limites a +- infinito

forma: $\dfrac{k}{\pm\infty} = 0, \; k \in \mathbb{R}$

forma: $\dfrac{k}{0} = \pm\infty, \; k \in \mathbb{R}, \; k \neq 0$

AV

↳ Requiere de analisis de limites laterales

$\dfrac{k}{0^+} = +\infty$ if $k > 0$ $\qquad \dfrac{k}{0^-} = -\infty$ if $k > 0$

$\dfrac{k}{0^+} = -\infty$ if $k < 0$ $\qquad \dfrac{k}{0^-} = +\infty$ if $k < 0$

teorema

$$\lim_{x \to \pm\infty} \frac{P(x)}{Q(x)} = \lim_{x \to \pm\infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x_0}{b_m x^m + b_{m-1} x^{n-1} \dots b_1 x_0}$$

$$\lim_{x \to \pm} \frac{a_n x^n}{b_m x^n}$$

Ejemplo $\displaystyle \lim_{x \to +\infty} = \frac{7}{x^2 - 3} = \frac{7}{(+\infty)^2 - 3} = \frac{7}{+\infty} \boxed{= 0}$

Ejemplo $\displaystyle \lim_{x \to +\infty} \frac{7x^2 - 2x + 1}{3x^2 + 3x + 5}$

$$\lim_{x \to +\infty} \frac{7\cancel{x^2}}{3\cancel{x^2}} = \boxed{\frac{7}{3}} = \text{AH}$$

Ejemplo

$$\lim_{x \to -\infty} \frac{5x^3 - 12x^2 + 25x - 30}{4x^2 - 10}$$

Si hubiera $-\frac{5}{2}$

seria $+\infty$ (depende si

$$\lim_{x \to -\infty} \frac{5x^3}{4x^2} = \frac{5x}{4} = \frac{5(-\infty)}{4} = \frac{5}{4}(-\infty) = \boxed{-\infty}$$

Ejemplo $\lim_{x \to 3} \frac{3x^5 - 7x}{x^8 - 2}$

No tiende a $\pm$ entonces no es valido

Ejemplo $\lim_{x \to +\infty} \frac{30x^{100} - 12}{x^{101} + 8}$

$$\lim_{x \to +\infty} \frac{30}{x^1} = \frac{30}{+\infty} = \boxed{0}$$

Ejemplo $\lim_{x \to -1} \frac{3x}{2x+2} = \frac{-3}{0}$ $\frac{k}{+-\infty}$

$$\lim_{x \to -1^-} \frac{3x}{2x+2} = \frac{-3}{0^-} = \frac{-3}{0^-} = \boxed{+\infty}$$

usar un valor menor a la izq de -1

tome $x = -1{,}01$

$2(-1{,}01) + 2 < 0$

se acerca por la izq al ser < 0

Solo en el denominador que es el que da problemas

$= AV$

$$\lim_{x \to -1^+} \frac{3x}{2x+2} = \frac{-3}{0^+} = \boxed{-\infty}$$

$\boxed{\nexists}$

tome $x = -0{,}99$

$2(-0{,}99)+2 > 0$

Ejemplo

$$\lim_{x \to 2} \frac{2x}{x-2} = \frac{4}{0}$$

$$\lim_{x \to 2^-} \frac{2x}{x-2} = \frac{4}{0^-} = \boxed{-\infty}$$

tome $x = 1{,}99$

$1{,}99 - 2 < 0$

$\neq \to \nexists$

$$\lim_{x \to 2^+} \frac{2x}{x-2} = \frac{4}{0^+} = \boxed{+\infty}$$

tome $x = 2{,}01$

$2{,}01 - 2 > 0$

Ejemplo

$$\lim_{x \to 3} \frac{3x}{(x-3)^2} = \frac{9}{0}$$

$$\lim_{x \to 3^-} \frac{3x}{(x-3)^2} = \frac{9}{0^+} = +\infty$$

tome $x = 2{,}99$

$(2{,}99-3)^2 > 0$

$$\lim_{x \to 3^+} \frac{3x}{(x-3)^2} = \frac{9}{0^+} = +\infty$$

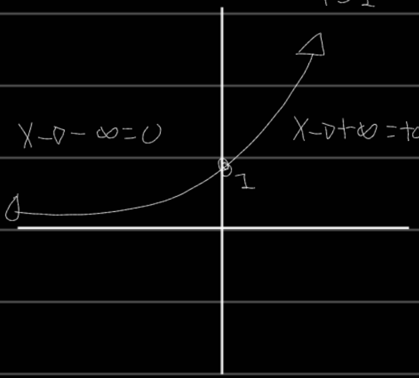
$x \to 3 \quad (x-3) \quad 0$

tome $x = 3{,}01$

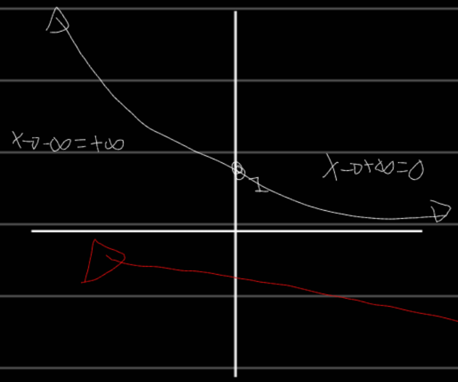
$(3{,}01-3)^2 > 0$

limites con funciones exponenciales y logaritmicas

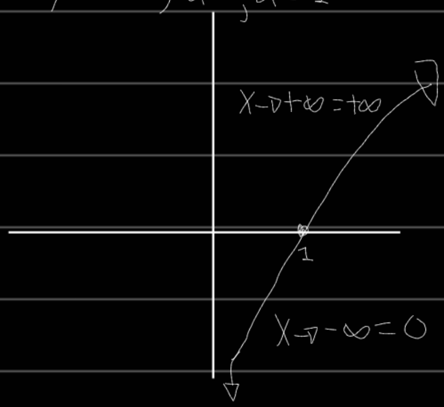
$y = a^x \quad a > 1$



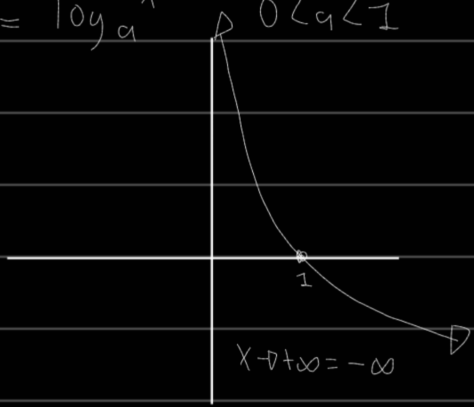
$y = a^x, \ 0 < a < 1$



$y = \log_a x, \ a > 1$



$y = \log_a x \quad 0 < a < 1$



$\lim_{x \to 0^+} \ln(x) = -\infty$ $\qquad$ $\lim_{x \to 0^+} \log_{\frac{1}{2}}(x) = +\infty$

$\lim_{x \to 1} \ln(x) = 0$ $\qquad$ $\lim_{x \to 1} \log_{\frac{1}{2}}(x) = 0$

$\lim_{x \to +\infty} \ln(x) = +\infty$ $\qquad$ $\lim_{x \to +\infty} \log_{\frac{1}{2}}(x) = -\infty$

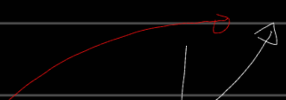
Por que se estabiliza en 0

Ejemplo $\lim_{x \to +\infty} (e^{-x}+1) = e^{-(+\infty)} + 1 = \cancel{e^{-\infty}}^{=0} + 1$

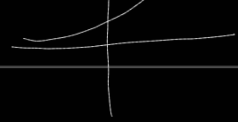
$= 1$

Ejemplo $\lim_{x \to 2} 3^{\frac{2}{2-x}} = 3^{\frac{2}{0}}$

NO se cumple
$0 < 3 < 1$



$$\lim_{x \to 2^-} \frac{\frac{2}{2-x}}{3} = \frac{\frac{2}{0^+}}{3} = \frac{+\infty}{3} \quad \boxed{= +\infty}$$



tome $x = 1{,}99$

$2 - 1{,}99 > 0$

$\nexists$

$$\lim_{x \to 2^-} \frac{\frac{2}{2-x}}{3} = \frac{\frac{2}{0^-}}{3} = \frac{-\infty}{3} \quad \boxed{= 0}$$

tome $x = 2{,}01$

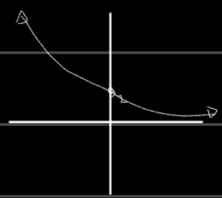
$2 - 2{,}01 < 0$

Se cumple

$0 < \frac{1}{4} < 1$

Tarea

$$\lim_{x \to -1} \left(\frac{1}{4}\right)^{\frac{2x}{x+1}} = \left(\frac{1}{4}\right)^{\frac{-2}{0}}$$



$$\lim_{x \to -1^-} \left(\frac{1}{4}\right)^{\frac{x}{x+1}} = \left(\frac{1}{4}\right)^{\frac{-2}{0^-}} = \left(\frac{1}{4}\right)^{+\infty} \boxed{= 0}$$

tome $x = -1{,}01$

$-1{,}01 + 1 < 0$

$\nexists$

$$\lim_{x \to -1^+} \left(\frac{1}{4}\right)^{\frac{x}{x+1}} = \left(\frac{1}{4}\right)^{\frac{-2}{0^+}} = \left(\frac{1}{4}\right)^{-\infty} \boxed{= +\infty}$$

tome $x = -0{,}99$

$-0{,}99 + 1 > 0$