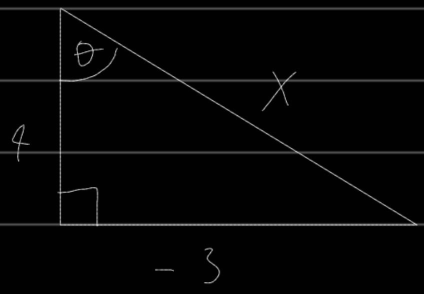
Práctica de Trigonometría

1) Si θ es un ángulo en posición estándar, tal que $\tan\theta = -\dfrac{3}{4}$ y $\cos\theta > 0$, determine el valor exacto de $\cos\left(2\theta - \dfrac{\pi}{3}\right)$.

$$\tan(\theta) = -\frac{3}{4} \rightarrow \tan = \frac{\text{Lado Opuesto}}{\text{Lado Adyacente}}$$



$$x^2 = 4^2 + 3^2$$
$$x^2 = 16 + 9$$
$$x^2 = 25$$
$$x = 5 \rightarrow \text{Hip} = 5$$

$$\boxed{\operatorname{sen}(\theta) = -\frac{3}{5}} \qquad \boxed{\cos(\theta) = \frac{4}{5}}$$

$$\cos\left(2\theta - \frac{\pi}{3}\right)$$

$$\cos(2\theta)\cdot\cos\left(\frac{\pi}{3}\right) + \operatorname{sen}(2\theta)\cdot\operatorname{sen}\left(\frac{\pi}{3}\right)$$

$$\cos(2\theta) = 2\cos^2(\theta) - 1 \qquad \cos\left(\frac{\pi}{3}\right)$$

$$2\left(\frac{4}{5}\right)^2 - 1$$

$$\boxed{\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}}$$

$$\boxed{\cos(2\theta) = \frac{7}{25}}$$

$\text{Sen}(2\theta) = 2\,\text{Sen}(\theta)\cdot\text{Cos}(\theta)$

$\text{Sen}\left(\frac{\pi}{3}\right)$

$2\left(-\frac{3}{5}\right)\cdot\left(\frac{4}{5}\right)$

$\boxed{\text{Sen}\,2(\theta) = -\frac{24}{25}}$

$\boxed{\text{Sen}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}}$

$\text{Cos}\left(2\theta - \frac{\pi}{3}\right) = \frac{7}{25}\cdot\frac{1}{2} + -\frac{24}{25}\cdot\frac{\sqrt{3}}{2}$

$\boxed{\text{R/}\ \text{Cos}\left(2\theta - \frac{\pi}{3}\right) = \frac{7 - 24\sqrt{3}}{50}}$

2) Verifique las siguientes identidades:

a) $\dfrac{1+\cos(3t)}{\operatorname{sen}(3t)} + \dfrac{\operatorname{sen}(3t)}{1+\cos(3t)} = 2\csc(3t)$

b) $\left(\dfrac{\tan\beta+1}{\tan\beta-1}\right)^2 = \dfrac{1+\operatorname{sen}(2\beta)}{1-\operatorname{sen}(2\beta)}$

c) $\dfrac{\operatorname{sen}\,\alpha+\tan\alpha}{\cot\alpha+\csc\alpha} = \sec\alpha - \cos\alpha$

d) $\dfrac{\tan x - \operatorname{sen}\,x}{\operatorname{sen}^3 x} = \dfrac{\sec x}{1+\cos x}$

$X = t$ (Para no confundir la letra t y el mas +

A) $\dfrac{1+\text{Cos}(3X)}{\text{Sen}(3X)} + \dfrac{\text{Sen}(3X)}{1+\text{Cos}(3X)} = 2\,\text{Csc}(3X)$

$\dfrac{(1+\text{Cos}(3X))^2 + (\text{Sen}(3X))^2}{\text{Sen}(3X)\cdot(1+\text{Cos}(3X))}$

$1 + 2\,\text{Cos}(3X) + \text{Cos}^2(3X) + \text{Sen}^2(3X)$

$$\mathrm{sen}(3x)\cdot(1+\cos(3x))$$

$$\frac{1+2\cos(3x)+1}{\mathrm{sen}(3x)\cdot(1+\cos(3x))} \to \mathrm{sen}^2(3x)+\cos^2(3x)=1$$

$$\frac{2+2\cos(3x)}{\mathrm{sen}(3x)\cdot(1+\cos(3x)}$$

$$\frac{2\,\cancel{(1+\cos(3x))}}{\mathrm{sen}(3x)\cdot\cancel{(1+\cos(3x))}}$$

$$\frac{2}{\mathrm{sen}(3x)}$$

$$\boxed{R/\quad 2\csc(3x)} \to \csc = \frac{1}{\mathrm{sen}}$$

b) $$\left(\frac{\tan\beta+1}{\tan\beta-1}\right)^2 = \frac{1+\mathrm{sen}(2\beta)}{1-\mathrm{sen}(2\beta)}$$

$x=\beta$, para no confundir symbolos

$$\left(\frac{\tan(x)+1}{\tan(x)-1}\right)^2 = \frac{1+\mathrm{sen}(2x)}{1-\mathrm{sen}(2x)}$$

$$\frac{(\tan(x)+1)^2}{(\tan(x)-1)^2} \qquad \frac{1+2\,\mathrm{sen}(x)\cdot\cos(x)}{1-2\,\mathrm{sen}(x)\cdot\cos(x)}$$

$$\frac{\tan^2(x)+2\tan(x)+1}{\tan^2(x)-2\tan(x)+1}$$

$$\frac{\frac{\operatorname{sen}^2(x)}{\cos^2(x)} + 2\left(\frac{\operatorname{sen}(x)}{\cos(x)}\right)+1}{\frac{\operatorname{sen}^2(x)}{\cos^2(x)} - 2\left(\frac{\operatorname{sen}(x)}{\cos(x)}\right)+1}$$

$$\frac{\frac{\operatorname{sen}^2(x) + 2\operatorname{sen}(x)\cdot\cos(x) + \cos^2(x)}{\cancel{\cos^2(x)}}}{\frac{\operatorname{sen}^2(x) - 2\operatorname{sen}(x)\cdot\cos(x) + \cos^2(x)}{\cancel{\cos^2(x)}}}$$

$$\frac{\operatorname{sen}^2(x) + 2\operatorname{sen}(x)\cdot\cos(x) + \cos^2(x)}{\operatorname{sen}^2(x) - 2\operatorname{sen}(x)\cdot\cos(x) + \cos^2(x)}$$

$$\frac{\operatorname{sen}^2(x)+\cos^2(x) + 2\operatorname{sen}(x)\cdot\cos(x)}{\operatorname{sen}^2(x)+\cos^2(x) - 2\operatorname{sen}(x)\cdot\cos(x)}$$

$$\frac{1 + 2\operatorname{sen}(x)\cdot\cos(x)}{1 - 2\operatorname{sen}(x)\cdot\cos(x)}$$

$$\boxed{R/ \quad \frac{1+\operatorname{sen}(2x)}{1-\operatorname{sen}(2x)}}$$

c) $$\frac{\operatorname{sen}\alpha + \tan\alpha}{\cot\alpha + \csc\alpha} = \sec\alpha - \cos\alpha$$

$x = \alpha$

$$\frac{1}{\cos} - \cos$$

$$\frac{\operatorname{sen}(x) + \tan(x)}{\cot(x) + \csc(x)}$$

$1 - \cos^2$

$$\frac{\quad}{\cos}$$

$$\frac{\operatorname{sen}(x) + \dfrac{\operatorname{sen}(x)}{\cos(x)}}{\dfrac{\cos(x)}{\operatorname{sen}(x)} + \dfrac{1}{\operatorname{sen}(x)}}$$

$$\frac{\dfrac{\operatorname{sen}(x)\cdot\cos(x) + \operatorname{sen}(x)}{\cos(x)}}{\dfrac{\cos(x)+1}{\operatorname{sen}(x)}}$$

$$\frac{\dfrac{\operatorname{sen}(x)\left(\cos(x)+1\right)}{\cos(x)}}{\dfrac{\cos(x)+1}{\operatorname{sen}(x)}}$$

$$\frac{\operatorname{sen}^2(x)\ \left(\cancel{\cos(x)+1}\right)}{\cos(x)\ \left(\cancel{\cos(x)+1}\right)}$$

$$\frac{\operatorname{sen}^2(x)}{\cos(x)} \longrightarrow \quad \sec^2(x) + \cos^2(x) = 1$$

$$\sec^2(x) = 1 - \cos^2(x)$$

$$\frac{1-\cos^2}{\cos(x)}$$

$$\frac{1}{\cos(x)} - \frac{\cos^2}{\cos}$$

$$\frac{1}{\cos(x)} - \cos(x)$$

b)

$$\boxed{\sec(x) = \cos(x)}$$

d) $$\frac{\tan x - \operatorname{sen} x}{\operatorname{sen}^3 x} = \frac{\sec x}{1+\cos x}$$

$$\frac{\dfrac{\operatorname{sen}(x)}{\cos(x)} - \operatorname{sen}(x)}{\operatorname{sen}^3(x)}$$

$$\frac{\dfrac{\operatorname{sen}(x) - \operatorname{sen}(x)\cdot\cos(x)}{\cos(x)}}{\operatorname{sen}^3(x)}$$

$$\frac{\dfrac{\operatorname{sen}(x)(1-\cos(x))}{\cos(x)}}{\dfrac{\operatorname{sen}^3(x)}{1}}$$

$$\frac{\cancel{\operatorname{sen}(x)}(1-\cos(x))}{\cancel{\operatorname{sen}^3(x)}\cdot\cos(x)}$$

$$\frac{1-\cos(x)}{\operatorname{sen}^2(x)\cdot\cos(x)}$$

$$\frac{1-\cos(x)}{\cos(x)\cdot(1-\cos^2(x))}$$

Diferencia de cuadrados

$$\frac{\cancel{1-\cos(x)}}{\cos(x)\cdot(1+\cos(x))\cdot\cancel{(1-\cos(x))}}$$

$$\frac{1}{\cos(x)\cdot(1+\cos(x))}$$

$$\boxed{R/ \quad \frac{\sec(x)}{1+\cos(x)}}$$

3) Resuelva las siguientes ecuaciones:

a) $(2\ \text{sen}\ x - 4)(\sec x + 2) = 0$

b) $2\ \text{sen}(x) = \text{sen}(2x)$

c) $\left(2\cos(4x) + \sqrt{3}\right)(1 - \tan x) = 0$

d) $2\tan^2 x + 3\sec x = 0$, en el intervalo $[0, 3\pi[$.

A) $(2\,\text{sen}(x) - 4)(\sec(x) + 2) = 0$

$2\,\text{sen}(x) - 4 = 0$

$2\,\text{sen}(x) = 4$

$\text{sen}(x) = 2$

No solution

$\sec(x) + 2 = 0$

$\sec(x) = -2$

$\cos = -\frac{1}{2}$

$\arccos\left(-\frac{1}{2}\right)$

$= \frac{2\pi}{3}$

$x_1 = \pi - \frac{2\pi}{3} \qquad x_2 = \pi + \frac{2\pi}{3}$

$\boxed{x_1 = \frac{\pi}{3}} \qquad \boxed{x_2 = \frac{5\pi}{3}}$

$$\boxed{R/ \quad S = \left\{x_1 = \frac{\pi}{3} + 2\pi k,\ x_2 = \frac{5\pi}{3} + 2\pi k,\ k \in \mathbb{Z}\right\}}$$

b) $2\ \mathrm{sen}(x) = \mathrm{sen}(2x)$

$$2\ \mathrm{sen}(x) = \mathrm{sen}(2x)$$
$$2\ \mathrm{sen}(x) = 2\ \mathrm{sen}(x) \cdot \cos(x)$$
$$1 = \cos(x)$$

$$\boxed{S = \{x = 2\pi k,\ k \in \mathbb{Z}\}}$$

c) $\left(2\cos(4x) + \sqrt{3}\right)(1 - \tan x) = 0$

$$2\cos(4x) + \sqrt{3} = 0$$
$$2\cos(4x) = -\sqrt{3}$$
$$\cos(4x) = -\frac{\sqrt{3}}{2}$$
$$\cos(\theta) = -\frac{\sqrt{3}}{2}$$
$$\arccos\left(-\frac{\sqrt{3}}{2}\right)$$
$$\boxed{x_1 = \frac{\pi}{6}}$$
$$\boxed{x_2 = \frac{11\pi}{6}}$$

$$1 - \tan(x) = 0$$
$$\tan(x) = 1$$
$$\arctan(1) = \frac{\pi}{4}$$
$$\boxed{x_1 = \frac{\pi}{4}}$$
$$\boxed{x_2 = \frac{5\pi}{4}}$$

$$S_1 = \left\{x_1 = \frac{\pi}{6} + 2\pi k,\ x_2 = \frac{11\pi}{6} + 2\pi k,\ k \in \mathbb{Z}\right\}$$

$$S_2 = \left\{x_1 = \frac{\pi}{4} + \pi k,\ x_2 = \frac{5\pi}{4} + \pi k,\ k \in \mathbb{Z}\right\}$$

d) $2\tan^2 x + 3\sec x = 0$, en el intervalo $[0, 3\pi[$.

$$2\left(\frac{sen^2(x)}{cos^2(x)}\right) + 3\left(\frac{1}{cos(x)}\right) = 0$$

$$\frac{2sen^2(x)}{cos^2(x)} + \frac{3}{cos(x)} = 0$$

$$\frac{2sen^2(x) + 3cos(x)}{cos^2(x)} = 0$$

$$2sen^2(x) + 3cos(x) = 0$$
$$2(1 - cos^2(x)) + 3cos(x) = 0$$
$$2 - 2cos^2(x) + 3cos(x) = 0$$
$$-2cos^2(x) + 3cos(x) + 2 = 0$$
$$u = cos(x)$$

$$-2u^2 + 3u + 2 = 0$$

$$\begin{array}{lll} 2u & 1 & = -u \\ -u & 2 & = \underline{4u} \\ & & \quad 4u \end{array}$$

$$(2u+1)(-u+2) = 0$$

$$2u + 1 = 0 \qquad -u + 2 = 0$$

$$u = -\frac{1}{2} \qquad u = 2$$

$$cos(x) = 2$$

$$cos(x) = -\frac{1}{2} \qquad \rightarrow \text{No Solution}$$

$$[-1, 1]$$

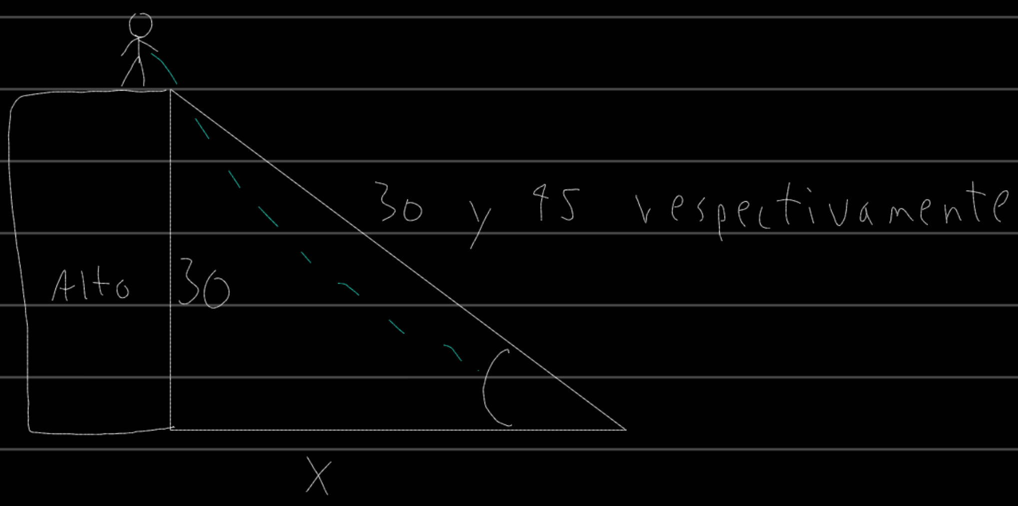
$\arccos\left(-\frac{1}{2}\right)$

$x_1 = \frac{\pi}{3}$

$x_2 = \frac{5\pi}{3}$

$$\boxed{S = \left\{ x_1 = \frac{\pi}{3} + 2\pi k,\ x_2 = \frac{5\pi}{3} + 2\pi k,\ k \in \mathbb{Z} \right\}}$$

4) Desde lo alto de una roca de 30 metros de alto, los ángulos de depresión de dos botes que están en el mar, ambos hacia el oeste de un observador son 45° y 30° respectivamente. ¿Qué distancia hay entre los botes?



Para calcular ambos botes

$\tan(45) = \frac{\text{opuesto}}{\text{Adyacente}}$

$\text{Adyacente} = \frac{\text{opuesto}}{\tan(45)}$

Bote 2

$\tan(30) = \frac{\sqrt{3}}{3} \Rightarrow 0{,}577$

$\frac{30}{0{,}577} = 51{,}96$ metros

Bote 1

$\tan(45) = 1$

$\frac{0}{1} = 30$ metros

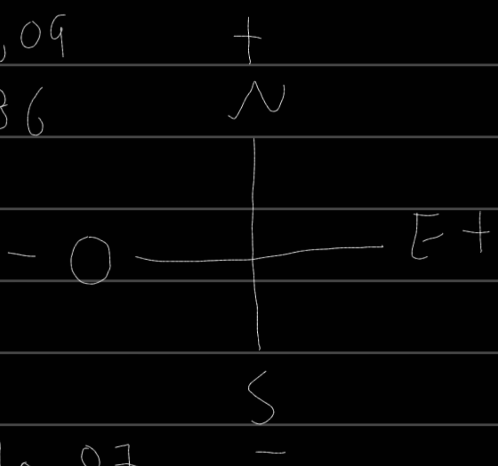
Distancia entre los botes:

$51,96 - 30 = 21,96$ metros

5) Un barco zarpó de un muelle y navegó 5 km en dirección N 35° O. A continuación, el barco viró a un rumbo de S 25° O y navegó 12 km. ¿A qué distancia está el barco del muelle?

No Oeste 35°

Oeste $(x) = -5 \cdot \cos(35) \rightarrow -4,09$
Norte $(y) = 5 \cdot \cos(35) \rightarrow 2,86$



Nor Oeste 25°

Oeste $(x) = -12 \cdot \cos(25) \rightarrow -10,87$
Norte $(y) = 12 \cdot \operatorname{sen}(25) \rightarrow 5,07$

Total $x = -4,09 + -10,87 \rightarrow -14,96$

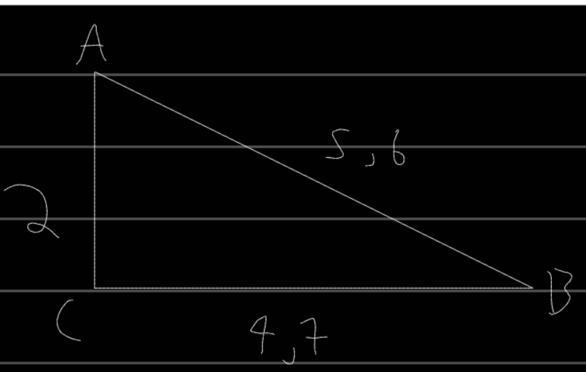
Total $y = 2,86 + 5,07 \rightarrow 7,93$

Distancia $= \sqrt{(-14,96)^2 + (7,93)^2}$

Distancia $= 16,93$

R/ El Barco esta a 16,93 Km del muelle

6) Considere el triángulo ABC, con D tal que $A - D - B$. Si $AC = 2$ cm, $CB = 4.7$ cm, $AB = 5.6$ cm y $\angle ACB = \angle CDB$, determine el perímetro del triángulo DCB.



$A - D - B$

D esta entre A y B

$AD = x$ $\quad DB = 5{,}6 - x$ ya que $AB = 5{,}6$

$$\frac{AD}{AC} = \frac{DB}{CB}$$

$$\frac{x}{2} = \frac{5{,}6 - x}{4{,}7}$$

$4{,}7x = 2(5{,}6 - x)$

$4{,}7x = 11{,}2 - 2x$

$6{,}7x = 11{,}2$

$x = 1{,}67 \longrightarrow \boxed{AD = 1{,}67}$

$$\boxed{\begin{aligned} DB &= 5{,}6 - 1{,}67 \\ &= 3{,}93 \end{aligned}}$$

$DC = x$ $\quad \dfrac{AC}{x} = \dfrac{AD}{DB}$

$$\frac{2}{x} = \frac{1,67}{3,93}$$

$$2(3,93) = 1,67x$$

$$7,86 = 1,67x$$

$$x = 4,70 \rightarrow \boxed{DC = 4,7}$$

Perimetro $DCB \triangle$

$$DC + CB + DB$$

$$4,7 + 4,7 + 3,93$$

$$= 13,33$$

El perimetro de de $DCB\triangle$ es 13,33

Tarea realizada por
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