(1) [5 puntos] Sea X una variable aleatoria continua, cuya distribución de probabilidad es:

$$f_X(x) = \begin{cases} ke^{-\frac{x}{4}+1} & \text{si } \ x \geq \frac{1}{2} \\ 0 & \text{si } \ x < \frac{1}{2} \end{cases}$$

Determine el valor de k.

$$\int_{\frac{1}{2}}^{+\infty} k \cdot e^{-\frac{x}{4}+1} = 1$$

$$\int_{\frac{1}{2}}^{+\infty} k \cdot e^{-\frac{x}{4}} \cdot e^{1} = 1$$

$$k \cdot e^{1} \int_{\frac{1}{2}}^{+\infty} e^{\frac{-x}{4}} = 1$$

$$\int e^{\frac{-x}{4}} dx \qquad u = \frac{-x}{4} \qquad du = \frac{4^{-1} \cdot -x}{1} \rightarrow du = \frac{-1}{4} dx$$

$$4 du = -1 dx$$

$$-4 du = dx$$

$$-4 \int e^{u} \rightarrow -4e^{u} \rightarrow \boxed{-4e^{\frac{-x}{4}}}$$

$$\int_{\frac{1}{2}}^{+\infty} -4 \cdot e^{\frac{-x}{4}} \rightarrow -4 \cdot e^{\frac{-x}{4}} \Big|_{\frac{1}{2}}^{+\infty}$$

$$\rightarrow \cancel{-4 \cdot e^{\frac{-\infty}{4}}} - -4 \cdot e^{\frac{-\frac{1}{2}}{4}}$$

$$k \cdot e^{1} \cdot 4 \cdot e^{\frac{-1}{8}} = 1$$

$$4 k \cdot e^{\frac{7}{8}} = 1$$

$$k \cdot e^{\frac{7}{8}} = \frac{1}{4}$$

$$k = \frac{\frac{1}{4}}{e^{\frac{7}{8}}} \rightarrow k = \frac{1}{4 \cdot e^{\frac{7}{8}}} \rightarrow k = \boxed{0,104215504 9}$$

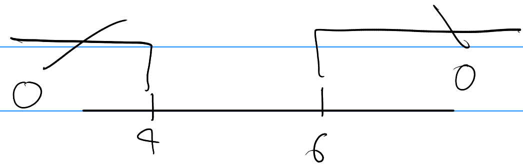
$$= \boxed{0,104}$$

Sea X una variable aleatoria continua con distribución de probabilidad dada por:

$$f_X(x) = \begin{cases} \dfrac{k}{x^2} & \text{si } \ 4 \leq x \leq 6 \\ 0 & \text{en caso contrario} \end{cases}$$

donde $k = 12$.

(2) [2 puntos] Determine $P\left[2 < X \leq \dfrac{24}{5}\right]$.



$$12 \int_4^{\frac{24}{5}} x^{-2} \to \int x^{-2} \to \frac{-1}{x}$$

$$12 \cdot \left. \frac{-1}{x} \right|_4^{\frac{24}{5}}$$

$$12 \cdot \left[\frac{-1}{\frac{24}{5}} - -\frac{1}{4} \right]$$

$$\frac{12}{24} = \frac{1}{2} = \boxed{0{,}5}$$

(3) [2 puntos] Determine $E(X)$.

$$12 \cdot \int_4^6 \cancel{x} \cdot \frac{1}{\cancel{x^2}} , \int \frac{1}{x} = \ln|x|$$

$$12 \cdot \ln|x| \Big|_4^6$$

$$12 \left[\ln(6) - \ln(4) \right]$$

$$4{,}865581297$$

$$\boxed{4{,}866}$$

(4) [2 puntos] Determine $E(X^2)$.

$$12 \cdot \int_4^6 \cancel{x^2} \cdot \frac{1}{\cancel{x^2}}$$

$$12x \Big|_4^6 = 12 \cdot 6 - 12 \cdot 4 = \boxed{24}$$

(5) [2 puntos] Determine la desviación estándar de X.

$$\sqrt{Var(x)} = \sqrt{E(x^2) - [E(x)]^2}$$

$$\sqrt{24 - [12[\ln(6) - \ln(4)]^2}$$

$$= 0{,}5710679814$$

$$= \boxed{0{,}571}$$

Sea X una variable aleatoria, cuya función de distribución acumulada está dada por:

$$F_X(x) = \begin{cases} 0 & \text{si } \; x \leq 2 \\ 1 - \dfrac{2}{x} & \text{si } \; x > 2 \end{cases}$$

(6) **[3 puntos]** Determine $P[6 < X < 13]$.

$$F_x(13) - F_x(6)$$

$$\left(1 - \frac{2}{13}\right) - \left(1 - \frac{2}{6}\right)$$

$$\frac{7}{39} \approx \boxed{0{,}179}$$

(7) **[4 puntos]** Si $P[X > \omega] = 0.146$, determine el valor de ω.

$$P(x > w) = 1 - F_x(w)$$

$$1 - F_x(w) = 0{,}146$$

$$F_x(w) = 0{,}854$$

$$1 - \frac{2}{w} = 0{,}854$$

$$0{,}146 = \frac{2}{w}$$

$$w \cdot 0{,}146 = 2$$

$$w = \frac{2}{0{,}146}$$

$$\boxed{13{,}698}$$