cap 6

6.1

1

$$f(x) = x$$

$$F(x) = ?$$

$$F(x) = \int f(x)\, dx$$

$$F(x) = \int x\, dx = \frac{x^2}{2} + C$$

2

$$f(x) = 9x^8$$

$$F(x) = \int f(x)\, dx = \int 9x^8\, dx =$$

$$= 9 \int x^8\, dx = 9(\frac{x^9}{9}) + C =$$

$$= \frac{9x^9}{9} + C = x^9 + C$$

3

$$f(x) = e^{3x}$$

$$F(x) = \int e^{3x}\, dx = \frac{e^{3x}}{3} + C$$

4

$$f(x) = e^{-3x}$$

$$F(x) = \int e^{-3x}\, dx = \frac{e^{-3x}}{-3} + C = -\frac{e^{-3x}}{3} + C$$

5

$$f(x) = 3$$

$$F(x) = \int f(x)\, dx = \int 3\, dx = 3 \int dx = 3x + C$$

6

$$f(x) = -4x$$

$$F(x) = \int f(x)\, dx = \int -4x\, dx =$$

$$= -4 \int x\, dx = -4\frac{x^2}{2} + C = -2x^2 + C$$

7

$$f(x) = 4x^3$$

$$F(x) = \int 4x^3\, dx = 4 \int x^3\, dx = 4\frac{x^4}{4} + C = x^4 + C$$

8

$$f(x) = \frac{x}{3}$$

$$F(x) = \int f(x)\, dx = \int \frac{x}{3}\, dx = \frac{1}{3} \int x\, dx = \frac{1}{3}\frac{x^2}{2} + C =$$

$$= \frac{x^2}{6} + C$$

9

$$f(x) = 7$$

$$F(x) = \int f(x)\,dx = \int 7\,dx = 7\int dx = 7x + C$$

10

$$f(x) = k^2$$

$$F(x) = \int f(x)\,dx = \int k^2\,dx = k^2\int dx = k^2x + C$$

11

$$f(x) = \frac{x}{c}$$

$$F(x) = \int f(x)\,dx = \int \frac{x}{c}\,dx = \frac{1}{c}\int x\,dx = \frac{1}{c}\frac{x^2}{2} + C =$$

$$= \frac{x^2}{2c} + C$$

12

$$f(x) = x\,x^2 = x^3$$

$$F(x) = \int f(x)\,dx = \int x^3\,dx = \frac{x^4}{4} + C$$

13

$$f(x) = \frac{2}{x} + \frac{x}{2}$$

$$F(x) = \int f(x)\,dx = \int \left(\frac{2}{x} + \frac{x}{2}\right) dx =$$
$$= \int \frac{2}{x}\,dx + \int \frac{x}{2}\,dx = 2\int \frac{1}{x}\,dx + \frac{1}{2}\int x\,dx =$$
$$= 2\,(ln|x| + C) + \frac{1}{2}\left(\frac{x^2}{2} + C\right) = 2\,ln|x| + 2C + \frac{x^2}{4} + \frac{C}{2} =$$
$$= 2\,ln|x| + \frac{x^2}{4} + C$$

14

$$f(x) = \frac{1}{7x}$$

$$F(x) = \int f(x)\,dx = \int \frac{1}{7x}\,dx =$$
$$= \frac{1}{7}\int \frac{1}{x}\,dx = \frac{1}{7}(ln|x| + C) = \frac{ln|x|}{7} + \frac{C}{7} = \frac{ln|x|}{7} + C$$

15

$$f(x) = x\sqrt{x} = x\,x^{\frac{1}{2}} = x^{\frac{2}{2}+\frac{1}{2}} = x^{\frac{3}{2}}$$

$$F(x) = \int f(x)\,dx = \int x^{\frac{3}{2}}\,dx =$$
$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2x^{\frac{5}{2}}}{5} + C$$

16

$$f(x) = \left(\frac{2}{\sqrt{x}} + 2\sqrt{x}\right) = 2x^{-\frac{1}{2}} + 2x^{\frac{1}{2}}$$

$$F(x) = \int f(x)\,dx = \int \left(2x^{-\frac{1}{2}} + 2x^{\frac{1}{2}}\right) dx =$$
$$= \int 2x^{-\frac{1}{2}}\,dx + \int 2x^{\frac{1}{2}}\,dx =$$

$$= 2\int x^{-\frac{1}{2}}\,dx + 2\int x^{\frac{1}{2}}\,dx =$$

$$= 2\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C\right) + 2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C\right)$$

$$= 4x^{\frac{1}{2}} + 2C + \frac{4}{3}x^{\frac{3}{2}} + 2C = 4x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + C$$

17

$$f(x) = \left(x - 2x^2 + \frac{1}{3x}\right)$$

$$F(x) = \int f(x)\,dx = \int \left(x - 2x^2 + \frac{1}{3x}\right) dx =$$
$$= \int x\,dx - 2\int x^2\,dx + \frac{1}{3}\int \frac{1}{x}\,dx =$$

$$= \frac{x^2}{2} + C - 2\left(\frac{x^3}{3} + C\right) + \frac{1}{3}(ln|x| + C) =$$

$$= \frac{x^2}{2} - \frac{2}{3}x^3 + \frac{ln|x|}{3} + C$$

18

$$f(x) = \left(\frac{7}{2x^3} - \sqrt[3]{x}\right) = \frac{7}{2}x^{-3} - x^{\frac{1}{3}}$$

$$F(x) = \int f(x)\,dx = \int \left(\frac{7}{2}x^{-3} - x^{\frac{1}{3}}\right) dx =$$
$$= \frac{7}{2}\int x^{-3}\,dx - \int x^{\frac{1}{3}}\,dx =$$

$$= \frac{7}{2}\left(\frac{x^{-2}}{-2} + C\right) - \frac{x^{\frac{4}{3}}}{\frac{4}{3}} - C =$$

$$= -\frac{7}{4}x^{-2} - \frac{3}{4}x^{\frac{4}{3}} + C$$

19

$$f(x) = 3e^{-2x}$$

$$F(x) = \int f(x)\,dx = \int 3e^{-2x}\,dx =$$
$$= 3\int e^{-2x}\,dx = 3\left(\frac{e^{-2x}}{-2} + C\right) =$$

$$= -\frac{3}{2}e^{-2x} + C$$

20

$$f(x) = e^{-x}$$

$$F(x) = \int f(x)\,dx = \int e^{-x}\,dx =$$
$$= -e^{-x} + C$$

21

$$f(x)=e$$

$$F(x)=\int f(x)\,dx=\int e\,dx=e\int dx$$
$$=ex+C$$

22

$$f(x)=\frac{7}{2e^{2x}}$$

$$F(x)=\int f(x)\,dx=\int \frac{7}{2e^{2x}}\,dx=$$
$$=\frac{7}{2}\int e^{-2x}\,dx=\frac{7}{2}\left(\frac{e^{-2x}}{-2}+C\right)=$$

$$=-\frac{7}{4}e^{-2x}+C$$

23

$$f(x)=-2\left(e^{2x}+1\right)=-2e^{2x}-2$$

$$F(x)=\int f(x)\,dx=\int \left(-2e^{2x}-2\right)\,dx=$$
$$=-2\int e^{2x}-2\int\,dx=-2\left(\frac{e^{2x}}{2}\right)-2\,(x)+C=$$

$$=-e^{2x}-2x+C$$

24

$$f(x) = \left(-3e^{-x} + 2x - \frac{e^{0.5x}}{2}\right)$$

$$F(x) = \int f(x)\,dx = \int \left(-3e^{-x} + 2x - \frac{e^{0.5x}}{2}\right) dx =$$
$$= -3\int e^{-x}\,dx + 2\int x\,dx - \frac{1}{2}\int e^{0.5x}\,dx =$$

$$= -3\left(-e^{-x}\right) + 2\left(\frac{x^2}{2}\right) - \frac{1}{2}\left(\frac{e^{0.5x}}{0.5}\right) + C =$$

$$= 3e^{-x} + x^2 - e^{\frac{1}{2}x} + C$$

25

$$\int 5e^{-2t}\,dt = ke^{-2t} + C$$

$$f(x) = 5e^{-2t}$$

$$F(x) = \int f(x)\,dx = \int 5e^{-2t}\,dx =$$
$$= 5\int e^{-2t}\,dx = 5\left(\frac{e^{-2t}}{-2}\right) + C$$

$$= -\frac{5}{2}e^{-2t} + C$$

Portanto,

$$-\frac{5}{2}e^{-2t} + C = ke^{-2t} + C$$

$$k = -\frac{5}{2}$$

Derivando $F(x)$:

$$F(x) = -\frac{5}{2}e^{-2t} + C$$

$$F'(x) = f(x) = 5e^{-2t}$$

26

$$\int 3e^{\frac{t}{10}}\,dt = ke^{\frac{t}{10}} + C$$

$$f(x) = 3e^{\frac{t}{10}}$$

$$\begin{aligned} F(x) &= \int f(x)\,dx = \int 3e^{\frac{t}{10}}\,dx = \\ &= 3\int e^{\frac{t}{10}}\,dx = 3\left(\frac{e^{\frac{t}{10}}}{\frac{1}{10}}\right) + C \\ &= 30e^{\frac{t}{10}} + C \end{aligned}$$

Portanto,

$$30e^{\frac{t}{10}} + C = ke^{\frac{t}{10}} + C$$

$$k = 30$$

27

$$\int 2e^{4x-1}\,dt = ke^{\frac{t}{10}} + C$$

$$f(x) = 2e^{4x-1}$$

$$\begin{aligned} F(x) &= \int f(x)\,dx = \int 2e^{4x-1}\,dx = \\ &= 2\int e^{4x-1}\,dx = 2\int e^{4x}e^{-1}\,dx = \\ &= 2e^{-1}\int e^{4x}\,dx = 2e^{-1}\left(\frac{e^{4x}}{4}\right) + C \\ &= \frac{e^{4x}}{2e} + C = \frac{e^{4x-1}}{2} + C \end{aligned}$$

28

$$\int \frac{4}{e^{3x+1}}\, dt = \frac{k}{e^{3x+1}} + C$$

$$f(x) = \frac{4}{e^{3x+1}}$$

$$f(x) = \frac{d}{dx}\left(\frac{k}{e^{3x+1}} + C\right) =$$

$$= \frac{d}{dx}\left(k\, e^{-3x-1}\right) = k\, \frac{d}{dx}\left(e^{-3x-1}\right) =$$

$$= ke^{-3x-1}(-3) = -3ke^{-3x-1}$$

Portanto,

$$\frac{4}{e^{3x+1}} = -3ke^{-3x-1}$$

$$\frac{4}{e^{3x+1}} = \frac{-3k}{e^{3x+1}}$$

$$k = -\frac{4}{3}$$

37

$$f'(t) = t^{\frac{3}{2}}$$

$$f(t) = \int f'(t)\, dt = \int t^{\frac{3}{2}}\, dt = \frac{5}{2} t^{\frac{5}{2}} + C$$

42

$$f'(x) = 2x - e^{-x}, \quad f(0) = 1$$

$$\int f'(x)\, dt = f(x) = \int \left(2x - e^{-x}\right)\, dx =$$

$$= \int 2x\, dx - \int e^{-x}\, dx =$$

$$= 2\int x\, dx - \int e^{-x}\, dx =$$

$$= 2\frac{x^2}{2} + e^{-x} + C = x^2 + e^{-x} + C$$

Aplicando valor inicial:

$$f(x) = x^2 + e^{-x} + C$$

$$f(0) = 1$$

$$0^2 + e^{-0} + C = 1$$

$$1 + C = 1$$

$$C = 0$$

Portanto,

$$f(x) = x^2 + e^{-x}$$

6.3

1

$$\text{Área entre a e b} = \int_{\frac{1}{2}}^{2} \frac{1}{x}\, dx$$

2

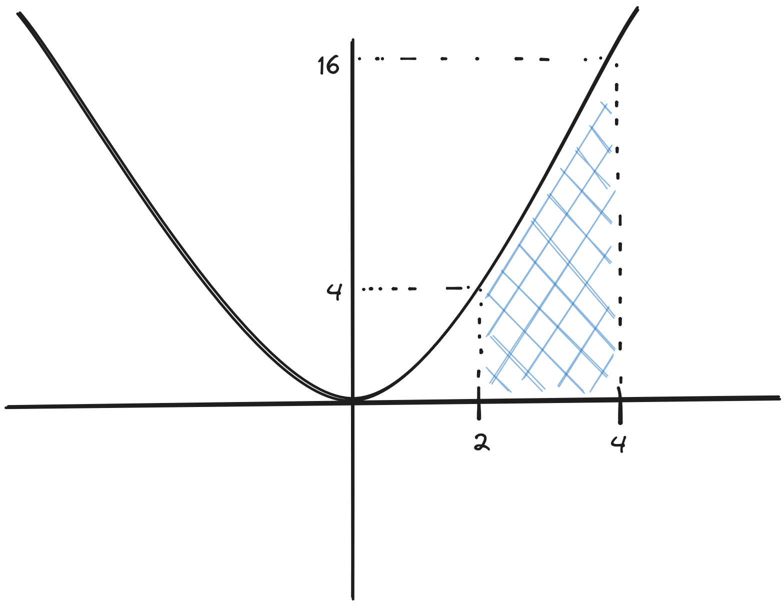
$$\text{Área entre a e b} = \int_{1}^{3} x + \frac{1}{x}\, dx$$

3

$$\text{Área entre a e b} = \int_1^3 ((1-x)(x-3))\,dx$$

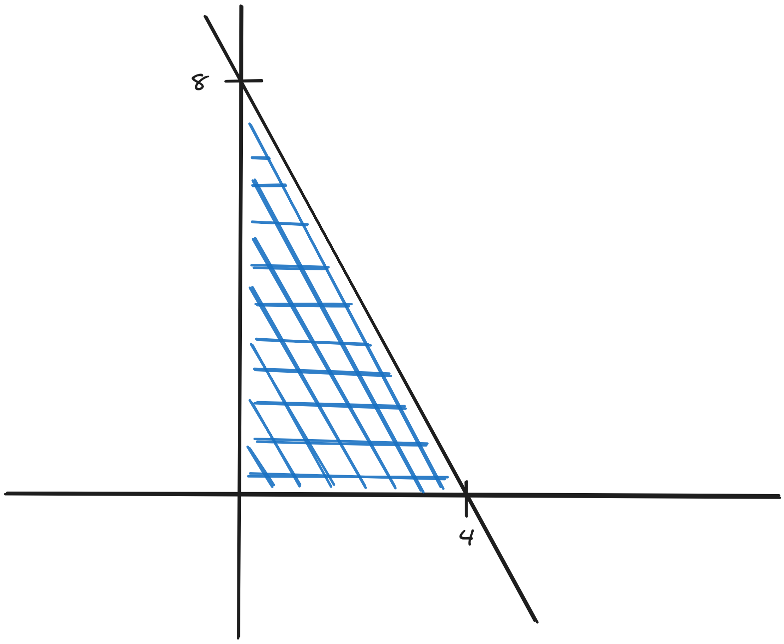
4

$$\int_2^4 x^2\,dx$$



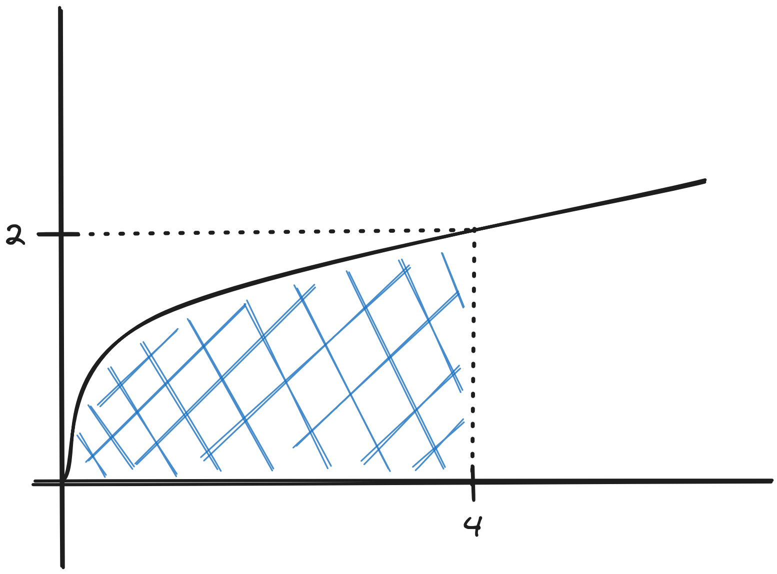
5

$$\int_0^4 (8-2x)\,dx$$



6

$$\int_0^4 \sqrt{x}\,dx$$



7

$$\int_{-1}^{1} x\,dx = \left.\frac{x^2}{2}\right|_1 - \left.\frac{x^2}{2}\right|_{-1} =$$

$$= \frac{1}{2} - \frac{1}{2} = 0$$

8

$$\int_0^1 \left(x+\frac{1}{2}\right) dx = \int_0^1 x\,dx + \frac{1}{2}\int_0^1 dx =$$

$$= \left[\frac{x^2}{2}\bigg|_1 - \frac{x^2}{2}\bigg|_0\right] + \frac{1}{2}[x|_1 - x|_0] =$$

$$= \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$$

9

$$\int_5^9 5\,dx = 5\int_5^9 \,dx = 5[x|_9 - x|_5] = 5(9-5) = 20$$

10

$$\int_1^4 x^2\sqrt{x}\,dx = \int_1^4 x^2 x^{\frac{1}{2}}\,dx = \int_1^4 x^{2+\frac{1}{2}}\,dx = \int_1^4 x^{\frac{5}{2}}\,dx =$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}}\Bigg|_1^4 = \left[\frac{2}{7}x^{\frac{7}{2}}\right]\Bigg|_1^4 = \left[\frac{2}{7}(4)^{\frac{7}{2}}\right] - \left[\frac{2}{7}(1)^{\frac{7}{2}}\right] =$$

$$= \frac{2}{7}\,128 - \frac{2}{7} \approx 36.2857$$

11

$$\int_1^4 \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2}\int_1^4 x^{-\frac{1}{2}}\,dx = \frac{1}{2}\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\Bigg|_1^4\right] =$$

$$= \frac{1}{\cancel{2}}\left[\cancel{2}x^{\frac{1}{2}}\Big|_1^4\right] = (4)^{\frac{1}{2}} - 1^{\frac{1}{2}} = 2 - 1 = 1$$

12

$$\int_0^1 \left(4x^3 - 1\right) dx = \int_0^1 4x^3 \, dx - \int_0^1 \, dx = 4\int_0^1 x^3 \, dx - \int_0^1 \, dx =$$

$$= 4\left[\frac{x^4}{4}\bigg|_0^1\right] - x\bigg|_0^1 = \cancel{4}\left[\frac{x^4}{\cancel{4}}\bigg|_0^1\right] - x\bigg|_0^1 = x^4\bigg|_0^1 - x\bigg|_0^1 =$$

$$= (1)^4 - (0)^4 - ((1) - 0) = 1 - 1 = 0$$

13

$$\int_0^1 4e^{-3x} \, dx = 4\int_0^1 e^{-3x} \, dx = 4\left[\frac{e^{-3x}}{-3}\bigg|_0^1\right] =$$

$$= -4\left[\frac{e^{-3}}{3} - \frac{e^0}{3}\right] = -4\left[\frac{e^{-3} - 1}{3}\right] \approx 1.267$$

27

$$\int_2^3 (5 - 2t)^4 \, dt = \frac{(5 - 2t)^5}{5(-2)}\bigg|_2^3$$

28

$$\int_4^9 \frac{3}{t - 2} \, dt = 3\int_4^9 \frac{1}{t - 2} \, dt = 3\left[ln(t - 2)\bigg|_4^9\right]$$

6.4

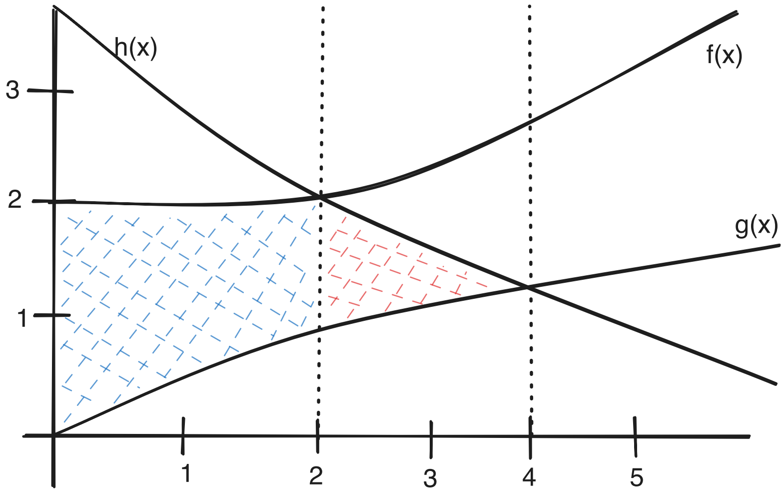
1

$$\int_1^2 f(x) \, dx - \int_3^4 f(x) \, dx$$

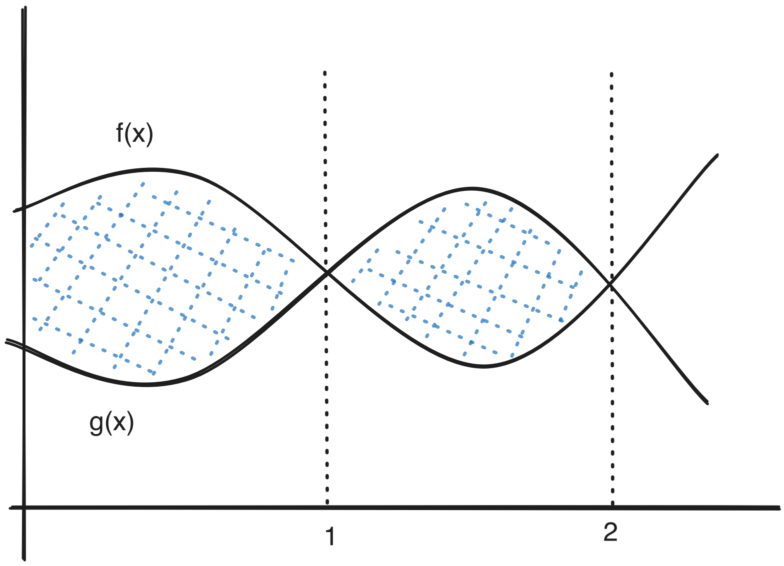
2

$$\int_2^3 [f(x) - g(x)] \, dx = \int_2^3 f(x) \, dx - \int_2^3 g(x) \, dx$$

3



4



5

$$y_1 = 2x^2$$
$$y_2 = 8$$
$$-2 \leq x \leq 2$$

$$\int_{-2}^{2} (y_1 - y_2)\, dx = \int_{-2}^{2} \left(2x^2 - 8\right)\, dx =$$

$$= 2\int_{-2}^{2} x^2\, dx - 8\int_{-2}^{2}\, dx = 2\left[\frac{x^3}{3}\Big|_{-2}^{2}\right] - 8\left[x\Big|_{-2}^{2}\right] =$$

$$= 2\left(\frac{2^3}{3} - \frac{(-2)^3}{3}\right) - 8\,(2 - (-2)) = 2\left(\frac{8}{3} + \frac{8}{3}\right) - 8 * 4 =$$

$$= \frac{32}{3} - 32 = -\frac{64}{3} \approx -21.333$$

6

$$y_1 = x^2 + 1$$
$$y_2 = -x^2 - 1$$
$$-1 \leq x \leq 1$$
$$\int_{-1}^{1} \left(x^2 + 1 - (-x^2 - 1)\right)\, dx = \int_{-1}^{1} 2x^2 + 2\, dx =$$

$$= 2\int_{-1}^{1} x^2\, dx + 2\int_{-1}^{1}\, dx = 2\left[\frac{x^3}{3}\Big|_{-1}^{1}\right] + 2\left[x\Big|_{-1}^{1}\right] =$$

$$= 2\left(\frac{1^3}{3} - \frac{(-1)^3}{3}\right) + 2\,(1 - (-1)) =$$

$$= 2\left(\frac{1}{3} + \frac{1}{3}\right) + 4 = \frac{16}{3} \approx 5.33$$

7

$$y_1 = x^2 - 6x + 12$$
$$y_2 = 1$$
$$0 \leq x \leq 4$$

$$\int_0^4 \left(x^2 - 6x + 12 - 1\right)\, dx = \int_0^4 \left(x^2 - 6x + 11\right)\, dx =$$

$$= \int_0^4 x^2\, dx - 6\int_0^4 x\, dx + 11\int_0^4 dx =$$

$$= \left(\frac{x^3}{3}\Big|_0^4\right) - 6\left(\frac{x^2}{2}\Big|_0^4\right) + 11\left(x\Big|_0^4\right) =$$

$$= \left(\frac{4^3}{3} - 0\right) - 6\left(\frac{4^2}{2} - 0\right) + 11\,(4 - 0) =$$

$$= \frac{64}{3} - 48 + 44 = \frac{52}{3} \approx 17.33$$

11

Encontrar pontos de intersecção entre as duas funções y_1 e y_2

$$y_1 = y_2$$
$$x^2 = x$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = \{0, 1\}$$

Reescrever com os limites de integração:

$$\int_0^1 x^2 - x\, dx = \int_0^1 x\, dx - \int_0^1 x^2\, dx$$

$$= \frac{x^2}{2}\Big|_0^1 - \frac{x^3}{3}\Big|_0^1 = \frac{1^2}{2} + \frac{0^2}{2} - \frac{1^3}{3} + \frac{0^3}{3} =$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6} \approx 0.16$$

12

Intersecção:

$$\begin{aligned}
&y_1 = 4x(1-x) \\
&y_2 = \frac{3}{4} \\
&y_1 = y_2 \\
&4x(1-x) = \frac{3}{4} \\
&-4x^2 + 4x - \frac{3}{4} = 0 \\
&x = \{0.25, 0.75\}
\end{aligned}$$

Teste:

$$\begin{aligned}
&y_1(0.5) = 1 \\
&y_2(0.5) = \frac{3}{4} \\
&y_1 > y_2, \quad 0.25 < x < 0.75
\end{aligned}$$

Integral:

$$\int_{0.25}^{0.75} -4x^2 + 4x - \frac{3}{4}\, dx = -4\int_{0.25}^{0.75} x^2\, dx + 4\int_{0.25}^{0.75} x - \frac{3}{4}\int_{0.25}^{0.75} dx =$$

$$= -4\left(\frac{x^3}{3}\Bigg|_{0.25}^{0.75}\right) + 4\left(\frac{x^2}{2}\Bigg|_{0.25}^{0.75}\right) - \frac{3}{4}(x\Big|_{0.25}^{0.75}) =$$

$$= -4\left(\frac{0.75^3}{3} - \frac{0.25^3}{3}\right) + 4\left(\frac{0.75^2}{2} - \frac{0.25^2}{2}\right) - \frac{3}{4}(0.75 - 0.25) =$$

$$\approx 0.0833$$

13

Intersecção:

$$
\begin{aligned}
&y_1 = -x^2 + 6x - 5 \\
&y_2 = 2x - 5 \\
&y_1 = y_2 \\
&-x^2 + 6x - 5 = 2x - 5 \\
&-x^2 + 4x = 0 \\
&x(-x + 4) = 0 \\
&x = \{0, 4\}
\end{aligned}
$$

Teste:

$$
\begin{aligned}
&y_1(2) = 3 \\
&y_2(2) = -1 \\
&y_1 > y_2, \quad 0 < x < 4
\end{aligned}
$$

Integral:

$$
\int_0^4 -x^2 + 6x - 5 - (2x - 5)\, dx = \int_0^4 -x^2 + 4x\, dx =
$$

$$
= -\int_0^4 x^2 + 4\int_0^4 x\, dx = -\left(\frac{x^3}{3}\bigg|_0^4\right) + 4\left(\frac{x^2}{2}\bigg|_0^4\right) =
$$

$$
= -\left(\frac{4^3}{3} - \frac{0^3}{3}\right) + 4\left(\frac{4^2}{2} - \frac{0^2}{2}\right) = -\frac{64}{3} + \frac{64}{2}
$$

$$
= \frac{32}{3} \approx 10.66
$$

19

$$
\begin{aligned}
&y_1 = x^2 - 3x \\
&y_2 = 0
\end{aligned}
$$

Intersecção:

$$
\begin{aligned}
&y_1 = y_2 \\
&x^2 - 3x = 0 \\
&x(x - 3) = 0 \\
&x = \{0, 3\}
\end{aligned}
$$

a

$$0 \leq x \leq 3$$

Testando em $x = 1$:

$$\begin{aligned} &y_1(1) = 1^2 - 3 * 1 = 1 - 3 = -2 \\ &y_2(1) = 0 \\ &y_2 > y_1, \quad 0 < x < 3 \end{aligned}$$

$$\int_0^3 y_2 - y_1 \, dx = \int_0^3 -(x^2 - 3x) \, dx =$$

$$= \int_0^3 -x^2 + 3x \, dx = -\int_0^3 x^2 \, dx + 3 \int_0^3 x \, dx =$$

$$= -\left(\frac{x^3}{3} \bigg|_0^3 \right) + 3 \left(\frac{x^2}{2} \bigg|_0^3 \right) = -\frac{3^3}{3} + \frac{0^3}{3} + 3 \left(\frac{3^2}{2} - \frac{0^2}{2} \right) =$$

$$= -9 + 3 \left(\frac{9}{2} \right) = -9 + \frac{27}{2} = \frac{9}{2}$$

b

$$0 \leq x \leq 4$$

Limites:

$$\begin{aligned} &y_2 > y_1, \quad 0 < x < 3 \\ &y_1 > y_2, \quad 3 < x < 4 \end{aligned}$$

Portanto,

$$\int_0^3 y_2 - y_1 \; dx + \int_3^4 y_1 - y_2 \; dx =$$

$$= \int_0^3 -x^2 + 3x \; dx + \int_3^4 x^2 - 3x \; dx =$$

$$= -\int_0^3 x^2 \; dx + 3\int_0^3 x \; dx + \int_3^4 x^2 \; dx - 3\int_3^4 x \; dx =$$

$$= -\left(\frac{x^3}{3}\bigg|_0^3\right) + 3\left(\frac{x^2}{2}\bigg|_0^3\right) + \frac{x^3}{3}\bigg|_3^4 - 3\left(\frac{x^2}{2}\bigg|_3^4\right) =$$

$$= -\frac{3^3}{3} + 3\frac{3^2}{2} + \frac{4^3}{3} - \frac{3^3}{3} - 3\frac{4^2}{2} + 3\frac{3^2}{2} =$$

$$= \frac{19}{3}$$

c

$$-2 \le x \le 3$$

$$\begin{aligned} y_2 > y_1, &\quad 0 < x < 3 \\ y_1 > y_2, &\quad -2 < x < 0 \end{aligned}$$

$$\int_{-2}^{0} y_1 - y_2 \; dx + \int_{0}^{3} y_2 - y_1 \; dx =$$

$$= \int_{-2}^{0} x^2 - 3x \; dx + \int_{0}^{3} -x^2 + 3x \; dx =$$

$$= \int_{-2}^{0} x^2 \; dx - 3\int_{-2}^{0} x \; dx - \int_{0}^{3} x^2 \; dx + 3\int_{0}^{3} x \; dx =$$

$$= \left(\frac{x^3}{3} \bigg|_{-2}^{0} \right) - 3\left(\frac{x^2}{2} \bigg|_{-2}^{0} \right) - \frac{x^3}{3} \bigg|_{0}^{3} + 3\left(\frac{x^2}{2} \bigg|_{0}^{3} \right) =$$

$$= \left(-\frac{(-2)^3}{3} \right) - 3\left(-\frac{(-2)^2}{2} \right) - \frac{3^3}{3} + 3\left(\frac{3^2}{2} \right) =$$

$$= \frac{8}{3} + \frac{12}{2} - 9 + \frac{27}{2} = \text{-}$$