CSE530: Algorithms & Complexity
Exercise Set 3

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This is not an assignment. These exercises will not be graded.

1. Let $A[1 \dots n]$ be an array of n numbers. A *maximum sum subarray* of A is a subarray $A[p \dots q]$ such that the sum $\sum_{i=p}^{q} A[i]$ of its elements is maximum. For instance, if $A = [-2, 1, -3, 4, -1, 2, 1, -5, 4]$, then a maximum sum subarray is $[4, -1, 2, 1]$, and its sum is 6.

Give an algorithm that computes the sum of a maximum sum subarray in $O(n^2)$ time. For instance, in the example above, your algorithm should return 6.

2. You are given k types of coins with values $v_1, \dots, v_k \in \mathbb{N}$ and a cost $C \in \mathbb{N}$. You may assume $v_1 = 1$ so that it is always possible to make any cost. You want to find the smallest number of coins required to sum to C exactly. For example, assume you have coins of values 1, 5, and 10. Then the smallest number of coins to make 26 is 4: take 2 coins of value 10, 1 coin of value 5, and 1 coin of value 1.

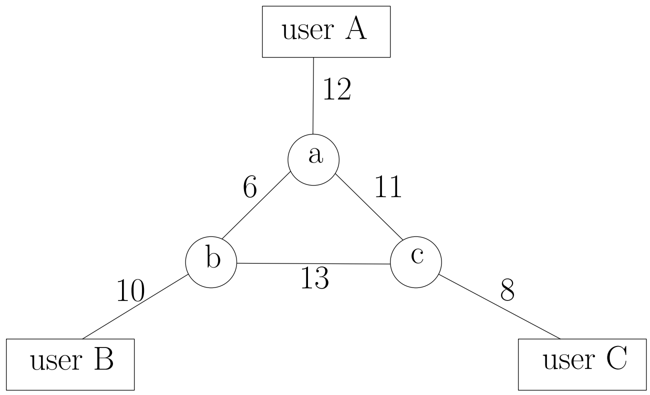
Give an algorithm for this problem. Its running time should be polynomial in $n = \max(k, C)$.

3. Suppose General Motors makes a profit of \$100 on each Chevrolet, \$200 on each Buick and \$400 on each Cadillac. These cars get 20, 17, and 14 miles a gallon respectively, and it takes respectively 1, 2, and 3 minutes to assemble one Chevrolet, one Buick, and one Cadillac. The government requires that the fuel efficiency of the cars produced by General Motors be on average at least 18 miles a gallon. General Motors needs to determine the optimal number of cars that can be assembled in an 8-hours day, so that the profit is maximized.

(a) Formulate this problem as a linear program.

(b) Solve this problem numerically using a linear program solver.

4. Suppose that you are managing a network whose lines have the bandwidths shown in the figure below, and you need to establish three connections: between users A and B, between B and C, and between A and C. Each connection requires at least two units of bandwidth, but can be assigned more. Connection AB pays \$3 per unit of bandwidth, and connections BC and AC pay \$2 and \$4, respectively. Each connection can be routed in two ways, a long path and a short path, or by a combination: for instance, between A and B, two units of bandwidth via the short route AabB, and one via the long route AacbB. Your goal is to route these connections so as to maximize the network's revenue. Formulate this problem as a linear program.



5. We consider a line-fitting problem similar with the one studied in class. The difference is that, instead of minimizing the sum of the vertical distances to the points, we want to minimize the maximum vertical distance. So the input is still a set of n points (x_i, y_i), and the output should be the coefficients a and b of a line $y = ax + b$ such that

$$\max_{i \in \{1,\ldots,n\}} |ax_i + b - y_i|$$

is minimized.

Formulate this problem as a linear program.

6. Show that the set of optimal solutions to a linear program is a convex set.

7. Consider the linear program below.

$$\begin{array}{lrcrcrl} \text{maximize} & 2x_1 & + & 3x_2 & + & 2x_3 & \\ \text{subject to} & x_1 & & & + & x_3 & \leqslant 1 \\ & & & x_2 & + & x_3 & \leqslant 6 \\ & x_1 & + & x_2 & - & x_3 & \leqslant 5 \\ & & & & & x_1, x_2, x_3 & \geqslant 0 \end{array}$$

Prove that $(x_1^*, x_2^*, x_3^*) = \left(\frac{1}{3}, \frac{16}{3}, \frac{2}{3}\right)$ is an optimal solution to this linear program.

8. Solve the linear program below using the simplex algorithm.

$$\begin{array}{lrcrcrl} \text{maximize} & 42x_1 & + & 39x_2 & + & 52x_3 & \\ \text{subject to} & 9x_1 & + & 5x_2 & + & 6x_3 & \leqslant 600 \\ & 2x_1 & + & x_2 & + & 2x_3 & \leqslant 150 \\ & & & & & x_3 & \leqslant 60 \\ & x_1 & + & x_2 & + & x_3 & \leqslant 90 \\ & & & & & x_1, x_2, x_3 & \geqslant 0 \end{array}$$

9. Solve the linear program below using the simplex algorithm.

$$\begin{array}{lrl} \text{Maximize} & x_1 - 2x_3 & \\ \text{subject to} & x_1 - x_2 & \leqslant 1 \\ & 2x_2 - x_3 & \leqslant 1 \\ & x_1, x_2, x_3 & \geqslant 0 \end{array}$$