Ejercicio 41

Demuestre que $\csc^2(\theta) - \cot^2(\theta) = 1$

Nota:
Recuerde que:

$$\tan(\theta) = \frac{\operatorname{sen}(\theta)}{\cos(\theta)} \qquad \csc(\theta) = \frac{1}{\operatorname{sen}(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\operatorname{sen}(\theta)}$$

Otras identidades trigonométricas

Función par e impar

1. $\cos(-\alpha) = \cos\alpha$
2. $\operatorname{sen}(-\alpha) = -\operatorname{sen}\alpha$

Suma de ángulos

1. $\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \operatorname{sen}\beta\cos\alpha$
2. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$
3. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

Diferencia de ángulos

1. $\operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha\cos\beta - \operatorname{sen}\beta\cos\alpha$
2. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta$
3. $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Ángulos dobles

1. $\operatorname{sen}(2\alpha) = 2\operatorname{sen}\alpha\cos\alpha$
2. $\cos(2\alpha) = \cos^2\alpha - \operatorname{sen}^2\alpha = 1 - 2\operatorname{sen}^2\alpha = 2\cos^2\alpha - 1$

Razones al cuadrado

1. $\operatorname{sen}^2\alpha + \cos^2\alpha = 1$
2. $\sec^2\alpha - \tan^2\alpha = 1$
3. $\csc^2\alpha - \cot^2\alpha = 1$

Sumas como productos

1. $\operatorname{sen}(\alpha) + \operatorname{sen}(\beta) = 2\operatorname{sen}\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
2. $\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Diferencias como productos

1. $\operatorname{sen}(\alpha) - \operatorname{sen}(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\operatorname{sen}\left(\frac{\alpha-\beta}{2}\right)$
2. $\cos(\alpha) - \cos(\beta) = 2\operatorname{sen}\left(\frac{\alpha+\beta}{2}\right)\operatorname{sen}\left(\frac{\alpha-\beta}{2}\right)$

Funciones al cuadrado

1. $\operatorname{sen}^2(\alpha) = \frac{1-\cos(2\alpha)}{2}$
2. $\cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2}$
3. $\tan^2(\alpha) = \frac{1-\cos(2\alpha)}{1+\cos(2\alpha)}$

	30°	45°	60°
sen	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

$$\csc^2(\theta) - \cot^2(\theta) = 1$$

same deno

$$\left(\frac{1}{\operatorname{sen}^2(\theta)}\right) - \left(\frac{\cos^2(\theta)}{\operatorname{sen}^2(\theta)}\right)$$

$$\frac{1 - \cos^2(\theta)}{\operatorname{sen}^2(\theta)} \rightarrow 1 - \cos^2 = \operatorname{sen}^2$$

Despejando

$$\frac{\operatorname{sen}^2(\theta)}{\operatorname{sen}^2(\theta)}$$

$$\boxed{=1}$$

Ejercicio 40

$$\sec^2(\theta) - \tan^2(\theta) = 1$$

$$\frac{1}{\cos^2(\theta)} - \frac{\operatorname{sen}^2(\theta)}{\cos^2(\theta)}$$

$$\frac{1 - \operatorname{sen}^2(\theta)}{\cos^2(\theta)} \rightarrow \operatorname{sen}^2 + \cos^2 = 1$$

$$\cos^2 = 1 - \operatorname{sen}^2$$

$\cos^2$

$$\cos^2$$

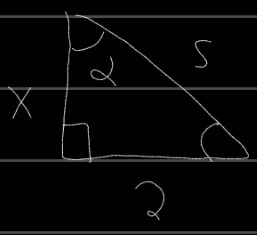
$$= 1$$

Ejemplo 219

Se sabe que $\operatorname{sen}\alpha = -\frac{2}{5}$ *y además* $\cos\beta = \frac{1}{5}$, *con* α *y* β *ángulos en posición estándar, cuyo lado terminal se ubica en el tercer y cuarto cuadrante, respectivamente. Calcule el valor exacto de* $\operatorname{sen}(\alpha - \beta)$

Eje x coseno   Eje y seno

$$\text{Sen}\,\alpha = \frac{2}{5} \to \frac{LO}{HI}$$



Pitagoras

$$5^2 = x^2 + 2^2$$

$$25 - 4 = x^2$$

$$21 = x^2$$

$$x = \sqrt{21}$$

$$\cos\alpha = \frac{LA}{HI} = \frac{\sqrt{21}}{5}$$

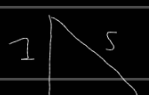
$$\text{Sen}(\alpha - \beta)$$

$$= \text{Sen}\,\alpha \cdot \cos\beta - \text{sen}\,\beta \cdot \cos\alpha$$

$$-\frac{2}{5} \cdot \frac{1}{5} - \text{sen}\,\beta \cdot \cos\alpha$$

III ∧ IV Cuadrante

$$-\frac{2}{5} \cdot \frac{1}{5} - \text{sen}\,\beta - \frac{\sqrt{21}}{5}$$



$$\cos\beta = \frac{LO}{HIP}$$

$$-\frac{2}{5} , \frac{1}{5} - \frac{24}{5} - \frac{21}{5}$$

$$= 0,81$$

y

$$\operatorname{sen}\beta = \frac{\sqrt{24}}{5}$$

Pita

$5^2 = 1^2 + y^2$

$24 = y^2$

$y = \sqrt{24}$

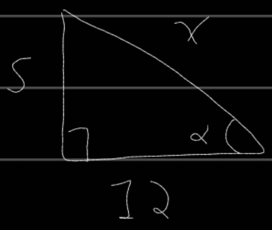
Ejercicio 42

Si α es un ángulo en posición estándar, cuyo lado terminal está ubicado en el segundo cuadrante tal que $\tan\alpha = -\frac{5}{12}$. Halle el valor exacto de la expresión $\frac{\cos(2\alpha)}{\cos\alpha}$

$$\frac{\cos^2(\alpha) - \operatorname{sen}^2(\alpha)}{\cos}$$

$$\tan\alpha = \frac{LO}{LA}$$

$$\tan\alpha = \frac{-5}{12}$$



Pita $\rightarrow$ sen

$x^2 = 5^2 + 12^2$

$x^2 = 25 + 144$

$x^2 = 169$

$x = 13$

$$\cos\alpha = \frac{12}{13}$$

$$\operatorname{sen}\alpha = \frac{5}{13}$$

$$\frac{\cos^2(\alpha) - \operatorname{sen}^2(\alpha)}{\cos}$$

$$\frac{\left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{-\frac{12}{13}}$$

$\rightarrow$ Symbols con cuadrantes

$$= -0,76$$

Ejemplo 227

Verifique la siguiente identidad

$$\frac{\operatorname{sen}(x)\cdot\cos(x)}{1-\cos(x)}-\cot(x)=\csc(x)-\operatorname{sen}(x)$$

Recuerde que $\csc(x)=\dfrac{1}{\operatorname{sen}(x)}$

$$\frac{\operatorname{sen}(x)\cdot\cos(x)}{1-\cos(x)}-\cot(x)$$

$$\frac{\operatorname{sen}(x)\cdot\cos(x)}{1-\cos(x)}-\frac{\cos(x)}{\operatorname{sen}(x)}$$

$$\frac{\operatorname{sen}^2(x)\cdot\cos(x)-\cos(x)\cdot(1-\cos(x))}{(1-\cos(x)\cdot\operatorname{sen}(x)}$$

$$\frac{\operatorname{sen}^2(x)\cdot\cos(x)-\cos(x)+\cos^2(x)}{(1-\cos(x))\cdot\operatorname{sen}(x)}$$

$$\frac{\cos(x)(\operatorname{sen}^2(x)-1+\cos(x))}{(1-\cos(x))\cdot\operatorname{sen}(x)}$$

$$\frac{\cos(x)\cdot[(\cancel{1}-\cos^2(x)+\cos(x)\cancel{-1}]}{(1-\cos(x)\cdot\operatorname{sen}(x)}$$

$$\frac{\cos(x)[-\cos^2(x)+\cos(x)]}{(1-\cos(x)\cdot\operatorname{sen}(x))}$$

$$\frac{\cos(x)[\cos(x)(\cancel{-\cos(x)+1})]}{(\cancel{1-\cos(x)})\cdot\operatorname{sen}(x)}$$

$$\frac{\cos^2(x)}{\operatorname{sen}(x)}$$

$$\frac{1-\operatorname{sen}^2(x)}{\operatorname{sen}(x)}$$

Separarlo en fracciones

$$\frac{1}{\operatorname{sen}(x)} - \frac{\operatorname{sen}^2}{\operatorname{sen}}$$

$$\csc(x) - \operatorname{sen}^2(x)$$

Ejemplo 228

Verifique la siguiente identidad

$$\cos^2(\theta) - \operatorname{sen}^2(\theta) = \frac{1-\tan^2(\theta)}{1+\tan^2(\theta)}$$

$$\frac{1 - \frac{\operatorname{sen}^2(x)}{\cos^2(x)}}{1 + \frac{\operatorname{sen}^2(x)}{\cos^2(x)}}$$

$$= \frac{\frac{\cos^2 - \operatorname{sen}^2(x)}{\cancel{\cos^2(x)}}}{\frac{\cos^2(x) + \operatorname{sen}^2(x)}{\cancel{\cos^2(x)}}}$$

$$\frac{\cos^2(x) - \operatorname{sen}^2(x)}{\cos^2(x) + \operatorname{sen}^2(x)} = 1$$

$$\cos^2(x) - \operatorname{sen}^2(x)$$

Ejemplo 230

Verifique la siguiente identidad

$$\frac{1-\operatorname{sen}(\alpha)-\cos(2\alpha)}{\operatorname{sen}(2\alpha)-\cos(\alpha)}=\tan(\alpha)$$

$$\frac{1-\operatorname{sen}(x)-\left[\cos^2(x)-\operatorname{sen}^2(x)\right]}{2\operatorname{sen}(x)\cdot\cos(x)-\cos(x)}$$

$$\frac{1-\operatorname{sen}(x)-\cos^2(x)+\operatorname{sen}^2(x)}{\cos(x)\left[2\operatorname{sen}(x)-1\right]}$$

$$\frac{\cancel{1}-\operatorname{sen}(x)\cancel{-1}+\operatorname{sen}^2(x)}{\cos(x)\left[2\operatorname{sen}(x)-1\right]}$$