UNIST
Department of Mechanical Engineering

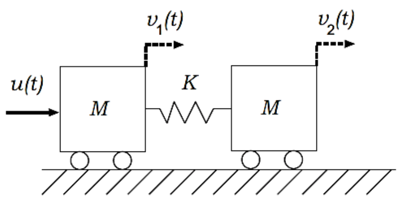
MEN 573: Advanced Control Systems I **Spring, 2016**

Homework #9 Assigned: Friday, May 27, 2016
Due: Wednesday, June 8, 2016 (in class)

Problem 1.

Consider the two-mass system sketched below and with the following realization

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ v_2 \\ x \end{bmatrix} = \begin{bmatrix} 0 & 0 & -100 \\ 0 & 0 & 100 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ x \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \qquad \Delta(s) = s(s^2 + 200)$$



(a) Show that this system is controllable. You are encouraged to perform all necessary matrix computation using Matlab.

(b) Transform the system into the controllable canonical form.

(c) Obtain the state feedback control to place the closed loop eigenvalues at -20$\pm$20j and -40.

Problem 2.

Consider now the LTI discrete time SISO system

$$x(k+1) = \begin{bmatrix} -1 & -2 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u(k)\,, \qquad y(k) = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x(k)$$

under the state variable feedback control law

$$u(k) = -K\,x(k) + F\,v$$

(a) Obtain the state variable feedback gain $K^T \in R^3$ so that all of the eigenvalues of $A_C = A - BK$ are at the origin using the command acker in matlab, which uses an algorithm similar to the one discussed in the class, and was originally developed by Prof. Ackermann. The matlab command place is more robust, but it will not place eigenvalues of a multiplicity that is greater than the rank of B

(b) Calculate using matlab the gain F so that the steady state output of the closed loop system is equal to the constant exogenous input v.

(c) Design a discrete time a-priori observer

$$\hat{x}(k+1) = A\,\hat{x}(k) + B\,u(k) + L\,[y(k) - C\hat{x}(k)] \tag{2}$$

and calculate using `matlab` the gain $L \in \mathcal{R}^3$ so that all eigenvalues of $A_e = A - L\,C$ are at the origin.

(d) Simulate using `matlab` the response of the system under the state observer feedback control

$$u(k) = -K\,\hat{x}(k) + F\,v \tag{3}$$

where $\hat{x}(k)$ is the a-priori observer designed in (d), and the gains K and F are as selected in item (a) and (c), under the following two cases:

- $v = 5$, $\hat{x}(0) = x(0) = 0$
- $v = 5$, $\hat{x}(0) = 0$ and $x(0) = \begin{bmatrix} -2 & 2 & 3 \end{bmatrix}^T$

Plot the responses of $x(k)$, $\hat{x}(k)$ and $y(k)$ for both cases.

Hint: The a-priori observer Eq. (2) and control law Eq. (3) can be implemented using the following state and output equations

$$\begin{aligned} \hat{x}(k+1) &= [A - LC - BK]\,\hat{x}(k) + B\,Fv + L\,y(k) \\ u(k) &= -K\,\hat{x}(k) + F\,v \end{aligned}$$

(e) Design a discrete time a-posteriori observer

$$\hat{x}^o(k+1) = A\,\hat{x}(k) + B\,u(k) \tag{4}$$

$$\hat{x}(k+1) = \hat{x}^o(k+1) + L\,[y(k+1) - C\hat{x}^o(k+1)] \tag{5}$$

and calculate using `matlab` the gain $L \in \mathcal{R}^3$ so that all eigenvalues of $A_e = (I - LC)\,A$ in

$$\hat{x}(k+1) = A_e\,\hat{x}(k) + B\,u(k) + L\,y(k+1) \tag{6}$$

are at the origin.

(f) Simulate using `matlab` the response of the system under the state observer feedback control

$$u(k) = -K\,\hat{x}(k) + F\,v \tag{7}$$

where $\hat{x}(k)$ is the a-posteriori observer designed in (e), and the gains K and F are as selected in item (a) and (e), under the following conditions:

- $v = 5$, $\hat{x}^o(0) = x(0) = 0$
- $v = 5$, $\hat{x}^o(0) = 0$ and $x(0) = \begin{bmatrix} -2 & 2 & 3 \end{bmatrix}^T$.

Comments:

- Notice that Eq. (6) is not in a state equation format (why?).

- In an actual discrete-time controller implementation the a-posteriori observer equations (4)-(5) and feedback law (7) would be implemented as follows:

  (i) Update $\hat{x}(k)$ and $u(k)$ at the sample index k:

$$\begin{aligned} \hat{x}(k) &= (I - LC)\,\hat{x}^o(k) + L\,y(k) \\ u(k) &= -K\,\hat{x}(k) + F\,v\,. \end{aligned}$$

  (ii) Update $\hat{x}^o(k+1)$ for the next sample index $(k+1)$:

$$\hat{x}^o(k+1) = A\,\hat{x}(k) + B\,u(k)\,.$$

  In this manner, there is a minimum of computation delay between the instant when the output $y(k)$ is measured and the instant when the control $u(k)$ is generated. Notice that, strictly speaking, there will always be a small computational delay.

- In order to implement the a-posteriori observer in Eqs. (4) and (5) and control law (7) using state and output equations, it is necessary to delay Eq. (5) by one step

$$\begin{aligned} \hat{x}(k) &= \hat{x}^o(k) + L\,[y(k) - C\hat{x}^o(k)] \\ \hat{x}(k) &= (I - LC)\,\hat{x}^o(k) + L\,y(k) \end{aligned} \qquad (8)$$

  and insert Eq. (8) into Eqs. (4) and (7)to obtain

$$\hat{x}^o(k+1) = A\,(I - LC)\,\hat{x^o}(k) + B\,u(k) + A\,L\,y(k) \qquad (9)$$

$$u(k) = -K\,[(I - LC)\,\hat{x}^o(k) + L\,y(k)] + F\,v \qquad (10)$$

  Then we insert Eq. (10) into (9) to obtain

$$\begin{aligned} \hat{x}^o(k+1) &= (A - BK)\,(I - LC)\,\hat{x^o}(k) + BF\,v + (A - BK)L\,y(k) \\ u(k) &= -K(I - LC)\,\hat{x}^o(k) - KL\,y(k) + F\,v \end{aligned}$$