Lec 6:

① Not solved

② ~~for all~~ if the grah is complete & there is no edge feature, the convolutional GNN ~~reduce~~ are equivalent to deep sets.

For CGNN :-

$$h_i = \phi(x_i, \bigoplus_{j \in N_i} c_{ij} \psi(x_j))$$

for a complete graph

$$c_{ij} = c$$

$$h_i = \phi(x_i, \bigoplus_{j \in V} c \psi(x_j))$$

let $\bigoplus_{j \in V} c\psi(x_j) = B \quad \forall i$

$h_i = \phi(x_i, B) \Rightarrow \phi_2(x_i) \Rightarrow$ deep set

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Lec 7:

If $g \in G$, $g = k$ $\quad g \cdot x = S^k x$

where S_k is shift matrix

$$= \begin{bmatrix} 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ & & & \\ 1 & \cdots & & 0 \end{bmatrix}^k$$

proof $\forall g, h \in G$

$$g \cdot h = h \cdot g$$

solution - let $g = k_1$, $h = k_2$

$\Rightarrow g.h = S^{k_1} S^{k_2} = S^{k_1 + k_2} = S^{k_2 + k_1} = S^{k_2} S^{k_1}$

$= (h.g)$

2) if $\langle x, (S^k)^* v \rangle = \langle x, v \rangle$

proof: $v = Sv$

$(S^k)^* v = v$

also $(S^k)^* = (S^k)^{-1} = (S^{-1})^k$

$(S^{-1})^k v = v$

$v = S^k v$

this must be satisfied for all k

we must prove that for all k: $v = Sv$

now by induction

let $k = 1$

$\Rightarrow v = Sv$ (proofed)

~~to prove~~ for $k = n$:

~~let~~ $k = n-1$ ~~and~~ we assume it is ture ~~for~~

$\Rightarrow v = S^{n-1} v \Leftrightarrow v = Sv$

~~then~~ and for $k = n$:

$v = S^n v = S(S^{n-1} v) = Sv$

~~then~~ thus we proved for $k = n$ the statement

(2) for $CS^k x = S^k C x$ C must satisfy $CS = SC$ prove?

for $k=1$ the statement holds

$CS = SC$

to prove for $k = n$ we use induction,

so we assume that it is true for $k = n-1$

$\therefore CS^{n-1} = S^{n-1} S \Leftrightarrow CS = SC$

for $k = n$

$\cancel{CS^n =} \quad \cancel{CS(S^{n-1})} = \cancel{S(C\,S^{n-1})}$

$\cancel{= S\{S^{n-1}C\} = S^nC}$

ie: $CS^n = S^n C$

$(CS^{n-1}) S = S^n C$

$S^{n-1} CS = S^n C$

$S^{n-1}(CS) = S^{n-1}(SC)$

$\Rightarrow CS = CS$ #

(3)

③ verify ~~the eigen vector~~ S is v_k

S must have a set of d linearly independent eigen vectors.

k takes values from 1 to d so we already have all the d eigen vectors

④ Since $(Cx)_j = \sum_{L} \alpha_{j-L}\, a_L = \sum (a_L S^{-L} x)_j$ any convolution ~~commutes~~ - commutes with S

$$Cx = \sum_{L} (\alpha_L S^{-L} x)$$

$$CSx = \sum_{L} \alpha_L S^{-L} S x = \sum_{L} \alpha_L S^{-L+1} x$$

$$SCx = \sum_{L} \alpha_L S^{-L+1} x$$

$$CSx = SCx$$