Sucesiones

Si tiene $\boxed{!, \ldots}$

$\frac{a_{n+1}}{a_n} \geq 1$, a_n es una sucesion

Asumir, que es creciente

$2 \geq 1$ ✓ Creciente
$-1 \geq 1$ ✗ Decreciente

$a_n = n!$ $a_{n+1} = (n+1)!$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{n!}$$

Derivables NO tiene $!, \ldots$

$f(x) = a_n$ $f'(x) > 0$, es creciente
$f'(x) < 0$, es decreciente

1. $p_q = 14q - q^2, \forall q \geq 7$

$f(x) = 14x - x^2$ $\qquad\qquad x \cdot \pi = \pi$

$f'(x) = 14 - 2x = 0$

$2(7 - x) = 0$

$7 - x = 0$

$x = 7$

	$-\infty$	7	$+\infty$
$14-2x$	$+$	0	$-$
$f'(x)$	$+$		$-$
$f(x)$	$\nearrow$		$\searrow$

$\therefore$ Decrece

			0		
x	$-$	$- -$	0	$+ +$	
$-x$	$+$	$+ +$	0	$- -$	$-$

2. $f_m = m^3 - 5m^2 - 25m, \forall m \geq 5$

$f(x) = x^3 - 5x^2 - 25x$

$f'(x) = 3x^2 - 10x - 25 = 0$

$3x \quad 5 = 5x$

$x \quad -5 = -15x$

$-10x$

$(3x + 5)(x - 5) = 0$

$3x + 5 = 0 \qquad x - 5 = 0$

$x = -\frac{5}{3} \qquad x = 5$

$(3x+5)(x-5)$

$x = \frac{-5}{3}$ $\qquad$ $x = 5$

	$-\infty$	$-\frac{5}{3}$		5		$+\infty$
$3x+5$	$-$	0	$+$		$+$	
$x-5$	$-$		$-$	0	$+$	
$f'(x)$	$+$		$-$		$+$	
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$	

$\boxed{\text{Crece}}$

9. $a_m = \dfrac{m!}{m^2}$ $\qquad \dfrac{a_{n+1}}{a_n} \geq 1$ $\quad$ ✓ Crece

✗ Decre

Se asume que a_m es creciente

$$a_{n+1} = \frac{(n+1)!}{(n+1)^2} \qquad a_n = \frac{n!}{n^2}$$

$$\frac{\frac{(n+1)!}{(n+1)^2}}{\frac{n!}{n^2}} \geq 1$$

$$\frac{(n+1)! \cdot n^2}{(n+1)^2 \cdot n!} \geq 1$$

$(p+1)!$

$(p+1) \cdot (p+1-1)!$

$(p+1) \cdot p!$

$(p+1) \cdot p \cdot (p-1)!$

$$\frac{\cancel{(n+1)} \cdot \cancel{n!} \cdot n^2}{\cancel{(n+1)^2} \cdot \cancel{n!}} \geq 1$$

$$\frac{n^2}{n+1} \geq 1$$

$$n^2 \geq n+1$$

$$n^2 - n - 1 \geq 0$$

$$\frac{1+\sqrt{5}}{2} \checkmark \qquad \frac{1-\sqrt{5}}{2}$$

Crece

$$a_n = \frac{3^n \cdot n!}{2 \cdot 4 \cdot 6 \dots (2n)}$$

$$\boxed{\frac{a_{n+1}}{a_n} \geq 1}$$

$$a_{n+1} = \frac{3^{n+1} \cdot (n+1)!}{2 \cdot 4 \cdot 6 \cdot 2n \cdot \underbrace{2(n+1)}_{2n+2}} = \frac{3^{n+1} \cdot (n+1)!}{2 \cdot 4 \cdot 6 \cdot 2n \cdot (2n+2)}$$

$$\frac{\frac{3^{n+1} \cdot (n+1)!}{\cancel{2 \cdot 4 \cdot 6 \cdot 2n} \cdot (2n+2)}}{\frac{3^n \cdot n!}{\cancel{2 \cdot 4 \cdot 6 \dots (2n)}}} \geq 1$$

$$\frac{3^{n+1} \cdot (n+1)!}{3^n \cdot n! \, (2n+2)} \geq 1$$

$$3^{n+1} = 3^n \cdot 3^1$$

$$\boxed{(n+1-1)!}$$

$$\frac{\cancel{3^n} \cdot 3 \cdot (n+1) \cdot \cancel{n!}}{\cancel{3^n} \cdot \cancel{n!} \, (2n+2)} \geq 1$$

$$\frac{3(n+1)}{2n+2} \geq 1$$

$$\frac{3 \cancel{(n+1)}}{2 \cancel{(n+1)}} \geq 1$$

$$\frac{3}{2} \geq 1$$

$$\boxed{\text{Creciente} \checkmark}$$

$$a_n = \frac{n^4}{e^n} \quad , \quad \begin{matrix} > 0 & \nearrow \\ < 0 & \searrow \end{matrix}$$

$$f(x) = \frac{x^4 \leftarrow a}{e^x \leftarrow b} \qquad \frac{a' \cdot b - a \cdot b'}{b^2}$$

$$\frac{4x^3 \cdot e^x - x^4 \cdot e^x}{e^{x^2}} = 0$$

$$\frac{x^3 \cdot \cancel{e^x}(4 - x)}{\cancel{e^{x^2}}} = 0$$

$$\frac{x^3 (4-x)}{e^x} = 0$$

$$x^3(4-x) = 0$$

$$x^3 = 0 \qquad 4 - x = 0$$
$$x = 0 \qquad x = 4$$

	$-\infty$	0		4	$+\infty$
x^3	$-$	$\bullet$	$+$		$+$
$4-x$	$+$		$+$	$\bullet$	$-$
$f'(x)$	$-$		$+$		$-$
$f(x)$	$\searrow$		$\nearrow$		$\searrow$

$\therefore$ Decrece

Induccion

$$\text{(g)} \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$n=1 \qquad 1^2 = \dfrac{1(1+1)(2\cdot 1+1)}{6}$

$1=1$ ✓

$n=p \qquad \displaystyle\sum_{i=1}^{p} i^2 = \frac{p(p+1)(2p+1)}{6}$, Hi

$p+1+1 = p+2$

$n=p+1 \qquad \displaystyle\sum_{i=1}^{p+1} i^2 = \frac{(p+1)(p+2)(2p+3)}{6}$

$2(p+1)+1$

$2p+2+1$

$2p+3$

$$\sum_{i=1}^{p+1} i^2$$

$$\sum_{i=1}^{p} i^2 + (p+1)^2$$

$$\frac{p(p+1)(2p+1)}{6} + (p+1)^2 \cdot \frac{6}{6}, \text{ Hi}$$

$$\frac{p(p+1)(2p+1)}{6} + \frac{(p+1)^2\, 6}{6}$$

$$\frac{p(p+1)(2p+1)}{6} + \frac{(p+1)^2\,6}{6}$$

$$\frac{p(p+1)(2p+1) + (p+1)^2\,6}{6}$$

$$\frac{(p+1)(p(2p+1) + 6(p+1))}{6}$$

$$\frac{(p+1)(2p^2+p+6p+6)}{6}$$

$$\frac{(p+1)(2p^2+7p+6)}{6}$$

$$2p^2+7p+6$$

$$\begin{array}{lll} 2p & 3 & = 3p \\ p & 2 & = 4p \\ & & = 7p \end{array}$$

$$\boxed{\frac{(p+1)(p+2)(2p+3)}{6}}$$

$$\frac{(p+1)(p+2)(2p+3)}{6}$$

(j) $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$

$n=1 \quad \frac{1}{(2\cdot 1-1)\cdot(2\cdot 1+1)} = \frac{1}{2\cdot 1+1}$

$\frac{1}{3}=\frac{1}{3}$ ✓

$n=p \quad \frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\ldots+\frac{1}{(2p-1)(2p+1)}=\frac{p}{2p+1}$, H.i

$n=p+1 \quad \frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\ldots+\frac{1}{(2p+1)(2p+3)}=\frac{p+1}{2p+3}$

$2(p+1)-1 \qquad 2(p+1)+1$

$2p+2-1 \qquad 2p+2+1$

$2p+1 \qquad 2p+3$

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\ldots+\frac{1}{(2p-1)(2p+1)}+\frac{1}{(2p+1)(2p+3)}$$

$$\frac{p}{2p+1}+\frac{1}{(2p+1)(2p+3)}, \text{ H.I.}$$

$$\frac{(2p+3)}{2p+3}\cdot\frac{p}{2p+1}+\frac{1}{(2p+1)(2p+3)}$$

$$\frac{(2p+3)p}{(2p+3)(2p+1)}+\frac{1}{(2p+1)(2p+3)}$$

$$\frac{(2p+3)p}{(2p+3)(2p+1)} + \frac{1}{(2p+1)(2p+3)}$$

$$\frac{2p^2+3p+1}{(2p+1)(2p+3)}$$

$$2p^2+3p+1$$

$$\begin{array}{ccc} 2p & & 1 = p \\ p & & 1 = 2p \\ & & \overline{3p} \end{array}$$

$$\frac{\cancel{(2p+1)}(p+1)}{\cancel{(2p+1)}(2p+3)}$$

$$\boxed{\frac{p+1}{2p+3}}$$

6. $\displaystyle\sum_{k=1}^{n} k \cdot k! = (n+1)! - 1$

$n=1 \qquad 1 \cdot 1! = (1+1)! - 1$

$1 = 1$

$n = p \qquad \displaystyle\sum_{k=1}^{p} k \cdot k! = (p+1)! - 1 \quad , \text{H.I.}$

$(p+1+1)! - 1$

$n = p+1 \qquad \displaystyle\sum_{k=1}^{p+1} k \cdot k! = (p+2)! - 1 \quad , \text{HQD}$

$\displaystyle\sum_{k=1}^{p+1} k \cdot k!$

$\displaystyle\sum_{k=1}^{p} k \cdot k! + (p+1) \cdot (p+1)!$

$(p+1)! - 1 + (p+1) \cdot (p+1)!$

$(p+1)! + (p+1) \cdot (p+1)! - 1$

$(p+1)! (1 + p + 1) - 1$

$(p+1)! (p+2) - 1$

$\boxed{(p+2)! - 1}$

$(p+1)! \quad (p+2)-1$

$$(p+2)! = (p+2)\cdot(p+2-1)!$$
$$(p+2)\cdot(p+1)!$$

$$(p+2)! = (p+2)\cdot(p+2-1)$$
$$(p+2)\cdot(p+1)\cdot p\cdot(p-1)\cdot(p-2)!$$

9. $\sum_{j=1}^{k}(2j-1)\cdot 3^j = 3+(k-1)\cdot 3^{k+1}$

$h=1 \qquad (2\cdot 1-1)\cdot 3^1 = 3+\overset{0}{\cancel{(1-1)}}\cdot 3^{1+1}$

$$3=3$$

$h=p \qquad \sum_{j=1}^{p}(2j-1)\cdot 3^j = 3+(p-1)\cdot 3^{p+1} \quad ;\ \text{H.i.}$

$$(p+1)+1=p+2$$

$h=p+1 \qquad \sum_{j=1}^{p+1}(2j-1)\cdot 3^j = 3+p\cdot 3^{p+2} \quad ;\ \text{H.Q.)}$

$$(p+1)-1$$
$$p$$

$$\sum_{j=1}^{p+1}(2j-1)\cdot 3^j$$

$$2(p+1)-1\cdot 3^{p+1}$$
$$(2p+2-1)\cdot 3^{p+1}$$
$$(2p+1)\cdot 3^{p+1}$$

$$\sum_{j=1}^{p}(2j-1)\cdot 3^j+(2p+1)\cdot 3^{p+1}$$

$$3+(p-1)\cdot 3^{p+1}+(2p+1)\cdot 3^{p+1}$$

$$3+3^{p+1}(p\cancel{-1}+2p\cancel{+1})$$

$$3+3^{p+1}\cdot 3^1 = \boxed{3+3^{p+2}}$$

10. $\sum_{k=1}^{n} \frac{k+2}{(k^2+k)\cdot 2^k} = 1 - \frac{1}{(n+1)\cdot 2^n}$

$n=1 \qquad \frac{1+2}{(1^2+1)\cdot 2^1} = 1 - \frac{1}{(1+1)\cdot 2^1}$

$$\frac{3}{4} = \frac{3}{4} \quad \checkmark$$

$n=p \qquad \sum_{k=1}^{p} \frac{k+2}{(k^2+k)\cdot 2^k} = 1 - \frac{1}{(p+1)\cdot 2^p}$, Hi

$p+1+1 = p+2$

$n=p+1 \qquad \sum_{k=1}^{p+1} \frac{k+2}{(k^2+k)\cdot 2^k} = 1 - \frac{1}{(p+2)\cdot 2^{p+1}}$, H0

$$\sum_{k=1}^{p+1} \frac{k+2}{(k^2+k)\cdot 2^k}$$

$p+1+2 = p+3$

$$\sum_{k=1}^{p} \frac{k+2}{(k^2+k)\cdot 2^k} + \frac{p+3}{[(p+1)^2+(p+1)]\cdot 2^{p+1}}$$

$$1 - \frac{1}{(p+1)\cdot 2^p} + \frac{p+3}{[(p+1)^2+p+1]\cdot 2^{p+1}}$$

$$1 - \frac{1}{(p+1).2^{p}} + \frac{p+3}{[(p+1)^2+(p+1)].2^{p+1}}$$

$$1 - \frac{1}{(p+1).2^{p}} + \frac{p+3}{(p+1)(p+1+1).2^{p+1}}$$

$$1 - \frac{1}{(p+1).2^{p}} + \frac{p+3}{(p+1).(p+2).2^{p+1}}$$

$$1 - \frac{1}{(p+1).2^{p}} . \frac{(p+2).2}{(p+2).2} + \frac{p+3}{(p+1).(p+2).2^{p+1}}$$

$$1 - \frac{p+3}{(p+1).(p+2).2^{p+1}} - \frac{1}{(p+1).2^{p}} . \frac{(p+2).2}{(p+2).2} \;]$$

$$1 - \frac{-p-3+2p+4}{(p+1).(p+2).2^{p+1}}$$

$$1 - \frac{\cancel{p+1}}{\cancel{(p+1)}.(p+2).2^{p+1}}$$

$$1 - \frac{1}{(p+2).2^{p+1}}$$

$$1 - \frac{1}{(p+2).2^{p+1}}$$

1. Demuestre utilizando el Principio de Inducción Matemática que $\sum_{k=1}^{n} \frac{2k-1}{2^k} = 3 - \frac{2n+3}{2^n}$, para todo entero $n \geq 1$. (4 pts)

$n=1 \quad \frac{2\cdot 1-1}{2^1} = 3 - \frac{2\cdot 1+3}{2^1}$

$\frac{1}{2} , \frac{1}{2}$ ✓

$n=p \quad \sum_{k=1}^{p} \frac{2k-1}{2^k} = 3 - \frac{2p+3}{2^p}$, H.I.

$2(p+1)+3$

$2p+2+3 = 2p+5$

$n=p+1 \quad \sum_{k=1}^{p+1} \frac{2k-1}{2^k} = 3 - \frac{2p+5}{2^{p+1}}$, HQD

$\sum_{k=1}^{p+1} \frac{2k-1}{2^k}$

$2(p+1)-1$

$2p+2-1 = 2p+1$

$\sum_{k=1}^{p} \frac{2k-1}{2^k} + \frac{2p+1}{2^{p+1}}$

$3 - \frac{2p+3}{2^p} + \frac{2p+1}{2^{p+1}}$

$3 - \frac{2p+3}{2^p} \cdot \frac{2}{2} + \frac{2p+1}{2^{p+1}}$

$3 - \frac{4p+6+2p+1}{2^{p+1}}$

$$3 + \frac{-4p - 6 + 2p + 1}{2^{p+1}}$$

$$3 - \frac{2p+5}{2^{p+1}}$$

$$3 + \frac{-2p-5}{2^{p+1}}$$

$$x+2 = -(-x-2)$$

$$3 + -\frac{2p+5}{2^{p+1}}$$