- **Ejercicios combinados:**

  Determine si las siguientes series convergen o divergen y calcule su suma si son convergentes.

1. $$\sum_{n=2}^{\infty} \frac{(-1)^n \cdot (n^2-n) \cdot 3^n + 5^n}{5^n \cdot (n^2-n)}$$

$$\sum_{n=2}^{\infty} \frac{(-1)^n \cdot \cancel{(n^2-n)} \cdot 3^n}{5^n \cdot \cancel{(n^2-n)}} + \sum_{n=2}^{\infty} \frac{\cancel{5^n}}{\cancel{5^n} \cdot (n^2-n)}$$

$x^a \cdot y^a = (x \cdot y)^a$

$$\sum_{n=2}^{\infty} \frac{(-3)^n}{5^n} + \sum_{n=2}^{\infty} \frac{1}{n^2-n}$$

R p

$$\sum_{n=2}^{\infty} \left(\frac{-3}{5}\right)^n + \sum_{n=2}^{\infty} \frac{1}{n(n-1)} = \frac{A}{n-1} + \frac{B}{n}$$

$\left|\frac{-3}{5}\right| = \frac{3}{5} < 1$

$n=1 \quad n=0$

$n=1 \to 1 = A(n) + B(n-1)$

$1 = A(1)$

$A = 1$

$$\frac{r^p}{1-r}$$

$$\frac{\left(\frac{-3}{5}\right)^2}{1 - -\frac{3}{5}}$$

$n=0 \to 1 = B(-1)$

$B = -1$

$$\frac{9}{40} + \sum_{2}^{\infty} \frac{1}{n-1} - \frac{1}{n}$$

$$\frac{1}{1} - \lim_{n \to +\infty} \cancel{\frac{1}{n}}\ 0$$

$$\frac{9}{40} + 1 \cdot \frac{40}{40}$$

$$\boxed{\frac{49}{40}}$$ //

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$\{a_n\} = \frac{2^n \cdot n}{n!}$, ¿Crece o decrece?

$\frac{a_{n+1}}{a_n} \geq 1$, true / Crece
False \ Decrece

$$\frac{\frac{2^{n+1} \cdot (n+1)}{(n+1)!}}{\frac{2^n \cdot n}{n!}} \geq 1$$

$$\frac{2^{n+1} \cdot (n+1) \cdot n!}{(n+1)! \cdot 2^n \cdot n} \geq 1$$

$$\frac{\cancel{2^n} \cdot 2^1 \cdot \cancel{(n+1)} \cdot \cancel{n!}}{\cancel{(n+1)} \cdot \cancel{n!} \cdot \cancel{2^n} \cdot n} \geq 1$$

$$\frac{2}{n} \geq 1$$

$2 \geq n$, $n \geq 3$

$n \leq 2$

$3 \leq 2$, Falso

$\therefore$ Decreciente

Por induccion

$$\sum_{i=1}^{n} (3i-2) = \frac{n(3n-1)}{2}$$

$n=1$ $$\frac{3\cdot 1-2}{1} = \frac{\cancel{1}\cdot(3\cdot 1-1)}{2}$$

$$1 = 1 \checkmark$$

$n=p$ $$\sum_{i=1}^{p} (3i-2) = \frac{p(3p-1)}{2}, \text{ Hi}$$

$3(p+1)-1$

$3p+3-1$

$3p+2$

$n=p+1$ $$\sum_{i=1}^{p+1} (3i-2) = \frac{(p+1)(3p+2)}{2}, \text{ H(Q1)}$$

Respuesta

$3p^2+2p+3p+2$

$3p^2+5p+2$

$$\sum_{i=1}^{p+1} (3i-2)$$

$3(p+1)-2$

$3p+3-2$

$3p+1$

$$\sum_{i=1}^{p} (3i-2) + 3p+1$$

$$\frac{p(3p-1)}{2} + 3p+1, \text{ Hi}$$

$$\frac{p(3p-1)}{2} + 3p+1 \cdot \frac{2}{2}$$

$$\frac{p(3p-1)}{2}+\frac{(3p+1)2}{2}$$

$$\frac{p(3p-1)+2(3p+1)}{2}$$

$$\frac{3p^2-p+6p+2}{2}$$

$$\frac{3p^2+5p+2}{2} \qquad 3p^2 \qquad 2$$

$$\begin{array}{cccl} 3p & & 2 & = 2p \\ p & & 1 & = +3p \\ \hline & & & \quad 5p \end{array}$$

$$\frac{(3p+2)(p+1)}{2} \quad //$$

Calcular sumas de estas series

$$\sum_{n=2}^{\infty} \frac{(-3)^{n+1}}{4^n}$$

$$(-3) \sum_{n=2}^{\infty} \left(\frac{-3}{4}\right)^n \qquad |r| < 1 \,?$$

$$\left|\frac{-3}{4}\right| = \frac{3}{4} < 1 \checkmark$$

$$(-3) \cdot \frac{\left(\frac{-3}{4}\right)^2}{1 - \frac{-3}{4}} = \boxed{\frac{-27}{28}}$$

$$\sum_{n=3}^{\infty} \frac{5n - (n+1)}{5^{n+1}}$$

$$\sum_{n=3}^{\infty} \left[\frac{5n}{5^{n+1}} - \frac{n+1}{5^{n+1}} \right]$$

$$\sum_{n=3}^{\infty} \left[\frac{\cancel{5}}{\cancel{5}} \cdot \frac{n}{5^n} - \frac{n+1}{5^{n+1}} \right]$$

$$\frac{3}{5^3} - \cancelto{0}{\lim_{n \to +\infty} \frac{n+1}{5^{n+1}}} \qquad n^p < a^n$$

$$\boxed{\frac{3}{125}}$$

Vea si estas series convergen o divergen

$$\sum_{n=1}^{\infty} \frac{1+\operatorname{sen}(n^2)}{n^3+n}$$

$$-1 \le \operatorname{sen}(n^2) \le 1$$

$$0 \le \frac{\operatorname{sen}(n^2)}{n^3+n} \le \frac{2}{n^3+\cancel{n}}$$

$$2\sum_{n=1}^{\infty} \frac{1}{n^3}$$, p serie, $e > 1$
Converge

Entonces por criterio de la comparacion directa

$$\sum_{n=1}^{\infty} \frac{1+\operatorname{sen}(n^2)}{n^3+n} \leftarrow \text{Converge}$$