HW4: Linear System Theory (ECE532)

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Due Date: April 11 at the beginning of the class

Problem 1. Problem 4.3 of the textbook

Problem 2.
Consider the following matrix

$$A_1 = \begin{pmatrix} 1/2 & -1/2 \\ 1/2 & -1/2 \end{pmatrix},\ A_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ A_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ -3 & 0 & -2 \end{pmatrix}$$

- Compute $e^{A_i t}$ for $i = 1, 2, 3$. Check your answer via MATLAB expm
- In each case, determine whether the solution of $\dot{x} = A_i x$ with $x(0) = x_0$ decay to 0, say bounded or go to ∞ for various choices of initial conditions
- Try to state the general rule which can be used to determine, by looking at the eigenstructure of A, whether the solutions of $\dot{x} = Ax$ decay to 0, stay bounded, or go to 1.

Problem 3. If A and B are constant square matrix, show that the state transition matrix for the time varying system described by

$$\dot{x}(t) = e^{-At} B e^{At} x(t)$$

is

$$\Phi(t, x) = e^{-At} e^{(A+B)(t-s)} e^{As}$$

Problem 4. If A is a square matrix, show that

$$\int_0^t e^{A\sigma} d\sigma = A^{-1}[e^{At} - I]$$

Now using this result, obtain the solution to the linear time-invariant system

$$\dot{x} = Ax + B\bar{u},$$

where $\bar{u}$ is a constant r-dimensional vector, B is $n \times r$, and A is $n \times n$