Potencias de i

$i^0 = 1$ Formula
$i^1 = i$ $n - 4 \cdot \text{entera}\left(\frac{n}{4}\right)$
$i^2 = -1$
$i^3 = -i$

i^{65} $65 - 4 \cdot \text{entera}\left(\frac{65}{4}\right)$

$65 - 4 \cdot \text{entera}(16{,}25)$
$65 - 4 \cdot 16$
$65 - 64$
1

$i^{65} = i^1 = i$

$i^{19} \to 19 - 4 \cdot \text{int}\left(\frac{19}{4}\right)$
$19 - 4 \cdot \text{int}(4{,}75)$
$19 - 4 \cdot 4$
$19 - 16$
3

$i^{19} = i^3 = -i$

Logaritmo principal

$$\ln^2(x) = [\ln(x)]^2 = n\ln(x)$$

$$\ln(x \cdot y) = \ln(x) + \ln(y)$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln^2\left(\frac{i^{2024}}{1+i}\right)$$

$$\left[\ln\left(\frac{i^{2024}}{1+i}\right)\right]^2$$

i^{2024}

$2024 - 4 \cdot \operatorname{int}\left(\frac{2024}{4}\right)$

$2024 - 4 \cdot 506$

0

$i^{2024} = i^0 = 1$

$$\left[\ln\left(\frac{\overbrace{1+0i}^{z_1}}{\underbrace{1+i}_{z_2}}\right)\right]^2$$

$z_1 = 1 + 0i$

$r = \sqrt{1^2 + 0^2} = 1$

$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$z_1 = 1 \cdot e^{i0}$

$$\left[\ln\left(\frac{1 \cdot e^{i0}}{\sqrt{2} \cdot e^{i\frac{\pi}{4}}}\right)\right]^2$$

$$\left[\ln\left(\frac{1}{\sqrt{2}} \cdot e^{i\left(0-\frac{\pi}{4}\right)}\right)\right]^2$$

$z_2 = 1 + i$

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\theta = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$z_2 = \underbrace{\sqrt{2} \cdot e^{i\frac{\pi}{4}}}_{r \cdot e^{i\theta}}$

$$\left[\ln\left(\frac{1}{\sqrt{2}}\, e^{-\frac{\pi}{4}i}\right)\right]^2$$

$$\frac{r_1 \cdot e^{i\theta_1}}{r_2 \cdot e^{i\theta_2}}$$

$$= \frac{r_1}{r_2} \cdot e^{i(\theta_1 - \theta_2)}$$

$a+bi$

$$\left[\ln\left(\frac{1}{\sqrt{2}}\,e^{-\frac{\pi}{4}i}\right)\right]^2 \qquad \ln(e)^x = x$$

$$\left[\ln\left(\frac{1}{\sqrt{2}}\right)+\cancel{\ln}\left(\cancel{e}^{-\frac{\pi}{4}i}\right)\right]^2$$

$$\left[\underbrace{\ln\left(\frac{1}{\sqrt{2}}\right)}_{a}-\underbrace{\frac{\pi}{4}i}_{b}\right]^2 \qquad \begin{aligned}&a^2-2ab+b^2\\&(a-b)^2\end{aligned}$$

$$\left(\frac{\pi i}{4}\right)^2$$

$$\ln^2\left(\frac{1}{\sqrt{2}}\right)-2\ln\left(\frac{1}{\sqrt{2}}\right)\cdot\frac{\pi}{4}i+\frac{\pi^2 i^2}{16} \qquad \frac{\pi^2 i^2}{4^2}$$

$$\ln^2\left(\frac{1}{\sqrt{2}}\right)\boxed{-2\ln\left(\frac{1}{\sqrt{2}}\right)\cdot\frac{\pi}{4}i}-\frac{\pi^2}{16}$$

$$\left[\ln^2\left(\frac{1}{\sqrt{2}}\right)-\frac{\pi^2}{16}\right]+\left[-2\ln\left(\frac{1}{\sqrt{2}}\right)\cdot\frac{\pi}{4}\right]i$$

$(-1+i)^i$

$z = (-1+i)^i$

$a = 1 \quad b = 1$

$-1 + i$

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{-1}\right) + \pi = \frac{3\pi}{4}$

$\sqrt{2} \cdot e^{i\frac{3\pi}{4}}$

$\ln(z) = \ln(-1+i)^i$

$= i \ln(-1+i)$

$= i\left[\ln\left(\sqrt{2} \cdot e^{\frac{3\pi}{4}i}\right)\right]$

$= i\left[\ln(\sqrt{2}) + \cancel{\ln}\left(\cancel{e}^{\frac{3\pi}{4}i}\right)\right]$

$= i\left[\ln(\sqrt{2}) + \frac{3\pi}{4}i\right]$

$= \ln(\sqrt{2})i + \frac{3\pi}{4}i^2$

$\ln(\sqrt{2})i - \frac{3\pi}{4}$

$$z = (2+i)^{1-i}$$

$$\ln(z) = \ln(2+i)^{1-i}$$

$2+i$

$r = \sqrt{2^2+1^2} = \sqrt{5}$

$\theta = \arctan\left(\frac{1}{2}\right)$

$\sqrt{5} \cdot e^{i\arctan\left(\frac{1}{2}\right)}$

$$= (1-i)\ln(2+i)$$

$$= (1-i)\ln\left(\sqrt{5} \cdot e^{i\arctan\left(\frac{1}{2}\right)}\right)$$

$$= (1-i)\left[\ln\left(\sqrt{5}\right) + \cancel{\ln}\left(\cancel{e}\right)^{i\arctan\left(\frac{1}{2}\right)}\right]$$

$$= (1-i)\left(\ln(\sqrt{5}) + \arctan\left(\tfrac{1}{2}\right)i\right]$$

$$\ln(\sqrt{5}) + \arctan\left(\tfrac{1}{2}\right)i - \ln(\sqrt{5})i - \arctan\left(\tfrac{1}{2}\right)i^2$$

$$\ln(\sqrt{5}) + \arctan\left(\tfrac{1}{2}\right)i - \ln(\sqrt{5})i + \arctan\left(\tfrac{1}{2}\right)$$

$$\left[\ln(\sqrt{5}) + \arctan\left(\tfrac{1}{2}\right)\right] + \left[-\ln(\sqrt{5}) + \arctan\left(\tfrac{1}{2}\right)\right]i$$

Magnitudes $|z| = r = \sqrt{a^2+b^2}$

$|w| = |z \cdot x|$ ⇐ magnitud del producto de 2 z s

$|w| = |z \cdot x| = |z| \cdot |x|$

$|\overline{w}| = w$ 　 ll a b → $\theta \cdot \frac{180}{\pi}$

Cuadrantes	a	b
1) $0 < \theta < \frac{\pi}{2}$ $0 < \theta < 90$	>0	>0
2) $\frac{\pi}{2} < \theta < \pi$ $90 < \theta < 180$	<0	>0
3) $\pi < \theta < \frac{3\pi}{2}$ $180 < \theta < 270$	<0	<0
4) $\frac{3\pi}{2} < \theta < 2\pi$ $270 < \theta < 360$	>0	<0

Casos especiales

$\theta = \frac{\pi}{2}$ 　 $a = 0$ 　 $b > 0$

$\theta = -\frac{\pi}{2}$ 　 $a = 0$ 　 $b < 0$

$\theta = \pm\pi$ 　 $a < 0$ 　 $b = 0$

En cualquier que el θ NO sea de los casos especiales

Arg(w), usar $\frac{b}{a} = \tan(\theta)$

3) Determine la forma polar de todos los números complejos que cumplen, simultáneamente, las condiciones que se muestran en cada caso:

a) $\begin{cases} |w| = |\overline{(3-i)\cdot 2i-1}| \\ \operatorname{Arg}(w) = \operatorname{Arg}\left[\dfrac{(-1+i)^5}{\sqrt{3}-i}\right] \end{cases}$

$r \operatorname{cis}(\theta)$

$\operatorname{Arg}(w) = \operatorname{Arg}\left(\dfrac{(-1+i)^5}{\sqrt{3}-i}\right)$

$(r \operatorname{cis}(\theta))^n$

$r^n \operatorname{cis}(\theta \cdot n)$

$-1+i$

$\theta = \arctan\left(\frac{1}{-1}\right) + \pi$

$\theta = \frac{-3\pi}{4}$

$\operatorname{cis}\left(\frac{-3\pi}{4}\right)$

$\sqrt{3}-i$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

$\operatorname{cis}\left(-\frac{\pi}{6}\right)$

$\operatorname{Arg}(w) = \left(\dfrac{\operatorname{cis}\left(\frac{-3\pi}{4}\right)^5}{\operatorname{cis}\left(-\frac{\pi}{6}\right)}\right)$

$\operatorname{Arg}(w) = \left(\dfrac{\operatorname{cis}\left(\frac{-5\cdot 3\pi}{4}\right)}{\operatorname{cis}\left(-\frac{\pi}{6}\right)}\right)$

$\operatorname{Arg}(w) = \left(\dfrac{\operatorname{cis}\left(\frac{-15\pi}{4}\right)}{\operatorname{cis}\left(-\frac{\pi}{6}\right)}\right)$

$= \operatorname{cis}\left(\frac{-15\pi}{4} - -\frac{\pi}{6}\right)$

$\operatorname{cis}\left(\frac{97\pi}{2}\right)$

$|w| = |\overline{(3-i)\cdot\overline{2i-1}}|$

$|w| = |(3-i)(2i-1)|$

$|3-i|\cdot|-1+2i|$

$3-i \qquad\qquad -1+2i$

$r=\sqrt{3^2+(-1)^2}=\sqrt{10} \qquad r=\sqrt{(-1)^2+2^2}=\sqrt{5}$

$|3-i|\cdot|-1+2i|$

$\sqrt{10}\cdot\sqrt{5}$

$5\sqrt{2}$

$5\sqrt{2}\ \text{Cis}\left(\frac{97\pi}{2}\right)$

$5\sqrt{2}\ \text{Cis}\left(\frac{97\pi}{2}+2\pi k\right)$

Pide en forma rectangular

b) $\begin{cases} |z-3i| = 4 \\ \mathrm{Arg}(2-2z) = \pi/2 \end{cases}$ $z = a+bi$

$\mathrm{Arg}(2-2z) = \frac{\pi}{2}$

$2-2(a+bi) = \frac{\pi}{2}$ $a=0$ $b>0$

$2-2a-2bi = \frac{\pi}{2}$

$\underset{a}{(2-2a)} + \underset{b}{(-2b)}i = \frac{\pi}{2}$

$2-2a=0$ $-2b>0$

$-2a=-2$ $b<0$

$a=1$

$|z-3i| = 4$

$|a+bi-3i| = 4$

$|a+(b-3)i| = 4$

$\sqrt{a^2+(b-3)^2} = 4$ $a=1$

$a^2+(b-3)^2 = 16$ $(a-b)^2$

$1^2+b^2-6b+9 = 16$ $a^2-2ab+b^2$

$b^2-6b+10-16=0$

$b^2-6b-6=0$

$b = 3-\sqrt{15}$, $b = 3+\sqrt{15}$

$a=1$ $b=3-\sqrt{15}$

R/ $1+(3-\sqrt{15})i$ //

Determine $z \in \mathbb{C}$ que cumplan estoi

$|\bar{z}+(1-i)|=5$

$\operatorname{Arg}(z-(1+2i))=\frac{3\pi}{4}$

$\operatorname{Arg}(z-(1+2i))=\frac{3\pi}{4}$

$a+bi-1-2i=\frac{3\pi}{4}$

$(a-1)+(b-2)i=\frac{3\pi}{4}$

$\frac{b-2}{a-1}=\tan\left(\frac{3\pi}{4}\right) \longrightarrow$ $\frac{3\pi}{4} \cdot \frac{180}{\pi}$

135

$90<135<180$

$\frac{b-2}{a-1}=-1$ $\quad -1(a-1)$ $\quad a<0 \quad b>0$

$-a+1$

$b-2=1-a$ $\quad 1-a$

$b-3=-a$

$\boxed{a=3-b}$

$|\bar{z}+(1-i)=5$

$|\overline{a+bi}+1-i|=5$

$|a-bi+1-i|=5$

$|(a+1)+(-b-1)i|=5$

$(a+1)^2+(-b-1)^2=25$ $\qquad \boxed{a=3-b}$

$(3-b+1)^2+b^2-2b-1+1^2-25=0$

$(4-b)^2+b^2+2b+1-25=0$

$16-8b+b^2+b^2+2b+1-25=0$

$2b^2-6b+17=0$

$b \cancel{=} -1 \qquad b=4$

$a=3-b \qquad\qquad a<0 \quad b>0$

$b=4 \rightarrow a=3-4=-1$

$\boxed{-1+4i}$

$|z-2| = 5$

$\text{Arg}(z-1) = \frac{3\pi}{4}$

$a+bi-1 = \frac{3\pi}{4}$ $\qquad \frac{3\pi}{4} \cdot \frac{180}{\pi}$

$\frac{b}{a-1} = \tan\left(\frac{3\pi}{4}\right)$ $\qquad 135$

$90 < 135 < 180$

$a < 0 \quad b > 0$

$\frac{b}{a-1} = -1$

$b = 1-a$

$|z-2| = 5$

$a+bi-2 = 5$

$(a-2)^2 + b^2 = 5^2$

$a^2 - 4a + 4 + (1-a)^2 = 25$

$a^2 - 4a + 4 + 1 - 2a + a^2 = 25$

$2a^2 - 6a - 20 = 0$

$a_1 = 5 \quad a_2 = -2$ $\qquad a < 0 \quad b > 0$

$a = -2 \rightarrow b = 1 - -2$ $\qquad b = 1-a$

$b = 3$

$\boxed{-2+3i}$

Teorema fundamental del algebra

$P(x)$, $Q(x)$ tiene que hacer division sintetica

$$P(x) = x^4 - 12x + 8x^2 - 4x^3 + 15$$

$$Q(x) = 2+i \quad , \; 2+i \; , \; 2-i$$

$$x^4 - 12x + 8x^2 - 4x^3 + 15$$

$$x^4 - 4x^3 + 8x^2 - 12x + 15$$

$$\begin{array}{ccccc|l}
1 & -4 & 8 & -12 & 15 & 2+i \\
 & +2+i & -5 & 6+3i & -15 & \\
\hline
1 & -2+i & 3 & -6+3i & 0 & (x-(2+i))
\end{array}$$

$$(-2+i)(2+i)$$

$$-4 \cancel{-2i+2i} + i^2$$

$$-4-1$$

$$-5$$

$$x^3 + (-2+i)x^2 + 3x + (-6+3i)$$

$$\begin{array}{cccc|l}
1 & -2+i & 3 & -6+3i & 2-i \\
 & 2-i & 0 & 6-3i & \\
\hline
1 & 0 & 3 & 0 & (x-(2-i))
\end{array}$$

$$(2+i)3$$

$$6+3i$$

$$(x^2+3)$$

$$(2+i)(-6+3i)$$

$$-12 + \cancel{6i - 6i}\, 3i^2$$

$$-12 - 3$$

$$-15$$

$$(x-(2+i))\,(x-(2-i))\,(x^2+3) = 0$$

$$x-(2+i) = 0 \qquad x-(2-i) = 0 \qquad x^2+3 = 0$$

$$\boxed{x = 2+i} \qquad \boxed{x = 2-i} \qquad (x-\sqrt{3}i)(x+\sqrt{3}\,i) = 0$$

$$\boxed{x = \sqrt{3}\,i} \qquad \boxed{x = -\sqrt{3}\,i}$$

$$a^2+b^2 = (a-bi)(a+bi)$$

$x^4 - 6x^3 + 22x^2 - 54x + 117 \;\big)\; \underline{3-2i}$

$$\begin{array}{ccccc|c}
1 & -6 & 22 & -54 & 117 & \\
 & 3-2i & -13 & 27-18i & -117 & 3-2i \\
\hline
1 & -3-2i & 9 & -27-18i & 0 & (x-(3-2i))
\end{array}$$

$x^3 + x^2(-3-2i) + 9x + (-27+18)i$

$(3-2i)(-3-2i)$

$-9 \cancel{-6i} \cancel{+6i} + 4i^2$

$-9-4$

-13

$$\begin{array}{cccc|c}
1 & -3-2i & 9 & -27+18i & \\
 & 3+2i & 0 & 27+18i & 3+2i \\
\hline
1 & 0 & 9 & 0 & (x-(3+2i))
\end{array}$$

x^2+9

$9(3-2i)$

$27-18i$

$(x-3i)(x+3i)=0$

$\boxed{x=3i}$ $\boxed{x=-3i}$

$(3-2i)(-27-18i)$

$-81 \cancel{-54i} \cancel{+54i} + 36i^2$

$-81-36$

-117

$(x-(3-2i))=0$

$x = \boxed{3-2i}$

$(x-(3+2i))=0$

$x = \boxed{3+2i}$

$x^3 = z \quad k=0,1,2$