$

b) se demuestra para $n=p$

$$2p-1 = \frac{(2p)!}{2^p \cdot p!}, \text{ Hi}$$

c) Se demuestra para $n=p+1$

$$(2p-1) \cdot [2(p+1)-1] = \frac{[2(p+1)]!}{2^{p+1} \cdot (p+1)!}$$

$$(2p-1) \cdot (2p+1) = \frac{(2p+2)!}{2^{p+1} \cdot (p+1)!} \qquad \frac{\cancel{2(p+1)} \cdot (2p+1)(2p)!}{2^p \cdot 2 \cdot \cancel{(p+1)} \cdot p!}$$

uno antes

por el 1.3.5...

$$= \frac{(2p+1) \cdot (2p)!}{2^p \cdot p!} \to n \cdot (n+1)!$$

$$\boxed{\text{HQD} = \frac{(2p+1)!}{2^p \cdot p!}}$$

Demostración

$(2p-1) \cdot (2p+1)$

$$= \frac{(2p)!}{2^p \cdot p!} \cdot 2p+1 \quad Hi$$

$$\frac{(2p+1)!}{2^p \cdot p!}$$

$\therefore$ Lo propuesto es cierto

3) $\frac{4}{5^0} + \frac{4}{5^1} + \frac{4}{5^2} \ldots + \frac{4}{5^n} = 5 - \frac{1}{5^n}$

a) Demostrar para $n=0$

$$\frac{4}{5^0} = 5 - \frac{1}{5^0}$$

$$4 = 4 \checkmark$$

b) Demostrar para $n=p$

$$\frac{4}{5^p} = 5 - \frac{1}{5^p} \quad Hi$$

c) Demostrar para $n=p+1$

$$\frac{4}{5^p} + \frac{4}{5^{p+1}} = 5 - \frac{1}{5^{p+1}} \quad H(a)$$

Demostracion

$$\frac{4}{5^p} + \frac{4}{5^{p+1}}$$

$$5 - \frac{1}{5^p} + \frac{4}{5^{p+1}}$$

$$5 - \frac{5-4}{5^{p+1}}$$

$$5 - \frac{1}{5^{p+1}}$$

$\therefore$ Lo propuesto es cierto

4) a) $1\cdot 5^1 + 2\cdot 5^2 + 3\cdot 5^3 + \cdots + n\cdot 5^n = \dfrac{5+(4n-1)\cdot 5^{n+1}}{16}$

$n=1, \quad 1\cdot 5^1 = \dfrac{5+(4(1)-1)\cdot 5^{1+1}}{16}$

$5=5$ ✓

$n=p, \quad p\cdot 5^p = \dfrac{5+(4p-1)\cdot 5^{p+1}}{16}$, Hi

HQD

$n=p+1, \quad (p+1)\cdot 5^{p+1} = \dfrac{5+(4p+3)\cdot 5^{p+2}}{16}$

Demostración

$1\cdot 5^1+2\cdot 5^2 \ldots + \underbrace{p\cdot 5^p}_{\text{1 antes siempre}} + \underbrace{(p+1)\cdot 5^{p+1}}_{p+1}$

$\dfrac{5+(4p-1)\cdot 5^{p+1}}{16} + (p+1)\cdot 5^{p+1}$, Hi

$\dfrac{5+(4p-1)\cdot 5^{p+1} + 16(p+1)\cdot 5^{p+1}}{16}$

$\dfrac{5+5^{p+1}(4p-1+16p+16)}{16}$

$\dfrac{5+5^{p+1}(20p+15)}{16}$

$\dfrac{5+5^{p+1}\cdot 5(4p+3)}{16}$

$\dfrac{5+(4p+3)\cdot 5^{p+2}}{16}$

5) b) $\displaystyle\sum_{k=1}^{n} \frac{2k-1}{2^k} = 3 - \frac{2n+3}{2^n}.$

$n=1 \quad \frac{2-1}{2} = 3 - \frac{2+3}{2}$ ✓

$n=p \quad \displaystyle\sum_{k=1}^{p} \frac{2k-1}{2^k} = 3 - \frac{2p+3}{2^p}$, Hi

$n=p+1 \quad \displaystyle\sum_{k=1}^{p+1} \frac{2k-1}{2^k} = 3 - \frac{2p+5}{2^{p+1}}$

Demostracion

$\displaystyle\sum_{k=1}^{p+1} \frac{2k-1}{2^k}$

$\displaystyle\sum_{k=1}^{p} \frac{2k-1}{2^k} + \frac{2p+1}{2^{p+1}}$

$3 - \frac{2p+3}{2^p} + \frac{2p+1}{2^{p+1}}$, Hi

$3 + \frac{2p+1}{2^{p+1}} - \frac{2p+3}{2^p}$

$3 + \frac{2p+1-4p-6}{2^{p+1}}$

$3 - \frac{2p+5}{2^{p+1}}$

6) c) $\displaystyle\sum_{i=1}^{n} (2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$

$n=1 \quad (2-1)^2 = \frac{(2-1)(2+1)}{3}$

$1=1$ ✓

$n=p \quad \sum_{i=1}^{p}(2i-1)^2 = \frac{p(2p-1)(2p+1)}{3}$, Hi

H.QD

$n=p+1 \quad \sum_{i=1}^{p+1}(2i-1)^2 = \frac{(p+1)(2p+1)(2p+3)}{3}$

Demo

$\sum_{i=1}^{p+1}(2i-1)^2$

$\sum_{i=1}^{p}(2i-1)^2 + (2p+1)^2$

$\frac{p(2p-1)(2p+1)}{3} + (2p+1)^2$

$\frac{p(2p-1)(2p+1)+12p^2+12p+3}{3}$

$\frac{4p^3+12p^2+11p+3}{3}$

$\frac{(2p+1)(2p+3)(p+1)}{3}$

$4p^3+12p^2+11p+3$

4	12	11	3	
	-4	-8	-3	-1
4	8	3	0	

$(4p^2+8p+3)(p+1)$

$2p \quad 1 = 2p$

$2p \quad 3 = 6p$

$8p$

$(2p+1)(2p+3)(p+1)$

7) d) $\sum_{k=1}^{n} k \cdot k! = (n+1)! - 1.$

$n=1 \quad 1 \cdot 1! = (1+1)! - 1$

$1 = 1$ ✓

$n = p \quad \sum_{k=1}^{p} k \cdot k! = (p+1)! - 1$, Hi

$n = p+1 \quad \sum_{k=1}^{p+1} k \cdot k! = (p+2)! - 1$

Demo

$\sum_{k=1}^{p+1} k \cdot k!$

$\sum_{k=1}^{p} k \cdot k! + (p+1)(p+1)!$

$(p+1)! - 1 + (p+1)(p+1)!$, Hi

$(p+1)! + (p+1)(p+1)! - 1$

$(p+1)!(1 + p+1) - 1$

$(p+2)(p+1)! - 1$

$(p+2)! - 1$