L'hopital para calcular limites indeterminados por medio de derivadas

Derivables $\left(\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}\right)$

Sea $f(x), g(x)$ funciones donde $g(x) \neq 0$ se cumple:

$$\lim_{x\to a} \frac{f(x)}{g(x)}, \quad \text{Forma } \frac{0}{0} \quad \frac{0}{-+\infty}$$

-Es cierto que $\boxed{\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}}$

Nota, l'hopital puede tener que aplicarse varias veces, por ejemplo si en una de las veces queda $\frac{0}{0}$ V $\frac{+\infty}{+\infty}$

Indet. $0\cdot\infty$, $+\infty - \infty$, ∞^0, 1^∞, 0^0 } No son l'hopital pero se pueden convertir en

1) $\lim_{x\to 0} \frac{x\cos(x)-\operatorname{sen}(x)}{x^3}$, f $\frac{0}{0}$, L'hopital

$$\lim_{x\to 0} \frac{\cos(x)-x\operatorname{sen}(x)-\cos(x)}{3x^2}$$

No es Regla de cociente

$$\lim_{x\to 0} \frac{-x\operatorname{sen}(x)}{3x^2}, \text{ L'hopital}$$

Truco especial

$$\lim_{x\to 0} \frac{-\operatorname{sen}(x)}{3x} = \frac{-1}{3}$$

Ambas sirven

$$\lim_{x\to\infty} \frac{-\cos(x)}{3} = \frac{-1}{3}$$, L, hôpital

2) $$\lim_{x\to\infty} \frac{e^{x}-e^{-x}}{\operatorname{sen}(x)}$$, $\frac{0}{0}$, L'hopital

(cadena)

$$\lim_{x\to\infty} \frac{e^{x}-e^{-x}\cdot -1}{\cos(x)} = \frac{2}{1} = 2$$

3) $$\lim_{x\to\infty} \frac{x-\tan(x)}{x-\operatorname{sen}(x)}$$, f $\frac{0}{0}$, l'hopital

$$\lim_{x\to\infty} \frac{1-\sec^2(x)}{1-\cos(x)}$$, l'hôpital, f $\frac{0}{0}$

$$\lim_{x\to\infty} \frac{\cos^2(x)-1}{\cos^2(x)(1-\cos^2(x)}$$

$$\lim_{x\to\infty} \frac{-(1-\cos(x))(1+\cos(x))}{\cos^2(x)(1-\cos(x))}$$

$$\lim_{x\to\infty} \frac{-(1+\cos(x))}{\cos^2(x)} = \frac{-2}{1} = \boxed{-2}$$

Otras formas indeterminadas de límites

Forma $\pm\infty \,(\mp)\, \pm\infty$

$$\lim f(x) - g(x) = \infty - \infty$$

$x \to a$

1) Simplificar $f(x) - g(x)$ e intentar de nuevo
   - Llevar a formas $\frac{0}{0}, \frac{\pm\infty}{+\infty}$
   - L'hôpital

2) Escribir $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x)\left(1 - \frac{g(x)}{f(x)}\right)$
   - Factor común de $f(x)$

Ejemplos

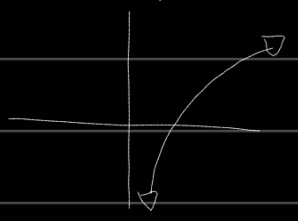
1) $\lim_{x\to 0} \left[\frac{1}{x} - \frac{1}{\operatorname{sen}(x)}\right] = f\ \frac{1}{0} - \frac{1}{0}$
$= \infty - \infty$ Indeterminado

$$\lim_{x\to 0} \frac{\operatorname{sen}(x) - x}{x \operatorname{sen}(x)}$$

$$\lim_{x\to 0} \frac{\cos(x) - 1}{\operatorname{sen}(x) + x\cos(x)}, \ f\ \frac{0}{0}, \text{ L'hôpital}$$

$$\lim_{x\to 0} \frac{-\operatorname{sen}(x)}{\cos(x) + \cos(x) - x\operatorname{sen}(x)} = \frac{0}{2} = \boxed{0}$$

2) $\lim_{x\to +\infty} [x - \ln(x^2 - 1)]$ 6 de log



$$\lim_{x\to +\infty} x\left(1 - \frac{\ln(x^2-1)}{x}\right)$$

$$\lim_{x\to +\infty} x\left(\frac{1 - \ln(x^2-1)}{x}\right)$$

$\lim_{x \to 0^+} \left(\quad \right)$

$\lim_{x \to 0^+} x \cdot \lim_{x \to 0^+} \frac{1 - \ln(x^2 - 1)}{x}$

$\lim_{x \to 0^+} \frac{1 - \frac{2x}{x^2 - 1}}{1}$

$1 - \lim_{x \to 0^+} \frac{2x}{x^2}$ Teorema de polinomio mayor cuando $x \to 0^+$

$1 - \lim_{x \to 0^+} \frac{2}{x}$

$\lim_{x \to 0^+} x \cdot \lim_{x \to 0^+} 1 - \frac{2}{x} = 1 - \frac{2}{+\infty} = 1$

$\boxed{+\infty \cdot 1 = +\infty}$

Forma $0 \cdot \infty$

$\lim_{x \to a} f(x) = 0 = 1 \lim_{x \to a} g(x) = \infty$

a) $\lim_{x \to a} \frac{f(x)}{\frac{1}{g(x)}}$, F $\frac{0}{0}$ — f arriba

b) $\lim_{x \to a} \frac{g(x)}{\frac{1}{f(x)}}$, F $\frac{\pm\infty}{\pm\infty}$ — g arriba

$\lim_{x \to a} f(x) \cdot g(x) = 0 \cdot \infty$

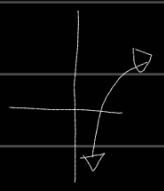
Nota $a \cdot b = \frac{a}{\frac{1}{b}} = a \cdot b \quad \wedge \quad a \cdot b = \frac{b}{\frac{1}{a}} = a \cdot b$

* Tratar tener funcion con derivada

mas complicada en forma $\frac{1}{f(x)}$

6 de log

Ejemplo $\lim_{x\to 2^+} (x-2)\ln(x-2)$, $f: 0\cdot\infty$



2 options

$\lim_{x\to 2^+} \frac{(x-2)}{\frac{1}{\ln(x-2)}}$ mas difícil

$\lim_{x\to 2^+} \frac{\ln(x-2)}{\frac{1}{(x-2)}}$ mas facil

ILATE

- Inversa trigo (arcsen,...)
- Logaritmica
- Algebraicas
- Trigonometricas
- Exponenciales

arriba la que este mas alto en ilate

Ej. $\arctan(x) = \frac{\arctan(x)}{\frac{1}{x}}$

$\lim_{x\to 2^+} \frac{\ln(x-2)}{\frac{1}{(x-2)}}$

$\lim_{x\to 2^+} \frac{\ln(x-2)}{\frac{1}{(x-2)}}$, $F: \frac{+\infty}{+\infty}$

$\lim_{x\to 2^+} \frac{\frac{1}{x-2}}{\frac{-1}{(x-2)^2}}$ (cadena $\frac{-1}{f^2(x)}\cdot f'(x)$)

$\lim_{x\to 2^+} \frac{(x-2)^2}{-(x-2)}$

$\lim_{x\to 2^+} \frac{x-2}{-1} = \boxed{2-2=0}$

1. $\lim_{x\to 0} \frac{e^{ax}-ax-1}{1-\cos x}$ R/ a^2
2. $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$ R/ $\frac{1}{2}$

Practica

1) $\lim_{x \to 0} \frac{e^{ax} - ax - 1}{1 - \cos(x)} = \frac{0}{0}$, L'hopital

$\lim_{x \to 0} \frac{e^{ax} \cdot a - a}{\operatorname{sen}(x)}$

$\lim_{x \to 0} \frac{a^2 e^{ax}}{\cos(x)} = \boxed{a^2}$

3. $\lim_{x \to 0} \left[\frac{1}{e^x - 1} - \frac{1}{x}\right]$ R/ $\frac{1}{2}$

4. $\lim_{x \to 0^+} x e^{\frac{1}{x}}$ R/ $+\infty$

5. $\lim_{x \to -\infty} x^2 \ln\left(1 + \frac{1}{x}\right)$ R/ $-\infty$

6. $\lim_{x \to +\infty} \left[(x^2+1) \cdot \operatorname{sen}\left(\frac{1}{x}\right)\right]$ R/ $+0$

7. $\lim_{x \to 1} \frac{2 - \sqrt[4]{17 - x}}{\sqrt{3x+1} - 2}$ R/ $\frac{1}{24}$

8. $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$ R/ $\frac{1}{2}$

9. $\lim_{x \to 0} \left(\frac{1}{e^x - 1} - \frac{1}{x}\right)$ R/ $\frac{-1}{2}$

10. $\lim_{x \to -\infty} \frac{\frac{2}{x}}{e^{\frac{1}{x}} - 1}$ R/ 2

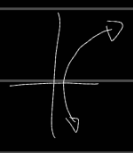
11. $\lim_{x \to +\infty} \frac{\ln\left(1 - \frac{6}{x}\right)}{\frac{1}{x}}$ R/ -6

12. $\lim_{x \to 0} \arcsen(x) \cdot \csc x$ R/ 1

2) $\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\ln(x)}\right)$, f $\frac{1}{0} - \frac{1}{0}$

$+\infty - -\infty$

L'hopital



$\lim_{x \to 1} \frac{x \ln(x) - (x-1)}{(x-1)\ln(x)}$

$\lim_{x \to 1} \frac{1 \cdot \ln(x) + x \cdot \frac{1}{x} \cdot x' - 1}{1 \cdot \ln(x) + (x-1) \cdot \frac{1}{x} \cdot 1}$

$\lim_{x \to 1} \frac{\ln(x)}{\ln(x) + \frac{x-1}{x}}$

$\lim_{x \to 1} \frac{\frac{1}{x}}{\frac{1}{x} + 1 - \frac{1}{x}}$, f $\frac{0}{0}$, l'hopital

$\lim_{x \to 1} \frac{\frac{1}{x}}{\frac{1}{x} + 0 - \left(\frac{-1}{x^2}\right)}$

$\lim_{x \to 1} \frac{\frac{1}{x}}{\frac{1}{x} + \frac{1}{x^2}} = \boxed{\frac{1}{2}}$

3) $\lim_{x \to 0} \left[\frac{1}{e^x - 1} - \frac{1}{x} \right]$, $\infty - \infty$, l'hôpital

$$\lim_{x \to 0} \frac{x - e^x + 1}{(e^x - 1)(x)}$$

$$\lim_{x \to 0} \frac{1 - e^x}{xe^x + e^x - 1}, \quad \frac{0}{0}, \text{ l'hôpital}$$

$$\lim_{x \to 0} \frac{-e^x}{e^x + xe^x + e^x}$$

$$\lim_{x \to 0} \frac{e^x(-1)}{xe^x + 2e^x}$$

$$\lim_{x \to 0} \frac{e^x(-1)}{e^x(x+2)} = \boxed{\frac{-1}{2}}$$

4) $\lim_{x \to 0^+} xe^{\frac{1}{x}}$, $0 \cdot +\infty$, ilate

$$\lim_{x \to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}}, \quad \frac{0}{0}, \text{ l'hôpital}$$

$$\lim_{x \to 0^+} \frac{e^{\frac{1}{x}} \cdot \frac{-1}{x^2}}{\frac{-1}{x^2}}$$

$$\lim_{x \to 0^+} e^{\frac{1}{0^+}} = \boxed{+\infty}$$

5) $\lim_{x \to -\infty} x^2 \ln\left(1+\frac{1}{x}\right) = -\infty \cdot \ln\left(1+\frac{1}{-\infty}\right) \to \frac{\infty}{-\infty} = 0$

$-\infty \cdot \ln(1+0)$

$-\infty \cdot 0$

$$\lim_{x \to -\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x^2}}$$

$$\lim_{x \to -\infty} \frac{\frac{-1}{x^2}}{\frac{1+\frac{1}{x}}{x^{-2}}}$$

$$\lim_{x \to -\infty} \frac{\frac{-1}{x^2}}{\frac{1+\frac{1}{x}}{-2 \cdot x^{-3}}}$$

$$\lim_{x \to -\infty} \frac{\frac{+1}{x^2(1+\frac{1}{x})}}{\frac{+2}{x^3}}$$

$$\lim_{x \to -\infty} \frac{x^3}{2x^2\left(1+\frac{1}{x}\right)}$$

$$\lim_{x \to -\infty} \frac{x}{2+\frac{2}{x}} = \frac{-\infty}{2} = \boxed{-\infty}$$

1. $\lim_{x \to 0} \frac{e^{ax}-ax-1}{1-\cos x}$ R/ a^2
2. $\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$ R/ $\frac{1}{2}$
3. $\lim_{x \to 0} \left[\frac{1}{e^x-1} - \frac{1}{x}\right]$ R/ $\frac{1}{2}$
4. $\lim_{x \to 0^+} xe^{\frac{1}{x}}$ R/ $+\infty$
5. $\lim_{x \to -\infty} x^2 \ln\left(1+\frac{1}{x}\right)$ R/ $-\infty$
6. $\lim_{x \to +\infty} \left[(x^2+1) \cdot \operatorname{sen}\left(\frac{1}{x}\right)\right]$ R/ $+0$
7. $\lim_{x \to 1} \frac{2-\sqrt[4]{17-x}}{\sqrt{3x+1}-2}$ R/ $\frac{1}{24}$
8. $\lim_{x \to 0} \frac{e^x-x-1}{x^2}$ R/ $\frac{1}{2}$
9. $\lim_{x \to 0} \left(\frac{1}{e^x-1} - \frac{1}{x}\right)$ R/ $\frac{-1}{2}$
10. $\lim_{x \to -\infty} \frac{\frac{2}{x}}{e^{\frac{1}{x}}-1}$ R/ 2
11. $\lim_{x \to +\infty} \frac{\ln\left(1-\frac{6}{x}\right)}{\frac{1}{x}}$ R/ -6
12. $\lim_{x \to 0} \operatorname{arcsen}(x) \cdot \operatorname{cs} cx$ R/ 1

6) $\lim_{x \to +\infty} (x^2+1) \cdot \operatorname{Sen}\left(\frac{1}{x}\right)$, f $+\infty \cdot 0$

$$\lim_{x \to +\infty} \frac{\operatorname{Sen}\left(\frac{1}{x}\right)}{\frac{1}{x^2+1}}, \text{ l'Ôpital}$$

$$\lim_{x \to 0^+ \infty} \frac{\frac{-1}{x^2}\cos\left(\frac{1}{x}\right)}{\frac{-2x}{(x^2+1)^2}}$$

$$\lim_{x \to 0^+ \infty} \frac{\frac{\cos\left(\frac{1}{x}\right)}{x^2}}{\frac{2x}{(x^2+1)^2}}$$

$$\lim_{x \to 0^+ \infty} \frac{\cos\left(\frac{1}{x}\right)(x^2+1)^2}{2x^3}$$

$$\lim_{x \to 0^+ \infty} \cos\left(\frac{1}{x}\right) \cdot \frac{x^4+2x^2+1}{2x^3}$$ → Teorema Polinomial

$$\lim_{x \to 0^+ \infty} \frac{x^4 \cos\left(\frac{1}{x}\right)}{2x^3}$$

$$\lim_{x \to 0^+ \infty} \frac{x\cos\left(\frac{1}{x}\right)}{2} = \frac{+\infty}{2} = \boxed{+\infty}$$

7) $\lim_{x \to 1} \frac{2-\sqrt[4]{7-x}}{\sqrt{3x+1}-2}$, f $\frac{0}{0}$, l'hôpital

$$\lim_{x \to 1}$$

$$\lim_{x \to 1}$$

$$\lim_{x \to 1}$$

$$\lim_{x \to 1}$$

$$\lim_{x \to 1}$$

$$\lim_{x \to 1} \frac{(3x+1)^{\frac{1}{2}}}{6(11-x)^{\frac{3}{4}}} = \boxed{\frac{1}{24}}$$

8) $$\lim_{x \to 0} \frac{e^x - x - 1}{x^2} = \frac{0}{0}, \text{ l'hôpital}$$

$$\lim_{x \to 0} \frac{e^x - 1 - 0}{2x}, \frac{0}{0}, \text{ l'hôpital}$$

$$\lim_{x \to 0} \frac{e^x - 0}{2} = \boxed{\frac{1}{2}}$$

9) $$\lim_{x \to 0} \frac{1}{e^x - 1} - \frac{1}{x}, \text{ f } \infty - \infty$$

$$\lim_{x \to 0} \frac{x - e^x + 1}{(e^x - 1)(x)} = \frac{0}{0}, \text{ l'hôpital}$$

$$\lim_{x \to 0} \frac{1 - e^x}{xe^x + e^x - 1}, \frac{0}{0}, \text{ l'hôpital}$$

$$\lim_{x \to 0} \frac{-e^x}{x \quad x \quad x}$$

$e^x + xe^x + e^x$

$$\lim_{x \to 0} \frac{-e^x}{2e^x + xe^x}$$

$$\lim_{x \to 0} \frac{-e^x}{e^x(2+x)} = \boxed{\frac{-1}{2}}$$

10) $$\lim_{x \to -\infty} \frac{\frac{2}{x}}{e^{\frac{1}{x}} - 1} = \frac{0}{0}, \text{ l'Hôpital}$$

$$\lim_{x \to -\infty} \frac{2 \cdot \frac{-1}{x^2}}{e^{\frac{1}{x}} \cdot \frac{-1}{x^2} - 0}$$

$$\lim_{x \to -\infty} \frac{\frac{-2}{x^2}}{\frac{-e^{\frac{1}{x}}}{x^2}}$$

$$\lim_{x \to -\infty} \frac{2}{e^{\frac{1}{x}}} = \frac{2}{e^{\frac{1}{-\infty}}} = \frac{2}{e^0} = \boxed{2}$$

11) $$\lim_{x \to +\infty} \frac{\ln\left(7 - \frac{6}{x}\right)}{\frac{1}{x}}, \quad \frac{+\infty}{+\infty}, \text{ l'Hôpital}$$

$$\lim_{x \to +\infty} \frac{\frac{1}{7 - \frac{6}{x}} \cdot 0 - \frac{-6}{x^2}}{\frac{-1}{x^2}}$$

$$\lim_{x \to +\infty} \frac{\frac{6}{x^2}}{\frac{x-6}{x}} =$$

$$\lim_{x \to +\infty} \frac{\frac{6}{x^2}}{\frac{-1}{x^2}}$$

$$\lim \frac{6}{}$$

$$\lim_{x \to +\infty} \frac{x^2 + 6x}{-\frac{1}{x^2}}$$

$$\lim_{x \to +\infty} \frac{-6x^2}{x^2 - 6x}$$

$$\lim_{x \to +\infty} \frac{-6x^2}{x^2} = \boxed{-6}$$

12) $\lim_{x \to 0} arc(sen(x)) \cdot cos(x)$, $0 \cdot +\infty$

$$\lim_{x \to 0} \frac{arc(sen(x))}{\frac{1}{1}}, \frac{0}{0}, \text{l'hopital}$$
$$sen(x)$$

$$\lim_{x \to 0} \frac{arc(sen(x))}{sen(x)}$$

$$\lim_{x \to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{cos(x)} = \frac{\frac{1}{\sqrt{1-0^2}}}{cos(0)} = 1$$