Sucesiones

Cálculo de términos

Escriba los primeros cinco términos para las siguientes sucesiones:

1. $b_n = \dfrac{(-1)^{n+1}(n^2-4)}{3^{n-4}}$, para $n \geq 0$ — R/ 324, −81, 0, 15, −12
2. $c_k = \left(\dfrac{-1}{2}\right)^k \cdot 2^{k^2}$, para $k \geq 0$ — R/ 1, −1, 4, −64, 4096
3. $b_n = \cos\left(\dfrac{\pi n}{2}\right)$, para $n \geq 0$ — R/ 1, 0, −1, 0, 1
4. $m_p = |4\ \ln(p) - p|$, para $p \geq 1$ — R/ 1; 1,306; 1,901; 2,613; 3,390
5. $a_i = (-1)^{i+1}(i+1) + i^2$, para $i \geq 0$ — R/ −1, 3, 1, 13, 11
6. $a_k = \dfrac{(-1)^k}{k}$, para $k \geq 1$ — R/ $-1, \dfrac{1}{2}, \dfrac{-1}{3}, \dfrac{1}{4}, \dfrac{-1}{5}$
7. $r_m = \dfrac{(-2)^m}{m^3+1}$, para $m \geq 0$ — R/ $1, -1, \dfrac{4}{9}, \dfrac{-2}{7}, \dfrac{16}{65}$
8. $\dfrac{7^k}{k!}$, para $k \geq 0$ — R/ $1, 7, \dfrac{49}{2}, \dfrac{343}{6}, \dfrac{2401}{24}$
9. $a_k = \dfrac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2k+1)}{(2k)!}$, a partir de $k = 0$ — R/ $1, \dfrac{3}{2}, \dfrac{5}{8}, \dfrac{7}{48}, \dfrac{3}{128}$
10. $\dfrac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2j)}{2^j \cdot j!}$, para $j \geq 1$ — R/ $1, 1, 1, 1, 1$
11. $\dfrac{4^q \cdot q!}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2q)}$, para $q \geq 1$ — R/ $2, 4, 8, 16, 32$
12. $\dfrac{1 \cdot 4 \cdot 7 \cdot \ldots \cdot (3i+1)}{2^{i+1}(i+1)!}$, para $i \geq 0$ — R/ $\dfrac{1}{2}, \dfrac{1}{2}, \dfrac{7}{12}, \dfrac{35}{48}, \dfrac{91}{96}$
13. $x_n = 2x_{n-1} - 3x_{n-2}$, para $n \geq 3$ y con $x_1 = 0$, $x_2 = -1$ — R/ $0, -1, -2, -1, 4$
14. $y_n = 3y_{n-1} + 2y_{n-2} - y_{n-3}$, para $n \geq 4$ y con $y_1 = -2$, $y_2 = 0$, $y_3 = 1$ — R/ $-2, 0, 1, 5, 17$

15. $s_n = \dfrac{1}{1+s_{n-1}}$ y con $s_1 = 1$ R/ $1, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}$

16. $b_n = \dfrac{1}{2}(b_{n-1} + b_{n-2})$ y con $b_1 = 0$, $b_2 = 3$ R/ $0, 3, \frac{3}{2}, \frac{9}{4}, \frac{15}{8}$

17. $a_k = 4a_{k-1} - 3a_{k-2}$ y con $a_0 = 1$, $a_1 = -1$ R/ $1, -1, -7, -25, -79$

18. $a_j = 2a_{j-1} + a_{j-2}$ y con $a_1 = 2$, $a_2 = 6$ R/ $2, 6, 14, 34, 82$

19. $x_n = \dfrac{1}{2}(x_{n-2} - x_{n-1})$ y con $x_0 = -1$, $x_1 = -2$ R/ $-1, -2, \frac{1}{2}, \frac{-5}{4}, \frac{7}{8}$

20. $p_j = 2p_{j-3} + p_{j-2} - 2p_{j-1}$ y con $p_0 = -1$, $p_1 = 0$, $p_2 = -4$ R/ $-1, 0, -4, 6, -16$

Sucesiones Monótonas

Derivación

Para cada una de las siguientes sucesiones, determine si son crecientes, decrecientes o no son monótonas.

1. $p_q = 14q - q^2, \forall q \geq 7$ R/ Decrece
2. $f_m = m^3 - 5m^2 - 25m, \forall m \geq 5$ R/ Crece
3. $m_n = n^4 - 2n^2 - 3, \forall n \geq 1$ R/ Crece
4. $a_p = 2400p - 50p^2, \forall p \leq 24$ R/ Crece
5. $b_n = 3n^4 - 4n^3 + 4, \forall n \leq 1$ R/ Decrece
6. $t_r = 10r^2 - \dfrac{5r^3\pi}{3}, \forall r \leq 0$ R/ Decrece
7. $a_n = \dfrac{n}{n-3}, \forall n \in \mathbb{R}$ R/ Decrece
8. $b_n = \dfrac{n}{n+1}, \forall n \geq 1$ R/ Crece
9. $b_p = \dfrac{p^2 - 3p + 6}{p-2}, \forall p \geq 4$ R/ Crece
10. $m_p = \dfrac{p^2}{p^2 - 4}, \forall p \geq 0$ R/ Decrece
11. $d_p = \dfrac{p^3 + 16000}{p}, \forall p \leq 20$ R/ Decrece
12. $a_n = \dfrac{2x^2}{x+1}, \forall n \geq 1$ R/ Crece
13. $a_k = \dfrac{k}{k^2 + 25}, \forall k \geq 5$ R/ Decrece
14. $c_p = \dfrac{(5/4)^p}{p}, \forall p \geq 4$ R/ Crece
15. $d_i = \dfrac{i^3}{2^i}, \forall i \geq 4$ R/ Decrece
16. $a_n = \dfrac{2n}{n^2 - 1}, \forall n \in \mathbb{R}$ R/ Decrece
17. $c_n = \dfrac{n^4}{e^n}, \forall n \geq 4$ R/ Decrece
18. $b_n = \dfrac{n}{1 + \ln(n)}, \forall n \geq 1$ R/ Crece
19. $d_i = \dfrac{i}{\ln(i)}, \forall i \geq 3$ R/ Crece
20. $m_j = \dfrac{1}{j \cdot \ln(j)}, \forall j \geq 2$ R/ Decrece

Términos consecutivos

Para cada una de las siguientes sucesiones, determine si son crecientes, decrecientes o no son monótonas.

1. $a_n = n^2 - 3n$ R/ Crece
2. $a_n = \dfrac{3n}{n+2}$ R/ Crece
3. $s_n = 2 - \dfrac{1}{n-3}$ R/ Crece
4. $b_i = \dfrac{i}{i+1}$ R/ Crece
5. $w_j = \dfrac{1+j}{2j}$ R/ Decrece
6. $c_n = \dfrac{-2n}{1+n}$ R/ Decrece
7. $b_n = \dfrac{2n^2}{n+1}$ R/ Crece
8. $a_n = \dfrac{(n+1)^2}{n^2}$ R/ Decrece
9. $a_m = \dfrac{m!}{m^2}$ R/ Crece
10. $p_k = \dfrac{7^k}{k!}$ R/ Decrece
11. $c_k = \dfrac{k!}{5^k}$ R/ Crece
12. $c_n = \dfrac{3n!}{n \cdot 7^{n+1}}$ R/ Crece
13. $a_n = \dfrac{2^n \cdot n}{3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n+1)}$ R/ Decrece
14. $a_t = \dfrac{1 \cdot 4 \cdot 7 \cdot \ldots \cdot (3t+1)}{3^{t-1} \cdot (2t)!}$ R/ Decrece
15. $m_j = \dfrac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2j)}{j!}$ R/ Crece
16. $a_n = \dfrac{24 \cdot 30 \cdot 36 \cdot \ldots \cdot (6n+6)}{n!}$ R/ Crece
17. $a_n = \dfrac{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n+2)}{(2n+4)!}$ R/ Crece
18. $t_n = \dfrac{n!}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}$ R/ Decrece
19. $a_m = \dfrac{(2m)!}{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2m+1)}$ R/ Crece
20. $t_n = \dfrac{3^n \cdot n!}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}$ R/ Crece
21. $a_n = \dfrac{[1 \cdot 5 \ldots (4n+1)]\,(2n)!}{(3n)!}$ R/ Decrece
22. $c_n = \dfrac{2^n}{(n+1)(n+1)!}$ R/ Decrece

Sucesiones Convergentes

Regla de L'Hôpital

Para cada una de las siguientes sucesiones, determine si son convergentes o divergentes. En caso de ser convergentes, indique su valor de convergencia.

- **Caso #1: formas indeterminadas $\frac{0}{0}$ y $\frac{\pm\infty}{\pm\infty}$:**

1. $a_n = \dfrac{-5n^2+6n+3}{4n^2-3n+1}$ R/ $\dfrac{-5}{4}$
2. $a_t = \dfrac{6t^2+8t-8}{4-9t^2}$ R/ $\dfrac{-2}{3}$
3. $b_k = \dfrac{2k^2+2k-12}{3k^2-5k-2}$ R/ $\dfrac{2}{3}$
4. $m_p = \dfrac{2p^3-54}{3p-p^2}$ R/ Diverge
5. $a_n = \dfrac{20n^2-29n^3-6+11n}{7n^2-4+2n^3}$ R/ $\dfrac{-29}{2}$
6. $b_p = \dfrac{p^3-3p+2}{p^4-4p+3}$ R/ 0
7. $a_t = \dfrac{2t^4+11t^3t+84}{t^3+t^2+8}$ R/ Diverge
8. $a_n = \dfrac{(n+8)(n+1)}{n(n-10)}$ R/ 1
9. $r_k = \dfrac{k^2+k-1}{e^k+e^{-k}}$ R/ 0
10. $a_n = \dfrac{e^n}{2e^n+5n^2}$ R/ $\dfrac{1}{2}$
11. $a_n = \dfrac{2^n}{n^2}$ R/ Diverge
12. $a_n = \dfrac{7^n}{3n^2}$ R/ Diverge
13. $s_n = \dfrac{n+1}{6^{n+1}-1/3}$ R/ $\dfrac{-1}{3}$
14. $a_n = \dfrac{4n-3}{\sqrt{5n^2-n+1}}$ R/ $\dfrac{4\sqrt{5}}{5}$
15. $b_p = \dfrac{p}{\ln^2(p)+2p}$ R/ $\dfrac{1}{2}$
16. $b_m = \dfrac{m+\ln(m)}{m}$ R/ 1
17. $m_p = \dfrac{\ln(\ln(p))}{p}$ R/ 0
18. $a_n = \ln\left(1-\dfrac{6}{n}\right) \div \dfrac{1}{n}$ R/ -6
19. $y_n = \dfrac{\ln(n^4+2n^3-6n)}{\ln(2n^3-4n+1)}$ R/ $\dfrac{4}{3}$
20. $z_n = \left(\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)$ R/ b

- **Caso #2: formas indeterminadas $0 \cdot \pm\infty$ y $\pm\infty \cdot 0$:**

1. $a_m = m^2 - e^{3m}$ R/ Diverge
2. $c_k = e^k - \sqrt{k}$ R/ Diverge
3. $f_b = b \cdot e^{1/b} - b$ R/ 1
4. $x_n = \sqrt{4n^2 + n} - 2n$ R/ $\frac{1}{4}$
5. $s_n = \sqrt{n + \sqrt{n^4 + 1}} - 2n$ R/ Diverge
6. $a_n = n - \sqrt{4n^2 + 4n}$ R/ Diverge
7. $a_n = \sqrt{n^2 + n} - \sqrt{n^2 + 9}$ R/ $\frac{1}{2}$
8. $a_n = \sqrt{n^2 + n + 1} - \sqrt{n^2 + 1}$ R/ $\frac{1}{2}$
9. $b_n = \sqrt{n + 4} - \sqrt{n}$ R/ 0
10. $c_n = \sqrt{n}\,(\sqrt{n + 3} - \sqrt{n + 2})$ R/ $\frac{1}{2}$

- **Caso #3: formas indeterminadas $+\infty - +\infty$ y $-\infty - +\infty$:**

1. $a_n = n \cdot 2^{-n}$ R/ 0
2. $b_n = n \cdot e^{-n}$ R/ 0
3. $b_p = 3^p \cdot p$ R/ 0
4. $a_n = n^2 \cdot e^n$ R/ 0
5. $m_p = p^p \cdot 5^{-p}$ R/ Diverge
6. $a_n = (1 - 3n)(1 - e^{\frac{4}{n}})$ R/ 12
7. $b_n = n \cdot \ln\left(1 - \dfrac{6}{n}\right)$ R/ -6
8. $d_p = p \cdot \ln\left(1 - \dfrac{23}{2p}\right)$ R/ $\dfrac{-23}{2}$
9. $a_p = p \cdot \ln\left(\dfrac{p+1}{p}\right)$ R/ 1
10. $a_n = n \cdot \ln\left(1 + \dfrac{3}{n}\right)$ R/ 3
11. $b_n = n^2 \cdot \ln\left(1 + \dfrac{1}{n}\right)$ R/ Diverge
12. $b_k = k^3 \cdot \ln\left(3 + \dfrac{5}{k}\right)$ R/ Diverge
13. $a_n = n^3 \cdot \ln\left(\dfrac{4}{n}\right)$ R/ Diverge
14. $a_n = \ln\left(1 - \dfrac{1}{n}\right) \cdot \ln(n)$ R/ 0
15. $a_n = \left(\dfrac{n}{n^3+3}\right) \cdot \ln(n^2)$ R/ 0
16. $a_n = (2 + 3n^2) \cdot \sqrt{\dfrac{n}{n^5 - 3}}$ R/ 3
17. $a_n = n \cdot \sqrt{\dfrac{n+1}{n^3+2}}$ R/ 1
18. $a_n = n \cdot \operatorname{sen}\left(\dfrac{1}{n}\right)$ R/ 1
19. $c_n = (n-1) \cdot \operatorname{sen}\left(\dfrac{\pi n}{n-1}\right)$ R/ $-\pi$
20. $m_n = (n+1) \cdot \operatorname{sen}\left(\dfrac{\pi n}{n+1}\right)$ R/ π

- **Caso #4: formas indeterminadas $1^{\pm\infty}$, $(\pm\infty)^0$, $(\pm\infty)^{\pm\infty}$ y 0^0:**

1. $a_n = (2n+1)^{3/n}$ R/ 1
2. $b_n = (e^n + n)^{1/n}$ R/ e
3. $a_n = (1+4^n)^{\left(\frac{1}{1-n}\right)}$ R/ $\frac{1}{4}$
4. $m_p = (1+2p^3)^{\left(\frac{1}{1+\ln(p)}\right)}$ R/ e^3
5. $a_n = \left(1+\frac{2}{3n}\right)^{3n}$ R/ e^2
6. $a_n = \left(1+\frac{\ln(5)}{2n}\right)^{2n+12}$ R/ 5
7. $r_n = \left(\frac{n-2}{n}\right)^{3n}$ R/ e^{-6}
8. $a_m = \left(\frac{m-1}{m+1}\right)^{m}$ R/ e^{-2}
9. $b_n = \left(\frac{2n-3}{2n+5}\right)^{2n+1}$ R/ e^{-8}
10. $m_p = \left(1+\frac{a}{p}\right)^{bp}$ R/ e^{ab}

Teorema del Encaje o del Encuadramiento

Para cada una de las siguientes sucesiones, determine si son convergentes o divergentes. En caso de ser convergente, indique su valor de convergencia.

1. $a_t = \dfrac{\operatorname{sen}(t)}{t}$ R/ 0
2. $a_p = \dfrac{\cos(p)}{p^2}$ R/ 0
3. $a_j = j + \cos(j)$ R/ Diverge
4. $c_n = \dfrac{n + \cos(n)}{6n + 2}$ R/ $\dfrac{1}{6}$
5. $a_n = \dfrac{n}{4 + \cos(2n)}$ R/ Diverge
6. $b_n = \dfrac{2 + \cos(n)}{n^3}$ R/ 0
7. $a_n = \dfrac{\operatorname{sen}^3(n)}{n^4}$ R/ 0
8. $b_k = \dfrac{3^k - \operatorname{sen}(k)}{5^k}$ R/ 0
9. $a_n = \dfrac{\operatorname{sen}(n)}{3^n + 1}$ R/ 0
10. $a_n = \dfrac{3n^2 - \operatorname{sen}(2n)}{n^2} + 1$ R/ 4
11. $a_n = \dfrac{2^n + \cos(3n)}{3^{n+2}}$ R/ 0
12. $a_n = 1 + \dfrac{\cos(n)}{\sqrt{n}}$ R/ 1
13. $a_n = \sqrt{\dfrac{2 + \cos(n)}{n}}$ R/ 0
14. $a_n = \dfrac{2n^3 + n^2 - \arctan(n)}{3n^2 - n^3}$ R/ -2
15. $a_n = \dfrac{1}{2^n} \cdot \ln(2^n + 1)$ R/ 0
16. $c_n = \dfrac{5 + (-2)^n}{3^n}$ R/ 0
17. $f_k = \dfrac{2k^2 + (-1)^k \cdot 3k}{5k^2 - 2k + 2}$ R/ $\dfrac{2}{5}$
18. $a_n = \displaystyle\sum_{k=1}^{n} \dfrac{n + k}{n^2 + k}$ R/ $\dfrac{3}{2}$
19. $a_n = \displaystyle\sum_{k=1}^{n} \dfrac{n + k}{\sqrt{n^4 + k}}$ R/ $\dfrac{3}{2}$
20. $b_n = \displaystyle\sum_{k=1}^{n} \dfrac{3k^2}{k^2 + 2n^3}$ R/ $\dfrac{1}{2}$

Criterio de la Razón y Teorema del Valor Absoluto

Para cada una de las siguientes sucesiones, determine si son convergentes o divergentes.

1. $c_m = \dfrac{(-1)^{m+2}}{m}$ R/ 0
2. $c_m = \dfrac{(-1)^m \cdot m}{m^2+13}$ R/ 0
3. $b_n = \dfrac{(-1)^n}{n} + 2$ R/ 2
4. $b_k = 3 + \dfrac{(-1)^k}{k^2}$ R/ 3
5. $x_n = \dfrac{(-1)^n \cdot n}{n^3+3n}$ R/ 0
6. $d_p = 4 + \dfrac{(-1)^p \cdot p}{2p^2+3}$ R/ 4
7. $a_n = \dfrac{(-1)^n \cdot n^2}{1+n^3}$ R/ 0
8. $a_n = \dfrac{n^6 \cdot (-2)^n}{3+4n^8}$ R/ 0
9. $a_n = \dfrac{(-1)^{n-1} \cdot n^4}{1+n^2+n^3}$ R/ Diverge
10. $b_n = \dfrac{(-1)^{n-1} \cdot 5n}{e^n+\pi}$ R/ 0
11. $a_n = (-1)^n \cdot \text{sen}\left(\dfrac{1}{n}\right)$ R/ 0
12. $b_m = \dfrac{m^3}{m!}$ R/ 0
13. $a_n = \dfrac{n!}{n^n}$ R/ 0
14. $a_n = \dfrac{n!}{2^n}$ R/ Diverge
15. $a_n = \dfrac{4^{n-1}}{n!}$ R/ 0
16. $a_n = \dfrac{(n-2)!}{n!}$ R/ 0
17. $d_n = \dfrac{(n+2)!}{n!}$ R/ Diverge
18. $d_k = \dfrac{(-3)^k}{k!}$ R/ 0
19. $x_n = \dfrac{(-1)^{n+1}}{n!}$ R/ 0
20. $a_n = \dfrac{(-1)^n \cdot 3^n \cdot n!}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}$ R/ Diverge