Partial Least Squares Regression with Multiple Functional and Scalar Predictors

Jongmin Mun [1] and Jeong Hoon Jang[2]

University of Southern California[1] and University of Texas Medical Branch[2]

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Abstract

abstract.

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1 Introduction

Functional regression settings involving a large number of predictors, $X_{i1}(t), X_{i2}(t), \ldots, X_{iK}(t)$, are becoming increasingly common. For example, **?** analyzes two gene expression datasets measured over time, which involve only a small number of patients but tens of thousands of functional predictors.

Infinite dimensional beta estimation problem (ill-posed problem)

Multicollinearity and high-dimensionality in scalar predictors are not handled.

Also, there may be high correlation between functional and scalar predictors.

Many approaches have been proposed 1. Roughness Penalty 2. basis approach (power-series, B-splines, wavelets) - Power Series splines (Goldsmith et al., 2011, Journal of Computational Statistics) - B-splines (H. Cardot, F. Ferraty, P. Sarda, Spline estimators for the functional linear model, Statistica Sinica 13 (2003) 571–591. T.T.Cai,P.Hall,Prediction in functional linear regression, Ann.Stat. 34(5) (2006) 2159–2179.) - Wavelet (Y.Zhao,R.T.Ogden,P.T.Reiss,W based lasso in functional linear regression, J.Comput.Graph.Stat.21(3)(2012)600–61.)7

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