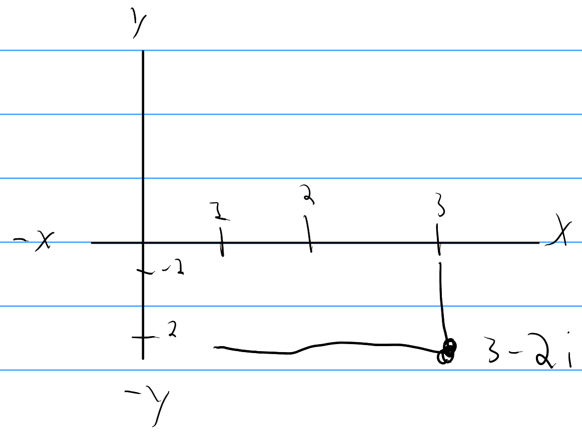
Rectangular $z = a + bi$

Real Imaginario



$z = 3 - 2i$

x y

$x = 3$ $y = -2$

Forma polar $r \cdot cis(\theta)$ $z = a + bi$

$r = |z|$, $|z| = \sqrt{a^2 + b^2}$

Tener la calculadora en Grados

$\theta = \arctan\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{b}{a}\right)$

$cis = \cos(\theta) + i\,sen(\theta)$

1) $z = 3 - 2i$ a forma polar $r\,cis(\theta)$

$R(z) = 3$ $Img(z) = -2$

$z = r\,cis(\theta)$

$\sqrt{3^2 + (-2)^2}$ $\arctan\left(\frac{-2}{3}\right)$

$\boxed{3,60 \cdot cis(-33,69)}$ ← Forma polar

De Polar a rectangular

$z = 3{,}60 \cdot cis(-33{,}69)$, $cis(\theta) = cos(\theta) + i\,sen(\theta)$

$3{,}60(cos(-33{,}69) + i\,sen(-33{,}69))$

$3{,}60(0{,}83 - 0{,}55i)$

$\boxed{3 - 2i}$ ← Rectangular

Forma exponencial , TIENE QUE ESTAR EN RADIANES

Rectangular $= a + bi$

Polar $= r\,cis(\theta)$

Exponencial $= r \cdot e^{i\theta}$

$z = 3 - 2i$ ← Rectangular

$r = \sqrt{3^2 + (-2)^2} = 3{,}60$

$\theta = arctan\left(\frac{-2}{3}\right) = -0{,}58$

$3{,}60 \cdot e^{-0{,}58i}$

$z = \underset{a}{7} + \underset{b}{4}i \leftarrow$ Rect

$$z = r \, \text{cis}(\theta)$$

$$r = \sqrt{a^2 + b^2} \qquad \theta = \arctan\left(\frac{b}{a}\right)$$

De rect a polar Grados

$r = \sqrt{7^2 + 4^2} = \sqrt{65}$

$\theta = \tan^{-1}\left(\frac{4}{7}\right) = 29{,}74$

$z = \sqrt{65} \cdot \text{cis}(29{,}74) \leftarrow$ Polar

De rect a expo Radianes

$z = 7 + 4i \qquad r \cdot e^{i\theta}$

$r = \sqrt{7^2 + 4^2} = \sqrt{65}$

$\theta = \tan^{-1}\left(\frac{4}{7}\right) = 0{,}52$

$z = \sqrt{65} \cdot e^{0{,}52 i} \leftarrow$ Exponencial

Teorema o formula de moivre

$z_1 \cdot z_2$, ambos en forma polar o exponencial

$$\left.\begin{array}{l} r_1 \cdot cis(\theta_1) \cdot r_2\, cis(\theta_2) \\ r_1 \cdot r_2 \cdot cis(\theta_1 + \theta_2) \end{array}\right\} \text{Polar}$$

$$\left.\begin{array}{l} r_1 \cdot e^{i\theta_1} \cdot r_2 \cdot e^{i\theta_2} \\ r_1 \cdot r_2 \cdot e^{i(\theta_1+\theta_2)} \end{array}\right\} \text{Exponencial}$$

Sumas en la multiplicacion

$$\left.\begin{array}{l} \dfrac{r_1 \cdot cis(\theta_1)}{r_2 \cdot cis(\theta_2)} \\ = \dfrac{r_1}{r_2} \cdot cis(\theta_1 - \theta_2) \end{array}\right\} \text{Polar}$$

$$\left.\begin{array}{l} \dfrac{r_1 \cdot e^{i\theta_1}}{r_2 \cdot e^{i\theta_2}} \\ = \dfrac{r_1}{r_2} \cdot e^{(\theta_1-\theta_2)} \end{array}\right\} \text{Exponencia}$$

restas en la division

$$\left.\begin{array}{l} (r\, cis(\theta))^n \\ r^n \cdot cis(\theta \cdot n) \end{array}\right\} \text{Polar}$$

$$\left.\begin{array}{l} (r \cdot e^{i\theta})^n \\ r^n \cdot e^{i\theta n} \end{array}\right\} \text{Exponencial}$$

Exponente r se eleva y cita se multiplica

Exprese esto en forma rectangular

$$\frac{(1-i)^6(1-i\sqrt{3})}{(2i-2\sqrt{3})^7}$$

Notas

1) Pasar todo a forma polar

2) Dependiendo sumar o restar π al angulo θ dependiendo del cuadrante

Cuadrante	a	b	Rango	Que hago?
I	+	+	Ya esta	Nada
II	−	+	Esta fuera	$+\pi$ al θ
III	−	−	Esta fuera	$-\pi$ al θ
IV	+	−	Ya esta	Nada

$\theta = \frac{\pi}{2}$ si $a=0$ y $b>0 \rightarrow$ Positivo

$\theta = \frac{-\pi}{2}$ si $a=0$ y $b<0 \rightarrow$ Negativo

$$\frac{\overbrace{(1-i)}^{z_1}{}^6\,\overbrace{(1-i\sqrt{3})}^{z_2}{}^3}{\underbrace{(2i-2\sqrt{3})}_{z_3}{}^7}$$

$z_1 = 1 - i$ $\quad a=1 \quad b=-1$

$r = \sqrt{1^2+(-1)^2} = \sqrt{2}$

$\theta = \arctan\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$

$z_1 = \sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$ ← Polar

$z_2 = 1 - i\sqrt{3}$ $\quad a=1 \quad b=-\sqrt{3}$

$r = \sqrt{1^2+(-\sqrt{3})^2} = 2$

$\theta = \arctan\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$

$z_2 = 2\cdot\operatorname{cis}\left(-\frac{\pi}{3}\right)$

$z_3 = -2\sqrt{3} + 2i$ $\quad a=-2\sqrt{3} \quad b=2$ $\quad \Delta$

$r = \sqrt{(-2\sqrt{3})^2+2^2} = 4$ ↙ Afuera del $\arctan\left(\frac{b}{a}\right)$

$\theta = \arctan\left(\frac{2}{-2\sqrt{3}}\right) + \pi$

$\theta = \frac{5\pi}{6}$

$z_3 = 4\cdot\operatorname{cis}\left(\frac{5\pi}{6}\right)$

$$\frac{\overbrace{(1-i)}^{z_1}{}^{6}\,\overbrace{(1-i\sqrt{3})}^{z_2}{}^{3}}{\underbrace{(2i-2\sqrt{3})}_{z_3}{}^{7}}$$

$z_1 = \sqrt{2}\ \text{cis}\left(-\frac{\pi}{4}\right)$

$z_2 = 2 \cdot \text{cis}\left(-\frac{\pi}{3}\right)$

$z_3 = 4 \cdot \text{cis}\left(\frac{5\pi}{6}\right)$

$$\frac{\left(\sqrt{2}\ \text{cis}\left(\frac{-\pi}{4}\right)\right)^{6} \cdot \left(2\ \text{cis}\left(\frac{-\pi}{3}\right)\right)^{3}}{\left(4\ \text{cis}\left(\frac{5\pi}{6}\right)\right)^{7}}$$

$\left.\begin{array}{l}(r\ \text{cis}(\theta))^{n} \\ r^{n} \cdot \text{cis}(\theta \cdot n)\end{array}\right\}$ Polar

$$\frac{\overset{\downarrow r}{(\sqrt{2})^{6}} \cdot \text{cis}\left(\frac{-\pi}{4} \cdot 6\right) \cdot \overset{\downarrow r}{2^{3}} \cdot \text{cis}\left(\frac{-\pi}{\cancel{3}} \cdot \cancel{3}\right)}{r \to 4^{7} \cdot \text{cis}\left(\frac{5\pi}{6} \cdot 7\right)}$$

$\frac{r_1}{r_2} \cdot \text{cis}(\theta_1 - \theta_2)$

$$\frac{2^{3} \cdot 2^{3}}{2^{14}} \cdot \frac{\text{cis}\left(\frac{-6\pi}{4}\right) \cdot \text{cis}(-\pi)}{\text{cis}\left(\frac{35\pi}{6}\right)}$$

$r_1 \cdot \text{cis}(\theta_1) \cdot r_2\ \text{cis}(\theta_2)$

$r_1 \cdot r_2 \cdot \text{cis}(\theta_1 + \theta_2)$

$$\frac{1}{2^{8}} \cdot \frac{\text{cis}\left(\frac{-6\pi}{4} + -\pi\right)}{\text{cis}\left(\frac{35\pi}{6}\right)}$$

$$\frac{1}{2^{8}} \cdot \frac{\text{cis}\left(\frac{-5\pi}{2}\right)}{\text{cis}\left(\frac{35\pi}{6}\right)}$$

$\frac{r_1 \cdot \text{cis}(\theta_1)}{r_2 \cdot \text{cis}(\theta_2)} = \frac{r_1}{r_2} \cdot \text{cis}(\theta_1 - \theta_2)$

$$\frac{1}{2^{8}} \cdot \text{cis}\left(\frac{-5\pi}{2} - \frac{35\pi}{6}\right)$$

$$\frac{1}{2^{8}} \cdot \text{cis}\left(\frac{-25\pi}{3}\right)$$

$r\ \text{cis}(\theta) \leftarrow$ Polar

$$\frac{1}{2^{8}} \cdot \left[\cos\left(\frac{-25\pi}{3}\right) + i\ \text{sen}\left(\frac{-25}{3}\right)\right]$$

$$\boxed{\frac{1}{2^{8}} \cos\left(\frac{-25}{3}\right) + \frac{1}{2^{8}}\ \text{sen}\left(\frac{-25}{3}\right) i}$$

Recordar

Rect = $a+bi$

Polar = $r\,\mathrm{cis}(\theta)$ → $r = \sqrt{a^2+b^2}$

→ $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ RADIANES

Nota Para $z = a+bi$ al calcular θ

$\theta = \frac{\pi}{2}$ si $a = 0 \wedge b > 1$

$\theta = \frac{-\pi}{2}$ si $a = 0 \wedge b < 1$

Raices Complejas → Se usan para resolver $x^n = z$, z complejo

a) Se factoriza

b) Se escribe en forma polar $z = r\,cis(\theta) = r \cdot e^{i\theta}$

c) Usar formula de moivre

Cuadrante	a	b	Rango	Que hago?
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$\theta = \frac{\pi}{2}$ si $a = 0$ y $b > 0$ → Positivo

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3) Resuelva en $\mathbb{C}$

$x^4 = 1 \leftarrow 1 + 0i \quad \swarrow \operatorname{Img}(z) = 0$

$x^4 - 1 = 0 \quad \leftarrow$ Igualar a 0

$(x^2 - 1)(x^2 + 1) = 0 \qquad a^2 - b^2 = (a-b)(a+b)$

$(x-1)(x+1)(x^2+1) = 0$

$(x-1)(x+1)(x-i)(x+i) = 0 \qquad a^2 + b^2 = (a-bi)(a+bi)$

$x - 1 = 0 \quad x + 1 = 0 \quad x - i = 0 \quad x + i = 0$

$x = 1 + 0i \quad x = -1 + 0i \quad x = i + 0 \quad x = -i + 0 \rightarrow a + bi$

$\quad a \;\; b \qquad a \;\; b \qquad b \;\; a \qquad b \;\; a \qquad bi +$

$x = 1 + 0i$

$r = \sqrt{1^2 + 0^2} = 1$

$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$z_1 = 1 \cdot \text{cis}(0)$

$x = -1 + 0i$

$r = \sqrt{(-1)^2 + 0^2} = 1$

$\theta = \tan^{-1}\left(\frac{0}{-1}\right) + \pi = \pi$

$z_2 = 1 \cdot \text{cis}(\pi)$

$x = 0 + i$ ← $a = 0, b = 1$

$r = \sqrt{0^2 + 1^2} = 1$

$\theta = \frac{\pi}{2}$

$z_3 = 1 \cdot \text{cis}\left(\frac{\pi}{2}\right)$

$x = 0 - i$ ← $a = 0, b = -1$

$r = \sqrt{0^2 + (-1)^2} = 1$

$\theta = \frac{-\pi}{2}$

$z_4 = 1 \cdot \text{cis}\,(-"$