cap 9

9.1

1

$$\int 2x ig(x^2+4ig)^5 \, dx \ u = x^2+4 \ rac{du}{dx} = 2x \ du = 2x \, dx \ \int (x^2+4)^5 \, 2x \, dx = \int u^5 \, du = \ = rac{u^6}{6} + C = rac{(x^2+4)^6}{6} + C$$

$$\int 2(2x-1)^7 dx$$
 $u = 2x-1$
 $\frac{du}{dx} = 2$
 $du = 2 dx$

$$\int (2x-1)^7 2 dx = \int u^7 du =$$
 $= \frac{u^8}{8} + C = \frac{(2x-1)^8}{8} + C$

$$\int rac{2x+1}{\sqrt{x^2+x+3}} \; dx = \int (2x+1)(x^2+x+3)^{-rac{1}{2}} \, dx$$

$$u=x^2+x+3$$
 $\dfrac{du}{dx}=2x+1$ $du=(2x+1)\ dx$

$$\int (x^2+x+3)^{-rac{1}{2}}(2x+1)\ dx = \int u^{-rac{1}{2}}\ du =$$

$$=rac{u^{rac{1}{2}}}{rac{1}{2}}+C=2\sqrt{u}+C=2\sqrt{x^2+x+3}+C=$$

$$\int (x^2 + 2x + 3)^6 (x+1) \ dx$$

$$u = x^2 + 2x + 3$$

$$\frac{du}{dx} = 2x + 2$$

$$du = (2x+2) dx$$

$$du = 2(x+1) dx$$

$$\int (x^2+2x+3)^6 \; rac{2}{2} \; (x+1) \; dx =$$

$$x=rac{1}{2}\int (x^2+2x+3)^6\ 2\ (x+1)\ dx=0$$

$$u=rac{1}{2}\int u^6\ du=rac{1}{2}rac{u^7}{7}+C=0$$

$$=rac{u^7}{14}+C=rac{(x^2+2x+3)^7}{14}+C$$

$$\int 3x^2 e^{x^3 - 1} \ dx$$

$$u=x^3-1 \ rac{du}{dx}=3x^2 \ du=3x^2 \ dx$$

$$\int e^{x^3-1} \ 3x^2 \ dx = \int e^u \ du = e^u + C =$$
 $= e^{x^3-1} + C$

$$\int 2xe^{-x^2}\ dx$$

$$u = -x^2$$

$$\frac{du}{dx} = -2x$$

$$du = -2x \ dx$$

$$-\,du=2x\;dx$$

$$\int e^{-x^2} \ 2x \ dx = \int e^u (-1) \ dx = - \int e^u \ dx = 0$$

$$=-e^{u}+C=-e^{-x^{2}}+C$$

$$\int x \sqrt{4-x^2} \ dx = \int (4-x^2)^{\frac{1}{2}} \ x \ dx$$

$$u=4-x^2 \ rac{du}{dx}=-2x \ du=-2x\ dx \ -rac{du}{2}=x\ dx$$

$$\int u^{rac{1}{2}} \left(-rac{1}{2}
ight) \, du = -rac{1}{2} \int u^{rac{1}{2}} \, du = -rac{1}{2} rac{u^{rac{3}{2}}}{rac{3}{2}} + C =$$

$$=-rac{(4-x^2)^{rac{3}{2}}}{3}+C$$

 $\int rac{(1+ln(x))^3}{x} \, dx$

$$egin{aligned} u &= 1 + ln(x) \ rac{du}{dx} &= rac{1}{x} \ du &= rac{1}{x} \, dx \end{aligned}$$

$$\int u^3 \; rac{1}{x} \; dx = \int u^3 \; du =$$

$$=rac{u^4}{4}+C=rac{(1+ln(x))^4}{4}+C$$

$$\int rac{1}{\sqrt{2x+1}} \; dx = \int (2x+1)^{-rac{1}{2}} \; dx$$

$$u=2x+1 \ rac{du}{dx}=2 \ du=2 \ dx \ rac{du}{2}=dx$$

$$\int u^{-rac{1}{2}}rac{du}{2}=rac{1}{2}\int u^{-rac{1}{2}}\,du=rac{1}{2}rac{u^{rac{1}{2}}}{rac{1}{2}}+C=$$

$$= u^{\frac{1}{2}} + C = \sqrt{2x+1} + C$$

$$\int (x^3 - 6x)^7 (x^2 - 2) \ dx$$

$$u = x^3 - 6x$$

$$\frac{du}{dx} = 3x^2 - 6$$

$$du = (3x^2 - 6) \ dx$$

$$du=3(x^2-2)\ dx$$

$$\frac{du}{3} = (x^2 - 2) dx$$

$$\int u^7 \; rac{du}{3} = rac{1}{3} \int u^7 \; du =$$

$$rac{1}{3}rac{u^8}{8} + C = rac{(x^3 - 6x)^8}{24} + C$$

$$\int xe^{x^2}\ dx$$

$$u=x^2$$
 $\dfrac{du}{dx}=2x$ $du=2x\ dx$ $\dfrac{du}{2}=x\ dx$

$$\int e^u \; rac{du}{2} = rac{1}{2} \int e^u \; du = rac{1}{2} e^u + C =$$

$$=rac{e^{x^2}}{2}+C$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \ dx$$

$$u=\sqrt{x}=x^{rac{1}{2}}$$
 $rac{du}{dx}=rac{x^{-rac{1}{2}}}{2}=rac{1}{2\sqrt{x}}$

$$du=rac{1}{2\sqrt{x}}dx$$

$$2\ du = rac{1}{\sqrt{x}}\ dx$$

$$\int e^u \ 2 \ du = 2 \int e^u \ du = 2 e^u + C = 2 e^{\sqrt{x}} + C$$

$$\int \frac{\ln(2x)}{x} \, dx$$

$$u=ln(2x) \ rac{du}{dx}=rac{2}{2x}=rac{1}{x} \ du=rac{1}{x}dx$$

$$\int u \ du = rac{u^2}{2} + C = rac{ln(2x)^2}{2} + C$$

$$\int \frac{\sqrt{ln(x)}}{x} \ dx$$

$$u=ln(x) \ rac{du}{dx}=rac{1}{x} \ du=rac{1}{x}dx$$

$$\int \sqrt{u}\ du = \int u^{rac{1}{2}}\ du = rac{u^{rac{3}{2}}}{rac{3}{2}} + C =$$

$$=rac{2u^{rac{3}{2}}}{3}+C=rac{2(ln(x))^{rac{3}{2}}}{3}+C$$

$$\int \frac{x^4}{x^5 + 1} \ dx$$

$$u=x^5+1 \ rac{du}{dx}=5x^4 \ du=5x^4 \ dx \ rac{du}{5}=x^4 \ dx$$

$$\int rac{1}{u} rac{du}{5} = rac{1}{5} \int rac{1}{u} du =$$

$$=rac{1}{5}ln|u|+C=rac{ln|x^{5}+1|}{5}+C$$

$$\int \frac{x}{\sqrt{x^2 + 1}} \, dx$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = x \, dx$$

$$\int (x^2+1)^{-rac{1}{2}} \ x \ dx = \int u^{-rac{1}{2}} rac{du}{2} =$$

$$u=rac{1}{2}\int u^{-rac{1}{2}}\ du=rac{1}{2}rac{u^{rac{1}{2}}}{rac{1}{2}}+C=0$$

$$=u^{rac{1}{2}}+C=\sqrt{x^2+1}+C$$

$$\int \frac{x-3}{(1-6x+x^2)^2} \ dx$$

$$u = 1 - 6x + x^2$$
 $\dfrac{du}{dx} = 2x - 6$ $du = (2x - 6) \ dx$ $du = 2(x - 3) \ dx$ $\dfrac{du}{2} = (x - 3) \ dx$

$$rac{1}{2} \int u^{-2} \ du = -rac{1}{2} u^{-1} + C =$$

$$= -\frac{1}{2 - 12x + 2x^2} + C$$

9.2

$$\int xe^{5x}\ dx$$

$$f(x) = x$$

$$g(x) = e^{5x}$$

$$G(x)=\int e^{5x}\ dx=rac{e^{5x}}{5}$$

$$f'(x) = 1$$

$$\int xe^{5x}\ dx = rac{xe^{5x}}{5} - \int rac{e^{5x}}{5}\ dx =$$

$$=rac{xe^{5x}}{5}-rac{e^{5x}}{25}+C=rac{e^{5x}}{5}igg(x-rac{1}{5}igg)+C$$

$$egin{align} \int xe^{rac{x}{2}} \ dx \ & f(x) = x \ g(x) = e^{rac{x}{2}} \ & G(x) = \int e^{rac{x}{2}} \ dx = 2e^{rac{x}{2}} \ & f'(x) = 1 \ & \int xe^{rac{x}{2}} \ dx = 2xe^{rac{x}{2}} - \int 2e^{rac{x}{2}} \ dx = \ & = 2xe^{rac{x}{2}} - 4e^{rac{x}{2}} + C = 2e^{rac{x}{2}}(x-2) + C \ \end{aligned}$$

$$\int x(x+7)^4 dx$$

$$f(x) = x$$
 $g(x) = (x+7)^4$
 $G(x) = \int (x+7)^4 dx$

$$u=x+7$$

$$\dfrac{du}{dx}=1$$

$$du=dx$$

$$\int u^4 \ du=\dfrac{u^5}{5}=$$

$$=\dfrac{(x+7)^5}{5}$$

$$G(x)=rac{(x+7)^5}{5} \ f'(x)=1$$

$$\int x(x+7)^4 dx = x rac{(x+7)^5}{5} - \int rac{(x+7)^5}{5} dx =$$
 $= rac{x(x+7)^5}{5} - rac{(x+7)^6}{30} + C$