Delft University of Technology - Applied Physics - ICCP

Lattice Boltzmann CFD in a horizontal pipe

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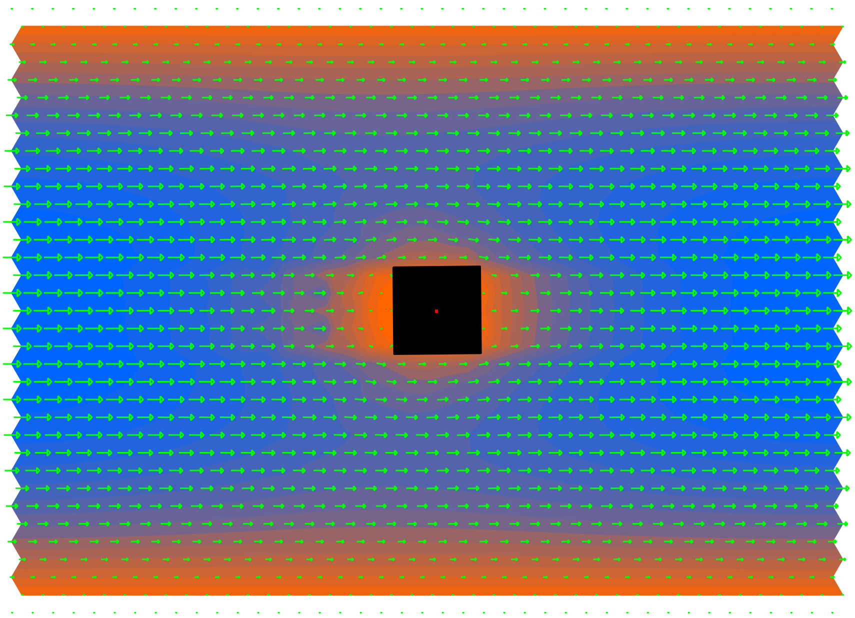


Introduction

In this short report a method is described for simulating the flow of an incompressible fluid in a horizontal two dimensional pipe. The lattice boltzmann model is used to simulate the flow characteristics. A hexagonal (d2q7) lattice is used in the model and the pipe has a constant pressure gradient, periodic boundary conditions in the direction of the flow and sticky boundaries in the perpendicular direction.

As a first step and to verify the working of the model, a simple simulation is run for a pipe without any objects. After checking that the velocity profile is parabolic as expected, objects that are able to interact with the fluid are added to the pipe. In a first stage these objects are stationary and in the final stage these objects are able to move (both translation and rotation), giving the simulation a dynamic component.

Lattice Boltzmann Model

It is not the aim of this report to explain the lattice boltzmann model in detail, rather this section will briefly discuss its main features. Key feature of this model is that it limits the microscopic details. Particles are placed on fixed lattice sites and can only move to neighbouring sites. The particles interact in the form of collisions and a relaxation effect is introduced. Note that both space and time are treated as being discrete. The 'particles' however are replaced by densities of the fluid. The boltzmann equation reads:

$$\frac{\partial n}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} n = \left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_{collisions}. \tag{1}$$

The left hand side describes the moving of densities and the right hand side describes the collision interaction. This effect of collision can be further written as:

$$\left(\frac{\mathrm{d}n\left(\mathbf{n},\mathbf{v},t\right)}{\mathrm{d}t}\right)_{collisions} = -\frac{n\left(\mathbf{n},\mathbf{v},t\right) - n^{eq}\left(\mathbf{n},\mathbf{v}\right)}{\tau}, \tag{2}$$

where the current distribution of densities is compared with the equilibrium distribution and relaxation to this equilibrium happens with a time constant τ.

Combining the boltzmann model with a hexagonal lattice comes down to defining 7 different densities for each lattice site, corresponding to each of the six velocities (in the six directions) and to the 7th zero-velocity. A time step in this model now becomes: move the densities according to their velocity and relax all the densities towards equilibrium. Finally, in equation form this becomes:

$$n_i\left(\mathbf{r} + c\Delta t\mathbf{e}_i, t + \Delta t\right) = n_i\left(\mathbf{r},t\right) - \frac{1}{\tau}\left[n_i\left(\mathbf{r},t\right) - n_i^{eq}\left(\mathbf{r},t\right)\right] \tag{3}$$

Algorithm

The algorithm steps are as follows:

1. Move density n_i to the appropriate neighbour
2. Handle boundary conditions correctly (bounce-back / periodic)
3. Calculate velocities at each point (Cartesian components)
4. Add effect of pressure gradient
5. Calculate equilibrium density distribution at each point
6. Relax the densities of fluid points
7. Move the solid object
8. Update wall points and fluid points

Results

In Figure 1 the fluid flow velocity is plotted. A parabolic profile is observed, as expected. Now let's see what happens to the flow profile if objects are placed in the flow.

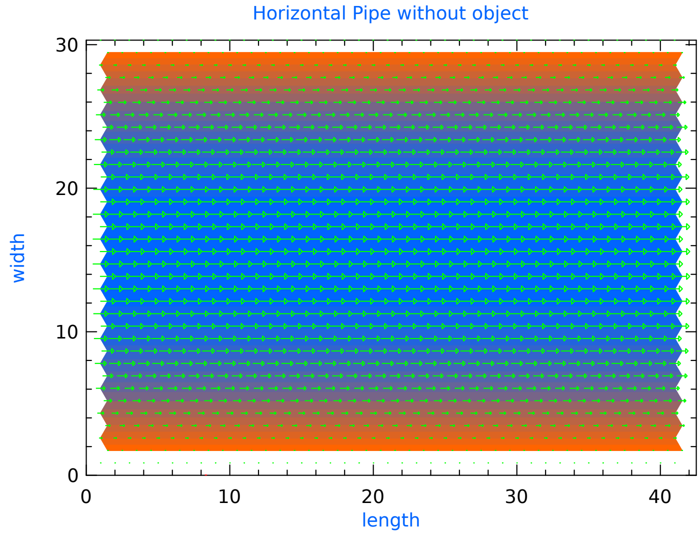
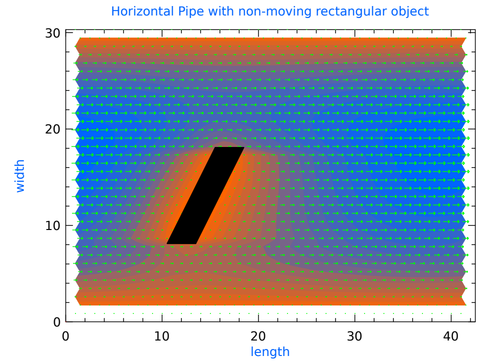


Figure 1: Horizontal flow in a pipe. The arrows give an indication of the velocity at that point, the colour indicates the horizontal velocity. Low velocities correspond to orange, high velocities to blue.

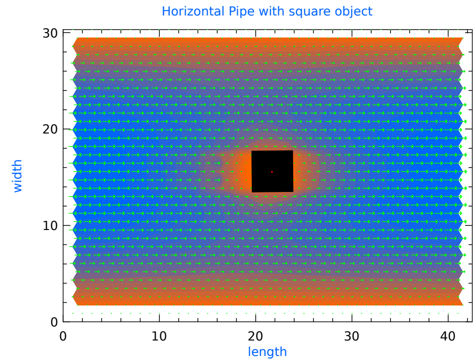
In Figure 2 snapshots are seen depicting various (non) moving objects in the horizontal pipe flow. The flow neatly goes around the objects. It should be noted that the objects are moving, except in Figure 2a.

Conclusion

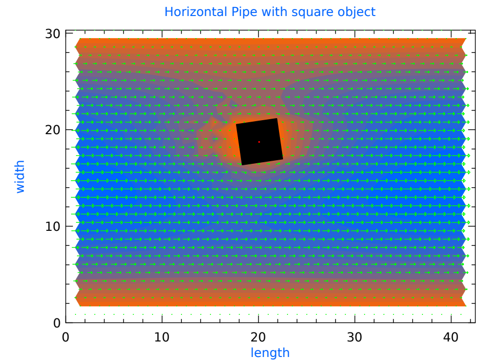
It can be concluded that the lattice boltzmann method used for simulating the flow of an incompressible fluid works properly. Even after introducing moving objects into the flow, the algorithm performs quite well. Improvements can be made to further stabilise the fluid-object interaction and also different pipe geometries can be studied. It would be nice to also look at the behaviour of multiple objects and introduce an object-object interaction.



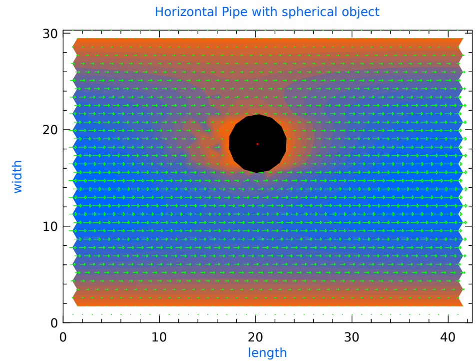
(a) Non-moving rectangle placed in the laminar flow.



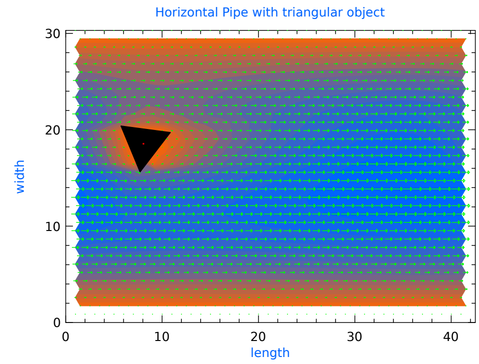
(b) Moving square placed in the laminar flow. It is placed near the middle of the flow so it hardly rotates.



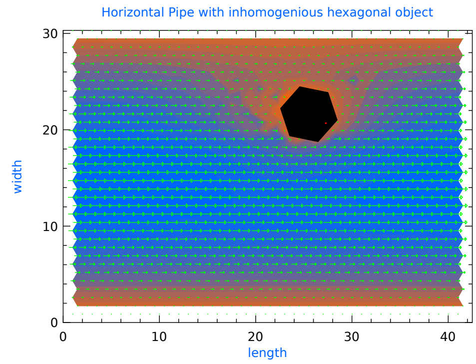
(c) Moving square placed in the laminar flow. It is placed near the edge of the flow so it rotates.



(d) Moving sphere placed in the laminar flow.



(e) Moving triangle placed in the laminar flow. Object is moving to the left due to initial applied velocity, eventually it will follow the flow and move rightwards.



(f) Moving hexagon placed in the laminar flow. The mass distribution is non uniform which influences its rotation.

Figure 2: Objects placed in laminar flow. The arrows give an indication of the velocity at that point, the colour indicates the horizontal velocity. Low velocities correspond to orange, high velocities to blue. The red dot corresponds to the position of the centre of mass of the object.