cap 9

9.1

1

$$\int 2x\left(x^2+4\right)^5\,dx$$

$$u = x^2 + 4$$
$$\frac{du}{dx} = 2x$$
$$du = 2x\,dx$$

$$\int (x^2+4)^5\,2x\,dx = \int u^5\,du =$$

$$= \frac{u^6}{6} + C = \frac{(x^2+4)^6}{6} + C$$

2

$$\int 2(2x-1)^7\,dx$$

$$u = 2x - 1$$
$$\frac{du}{dx} = 2$$
$$du = 2\,dx$$

$$\int (2x-1)^7\,2\,dx = \int u^7\,du =$$

$$= \frac{u^8}{8} + C = \frac{(2x-1)^8}{8} + C$$

3

$$\int \frac{2x+1}{\sqrt{x^2+x+3}}\,dx = \int (2x+1)(x^2+x+3)^{-\frac{1}{2}}\,dx$$

$$u = x^2+x+3$$
$$\frac{du}{dx} = 2x+1$$
$$du = (2x+1)\,dx$$

$$\int (x^2+x+3)^{-\frac{1}{2}}(2x+1)\,dx = \int u^{-\frac{1}{2}}\,du =$$

$$= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{u} + C = 2\sqrt{x^2+x+3} + C =$$

4

$$\int (x^2+2x+3)^6(x+1)\,dx$$

$$u = x^2+2x+3$$
$$\frac{du}{dx} = 2x+2$$
$$du = (2x+2)\,dx$$
$$du = 2\,(x+1)\,dx$$

$$\int (x^2+2x+3)^6\,\frac{2}{2}\,(x+1)\,dx =$$

$$= \frac{1}{2}\int (x^2+2x+3)^6\,2\,(x+1)\,dx =$$

$$= \frac{1}{2}\int u^6\,du = \frac{1}{2}\,\frac{u^7}{7} + C =$$

$$= \frac{u^7}{14} + C = \frac{(x^2+2x+3)^7}{14} + C$$

5

$$\int 3x^2 e^{x^3-1}\,dx$$

$$u = x^3 - 1$$
$$\frac{du}{dx} = 3x^2$$
$$du = 3x^2\,dx$$

$$\int e^{x^3-1}\,3x^2\,dx = \int e^u\,du = e^u + C =$$

$$= e^{x^3-1} + C$$

6

$$\int 2xe^{-x^2}\,dx$$

$$u = -x^2$$
$$\frac{du}{dx} = -2x$$
$$du = -2x\,dx$$
$$-\,du = 2x\,dx$$

$$\int e^{-x^2}\,2x\,dx = \int e^u(-1)\,dx = -\int e^u\,dx =$$

$$= -e^u + C = -e^{-x^2} + C$$

7

$$\int x\sqrt{4-x^2}\,dx = \int (4-x^2)^{\frac{1}{2}}\ x\,dx$$

$$u = 4 - x^2$$
$$\frac{du}{dx} = -2x$$
$$du = -2x\ dx$$
$$-\frac{du}{2} = x\ dx$$

$$\int u^{\frac{1}{2}}\left(-\frac{1}{2}\right) du = -\frac{1}{2}\int u^{\frac{1}{2}}\ du = -\frac{1}{2}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= -\frac{(4-x^2)^{\frac{3}{2}}}{3} + C$$

8

$$\int \frac{(1+ln(x))^3}{x}\,dx$$

$$u = 1 + ln(x)$$
$$\frac{du}{dx} = \frac{1}{x}$$
$$du = \frac{1}{x}\ dx$$

$$\int u^3\,\frac{1}{x}\,dx = \int u^3\ du =$$

$$= \frac{u^4}{4} + C = \frac{(1+ln(x))^4}{4} + C$$

9

$$\int \frac{1}{\sqrt{2x+1}}\,dx = \int (2x+1)^{-\frac{1}{2}}\,dx$$

$$u = 2x+1$$
$$\frac{du}{dx} = 2$$
$$du = 2\,dx$$
$$\frac{du}{2} = dx$$

$$\int u^{-\frac{1}{2}}\frac{du}{2} = \frac{1}{2}\int u^{-\frac{1}{2}}\,du = \frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C =$$

$$= u^{\frac{1}{2}} + C = \sqrt{2x+1} + C$$

10

$$\int (x^3-6x)^7(x^2-2)\,dx$$

$$u = x^3 - 6x$$
$$\frac{du}{dx} = 3x^2 - 6$$
$$du = (3x^2-6)\,dx$$
$$du = 3(x^2-2)\,dx$$
$$\frac{du}{3} = (x^2-2)\,dx$$

$$\int u^7\,\frac{du}{3} = \frac{1}{3}\int u^7\,du =$$

$$\frac{1}{3}\frac{u^8}{8} + C = \frac{(x^3-6x)^8}{24} + C$$

11

$$\int xe^{x^2}\,dx$$

$$u = x^2$$
$$\frac{du}{dx} = 2x$$
$$du = 2x\,dx$$
$$\frac{du}{2} = x\,dx$$

$$\int e^u\,\frac{du}{2} = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C =$$

$$= \frac{e^{x^2}}{2} + C$$

12

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$$u = \sqrt{x} = x^{\frac{1}{2}}$$
$$\frac{du}{dx} = \frac{x^{-\frac{1}{2}}}{2} = \frac{1}{2\sqrt{x}}$$
$$du = \frac{1}{2\sqrt{x}}dx$$
$$2\,du = \frac{1}{\sqrt{x}}\,dx$$

$$\int e^u\,2\,du = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C$$

13

$$\int \frac{ln(2x)}{x}\,dx$$

$$u = ln(2x)$$
$$\frac{du}{dx} = \frac{2}{2x} = \frac{1}{x}$$
$$du = \frac{1}{x}dx$$

$$\int u\,du = \frac{u^2}{2} + C = \frac{ln(2x)^2}{2} + C$$

14

$$\int \frac{\sqrt{ln(x)}}{x}\,dx$$

$$u = ln(x)$$
$$\frac{du}{dx} = \frac{1}{x}$$
$$du = \frac{1}{x}dx$$

$$\int \sqrt{u}\,du = \int u^{\frac{1}{2}}\,du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= \frac{2u^{\frac{3}{2}}}{3} + C = \frac{2(ln(x))^{\frac{3}{2}}}{3} + C$$

15

$$\int \frac{x^4}{x^5+1}\,dx$$

$$u = x^5+1$$
$$\frac{du}{dx} = 5x^4$$
$$du = 5x^4\,dx$$
$$\frac{du}{5} = x^4\,dx$$

$$\int \frac{1}{u}\,\frac{du}{5} = \frac{1}{5}\int \frac{1}{u}\,du =$$

$$= \frac{1}{5}ln|u| + C = \frac{ln|x^5+1|}{5} + C$$

16

$$\int \frac{x}{\sqrt{x^2+1}}\,dx$$

$$u = x^2+1$$
$$\frac{du}{dx} = 2x$$
$$\frac{du}{2} = x\,dx$$

$$\int (x^2+1)^{-\frac{1}{2}}\;x\,dx = \int u^{-\frac{1}{2}}\frac{du}{2} =$$

$$= \frac{1}{2}\int u^{-\frac{1}{2}}\;du = \frac{1}{2}\,\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C =$$

$$= u^{\frac{1}{2}} + C = \sqrt{x^2+1} + C$$

17

$$\int \frac{x-3}{(1-6x+x^2)^2}\,dx$$

$$\begin{aligned}
&u = 1-6x+x^2\\
&\frac{du}{dx} = 2x-6\\
&du = (2x-6)\,dx\\
&du = 2(x-3)\,dx\\
&\frac{du}{2} = (x-3)\,dx
\end{aligned}$$

$$\frac{1}{2}\int u^{-2}\,du = -\frac{1}{2}u^{-1} + C =$$

$$= -\frac{1}{2-12x+2x^2} + C$$

9.2

1

$$\int xe^{5x}\,dx$$

$$\begin{aligned}
&f(x) = x\\
&g(x) = e^{5x}\\
&G(x) = \int e^{5x}\,dx = \frac{e^{5x}}{5}\\
&f'(x) = 1
\end{aligned}$$

$$\int xe^{5x}\,dx = \frac{xe^{5x}}{5} - \int \frac{e^{5x}}{5}\,dx =$$

$$= \frac{xe^{5x}}{5} - \frac{e^{5x}}{25} + C = \frac{e^{5x}}{5}\left(x - \frac{1}{5}\right) + C$$

2

$$\int xe^{\frac{x}{2}}\,dx$$

$$f(x) = x$$
$$g(x) = e^{\frac{x}{2}}$$
$$G(x) = \int e^{\frac{x}{2}}\,dx = 2e^{\frac{x}{2}}$$
$$f'(x) = 1$$

$$\int xe^{\frac{x}{2}}\,dx = 2xe^{\frac{x}{2}} - \int 2e^{\frac{x}{2}}\,dx =$$

$$= 2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}} + C = 2e^{\frac{x}{2}}(x-2) + C$$

3

$$\int x(x+7)^4\,dx$$

$$f(x) = x$$
$$g(x) = (x+7)^4$$
$$G(x) = \int (x+7)^4\,dx$$

$$u = x+7$$
$$\frac{du}{dx} = 1$$
$$du = dx$$
$$\int u^4\,du = \frac{u^5}{5} =$$
$$= \frac{(x+7)^5}{5}$$

$$G(x) = \frac{(x+7)^5}{5}$$
$$f'(x) = 1$$

$$\int x(x+7)^4\,dx = x\frac{(x+7)^5}{5} - \int \frac{(x+7)^5}{5}\,dx =$$

$$= \frac{x(x+7)^5}{5} - \frac{(x+7)^6}{30} + C$$

4