I will be using the Lagrangian multipliers method.As the name implies, this method relies on the Lagrangian: $\mathcal{L} = f(\vec{x}) - \lambda g(\vec{x})$. We know the vaccine coverage, $VacCov$, as well as the population, Pop. We also know the case (c), deaths (d), and DALYs (D) averted. Our goal is to optimize the number of vaccines (v), which is the only value we can control.

Let's first look at cases averted. We know from the problem description that $c = Pop * VacCov * VacEff_c * \frac{100{,}000 * cases}{Pop}$, where $VacEff_c$ is the vaccine efficacy for cases (it doesn't matter, because, as we will soon see, we can eliminate it). Now, rearranging this, we see that the number of cases averted per fully vaccinated person (here, I will refer to it as "CPF") is given by: $CPF = VacEff_c * \frac{100{,}000 * cases}{Pop}$. Now, we will simply back-substitute CPF into our first equation to get

$$c = CPF * VacCov * Pop$$

So, we now have an equation for the number of cases averted, c in terms of information given in the data. Now, the problem is that CFP will change as we change $VacCov$ (the population may also change slightly, but we are assuming we're not preventing enough deaths to make that change matter for this calculation). So, we can think of $VacCov * Pop$ being the number of vaccines we will send, and optimizing it is the same as optimizing vaccine coverage.

This means (and I didn't realize this earlier), that we will have to build a model relating CPF to $VacCov$ before doing the optimization. I am planning to do so using linear regression, but it will depend on how good that model is.

At any rate, once we have done that, we get the function to optimize: $c(VacCov) = CPF(VacCov) * VacCov * Pop$, or, changing our variable to number of vaccines:

$$c(v) = CPF(\frac{v}{Pop}) * v$$

,

where v is the number of vaccines. We can do the same thing with deaths and DALYs to get very similar equations, giving us the final model to optimize:

$$f(v) = CPF(\frac{v}{Pop}) * v + dPF(\frac{v}{Pop}) * v + DPF(\frac{v}{Pop}) * v$$

.

If we wanted to, we could also easily change the importance of each term by scaling it by α:

$$f(v) = \alpha_1 c(v) + \alpha_2 d(v) + \alpha_3 D(v)$$

This is subject to the following constraint: $g(v) = Av - N$ i.e. we only have N vaccines to give. Now, using the Lagrangian multipliers method for optimizing this problem:

$$\frac{\partial f}{\partial v} = \lambda \frac{\partial g}{\partial v}$$

. This gives us our optimal solution. One final note is that this equation only gives us an optimization for one vaccine. I've written it like this to simplify the notation and explain the concepts.In reality, v is a vector, which makes things far more complicated to do by hand, but not much more complicated numerically.