UNIST
Department of Mechanical Engineering

MEN 573: Advanced Control Systems I		**Spring, 2016**
Homework #8	Assigned: Monday, May 16, 2016 Due: Wednesday, May 25, 2016 (in class)	

Problem 1.

Consider the LTI discrete time system

$$\begin{aligned}\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} &= \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x(k)\end{aligned}$$

(a) Is it possible to determine the initial condition $x(0)$ from the sequence of outputs $y(0)$, $y(1)$ and $y(2)$, and inputs $u(0)$, $u(1)$ and $u(2)$?

(b) If not, determine the Span of the set of vectors $v \in \mathcal{R}^3$ such that any two initial conditions which satisfy

$$x_1(0) = x_2(0) + v$$

cannot be uniquely determine from each other.

Problem 2.

Consider the transfer function

$$G(z) = \frac{B(z)}{A(z)} = \frac{2z^2 - 0.2z - 0.12}{z^3 + 0.8z^2 + 0.17z + 0.01} \tag{1}$$

(a) Obtain the controllable canonical state space realization of $G(z)$. We will call these matrices A_c, B_c, and C_c. Determine the observability of the pair $\{A_c, C_c\}$.

(b) Obtain the observable canonical state space realization of $G(z)$. We will call these matrices A_o, B_o, and C_o. Determine the controllability of the pair $\{A_o, B_o\}$.

(c) Use matlab to find the roots of the polynomials $B(z)$ and $A(z)$ and cancel common zeros and poles from the transfer function $G(z)$. Then, repeat parts a) and b) using the simplified transfer function.

Problem 3.

Given the LTI system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} -1.1 & -0.34 & -0.32 \\ 1 & 0 & 0 \\ 0 & 0.25 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} u(k)$$

(a) Determine if the initial state $x(0) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ can be transferred to the origin in 5 steps.

(b) If your answer to part a) is yes, determine the control sequence $\{u(k)\ ,\ k = 0, 1, 2, 3, 4\}$ that transfers $x(0) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ to $x(5) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$.

Problem 4.

Consider an LTI system under state variable feedback control. Assume that the n-th order system

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) \\ y(k) &= C\,x(k) \end{aligned}$$

where $y(k) \in \mathcal{R}$ and $u(k) \in \mathcal{R}$, is both controllable and observable. Lets use the control law

$$u(k) = -K\,x(k) + v(k)$$

where $K \in \mathcal{R}^{1\times n}$ is a constant gain matrix and $v(k)$ is the new input. The close loop system is now

$$\begin{aligned} x(k+1) &= A_c\,x(k) + B\,v(k) \\ y(k) &= C\,x(k) \end{aligned}$$

where $A_c = A - B\,K$.

(a) Show that this system is also controllable. I.e., for any initial condition $x(0)$ and target state x_1 there exists some finite integer N and control sequence $\{v(k),\ k = 0, \cdots N$ which will achieve $x(N) = x_1$. (Hint: consider the control $v(k) = u(k) + Kx(k)$.)

Problem 5.

Consider n_1 -th and n_2 -th order SISO LTI systems

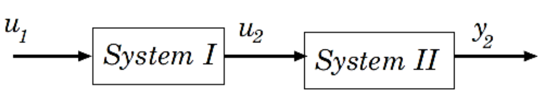
System I:

$$\begin{aligned} \dot{x}_1 &= A_1 x_1 + B_1 u_1 \\ y_1 &= C_1 x_1 + D_1 u_1 \qquad G_1(s) = C_1(sI - A_1)B_1 + D_1 = \frac{B_1(s)}{A_1(s)} \end{aligned}$$

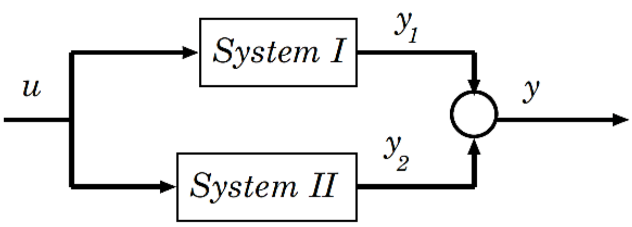
System II:

$$\begin{aligned} \dot{x}_2 &= A_2 x_2 + B_2 u_2 \\ y_2 &= C_2 x_2 + D_2 u_2 \qquad G_2(s) = C_2(sI - A_2)B_2 + D_2 = \frac{B_2(s)}{A_2(s)} \end{aligned}$$

and assume that both systems are controllable and observable.



Series Combination



Parallel Combination

(a) State the condition(s) so that the series combination of the two systems is controllable and observable.

(b) Assume now that the systems are connected in parallel. State the condition(s) so that the parallel combinations of the two systems is controllable and observable.

Problem 6.

Consider the following third order LTI system

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 4 & 1 & 6 \\ -3 & 0 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(a) Determine the controllable subspace $\mathcal{X}_c$.

(b) Determine the unobservable subspace $\mathcal{X}_{uo}$.

(c) Determine if this system is stabilizable. [1]

(d) Determine if the system is detectable. [2]

(e) Using `matlab`, calculate the transfer function $G(s) = \frac{Y(s)}{U(s)}$ and determine its poles and zeros.