

o) $P(x) = x^4 - 6x^3 + 22x^2 - 54x + 117$ sabiendo que $3 - 2i$ es uno de sus ceros. el otro es $3 + 2i$

$$\begin{array}{r|rrrr} x^4 & x^3 & x^2 & x & k \\ \hline 1 & -6 & 22 & -54 & 117 \\ & 3-2i & -13 & 27-18i & -217 \\ \hline 1 & -3-2i & 9 & -27+18i & 0 & (x-(3-2i)) \end{array}$$

$$x^3 + (-3-2i)x^2 + 9x + (-27+18i) \quad (3-2i)(-3-2i)$$

$$-9-6i+6i+9i^2$$

$$\begin{array}{r|rrr} 1 & -3-2i & 9 & -27+18i \\ & 3+2i & 0 & 27+18i \\ \hline 1 & 0 & 9 & 0 & (x-(3+2i)) \end{array}$$

$$(3-2i)(-27-18i)$$

$$-81-84i+89i+36i^2$$

$$-81-36$$

$$-217$$

$$(x^2+a) = (x-3i)(x+3i)$$

$$\boxed{(x-3i)(x+3i)(x-(3-2i))(x-(3+2i))}$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^2 + b^2 = (a-bi)(a+bi)$$

r) $P(x) = 4x^4 - 12x^3 + 22x^2 - 16x + 6$ sabiendo que $\frac{1}{2} - \frac{i}{2}$ es un cero de $P(x)$. El otro es $\frac{1}{2} + \frac{i}{2}$

$$\begin{array}{r} 4 \quad -12 \quad 22 \quad -16 \quad 6 \\ \underline{2-2i \quad -6+4i \quad 10-6i} \\ 4 \quad -10-2i \quad 16+4i \quad 6-6i \quad -6i \end{array} \left| \begin{array}{c} \frac{1}{2} - \frac{i}{2} \\ \frac{1}{2} \end{array} \right. \left(x - \left(\frac{1}{2} - \frac{i}{2} \right) \right)$$

$$(16+4i)\left(\frac{1}{2} - \frac{i}{2}\right)$$

$$8 - 8i + 2i - 2i^2$$

$$8 - 6i + 2$$

$$10 - 6i$$

$$4\left(\frac{1}{2} - \frac{i}{2}\right)$$

$$4 - \frac{4}{2} - 4 - \frac{4}{2}$$

$$(6-6i)\left(\frac{1}{2} - \frac{i}{2}\right)$$

$$3 - 3i - 3i + 3i^2$$

$$3 - 6i \rightarrow$$

$$-6i$$

$$2 - 2i$$

$$(-10-2i)\left(\frac{1}{2} - \frac{i}{2}\right)$$

$$-10 \cdot \frac{1}{2} + -10 \cdot -\frac{i}{2} + -2i \cdot \frac{1}{2} + -2i \cdot -\frac{i}{2}$$

$$-5 + 5i - i + i^2$$

$$-5 + 4i - 1$$

$$-6 + 4i$$

13. Determine los números complejos z, w que satisfagan simultáneamente las condiciones siguientes:

$$\begin{cases} z - w \in \mathbb{R} \\ \operatorname{Re}(z + w) = 1 \\ z \cdot w = -7 + i \end{cases}$$

$$z = a_1 + b_1 i \quad w = a_2 + b_2 i$$

$$\begin{aligned} z - w &= (a_1 - a_2) + (b_1 - b_2)i, \quad \text{K constante} \\ &\quad \begin{matrix} a & b \\ c & d \end{matrix} \\ z - w &= (a_1 - a_2) + (b_1 + b_2)i \end{aligned}$$

$$\operatorname{Im}(z - w) = 0 \quad \begin{matrix} c & d \\ b_1 - b_2 = 0 & b_1 = b_2 \end{matrix}$$

$$\begin{aligned} \operatorname{Re}(z + w) &= 1 \quad z = a_1 + b_1 i \quad w = a_2 + b_2 i \\ a_1 + a_2 &= 1 \\ b_1 + b_2 &= 0 \quad b_1 = b_2 \\ b_1 &= 1 - a_1 \end{aligned}$$

$$\begin{aligned} z \cdot w &= -7 + i \quad z = a_1 + b_1 i \quad w = a_2 + b_2 i \\ (a_1 + b_1)(a_2 + b_2) &= 1 - a_1 \\ a_1 a_2 + a_1 b_2 + b_1 a_2 + b_1 b_2 &= 1 - a_1 \\ a_1 a_2 + b_1 b_2 &= 1 - a_1 \\ a_1 a_2 - b_1 b_2 &= 1 - a_1 \\ (a_1 - b_1)(a_1 + b_1) &= 1 - a_1 \end{aligned}$$

$$a_1 c + a_1 d + b_1 c + b_1 d$$

$$(a_1 c - b_1 d) + (a_1 d + b_1 c) i$$

$$(a_1 c - b_1^2) + (b_1 a_1 + b_1 c)$$

$$(a_1 c - b_1^2) + (b_1(a_1 + c))$$

$$(a(-b^2) + b(a+c)) = -7+i$$

$$\begin{aligned} ac - b^2 &= -7 \\ b(a+c) &= 1 \\ \begin{cases} b_1 = 1 - a_1 \\ b = 1 \end{cases} \end{aligned}$$

$$\begin{aligned} ac - b^2 &= -7 \\ a(1-a) - 1^2 &= -7 \\ a - a^2 - 1 &= -7 \\ -a^2 + a + 6 &= 0 \quad \Rightarrow a = 3 \quad a = -2 \end{aligned}$$

$$b = d, \quad b = 1, \quad d = 1$$

$$z = a_1 + b_1 i \quad w = a_2 + b_2 i$$

$$\begin{array}{ccc} 3+i & -2+i & |(z+w) = 1 \\ -2+i & 3+i & \end{array}$$