

Graph Theory Knowledge Assignment

1

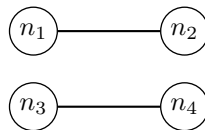
Let G be a simple graph with n nodes. Let k be the number of edges of G . Prove (or disprove)

$$k \leq \frac{n(n-1)}{2}$$

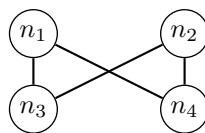
Row	Statement	Comment
1.	Let $n := 1$	Base case
2.	G_1 has 0 edges and $\frac{1(1-1)}{2} = 0$	by example 10.1.9, and subs.
3.	$k \leq \frac{n(n-1)}{2}$	inductive hyp.
4.	$k = \frac{n(n-1)}{2} = \frac{(n+1)((n+1)-1)}{2}$	
5.	$= \frac{n(n+1)}{2} + n$	
6.	$= \frac{n(n+1)}{2} + \frac{2n}{2}$	
7.	$= \frac{n^2 - n + 2n}{2}$	
8.	$= \frac{n(n+1)}{2}$	
9.	$\therefore k \leq \frac{n(n-1)}{2}$	QED

2

Let G be the graph:



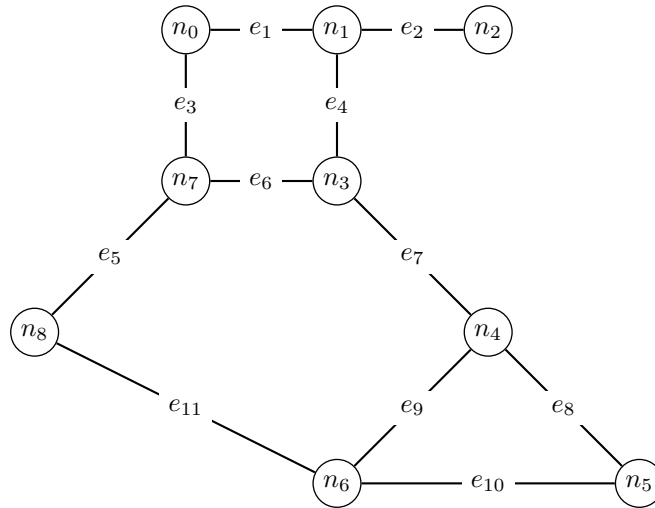
What is the complement of G ?



3 List a simple graph that has 4 nodes of different degrees, or prove that no such graph exists.

4 What is the maximum number of edges possible in a disconnected graph with n nodes and no loops or parallel edges? Explain your answer. (No proof needed)

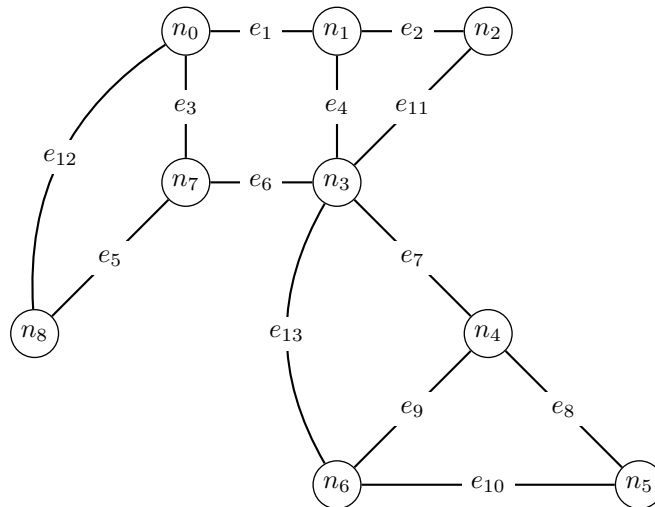
5 Let G be the graph:



(a) List the Adjacency matrix for this graph

(b) List the Incidence matrix for this graph

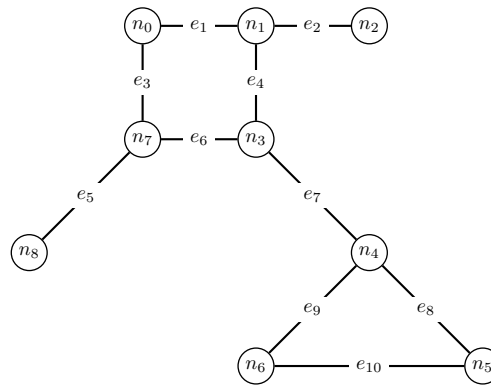
6 Let G be the graph:



(a) List the Laplacian matrix for this graph

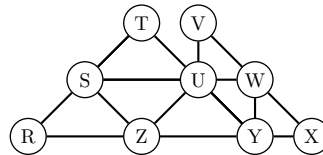
(b) List the eigenvalues for the Laplacian matrix for this graph

7 Let G be the graph:



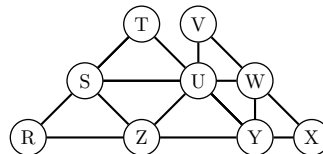
- List the Degree matrix for this graph
- List the Adjacency matrix for this graph
- Identify the bridges (if any) of this graph. If there are no bridges, write “none”.

8 Let G be the graph:



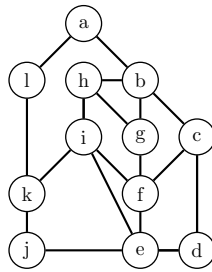
- List the Laplacian matrix for this graph
- List the Incidence matrix for this graph
- Identify an Euler circuit for this graph, or prove no such circuit exists

9 Let G be the graph:



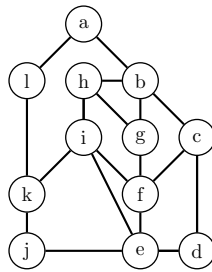
- List the Adjacency matrix for this graph
- List the Degree matrix for this graph
- Identify a Hamiltonian circuit for this graph, or prove no such circuit exists

10 Let G be the graph:

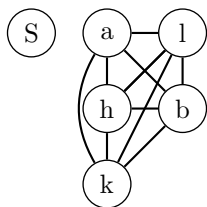


- List the Adjacency matrix for this graph
- List the Degree matrix for this graph
- Identify an Euler circuit for this graph, or prove no such circuit exists

11 Let G be the graph:



- List the Laplacian matrix for this graph
- List the Incidence matrix for this graph
- Identify a Hamiltonian circuit for this graph, or prove no such circuit exists



$$k \leq \frac{(n-1)((n-1)-1)}{2}$$