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CS 251
Graph Theory Knowledge
June 6, 2015
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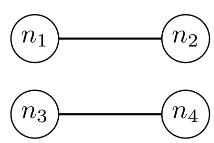
Graph Theory Knowledge Assignment

1
Let G be a simple graph with n nodes. Let k be the number of edges of G. Prove (or disprove)

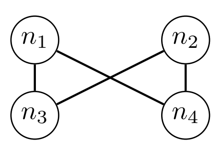
$$k \leq \frac{n(n-1)}{2}$$

Row	*Statement*	*Comment*
1.	*Let* $n := 1$	Base case
2.	G_1 *has* 0 *edges and* $\frac{1(1-1)}{2} = 0$	by example 10.1.9,and subs.
3.	$k \leq \frac{n(n-1)}{2}$	inductive hyp.
4.	$k = \frac{n(n-1)}{2} = \frac{(n+1)((n+1)-1)}{2}$	
5.	$= \frac{n(n+1)}{2} + n$	
6.	$= \frac{n(n+1)}{2} + \frac{2n}{2}$	
7.	$= \frac{n^2-n+2n}{2}$	
8.	$= \frac{n(n+1)}{2}$	
9.	$\therefore k \leq \frac{n(n-1)}{2}$	QED

2
Let G be the graph:



What is the complement of G?



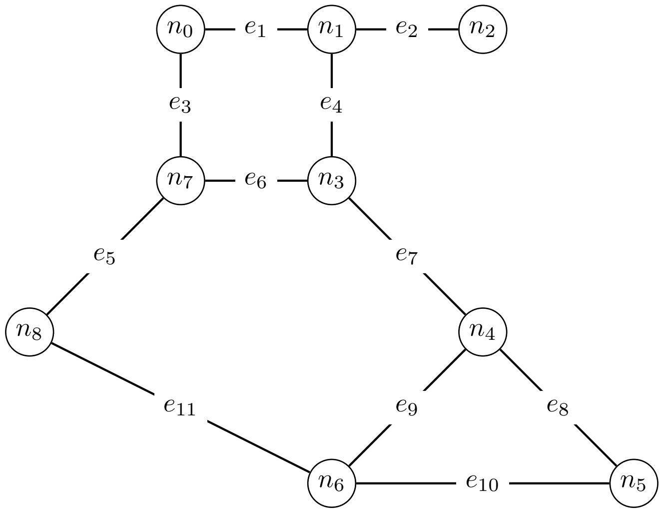
3 List a simple graph that has 4 nodes of different degrees, or prove that no such graph exists.

The proof to question 1 proves a contradiction

4 What is the maximum number of edges possible in a disconnected graph with n nodes and no loops or parallel edges? Explain your answer. (No proof needed)

$k_{max} = \frac{n(n-1)}{2}$

5 Let G be the graph:



(a) List the Adjacency matrix for this graph

(b) List the Incidence matrix for this graph

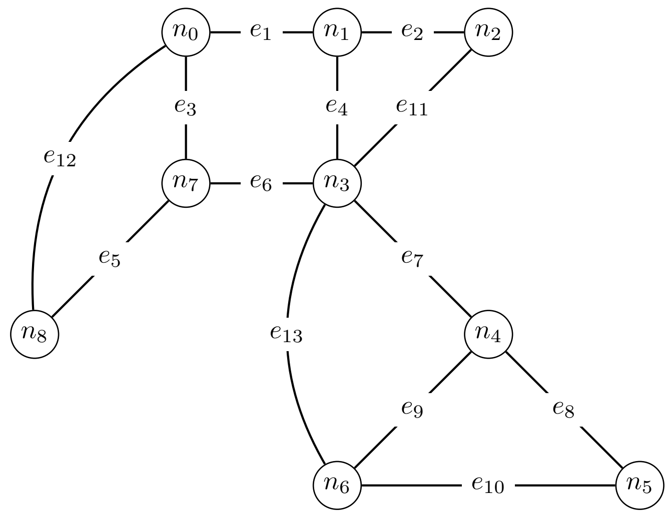
(a)

$$\mathrm{AM}_G = \begin{pmatrix} n_i & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

(b)

$$\mathrm{LM}_G = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

6 Let G be the graph:



(a) List the Laplacian matrix for this graph

(b) List the eigenvalues for the Laplacian matrix for this graph

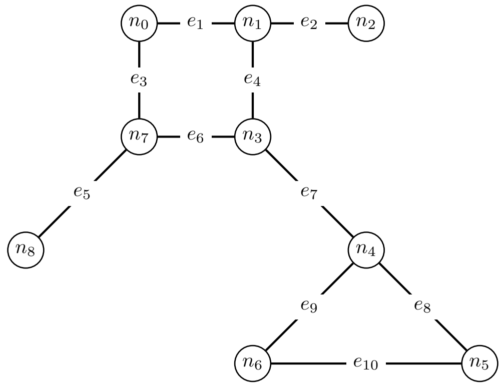
(a)

$$\mathrm{LM}_G = \begin{pmatrix} 3 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & 3 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 5 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 3 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 & 3 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 3 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

(b)

$$EV_{LM} = \begin{bmatrix} 6.3586 \\ 4.3977 \\ 4.0000 \\ 3.5651 \\ 3.0000 \\ 3.0000 \\ 1.1760 \\ 0.5126 \\ 0.0000 \end{bmatrix}$$

7 Let G be the graph:



(a) List the Degree matrix for this graph

(b) List the Adjacency matrix for this graph

(c) Identify the bridges (if any) of this graph. If ther are no bridges, write "none".

(a)

$$\mathrm{DM}_G = \begin{array}{cc} n_i & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} & \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

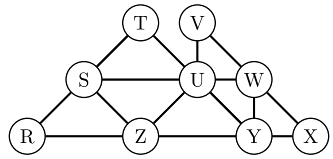
(b)

$$\mathrm{DM}_G = \begin{array}{cc} n_i & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(c)

$$\textit{bridges}: \ \{e_2, e_5, e_7\}$$

8 Let G be the graph:



(a) List the Laplacian matrix for this graph

(b) List the Incidence matrix for this graph

(c) Identify an Euler circuit for this graph, or prove no such circuit exists

(a)

$$\mathrm{LM}_G = \begin{array}{c@{}c} & \begin{array}{ccccccccc} R & S & T & U & V & W & X & Y & Z \end{array} \\ \begin{array}{c} R \\ S \\ T \\ U \\ V \\ W \\ X \\ Y \\ Z \end{array} & \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 4 & -1 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 6 & -1 & -1 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 4 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & -1 & 4 & -1 \\ -1 & -1 & 0 & -1 & 0 & 0 & 0 & -1 & 4 \end{pmatrix} \end{array}$$

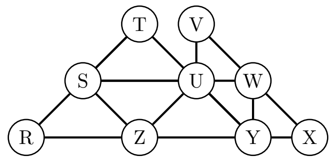
(b)

$$\mathrm{IM}_G = \begin{array}{c@{}c} & \begin{array}{ccccccccccccccc} \substack{R\\S} & \substack{R\\Z} & \substack{S\\T} & \substack{S\\U} & \substack{S\\Z} & \substack{T\\U} & \substack{U\\V} & \substack{U\\W} & \substack{U\\Y} & \substack{U\\Z} & \substack{V\\W} & \substack{W\\X} & \substack{W\\Y} & \substack{X\\Y} & \substack{Y\\Z} \end{array} \\ \begin{array}{c} R \\ S \\ T \\ U \\ V \\ W \\ X \\ Y \\ Z \end{array} & \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

(c)

$$\textit{euler circuit}: \ \{T, U, Y, X, W, Y, Z, R, S, Z, U, W, V, U, S, T\}$$

9 Let G be the graph:



(a) List the Adjacency matrix for this graph

(b) List the Degree matrix for this graph

(c) Identify a Hamiltonian circuit for this graph, or prove no such circuit exists

(a)

$$\mathrm{AM}_G = \begin{array}{cc} & \begin{array}{ccccccccc} R & S & T & U & V & W & X & Y & Z \end{array} \\ \begin{array}{c} R \\ S \\ T \\ U \\ V \\ W \\ X \\ Y \\ Z \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

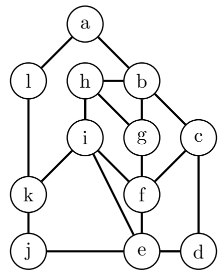
(b)

$$\mathrm{AM}_G = \begin{array}{cc} & \begin{array}{ccccccccc} R & S & T & U & V & W & X & Y & Z \end{array} \\ \begin{array}{c} R \\ S \\ T \\ U \\ V \\ W \\ X \\ Y \\ Z \end{array} & \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix} \end{array}$$

(c)

$$\textit{hamiltonian circuit}: \quad \{T, S, R, Z, Y, Y, X, W, V, U, T\}$$

10 Let G be the graph:



(a) List the Adjacency matrix for this graph

(b) List the Degree matrix for this graph

(c) Identify an Euler circuit for this graph, or prove no such circuit exists

(a)

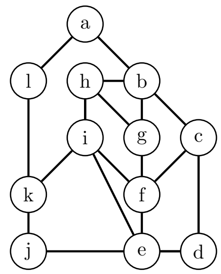
$$\mathrm{AM}_G = \begin{array}{cc} & \begin{array}{cccccccccccc} A & B & C & D & E & F & G & H & I & J & K & L \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \\ K \\ L \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(b)

$$\mathrm{AM}_G = \begin{array}{cc} & \begin{array}{cccccccccccc} A & B & C & D & E & F & G & H & I & J & K & L \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \\ K \\ L \end{array} & \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} \end{array}$$

(c) By the contrapositive version of theorem 10.2.2 in the epp text:
If some vertex of a graph has odd degree, then the graph does not have an euler circuit.

11 Let G be the graph:



(a) List the Laplacian matrix for this graph

(b) List the Incidence matrix for this graph

(c) Identify a Hamiltonian circuit for this graph, or prove no such circuit exists

(a)

$$\mathrm{LM}_G = \begin{array}{cc} & \begin{array}{cccccccccccc} A & B & C & D & E & F & G & H & I & J & K & L \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \\ K \\ L \end{array} & \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 4 & -1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 & 4 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 3 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix} \end{array}$$

(b)

$$\mathrm{IM}_G = \begin{array}{cc} & \begin{array}{cccccccccccccccccc} A & A & B & B & B & C & C & D & E & E & E & F & F & G & H & I & J & K \\ B & L & C & G & H & D & F & E & F & I & J & G & I & H & I & K & K & L \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \\ K \\ L \end{array} & \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

(c)

$$Hamiltonian\ circuit : \{b, a, l, k, j, e, d, c, f, i, h, g, b\}$$

12
Let

$$\mathrm{M} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

List two 2 x 2 matrices A and B that sitisfy *all* of the following conditions:

- $A \neq M$

- $B \neq M$

- $AB \neq M$

- $BA \neq M$

$$\mathrm{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathrm{B} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$
$$\mathrm{A} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \mathrm{B} = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

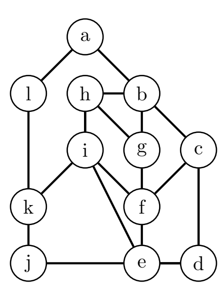
13 Let

$$M = \begin{pmatrix} 4 & 12 & 7 & 3 & 20 \\ 12 & 4 & 20 & 21 & 7 \\ 10 & 7 & 6 & 9 & 13 \\ 1 & 2 & 3 & 4 & 5 \\ 10 & 8 & 15 & 13 & 12 \end{pmatrix}$$

List the matrix m^6

$$M^6 = \begin{pmatrix} 1574709790 & 1574709790 & 2106725231 & 2076398918 & 2415751821 \\ 1643310630 & 1463710842 & 2196967681 & 2166165175 & 2521712946 \\ 1366176423 & 1216993970 & 1826701919 & 1800937567 & 2096602872 \\ 426094239 & 379500949 & 56993949 & 561795111 & 653713575 \\ 1678495370 & 1495020788 & 2244576049 & 2212798516 & 2575460889 \end{pmatrix}$$

14 Let G be the graph:



(a) List the Adjacency matrix for this graph

(b) List the Degree matrix for this graph

(c) How many walks of length 25 or greater are there from node a to node b

(a)

$$\mathrm{AM}_G = \begin{array}{cc} & \begin{array}{cccccccccccc} A & B & C & D & E & F & G & H & I & J & K & L \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \\ K \\ L \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(b)

$$\mathrm{AM}_G = \begin{array}{cc} & \begin{array}{cccccccccccc} A & B & C & D & E & F & G & H & I & J & K & L \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \\ K \\ L \end{array} & \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} \end{array}$$

(c) There are 362967939144 walks.

15 List all non-isomorphic graps with 4 nodes and no more than 2 edges.

Listing these becuase I can't get loops to work with tikz-graph latex package.
$\{0-0-0-0\}, \{1-0-1-0\}, \{1-1-1-1\}$
$\{1-0-2-1\}, \{0-2-2-0\}, \{0-2-0-0\}$
$\{0-4-0-0\}, \{0-3-1-0\}$