Propiedades
$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$ $\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$
$\log_a(x) = \frac{\log_b(x)}{\log_b(a)},\ \forall b \in \mathbb{R}^+ - \{1\}$ $\ln(x) = \frac{\log_b(x)}{\log_b(e)},\ \forall b \in \mathbb{R}^+ - \{1\}$
$\log_a(1) = 0$ $\ln(1) = 0$

Propiedades
$\log_a(x \cdot y) = \log_a(x) + \log_a(y)$ $\ln(x \cdot y) = \ln(x) + \ln(y)$
$\log_a(x^n) = n \cdot \log_a(x)$ $\ln(x^n) = n \cdot \ln(x)$
$\log_a(a) = 1$ $\ln(e) = 1$

$$\boxed{\ln\left(x\sqrt[3]{2x+1}\sqrt[5]{(3x+1)^2}\right) = \ln(x) + \frac{1}{3}\ln(2x+1) + \frac{2}{5}\ln(3x+1)}$$

$$\ln\left(x\sqrt[3]{2x+1}\sqrt[5]{(3x+1)^2}\right)$$

$$= \ln(x) + \ln\left(\sqrt[3]{2x+1}\right) + \ln\left(\sqrt[5]{(3x+1)^2}\right)$$

$$= \ln(x) + \ln\left[(2x+1)^{\frac{1}{3}}\right] + \ln\left[(3x+1)^{\frac{2}{5}}\right]$$

$$\boxed{= \ln(x) + \frac{1}{3}\ln(2x+1) + \frac{2}{5}\ln(3x+1)}$$

$$\boxed{\frac{1}{4}\log(x^2+4x+3) + \frac{1}{4}\log\left(\frac{x+1}{x+3}\right) = \log\sqrt{x+1}}$$

$$\begin{array}{ccc} x & & 3 = 3x \\ x & & 1 = x \\ & & \overline{4x} \end{array}$$

$$\frac{1}{4}\log\left[(x+3)(x+1)\right] + \frac{1}{4}\log\left(\frac{x+1}{x+3}\right)$$

$$\frac{1}{4}\left[\log(x+3) + \log(x+1)\right] + \frac{1}{4}\left[\log(x+1) - \log(x+3)\right]$$

$$\cancel{\frac{1}{4}\log(x+3)} + \underbrace{\frac{1}{4}\log(x+1) + \frac{1}{4}\log(x+1)}\ \cancel{-\frac{1}{4}\log(x+3)}$$

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\frac{1}{2}\log(x+1)$$

$$\boxed{\log\sqrt{x+1}}$$

$$\log\left(\frac{x\sqrt{x+1}}{x^2-1}\right) = \log(x) - \frac{1}{2}\log(x+1) - \log(x-1)$$

$$\log\left(\frac{x\sqrt{x+1}}{x^2-1}\right)$$

$$\log(x\sqrt{x+1}) - \log(x^2-1)$$

$$\log(x) + \log(\sqrt{x+1}) - [\log(x-1) + \log(x+1)]$$

$$\log(x) + \log(x+1)^{\frac{1}{2}} - \log(x-1) - \log(x+1)$$

$$\log(x) + \underbrace{\frac{1}{2}\log(x+1)} - \log(x-1) \underbrace{- \log(x+1)}$$

$$\boxed{\log(x) - \frac{1}{2}\log(x+1) - \log(x-1)}$$