Definición formal

$$f'(x) = \frac{dy}{dx} = [f(x)]' = \frac{d(f(x))}{dx}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{ó utilizar esta}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Calcule $f'(x)$ para $f(x) = \frac{1}{x}$ usando definición formal de la derivada

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}, \quad \text{Forma } \frac{0}{0}$$

$$\lim_{h \to 0} \frac{\frac{\cancel{x} - \cancel{x} - h}{x^2 + hx}}{h}$$

$$\lim_{h \to 0} \frac{\frac{-\cancel{h}}{x^2 + hx}}{\frac{\cancel{h}}{1}}$$

$$\lim_{h \to 0} \frac{-1}{x^2 + hx} = \frac{-1}{x^2 + 0(x)} = \boxed{\frac{-1}{x^2}}$$

En la tabla ... osea usar def formal de derivada

En la tabla ..., usar def formal de derivada

$\left[\frac{1}{x}\right]' = \frac{-1}{x^2}$ es como para demostrarlo

Ejemplo

usando def, calcule $f'(x)$ para $f(x) = \sqrt{x}$

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$\lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \to 0} \frac{\cancel{x}+h - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

Ejemplo

usando def, calcular $f'(x)$ de $f(x) = x^2 - 2x + 7$

$$\lim_{h \to 0} \frac{[(x+h)^2 - 2(x+h) + 7] - [x^2 - 2x + 7]}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-2x} - 2h \cancel{+7} \cancel{-x^2} \cancel{+2x} \cancel{-7}}{h}$$

$$\lim_{h \to 0} \frac{h^2 + 2xh - 2h}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(h+2x-2)}{\cancel{h}} =$$

$$\lim_{h\to 0} h+2x-2 = 0+2x-2 = 2x-2$$

$$\boxed{[x^2-2x+7]' = 2x-2}$$

Usando las reglas de derivacion para calcular la derivada anterior

$$f(x) = x^2 - 2x + 7 \qquad x^{2-1} \quad x'=1 \quad k'=0$$

$$2x - 2$$

Ejemplo

$$\left[x^3 - \frac{2}{x^4}\right]' = [x^3]' - [2x^{-4}]'$$

$$3x^2 - -8x^{-5} \leftarrow -4-1$$

$$\boxed{3x^2+8x^{-5}}$$

$$\left[\sqrt[3]{x^2}\right]' = x^{\frac{2}{3}} = \frac{2}{3}x^{-\frac{1}{3}}$$

Regla de la cadena

Se usa cuando hay una funcion dentro de otra

$$[\ln(x^2+3x)]' = \frac{1}{x^2+3x} \cdot 2x+3 = \frac{2x+3}{x^2+3x}$$

$$[\ln f(x)]' = \frac{1}{f(x)} f'(x)$$

Exponente derivado

Ejemplo $\left[a^{f(x)}\right]' = a^{f(x)} f'(x) \ln a$

$$\left[2^{7-3x}\right]' = 2^{7-3x} \cdot 0-3 \cdot \ln(2)$$

$$2^{7-3x} \cdot -3 \cdot \ln(2)$$

$$-3 \cdot 2^{7-3x} \cdot \ln(2)$$

Ejemplo

$$\left[\sqrt{\tan(x)}\right]'$$

usando $\left[(f(x))^n\right]' = n[f(x)]^{n-1} f'(x)$

$$= \tan(x)^{\frac{1}{2}}$$

$$= \frac{1}{2} \tan(x)^{\frac{-1}{2}} \cdot \sec^2(x)$$

usando $\left[\sqrt{f(x)}\right]' = \frac{1}{2\sqrt{f(x)}} f'(x)$

$$= \frac{1}{2 \cdot \sqrt{\tan(x)}} \cdot \sec^2(x)$$

$$= \frac{\sec^2(x)}{2\sqrt{\tan(x)}}$$

Ejemplo

$\left[(f(x))^n\right]' = n[f(x)]^{n-1} f'(x)$

$$\left[\ln^2(\operatorname{sen}(x) + 3^x)\right]' = 2\ln(\operatorname{sen}(x)+3^x) \cdot \left[\ln(\operatorname{sen}(x)+3^x)\right]'$$

3 funciones $(\ )^2$

$\ln(\ )$

$\operatorname{sen}(x)+3^x$

$$2\ln(\operatorname{sen}(x)+3^x) \cdot \frac{1}{\operatorname{sen}(x)+3^x} \cdot \cos(x) + 3^x \ln(3)$$

$$\frac{2\ln(\operatorname{sen}(x)+3^x)}{\operatorname{sen}(x)+3^x} \cdot \cos(x) + 3^x \ln(3)$$

Ejemplo

$$\left(\frac{e^{\sqrt{x}}}{x^3}\right)' = \frac{\left[e^{\sqrt{x}}\right]' x^3 - e^{\sqrt{x}} \cdot [x^3]'}{(x^3)^2}$$

$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$$= \frac{e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \cdot x^3 - e^{\sqrt{x}} \cdot 3x^2}{x^6}$$

$\left[e^{f(x)}\right]' = e^{f(x)} f'(x)$

Ejemplo

Sea $f: I \to \mathbb{R}$ derivable tal que (Intervalo)

$$x\,f(x) + f^2(x) = 1 \quad , \forall x \in I$$

Verifique

$$[x + 2 \cdot f(x)]\, f'(x) + f(x) = 0 \quad , \forall x \in I$$

Primero calcular $f'(x)$

Sea $x\,f(x) + f^2(x) = 1$

Derivar ambos lados $[x f(x)]' + [f^2(x)]' = 0$

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$[x]' f(x) + x[f(x)]' + 2f(x) \cdot f'(x) = 0$$

$$f(x) + x f'(x) + 2f(x) \cdot f'(x) = 0$$

$$x f'(x) + 2f(x) \cdot f'(x) = -f(x)$$

$$f'(x)(x + 2f(x)) = -f(x)$$

$$f'(x) = \frac{-f(x)}{x + 2f(x)}$$

$$[x + 2 \cdot f(x)]\, f'(x) + f(x) = 0$$

$$\cancel{\{x + 2f(x)\}} \cdot \frac{-f(x)}{\cancel{x + 2f(x)}} + f(x) = 0$$

$$-f(x) + f(x) = 0$$

$$0 = 0 \quad \checkmark$$