Derivacion de orden superior es la derivada de una derivada

Ejemplo $y = e^{3x} \cos(x)$, verifique

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 10 = 0$$

$$= f''(x) - 6f'(x) + 10f(x) = 0$$

$$f(x) = y = e^{3x} \cos(x)$$

$$\left[e^{3x} \cdot \cos(x)\right]' = \left[e^{3x}\right]' \cdot \cos(x) + \left[\cos(x)\right]' \cdot e^{3x}$$

$$f'(x) = 3 \cdot e^{3x} \cdot \cos(x) - \operatorname{sen}(x) \cdot e^{3x}$$

$$\left[3 \cdot e^{3x} \cdot \cos(x) - \operatorname{sen}(x) \cdot e^{3x}\right]'$$

$$\left[3e^{3x}\right]' \cdot \cos(x) + 3e^{3x} \cdot \left[\cos(x)\right]' - \left[\left[e^{3x}\right]' (x) + e^{3x} \cdot \left[\operatorname{sen}(x)\right]'\right]$$

$$3e^{3x} \cdot 3 \cdot \cos(x) + 3e^{3x} \cdot -\operatorname{sen}(x) - \left[e^{3x} \cdot 3 \cdot \operatorname{sen}(x) + e^{3x} \cdot \cos(x)\right]$$

$$9e^{3x}\cos(x) - 3e^{3x}\operatorname{sen}(x) - 3e^{3x}\operatorname{sen}(x) - e^{3x}\cos(x)$$

$$f''(x) = 8e^{3x}\cos(x) - 6e^{3x}\operatorname{sen}(x)$$

$$f''(x) - 6f'(x) + 10f(x) = 0$$

$$8e^{3x}\cos(x) - 9e^{3x}\operatorname{sen}(x) - 6\left(3 \cdot e^{3x} \cdot \cos(x) - \operatorname{sen}(x) \cdot e^{3x}\right) + 10\left(e^{3x} \cos(x)\right)$$

$$\cancel{8e^{3x}\cos(x)} \cancel{- 6e^{3x}\operatorname{sen}(x)} - \cancel{18e^{3x}\cos(x)} - \cancel{6\operatorname{sen}(x) \cdot e^{3x}} + \cancel{10e^{3x}\cos(x)}$$

$$\boxed{= 0}$$

Derivacion logaritmica

$$\log_a x^n = n \log_a(x)$$

$$\log_a(x \cdot y) = \log_a(x) + \log_a(y)$$

$$\log\left(\frac{x}{\ }\right) = \log$$

$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

Util en { productos — $h(x) = f(x) \cdot g(x)$
cocientes — $h(x) = \frac{f(x)}{g(x)}$ } $\Rightarrow h(x) = \frac{f(x) \cdot g(x)}{s(x)}$

Exponenciales

$\hookrightarrow h(x) = f(x)^{g(x)}$

De esta forma $s(x)$ es mas optimo $= f(x) \cdot g(x) - s(x)$

Objetivo: Transformar la funcion original usando props de logs y luego derivar

Ejemplo

Calcular $f(x) = \frac{(5x-1)^4 \cdot x^{sen(2x)}}{\sqrt[3]{x^2+4}}$

$$\ln(f(x)) = \ln\left(\frac{(5x-1)^4 \cdot x^{sen(2x)}}{(x^2+4)^{\frac{1}{3}}}\right)$$

$$\ln(f(x)) = \ln[(5x-1)^4] + \ln[x^{sen(2x)}] - \ln(x^2+4)^{\frac{1}{3}}$$

$$[\ln(f(x))]' = 4\ln(5x-1) + sen(2x)\ln(x) - \frac{1}{3}\ln(x^2+4)$$

$f'(x) = 2$ $\quad$ $f'(x) = \cos(2x)$

$[sen[f(x)]]' = \cos[f(x)]f'(x)$

$$\frac{1}{f(x)} \cdot f'(x) = 4 \cdot \frac{1}{5x-1} \cdot 5 + 2\cos(2x) \cdot \ln(x) + sen(2x)\frac{1}{x} - \frac{1}{3} \cdot \frac{1}{x^2+4} \cdot 2x$$

$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$

$$\frac{f'(x)}{f(x)} = \frac{20}{5x-1} + 2\cos(2x) \cdot \ln(x) + \frac{sen(2x)}{x} - \frac{2x}{3(x^2+4)}$$

$$f'(x) = f(x)\left(\frac{20}{5x-1} + 2\cos(2x) \cdot \ln(x) + \frac{sen(2x)}{x} - \frac{2x}{3(x^2+4)}\right)$$

$$f'(x) = \left(\frac{(5x-1)^4 \cdot x^{sen(2x)}}{\dots}\right)\left(\frac{20}{\dots} + 2\cos(2x) \cdot \ln(x) + \frac{sen(2x)}{\dots} - \frac{2x}{\dots}\right)$$

$$\left(\sqrt[3]{x^2+4}\right)\left(5x-1 \qquad x \qquad 3(x^2+4)\right)$$

Calcule $f'(x)$

$$f(x) = (x+\tan(x))^{x^2+3x}$$

$$\ln(f(x)) = \ln(x+\tan(x))^{x^2+3x}$$

$$\ln(f(x)) = (x^2+3x)\ln(x+\tan(x))$$

$$\ln(f(x)) = 2x+3x \cdot \ln(x+\tan(x) + x^2+3x \cdot \frac{1}{x+\tan(x)} \cdot 1+\sec^2(x)$$

$$\ln(f(x)) = (2x+3x) \cdot \ln(x+\tan(x)) + \frac{(x^2+3x)\cdot(1+\sec^2(x)}{x+\tan(x)}$$

$$\frac{f'(x)}{f(x)} = (2x+3x) \cdot \ln(x+\tan(x)) + \frac{(x^2+3x)\cdot(1+\sec^2(x)}{x+\tan(x)}$$

$$f'(x) = \left((x+\tan(x))^{x^2+3x}\right)\left((2x+3x) \cdot \ln(x+\tan(x)) + \frac{(x^2+3x)\cdot(1+\sec^2(x)}{x+\tan(x)}\right)$$

Derivacion implicita

Utilizada cuando la $\underbrace{\text{variable dependiente}}_{y}$ no puede ser despejada

$y = f(x) \to$ Explicita

$f(x,y)=0 \to$ Implicita

v dependiente (y) despejada

$xy^2+\operatorname{sen}(x)\cdot e^y = x^2-3$

No se puede despejar y

No se puedo o no esta despejada

Ejemplo

$x^2+y^2 = 9 \to$ Circulo de radio 3 (Graficado)

Forma implicita

$y = e^x \rightarrow$ Explicita

$y - e^x = 0 \rightarrow$ Implicita

Calcular $f'(x)$ con

$f(x) = xy^2 + sen(x) \cdot e^y = x^2 - 3$

Se derivan ambos lados

$[xy^2 + sen(x) \cdot e^y]' = [x^2 - 3]'$

Cada vez que se deriva "y", se usa regla de la cadena

pues y es una funcion sobre la cual no se

conoce su ecuacion en terminos de x

$y = f(x)$

$[xy^2 + sen(x) \cdot e^y]' = [x^2 - 3]'$

$1 \cdot y^2 + x(2y \cdot y') + cos(x) \cdot e^y + sen(x) \cdot (e^y \cdot y') = 2x - 0$

$[(f(x))^n]' = n[f(x)]^{n-1} f'(x)$

$[e^{f(x)}]' = e^{f(x)} f'(x)$

$1 \cdot y^2 + x \cdot 2y \cdot y' + cos(x) \cdot e^y + sen(x) \cdot e^y \cdot y' = 2x - 0$

$2xyy' + sen(x) \cdot e^y \cdot y' = 2x - y^2 + cos(x) \cdot e^y$

$y'(2xy + sen(x) \cdot e^y) = 2x - y^2 + cos(x) \cdot e^y$

$$\boxed{y' = \frac{2x - y^2 + cos(x) \cdot e^y}{2xy + sen(x) \cdot e^y}}$$

Ejemplo

$[\arctan[f(x)]]' = \frac{f'(x)}{1+f(x)^2}$ $\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ $[\ln f(x)]' = \frac{1}{f(x)} f'(x)$ $\left[\sqrt{f(x)}\right]' = \frac{1}{2\sqrt{f(x)}} f'(x)$

$\arctan\left(\frac{y}{x}\right) = \ln\left(\sqrt{x^2+y^2}\right)$, $\frac{dy}{dx}$

$$\frac{\frac{y' \cdot x - y \cdot 1}{x^2}}{1 + \left(\frac{y}{x}\right)^2} = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x + 2y \cdot y'$$

$y' \cdot x - y \cdot 1$

$$\frac{\cancel{x^2}}{\dfrac{x^2+y^2}{\cancel{x^2}}} = \frac{1}{2\left(\sqrt{x^2+y^2}\right)^2} \cdot 2x+2y\cdot y'$$

$$\frac{xy'-y}{x^2+y^2} = \frac{2x+2y\cdot y'}{2(x^2+y^2)}$$

$$\frac{xy'-y}{x^2+y^2} = \frac{\cancel{2}(x+y\cdot y'}{\cancel{2}(x^2+y^2)}$$

$$\frac{xy'-y}{x^2+y^2} = \frac{x+y\cdot y'}{x^2+y^2}$$

$$xy'-y = \frac{x+y\cdot y'}{\cancel{x^2+y^2}} \cancel{x^2+y^2}$$

$$xy'-y = x+y\cdot y'$$

$$xy'-y\cdot y' = x+y$$

$$y'(x-y) = x+y$$

$$\boxed{y' = \frac{x+y}{x-y}}$$