[MEN573] Advanced Control Systems I

Complement to Lec 17 & 18
State Variable and State Observer Feedback Control Design using Matlab

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Plant

Continuous time, second order unstable system **sysp**:

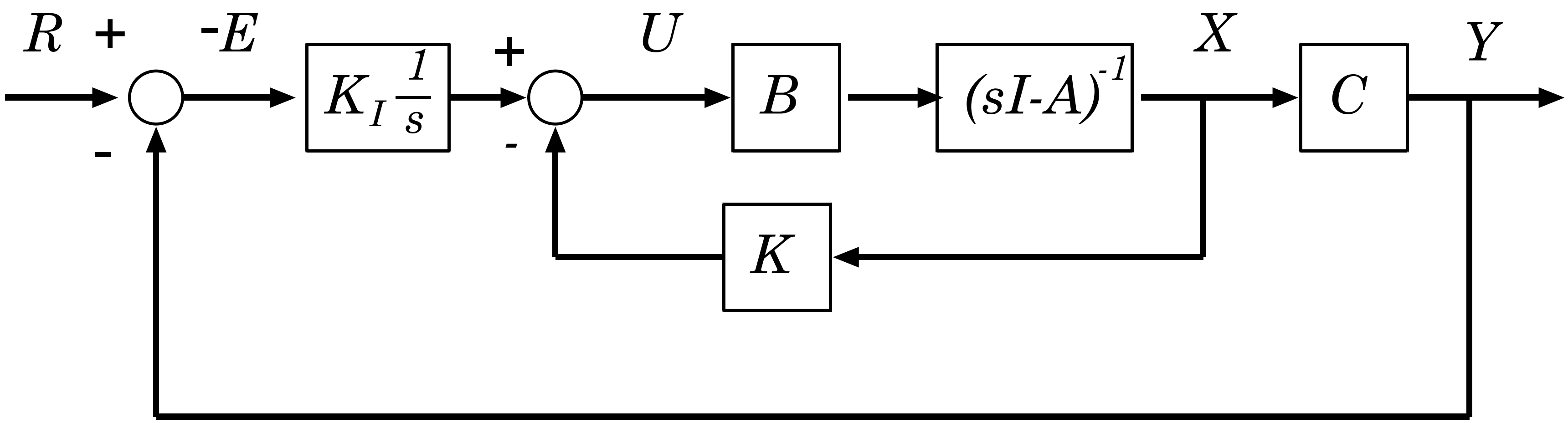
$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

```
A=[0  1 ;  1 0];

B=[0;1];

C=[1,0];

D=0;

sysp=ss(A,B,C,D)
```

```
P=ctrb(sysp)
P =
     0     1
     1     0
Q=obsv(sysp)
Q =
     1     0
     0     1
```

Control objective 1

- Design a state feedback with I-action with closed loop poles at: {-1, -1+j, -1-j}



$$\begin{aligned} u &= -K\,x + u_I \\ \dot{u}_I &= -K_I\,e \\ e &= y - r \end{aligned}$$

$$\begin{aligned} K &\in \mathcal{R}^{1\times 2} \\ K_I &\in \mathcal{R} \end{aligned}$$

Control design 1

- **Step 1**: Defined the augmented equivalent system to obtain the state and I-action feedback gains:

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}}_{\tilde{A}} \begin{bmatrix} e \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ B \end{bmatrix}}_{\tilde{B}} \tilde{u}$$

$$\dot{y} = \underbrace{\begin{bmatrix} 0 & C \end{bmatrix}}_{\tilde{C}} \underbrace{\begin{bmatrix} e \\ \dot{x} \end{bmatrix}}_{\tilde{x}}$$

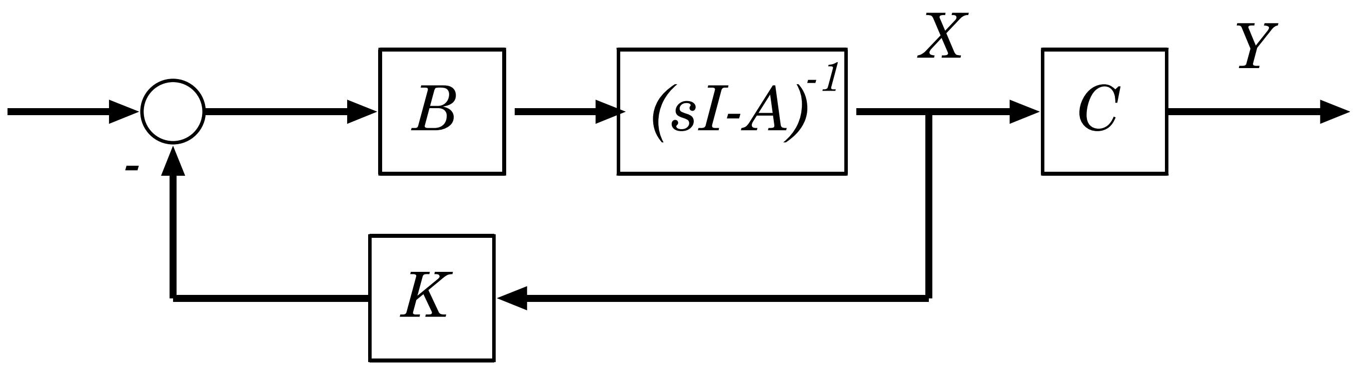
$$\tilde{u} = -\tilde{K}\tilde{x}$$

$$\begin{aligned} \tilde{K} &= \begin{bmatrix} K_i & K \end{bmatrix} \\ &= \begin{bmatrix} 2 & 5 & 3 \end{bmatrix} \end{aligned}$$

```
At=[ 0 C ; 0*B A];
Bt=[0;B];
Ct=[0 C];
syst=ss(At,Bt,Ct,D);

% Desired closed
%loop eigenvalues
p=[-1,-1+j,-1-j];
Kt=place(At,Bt,p)
Kt =
2     5     3
```

Control design 1

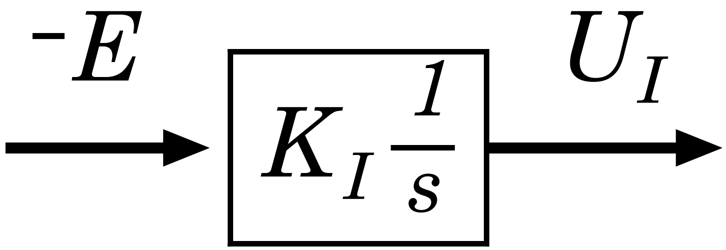
- **Step 2**: Implement state variable feedback :




```
% State variable feedback block syssvf
K=Kt(2:3);
syssvf=ss(A-B*K,B,C,D);
```

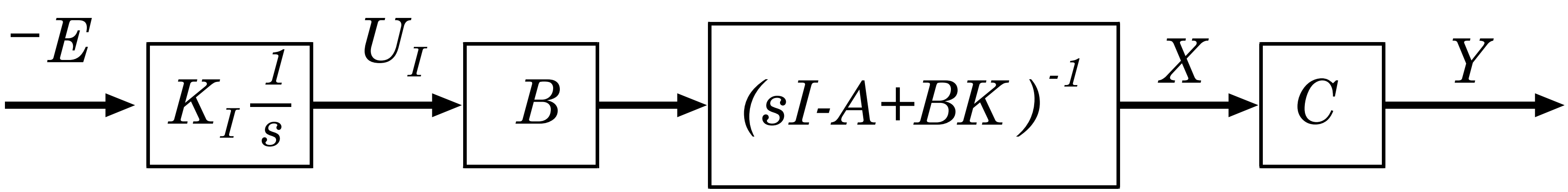
Control design 1

- **Step 3**: Create I-action block sysi:




```
% I-action block sysi:
sysi=ss(0,Kt(1,1),1,0);
```

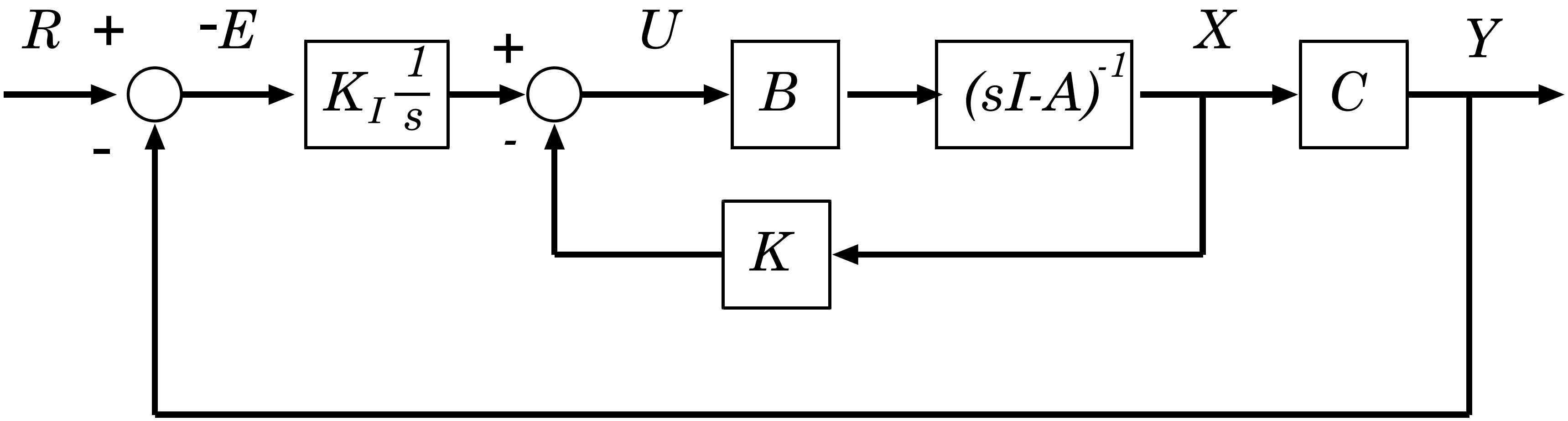
- **Step 4**: Create open-loop system syso:




```
% open-loop block syso:
syso=syssvf*sysi
```

Control design 1

- **Step 5**: Closed-loop syscl:

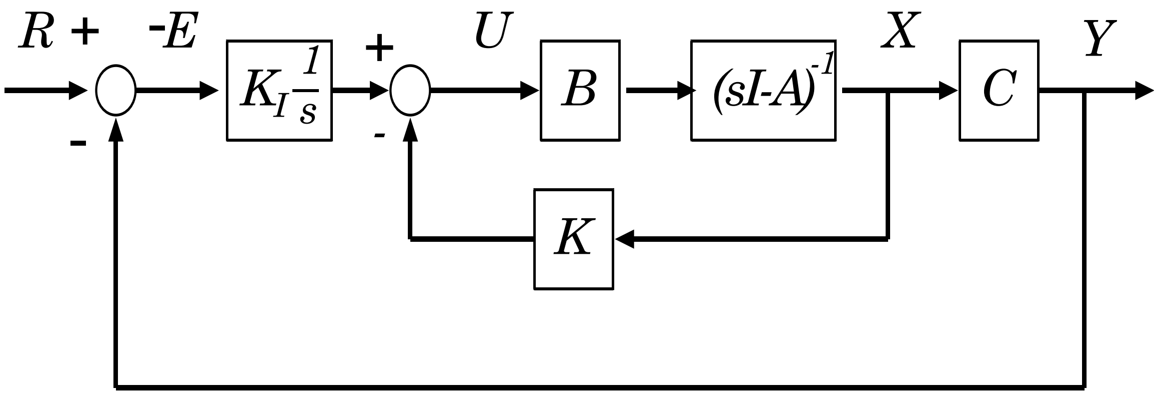



```
% closed-loop system syscl
syscl=feedback(syso,1)

% Check closed-loop eigenvalues
[acl,bcl,ccl,dcl] = ssdata(syscl);
eig(acl)
```

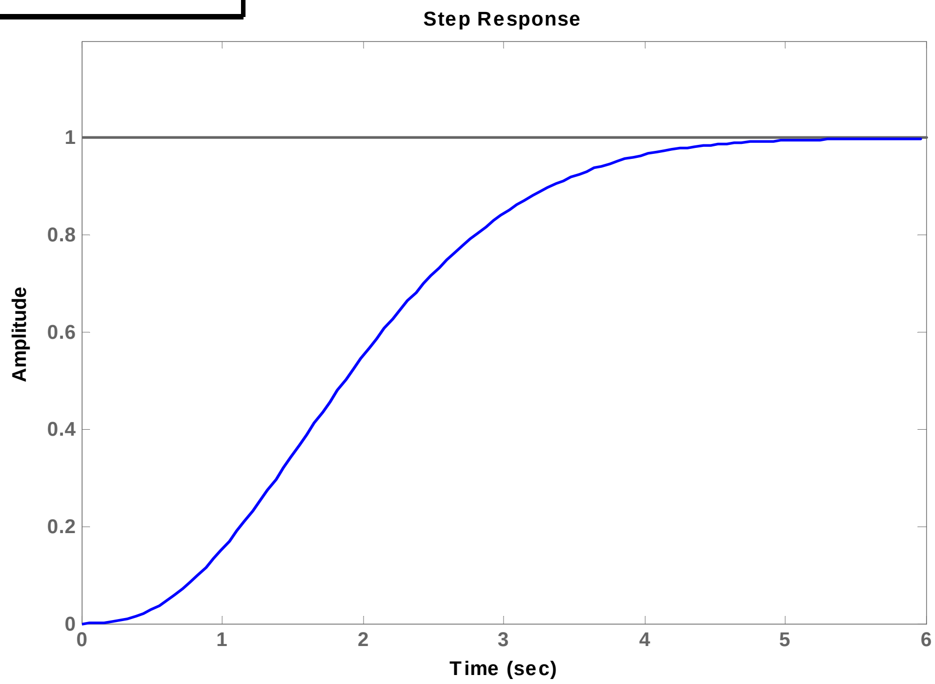
```
ans =

   -1  + 1i

   -1 - 1i

   -1.
```

Control design 1

- **Step 6**: Check step response R = unit step:




```
% step response
step(syscl)
```



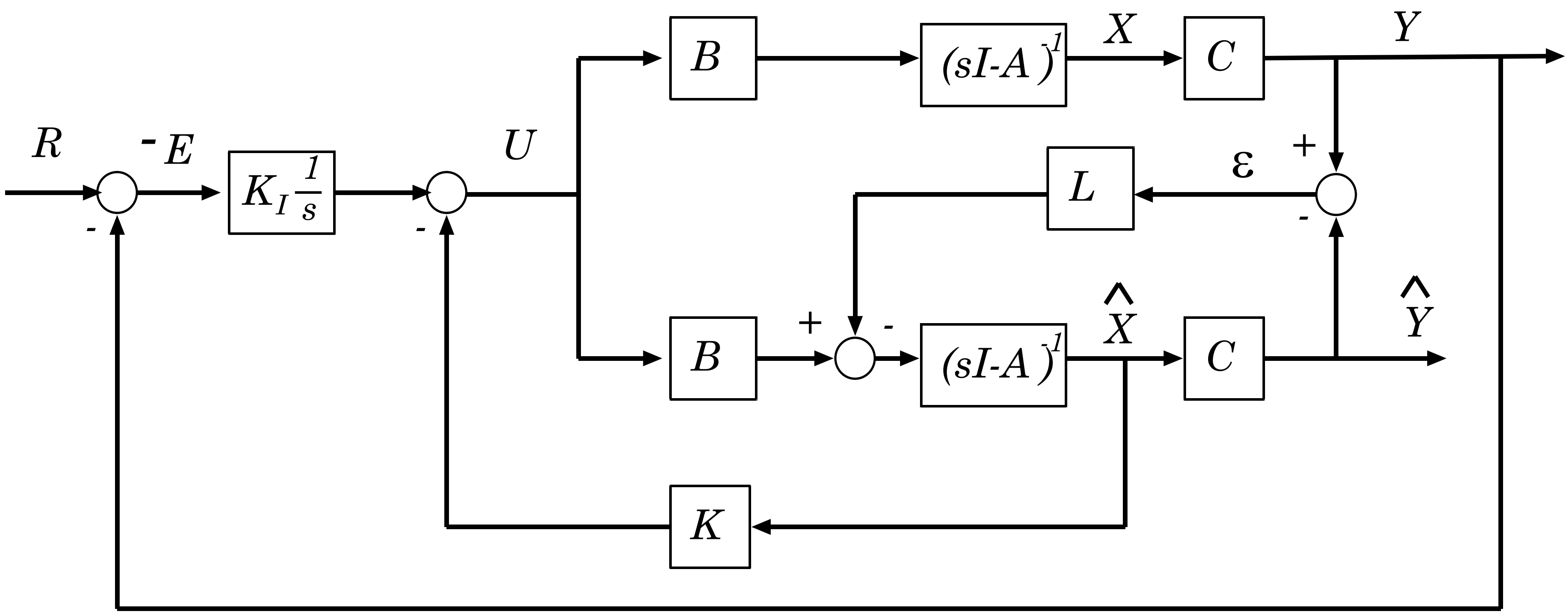
Control objective 2

- Design a state feedback with I-action with closed loop poles at: {-1, -1+j, -1-j}

- Use state observer feedback instead of state variable feedback with observer poles at:
- {-6+6j,-6-6j}.

$$
\begin{aligned}
u &= -K\,\hat{x} + u_I \\
\dot{u}_I &= -K_I\, e \\
e &= y - r
\end{aligned}
\qquad
\begin{aligned}
K &\in \mathcal{R}^{1\times 2} \\
L &\in \mathcal{R}^{2} \\
K_I &\in \mathcal{R}
\end{aligned}
$$

Control objective 2



$$u = -K\,\hat{x} + u_I$$

$$\dot{u}_I = -K_I\,e$$

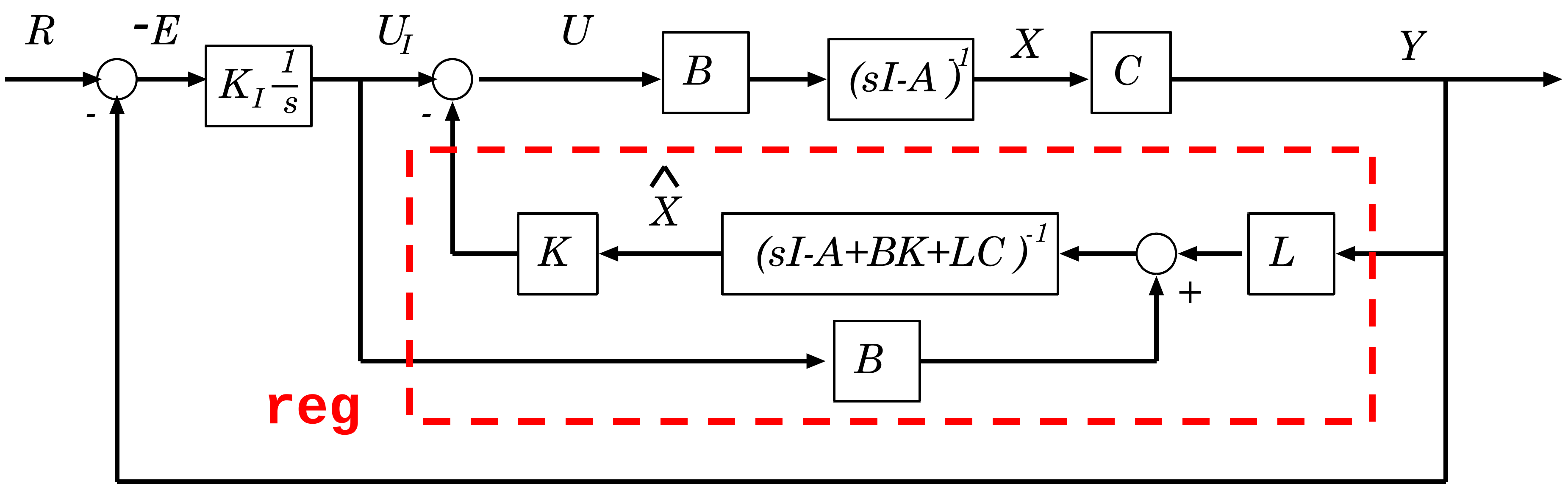
$$e = y - r$$

$$K \in \mathcal{R}^{1\times 2}$$

$$L \in \mathcal{R}^{2}$$

$$K_I \in \mathcal{R}$$

Control objective 2



$$
\begin{aligned}
u &= -K\,\hat{x} + u_I \\
\dot{u}_I &= -K_I\, e \\
e &= y - r
\end{aligned}
\qquad
\begin{aligned}
K &\in \mathcal{R}^{1\times 2} \\
L &\in \mathcal{R}^{2} \\
K_I &\in \mathcal{R}
\end{aligned}
$$

Control design 2

- **Step 1**: Defined the augmented equivalent system to obtain the state and I-action feedback gains:

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}}_{\tilde{A}} \begin{bmatrix} e \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ B \end{bmatrix}}_{\tilde{B}} \tilde{u}$$

$$\dot{y} = \underbrace{\begin{bmatrix} 0 & C \end{bmatrix}}_{\tilde{C}} \underbrace{\begin{bmatrix} e \\ \dot{x} \end{bmatrix}}_{\tilde{x}}$$

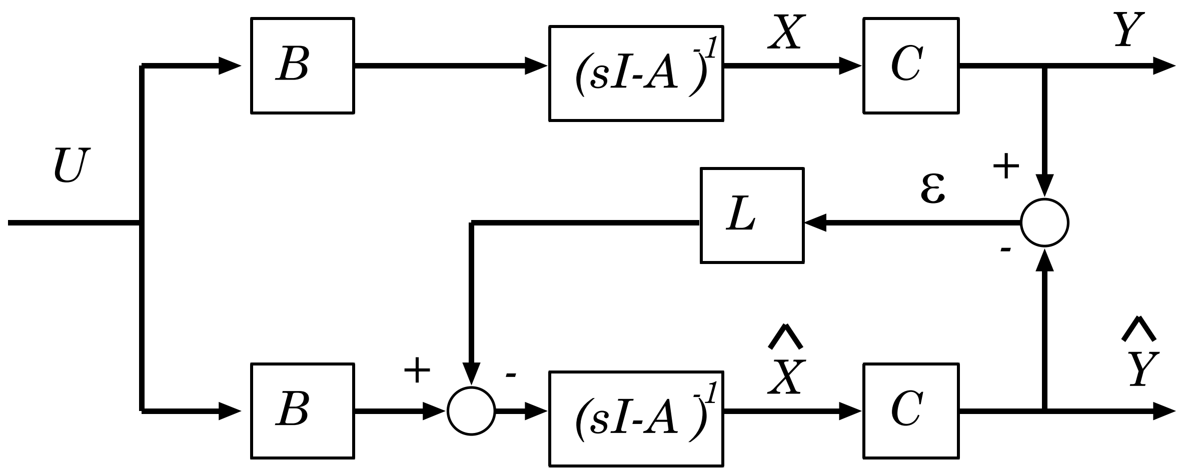
$$\tilde{u} = -\tilde{K}\,\tilde{x}$$

$$\begin{aligned} \tilde{K} &= \begin{bmatrix} K_i & K \end{bmatrix} \\ &= \begin{bmatrix} 2 & 5 & 3 \end{bmatrix} \end{aligned}$$

```
At=[ 0 C ; 0*B A];
Bt=[0;B];
Ct=[0 C];
syst=ss(At,Bt,Ct,D);

% Desired closed
%loop eigenvalues
p=[-1,-1+j,-1-j];
Kt=place(At,Bt,p)
Kt =
2     5     3
```

Control design 2

- **Step 2**: Calculate the observer gain :

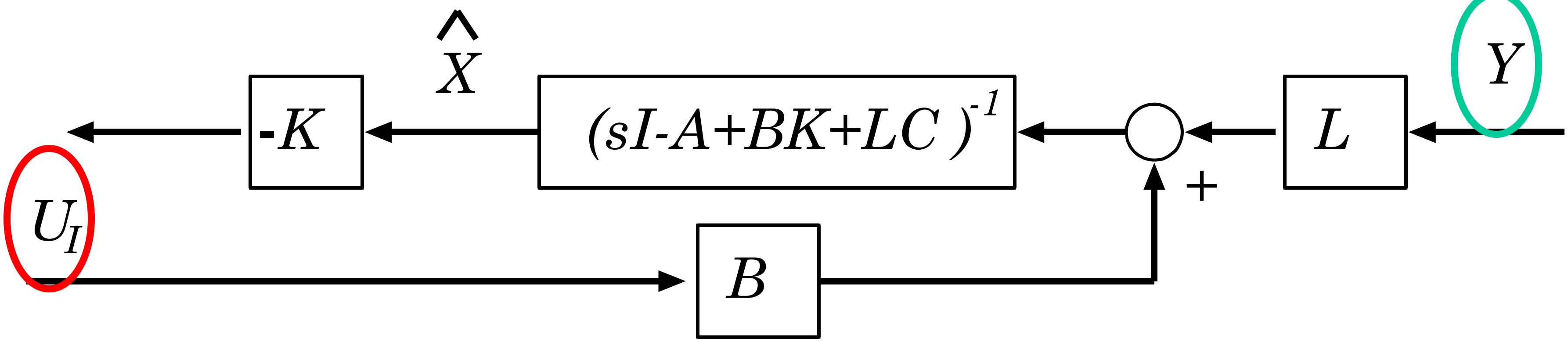



```
% Desired observer eigenvalues
po=[-6+6j,-6-6j];
% Calculate observer gain matrix
L=place(A',C',po)'
```

```
L =

    12

    73
```

Control design 2

- **Step 3**: Define reg block sysreg :

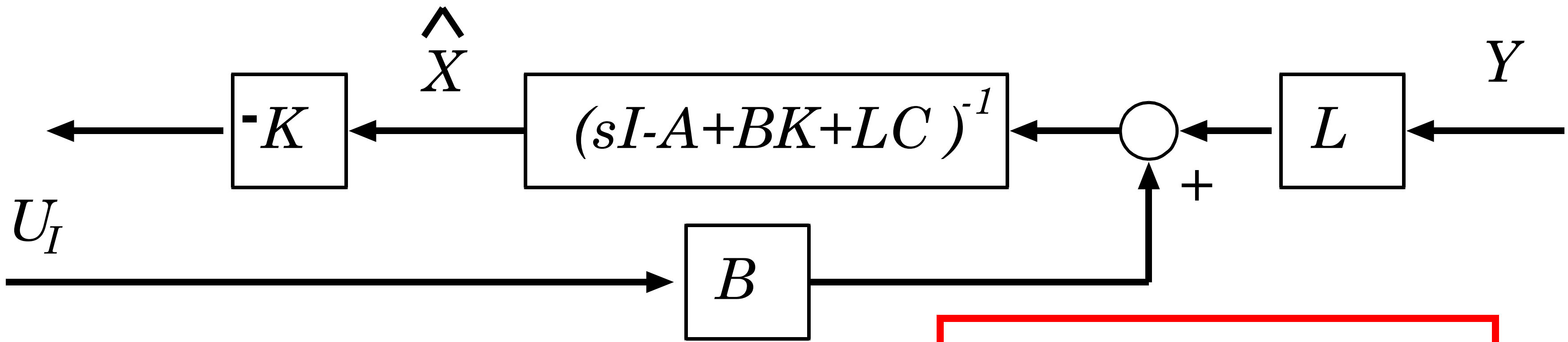



```
% Define regulator block
sysreg

Br=[B ,L]

K=Kt(2:3);
sysreg=ss(A-B*K-L*C,Br,-K,D)
```

```
a =
        x1   x2
  x1  -12    1
  x2  -77   -3

b =
       u1  u2
  x1    0  12
  x2    1  73

c =
        x1  x2
  y1  -5  -3

d =
        u1  u2
  y1    0   0
```

Control design 2

- **Step 3**: Another method to define the reg block sysreg1 :




```
B2=[B,B]
sysp2=ss(A,B2,C,D)
sysreg1 =
reg(sysp2,K,L,1,1,2)
```

a =

	x1	x2
x1	-12	1
x2	-77	-3

c =

	x1	x2
y1	-5	-3

b =

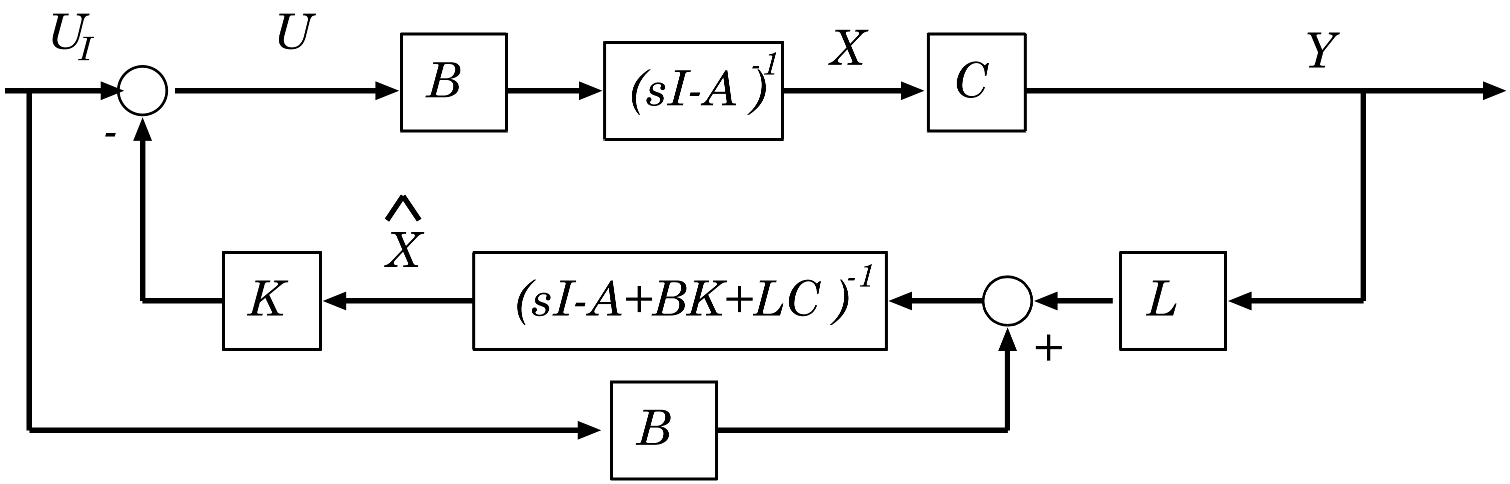
	u1	u2
x1	0	12
x2	1	73

d =

	u1	u2
y1	0	0

Control design 2

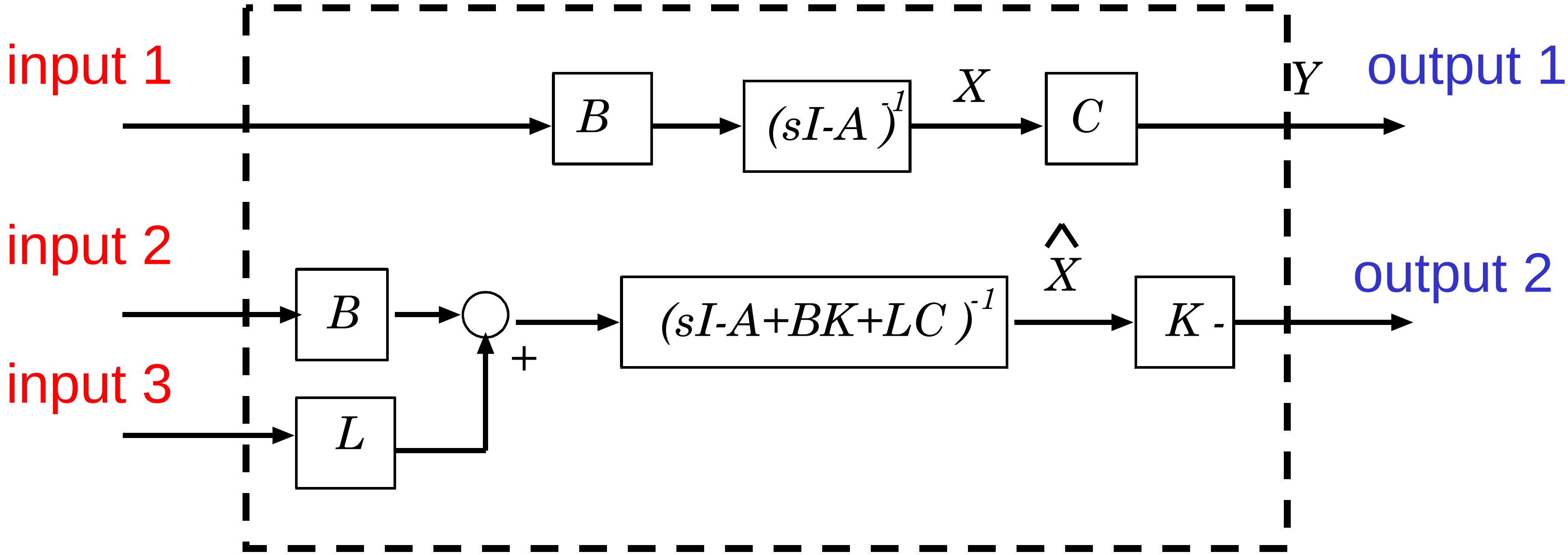
- **Step 4**: Perform state observer feedback syssof:




```
% Build regulator loop
sysrega=append(sysp,sysreg)
Q=[1 2; 3 1]
inputs=[1,2]
outputs=1;
sysregcl=connect(sysrega,Q,inputs,outputs);
syssof=sysregcl*[1;1];
```

Control design 2

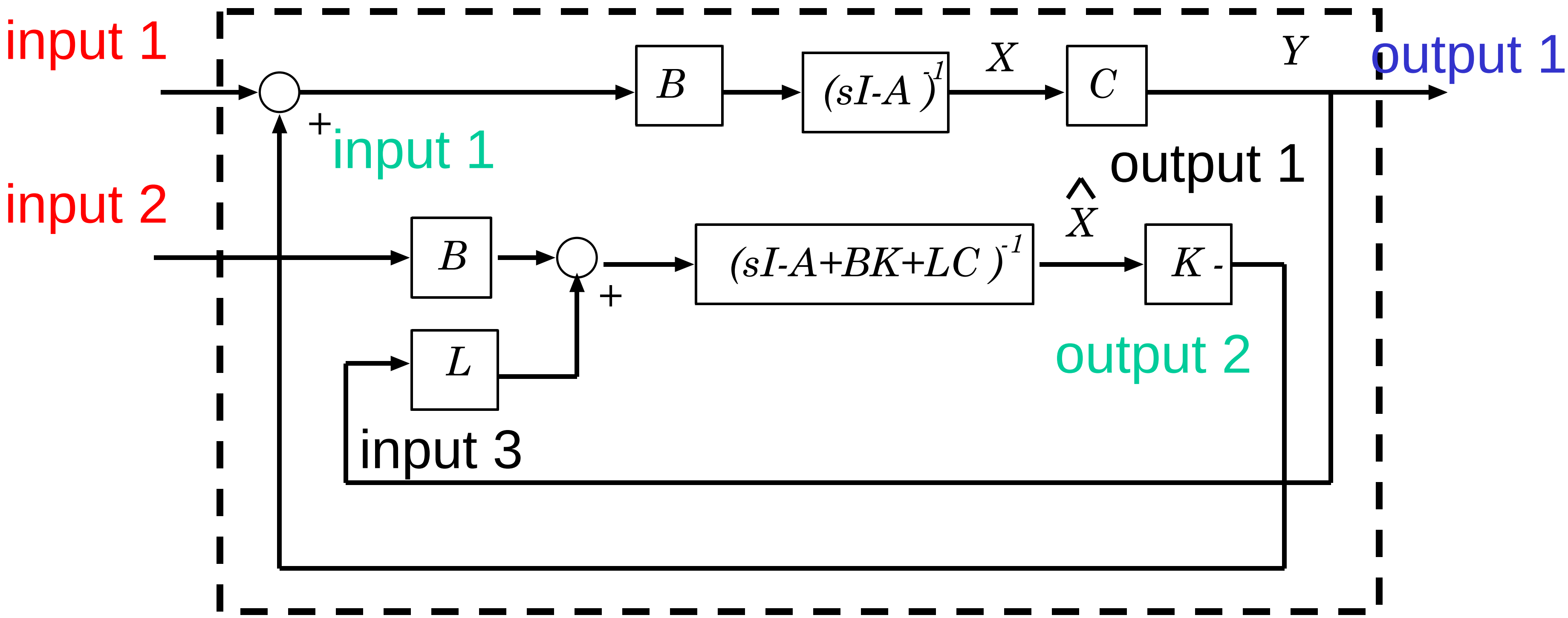
- **Step 4**:




```
% Build regulator loop
sysrega=append(sysp,sysreg)
```

Control design 2

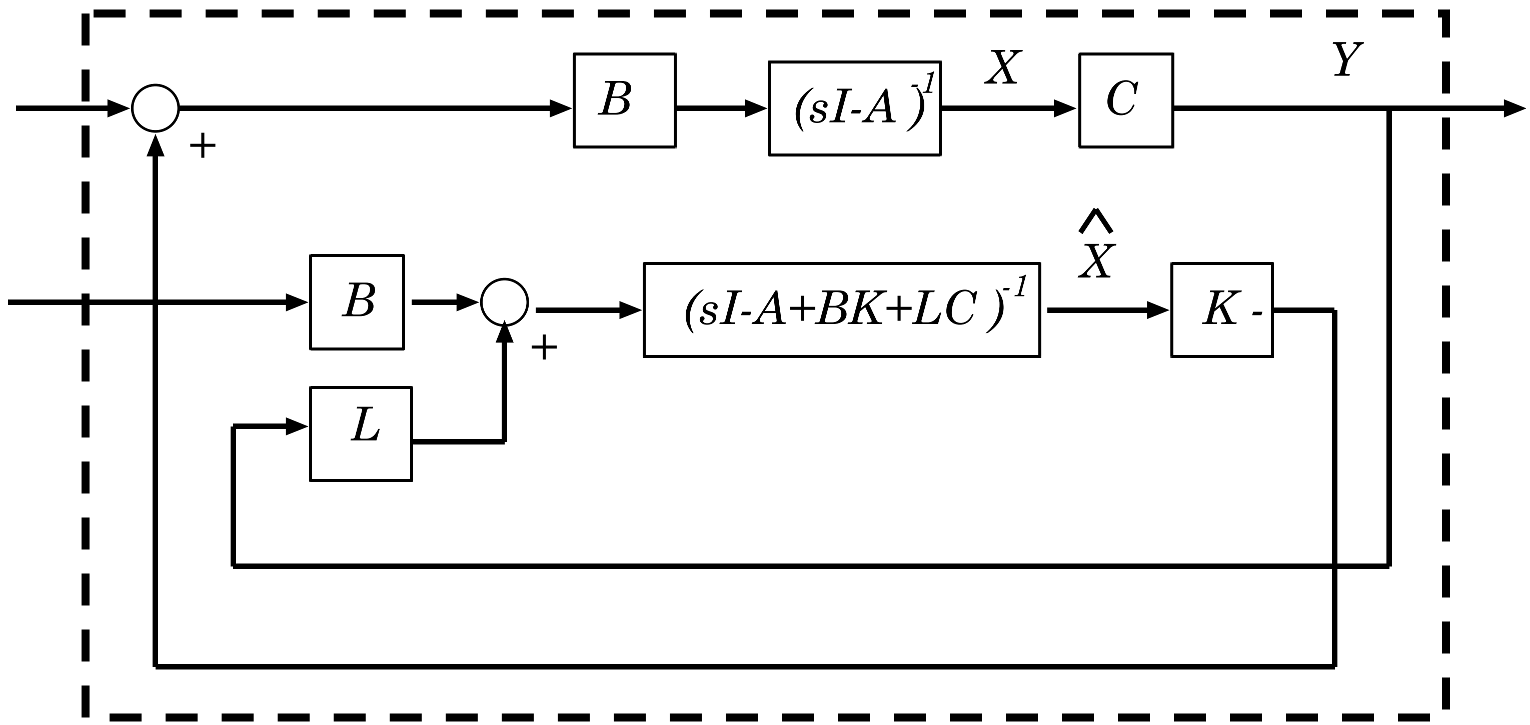
- **Step 4**:




```
inputs=[1,2];    outputs=1;
                                Q=[1 2; 3 1];
sysregcl=connect(sysrega,Q,inputs,outputs);
```

Control design 2

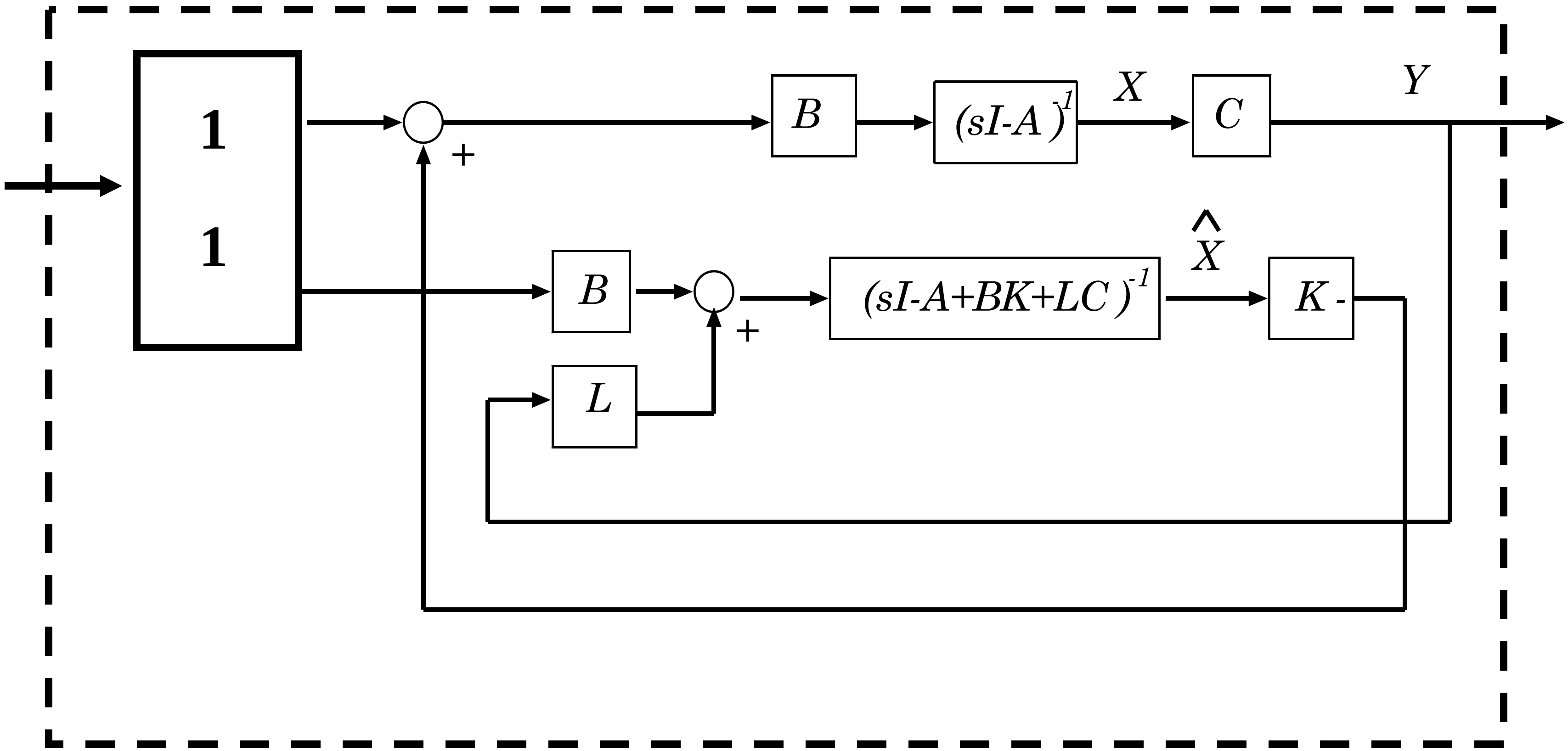
- **Step 4**:




```
inputs=[1,2];    outputs=1;
                              Q=[1 2; 3 1];
sysregcl=connect(sysrega,Q,inputs,outputs);
```

Control design 2

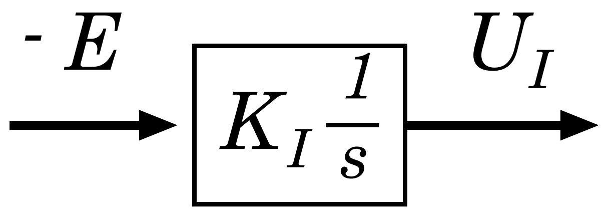
- **Step 4**:




```
syssof=sysregcl*[1;1];
```

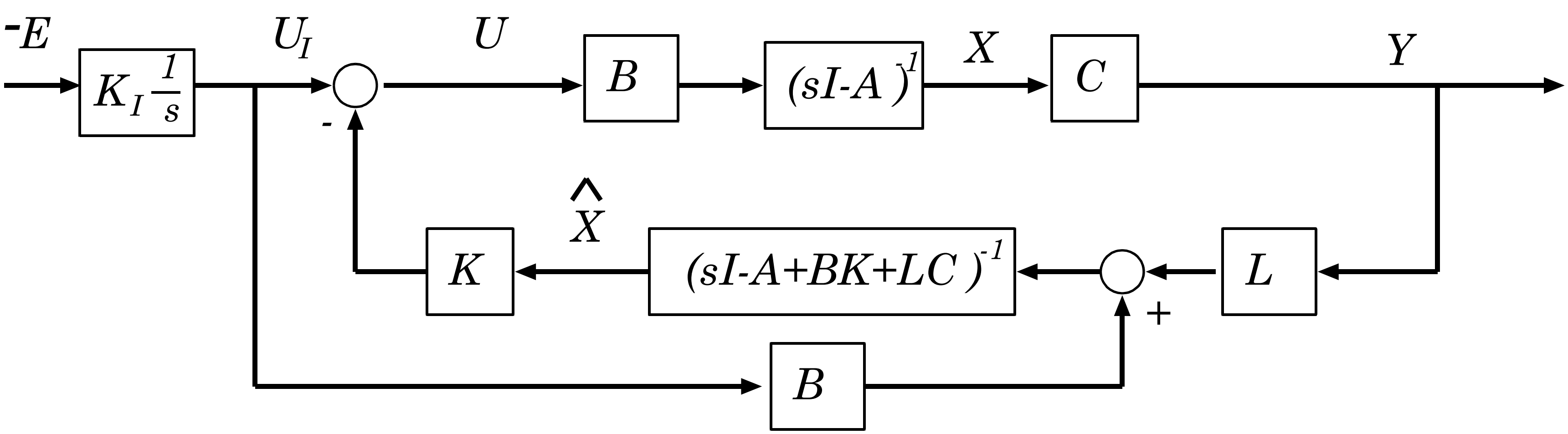
Control design 2

- **Step 5**: Create I-action block sysi:




```
% I-action block sysi2:
sysi2=ss(0,Kt(1,1),1,0);
```

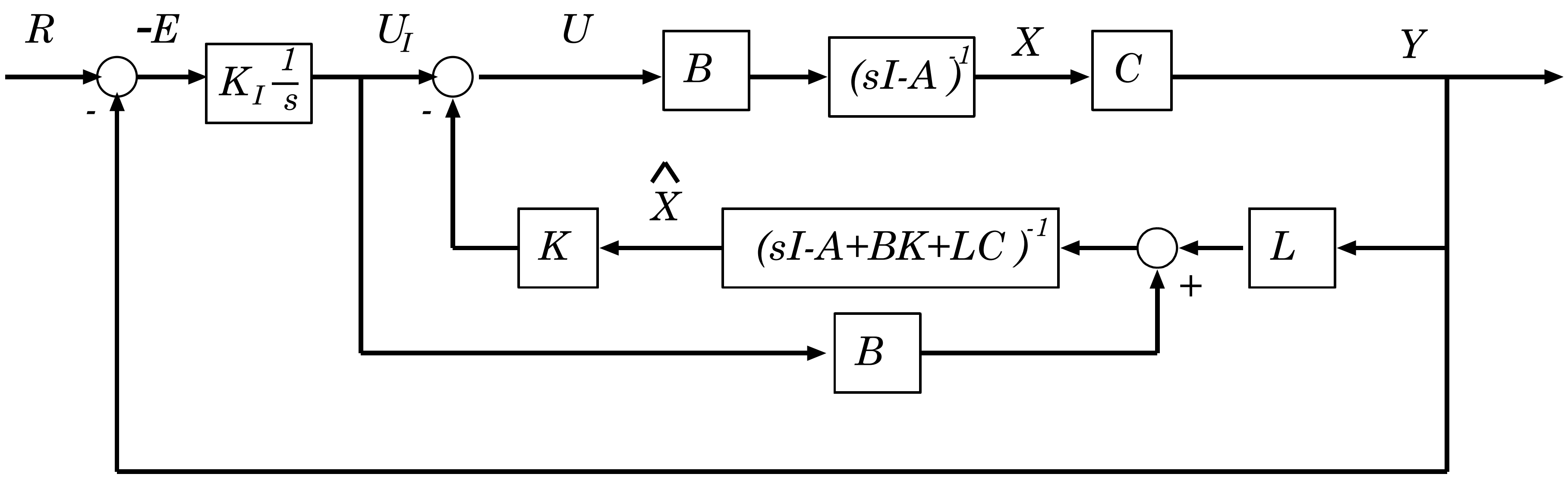
- **Step 6**: Create open-loop system syso2:




```
% open-loop block syso2:
syso2= syssof *sysi2
```

Control design 2

- **Step 7**: Closed-loop syscl2:

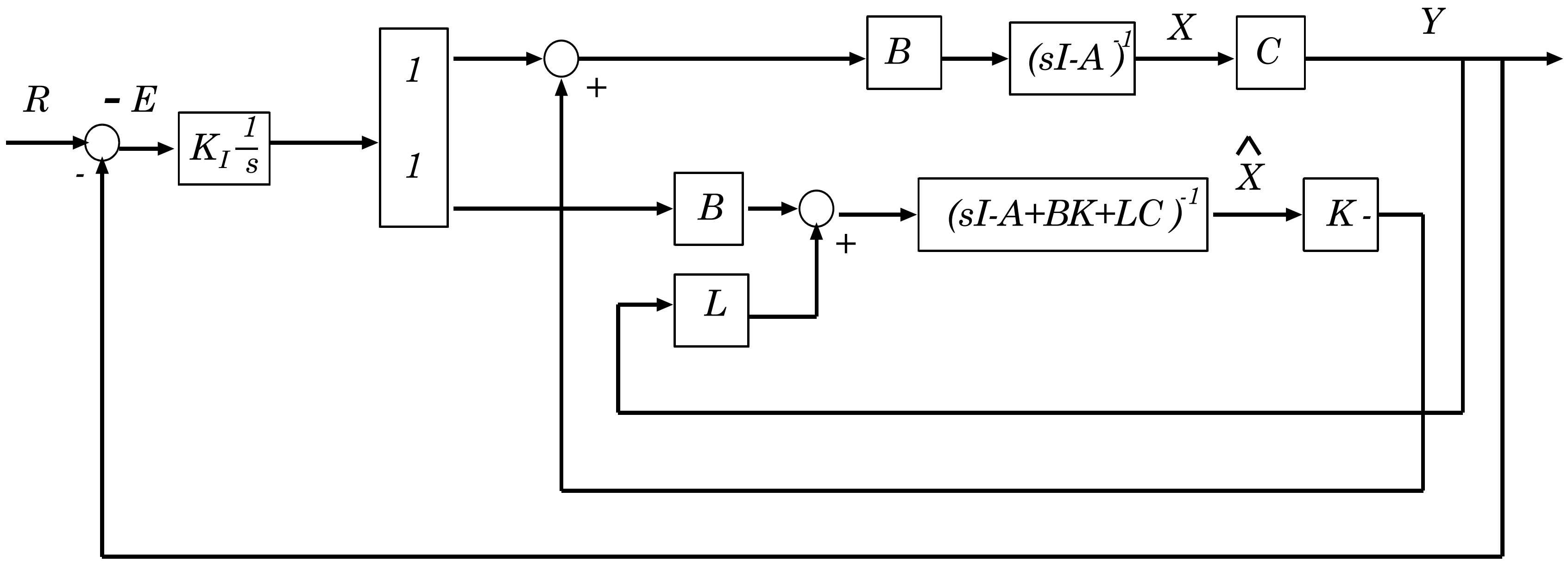



```
% closed-loop system syscl
syscl2=feedback(syso2,1)

% Check closed-loop eigenvalues
[acl2,bcl2,ccl2,dcl2] = ssdata(syscl2);
eig(acl2)
```

```
ans =

-6+ 6i

-6. – 6i

-1+ 1i

-1 – 1i

-1
```

Control design 2

- **Step 7**: Closed-loop syscl2:

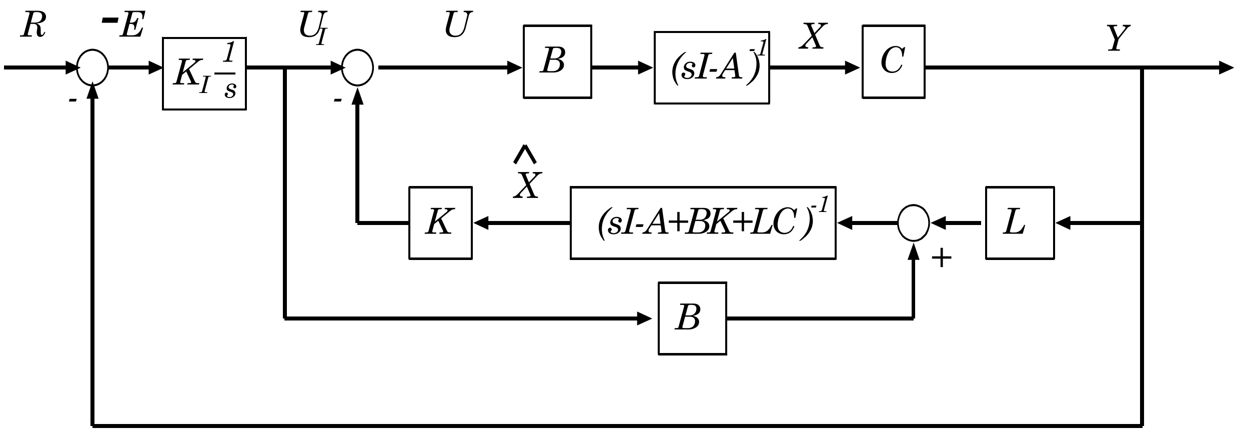



```
% closed-loop system syscl
syscl2=feedback(syso2,1)

% Check closed-loop eigenvalues
[acl2,bcl2,ccl2,dcl2] = ssdata(syscl2);
eig(acl2)
```

```
ans =
-6+ 6i
-6. – 6i
-1+ 1i
-1 – 1i
-1
```

Control design 2

- **Step 8**: Check step response R = unit step:




```
% step response
step(syscl2)
```