UNIST
Department of Mechanical Engineering

MEN 573: Advanced Control Systems I **Spring, 2016**

Homework #1 Assigned: Saturday, March 12, 2016
Due: Monday, March 21, 2016 (in class)

Problem 1.

Obtain the Laplace transform $F(s)$ for the time functions $f(t)$, or z transforms $F(z)$ for the sequences $f(k)$ given below. Assume that $f(t) = 0$ for $t < 0$ and $f(k) = 0$ for $k < 0$.

(i) $f(t) = \int_0^t e^{-5\tau}\, \sin(3\tau)d\tau$

(ii) $f(t) = t^2\, e^{-2t}$

(iii)

$$f(t) = \begin{cases} e^{(a\,t)} & \text{for} \quad 0 \le t < T \\ 0 & \text{for} \quad t \ge T \end{cases}$$

(iv)

$$f(k) = \begin{cases} 1 & \text{for} \quad \text{even } k \; (k = 0,\, 2,\, 4,\, \cdots) \\ 0 & \text{for} \quad \text{odd } k \; (k = 1,\, 3,\, 5,\, \cdots) \end{cases}$$

(v) $f(k) = f(k - N)$; where $\{f(j) \mid 0 \le j < N\}$ is given. Note that $f(k)$ is a periodic function with the period N.

(vi) The z-transform of $f(k)$ is given by $F(z)$. From $f(k)$, a new sequence $g(k)$ is generated as follows:

$$g(k) = \begin{cases} f(k) & \text{for} \quad k = 4k' \qquad k' = 0, 1, 2, \ldots \\ 0 & \text{for} \quad k \ne 4k' \qquad k' = 0, 1, 2, \ldots \end{cases}$$

Express that the z-transform of $g(k)$, $G(z)$, in terms of $F(z)$.

Problem 2.

The Laplace transform of$f(t)$ is expressed as

$$F(s) = \frac{(K_1 - K_2\,\tau)s + (K_1 - K_2)}{s(\tau s + 1)(s + 1)}$$

(a) Use the initial value and final value theorems to obtain the conditions so that $f(t)$ possesses a negative initial slope (derivative) and a positive final value.

(b) Note that $f(t)$ can be regarded as the unit step response of the system described by the transfer function

$$G(s) = \frac{(K_1 - K_2\tau)s + (K_1 - K_2)}{(\tau s + 1)(s + 1)}$$

Obtain the time plot for $f(t)$ using MATLAB for the following values of the system parameters:

$$K_1 = 2; K_2 = 1; \tau = 4$$

(c) Obtain $f(t)$ by the Laplace inverse transformation for the same parameter values as in b). Sketch the time response $f(t)$ by hand and confirm the MATLAB result. ($G(s)$ represents a reverse reaction process. Notice the presence of a zero in the right half side of s-plane.)

Problem 3.

Given a z transform

$$X(z) = \frac{z^{-1}}{(1 - z^{-1})(1 - 1.4z^{-1} + 0.48z^{-2})}$$

determine the initial and final values of $x(k)$.

(a) Find $x(k) = \mathcal{Z}^{-1}\{X(z)\}$ (the inverse z-transform of $X(z)$) in closed form using partial fractions expansion.

(b) Obtain the partial fractions expansion of $X(z)$ using the matlab command `residue` and verify your answer to part a).

(c) Plot $x(k)$ utilizing the matlab function `impulse`.

Problem 4.

The forced response of second order SISO discrete time system is given by

$$\begin{aligned} y(k) &= g(k) * u(k) = \sum_{j=0}^{k} g(k-j)u(j) \\ Y(z) &= G(z)U(z)\,, \end{aligned}$$

where $y(k)$ is the output and $Y(z) = Z\{y(k)\}$, $u(k)$ is the input and $U(z) = Z\{u(k)\}$, and the transfer function $G(z) = Z\{g(k)\}$ is given by

$$G(z) = \frac{0.8(z-1)}{z^2 + 0.2z - 0.15}\ .$$

(a) Obtain an expression for $g(k)$.

(b) Let

$$p(k) = \sum_{j=0}^{k} g(j)\,.$$

Using the final value theorem, compute

$$p_{ss} = \lim_{k \to \infty} p(k)\,.$$