HW6: Linear System Theory (ECE532)

Instructor: Jun Moon

Due Date: May 15 (Wed) at the beginning of the class.

Reading Assignment: Read Chapter 6

Problem 1:

Consider the controllability of the linear system discussed in class. Show that

- Controllability is equivalent to transferring any initial state to the origin (controllability is equivalent to controllability to the origin).
- Controllability is equivalent to transferring the origin to any final state (controllability is equivalent to reachability).

Problem 2:

Investigate the controllability of the LTI model $\dot{x} = Ax + Bu$, where

$$\text{(a)} \ \begin{pmatrix} -5 & 1 \\ 0 & 4 \end{pmatrix}, \ B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{(b)} \ \begin{pmatrix} 3 & 3 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Problem 3:

Consider the system

$$\dot{x} = -x + u, \ x \in \mathbb{R}$$

and the problem of steering from $x = 0$ at time 0 to $x = 1$ at some given time t.

(a) Since the system is controllable (why???), we know that this transfer is possible for every value of t. Verify this by giving an explicit formula for a control that solves the problem.

(b) Is the control you obtained in part (a) unique? If yes, prove it; if not, find another control that achieves the transfer (in the same time t).

(c) Now, suppose that the control values must satisfy the constraint $|u| \leq 1$ at all times. Is the above problem still solvable for every t? for at least some t? Prove or disprove.

(d) Answer the same question as in part (c) but for the system $\dot{x} = x + u$ (with $|u| \leq 1$ at all times).

Problem 4:

Consider the system $\dot{x} = Ax + Bu$ with

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(a) Verify its controllability by using 1) controllability matrix and 2) Hautus-Rosenbrock Test.

(b) Make the system not controllable by changing exactly one element of A.

Problem 5:

Consider the system $\dot{x} = Ax + Bu$ with

$$A = \begin{pmatrix} -1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \; B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Compute its Kalman decomposition. Identify controllable and uncontrollable modes.

Problem 6:

1. Problem 6.16 of the textbook.
2. Problem 6.19 of the textbook.