HW7: Linear System Theory (ECE532)

Instructor: Jun Moon

Due Date: May 23 (Wed) at the beginning of the class.

Reading Assignment: Read Chapter 6

Problem 1:

Construct minimal realizations of the following transfer functions:

$$G_1(s) = \frac{s-3}{s^2 - 5s + 6}$$
$$G_2(s) = \frac{s^2+1}{s^3 - 2s^2 + s}$$

Problem 2:

Consider

$$\dot{x} = Ax + Bu,\ A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Is the system controllable. If not, find the controllability form. Namely, perform the Kalman decomposition.

Problem 3:

- Consider the controllable form

  $$\dot{x} == Ax + Bu = \begin{bmatrix} A_c & A_{12} \\ 0 & A_{\bar{c}} \end{bmatrix} x + \begin{bmatrix} B_c \\ 0 \end{bmatrix} u$$

  The system or (A, B) is stabilizable if (A_c, B_c) is controllable and $A_{\bar{c}}$ is stable, that is, the eigenvalues of $A_{\bar{c}}$ are in LHS. Show that (A, B) is stabilizable if and only if

  $$rank[A - \lambda I \quad B] = n$$

  for any $\lambda \in \mathbb{C}$.

- Consider the controllable form

  $$\dot{x} == Ax + Bu,\ y = Cx$$
  $$\dot{x} = \begin{bmatrix} A_o & 0 \\ A_{21} & A_{\bar{o}} \end{bmatrix} x + Bu,\ y = \begin{bmatrix} C_o & 0 \end{bmatrix} x$$

The system or (C, A) is detactable if (C_o, A_o) is observable and $A_{\bar{o}}$ is stable, that is, the eigenvalues of $A_{\bar{o}}$ are in LHS. Show that (C, A) is detactable if and only if

$$rank \begin{bmatrix} A - \lambda I \\ C \end{bmatrix} = n$$

for any $\lambda \in \mathbb{C}$.

Problem 4: Construct the minimal realization for

$$G_1(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ \frac{1}{s^2} & \frac{1}{s} \end{bmatrix}, \; G_2(s) = \begin{bmatrix} 0 & \frac{1}{s} \\ \frac{1}{s^2} & \frac{1}{s} \end{bmatrix}.$$

Problem 5:

Suppose that β is a constant. Show that the system

$$\dot{x} = Ax + Bu$$

is controllable if and only if

$$\dot{x} = (A - \beta I)x + Bu$$

is controllable. Also, show that the system

$$\dot{x} = Ax + Bu$$

is controllable if and only if

$$\dot{x} = Ax + BB^T v$$

is controllable for a different input v.