1. **[5 puntos]** Considere el sistema de ecuaciones

$$\begin{cases} x+2y+3z=7 \\ x+z=y \\ x+5y+5z-14=0 \end{cases}$$

donde x, y, z son incógnitas. Utilice el método de Gauss-Jordan para determinar el conjunto solución del sistema. **(PREGUNTA PARA EVALUAR ATRIBUTO CI, NIVEL INICIAL)**

$$\begin{cases} x+2y+z=7 \\ x-y+z=0 \\ x+5y+5z=14 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 1 & -1 & 1 & 0 \\ 1 & 5 & 5 & 14 \end{array}\right)$$

$$\begin{array}{l} \\ -F1+\widetilde{F2} \\ -F1+\widetilde{F3} \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & -3 & -2 & -7 \\ 0 & 3 & 2 & 7 \end{array}\right)$$

$$\begin{array}{l} \\ -2\cdot F2+\widetilde{F1} \\ -\frac{1}{3}\cdot\widetilde{F2} \\ -3\cdot F2+\widetilde{F3} \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & 0 & \frac{5}{3} & \frac{7}{3} \\ 0 & 1 & \frac{2}{3} & \frac{7}{3} \\ 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

$$x+\frac{5}{3}z=\frac{7}{3} \longrightarrow x=\frac{7}{3}-\frac{5}{3}z$$

$$y+\frac{2}{3}z=\frac{7}{3} \longrightarrow y=\frac{7}{3}-\frac{2}{3}z$$

$$S=\left\{\left(\frac{7}{3}-\frac{5}{3}z, \frac{7}{3}-\frac{2}{3}z, z\right), z\in\mathbb{R}\right\}$$

2. **[4 puntos]** Considere las matrices $A = \begin{pmatrix} 3 & 0 & 2 \\ 2 & -1 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 2 & -3 \\ -4 & 0 \\ 2 & -1 \end{pmatrix}$ y $D = \begin{pmatrix} 6 & 1 \\ 3 & 1 \\ 3 & 0 \end{pmatrix}$.

Determine la matriz B tal que $A^T \cdot B - C = D$.

$$A^T \cdot B - C = D$$
$$A^T \cdot B = D + C$$

$$A^T = \begin{pmatrix} 3 & 2 \\ 0 & -1 \\ 2 & 1 \end{pmatrix}$$

$$D + C = \begin{pmatrix} 6 & 1 \\ 3 & 1 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 2 & -3 \\ -4 & 0 \\ -2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 \\ 0 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^T \cdot B = D + C$$

$$\begin{pmatrix} 3a + 2c & 3b + 2d \\ 0 - c & 0 - d \\ 2a + c & 2b + d \end{pmatrix} = \begin{pmatrix} 8 & -2 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\left\{ \begin{matrix} 3a + 2c & 3b + 2d \\ 0 - c & 0 - d \\ 2a + c & 2b + d \end{matrix} \right. = \begin{pmatrix} 8 & -2 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$$

$3a + 2c = 8 \rightarrow 3a + 2 \cdot 1 = 8 \rightarrow 3a = 6$

$\boxed{a = 2}$

$3b + 2d = -2$

$-c = -1 \rightarrow \boxed{c = 1}$

$-d = 1 \rightarrow \boxed{d = -1}$

$2a + c = 1$

$2b + d = -1 \rightarrow 2 \cdot b + \cancel{-1} = \cancel{-1}$

$\boxed{b = 0}$

$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \boxed{\begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}}$$

4. **[5 puntos]** Sean A, B y C matrices cuadradas 3×3 invertibles. Si se sabe que $\frac{1}{2}A = B^{-1}C$ donde $B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ y $A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, calcule C^{-1}.

Sugerencia: primero utilice propiedades matriciales para determinar C^{-1}.

$$\frac{1}{2}A = B^{-1}\cdot C$$

$$A = 2\cdot B^{-1}\cdot C$$

$$C^{-1} = 2A^{-1}\cdot B^{-1}$$

$$B^{-1} = \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \checkmark$$

$$\begin{array}{l} -F2+\widetilde{F1} \\ \\ \frac{1}{2}\cdot\widetilde{F3} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right)$$

$$C^{-1} = 2A^{-1}\cdot B^{-1}$$

$$A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$A^{-1}\cdot B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\boxed{2A^{-1}\cdot B^{-1} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{pmatrix}}$$

5. **[5 puntos]** Determine $z \in \mathbb{C}$ que satisface simultaneámente las siguientes condiciones :

- $|\bar{z} + (1-i)| = 5$
- $Arg(z-(1+2i)) = \dfrac{3\pi}{4}$

$a+bi-1-2i = \frac{3\pi}{4}$ $\qquad \frac{3\pi}{4} \cdot \frac{180}{\pi}$

$(a-1)+(b-2)i = \frac{3\pi}{4}$

$\frac{b-2}{a-1} = \tan\left(\frac{3\pi}{4}\right)$ $\qquad 135$

$90 < 135 < 180$

$\frac{b-2}{a-1} = -1$ $\qquad a<0 \quad b>0$

$b-2 = 1-a$

$b-2-1 = -a$

$-a = b-3$

$a = 3-b$

$|\bar{z}+1-i| = 5$

$|a+bi+1-i| = 5$

$|(a+1)+(b-1)| = 5$

$(a+1)^2+(b-1)^2 = 5^2$

$(3-b+1)^2+(b-1)^2 = 25$

$(4-b)^2+(b-1)^2 = 25$

$16-8b+b^2+b^2-2b+1 = 25$

$2b^2-10b+17-25=0$

$b^2-10b-8=0$

$b = 5 \pm \sqrt{33}$ $\qquad a = 3-5+\sqrt{33}$

$b = 5+\sqrt{33}$ $\qquad -2+\sqrt{33}$

$\boxed{R/ \quad -2+\sqrt{33}+(5+\sqrt{33})i}$