

o)  $P(x) = x^4 - 6x^3 + 22x^2 - 54x + 117$  sabiendo que  $3 - 2i$  es uno de sus ceros. el otro es  $3 + 2i$

$$\begin{array}{r|rrrrr}
 x^4 & x^3 & x^2 & x & k & \\
 1 & -6 & 22 & -54 & 117 & \\
 & 3-2i & -13 & 27-18i & -217 & 3-2i \\
 \hline
 1 & -3-2i & 9 & -27-18i & 0 & (x-(3-2i))
 \end{array}$$

$$x^3 + (-3-2i)x^2 + 9x + (-27-18i) \quad (3-2i)(-3-2i)$$

$$\begin{array}{r|rrrr}
 1 & -3-2i & 9 & -27-18i & \\
 & 3+2i & 0 & 27+18i & 3+2i \\
 \hline
 1 & 0 & 9 & 0 & (x-(3+2i))
 \end{array}$$

$$(x^2 + 9) = (x - 3i)(x + 3i)$$

$$\boxed{(x - 3i)(x + 3i)(x - (3 - 2i))(x - (3 + 2i))}$$

$$\begin{aligned}
 &(3-2i)(-27-18i) \\
 &-81-54i+54i+36i^2 \\
 &-81-36 \\
 &-117
 \end{aligned}$$

$$\begin{aligned}
 a^2 - b^2 &= (a-b)(a+b) \\
 a^2 + b^2 &= (a-bi)(a+bi)
 \end{aligned}$$

r)  $P(x) = 4x^4 - 12x^3 + 22x^2 - 16x + 6$  sabiendo que  $\frac{1}{2} - \frac{i}{2}$  es un cero de  $P(x)$ . el otro es  $\frac{1}{2} + \frac{i}{2}$

$$\begin{array}{r|rrrrr} 4 & -12 & 22 & -16 & 6 & \\ & 2-2i & -6+9i & 10-6i & & \\ \hline 4 & -10-2i & 16+9i & 6-6i & -6i & \end{array} \quad \begin{array}{l} \frac{1}{2} - \frac{i}{2} \\ \frac{1}{2} - \frac{i}{2} \\ \left(x - \left(\frac{1}{2} - \frac{i}{2}\right)\right) \end{array}$$

$$(16+9i)\left(\frac{1}{2} - \frac{i}{2}\right)$$

$$8 - 8i + 2i - 2i^2$$

$$9 - 6i + 2$$

$$10 - 6i$$

$$(6-6i)\left(\frac{1}{2} - \frac{i}{2}\right)$$

$$3 - 3i - 3i + 3i^2$$

$$3 - 6i \rightarrow$$

$$-6i$$

$$4\left(\frac{1}{2} - \frac{i}{2}\right)$$

$$4 \cdot \frac{1}{2} - 4 \cdot \frac{i}{2}$$

$$2 - 2i$$

$$(-10-2i)\left(\frac{1}{2} - \frac{i}{2}\right)$$

$$-10 \cdot \frac{1}{2} + 10 \cdot \frac{i}{2} + -2i \cdot \frac{1}{2} + -2i \cdot \frac{-i}{2}$$

$$-5 + 5i - i + i^2$$

$$-5 + 4i - 1$$

$$-6 + 4i$$

13. Determine los números complejos  $z, w$  que satisfagan simultáneamente las condiciones siguientes:

$$\begin{cases} z - w \in \mathbb{R} \\ \operatorname{Re}(z + w) = 1 \\ z \cdot w = -7 + i \end{cases}$$

$$z = a_1 + b_1 i \quad w = a_2 + b_2 i$$

$$z - w = k + 0i, \quad k \text{ constante}$$

$$z - w = (a_1 - a_2) + (b_1 - b_2)i$$

$$\operatorname{Im}(z - w) = 0$$

$$b_1 - b_2 = 0, \quad b_1 = b_2$$

$$\operatorname{Re}(z + w) = 1 \quad z = a_1 + b_1 i \quad w = a_2 + b_2 i$$

$$a_1 + a_2 = 1$$

$$b_1 = 1 - a_1$$

$$z \cdot w = -7 + i \quad z = a_1 + b_1 i \quad w = a_2 + b_2 i$$

$$(a_1 + b_1 i)(a_2 + b_2 i)$$

$$b_1 = b_2$$

$$a_1 a_2 + a_1 b_2 i + b_1 a_2 i + b_1 b_2 i^2$$

$$(a_1 a_2 - b_1^2) + (a_1 b_2 + b_1 a_2)i$$

$$(a_1 a_2 - b_1^2) + (b_1(a_1 + a_2))i$$

$$(a_1 a_2 - b_1^2) + (b_1(a_1 + a_2))i$$

$$(a - b^2) + ib(a + c) = -7 + i$$

$$a - b^2 = -7$$

$$b(a + c) = 1$$

$$\hookrightarrow b(1) = 1$$

$$b = 1$$

$$a \quad c$$

$$a_1 + b_1 = 1$$

$$c \quad a$$

$$b_1 = 1 - a_1$$

$$a - b^2 = -7$$

$$a(1 - a) - 1^2 = -7$$

$$a - a^2 - 1 = -7$$

$$-a^2 + a + 6 = 0 \quad \Rightarrow a = 3 \quad a = -2$$

$$b = 1, \quad b = 1, \quad d = 1$$

$$z = a_1 + b_1 i \quad w = a_2 + b_2 i$$

$$3 + i$$

$$-2 + i$$

$$\Re(z + w) = 1$$

$$-2 + i$$

$$3 + i$$