CS250 Methods Of Proof Knowledge Test

jonathon.sonesen

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1 $\forall x \in R,\ x > 1 \ \rightarrow \ x^2 > x$

Row	*Statement*	*Comment*
1	$x \in Z \ \mid \ x > 1$	*generic particular*
2	$x^2 \ = \ x \cdot x$	*multiplication*
3	$x \ = \ x \cdot 1$	*multiplicationR*
4	$x \cdot x > \ x \cdot 1$	*by multiplication*
5	$\forall x \in R,\ x > 1 \ \rightarrow \ x^2 > x$	*QED*

2 $\exists x \in R \ \mid \ 0 < x < 1 \ \wedge \ x^2 \geq x$

Row	*Statement*	*Comment*
1	$Suppose : \forall x \in R, 0 < x < 1 \ \rightarrow \ x^2 \ < \ x$	*negation*
2	$\forall x \in R \ \mid \ (0 < x < 1)$	*generic particular*
3	$\forall x \in R, \exists y \in R \mid y > 1 \wedge x \ = \ \frac{1}{y}$	$by\ definition$
4	$(\frac{1}{y})^2 < \frac{1}{y}$	*by substitution*
5	$\frac{1}{y^2} < \frac{1}{y}$	$multiplication\ of\ fractions\ T15$
6	$\forall x \in R, 0 < x < 1 \ \rightarrow \ x^2 \ < \ x$	*QED*

3 $\forall n \in Z,\ n > 0 \ \rightarrow \ 2^{2^n} \ + \ 1 \ \in Z^P$

Row	*Statement*	*Comment*
1	$Suppose : \exists n \in Z,\ n > 0 \wedge 2^{2^n} \ + \ 1 \ \in Z^C$	*negation*
2	$Let : n \ := \ 5$	*counterexample*
3	$2^{2^5} \ + \ 1 \ = \ 4,294,967,297 = (641) \cdot (6700417)$	$by\ definition$
4	$\exists n \in Z,\ n > 0 \wedge 2^{2^n} \ + \ 1 \ \in Z^C$	*QED*

$https : //www.sharelatex.com/project$

4 $\forall a,b,c,d \in Z,\ x \in R, \Big(\ a \neq c\ \wedge\ \left(\frac{ax\ +\ b}{cx\ +\ d}\ =\ 1\right)\Big)\ \rightarrow\ x \in Q$

Row	*Statement*	*Comment*
1	$\forall a,b,c,d \in Z,\ x \in R, \Big(\ a \neq c\ \wedge\ \left(\frac{ax\ +\ b}{cx\ +\ d}\ =\ 1\right)$	*generic particular*
2	$\forall a,b,c,d \in Z,\ x \in R, ax + b = cx + d$	*cross multiply*
3	$ax - cx = d - b$	*algebra*
4	$x(a - c) = d - b$	$by factorization$
5	$x = \frac{d-b}{a-c}, a \neq c$	*algebra*
6	$x \in Q$	$definition$
7	$\exists y \int Z \vert y = d - b$	$byclosure$
8	$\exists w \int Z \vert w = a - c \neq 0$	$byclosure`doesnt\ equal\ zero\ a \neq c$
9	$x = \frac{y}{w}$	*ratio of two ints*
10	$\forall a,b,c,d \in Z,\ x \in R, \Big(\ a \neq c\ \wedge\ \left(\frac{ax\ +\ b}{cx\ +\ d}\ =\ 1\right)\Big)\ \rightarrow\ x \in Q$	*QED*

5 $\forall x \in R,\ x \in Q\ \rightarrow\ 5x^3\ +\ 8x^2\ +\ 7\ \in Q$

Row	*Statement*	*Comment*
1	$Suppose : \exists\ c \in R \mid c = 5x^3 + 8x^2 + 7$	*generic particular*
2	$\forall x \in Q, \exists a, b \in Z\ \mid\ x = \frac{a}{b}$	$by\ definition\ of\ rational\ num$
3	$5x^3 + 8x^2 + 7 = c = \frac{5a^3}{b^3} + \frac{8a^2}{b^2} + \frac{7}{1}$	*by substitution*
4	$c = \frac{5a^3b^2 + 8a^2b^3 + 7b^3}{b^3 \cdot b^2}$	*by cross multiply*
5	$c = \frac{5a^3 + 8a^2b^3 + 7b^3}{b^3}$	$by\ division\ of\ b^2$
6	$Let : y = 5a^3 + 8a^2b^3 + 7b^3$	*by substitution*
7	$y \in Z$	*by closure on sum and products on int*
8	$Let : z = b^3$	*by substitution*
9	$y \in Z$	$by\ closure\ of\ products$
10	$c = \frac{y}{z}$	*by substitution*
11	$c \in Q$	*by closure*
12	$\forall x \in R,\ x \in Q\ \rightarrow\ 5x^3\ +\ 8x^2\ +\ 7\ \in Q$	*QED*

6 $\exists a,b,c \in Z\ \mid\ a \mid bc \wedge ab\ \wedge\ ac$

Row	*Statement*	*Comment*
1	$Let : a := 4$	*example*
2	$Let : b := 2$	*example*
3	$Let : c := 6$	*example*
4	$4 \vert 12$	*substitution*
5	42	*substitution*
6	46	*by substitution*
7	$\exists a,b,c \in Z\ \mid\ a \mid bc \wedge ab\ \wedge\ ac$	*QED*

7 $\forall a, n \in Z,\ a \mid n^2 \ \wedge\ a \leq n \ \rightarrow\ a \mid n$

Row	*Statement*	*Comment*
1	$\exists a, n \in Z, a \mid n^2 \wedge a \leq n \wedge an$	$negation for counter example$
2	$Let : a := 4$	$example$
3	$Let : n := 6$	$example$
4	$4 \mid 6^2 \wedge 4 \leq 6 \wedge 46$	$substitution$
5	$6^2 = 36$	$bymultiplication$
6	$9 \cdot 4 = 36$	$bymultiplication$
7	$\exists a, n \in Z, a \mid n^2 \wedge a \leq n \wedge an$	QED

8 $\forall m, n \in Z,\ m5\ =\ 2\ \wedge\ n5\ =\ 1\ \rightarrow\ nm5\ =\ 1$

Row	*Statement*	*Comment*
1	$\exists m, n \in Z,\ m5\ =\ 2\ \wedge\ n5\ = 1\ \wedge\ nm5 \neq 1$	*counter example*
2	$Let : m := 7$	$setmto7$
3	$Let : n := 6$	$setnto6$
4	$75 = 2$	$bymath$
5	$65 = 1$	$bymath$
6	$7 \cdot 6 = 42$	
7	$425 = 2$	
8	$\exists m, n \in Z,\ m5\ =\ 2\ \wedge\ n5\ = 1\ \wedge\ nm5 \neq 1$	QED

9 $\forall m, d, k \in Z,\ d > 0\ \rightarrow\ (m\ +\ dk)d\ =\ md$

Row	*Statement*	*Comment*
1	$\forall m, d, k \in Z,\ d > 0\ \rightarrow\ (m\ +\ dk)d\ =\ md$	*theorem*
2	$\forall m, d, k \in Z,\ d > 0$	*generic*
3	$Let : \exists x \in Z, x = (m + dk)d$	$bysubstitution$
4	$Let : \exists y \in Z\ \mid\ m + dk = yd + x$	*by quotient remainder theorem*
5	$m = (y - k)d + x$	*by quotientremainder r is remainder*
6	$md = r$	
7	$\forall m, d, k \in Z,\ d > 0\ \rightarrow\ (m\ +\ dk)d\ =\ md$	QED

10 $\forall n \in Z,\ n \equiv 1(2)\ \rightarrow\ \lceil \frac{n^2}{4} \rceil\ =\ \frac{n^2\ +\ 3}{4}$

Row	*Statement*	*Comment*
1	$Let : \forall n \in Z \exists r \in Z, n = 2k+1$	$definition\ of\ odd\ int, generic\ particular$
2	$\lceil \frac{n^2}{4} \rceil = \lceil \frac{(2k+1)^2}{4} \rceil$	$substitution$
3	$\lceil \frac{(2k+1)^2}{4} \rceil = \lceil \frac{(4k^2+1+4k}{4} \rceil$	$algebra$
4	$\lceil \frac{(2k+1)^2}{4} \rceil = \lceil \frac{(4k^2+4k}{4} + \frac{1}{4} \rceil$	$algebra$
5	$\lceil \frac{(4k^2+4k}{4} + \frac{1}{4} \rceil = \lceil \frac{(4(k^2+k}{4} + \frac{1}{4} \rceil$	$by\ math$
6	$\lceil \frac{(4(k^2+k}{4} + \frac{1}{4} \rceil = k^2+k+1$	
7	$\frac{n^2+3}{4} = \frac{(2k+1)^2+3}{4}$	$substitution\ for\ right\ side$
8	$\frac{(2k+1)^2+3}{4} = \frac{4k^2+4k+4}{4}$	$expand$
9	$\frac{4k^2+4k+4}{4} = \frac{4(k^2+k+1}{4}$	$factor$
10	$\frac{4(k^2+k+1}{4} = k^2+k+1$	$division$
11	$\forall n \in Z,\ n \equiv 1(2)\ \rightarrow\ \lceil \frac{n^2}{4} \rceil\ =\ \frac{n^2\ +\ 3}{4}$	QED

11 $\forall x \in R,\ \lfloor x^2 \rfloor\ =\ \lfloor x \rfloor^2$

Row	*Statement*	*Comment*
1	$\exists x \in R, \lfloor x^2 \rfloor \neq \lfloor x \rfloor^2$	$counterexample$
2	$Suppose : \exists x \in R, x := 1.5$	$counterexample$
3	$\lfloor 1.5^2 \rfloor = 2$	
4	$\lfloor 1.5 \rfloor = 1$	
5	$1^2 = 1$	
6	$\exists x \in R, \lfloor x^2 \rfloor \neq \lfloor x \rfloor^2$	

12 $\forall n \in Z,\ n6\ =\ 3\ \rightarrow\ n3\ \neq\ 2$

Row	*Statement*	*Comment*
1	$Assume : \exists a \in Z, a6 = 3 \wedge a3 = 2$	$proof\ by\ contradiction$
2	$\exists p, k\ \mid\ a = 3p+2 = 6k+3$	$quotientremainder$
3	$3(p+2k) = 1$	$algebra$
4	$p+2k = \frac{1}{3}$	$alwaysdividesbythree, andisacontradiction$
5	$\forall n \in Z,\ n6\ =\ 3\ \rightarrow\ n3\ \neq\ 2$	

13 $\forall a, b \in Z,\ gcd(a,b) \mid lcm(a,b)$

Row	*Statement*	*Comment*
1	$Let: \forall a, b \in Z, \exists x, y \in Z \mid x = gcd(a,b) \wedge y = lcm(a,b)$	
2	$x \mid a, x \mid b \ \wedge \ a \mid y, b \mid y$	*quotientremainder*
3	$\forall x \in Z, \exists p, q \in Z \mid a = x \cdot p \wedge b = x \cdot q$	*algebra*
4	$\forall y \in, \exists r, s \in Z \mid y = a \cdot r \wedge y = b \cdot s$	
5	$y = a \cdot r$	
6	$y = r \cdot x \cdot p$	*by substitution*
7	$y = x(rp)$	*commutative*
8	$lcm(ab) = gcm(ab) \cdot rp$	
9	$lcm(ab) \mid gcm(ab) = rp$	

A={ x $\in Z|\frac{3x^3+x^2-2x+4}{3x+4}| \leq (2^{50}-1)\}$