Series de potencias

Radio e intervalo de convergencia

Determine el intervalo de convergencia y el radio de convergencia para las siguientes series:

1. $\sum_{n=0}^{\infty}(x-3)^n$ $\quad r=1,\ I=]2,4[$

2. $\sum_{m=0}^{\infty}(4x+6)^m$ $\quad r=\frac{1}{4},\ I=\left]\frac{-7}{4},\frac{-5}{4}\right[$

3. $\sum_{n=0}^{\infty}n^n\cdot(1-x)^n$ $\quad r=0,\ I=\{1\}$

4. $\sum_{k=0}^{\infty}[-7(3-2x)]^k$ $\quad r=\frac{1}{14},\ I=\left]\frac{10}{7},\frac{11}{7}\right[$

5. $\sum_{n=1}^{\infty}[1-(-2)^n]\cdot x^n$ $\quad r=\frac{1}{2},\ I=\left]\frac{-1}{2},\frac{1}{2}\right[$

6. $\sum_{k=0}^{\infty}\frac{k}{k+1}\cdot(x-2)^k$ $\quad r=1,\ I=]1,3[$

7. $\sum_{k=0}^{\infty}\frac{(2y+4)^k}{k!}$ $\quad r=\infty,\ I=\mathbb{R}$

8. $\sum_{n=0}^{\infty}\frac{(t+1)^n}{n+1}$ $\quad r=1,\ I=[-2,0[$

9. $\sum_{n=0}^{\infty}\frac{(-1)^n\cdot(2x-1)^n}{n+1}$

10. $\sum_{n=0}^{\infty}(2-x)^{3n}\cdot\frac{n!}{2^n}$ $\quad r=0,\ I=\{2\}$

11. $\sum_{n=1}^{\infty}\frac{3^n\cdot(x-1)^n}{n}$ $\quad r=\frac{1}{3},\ I=\left[\frac{2}{3},\frac{4}{3}\right[$

12. $\sum_{n=0}^{\infty}\frac{3^n\cdot(x-1)^n}{(n+1)^{2n}}$ $\quad r=\infty,\ I=\mathbb{R}$

13. $\sum_{n=1}^{\infty}\frac{(x+5)^n}{2n\cdot 4^n}$ $\quad r=4,\ I=[-9,-1[$

14. $\sum_{n=1}^{\infty}\frac{(x-4)^n}{n\cdot 5^n}$

15. $\sum_{n=1}^{\infty}(-1)^{n-1}\cdot\frac{(x-5)^n}{n\cdot 3^n}$ $\quad$ R/ $]2,8[$

16. $\sum_{n=1}^{\infty}\frac{(x+3)^n}{(n+1)\cdot 2^n}$

17. $\sum_{n=1}^{\infty}\frac{(x-2)^n}{(n+1)\cdot 3^n}$

18. $\sum_{n=1}^{\infty}\frac{(3-2x)^{3n}}{n\cdot 8^n}$

19. $\sum_{n=1}^{\infty}\frac{(-1)^n}{2^n\cdot n}\cdot(x-1)^n$

20. $\sum_{n=0}^{\infty}\frac{n!\cdot(x+1)^n}{n+1}$ $\quad r=0,\ I=\{-1\}$

21. $\sum_{n=1}^{\infty}\frac{5^n\cdot(2-3x)^n}{n}$ $\quad r=\frac{1}{15},\ I=\left]\frac{3}{5},\frac{11}{15}\right[$

22. $\sum_{n=0}^{\infty}\frac{3^n\cdot n!\cdot(x+1)^n}{(n+1)!}$ $\quad r=\frac{1}{3},\ I=\left[\frac{-4}{3},\frac{-2}{3}\right[$

23. $\sum_{n=0}^{\infty}\frac{n\cdot(x+3)^n}{4n+1}$ $\quad r=1,\ I=]-4,-2[$

24. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n \cdot x^{2n+1}}{n+1}$ $\quad r = \frac{\sqrt{2}}{2},\ I = \left]\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right[$

25. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n - 7^n}{3^n + 7^n} \cdot (4x-5)^{2n}$ $\quad r = \frac{1}{4},\ I = \left]1, \frac{3}{2}\right[$

26. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!}$ $\quad r = \infty,\ I = \mathbb{R}$

27. $\displaystyle\sum_{j=0}^{\infty} \frac{(-1)^{j+1} \cdot (4t^2-1)^j}{(1-2t)^j \cdot (1+2j)!}$ $\quad r = \infty,\ I = \mathbb{R}$

28. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{n+1} \cdot (x-1)^n}{8^n}$ $\quad r = \frac{8}{3},\ I = \left]\frac{-5}{3}, \frac{4}{3}\right[$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot x^{2n-1}}{(2n-1)!}$ $\quad r = +\infty,\ I = \mathbb{R}$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{n \cdot (x+3)^n}{3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n+1)}$ $\quad r = +\infty,\ I = \mathbb{R}$

31. $\displaystyle\sum_{k=1}^{\infty} \frac{k!(1-2x)^k}{1 \cdot 4 \cdot 7 \cdot \ldots \cdot (3k-2)}$

32. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{(2x+1)^n}{n\sqrt{n}}$

33. $\displaystyle\sum_{n=2}^{\infty} \frac{(n-2)! \cdot (x-3)^n}{3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n+1)}$

34. $\displaystyle\sum_{n=1}^{\infty} \frac{n! \cdot (x-1)^n}{3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n+1)}$ $\quad$ R/ $]-1, 3[$

35. $\displaystyle\sum_{k=1}^{\infty} \frac{(k+3)! \cdot 3^k \cdot (2x-1)^k}{5 \cdot 7 \cdot 9 \cdot \ldots \cdot (2k+3)}$

36. $\displaystyle\sum_{m=2}^{\infty} \frac{(m-2)! \cdot (2-3x)^m}{3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2m-1)}$

37. $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1) \cdot (x+3)^n}{(n+1)!}$

38. $\displaystyle\sum_{n=0}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n+2) \cdot (x-3)^n}{(n+1)!}$ $\quad$ R/ $\left]\frac{5}{2}, \frac{7}{2}\right[$

39. $\displaystyle\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}{n!} \cdot (x-2)^{n-1}$

40. $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1) \cdot (x+3)^n}{(n+1)!}$

41. $\displaystyle\sum_{n=1}^{\infty} \frac{(2t+5)^{3n}}{\sqrt{n}}$

42. $\displaystyle\sum_{n=1}^{\infty} \frac{(2t-3)^n}{2^n \cdot \sqrt{n}}$

43. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3^n(n+2)}$

44. $\displaystyle\sum_{k=1}^{\infty} \frac{(x-2)^k}{2^{k+1}}$

45. $\displaystyle\sum_{n=1}^{\infty} \frac{4^n \cdot (n+2)! \cdot (x-3)^n}{5 \cdot 8 \cdot 11 \cdot \ldots \cdot (3n+2)}$ $\quad$ R/ $\left]\frac{9}{4}, \frac{15}{4}\right[$

46. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n \cdot (x+2)^n}{n+5}$

47. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+2)^n \cdot (x-1)^n}{3^{2n} \cdot n^n}$ $\quad$ R/ $]-8, 10[$

48. $\displaystyle\sum_{n=2}^{\infty} \left[\frac{(x+4) \cdot 3\ln(n+1)}{\ln(n)}\right]^n$ $\quad$ R/ $\left]\frac{-13}{3}, \frac{-11}{3}\right[$

49. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 \cdot (x-3)^n}{10^n}$

50. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3n+1} \cdot \left(x + \frac{1}{2}\right)^n$

51. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^{2n}}{(n+1) \cdot \ln(n+1)}$

52. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k \cdot (3x-1)^k}{k \cdot 2^k}$

53. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1} \cdot (-2x)^{n-1}$

Ejercicios especiales

1. Sabiendo que la serie $\displaystyle\sum_{k=2}^{\infty} \frac{2k \cdot (x+1)^k}{3^k\left(3k^2-4\right)}$ es absolutamente convergente en $]-4,2[$, determine si es convergente o divergente para $x=-4$

2. Suponiendo que $\displaystyle\lim_{x \to +\infty} \frac{\ln(x+1)}{\ln(x)} = 1$, halle el intervalo de convergencia de $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{4^n} \cdot (x+2)^n$. Debe estudiar los extremos de dicho intervalo. R/ $]-6,2[$

3. Determine el radio de convergencia de la serie $\displaystyle\sum_{n=0}^{\infty} \frac{n\,(2x-1)^n}{4^n}$ R/ $r=4$