HW1: Linear System Theory (ECE532)

Instructor: Jun Moon

Due Date: March 7 at the beginning of the class.

Reading Assignment: Read Chapters 1 and 2 of the textbook.

Note: You must use LaTeXto write your homework.

The following problems are related to mathematical analysis and linear algebra:

Problem 1

A sequence (s_n) (or $\{s_n\}$) of real numbers, where n is a natural number, i.e., $n \in \mathbb{N}$, is said to *converge* to the real number $s \in \mathbb{R}$, provided that

for each $\epsilon > 0$, there exists a number N such that $n > N$ implies $|s_n - s| < \epsilon$.

If (s_n) converges to s, we write $\lim_{n\to\infty} s_n = s$ or $s_n \to s$. The number s is called the limit of the sequence (s_n). A sequence that does not converge to some real number is said to *diverge*. A sequence (s_n) of real numbers is called nondecreasing (resp. nonincreasing) if $s_n \leq s_{n+1}$ (resp. $s_n \geq s_{n+1}$) for all n. A sequence that is nondecreasing or nonincreasing will be called a *monotonic sequence*. A sequence (s_n) is bounded if there exists a real number M such that $|s_n| \leq M$ for all n.

- By using the definition, prove that $\lim_{n\to\infty} \frac{4n^3+3n}{n^3-6} = 4$ and $s_n = (-1)^n$ does not converge.
- Show that all convergent sequences are bounded.
- Show that all bounded monotonic sequences converge.
- Let $s_1 = 1$ and $s_{n+1} = \frac{1}{3}(s_n + 1)$ for $n \leq 1$.
  - Find s_2, s_3 and s_4
  - Use induction to show that $s_n \geq \frac{1}{2}$ for all n
  - Show that (s_n) is a nondecreasing sequence
  - Show that the limit exists and find its limit.

Problem 2

Let f be a real-value function whose domain is a subset of the set of real numbers, $\mathbb{R}$, that is, $f : \mathrm{dom}(f) \to \mathbb{R}$ where $\mathrm{dom}(f) \subset \mathbb{R}$. The function f is *continuous at* $x_0 \in dom(f)$ if for every sequence $(x_n) \in \mathrm{dom}(f)$ converging to x_0, ($\lim_{n\to\infty} x_n = x_0$), we have $\lim_{n\to\infty} f(x_n) = f(x_0)$. The function is said to be *continuous* if it is continuous on every points in $\mathrm{dom}(f)$.

- Show that f is continuous at $x_0 \in \mathrm{dom}(f)$ *if and only if*

$$\text{for each } \epsilon > 0, \text{ there exists } \delta > 0 \text{ such that}$$
$$x \in \mathrm{dom}(f) \ \& \ |x - x_0| < \delta \text{ imply } |f(x) - f(x_0)| < \epsilon.$$

- Is $f(x) = x^2 \sin(\frac{1}{x})$ for $x \neq 0$ and $f(0) = 0$ continuous at 0?
- Is $f(x) = \frac{1}{x} \sin(\frac{1}{x^2})$ for $x \neq 0$ and $f(0) = 0$ continuous at 0?
- Let f be a continuous real-valued function whose domain is $[a, b] \subset \mathbb{R}$, where $[a, b]$ is a closed interval. Show that there exist $x_0, y_0 \in [a, b]$, such that $f(x_0) \leq f(x) \leq f(y_0)$ for all $x \in [a, b]$.

Problem 3

Let A be an $n \times n$ real-valued square matrix. Let $\det(A)$ be determinant of A, and $\mathrm{trace}(A)$ be trace of A. A square symmetric matrix A is positive definite if $x^\top Ax > 0$ for all $x \in \mathbb{R}^n$.

- A is nonsingular if $\det(A) \neq 0$. Show that A is invertible *if and only if* A is nonsingular.
- Show that $Ax = b$ has a unique solution for each $b \in \mathbb{R}^n$ if and only if A is nonsingular.
- Show that $\det(A) = \prod \lambda_i$ where λ_i is an eigenvalue of A
- Show that $\mathrm{trace}(A) = \sum \lambda_i$ where λ_i is an eigenvalue of A.
- Show that if A is positive definite, then each eigenvalue is a positive real number.

The following problems are related to undergraduate control theory. You need to use MATLAB to solve the problem.

Problem 4

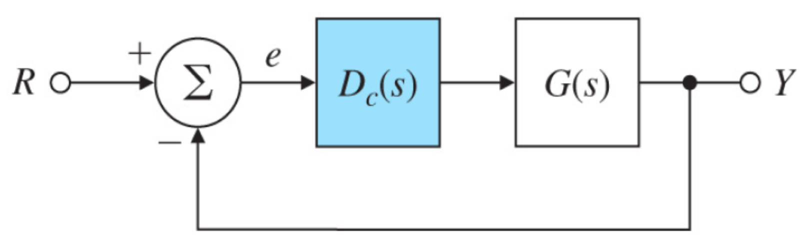

Figure 1: Block diagram of a standard feedback control system, where D_c is the compensator.

A DC motor with negligible armature inductance is to be used in a position control system. Its open-loop transfer function is given by

$$G(x) = \frac{50}{s(s/5 + 1)}$$

Use Bode plot to design a compensator for the feedback motor control system satisfies the following specifications:

- The steady state error to a unit-RAMP input is less than 1/200
- The unit-STEP response has an overshoot of less than 20%
- The bandwidth of the compensated feedback system is no less than that of the uncompensated open-loop system.

Verify your design using MATLAB. You must provide the time response plots.

Problem 5

Consider the feedback system in Figure 1 with the following system:

$$G(s) = \frac{2500K}{s(s+25)}.$$

- Design a lead compensator so that the phase margin of the system is more than 45°; the steady state error due to a unit RAMP should be less than or equal to 0.01. Verify your answer with the Bode plot and the time response via the Simulink.
- Using the plant transfer function from part (a), design a lead compensator so that the overshoot is less than 25% and the 2% settling time less than 0.1 sec. Verify your answer with the unit STEP response (MATLAB).