Set Relations Knowledge

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1.)

Let $A = \{1, 2, 3\}$ *and* $B = \{4, 5, 6\}$ *and let* $R = \{(x, y), such\ that\ x|y\}$

1.a) $S = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 6), (3, 6)\}$

1.b) $S^{-1} = \{(4, 1), (5, 1), (6, 1), (4, 2), (6, 2), (6, 3)\}$

2.)

$Let\ C : \mathbb{R} \to \mathbb{R}\ be\ a\ relation\ defined\ as$
$xCy \Leftrightarrow x^2 + y^2 = 1$

$C\ is\ symmetric$

Row	$Statement$
2.a.1)	$C\ is\ symmetric \Leftrightarrow \forall x, y \in \mathbb{R}, xCy \to yCx$
2.a.2)	$\forall\ x,\ y \in \mathbb{R},\ x^2 + y^2 = 1 \to y^2 + x^2 = 1$
2.b.3)	$\therefore \forall x, y \in \mathbb{R}, xCy \to yCx$

$C\ is\ not\ reflexive\ \ \Leftrightarrow \exists\ x, \in \mathbb{R} \mid x^2 + x^2 \neq 1$

Row	$Statement$
2.b.2)	$Let x = 1$
2.b.2)	$1^2 + 1^2 = 2x^2$
2.b.3)	$\therefore C\ is\ not\ reflexive$

$C\ is\ not\ transitive \Leftrightarrow \exists x,\ y,\ and\ z\ \in \mathbb{R}, (\ x^2 + y^2 = 1\ \wedge\ y^2 + z^2 = 1) \wedge x^2 + z^2 \neq 1$

Row	$Statement$
2.c.1)	$\exists x,\ y,\ and\ z\ \in \mathbb{R}, (\ x^2 + y^2 = 1\ \wedge\ y^2 + z^2 = 1) \wedge x^2 + z^2 \neq 1$
2.c.2)	$Let,\ x := 1,\ y := 0,\ z := 1$
2.c.3)	$x^2 + y^2 = 1^2 + 0^2 = 1 + 0 = 1$
2.c.4)	$y^2 + z^2 = 0^2 + 1^2 = 1$
2.c.6)	$x^2 + z^2 = 1^2 + 1^2 = 2$
2.c.7)	$2 \neq 1$
2.c.8)	$\therefore C\ is\ not\ transitive$

3.)

Let $C : \mathbb{Z} \to \mathbb{Z}$ *be a relation defined as*
$xCy \Leftrightarrow 7 \mid (x - y)$

C *is symmetric* $\Leftrightarrow \forall x, y \in \mathbb{R}, xCy \to yCx$

Row	*Statement*
3.a.1)	$\forall x, y \mid x,\ y \in \mathbb{Z} \to 7 \mid (x - y) = 7 \mid (y - x)$
3.a.2)	*Suppose* $\exists\ k\ \in\ \mathbb{Z} \mid x - y = k$
3.a.3)	$(y - x) = -7(-k) = -7k$
3.a.4)	$-1(x - y) = -7k$
3.a.5)	$\therefore C$ *is symmetric*

C *is reflexive* $\Leftrightarrow\ \forall\ x, \in \mathbb{R}, s.t\ 7|(x - x)$

Row	*Statement*
3.b.1)	$x - x = 0$
3.b.2)	$7|0$
3.b.3)	$\therefore C$ *is reflexive*

C *is transitive* $\Leftrightarrow\ \forall x,\ y,$ *and* $z\ \in \mathbb{Z}, (7|(x - y) \wedge\ 7|(y - z)) \to 7(x - z)$

3.c.1)	*Suppose* $\exists\ a, b\ \in \mathbb{Z}|\ 7|x - y = 7a \wedge 7|y - z = 7b$
3.c.2)	$(x - y) - (y - z) = 7a - 7b = 7(a - b)$
3.c.3)	$\therefore\ C$ *is transitive*

4.)

Let $C : \{4, 3, 2, 1, 0, 1, 2, 3, 4, 5\} \to \{4, 3, 2, 1, 0, 1, 2, 3, 4, 5\}$
be a relation defined as $xCy \Leftrightarrow 3|(x - y)$
List the equivalence classes of C
The equivalence classes are, $\{-2, 1, 4\}, \{-4, -1, 2, 5\}, \{-3, 0, 3\}$

5.)

Let $A = \{-1, 0, 1\}$, *and let* $C : \wp(A) \to \wp(A)$
be a relation on the power set of A *defined as*

$$xRy \Leftrightarrow \sum_{i=1}^{|x|} x_i = \sum_{i=1}^{|y|} y_i$$

The equivalence classes are,
$\{\{\emptyset\}, \{\{-1\}\{-1, 0\}\}, \{\{0\}, \{1, -1\}, \{-1, 0, 1\}, \{\{1\}\{0, 1\}\}$

6.)

$What\ is\ gcd(323421, 334355)?\ Express\ your\ answer\ as\ a\ linear\ combination\ of\ the\ two\ numbers$
$3 \cdot 7 \cdot 15401, 5 \cdot 7 \cdot 41 \cdot 233$

7.)

$The\ modular\ inverse\ of\ 323441211\ mod\ 23345\ is\ 9111$

8.)

8.a) $\forall a, b, c \in \mathbb{Z}, ab|c \rightarrow (gcd(a, b) = (1) \wedge (b) \wedge (b|c)$
8.b) $\exists a, b, c \in \mathbb{Z}, ab|c \wedge (gcd(a, b) \neq (1) \wedge (b) \wedge (b|c)$

Row	*Statement*
8.b.1)	$Let\ a := 2, b := 4,\ and\ c = 16$
8.b.2)	$\frac{a}{c} = \frac{2}{16}$
8.b.3)	$\frac{b}{c} = \frac{4}{16}$
8.b.4)	$\frac{ab}{c} = \frac{2 \cdot 4}{16} = \frac{8}{16}$
8.b.5)	$gcd(2, 4) = 2$
8.b.6)	$2 \neq$
8.b.7)	$\therefore \exists a, b, c \in \mathbb{Z}, ab|c \wedge (gcd(a, b) \neq (1) \wedge (b) \wedge (b|c)$

9.)

9.1)	$Let\ a := 5,\ and,\ p = 4$
9.2)	$gcd(5, 4) = 1$
9.3)	$a^{p-1} = 5^{4-1}$
9.4)	$= 5^3$
9.5)	$= 125$
9.6)	$125 mod 4 = 0$
9.7)	$\therefore a^{p-1}\ is\ not\ prime$

10.)

$Let\ A = \{a, b, c, d\}, and\ let\ C : \wp(A) \rightarrow \wp(A)$
$Define\ the\ realtion\ C\ as\ xCy \Leftrightarrow x \subset y$
$The\ topological\ sorting\ for\ C\ is$
$\{\{\emptyset\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\} \{b, c\},$
$\{b, d\}, \{c, d\} \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \{b, c, d\}, \{a, b, c, d\}$