

## 12 Background, Reading Part : How to use MATLAB to find a basis Null space of A

Enter your matrix  $A$  in MATLAB.

1. Enter the following matrix:

$$A1 = \begin{bmatrix} 1 & 3 & 0 & 2 & 6 & 3 & 1 \\ -2 & -6 & 0 & -2 & -8 & 3 & 1 \\ 3 & 9 & 0 & 0 & 6 & 6 & 2 \\ -1 & -3 & 0 & 1 & 0 & 9 & 3 \end{bmatrix}$$

2. Find  $rref(A1)$  by typing  $RRA1 = rref(A1)$  you will get

$$RRA1 = \begin{bmatrix} 1 & 3 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3. Solve for leading variables:

$$\begin{aligned} x_1 &= -3x_2 - 2x_5 \\ x_4 &= -2x_5 \\ x_6 &= -\frac{1}{3}x_7 \end{aligned}$$

Set  $x_2 = r, x_3 = s, x_5 = t$  and  $x_7 = w$  we obtain

$$\begin{bmatrix} x_1 = & -3r & -2t \\ x_4 = & & -2t \\ x_6 = & & & -\frac{1}{3}w \end{bmatrix}$$

In matrix form we can write this as

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -3r - 2t \\ r \\ s \\ -2t \\ t \\ \frac{1}{3}w \\ w \end{bmatrix} = r \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 0 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{3} \\ 1 \end{bmatrix}$$

So the vectors

$$\mathbf{v}_1 = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -2 \\ 0 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{3} \\ 1 \end{bmatrix}$$

form a basis for the solution space of  $AX = 0$ .