Serie Telescopica

$$\sum_{n=p}^{\infty} (a_n - a_{n+1})$$

SIEMPRE es una resta

Ejemplos

$2^n - 2^{n+1}$

$\tan(n) - \tan(n+1)$

$\frac{1}{n} - \frac{1}{n+1}$

La suma es $a_p - \lim_{n \to +\infty} a_{n+1}$

Ejemplo

$$\sum_{n=7}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

$$\frac{1}{7} - \cancel{\lim_{n \to +\infty} \frac{1}{n+1}} \; 0$$

$$\boxed{\frac{1}{7}}$$

1)
$$\sum_{n=3}^{\infty} \frac{1}{n^2+2n} \qquad \frac{A}{\text{Expresión}} + \frac{B}{\text{Expresión}} + \ldots$$

$$\sum_{n=3}^{\infty} \frac{1}{n(n+2)}$$

1) $$\frac{1}{\cancel{n(n+2)}} = \frac{A}{n} + \frac{B}{n+2}$$

Se borra y ya

2) $$1 = A(n+2) + Bn$$

$n+2=0$ $\qquad$ $n=0$

$n=-2$ $\qquad$ $n=0$

$n=0 \rightarrow 1 = A(0+2) + \cancel{B\cdot 0}$

$1 = 2A$

$A = \frac{1}{2}$

$n=-2 \rightarrow 1 = \underbrace{\cancel{A(-2+2)}}^{0} + B\cdot -2$

$1 = -2B$

$B = \frac{-1}{2}$

$$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

$$= \frac{\frac{1}{2}}{n} - \frac{\frac{1}{2}}{n+2}$$

$$\frac{1}{n(n+2)} = \frac{\frac{1}{2}}{n} - \frac{\frac{1}{2}}{n+2}$$

$$\sum_{n=3}^{\infty} \frac{\frac{1}{2}}{n} - \frac{\frac{1}{2}}{n+2}$$

$$\frac{1}{2}\sum_{n=3}^{\infty} \frac{1}{n} - \frac{1}{n+2}$$

Se agrega

$$\frac{1}{2}\sum_{n=3}^{\infty} \boxed{\frac{1}{n} - \frac{1}{n+1}} + \boxed{\frac{1}{n+1} - \frac{1}{n+2}}$$

$$\frac{1}{2}\left[\sum_{n=3}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right) + \sum_{n=3}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right]$$

$$\frac{1}{2}\left[\frac{1}{3} - \cancel{\lim_{n\to+\infty}\frac{1}{n+1}} + \frac{1}{3+1} - \cancel{\lim_{n\to+\infty}\frac{1}{n+2}}\right]$$

$0 \qquad 0$

$$\frac{1}{2}\left[\frac{1}{3} + \frac{1}{4}\right]$$

$$\frac{1}{6} + \frac{1}{8} = \boxed{\frac{7}{24}}$$

2) $\sum_{n=1}^{\infty} \dfrac{n}{(n+1)(n+2)(n+3)}$

$$\frac{n}{(n+1)(n+2)(n+3)} = \frac{A}{n+1} + \frac{B}{n+2} + \frac{C}{n+3}$$

$$n = A(n+2)(n+3) + B(n+1)(n+3) + C(n+1)(n+2)$$

$n+2=0$ $\quad$ $n+1=0$ $\quad$ $n+3=0$

$n=-2$ $\quad$ $n=-1$ $\quad$ $n=-3$

$n=-1$

$$-1 = A(-1+2)(-1+3)$$
$$-1 = A(1)(2)$$
$$-1 = 2A$$
$$A = \frac{-1}{2}$$

$n=-2$

$$-2 = B(-2+1)(-2+3)$$
$$-2 = B(-1)1$$
$$-2 = -B$$
$$B = 2$$

$n=-3$

$$-3 = C(-3+1)(-3+2)$$
$$-3 = C(-2)(-1)$$
$$-3 = 2C$$
$$C = \frac{-3}{2}$$

$$\frac{n}{(n+1)(n+2)(n+3)} = \frac{\frac{-1}{2}}{n+1} + \frac{2}{n+2} + \frac{\frac{-3}{2}}{n+3}$$

$$\sum_{n=1}^{\infty} \frac{\frac{-1}{2}}{n+1} + \frac{2}{n+2} + \frac{\frac{-3}{2}}{n+3}$$

$$\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

Se le cambian los signos

$$\sum_{n=1}^{\infty} \frac{\frac{-1}{2}}{n+1} + \frac{2}{n+2} + \frac{\frac{-3}{2}}{n+3}$$

$$\sum_{n=1}^{\infty} \frac{\frac{-1}{2}}{n+1} + \frac{\frac{1}{2} + \frac{3}{2}}{n+2} + \frac{\frac{-3}{2}}{n+3}$$

$$\sum_{n=1}^{\infty} \frac{\frac{-1}{2}}{n+1} + \frac{\frac{1}{2}}{n+2} + \frac{\frac{3}{2}}{n+2} + \frac{\frac{-3}{2}}{n+3}$$

$$\sum_{n=1}^{\infty} \frac{\frac{-1}{2}}{n+1} + \frac{\frac{1}{2}}{n+2} + \frac{\frac{3}{2}}{n+2} + \frac{\frac{-3}{2}}{n+3}$$

$$\sum_{n=1}^{\infty} \left(\frac{\frac{-1}{2}}{n+1} + \frac{\frac{1}{2}}{n+2} \right) + \sum_{n=1}^{\infty} \left(\frac{\frac{3}{2}}{n+2} - \frac{\frac{3}{2}}{n+3} \right)$$

$$-\sum_{n=1}^{\infty} \left(\frac{\frac{1}{2}}{n+1} - \frac{\frac{1}{2}}{n+2} \right) + \sum_{n=1}^{\infty} \left(\frac{\frac{3}{2}}{n+2} - \frac{\frac{3}{2}}{n+3} \right)$$

$$-\frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right) + \frac{3}{2} \sum_{n=1}^{\infty} \left(\frac{1}{n+2} - \frac{1}{n+3} \right)$$

$$\frac{-1}{2} \left[\frac{1}{1+1} - \cancel{\lim_{n \to +\infty} \frac{1}{n+2}} \right] + \frac{3}{2} \left[\frac{1}{1+1} - \cancel{\lim_{n \to +\infty} \frac{1}{n+3}} \right]$$

0 0

$$\frac{-1}{2} \cdot \frac{1}{2} + \frac{3}{2} \cdot \frac{1}{2}$$

$$\boxed{\frac{1}{4}}$$

$$\sum_{n=3}^{\infty} \frac{1}{3^{n-1}} - \frac{1}{3^{n+1}} \quad \to \quad \frac{1}{3^{n+1-1}} = \frac{1}{3^n}$$

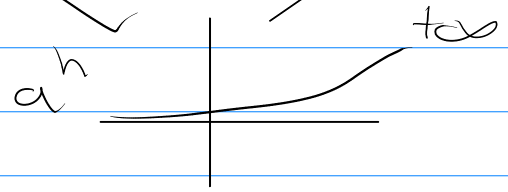
$\to a_{n+1}?$ $\quad \to a_{n+2}?$ $\quad \frac{1}{3^{n+1}}$

$$\sum_{n=3}^{\infty} \frac{1}{3^{n-1}} - \frac{1}{3^{n+1}}$$

$$\sum_{n=3}^{\infty} \frac{1}{3^{n-1}} - \frac{1}{3^{n}} + \frac{1}{3^{n}} - \frac{1}{3^{n+1}}$$

$$\sum_{n=3}^{\infty} \left(\frac{1}{3^{n-1}} - \frac{1}{3^{n}} \right) + \sum_{n=3}^{\infty} \left(\frac{1}{3^{n}} - \frac{1}{3^{n+1}} \right)$$

$$\frac{1}{3^{3-1}} - \boxed{\cancel{\lim_{n \to +\infty} \frac{1}{3^n}}}^{\,0} + \frac{1}{3^3} - \boxed{\cancel{\lim_{n \to +\infty} \frac{1}{3^{n+1}}}}^{\,0}$$



$$\frac{1}{3^2} + \frac{1}{3^3} = \boxed{\frac{4}{27}}$$

3. Calcule la suma de la serie $B = \sum_{k=3}^{\infty}\left[\frac{1}{2k-3} - \frac{1}{2k+1}\right]$. $n = k$

$a_n - a_{n+1}$, $a_n = \frac{1}{2n-3}$, $a_{(n+1)} = \frac{1}{2(n+1)-3}$

$$\sum_{n=3}^{\infty} \frac{1}{2n-3} - \frac{1}{2n-1} + \frac{1}{2n-1} - \frac{1}{2n+1} \qquad = \frac{1}{2n+2-3} = \frac{1}{2n-1}$$

$$\sum_{n=3}^{\infty}\left(\frac{1}{2n-3} - \frac{1}{2n-1}\right) + \sum_{n=3}^{\infty}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$$

$$\frac{1}{2\cdot 3-3} - \lim_{n\to+\infty} \cancel{\frac{1}{2n-1}}^{\,0} + \frac{1}{2\cdot 3-1} - \lim_{n\to+\infty} \cancel{\frac{1}{2n+1}}^{\,0}$$

$$\frac{1}{3} + \frac{1}{5} = \boxed{\frac{8}{15}}$$

P-series

$$\sum_{n=p}^{\infty} \frac{1}{n^p} \begin{cases} \text{Converge si } p > 1, & \frac{1}{n^6} \\ \text{Diverge si } p \le 1, & \frac{1}{n^1} \end{cases}$$

CADENA DE TÉRMINOS DOMINANTES

Sean $k \in \mathbb{R}, a, p \in \mathbb{R}^+, a > 1$ entonces para n suficientemente grande se tiene que:

$$k \ll \ln n \ll n^p \ll a^n \ll n! \ll n^n$$

$$\sum_{n=2}^{\infty} \frac{3}{n^2+1} = \frac{\overset{k=1}{k}}{\underbrace{n^p + k}_{k < n^p,\ k=0}}$$

Las sumadas o restadas = 0

Las multiplicadas = 1

$$\sum_{n=2}^{\infty} \frac{3}{n^2 \cancel{+1}} = 3 \boxed{\sum_{n=2}^{\infty} \frac{1}{n^2}}$$

$$\lim_{n \to +\infty} \frac{f(x)}{g(x)}, \quad f(x) > g(x), \ = +\infty$$

$$f(x) < g(x), \ = 0$$

$$\lim_{n \to +\infty} \frac{n^n}{n!}, \quad f(x) > g(x) = +\infty$$

$$\frac{n!}{n} \quad f(x) < g(x) = 0$$

Criterio de comparacion directa

SOLO PARA COSAS TRIGONOMETRICAS

Sen, Cos $\rightarrow$ $-1 \leq x \leq 1$

Sen^2, Cos^2 $\rightarrow$ $0 \leq x \leq 1$

Aplicar teorema del sandwich

$$\sum_{n=2}^{\infty} \frac{4-3 \cdot sen(n)}{n-1}$$

$-x \leq 2$

$x \geq 2$

$$-1 \leq sen(n) \leq 1$$
$$3 \geq -3\,sen(n) \geq -3$$
$$-3 \leq -3\,sen(n) \leq 3$$
$$4-3 \leq 4-3\,sen(n) \leq 3+4$$
$$1 \leq 4-3\,sen(n) \leq 7$$
$$\frac{1}{n-1} \leq \frac{4-3\,sen(n)}{n-1} \leq \frac{7}{n-1}$$

$$\sum_{n=2}^{\infty} \frac{1}{n-1}$$

$$\sum_{n=2}^{\infty} \frac{1}{n-1}$$

$$\sum_{n=2}^{\infty} \frac{1}{n^1} \;, \text{ Diverge}$$

$$\boxed{\frac{4-3\,\mathrm{Sen}(n)}{n-1} \;, \therefore \text{ Diverge}}$$

$$\sum_{n=0}^{\infty} \frac{2+\cos(n)}{3n+5}$$

$$-1 \leq \cos(n) \leq 1$$

$$2-1 \leq 2+\cos(n) \leq 2+1$$

$$1 \leq 2+\cos(n) \leq 3$$

$$\frac{1}{3n+5} \leq \frac{2+\cos(n)}{3n+5} \leq \frac{3}{3n+5}$$

$$\sum_{n=0}^{\infty} \frac{\cancel{3}^{\,1}}{\cancel{3}^{\,1}n+\cancel{5}^{\,0}} = \sum_{n=0}^{\infty} \frac{1}{n^1} \;, \text{ p serie,}$$

$$p = 1$$

$\therefore$ Diverge

$$\boxed{\therefore \frac{2+\cos(n)}{3n+5} \text{ Diverge}}$$