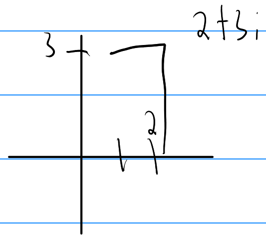
Numeros Complejos

$i^2 = -1$ $\qquad$ $i = \sqrt{-1}$

Rectangular	Polar	Exponencial
$a + bi$	$r\,cis(\theta)$	$r \cdot e^{i\theta}$



$a + bi$

$2 + 3i$

$\downarrow \quad \downarrow$

$x \quad y$

$z = a + bi$

$Re(z) = a$ $\qquad$ $2+3i$ $\qquad$ $Re(z) = 2$

$Im(z) = b$ $\qquad\qquad\qquad$ $Im(z) = 3$

Suma $\quad 3+2i \wedge 4+4i$ $\qquad$ restar

$3+2i+4+4i$

$7+6i$

$(3+2i) - (4+4i)$

$3+2i-4-4i$

$-1-2i$

Multiplicacion $\quad 3+2i \wedge 4+4i$

$(3+2i)(4+4i)$

$12 + 12 + 8i + 4i^2$

$24 + 8i + 4 \cdot -1$

$24 + 8i - 4$

$20 + 8i$

$z_1 = 2 - i$ y $z_2 = 3 + 2i$ $\frac{z1}{z2}$

$\frac{2-i}{3+2i} \cdot \frac{3-2i}{3-2i}$

$\frac{(2-i)(3-2i)}{(3+2i)(3-2i)}$ $a^2 - b^2 = (a-b)(a+b)$

$i \cdot i = i^2 = -1$

$\frac{6 - 4i - 3i + 2i^2}{9 - 4i^2}$

$\frac{6 - 4i - 3i + 2 \cdot -1}{9 - 4 \cdot -1}$

$\frac{6 - 7i - 2}{9 - -4}$

$\frac{4 - 7i}{13}$ $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

$\frac{4}{13} - \frac{7i}{13}$

$a \quad \pm bi$

$Re(z) = \frac{4}{13}$ $Im(z) = \frac{-7}{13}$

Cambia símbolo

$z = a+bi \qquad \overline{z} = a-bi \qquad \overline{\overline{z}} = z$

$$\overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}} \qquad \overline{z \cdot w} = \overline{z} \cdot \overline{w}$$

Pruebe $\dfrac{\overline{i+\overline{z}}}{i-z} = -1$

$$\frac{-i+z}{i-z}$$

$$\frac{-(\cancel{i-z})}{\cancel{i-z}} = -1 \checkmark$$

Determinar todos los números complejos x en forma rectangular que satisfacen lo siguiente

$$(ix^2+x)\left(\frac{ix}{1-8i}-1\right) = 0 \qquad a \cdot b = 0 \quad a=0 \quad b=0$$

$ix^2+x=0$

$x(ix+1)=0$

$x=\boxed{0} \quad ix+1=0$

$ix=-1$

$x = \dfrac{-1}{i} \cdot \dfrac{-i}{-i}$

$$x = \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{1} = \boxed{i}$$

$$\frac{ix}{1-4i} - 1 = 0$$

$$\frac{ix}{1-4i} = 1$$

$$ix = 1-4i$$

$$x = \frac{1-4i}{i} \cdot \frac{-i}{-i}$$

$$x = \frac{(1-4i)(-i)}{-i^2}$$

$$\frac{-i+4i^2}{-\,-1} = \frac{-i+4\cdot -1}{1} = \boxed{-4-i}$$

B/ $\{0+0i,\ 0+i,\ -4-i\}$

$$z = 3+0i$$

$$z = \frac{2-5\overset{a}{a}i}{1+2i}$$, determine todos los valores para el numero real a, de forma que $Im(z) \neq 0$

$$\frac{2-5ai}{1+2i} \cdot \frac{1-2i}{1-2i}$$

$$\frac{(2-5ai)(1-2i)}{(\underset{a}{1}+\underset{b}{2i})(\underset{a}{1}-\underset{b}{2i})} \longleftarrow a^2-b^2=(a-b)(a+b)$$

$$1^2-4i^2$$

$$\frac{2-4i-5ai+10ai^2 \rightarrow -1}{1-4\cdot i^2 \leftarrow -1}$$

$$\frac{2-4i-5ai-10a}{1--4}$$

$$\frac{2-4i-5ai-10a}{5}$$

$$\frac{(2-10a)+(-4-5a)i}{5}$$

$$Re(z)=\frac{2-10a}{5} \qquad Im(z)=\frac{-4-5a}{5}$$

$$\frac{-4-5a}{5} \neq 0$$

$$-4-5a \neq 0$$

$$-5a \neq 4$$

$$\boxed{a \neq -\frac{4}{5}}$$

$a = 2 - ix \qquad b = 3 - iy$, hallar $x, y \in \mathbb{R}$, tal que $a \cdot b = 8 + 4i$

$$\underbrace{(2-ix)}_{a}\underbrace{(3-iy)}_{b} = 8+4i$$

$$6 - 2iy - 3ix + i^2 xy = 8 + 4i$$

$$6 - 2iy - 3ix - xy = 8 + 4i$$

$$\underbrace{(6 - xy)}_{a} + \underbrace{(-2y - 3x)}_{b} i = \underbrace{8}_{a} + \underbrace{4}_{b} i$$

$$6 - xy = 8$$

$$-xy = 2$$

$$xy = -2$$

$$y = \frac{-2}{x}$$

$$-2y - 3x = 4$$

$$-2\left(\frac{-2}{x}\right) - 3x = 4$$

$$\frac{4}{x} - 3x \cdot \frac{x}{x} = 4$$

$$\frac{4 - 3x^2}{x} = 4$$

$$4 - 3x^2 = 4x$$

$$-3x^2 - 4x + 4 = 0$$

$$\begin{array}{lcl} -3x & 2 & = 2x \\ x & 2 & = -6x \\ & & \overline{-4x} \end{array}$$

$$(-3x+2)(x+2) = 0$$

$$x = \frac{2}{3} \qquad x = -2$$

$$y = \frac{-2}{x}$$

$$y = \frac{-2}{\frac{2}{3}} \qquad y = \frac{-2}{-2}$$

$$y = -3 \qquad y = 1$$

$$\boxed{\begin{array}{ll} x = \frac{2}{3}, & y = -3 \\ x = -2, & y = 1 \end{array}}$$

Encontrar $x, y \in \mathbb{R}$ tales que

$$\frac{43 + yi}{x - 5i} = 4 + 3i$$

$$43 + yi = (4+3i)(x-5i)$$
$$43 + yi = 4x - 20i + 3xi - 15i^2$$
$$43 + yi = 4x - 20i + 3xi + 15$$
$$\underbrace{43}_{\mathbb{R}} + \underbrace{yi}_{I} = \underbrace{(4x+15)}_{\mathbb{R}} + \underbrace{(-20+3x)i}_{I}$$

$$4x + 15 = 43 \qquad -20 + 3x = y$$
$$4x = 28 \qquad -20 + 3(7) = y$$
$$x = 7 \qquad y = 1$$

$$\boxed{x = 7 \wedge y = 1}$$

RADIANES

Forma Polar $\wedge$ exponencial

Rectangular	Polar	Exponencial
$a+bi$	$r\,cis(\theta)$	$r.e^{i\theta}$

$r = |z| = \sqrt{a^2+b^2}$ $\qquad cis(\theta) = cos(\theta) + i\,sen(\theta)$

$\theta = \arctan\left(\frac{b}{a}\right) \pm \pi$ o nada

Cuadrante	a	b	Rango	Que hago?
I	+	+	✓	NADA
II	−	+	X	Sumar π al θ
III	−	−	X	Resta π al θ
IV	+	−	✓	NADA

Casos especiales

$\theta = \frac{\pi}{2}$ si $a=0 \wedge b>0 \rightarrow$ Positivo

$\theta = -\frac{\pi}{2}$ si $a=0 \wedge b<0 \rightarrow$ Negativo

$z = 3-2i$, a $r\,cis(\theta) \wedge r.e^{i\theta}$

$r = \sqrt{3^2+(-2)^2} = \sqrt{13}$ $\qquad \sqrt{13}.e^{i \cdot -0{,}58}$ ✓

$\theta = \arctan\left(\frac{-2}{3}\right) = -0{,}58$

$\sqrt{13}.\,cis(-0{,}58)$ ✓

De vuelta a rectangular

$\sqrt{13}\cdot[\cos(-0{,}58) + i\,sen(-0{,}58)]$

$3-2i$ ✓

$z = -4 + 2i$, a $r\,\text{cis}(\theta)$

$r = \sqrt{(-4)^2 + 2^2} = 2\sqrt{5}$

$\theta = \arctan\left(\frac{2}{-4}\right) + \pi \simeq 2{,}67$

$2\sqrt{5} \cdot \text{cis}(2{,}67)$ ✓

Teorema de demoivre

Sean $z \wedge w$, 2 complejos en forma polar o exponencial, entonces se cumple

$$\left.\begin{array}{l}\left.\begin{array}{l} r_1 \cdot cis(\theta_1) \cdot r_2 \cdot cis(\theta_2) \\ r_1 \cdot r_2 \cdot cis(\theta_1 + \theta_2) \end{array}\right\} \text{Polar} \\ \\ \left.\begin{array}{l} r_1 \cdot e^{i\theta_1} \cdot r_2 \cdot e^{i\theta_2} \\ r_1 \cdot r_2 \cdot e^{i(\theta_1+\theta_2)} \end{array}\right\} \text{Exponencial} \end{array}\right\} \text{multiplicacion}$$

$$\left.\begin{array}{l} \dfrac{r_1 \cdot cis(\theta_1)}{r_2 \cdot cis(\theta_2)} = \dfrac{r_1}{r_2} \cdot cis(\theta_1 - \theta_2) \\ \\ \dfrac{r_1 \cdot e^{i\theta_1}}{r_2 \cdot e^{i\theta_2}} = \dfrac{r_1}{r_2} \cdot e^{i(\theta_1 - \theta_2)} \end{array}\right\} \text{Division}$$

$$\left.\begin{array}{l} (r\, cis(\theta))^x = r^x \cdot cis(\theta \cdot x) \\ \\ (r \cdot e^{i\theta})^x = r^x \cdot e^{i(\theta x)} \end{array}\right\} \text{Exponente}$$

1) Pasar todo a polar

2) Dependiendo sumar o restar π y tener en cuenta casos especiales del θ

Cuadrante	a	b	Rango	Que hago?
I	+	+	✓	NADA
II	−	+	X	Sumar π al θ
III	−	−	X	Resta π al θ
IV	+	−	✓	NADA

Casos especiales

$\theta = \frac{\pi}{2}$ si $a = 0 \wedge b > 0 \rightarrow$ Positivo

$\theta = -\frac{\pi}{2}$ si $a = 0 \wedge b < 0 \rightarrow$ Negativo

$$\frac{(\underbrace{1-i}_{z_1})^6\,(\underbrace{1-i\sqrt{3}}_{z_2})^3}{(\underbrace{-2\sqrt{3}+2i}_{z_3})^7}$$

$$z_3 = -2\sqrt{3}+2i$$
$$r = \sqrt{(-2\sqrt{3})^2+2^2} = 4$$
$$\theta = \arctan\left(\frac{2}{-2\sqrt{3}}\right)+\pi = \frac{5\pi}{6}$$
$$z_3 = 4 \cdot \operatorname{cis}\left(\frac{5\pi}{6}\right)$$

$$z_1 = 1-i$$
$$r = \sqrt{1^2+(-1)^2} = \sqrt{2}$$
$$\theta = \arctan\left(\frac{-1}{1}\right) = \frac{-\pi}{4}$$
$$z_1 = \sqrt{2} \cdot \operatorname{cis}\left(\frac{-\pi}{4}\right)$$

$$z_2 = 1-i\sqrt{3}$$
$$r = \sqrt{(1)^2+(-\sqrt{3})^2} = 2$$
$$\theta = \arctan\left(\frac{-\sqrt{3}}{1}\right) = \frac{-\pi}{3}$$
$$z_2 = 2 \cdot \operatorname{cis}\left(\frac{-\pi}{3}\right)$$

$$\frac{\left(\sqrt{2}\operatorname{cis}\left(\frac{-\pi}{4}\right)\right)^6 \cdot \left(2\operatorname{cis}\left(\frac{-\pi}{3}\right)\right)^3}{\left(4\operatorname{cis}\left(\frac{5\pi}{6}\right)\right)^7} \qquad \frac{(\underbrace{1-i}_{z_1})^6\,(\underbrace{1-i\sqrt{3}}_{z_2})^3}{(\underbrace{-2\sqrt{3}+2i}_{z_3})^7}$$

$$\frac{(\sqrt{2})^6 \cdot \operatorname{cis}\left(\frac{-\pi}{4}\cdot 6\right) 2^3 \operatorname{cis}\left(\frac{-\pi}{\cancel{3}}\cdot \cancel{3}\right)}{4^7 \operatorname{cis}\left(\frac{5\pi}{6}\cdot 7\right)}$$

$(r \operatorname{cis}(\theta))^x = r^x \cdot \operatorname{cis}(\theta \cdot x)$
$(r \cdot e^{i\theta})^x = r^x \cdot e^{i(\theta x)}$ } Exponente

$$\frac{2^3 \cdot 2^3}{\underbrace{2^{14}}_{4^7}} \cdot \frac{\operatorname{cis}\left(\frac{-\pi\, 6}{4}\right) \cdot \operatorname{cis}(-\pi)}{\operatorname{cis}\left(\frac{35\pi}{6}\right)}$$

$$\frac{1}{2^8} \cdot \frac{\operatorname{cis}\left(\frac{-6\pi}{4} + -\pi\right)}{\operatorname{cis}\left(\frac{35\pi}{6}\right)}$$

$$\frac{1}{2^8} \cdot \frac{\operatorname{cis}\left(\frac{-5\pi}{2}\right)}{\operatorname{cis}\left(\frac{35\pi}{6}\right)}$$

$$\frac{1}{2^8} \cdot \left(\operatorname{cis}\left(\frac{-5\pi}{2} - \frac{35\pi}{6}\right)\right)$$

$$\frac{1}{2^8} \cdot \left(\operatorname{cis}\left(\frac{-5\pi}{2} - \frac{35\pi}{6} \right) \right)$$

$$\frac{1}{2^8} \operatorname{cis}\left(\frac{-25\pi}{3} \right)$$

$$\frac{1}{2^8} \left[\cos\left(\frac{-25\pi}{3} \right) + i \operatorname{sen}\left(\frac{-25\pi}{3} \right) \right]$$

$$\frac{1}{2^8} \cdot \cos\left(\frac{-25\pi}{3} \right) + \frac{1}{2^8} \operatorname{sen}\left(\frac{-25\pi}{3} \right) i$$

$$\underbrace{(3i)}_{z_1}{}^{6}\ \underbrace{(-2+4i)}_{z_2}{}^{7}$$

$z_1 = 0 + 3i$

$r = \sqrt{0^2 + 3^2} = 3$

$\theta = \frac{\pi}{2}$

$z_1 = 3 \cdot \text{cis}\left(\frac{\pi}{2}\right)$

$z_2 = -2 + 4i$

$r = \sqrt{(-2)^2 + (4)^2} = 2\sqrt{5}$

$\theta = \arctan\left(\frac{4}{-2}\right) = -1{,}10$

$z_2 = 2\sqrt{5} \cdot \text{cis}(-1{,}10)$

$$\underbrace{(3i)}_{z_1}{}^{6}\ \underbrace{(-2+4i)}_{z_2}{}^{7}$$

$$\left[3\,\text{cis}\left(\frac{\pi}{2}\right)\right]^6 \cdot \left[2\sqrt{5} \cdot \text{cis}(-1{,}10)\right]^7$$

$$3^6 \cdot \text{cis}\left(\frac{6\pi}{2}\right) \cdot (2\sqrt{5})^7 \cdot \text{cis}(-1{,}10 \cdot 7)$$

$$3^6 \cdot (2\sqrt{5})^7 \cdot \text{cis}\left(\frac{6\pi}{2} + -7{,}70\right)$$

Raíces complejas

Se usa para resolver $x^n = z$, z complejo en forma polar

$x^n = r \operatorname{cis}(\theta)$ ← Polar

$n = 3$

$x = \sqrt[n]{r \operatorname{cis}(\theta)}$

$k = 0, 1, 2$

$= \sqrt[n]{r} \operatorname{cis}\left(\frac{\theta + 2\pi k}{n}\right)$, probando desde $k=0$ hasta $k = n-1$

Resuelva en $\mathbb{C}$, encontrar valores de z

$z^3 = -2-2i$

$-2-2i$

$z^3 = 2\sqrt{2} \operatorname{cis}\left(\frac{-3\pi}{4}\right)$

$r = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-2}{-2}\right) - \pi = \frac{-3\pi}{4}$

$z = \sqrt[3]{2\sqrt{2} \operatorname{cis}\left(\frac{-3\pi}{4}\right)}$

$2\sqrt{2} \operatorname{cis}\left(\frac{-3\pi}{4}\right)$

$z = \sqrt[3]{2\sqrt{2}} \operatorname{cis}\left(\frac{\frac{-3\pi}{4} + 2\pi k}{3}\right)$

$n = 3$

$k = 0, 1, 2$

$k=0 \rightarrow \sqrt[3]{2\sqrt{2}} \operatorname{cis}\left(\frac{\frac{-3\pi}{4} + 2\pi \cdot 0}{3}\right) = \sqrt[3]{2\sqrt{2}} \operatorname{cis}\left(\frac{\pi}{6}\right)$

$k=1 \rightarrow \sqrt[3]{2\sqrt{2}} \operatorname{cis}\left(\frac{\frac{-3\pi}{4} + 2\pi \cdot 1}{3}\right) = \sqrt[3]{2\sqrt{2}} \operatorname{cis}\left(\frac{5\pi}{6}\right)$

$k=2 \rightarrow \sqrt[3]{2\sqrt{2}} \operatorname{cis}\left(\frac{\frac{-3\pi}{4} + 2\pi \cdot 2}{3}\right) = \sqrt[3]{2\sqrt{2}} \operatorname{cis}\left(\frac{9\pi}{6}\right)$

Resolver en $\mathbb{C}$, encontrar complejos

$x^4 = 1 + 0i$

$x^4 - 1 = 0$ $\quad a^2 - b^2 = (a-b)(a+b)$

$(x^2)^2 - 1^2 = 0$ $\quad a^2 + b^2 = (a-bi)(a+bi)$

$(x^2-1)(x^2+1) = 0$

$(x-1)(x+1)(x-i)(x+i) = 0$

$x - 1 = 0 \quad x + 1 = 0 \quad x - i = 0 \quad x + i = 0$

$x = 1 \quad x = -1 \quad x = i \quad x = -i$

$x = 1 + 0i$
$r = \sqrt{1^2 + 0^2} = 1$
$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$
$1 \cdot Cis(0)$

$x = -1 + 0i$
$r = \sqrt{(-1)^2 + 0^2} = 1$
$\theta = \tan^{-1}\left(\frac{0}{-1}\right) + \pi = \pi$
$1 \cdot Cis(\pi)$

$x = 0 + i$
$r = \sqrt{0^2 + 1^2} = 1$
$\theta = \arctan\left(\frac{1}{0}\right) = \frac{\pi}{2}$
$1 \cdot Cis\left(\frac{\pi}{2}\right)$

$x = 0 - i$
$r = \sqrt{0^2 + (-1)^2} = 1$
$\theta = \arctan\left(\frac{-1}{0}\right) = -\frac{\pi}{2}$
$1\, Cis\left(\frac{-\pi}{2}\right)$

$\boxed{R/ \quad 1 \cdot Cis(0) \quad 1 \cdot Cis(\pi) \quad 1 \cdot Cis\left(\frac{\pi}{2}\right) \quad 1\, Cis\left(\frac{-\pi}{2}\right)}$