# CS 251 Graph Theory Knowledge Assignment

## 1 Instructions

1. Create a `.pdf` file named `knowledge.pdf` that contains the answers to the questions below.

2. Note that the file must be named `knowledge.pdf` and must be a `.pdf` file. Do not submit Word documents, Open Office documents, rich text or plain text files, or any format other than `.pdf`. Do not make up your own file name - use `knowledge.pdf`

3. If you create your `.pdf` file by scanning handwritten documents, make sure all pages are oriented properly, pages are in order, and all pages are legible

4. Use formal notation only in all proofs, number each step of the proof, and add a comment to your derivations identifying the rule(s) or axiom(s) used to justify the step, e.g "Commutativity of addition" or "closure of $\mathbb{Z}$ on addition" or "Exercise 5.4 of Textbook". Note that "algebra" and "obvious" are **not** rules or axioms that can justify a step. Your goal in all proofs is to demonstrate that you have mastered the techniques, logic and formal notation needed for the proof - I already know the answers to the questions.

5. Make friends with your computer! Some questions are intended to be completed with "electronic assistance" - software such as Mathlab or Maple, websites such as Wolfram Alpha, C++ programs using gnu/gmp, Python programs, or whatever you wish. This is a Computer Science class - let your inner geek run free.

6. You may work in groups if you wish. If you work in a group, please list the names of all group members in the first lines of your `knowledge.pdf` file.

7. Submit your `knowledge.pdf` file to the Desire2Learn dropbox. You must submit in order to receive a grade. If you worked in a group and your answers are the same as others in your group, you still must make your own submission in order to get credit for the assignment.

8. This assignment is graded on a 10-point scale (0 = very bad, 10 = very good). Each question is worth **0.75** points. Partial credit is given for partially correct answers.

9. The *Graph Theory* module is worth 25% of your final course grade.

10. This assignment includes extra credit. Since each question is worth 0.75 points, you need $13\frac{1}{3}$ correct answers for full credit. Any additional correct answers are extra credit (0.75 pt each). There is a maximum of 5 points of extra credit available.

11. Please review all items of this assignment as soon as possible, and if you have any questions please email me or post them as soon as you can. I don't think there are any misteaks in the assignment, but I could be mistaken.

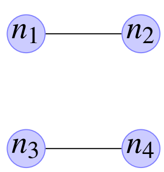
12. If an answer requires identifying a graph, you may either draw the graph or list the adjacency and incidence matrices for the graph.

## 2 Questions

1. Let $G$ be a simple graph with $n$ nodes. Let $k$ be the number of edges of $G$. Prove (or disprove)

$$k \leq \frac{n(n-1)}{2}$$

2. Let $G$ be the graph:

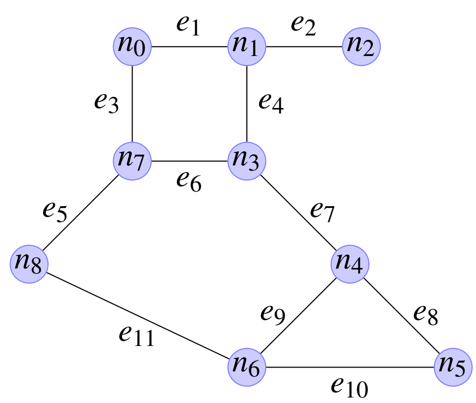



What is the complement of $G$?

3. List a simple graph that has 4 nodes of different degrees, or prove that no such graph exists.

4. What is the maximum number of edges possible in a disconnected graph with $n$ nodes and no loops or parallel edges? Explain your answer (informal is fine - no proof needed).

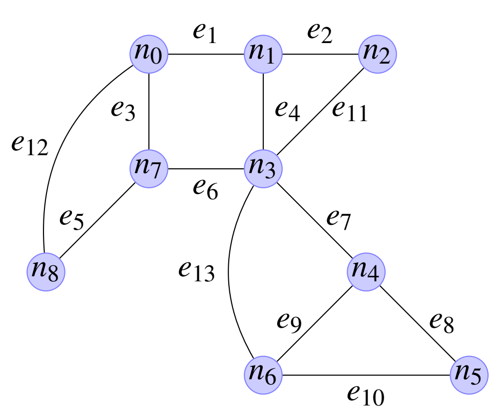
5. Let $G$ be the graph:




   (a) List the Adjacency matrix for this graph

   (b) List the Incidence matrix for this graph

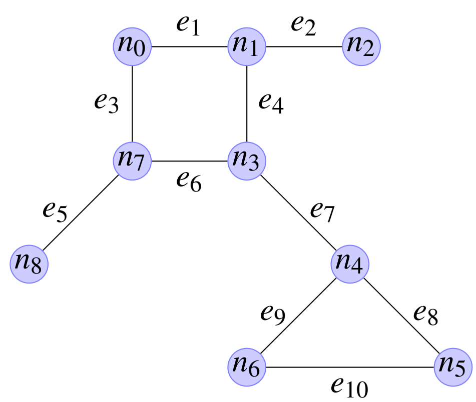
6. Let $G$ be the graph:




   (a) List the Laplacian matrix for this graph

   (b) List the eigenvalues for the Laplacian matrix for this graph

7. Let $G$ be the graph:

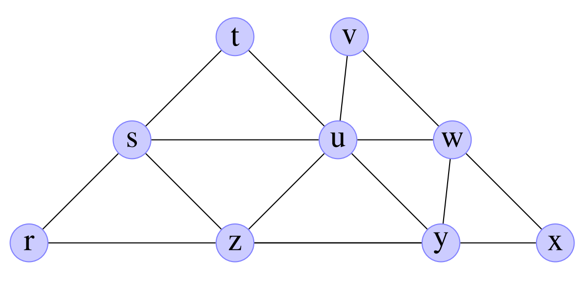



   (a) List the Degree matrix for this graph

   (b) List the Adjacency matrix for this graph

   (c) Identify the bridges (if any) of this graph. If there are no bridges, write ”none”.

8. Let $G$ be the graph:

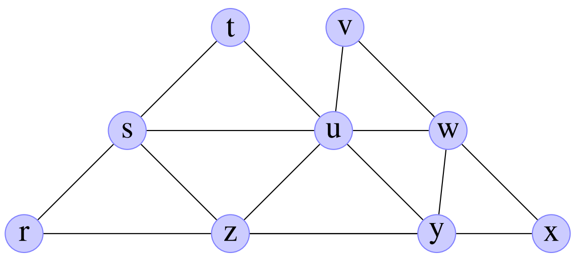



   (a) List the Laplacian matrix for this graph

   (b) List the Incidence matrix for this graph

   (c) Identify an Euler circuit for this graph, or prove no such circuit exists

9. Let $G$ be the graph:

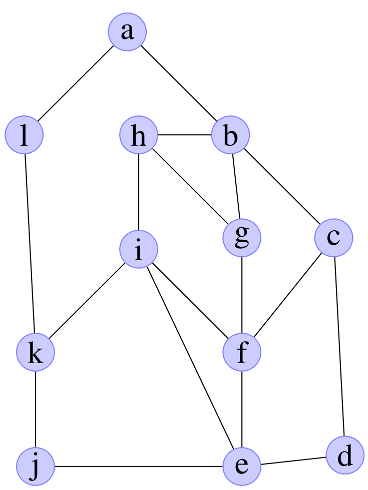



(a) List the Adjacency matrix for this graph

(b) List the Degree matrix for this graph

(c) Identify a Hamiltonian circuit for this graph, or prove no such circuit exists

10. Let $G$ be the graph:

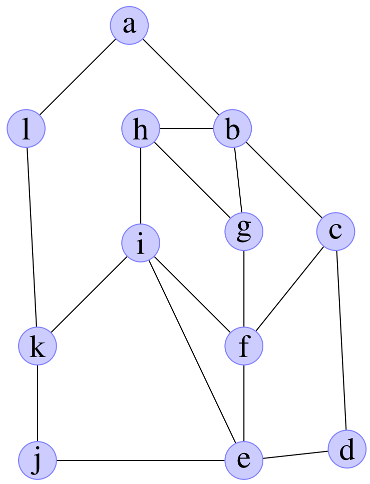



(a) List the Adjacency matrix for this graph

(b) List the Degree matrix for this graph

(c) Identify an Euler circuit for this graph, or prove no such circuit exists

11. Let $G$ be the graph:




 (a) List the Laplacian matrix for this graph
 (b) List the Incidence matrix for this graph
 (c) Identify a Hamiltonian circuit for this graph, or prove no such circuit exists

12. Let

$$M = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{1}$$

List two $2 \times 2$ matrices $A$ and $B$ that satisfy **all** of the following conditions:

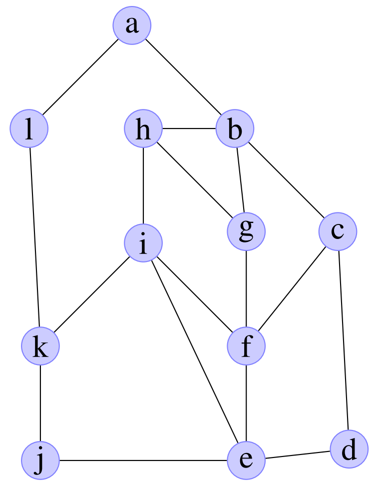
- $A \neq M$
- $B \neq M$
- $AB \neq M$
- $BA = M$

13. Let

$$M = \begin{pmatrix} 4 & 12 & 7 & 3 & 20 \\ 12 & 4 & 20 & 21 & 7 \\ 10 & 7 & 6 & 9 & 13 \\ 1 & 2 & 3 & 4 & 5 \\ 10 & 8 & 15 & 13 & 12 \end{pmatrix} \tag{2}$$

List the matrix $M^6$.

14. Let $G$ be the graph:



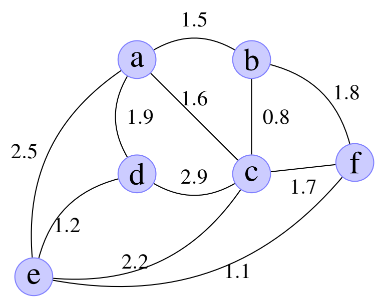

(a) List the Adjacency matrix for this graph
(b) List the Degree matrix for this graph
(c) How many walks of length 25 are there between node $a$ and node $e$?

15. List all **non**-isomorphic graphs with 4 nodes and no more than two edges.

16. List all **non**-isomorphic trees with 5 nodes.

17. List a full binary tree with 7 nodes or prove why no such tree exists.

18. Let $G$ be the graph:

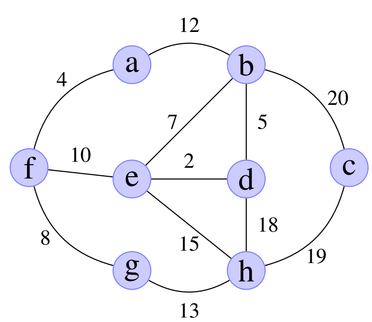



    (a) List the Adjacency matrix for this graph

    (b) List the Degree matrix for this graph

    (c) List a minimal spanning tree for this graph

19. Let $G$ be the graph:




    (a) List the Laplacian matrix for this graph

    (b) List the Incidence matrix for this graph

    (c) List a minimal spanning tree for this graph

20. Prove (or disprove) that any two spanning trees for a graph have the same number of edges. Note that this is part 2 of Proposition 10.7.1 on page 702 of the textbook.