

$$(a) 2 + 5 + 8 + \dots + (3n - 1) = \frac{1}{2}n(3n + 1), \forall n \geq 1$$

$$n=1 \rightarrow 2=2 \quad \checkmark$$

$$n=p \rightarrow 2+5+\dots+(3p-1) = \frac{p}{2} p(3p+2), \text{H:}$$

$$n=p+1 \rightarrow 2+5+\dots+(3p-1)+(3p+2) = \frac{p+1}{2} (p+2)(3p+4), \text{HQ)}$$

Demo

$$2+5+\dots+(3p+2)$$

$$2+5+\dots+(3p-1)+(3p+2)$$

$$\frac{p}{2} p(3p+2)+(3p+2)$$

$$\frac{p(3p+2)}{2} + \frac{2(3p+2)}{2}$$

$$\frac{3p^2+6p+4}{2}$$

$$\frac{3p^2+7p+4}{2} \quad \begin{array}{l} 3p \\ p \end{array} \times \begin{array}{l} 4 \\ 1 \end{array} = \frac{3p}{7p} \\ (3p+4)(p+2)$$

$$\boxed{\frac{1}{2} (p+1)(3p+4) //}$$

$$(b) 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n! = (n+1)! - 1, \forall n \geq 1$$

$$n=1 \rightarrow 1=1$$

$$n=p \rightarrow 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + p \cdot p! = (p+1)! - 1, \text{H:}$$

$$n=p+1 \rightarrow 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + (p+1) \cdot (p+1)! = (p+2)! - 1, \text{HQ)}$$

Demo

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + (p+1) \cdot (p+1)!$$

$$\boxed{[(p+1)! - 1] + (p+1) \cdot (p+1)!}, \text{H:}$$

$$(p+1)! + (p+1) \cdot (p+1)! - 1$$

$$\{(p+1)!(p+1)\} - 1$$

$$[(\rho+1)(\rho+2)] - 1$$

$$\boxed{(\rho+1)(-1) //}$$

$$(c) \quad 1^2 - 2^2 + 3^2 - \dots + (-1)^{n+1} n^2 = \frac{(-1)^{n+1} n(n+1)}{2}, \quad \forall n \geq 1$$

$$\begin{aligned} h=1 &\rightarrow l=1 \\ h=\rho & \quad 1^2 - 2^2 - 3^2 \dots + (-1)^{\rho+1} \cdot \rho^2 = \frac{(-1)^{\rho+1} \cdot \rho \cdot (\rho+1)}{2}, \text{ H1:} \\ h=\rho+1 & \quad 1^2 - 2^2 - 3^2 \dots + (-1)^{\rho+2} \cdot (\rho+1)^2 + (-1)^{\rho+1} \cdot (\rho+1)^2 = \frac{(-1)^{\rho+2} \cdot (\rho+1) \cdot (\rho+2)}{2}, \text{ H2D} \end{aligned}$$

Lemma

$$1^2 - 2^2 - 3^2 \dots + (-1)^{\rho+1} \cdot \rho^2 + (-1)^{\rho+1} \cdot (\rho+1)^2$$

$$\frac{(-1)^{\rho+1} \cdot \rho \cdot (\rho+1)}{2} + (-1)^{\rho+1} \cdot (\rho+1)^2, \text{ H1:}$$

$$\frac{(-1)^{\rho+1} \cdot \rho \cdot (\rho+1) + 2 \cdot (-1)^{\rho+2} \cdot (\rho+1)^2}{2}$$

$$\frac{(-1)^{\rho+1} \cdot \rho \cdot (\rho+1) + 2(-1)^{\rho+1} \cdot (-1) \cdot (\rho+1)^2}{2}$$

$$\frac{(-1)^{\rho+1} (\rho+1) \left[\rho + 2(-1) (\rho+1) \right]}{2}$$

$$\frac{(-1)^{\rho+1} (\rho+1) \left[\rho - 2\rho - 2 \right]}{2}$$

$$\frac{(-1)^{\rho+1} (\rho+1) \left[-\rho - 2 \right]}{2}$$

$$\frac{(-1)^{\rho+1} (\rho+1) (-1) (\rho+2)}{2}$$

$$\boxed{(-1)^{\rho+2} (\rho+1) (\rho+2) //}$$

$$(d) \frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + \dots + \frac{1}{(n+1)^2 - 1} = \frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}, \quad \forall n \geq 1$$

$$h=2 \rightarrow \frac{1}{3} = \frac{1}{3} \checkmark$$

$$h=p \quad \frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + \dots + \frac{1}{(p+1)^2 - 1} = \frac{3}{4} - \frac{1}{2(p+2)} - \frac{1}{2(p+3)}, \quad \text{H:}$$

$$h=p+1 \quad \frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + \dots + \frac{1}{(p+2)^2 - 1} + \frac{1}{(p+3)^2 - 1} = \frac{3}{4} - \frac{1}{2(p+2)} - \frac{1}{2(p+3)}, \quad \text{HQD}$$

Demo

$$\frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + \dots + \frac{1}{(p+1)^2 - 1} + \frac{1}{(p+2)^2 - 1}$$

$$\frac{3}{4} - \frac{1}{2(p+2)} - \frac{1}{2(p+3)} + \frac{1}{(p+2)^2 - 1} \quad \text{H:}$$

$$\frac{3}{4} - \frac{1}{2(p+2)} + \frac{1}{(p+2)^2 - 1} - \frac{1}{2(p+3)}$$

$$\frac{3}{4} - \frac{1}{2(p+2)} + \frac{1}{(p+2)(p+3)} - \frac{1}{2(p+3)}$$

$$\frac{3}{4} - \frac{1}{2(p+2)} + \frac{1}{(p+2)(p+3)} - \frac{1}{2(p+3)}$$

$$\frac{1}{(p+2)(p+3)} = \frac{A}{(p+2)} + \frac{B}{(p+3)}$$

$$1 = A(p+3) + B(p+2)$$

$$p = -2 \rightarrow 1 = 2A \rightarrow A = \frac{1}{2}$$

$$p = -3 \rightarrow 1 = -2B \rightarrow B = -\frac{1}{2}$$

$$\frac{1}{(p+2)(p+3)} = \frac{\frac{1}{2}}{(p+2)} - \frac{\frac{1}{2}}{(p+3)}$$

$$= \frac{1}{2(p+2)} - \frac{1}{2(p+3)}$$

$$\frac{3}{4} - \frac{1}{2(p+1)} + \frac{1}{2(p+1)} - \frac{1}{2(p+3)} - \frac{1}{2(p+1)}$$

$$\left[\frac{3}{4} - \frac{1}{2(p+1)} - \frac{1}{2(p+3)} \right] //$$

$$(e) 2 + 6 + 18 + \dots + 2 \cdot 3^{n-1} = 3^n - 1, \quad \forall n \geq 1$$

$$k=1 \rightarrow 2=2$$

$$k=p \quad 2 + 6 + 18 + \dots + 2 \cdot 3^{p-1} = 3^p - 1, \quad H:$$

$$k=p+1 \quad 2 + 6 + 18 + \dots + 2 \cdot 3^{p-1} + 2 \cdot 3^p = 3^{p+1} - 1, \quad H \text{ QD}$$

$$\begin{aligned} & \text{Durch } \\ & 2 \cdot 3^{p-1} + 2 \cdot 3^p \\ & 3^p - 1 + 2 \cdot 3^p \\ & 3^p(1+2) - 1 \\ & \boxed{3^{p+1} - 1} // \end{aligned}$$

$$(f) (-1)^1 + (-1)^2 + (-1)^3 + \dots + (-1)^n = \frac{1}{2}((-1)^n - 1), \quad \forall n \geq 1$$

$$k=1 \quad -1 = -1$$

$$k=p \quad (-1)^1 + (-1)^2 + (-1)^3 + \dots + (-1)^p = \frac{1}{2}((-1)^p - 1), \quad H:$$

$$k=p+1 \quad (-1)^1 + (-1)^2 + (-1)^3 + \dots + (-1)^p + (-1)^{p+1} = \frac{1}{2}((-1)^{p+1} - 1), \quad H \text{ QD}$$

$$\begin{aligned} & \text{Durch } \\ & (-1)^1 + (-1)^2 + (-1)^3 + \dots + (-1)^p + (-1)^{p+1} \\ & \frac{1}{2}((-1)^p - 1) + (-1)^{p+1}, \quad H: \\ & \frac{(-1)^p}{2} - \frac{1}{2} + \frac{2(-1)^{p+1}}{2} \\ & \underline{\underline{(-1)^p - 1 + 2(-1)^{p+1}}} \\ & \frac{1}{2} \left[(-1)^p - 1 + 2(-1)^{p+1} \right] \\ & \underline{\underline{1 \left((-1)^p + 2(-1)^{p+1} - 1 \right)}} \end{aligned}$$

$$\frac{1}{2} \left[(-1)^p (1+2(-1)) - 1 \right]$$

$$\frac{1}{2} \left[(-1)^p (-1) - 1 \right]$$

$$\boxed{\frac{1}{2} \left[(-1)^{p+1} - 1 \right]} //$$

$$(g) \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}, \quad \forall n \geq 1$$

$$h=1 \quad \frac{1}{6} = \frac{1}{6}$$

$$h=p \quad \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{p(p+1)(p+2)} = \frac{p(p+3)}{4(p+1)(p+2)}, \quad H:$$

$$h=p+1 \quad \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{p(p+1)(p+2)} + \frac{1}{(p+1)(p+2)(p+3)} = \frac{(p+1)(p+2)}{4(p+1)(p+2)}, \quad H(Q)$$

∴ RMO

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{p(p+1)(p+2)} + \frac{1}{(p+1)(p+2)(p+3)}$$

$$\frac{p(p+3)}{4(p+1)(p+2)} + \frac{1}{(p+1)(p+2)(p+3)}$$

$$\frac{p(p^2+6p+9)}{4(p+1)(p+2)(p+3)} + \frac{1}{(p+1)(p+2)(p+3)}$$

$$\frac{p(p^2+6p+9)}{4(p+1)(p+2)(p+3)} + \frac{1}{(p+1)(p+2)(p+3)}$$

$$\begin{array}{r} p^3 + 6p^2 + 9p + 8 \\ 4(p+1)(p+2)(p+3) \\ \hline -1 \quad -5 \quad -4 \quad -1 \end{array}$$

$$\frac{(p+4)(p+1)(p+2)}{4(p+1)(p+2)(p+3)} = \frac{(p^2+5p+4)(p+1)}{(p+1)(p+2)(p+3)}$$

$$\boxed{\frac{(p+1)(p+2)}{4(p+1)(p+2)(p+3)}} //$$

$$\frac{p}{p} \times \frac{4-p}{1-p} = \frac{4-p}{p}$$

$$(p+4)(p+1)(p+2)$$

$$(b) 2 + 5 + 12 + \dots + (2^{n-1} + 2^{n-1}) = \frac{1}{2} (2^{n+1} + 2^n - 2), \quad \forall n \geq 1$$

$$(n) \quad z + 3 + 13 + \dots + (z^{p-1} + 3^{p-1}) = \frac{1}{2}(z^p + 3^p - 3), \quad \forall n \geq 1$$

$$k=1 \quad j=2$$

$$k=p \quad 2+3+13+\dots+(z^{p-1}+3^{p-1}) = \frac{1}{2}(z^p + 3^p - 3), \quad \text{H:}$$

$$k=p+1 \quad 2+3+13+\dots+(z^{p-1}+3^{p-1}) + (z^p + 3^p) = \frac{1}{2}(z^{p+1} + 3^{p+1} - 3), \quad \text{HQA}$$

\Rightarrow $e_m o$

$$2+3+13+\dots+(z^{p-1}+3^{p-1}) + (z^p + 3^p)$$

$$\frac{1}{2}(z^{p+1} + 3^p - 3) + (z^p + 3^p), \quad \text{H:}$$

$$\frac{z^{p+1}}{2} + \frac{3^p}{2} - \frac{3}{2} + \frac{z^p + 3^p}{2}$$

$$\frac{z^{p+1}}{2} + \frac{3^p}{2} - \frac{3}{2} + \frac{z \cdot z^p}{2} + \frac{z \cdot 3^p}{2}$$

$$\frac{z^{p+2}}{2} + \frac{3^p}{2} - 3 + \frac{z^{p+1}}{2} + \frac{z \cdot 3^p}{2}$$

$$\frac{1}{2} \left(\underline{z^{p+2} + 3^p - 3} + \underline{z^{p+1} + z \cdot 3^p} \right)$$

$$\frac{1}{2} \left(\underline{z(z^{p+1})} + \underline{3^p(z+1)} - 3 \right)$$

$$\boxed{\frac{1}{2} (z^{p+2} + 3^{p+1} - 3)}$$

//

$$(i) \quad 1 \cdot 5 + 2 \cdot 5^2 + 3 \cdot 5^3 + \dots + n \cdot 5^n = \frac{5 + (4n-1)5^{n+1}}{16}, \quad \forall n \geq 1$$

$$k=1 \quad s=5$$

$$k=p \quad 1 \cdot 5 + 2 \cdot 5^2 + 3 \cdot 5^3 + \dots + p \cdot 5^p = \frac{5 + (4p-1) \cdot 5^{p+2}}{16}, \quad \text{H:}$$

$$k=p+1 \quad 1 \cdot 5 + 2 \cdot 5^2 + 3 \cdot 5^3 + \dots + (p+1) \cdot 5^{p+1} = \frac{5 + (4p+3) \cdot 5^{p+3}}{16}, \quad \text{H:}$$

\Rightarrow $e_m o$

$$1 \cdot 5 + 2 \cdot 5^2 + 3 \cdot 5^3 + \dots + (p+1) \cdot 5^{p+1}$$

$$\frac{5 + (4p-1) \cdot 5^{p+2}}{16} + (p+1) \cdot 5^{p+2} \quad \text{H:}$$

$$\frac{5 + (4p-1) \cdot 5^{p+2}}{16} + \frac{16(p+1) \cdot 5^{p+2}}{16}$$

$\frac{1}{16}$

$$5 + 5^{p+2} (4p-1 + 16)$$

76

$$\frac{s+s^{p+1}}{2} \left(2ap + 2s \right)$$

76

$$s + s^{p+1} (s(p+3))$$

76

$$s + s^{p+2} (sp+3)$$

76

$$\boxed{s + (sp+3) \cdot s^{p+2}}$$

76 //

$$(j) \quad a + (a+d) + (a+2d) + \dots + [a + (n-1)d] = \frac{n[2a + (n-1)d]}{2}, \quad \forall n \geq 1$$

$$l=1 \quad a = a$$

$$l=p \quad a + (a+d) + (a+2d) + \dots + [a + (p-1)d] = \frac{p(2a + (p-1)d)}{2}, \quad \text{H:}$$

$$l=p+1 \quad a + (a+d) + (a+2d) + \dots + [a + (p-1)d] + [a + (p)d] = \frac{(p+1)(2a + (p)d)}{2}, \quad \text{H(Q)}$$

) RMO

$$a + (a+d) + (a+2d) + \dots + [a + (p-1)d] + [a + (p)d]$$

$$\frac{p[(2a + (p-1)d)] + a + (p)d}{2}, \quad \text{H:}$$

$$\frac{p[2a + (p-1)d] + 2a + 2pd}{2}$$

$$\frac{p[2a + pd - d] + 2a + 2pd}{2}$$

$$\frac{2ap + p^2d - pd + 2a + 2pd}{2}$$

$$\frac{2ap + p^2d + 2a + pd}{2}$$

$$\frac{pd(p+1) + 2a(p+1)}{2}$$

$$\boxed{\frac{(p+1)(2a + pd)}{2}}$$

$$(k) \quad 1 \cdot 3 \cdot 5 \cdots (2n-1) = \frac{(2n)!}{2^n \cdot n!}, \quad \forall n \geq 1$$

$$k=1 \quad l=1$$

$$k=p \quad 1 \cdot 3 \cdot 5 \cdots (2p-1) = \frac{(2p)!}{2^p \cdot p!} \quad , \text{Hilf}$$

$$k=p+1 \quad 1 \cdot 3 \cdot 5 \cdots (2p-1) \cdot (2p+1) = \frac{(2p+2)!}{2^{p+1} \cdot (p+1)!} \quad , \text{HQD}$$

$$= \frac{(2p+2)(2p+1)(2p)!}{2^{p+1} (p+1)!}$$

$\left| \right\rangle_{\text{erm}} 0$

$$1 \cdot 3 \cdot 5 \cdots (2p-1) + (2p+1)$$

$$= 2(p+1)(2p+2)(2p)!$$

$$\frac{(2p)!}{2^p \cdot p!} \cdot (2p+1)$$

$$2 \cdot 2^p (p+1) \cdot$$

$$(p+1)(2p+1)(2p)!$$

$$2^p (p+1)(p)!$$

$$(2p+1)(2p)!$$

$$2^p \cdot p!$$

$\frac{2p!}{2^p \cdot p!} \cdot (2p+1)$

//

$$(l) \quad \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \cdots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}, \quad \forall n \geq 2$$

$$k=2 \quad \frac{3}{4} = \frac{3}{4}$$

$$k=p \quad \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \cdots \left(1 - \frac{1}{p^2}\right) = \frac{p+1}{2p}, \quad \text{Hilf}$$

$$k=p+1 \quad \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \cdots \left(1 - \frac{1}{p^2}\right) \left(1 - \frac{1}{(p+1)^2}\right) = \frac{(p+2)}{2(p+1)}, \quad \text{HQD}$$

$\left| \right\rangle_{\text{erm}} 0$

$$\left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \cdots \left(1 - \frac{1}{p^2}\right) \left(1 - \frac{1}{(p+1)^2}\right)$$

$$\left(\frac{p+1}{2p}\right) \left(1 - \frac{1}{(p+1)^2}\right) \quad , \quad \text{Hilf}$$

$$\left(\frac{p+1}{2p}\right) \left(\frac{(p+1)^2 - 1}{(p+1)^2}\right)$$

$$\underline{(p+1)^2 - 1}$$

$$\frac{((p+1)-1)(((p+1)+1)}{2p(p+1)}$$

$$2p(p+1)$$

$$\cancel{p}(p+2)$$

$$2\cancel{p}(p+1)$$

$$\boxed{\frac{p+2}{2(p+1)}} \quad //$$

$$(m) 4(1 + 5 + 5^2 + \dots + 5^n) + 1 = 5^{n+1}$$

$$k=1 \quad S=5$$

$$k=p \quad 1 + 5 + 5^2 + \dots + 5^p + 1 = 5^{p+1}, \text{ Hi:}$$

$$k=p+1 \quad 1 + 5 + 5^2 + \dots + 5^p + 5^{p+1} + 1 = 5^{p+2}, \text{ HQD}$$

$\prod_{k=0}^{\infty}$

$$1 + 5 + 5^2 + \dots + 5^k + 5^{k+1}$$

$$5^{k+1} + 5 \cdot 5^{k+1} \quad | \cdot 5^{k+1}$$

$$5^{k+2} \cdot 5$$

$$\boxed{5^{k+2}} \quad //$$

$$(n) \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \dots + \frac{2p-1}{2^p} = 3 - \frac{2p+3}{2^p}, \forall n \geq 1$$

$$k=1 \quad \frac{1}{2} = \frac{1}{2}$$

$$k=p \quad \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \dots + \frac{2p-1}{2^p} = 3 - \frac{2p+3}{2^p}, \text{ Hi:}$$

$$k=p+1 \quad \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \dots + \frac{2p-1}{2^p} + \frac{2p+1}{2^{p+1}} = 3 - \frac{2p+5}{2^{p+2}}, \text{ HQD}$$

$\prod_{k=0}^{\infty}$

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \dots + \frac{2p-1}{2^p} + \frac{2p+1}{2^{p+1}}$$

$$3 - \frac{2p+3}{2^p} + \frac{2p+1}{2^{p+2}}, \text{ Hi:}$$

$$3 + \frac{2p+1}{2^{p+2}} - \frac{2p+3}{2^p}$$

$$3 + \frac{2p+1}{2^{p+2}} - \frac{4p-6}{2^{p+1}}$$

$$3 - \frac{1}{2^{p+1}} \left(-2p-1 + 4p+6 \right)$$

$$3 - \frac{1}{2^{p+1}} \left(2p+5 \right)$$

$$\boxed{3 - \frac{2p+5}{2^{p+1}}} \quad //$$

4. Usando inducción matemática demuestre la validez de cada una de las siguientes igualdades.

$$(a) \sum_{i=0}^n \frac{1}{(2i+1)(2i+3)} = \frac{n+1}{2n+3}$$

$$k=0 \quad \frac{1}{3} = \frac{1}{3}$$

$$k=p \quad \sum_{i=0}^p \frac{1}{(2i+1)(2i+3)} = \frac{p+1}{2p+3}, \text{ H.I.}$$

$$k=p+1 \quad \sum_{i=0}^{p+1} \frac{1}{(2i+1)(2i+3)} = \frac{p+2}{2p+5}, \text{ H.Q.D}$$

Demo

$$\sum_{i=0}^{p+1} \frac{1}{(2i+1)(2i+3)}$$

$$\sum_{i=0}^p \frac{1}{(2i+1)(2i+3)} + \frac{1}{(2p+3)(2p+5)}$$

$$\frac{p+1}{2p+3} + \frac{1}{(2p+3)(2p+5)}, \text{ H.I.}$$

$$\frac{(2p+5)(p+1) + 1}{(2p+3)(2p+5)}$$

$$\frac{2p^2 + 2p + 5p + 5 + 1}{(2p+3)(2p+5)}$$

$$\frac{2p^2 + 7p + 6}{(2p+3)(2p+5)} \quad \frac{2p \times 3 = 6}{7p}$$

$$(2p+1)(2p+3)$$

$$\underline{(2p+3)(p+2)}$$

$$\cancel{(2p+3)}(2p+5)$$

$$\boxed{\begin{array}{c} p+2 \\ 2p+5 \end{array}} //$$

$$(b) \sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$$

$$k=1 \quad \frac{1}{2} = \frac{1}{2}$$

$$k=p \quad \sum_{i=1}^p \frac{1}{i(i+1)} = \frac{p}{p+1}, \text{ H:}$$

$$k=p \quad \sum_{i=1}^{p+1} \frac{1}{i(i+1)} = \frac{(p+1)}{p+2}, \text{ H Q D}$$

Demo

$$\sum_{i=1}^{p+1} \frac{1}{i(i+1)}$$

$$\leq \sum_{i=1}^p \frac{1}{i(i+1)} + \frac{1}{(p+1)(p+2)}$$

$$\frac{p(p+2) + 1}{(p+1)(p+2)}, \text{ H:}$$

$$\frac{p(p+2) + 1}{(p+1)(p+2)}$$

$$\frac{p^2 + 2p + 1}{(p+1)(p+2)}$$

$$\frac{(p+1)^2}{(p+1)(p+2)}$$

$$\boxed{\begin{array}{c} p+1 \\ p+2 \end{array}} //$$

$$(c) \sum_{i=1}^k \ln i = \ln(k!)$$

$$\begin{aligned} & k=2 \quad 0=0 \\ & k=p \quad \sum_{i=1}^p \ln(i) = \ln(p!) \quad , \text{H} \end{aligned}$$

$$k=p+1 \quad \sum_{i=1}^{p+1} \ln(i) = \ln((p+1)!) \quad , \text{H.Q.D}$$

$\boxed{\text{D}_{\text{emo}}}$

$$\sum_{i=1}^{p+1} \ln(i)$$

$$\sum_{i=1}^p \ln(i) + \ln(p+1)$$

$$\ln(p!) + \ln(p+1)$$

$$\ln(p! \cdot (p+1))$$

$$\boxed{\ln((p+1)!)} //$$

$$(d) \sum_{i=1}^n \frac{1}{i(i+1)(i+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

$$k=1 \quad \frac{1}{6} = \frac{1}{6}$$

$$k=p \quad \sum_{i=1}^p \frac{1}{i(i+1)(i+2)} = \frac{p(p+3)}{4(p+1)(p+2)} \quad , \text{H} \quad \boxed{i}$$

$$k=p+1 \quad \sum_{i=1}^{p+1} \frac{1}{i(i+1)(i+2)} = \frac{(p+1)(p+4)}{4(p+2)(p+3)} \quad , \text{H.Q.D}$$

$\boxed{\text{D}_{\text{emo}}}$

$$\sum_{i=1}^{p+1} \frac{1}{i(i+1)(i+2)}$$

$$\sum_{i=1}^p \frac{1}{i(i+1)(i+2)} + \frac{1}{(p+1)(p+2)(p+3)}$$

$$\frac{p(p+3)}{4(p+1)(p+2)} + \frac{1}{(p+1)(p+2)(p+3)}$$

$$\frac{p(p+3)^2 + 4}{4(p+1)(p+2)(p+3)}$$

$$\begin{array}{r} p^3 + 6p^2 + 9p + 4 \\ \hline 1 \ 6 \ 9 \ 4 \end{array}$$

$$\frac{(\rho^2 + 6\rho + 9) + 4}{4(\rho+1)(\rho+2)(\rho+3)} \quad \begin{matrix} -1 & -3 & -1 & -1 \\ 1 & 5 & 4 & 0 \end{matrix}$$

$$(\rho^2 + 5\rho + 9)(\rho + 1)$$

$$\frac{\rho^3 + 6\rho^2 + 10\rho + 8}{4(\rho+1)(\rho+2)(\rho+3)}$$

$$\frac{(\rho+4)(\rho+1)}{4(\rho+1)(\rho+2)(\rho+3)}$$

$$\boxed{(\rho+4)(\rho+1)} \\ 4(\rho+2)(\rho+3) \quad //$$

$$(e) \sum_{i=1}^n \log \left(\frac{i+1}{i} \right)^i = \log \frac{(n+1)^n}{n!}$$

$$\begin{aligned} h=1 & \log(2) = \log(2) \\ h=\rho & \leq \log \left(\frac{\rho+1}{\rho} \right)^{\rho} = \left| \log \left(\frac{(\rho+1)^{\rho}}{\rho!} \right) \right|, \text{ bei} \end{aligned}$$

$$h=\rho+1 \leq \log \left(\frac{\rho+2}{\rho+1} \right)^{\rho+1} = \left| \log \left(\frac{(\rho+2)^{\rho+1}}{(\rho+1)!} \right) \right|, \text{ HGD}$$

Damit

$$\leq \log \left(\frac{\rho+1}{\rho} \right)^{\rho}$$

$$\leq \log \left(\frac{\rho+1}{\rho} \right)^{\rho} + \left| \log \left(\frac{\rho+2}{(\rho+1)} \right)^{\rho+1} \right|, \text{ bei}$$

$$\left| \log \left(\frac{(\rho+1)^{\rho}}{\rho!} \right) + \left| \log \left(\frac{\rho+2}{(\rho+1)} \right)^{\rho+1} \right| \right|$$

$$\left| \log \left(\frac{(\rho+1)^{\rho}}{\rho!} \cdot \frac{(\rho+2)^{\rho+1}}{(\rho+1)^{\rho+1}} \right) \right|$$

$$\left| \log \left(\frac{(\rho+1)^{\rho}}{\rho!} \cdot (\rho+2)^{\rho+1} \right) \right|$$

