

CS 251 Counting and Probability Knowledge Assignment

1 Instructions

1. Create a `.pdf` file named `knowledge.pdf` that contains the answers to the questions below.
2. Note that the file must be named `knowledge.pdf` and must be a `.pdf` file. Do not submit Word documents, Open Office documents, rich text or plain text files, or any format other than `.pdf`. Do not make up your own file name - use `knowledge.pdf`
3. If you create your `.pdf` file by scanning handwritten documents, make sure all pages are oriented properly, pages are in order, and all pages are legible
4. Make friends with your computer! Some questions are intended to be completed with “electronic assistance” - software such as MATLAB or Maple, websites such as Wolfram Alpha, C++ programs using gnu/gmp, Python programs, or whatever you wish. This is a Computer Science class - let your inner geek run free.
5. You may work in groups if you wish. If you work in a group, please list the names of all group members in the first lines of your `knowledge.pdf` file.
6. Submit your `knowledge.pdf` file to the Desire2Learn dropbox. You must submit in order to receive a grade. If you worked in a group and your answers are the same as others in your group, you still must make your own submission in order to get credit for the assignment.
7. This assignment is graded on a 10-point scale (0 = very bad, 10 = very good). Each question is worth 1 point. Partial credit is given for partially correct answers.
8. The *Counting and Probability* module is worth 25% of your final course grade, so this assignment is worth 12.5% of your final course grade.
9. Please review all items of this assignment as soon as possible, and if you have any questions please email me or post them as soon as you can. I don't think there are any mistakes in the assignment, but I could be mistaken.

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2 Questions

You need 10 correct answers to receive full credit on this assignment. Any additional correct answers are extra-credit, and will be applied to the Application assignment for this module.

1. In a competition between players A and B, the first player to win 5 games in a row, or a total of 6 games, wins. How many ways can the competition be played if A wins the first game and B withs the second and third games?
2. If p , q , and r are distinct prime numbers, and a , b , and c are positive integers, how many distinct positive divisors does $p^a \cdot q^b \cdot r^c$ have?
3. At a certain university, passwords must be from 15 to 20 symbols long, and composed of the 26 letters of the alphabet, the ten digits 0 - 9, and 14 special symbols (for a total of 50 possible symbols). How many passwords contain no repeated symbols?
4. What is the probability that a randomly chosen string of 7 hexadecimal digits has at least one repeated digit? Assume equal likelihood.
5. Let S be the set of all strings of length 12 over the set $\{w, x, y, z\}$. In other words, S consists of all strings of length 12 composed of these characters. What is the probability that a randomly chosen element of S contains at least 1 pair of adjacent characters that are the same? In other words, what is the probability a string will contain a “ww” sequence, or “xx” sequence, or “yy” sequence, or “zz” sequence?
6. Consider the infinite decimal 12.112211122211112222..., where each group of 1's and 2's becomes longer in each repetition. Is this number rational or irrational? Explain your reasoning (no formal proof is needed).
7. Suppose that 5 computers in a production run of 65 are defective. A sample of 7 computers is to be selected to be checked for defects.
 - How many samples will contain at least one defective computer?
 - What is the probability that a randomly chosen sample of 7 contains at least 1 defective computer?

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8. A large pile of coins consists of pennies, nickels, dimes, and quarters. If the pile contains only 23 dimes, but at least 37 of each other kind of coin, how many collections of 37 coins can be chosen?
9. How many integers from 1×10^0 through 1×10^9 have the sum of their digits equal to 10? For example, the integers 9001 and 800002 have this property.
10. A fair coin is tossed until either 2 “heads” or 7 “tails” is obtained. What is the expected number of tosses?
11. An urn contains 10 balls numbered 1, 2, 3, 3, 5, 5, 7, 7, 7, 8. If a person selects a set of 4 balls at random (equal probability), what is the expected value of the sum of the numbers on the balls?
12. A company sends an entry form for a sweepstakes to a lot of people. The prizes for the contest are
 - One first place award of \$10,000,000.
 - Two second-place awards of \$1,000,000.
 - Three third-place awards of \$10,000.
 - Four fourth-place awards of \$1,000.

A person must spend \$0.75 to enter the sweepstakes, of which \$0.30 goes to the company to offset the prize money, and the other \$0.45 is postage.

- How many entries are required in order to offset the total amount of the prize money?
 - Assuming exactly that many entries are received, what is the expected gain or loss for a single entry?
13. A coin is loaded so that the probability of “heads” is 0.72, and the probability of “tails” is $1 - 0.72$. Suppose that the coin is tossed twice and the results of the tosses are independent. What is the probability of obtaining exactly two “heads”?
 14. Assume you are taking a multiple-choice exam that contains 10 questions. Each question has 5 possible answers. You are a good student, and are confident about your answers to 7 of the 10 questions. You have a 0.96

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probability that you answered correctly on each of these 7 questions. You are clueless about the remaining 3 questions, and randomly chose answers for these 3 questions.

- Assume each correct question is worth 1 point, and each incorrect question is worth 0 points. What is the expected value of your score on the exam?
 - Assume each correct question is worth 1 point, and each incorrect question is worth -0.25 points. What is the expected value of your score on the exam?
15. Assume that there is a 0.4976 probability that a newborn child is male. A family has 10 children, and the genders of the children are mutually independent. What is the probability that exactly 7 of the children are female?