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CS 251
Final Exam (Of Doom)
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PCC G number: G03584121

1 State precisley (but concisely) in your own words what it mean for an argument to be valid.

An argument is considered valid if it's premise and conclusion are logically sound.

2 Prove (or disprove).

$$\forall n \in \mathbb{Z}, n \geq 3 \to \sum_{i=3}^{n} i(i-1) = \frac{(n-2)(n^2+2n+3)}{2}$$

Proof:

$$\exists n \in \mathbb{Z}, n \geq 3 \wedge \sum_{i=3}^{n} i(i-1) \neq \frac{(n-2)(n^2+2n+3)}{2}$$

Row	*Statement*	*Comment*
1.	*Let* $n := 3$	counter example
2.	$\sum_{i=3}^{3} i(i-1) = 6$	
3.	$\frac{(3-2)(3^2+2\cdot 3+3)}{2} = 9$	
4.	$6 \neq 9$	
5.	$\therefore \exists n \in \mathbb{Z}, n \geq 3 \wedge \sum_{i=3}^{n} i(i-1) \neq \frac{(n-2)(n^2+2n+3)}{2}$	QED

3 Prove (or disprove)

$$\forall \text{ } sets \text{ } A, B, C \in U \mid A \cup (B \cap C) \subset (A \cup B) \cap C$$

Proof:

$$\exists \text{ } sets \text{ } A, B, C \in \mathbb{U} \mid A \cup (B \cap C) \not\subset (A \cup B) \cap C$$

Row	*Statement*	*Comment*
1.	*Let* $A = \{1, 2, 4, 5\}$	counter example
2.	*Let* $B = \{2, 3, 5, 6\}$	
3.	*Let* $C = \{4, 5, 6, 7\}$	
4.	*Say* $\exists$ *some set* $X \mid X = A \cup (B \cap C) = \{1, 2, 5, 4, 6\}$	
6.	*Say* $\exists$ *some set* $Y \mid Y = (A \cup B) \cap C = \{2, 5, 4, 6\}$	
7.	$1 \notin Y$	
8.	$X \not\subset Y$	
9.	$\therefore \exists$ *sets* $A, B, C \in \mathbb{U} \mid A \cup (B \cap C) \not\subset (A \cup B) \cap C$	QED

4 Let $F : \mathbb{R}\ x\ \mathbb{R} \rightarrow \mathbb{R}\ x\ \mathbb{R}$ be a relation defined as

$$F(x, y) = (3y - 1, 1 - x)$$

(a) Prove or disprove that F is a function.

(b) Prove (or disprove) F^{-1} is a function

(a)

Row	*Statement*	*Comment*
1.	$\forall\ F(x, y)\ \exists\ z \in \mathbb{R} \mid z = F(x, y) \rightarrow z \in \mathbb{R}$	By closure on addition, and, subtraction
2.	$\therefore F(x, y)\ is\ a\ function$	*QED*

(b)

Row	*Statement*	*Comment*
1.	$F(x, y) = (3y - 1, 1 - x) \rightarrow F^{-1} = (1 - y, \frac{x+1}{3}$	By definition of inverse function
2.	$\forall\ F(x, y)^{-1}\ \exists\ z \in \mathbb{R} \mid z = F^{-1}(x, y) \rightarrow z \in \mathbb{R}$	By closure on subtraction, division
3.	$\therefore\ F^{-1}\ is\ a\ function$	*QED*

5 Let:

$$A = \left\{x \in \mathbb{Z} \mid \exists\ k \in \mathbb{Z} \wedge x = k^2\right\}$$

What is $\mid A \mid$? Explain your reasoning and justify your answer.

$\mid A \mid$ is countably infinite. This is because for every integer in the set of all integers that exists there is a square that also exists. In other words, can be put into a one to one correspondence with the set of all integer squares

6 Define A to be the set of unique digits on your PCC G-Number, and let R $: A \rightarrow A$ be deifined as

$$xRy \leftrightarrow 2 \mid (x - y)$$

List the equivalence classes of R or prove no such classes exist.

$$A = \{0, 1, 2, 3, 4, 5, 8\}$$

The equivalence classes are:

$\{0, 2, 4, 8\}, \{1, 3, 5\}, \{\emptyset\}$

7 Let S be the set of all strings of 0's and 1's of length 3. Define $R : S \to S$ as $$sRt \leftrightarrow \begin{array}{r} the\ two\ left\ most\ characters \\ of\ s\ are\ the\ same\ as\ the\ two \\ left\ most\ characters\ of\ t \end{array}$$ List the equivalence classes of R or prove no such classes exist.

$$S = \{000, 100, 010, 001, 111, 011, 101, 110\}$$

The equivalence classes are:

$$\{000, 001\}, \{001, 010\}, \{101, 100\}, \{110, 111\}, \{\emptyset\}$$

8 Let x be you PCC G number without the leading G. What is the inverse modulo of x modulo 8831?

The inverse modulo does not exist. My G number and 8331 are not coprime.

9 An RSA Cipher has the public key pq=65 and e=7. What is the encrypted value of the last 3 digits of your PCC G number?

The encrypted values of 1, 2, 1 are: 1, 63, 1

10 Three quizzes are given to a class of 30 students, and all student submitted all quizzes. Given: - 15 students scored 12 or more on quiz 1 - 12 students scored 12 or more on quiz 2 - 18 students scored 12 or more on quiz 3 - 7 students scored 12 or more on quizzes 1 and 2 - 11 students scored 12 or more on quizzes 1 and 3 - 8 students scored 12 or more on quizzes 2 and 3 - 4 students scored 12 or more on quizzes 1, 2, and ,3 How many students scored 12 or more on quizzes 1 and 2 but not 3?

Three students scored 12 or more on quzzes 1 and 2 but not 3.

11 An urn contains four balls, each numbered with one of the last 4 digits of your PCC G-Number. If a person selects two balls at random (equal probability), what is the expected value of the product of the numbers on the balls?

The expected value of the product of the numbers on the balls is about 5.1667.

12 If a graph has nodes of degrees 1, 1, 2, 3, and 3, how many edges does it have? Explain your reasoning and justify your answer, although a formal proof is not needed.

$$1 + 1 + 2 + 3 + 3 + 3 = 10$$

By thm. 10.1.1 the sum of the degrees is equal to the twice the number of edges.

$10 = 2 \cdot k$ where k is the number of edges so $k = 5$

13 Specify (drawing or adjacency matrix) a full binary tree with 16 nodes, of which 6 are internal nodes, or prove no such graph exists.

Suppose not:

Row	*Statement*	*Comment*
1.	*A full binary tree with k nodes has $2k+1$ internal nodes*	thm 10.6.1 Susanna Epp
2.	$k = 6$	
3.	$2 \cdot 6 + 1 = 13$	
4.	*A full binary tree with 6 internal nodes would have a total of 13 nodes*	
5.	$\therefore$ *no such graph exists*	QED

14 Must a graph with 68 nodes and 72 edges have a circuit? Explain your reasoning and justify your answer, although a formal proof is not needed.

,No, it could be a tree.

15 Given the adjacency matrix:

$$\text{AM} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Assume the ows correspond to nodes 1, 2,3, and 4 of the graph. How many walks of length 15 are there from node 3 to node 1?

Zero Walks.