Algorithms and Complexity

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Things we'll study from now on

- Computational models (Mostly Turing machine)
- A little bit of computability
- P versus NP problem
- NP-complete problems
- Why P versus NP is so hard?

Textbook

- Oded Goldreich, "P, NP, and NP-completeness"
  - Freely available on the internet
  - I'll also upload a copy on the Blackboard
  - We'll only follow this somewhat loosely

Today

» Turing machines

Turing machine

Alan Turing

What is computation?

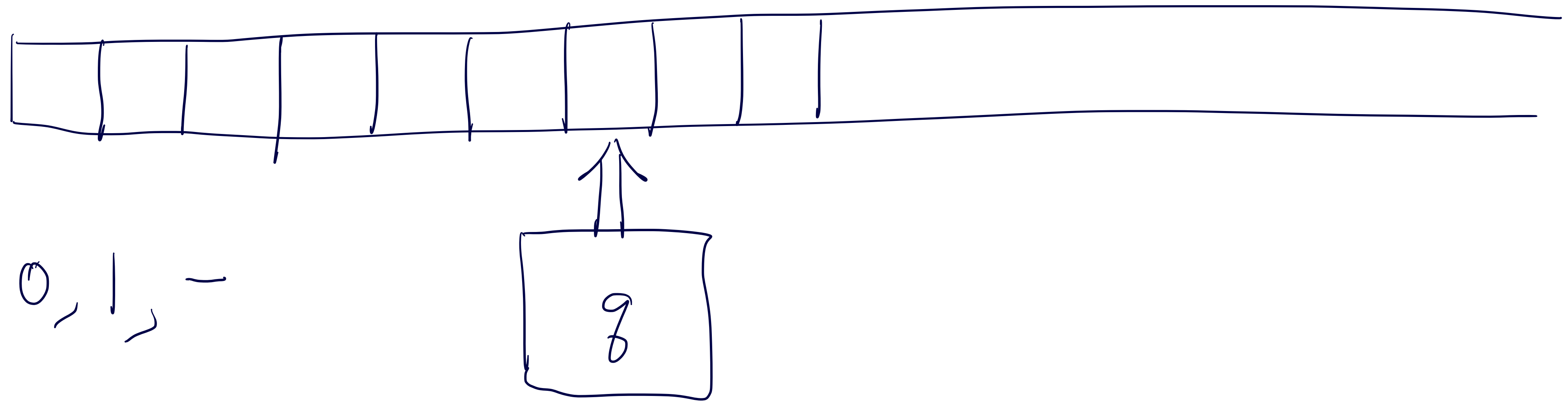
- Or, what is a computer?
- Not some particular implementation, but the essence?
- There are at least three satisfying, equivalent answers
  - Alonzo Church: Lambda calculus
  - Kurt Gödel: Recursive function theory
  - Alan Turing: Turing machine

Turing machine

- Turing machine: an abstract formalism of computer
- We will define it and study it
- Eventually, we can define computation as what Turing machines can do
  - Computability = Turing computability
- And we'll learn why any particular formalism isn't that important

Turing machine

»



$\Sigma \ni 0, 1, \_$

$Q \ni q$

More formally

» A turing machine M can be specified by

$M = (Q, \Sigma, \delta, q_{start}, q_{halt})$

– Q : a finite set called the state space

– Σ : " the tape alphabet

$0, 1, \_ \in \Sigma$

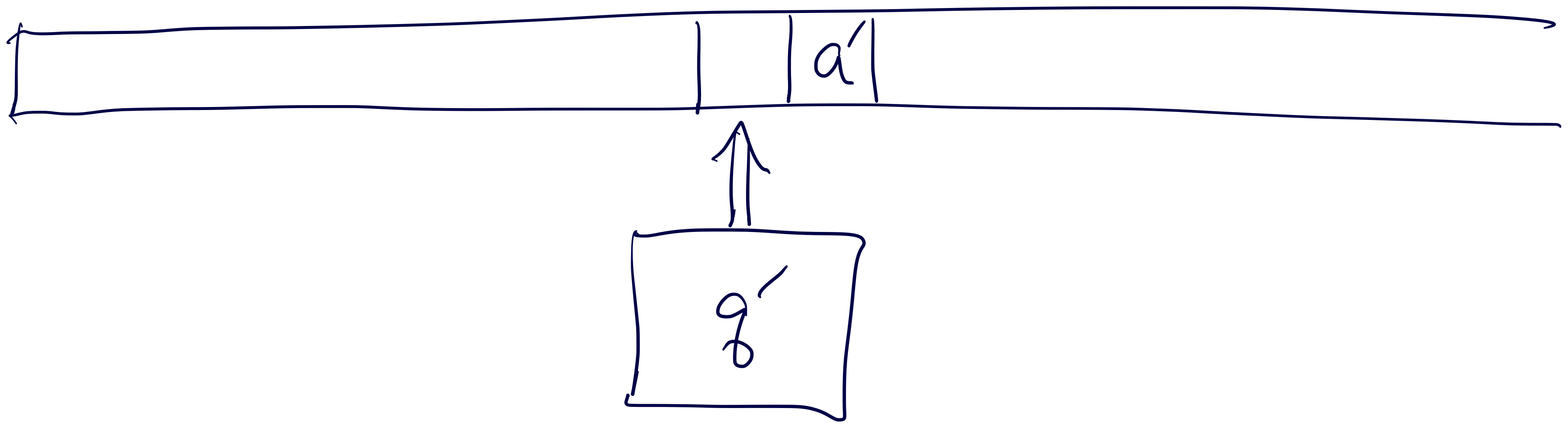
– $\delta : Q \times \Sigma \to Q \times \Sigma \times \{-1, 0, 1\}$

– $q_{start}, q_{halt} \in Q$

How does it operate

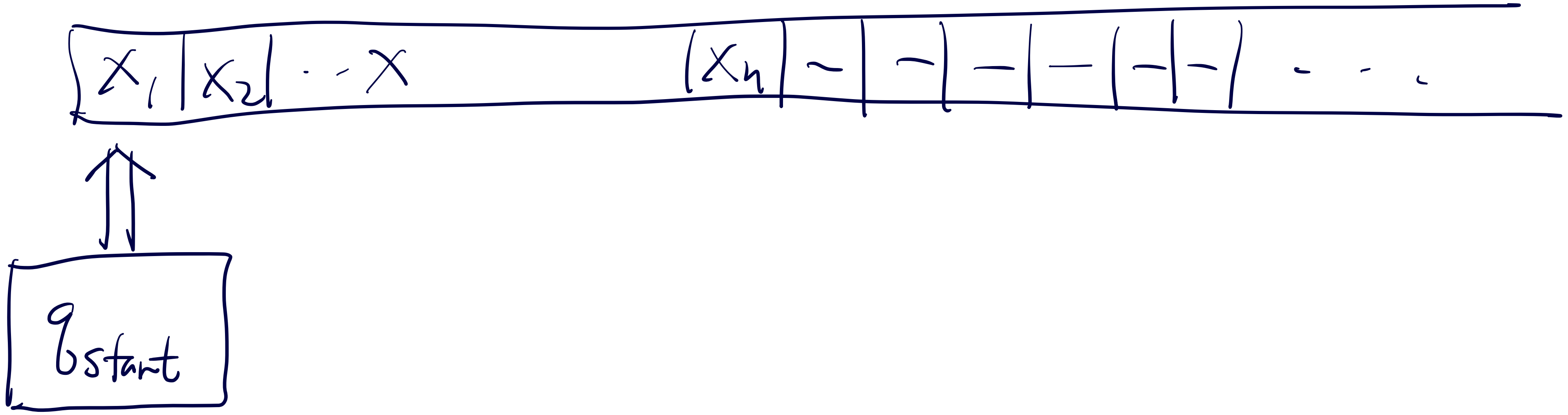
» $\delta : Q \times \Sigma \to Q \times \Sigma \times \{-1, 0, 1\}$

$$\delta(q, a) = (q', a', -1)$$



How does it operate

>> Input $x \in \{0,1\}^*$ $x = x_1 x_2 \cdots x_n$, $x_i \in \{0,1\}$



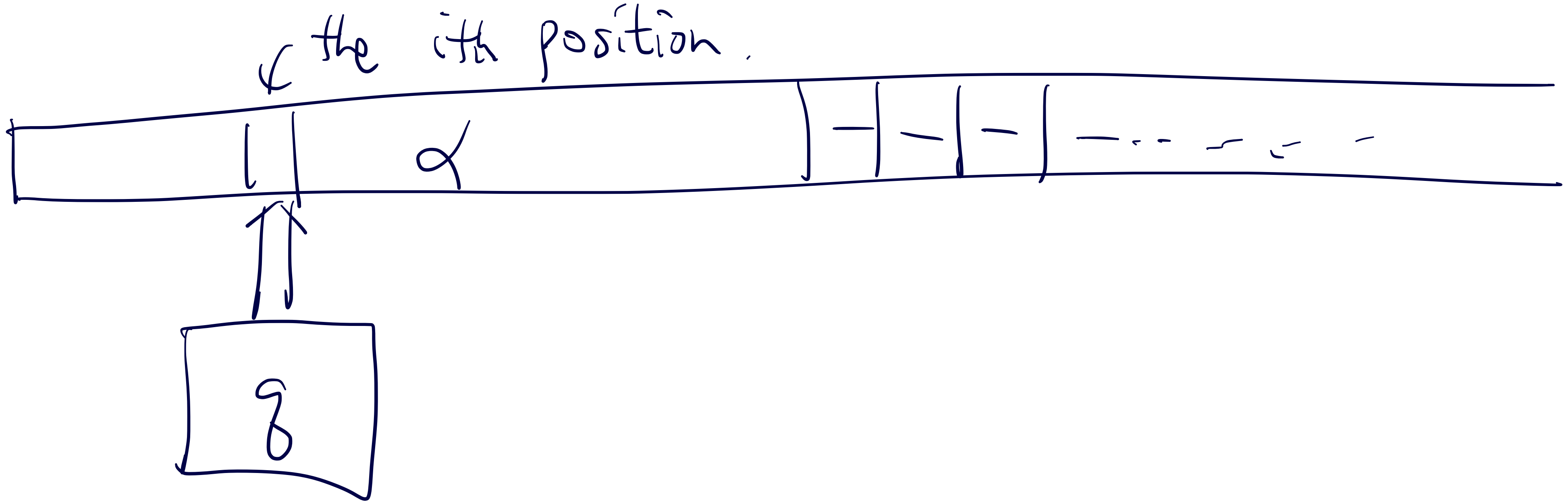
How does it operate

» An instantaneous configuration is (α, q, i) where

$\alpha \in \Sigma^*$

$q \in Q$

$i \in \mathbb{N}$ with $1 \leq i \leq |\alpha|$.



How does it operate

>> The machine is initialized as $(x, q_{start}, 1)$

where x is the input string.

In most cases, (α, q, i) is updated to $(\alpha', q', i+d)$

if $\delta(q, a) = (q', a', d)$

and $\alpha[i] = a$

(α' is the same as α, except that $\alpha'[i] = a'$)

How does it operate

» At the leftmost position, if it tries to move left, it halts.

$$\delta(q, a) = (q', a', -1)$$

and $\alpha[1] = a$,

then, $(\alpha, q, 1)$ halts the computation.

How does it operate

» If $\delta(q, a) = (q', a', 1)$ then,

$(\alpha, q, |\alpha|)$ is updated to $(\alpha', q', |\alpha|+1)$

if $\alpha[|\alpha|] = a$

, where $\alpha' = \alpha[1] \cdots \alpha[|\alpha|-1]\, a'$ —