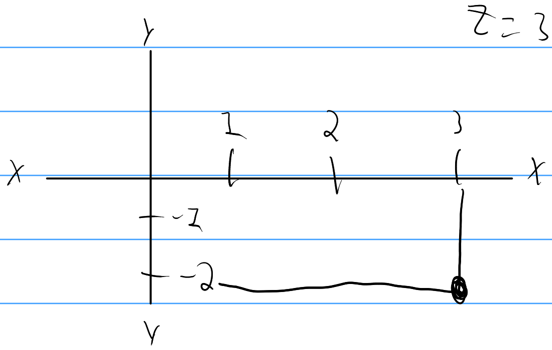
Rectangular $z = a + bi$

$z = 3 - 2i$



RADIANES

Forma polar $r \cdot cis(\theta)$

$r = \sqrt{a^2 + b^2}$ $\qquad cis(\theta) = \cos(\theta) + sen(\theta)i$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Cuadrante	a	b	Rango	Que hago
	Re(z)	Im(z)		
I	+	+	Ya esta	NADA
II	−	+	Esta fuera	$+\pi$ al θ
III	−	−	Esta fuera	$-\pi$ al θ
IV	+	−	Ya esta	NADA

$\theta = \frac{\pi}{2}$ si $a = 0$ y $b > 0 \rightarrow$ Positivo

$\theta = -\frac{\pi}{2}$ si $a = 0$ y $b < 0 \rightarrow$ Negativo

$z = \overset{a}{3} \overset{b}{-2}i$ ← Rectangular $\quad r\,cis(\theta)$

$r = \sqrt{3^2 + (-2)^2} = 3{,}60$

$\theta = \arctan\left(\frac{-2}{3}\right) = -0{,}58$

$3{,}60\ cis(-0{,}58)$ ← Polar

De Polar a rect

$3{,}60 \cdot [\cos(-0{,}58) + sen(-0{,}58)i$

$3 - 2i$

Exponencial $r \cdot e^{i\theta}$

$r\,cis(\theta) \Leftarrow$ Rect

$r\,e^{i\theta} \Leftarrow$ Exponencial

$3 - 2i$

$r\sqrt{3^2+(-2)^2} = 3{,}60$

$\theta = \arctan\left(\frac{-2}{3}\right) = -0{,}58$

$3{,}60 \cdot e^{i \cdot -0{,}58}$

Teorema de de moivre

$z_1 \cdot z_2$ $\quad \frac{z_1}{z_2}$ ← Aplica para ambas

$$\left.\begin{array}{l} r_1 \cdot cis(\theta_1) \cdot r_2\, cis(\theta_2) \\ r_1 \cdot r_2 \cdot cis(\theta_1 + \theta_2) \\ \\ r_1 \cdot e^{i\theta_1} \cdot r_2 \cdot e^{i\theta_2} \\ r_1 \cdot r_2 \cdot e^{i(\theta_1 + \theta_2)} \end{array}\right\} \text{Se suman } \theta$$

$$\left.\begin{array}{l} \frac{r_1 \cdot cis(\theta_1)}{r_2 \cdot cis(\theta_2)} = \frac{r_1}{r_2} \cdot cis(\theta_1 - \theta_2) \\ \\ \frac{r_1 \cdot e^{i\theta_1}}{r_2 \cdot e^{i\theta_2}} = \frac{r_1}{r_2} \cdot e^{i(\theta_1 - \theta_2)} \end{array}\right\} \text{Se restan } \theta$$

$$\left.\begin{array}{l} (r\, cis(\theta))^n = r^n \cdot cis(\theta \cdot n) \\ (re^{i\theta})^n = r^n \cdot e^{i\theta \cdot n} \end{array}\right\} \text{Se eleva y multiplica}$$

$z = 2$ ✓ → $2+0i$

$z = i$ ✓ → $0+1i$

Expresar esto en rectangular

$$\frac{(\overbrace{1-i}^{z_1})^6\,(\overbrace{1-i\sqrt{3}}^{z_2})^3}{(\underbrace{2i-2\sqrt{3}}_{z_3})^7}$$

1. Pasar todo a polar
2. Recordar tabla de θ
3. Usar exponentes

$z_1 = 1 - i$

$a = 1 \quad b = -1$

$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\theta = \arctan\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$

$z_1 = \sqrt{2}\ \text{cis}\left(-\frac{\pi}{4}\right)$

Cuadrante	Re(z) a	Im(z) b	Rango	Que hago
I	+	+	Ya esta	NADA
II	−	+	Esta fuera	$+\pi$ al θ
III	−	−	Esta fuera	$-\pi$ al θ
IV	+	−	Ya esta	NADA

$\theta = \frac{\pi}{2}$ si $a = 0$ y $b > 0 \to$ Positivo

$\theta = -\frac{\pi}{2}$ si $a = 0$ y $b < 0 \to$ Negativo

$z_2 = 1 - i\sqrt{3}$

$a = 1 \quad b = -\sqrt{3}$

$r = \sqrt{1^2 + (-\sqrt{3})^2} = 2$

$\theta = \arctan\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$

$z_2 = 2 \cdot \text{cis}\left(-\frac{\pi}{3}\right)$

$z_3 = -2\sqrt{3} + 2i$

$a = -2\sqrt{3} \quad b = 2$

$r = \sqrt{(-2\sqrt{3})^2 + 2^2} = 4$

$\theta = \arctan\left(\frac{2}{-2\sqrt{3}}\right) + \pi = \frac{5\pi}{6}$

$z_3 = 4 \cdot \text{cis}\left(\frac{5\pi}{6}\right)$

$$\frac{(\overbrace{1-i}^{z_1})^6\,(\overbrace{1-i\sqrt{3}}^{z_2})^3}{(\underbrace{2i-2\sqrt{3}}_{z_3})^7}$$

$(r\,\text{cis}(\theta))^n = r^n \cdot \text{cis}(\theta \cdot n)$

$$\frac{\left(\sqrt{2}\ \text{cis}\left(-\frac{\pi}{4}\right)\right)^6 \cdot \left(2\ \text{cis}\left(-\frac{\pi}{3}\right)\right)^3}{\left(4\ \text{cis}\left(\frac{5\pi}{6}\right)\right)^7}$$

$$\frac{(\sqrt{2})^6 \cdot \text{cis}\left(-\frac{\pi}{4}\cdot 6\right) \cdot 2^3 \cdot \text{cis}\left(-\frac{\pi}{3}\cdot 3\right)}{4^7 \cdot \text{cis}\left(\frac{-5\pi}{6}\cdot 7\right)}$$

$$\frac{r_1 \cdot cis(\theta_1)}{r_2 \cdot cis(\theta_2)} = \frac{r_1}{r_2} \cdot cis(\theta_1 - \theta_2)$$

$$\frac{(2\sqrt{2})^6 \cdot cis\left(-\frac{\pi}{4} \cdot 6\right) \cdot 2^3 \cdot cis\left(-\frac{\pi}{\cancel{3}} \cdot \cancel{3}\right)}{4^7 \cdot cis\left(\frac{-5\pi}{6} \cdot 7\right)}$$

$$\frac{2^3 \cdot 2^3}{2^{14}} \cdot \frac{cis\left(\frac{-\pi\, 6}{4} + -\pi\right)}{cis\left(\frac{35\pi}{6}\right)}$$

$$\frac{1}{2^8} \cdot \frac{cis\left(\frac{-5\pi}{2}\right)}{cis\left(\frac{35\pi}{6}\right)}$$

$$\frac{1}{2^8} \cdot cis\left(\frac{-5\pi}{2} - \frac{35}{6}\right)$$

$$\frac{1}{2^8} \cdot cis\left(\frac{-25\pi}{3}\right) \leftarrow \text{Polar}$$

$$\frac{1}{2^8} \cdot \left[\cos\left(\frac{-25\pi}{3}\right) + sen\left(\frac{-25\pi}{3}\right) i\right]$$

$$\boxed{\frac{1}{2^8} \cdot \cos\left(\frac{-25\pi}{3}\right) + \frac{1}{2^8}\, sen\left(\frac{-25\pi}{3}\right) i}$$

Raíces complejas, se usa para resolver
$x^n = z$, z complejo

a) se factoriza √
b) se pasa a polar
c) se usa de moivre

Resuelva en $\mathbb{C}$ $\qquad a^2-b^2=(a-b)(a+b)$

$x^4 = 1 \longrightarrow 1+0i \qquad a^2+b^2=(a-bi)(a+bi)$

$x^4-1=0 \qquad k=0,1,2,$

$(x^2)^2-(1)^2=0$

$(x^2-1)(x^2+1)=0$

$(x-1)(x+1)(x-i)(x+i)=0$

$x-1=0 \quad x+1=0 \quad x-i=0 \quad x+i=0$

$x=1 \quad x=-1 \quad x=i \quad x=-i$

$1+0i \quad -1+0i \quad 0+i \quad 0-i$

$z1 = 1+0i$
$r=\sqrt{1^2+0^2}=1$
$\theta=\tan^{-1}\left(\frac{0}{1}\right)=0$
$1\cdot \text{Cis}(0)$

$z2=-1+0i$
$r=\sqrt{(-1)^2+0^2}=1$
$\theta=\tan^{-1}\left(\frac{0}{-1}\right)+\pi=\pi$
$1\cdot\text{Cis}(\pi)$

$z_3=0+i$
$r=\sqrt{0^2+1^2}=1$
$\theta=\frac{\pi}{2}$
$1\cdot\text{Cis}\left(\frac{\pi}{2}\right)$

$z4=0-i$
$r=\sqrt{0^2+(-1)^2}=1$
$\theta=\frac{-\pi}{2}$
$1\cdot\text{Cis}\left(\frac{-\pi}{2}\right)$

$x= 1\text{Cis}(0),\ 1\text{Cis}(\pi),\ 1\text{Cis}\left(\frac{\pi}{2}\right),\ 1\text{Cis}\left(-\frac{\pi}{2}\right)$

$$x^n = r\,cis(\theta) \qquad k = 0 \ldots n-1$$

$$x = \sqrt[n]{r\,cis(\theta)} = \sqrt[n]{r} \cdot cis\left(\frac{\theta + 2\pi k}{n}\right)$$

$z^3 = -2-2i$

$r = \sqrt{(-2)^2+(-2)^2} = 2\sqrt{2}$

$\theta = \arctan\left(\frac{-2}{-2}\right) - \pi = \frac{-3\pi}{4}$

$2\sqrt{3} \cdot cis\left(\frac{-3\pi}{4}\right)$

$$z^3 = 2\sqrt{2} \cdot cis\left(\frac{-3\pi}{4}\right)$$

$$z = \sqrt[3]{2\sqrt{2}\ cis\left(\frac{-3\pi}{4}\right)}$$

$$= \sqrt[3]{2\sqrt{2}} \cdot cis\left(\frac{\frac{-3\pi}{4} + 2\pi k}{3}\right)$$

$$k=0 \Rightarrow \sqrt[3]{2\sqrt{2}} \cdot cis\left(\frac{\frac{-3\pi}{4} + 2\pi \cdot 0}{3}\right)$$

$$x_1 = \sqrt[3]{2\sqrt{2}}\ cis\left(\frac{\pi}{6}\right)$$

$$k=1 \Rightarrow \sqrt[3]{2\sqrt{2}} \cdot cis\left(\frac{\frac{-3\pi}{4} + 2\pi \cdot 1}{3}\right)$$

$$x_2 = \sqrt[3]{2\sqrt{2}} \cdot cis\left(\frac{5\pi}{6}\right)$$

$$k=2 \Rightarrow \sqrt[3]{2\sqrt{2}} \cdot cis\left(\frac{\frac{-3\pi}{4} + 2\pi \cdot 2}{3}\right)$$

$$x_3 = \sqrt[3]{2\sqrt{2}} \cdot cis\left(\frac{9\pi}{6}\right)$$