Induccion matematica

1) $n = 1$

2) $n = p \leftarrow$ Hipotesis de induccion

3) $n = p+1$

(a) [*] $1 + 4 + 7 + \cdots + (3n-2) = \dfrac{3n^2 - n}{2}$

Se asume cierto para $n=1$

$$3 \cdot 1 - 2 = \frac{3 \cdot 1^2 - 1}{2}$$

$$1 = 1 \quad \checkmark$$

Se asume cierto para $n=p$

$$1+4+7+\ldots+(3p-2) = \frac{3p^2-p}{2}, \quad H.I.$$

Se asume cierto para $n=p+1$

$$1+4+7+\ldots+3(p+1)-2 = \frac{3(p+1)^2-(p+1)}{2}$$

$\hookrightarrow$ Respuesta

$$1+4+7+\dots+3(p+1)-2 = \frac{3(p+1)^2-(p+1)}{2}$$

$$3p+3-2$$

$$3p+1$$

$$(a+b)^2 = a^2+2ab+b^2$$

$$\frac{3(p+1)^2-(p+1)}{2}$$

$$\frac{3(p^2+2p+1^2)-p-1}{2}$$

$$\frac{3p^2+6p+3-p-1}{2} = \frac{3p^2+5p+2}{2}$$

Demostracion

$$1+4+7+\dots+(3p-2)+3(p+1)-2$$

$$1+4+7+\dots+(3p-2)+3p+3-2$$

$$\boxed{1+4+7+\dots+(3p-2)}+3p+1$$

$$\frac{3p^2-p}{2}+3p+1\cdot\frac{2}{2} \qquad \frac{a}{b}+c\cdot\frac{b}{b}$$

$$\frac{3p^2-p}{2}+\frac{(3p+1)\cdot 2}{2} \qquad \frac{a+cb}{b}$$

$$\frac{3p^2-p}{2}+\frac{(3p+1)\cdot 2}{2}$$

$$\frac{3p^2-p+6p+2}{2}$$

$$\frac{3p^2+5p+2}{2}$$

$$\frac{3(p+1)^2-(p+1)}{2}$$

(b) [*] $\frac{2}{3^2} + \frac{2^2}{3^3} + \frac{2^3}{3^4} + \cdots + \frac{2^{n-1}}{3^n} = \frac{2}{3} - \left(\frac{2}{3}\right)^n.$

$$\sum_{i=2}^{n} \frac{2^{n-1}}{3^n} = \frac{2}{3} - \left(\frac{2}{3}\right)^n$$

$$n = 2 \qquad \frac{2^{2-1}}{3^2} = \frac{2}{3} - \left(\frac{2}{3}\right)^2$$

$$\frac{2}{9} = \frac{2}{9} \checkmark$$

$$n = p \qquad \sum_{i=2}^{p} \frac{2^{p-1}}{3^p} = \frac{2}{3} - \left(\frac{2}{3}\right)^p, \text{ H.I.}$$

$$p+1-1 = p$$

$$n = p+1 \qquad \sum_{i=2}^{p+1} \frac{2^{(p+1)-1}}{3^{p+1}} = \frac{2}{3} - \left(\frac{2}{3}\right)^{p+1}$$

$$n = p+1 \qquad \sum_{i=2}^{p+1} \frac{2^{p}}{3^{p+1}} = \frac{2}{3} - \left(\frac{2}{3}\right)^{p+1}.$$

H.Q.D

$$\sum_{i=k}^{p+1} \frac{n}{n+1} \qquad \sum_{i=k}^{p} \frac{n}{n+1} + \frac{n+1}{(n+1)+1}$$

$$\sum_{i=k}^{p+1} n = \sum_{i=k}^{p} n + (n+1)$$

Demostracion

$$\sum_{i=2}^{p} \frac{2^{p-1}}{3^p} + \frac{2^p}{3^{p+1}} \qquad \sum_{i=2}^{p+1} \frac{2^{p-1}}{3^p}$$

$$\sum_{i=2}^{p} \frac{2^{p-1}}{3^p} + \frac{2^p}{3^{p+1}}$$

$$\frac{2}{3} - \left(\frac{2}{3}\right)^p + \frac{2^p}{3^{p+1}} \qquad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$\frac{2}{3} - \frac{2^p}{3^p} + \frac{2^p}{3^{p+1}}$$

$$3^p \cdot 3^1 = 3^{p+1}$$

$$\frac{2}{3} - \frac{2^p}{3^p} + \frac{2^p}{3^p \cdot 3}$$

$$\frac{2}{3} - \frac{2^p}{3^p} + \frac{2^p}{3^p \cdot 3}$$

$$\frac{2}{3} - \left(\frac{2^p}{3^p} + \frac{2^p}{3^p} \cdot \frac{1}{3} \right)$$

$$\frac{2}{3} - \frac{2^p}{3^p} \left(1 - \frac{1}{3} \right)$$

$$x = \frac{2}{3}$$

$$\frac{2}{3} - \left(\frac{2}{3}\right)^p \cdot \left(\frac{2}{3}\right)^1$$

$$x^p \cdot x^1 = x^{p+1}$$

$$\frac{2}{3} - \left(\frac{2}{3}\right)^{p+1}$$

$$\left(\frac{2}{3}\right)^{p+1}$$

$n! = n(n-1)!$

$n(n-1) \cdot (n-2) \cdot (n-3)!$

(d) [*] $\displaystyle\sum_{k=1}^{n} \frac{k-(k-1)^2}{k!} = 1 + \frac{n-1}{n!}$

$n = 1$ $\qquad \dfrac{1 - \cancel{(1-1)^2}\, 0}{1!} = 1 + \cancel{\dfrac{1-1}{1!}}\ 0$

$1 = 1 \checkmark$

$n = p$ $\qquad \displaystyle\sum_{k=1}^{p} \frac{k-(k-1)^2}{k!} = 1 + \frac{p-1}{p!}$, Hi

$n = p+1$ $\qquad \displaystyle\sum_{k=1}^{p+1} \frac{k-(k-1)^2}{k!} = 1 + \frac{p+1-1}{(p+1)!}$

$$= 1 + \frac{p}{(p+1)!}$$

Demostracion

$$\frac{k-(k-1)^2}{k!}$$

$$\sum_{k=1}^{p+1} \frac{k-(k-1)^2}{k!}$$

$$\frac{(p+1)-(p+1-1)^2}{(p+1)!}$$

$$\sum_{k=1}^{p} \frac{k-(k-1)^2}{k!} + \frac{(p+1)-(p\cancel{+1-1})^2}{(p+1)!}$$

$p+1$

$$1+\frac{p-1}{p!}+\frac{p+1-p^2}{(p+1)!}$$

$$\frac{D}{D}\cdot\frac{a}{b}+\frac{c}{D}\cdot\frac{b}{b}$$

$$\frac{D\cdot a+cb}{D\cdot b}$$

$$1+\frac{p-1}{p!}+\frac{p+1-p^2}{(p+1)\cdot p!}$$

$$(p+1)! = (p+1)\cdot(p+1-1)!$$
$$= (p+1)\cdot p!$$

$$1+\frac{(p+1)}{p+1}\frac{p-1}{p!}+\frac{p+1-p^2}{(p+1)\cdot p!}$$

$$a^2-b^2$$
$$=(a-b)(a+b)$$

$$1+\frac{(p+1)(p-1)+p+1-p^2}{(p+1)\cdot p!}$$

$$p^2-1^2$$

$$1+\frac{p^2-1^2+p+1-p^2}{(p+1)\cdot p!}$$

$$(p+1)(p-1)$$

$$1 + \frac{\cancel{p^2} \cancel{- 1^2} + p \cancel{+ 1} \cancel{- p^2}}{(p+1)\cdot p!}$$

$$\frac{p+2!}{(p+2)\cdot(p+1)\cdot p!}$$

$$(p+1)! = (p+1)\cdot p!$$

$$1 + \frac{p}{(p+1)\cdot p!}$$

$$\boxed{1 + \frac{p}{(p+1)!}}$$

Pruebe la validez, de la igualdad, mediante el principio de inducción matemática

$$1 \cdot 5 + 2 \cdot 5^2 + 3 \cdot 5^3 + \cdots + n \cdot 5^n = \frac{5 + (4n-1)5^{n+1}}{16}$$

$\forall n \in \mathbb{N}, n \geq 1.$

$n = 1 \qquad 1 \cdot 5^1 = \dfrac{5 + (4 \cdot 1 - 1) \cdot 5^{1+1}}{16}$

$5 = 5$ ✓

$n = p \quad 1 \cdot 5 + 2 \cdot 5^2 + 3 \cdot 5^3 + \ldots p \cdot 5^p = \dfrac{5 + (4p-1) \cdot 5^{p+1}}{16}$

$n = p+1 \quad 1 \cdot 5 + 2 \cdot 5^2 + 3 \cdot 5^3 + (p+1) \cdot 5^{p+1} = \dfrac{5 + (4(p+1)-1) \cdot 5^{p+1+1}}{16}$

$\to \dfrac{5 + (4p+4-1) \cdot 5^{p+2}}{16}$

$\dfrac{5 + (4p+3) \cdot 5^{p+2}}{16}$

$1 \cdot 5 + 2 \cdot 5^2 + 3 \cdot 5^3 + \ldots p \cdot 5^p + (p+1) \cdot 5^{p+1}$

$\dfrac{5 + (4p-1) \cdot 5^{p+1}}{16} + (p+1) \cdot 5^{p+1} \cdot \dfrac{16}{16} \quad \circ \; Hi$

$\dfrac{5 + (4p-1) \cdot 5^{p+1}}{16} + \dfrac{(p+1) \cdot 5^{p+1} \cdot 16}{16}$

$$\frac{5+(4p-1)\cdot 5^{p+1}}{16} + \frac{(p+1)\cdot 5^{p+1}\cdot 16}{16}$$

$$\frac{5+(4p-1)\cdot \boxed{5^{p+1}}+(p+1)\cdot \boxed{5^{p+1}}\cdot 16}{16}$$

$$\frac{5+5^{p+1}\cdot(4p-1+16\cdot(p+1))}{16}$$

$$\frac{5+5^{p+1}\cdot(4p-1+16p+16)}{16}$$

$$\frac{5+5^{p+1}\cdot(20p+15)}{16} \qquad 5^{p+1}\cdot 5^1$$

$$\frac{5+5^{p+1}(5^1(4p+3))}{16}$$

$$\frac{5+5^{p+2}(4p+3)}{16}$$

$$\frac{5+(4(p+1)-1)\cdot 5^{p+1+1}}{16} \quad \checkmark$$

$$\sum_{k=2}^{n} \frac{2k-3}{3^k} = \frac{1}{3} - \frac{n}{3^n}$$

$n=2$ $$\frac{2\cdot 2-3}{3^2} = \frac{1}{3} - \frac{2}{3^2}$$

$$\frac{1}{9} = \frac{1}{9} \checkmark$$

$n=p$ $$\sum_{k=2}^{p} \frac{2k-3}{3^k} = \frac{1}{3} - \frac{p}{3^p} \text{ . Hi}$$

$n=p+1$ $$\sum_{k=2}^{p+1} \frac{2k-3}{3^k} = \frac{1}{3} - \frac{p+1}{3^{p+1}}$$ ¿HQD

$$\sum_{k=2}^{p+1} \frac{2k-3}{3^k}$$

$$2p+2-3$$
$$2p-1$$

$$\sum_{k=2}^{p} \frac{2k-3}{3^k} + \frac{\overbrace{2(p+1)-3}}{3^{p+1}}$$

$$\sum_{k=2}^{p} \frac{2k-3}{3^k} + \frac{2p-1}{3^{p+1}}$$

$$\sum_{k=2}^{p} \frac{2k-3}{3^k} + \frac{2p-1}{3^{p+1}}$$

$$\frac{1}{3} - \frac{p}{3^p} \cdot \frac{3}{3} + \frac{2p-1}{3^{p+1}}$$

$$\frac{1}{3} - \frac{3p}{3^{p+1}} + \frac{2p-1}{3^{p+1}}$$

$$\frac{1}{3} + \frac{-3p + 2p - 1}{3^{p+1}}$$

$$\frac{1}{3} - \frac{p+1}{3^{p+1}}$$