Inverse Transform Method

Let $F(x)$, CDF. Let $U \sim \text{Uniform}(0,1)$.
We have that $F^{-1}(U)$ is distributed as F.

Proof: $P[F^{-1}(U) \leq x]$

$$= P[U \leq F(x)], \quad \text{by monotonicity}$$

$$= F(x).$$

Ex: Exponential (λ)

$$f(x) = \frac{1}{\lambda} e^{-x/\lambda} \quad ; \quad F(x) = 1 - e^{-x/\lambda}$$

$$u = 1 - e^{-x/\lambda}$$

$$\Leftrightarrow \quad 1 - u = e^{-x/\lambda}$$

$$\Leftrightarrow \quad -\log(1-u) = x/\lambda$$

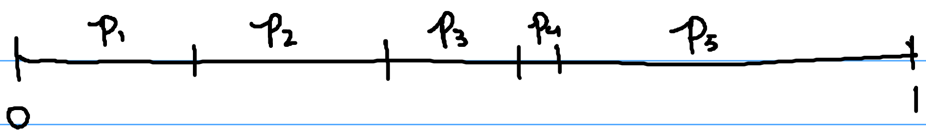
$$\Leftrightarrow \quad -\lambda \log(1-u) = x$$

$$\boxed{F^{-1}(x) = -\lambda \log(1-x)}$$

Discrete random variables

$$f(x) = \begin{cases} x_1 & \text{with prob. } p_1 \\ \vdots & \\ x_n & \text{with prob. } p_n \end{cases}$$

$$\sum_i p_i = 1$$



Generate $u \sim U(0,1)$

$$\begin{cases} x_1 & \text{if} \quad 0 < u \leq p_1 \\ x_2 & \text{if} \quad p_1 < u \leq p_1 + p_2 \\ \vdots & \\ x_n & \text{if} \quad \sum_{i=1}^{n-1} p_i < u \leq \sum_{i=1}^{n} p_i = 1 \end{cases}$$

Acceptance - Rejection Method
Accept - Reject Method

Let $f(x)$, pdf. of a r.v. we want to generate

$g(x)$, pdf. of a r.v. we use as an instrumental dist.

Same support

$c > 0$ such that

$$\frac{f(x)}{g(x)} \leq c .$$

Then we can generate realisations from f using this algorithm:

1. Find the value c smallest c possible
2. Generate $y \sim g(\cdot)$
3. Generate $u \sim \text{Uniform}(0,1)$
4. if $u \leq \dfrac{f(y)}{c\, g(y)}$, then $x = y$ highest value

otherwise, we repeat from step 2.