UNIST
Department of Mechanical Engineering

MEN 573: Advanced Control Systems I		**Spring, 2016**
Homework #2	Assigned: Saturday, March 19, 2016 Due: Monday, March 28, 2016 (in class)	

Problem 1. Linear Algebra

1) Explain why the $R^{2\times 2}$, the set of 2×2real matrices, is not a field. Is $(R^{2\times 2}, R)^1$ a vector space?

2) Let $R(s)$ by the field of rational functions of polynomials in $s \in C$ with real coefficients. Explain why the following two vectors

$$v_1 = \begin{bmatrix} \frac{1}{s+1} \\ \frac{1}{s+2} \end{bmatrix} \qquad v_2 = \begin{bmatrix} \frac{s+2}{s^2+4s+3} \\ \frac{1}{s+3} \end{bmatrix}$$

are linearly independent in $(R^2(s), R)$ but linearly dependent in $(R^2(s), R(s))$.

3) Find the ranks and nullities and provide bases for the range and null spaces of the following matrices:

$$\text{(a)} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix} \quad \text{(b)} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 4 & 0 \\ 1 & 0 \end{bmatrix} \quad \text{(c)} \begin{bmatrix} 0 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 \\ 11 & 0 & 0 & 0 \end{bmatrix}$$

4) Let $\mathcal{A} : \mathcal{R}^3 \to \mathcal{R}^3$ be a linear operator. Consider the two sets $B = \{b_1, b_2, b_3\}$ and $C = \{c_1, c_2, c_3\}$ below

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}, \; C = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

It should be clear to you that these are basis for $\mathcal{R}^3$. Suppose the linear operator $\mathcal{A}$ maps

$$\mathcal{A}(b_1) = 2\,b_1 - b_2, \; \mathcal{A}(b_2) = 0, \; \mathcal{A}(b_3) = 4\,b_2 + 2\,b_3$$

where 0 is the additive identity (e.g. $v + 0 = v$ for all $v \in \mathcal{R}^3$.

(i) Write down the matrix representation of $\mathcal{A}$ with respect to the basis B.
(ii) Write down the matrix representation of $\mathcal{A}$ with respect to the basis C.

5) Let $S \subset \mathcal{R}^3$ be given by

$$S = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -6 \\ 3 \end{bmatrix} \right\}$$

(a) Determine is the dimension of S.
(b) Determine the orthogonal complement of S, $S^\perp$.

6)

Let $A \in \mathcal{C}^{m\times n}$. Prove that

$$\text{Rank}(A) \leq \min\{m, n\}\,.$$

7)

For A, B, $T \in R^{n\times n}$, where T is nonsingular and

$$B = T^{-1}\,A\,T$$

Prove that B and A have the same eigenvalues.

8) Prove that if $\{v_1, v_2, \cdots, v_n\}$ spans V, then so does the set $\{v_1 - v_2, v_2 - v_3, \cdots, v_{n-1} - v_n, v_n\}$.