Linear System Theory

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Chapter 6: Controllability & Observability

Chapter 7: Minimal Realizations

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Recap

- State space equation
- Linear Algebra
- Solutions of LTI and LTV system
- Stability

We will study

- Controllability & Observability
- Kalman Decomposition
- Minimal realizations

Controllability & Observability

The first chapter dealing with the control input and output variable in the system

$$\dot{x} = Ax + Bu, \;\; y = Cv$$

- x: state
- u: control
- y: output

Controllability & Observability

Controllability (informal): we want to know whether the state of the system is controllable or not from the input

- Analyze the system structure from the input
- With the input, we want to move the state to the desired point in a finite time.

Controllability & Observability

Observability (informal): we want to observe the initial state of the system from the output and input to quantify the behavior of the system

- State: position, velocity, acceleration, etc
- Sensors are required to measure the state. We are not able to use many sensors in real applications.

Controllability & Observability

Controllability & Observability

- Important concepts in control, estimation, and filtering problems
- Optimal control (LQG, Kalman filtering, etc.)

Controllability & Observability

Example

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix} x + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} u, \;\; x = (x_1 \;\; x_2)^T$$

- $(b_1, b_2)^T = (-1, 1)^T$: can move both eigenvalues $\Leftrightarrow$ can control the state x_1 and x_2
- $(b_1, b_2)^T = (1, 0)^T$: cannot move the eigenvalue 3 $\Leftrightarrow$ cannot control state x_2
- $(b_1, b_2)^T = (1, 0)^T$: No matter input, x_2 diverges $\Leftrightarrow$ we cannot control x_2

Controllability & Observability

Example

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix} x, \;\; y = \begin{pmatrix} c_1 & c_2 \end{pmatrix} x$$

- $(c_1, c_2) = (1, 1)$: can observe the state x_1 and x_2
- $(c_1, c_2) = (1, 0)$: cannot observe the state x_2
- $(c_1, c_2) = (1, 0)$: Output is always stable, but the system is internally unstable

Controllability

$$\dot{x}(t) = Ax(t) + Bu(t),\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m$$

Definition (Definition 6.1)

The state equation with the pair (A, B) is said to be controllable if for any initial state $x(0) = x_0$, any final state x_1, there exists an input that transfers x_0 to x_1 in a finite time.

Controllability

Equivalent Definition:
A system is controllable at time t_0 if there exists a finite time t_f such that for any initial condition x_0, and any final state x_f, there is a control input u defined on $[t_0, t_f]$ such that $x(t_f) = x_f$.

- We need an input u to transfer the state from the initial to the final state
- Given initial and finial state conditions in $\mathbb{R}^n$, is it possible to steer $x(t)$ to the final state by choosing an appropriate input $u(t)$?

Controllability: A Preview

Discrete-time LTI system

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k),\; x(0) = 0,\; x \in \mathbb{R}^n,\; u \in \mathbb{R}^m \\
x(1) &= Bu(0) \\
x(2) &= Ax(1) + Bu(1) = ABu(0) + Bu(1) \\
x(3) &= A^2Bu(0) + ABu(1) + Bu(2) \\
&\vdots \\
x(r) &= A^{r-1}Bu(0) + A^{r-1}Bu(1) + \cdots + Bu(r-1) \\
x(r) &= \begin{pmatrix} B & AB & \cdots A^{r-1}B \end{pmatrix} \begin{pmatrix} u(r-1) \\ u(r-2) \\ \vdots \\ u(0) \end{pmatrix}
\end{aligned}
$$

Controllability: A Preview

$$x(r) = \begin{pmatrix} B & AB & \cdots A^{r-1}B \end{pmatrix} \begin{pmatrix} u(r-1) \\ u(r-2) \\ \vdots \\ u(0) \end{pmatrix}$$

$$R((B\ AB\ \cdots\ A^{r-1}B)) = \{z \in \mathbb{R}^n,\ z = (B\ AB\ \cdots\ A^{r-1}B)p,\ p \in \mathbb{R}^{nm}\}$$

If $x_f \in R((B\ AB\ \cdots\ A^{r-1}B))$, then x_f is reachable

Controllability: A Preview

This implies that we can reach arbitrary $x_f \in \mathbb{R}^n$ at time $t_f = r$ if and only if $R((B \ AB \ \cdots \ A^{r-1}B)) = \mathbb{R}^n$ that is equivalent to $rank((B \ AB \ \cdots \ A^{r-1}B)) = n$

Rank of $(B \ AB \ \cdots \ A^{r-1}B))$

- By C-H theorem, A^k is a linear combination of $\{I, A, \ldots, A^{n-1}\}$
- For $r \geq n$, the rank of $(B \ AB \ \cdots \ A^{r-1}B))$ cannot increase

Hence, if $rank((B \ AB \ \cdots \ A^{n-1}B)) = n$, then we can find u for an arbitrary $x_f \in \mathbb{R}^n$ for any finite time

Controllability

$$\dot{x}(t) = Ax(t) + Bu(t), \;\; x \in \mathbb{R}^n$$
$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

The system is controllable (page 213 of the textbook)

- $\Leftrightarrow$ for any x_0, there exists $u(t)$ on $[t_0, t_f]$ that transfers x_0 to the origin at t_f (controllability to the origin)
- $\Leftrightarrow$ there exists $u(t)$ on $[t_0, t_f]$ that transfers state from the origin to any final state x_f at t_f (reachability)

Proof: Exercise!! (note that $e^{A(t-t_0)}$ is always invertible!)

Basically, we need the surjectivity of $\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$

Controllability

$$\dot{x}(t) = Ax(t) + Bu(t),\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ x_0 = 0$$

- Set of reachable state for a fixed time t:

$$\mathcal{R}_t = \{\xi \in \mathbb{R}^n,\ \text{there exists } u \text{ such that } x(t) = \xi\}$$

Note that $\mathcal{R}_t$ is a subspace of $\mathbb{R}^n$

Controllability

- Controllability matrix and controllability subspace

$$\mathcal{C}_{AB} = \{\xi \in \mathbb{R}^n : \ \xi = \begin{pmatrix} B & AB & \cdots & A^{n-1}B \end{pmatrix} z, \ z \in \mathbb{R}^{nm}\}$$

$\mathcal{C}_{AB}$: range space of $\mathcal{C}$, where $\mathcal{C} = (B \ AB \ \cdots \ A^{n-1}B) \in \mathbb{R}^{n \times nm}$

Controllability

- Controllability Gramian

$$W_t = \int_0^t e^{A(t-\tau)} BB^T e^{A^T(t-\tau)} d\tau = \int_0^t e^{A\tau} BB^T e^{A^T\tau} d\tau \geq 0$$

$R(W_t)$: the range space of W_t, W_t is a symmetric positive semi-definite matrix

Controllability

Theorem: Controllability (Theorem 6.1 of the textbook)
For each time $t > 0$, the following set equality holds:

$$\mathcal{R}_t = \mathcal{C}_{AB} = R(W_t).$$

Controllability

- $\mathcal{C} = (B \ AB \ \cdots \ A^{n-1}B)$: controllability matrix
- Hence if $\dim \mathcal{C}_{AB} = rank((B \ AB \ \cdots \ A^{n-1}B)) = n$, the system is controllable
- Due to $\mathcal{C}_{AB}$, the controllability is independent of the time
- If the system is controllable, then $\mathcal{R}_t = \mathbb{R}^n$, all the states are reachable by an appropriate choice of the control u

We will show that

- $\mathcal{R}_t \subset \mathcal{C}_{AB}$, $\mathcal{C}_{AB} \subset R(W_t)$, $R(W_t) \subset \mathcal{R}_t$

Required tools

- C-H theorem (Chapter 3), $R(A^T) = (N(A))^\perp$: Problem 1 in HW3

Controllability

Theorem: $\mathcal{R}_t \subset \mathcal{C}_{AB}$

Proof:
Fix $t > 0$, and choose any reachable state $\xi \in \mathcal{R}_t$. We need to show that $\xi \in \mathcal{R}_t$ implies $\xi \in \mathcal{C}_{AB}$.

We have $\xi \in \mathcal{R}_t$, which implies $\xi = \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau$. Then by C-H theorem, $e^{At} = \beta_0(t) I + \cdots + \beta_{n-1}(t) A^{n-1}$ ($\beta_i(t)$: scalar function).

Controllability

Hence

$$
\begin{aligned}
\xi =& B\int_0^t \beta_0(t-\tau)u(\tau)d\tau + \cdots + A^{n-1}B\int_0^t \beta_{n-1}(t-\tau)u(\tau)d\tau \\
=& \begin{pmatrix} B & AB & \cdots & A^{n-1}B \end{pmatrix} \underbrace{\begin{pmatrix} \int_0^t \beta_0(t-\tau)u(\tau)d\tau \\ \vdots \\ \int_0^t \beta_{n-1}(t-\tau)u(\tau)d\tau \end{pmatrix}}_{\in \mathbb{R}^{nm}}
\end{aligned}
$$

Hence, $\xi \in \mathcal{C}_{AB}$

Controllability

Theorem: $\mathcal{C}_{AB} \subset R(W_t)$

Proof:

Since $\mathcal{C}_{AB} \subset R(W_t)$ is equivalent to $\mathcal{C}_{AB}^{\perp} \supset R(W_t)^{\perp}$ (proof: exercise!!), we will show that $\mathcal{C}_{AB}^{\perp} \supset R(W_t)^{\perp}$.

From Problem 1 in HW3, $R(W_t) = (N(W_t))^{\perp}$, which is equivalent to $(R(W_t))^{\perp} = N(W_t)$, and similarly, $\mathcal{C}_{AB}^{\perp} = N((B \ AB \ \cdots \ A^{n-1}B))$.

Hence we need to show that if $\xi \in N(W_t)$, then $\xi \in N((B \ AB \ \cdots \ A^{n-1}B))$.

Controllability

Let $\xi \in N(W_t)$, then $W_t\xi = 0 \in \mathbb{R}^n$, which also implies $\xi^T W_t \xi = 0 \in \mathbb{R}$. Then

$$0 = \xi^T \int_0^t e^{A\tau} B B^T e^{A^T\tau} d\tau \xi = \int_0^t \|B^T e^{A^T\tau}\xi\|^2 d\tau \Leftrightarrow B^T e^{A^T\tau}\xi = 0, \ \forall \tau \in [0, t]$$

Since $y(\tau) = \xi^T e^{A\tau} B = 0$, $\forall \tau \in [0, t]$, we have

$$\xi^T \left(\frac{d^k}{d\tau^k} e^{A\tau} \right) \Big|_{\tau=0} B = \xi^T A^k B = 0, \ \forall k \geq 0$$
$$\Rightarrow \xi^T \begin{pmatrix} B & AB & \cdots & A^{n-1}B \end{pmatrix} = 0 \Rightarrow \xi \in N((B \ AB \ \cdots \ A^{n-1}B)) = \mathcal{C}_{AB}^{\perp}$$

Controllability

Theorem: $R(W_t) \subset \mathcal{R}_t$

Proof:
Let $\xi \in R(W_t)$. Then there exists $v \in \mathbb{R}^n$ such that

$$\xi = W_t v = \int_0^t e^{A\tau} B B^T e^{A^T \tau} v d\tau$$

Define $u(\tau) = B^T e^{A^T (t-\tau)} v, \ \tau \in [0, t]$

Controllability

Then, since $\dot{x} = Ax + Bu$ with $x(0) = 0$, we have

$$\begin{aligned} x(t) &= \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau \\ &= \int_0^t e^{A(t-\tau)} BB^T e^{A^T(t-\tau)} v d\tau = W_t v = \xi \end{aligned}$$

This means that $\xi \in \mathcal{R}_t$, since we have found the control u that steers the state to ξ from the origin.

Controllability

Theorem (Theorem 6.1 of the textbook)
If (A, B) is controllable, and A is stable (eigenvalues of A have negative real parts), then there exists a unique solution of

$$AP + PA^T = -BB^T,$$

where $P = \int_0^\infty e^{A\tau} BB^T e^{A^T\tau} d\tau > 0$

- Note that $BB^T \geq 0$
- In Chapter 5, $AP + PA^T = -Q$ where $Q > 0$

Controllability

Theorem (Theorem 6.1 of the textbook)
(A, B) is controllable if and only if $rank((A - \lambda I \;\; B)) = n$ for all eigenvalues, λ, of A.

- Hautus-Rosenbrock test

Controllability

Theorem (Theorem 6.1 of the textbook)
(A, B) is controllable if and only if $W_t > 0$, that is, the controllability Gramian is non-singular

Theorem (Theorem 6.2 of the textbook)
Let $\bar{A} = PAP^{-1}$ and $\bar{B} = PB$. Then (A, B) is controllable if and only if $(\bar{A}, \bar{B})$ is controllable

- Controllability is invariant under the similarity transformation

Fact: The state space equation with the controllable canonical form is always controllable.

Controllability

$$\dot{x} = Ax + Bu, \;\; G(s) = \frac{X(s)}{U(s)} = (sI - A)^{-1}B$$

Kalman Decomposition Theorem (Theorem 6.6 of the textbook):
Suppose that $\mathcal{C}_{AB} = r < n$. Let

$$P = (v_1, \ldots, v_r, v_{r+1}, \ldots, v_n)$$

where v_i, $i = 1, 2, \ldots, r$ is eigenvectors of $\mathcal{C}$, and $v_{r+1}, \ldots, v_n$ are arbitrary vectors that guarantees P being nonsingular. Let $z = Px$.

Controllability

Then

$$\dot{z} = PAP^{-1}z + PBu$$
$$\bar{A} = PAP^{-1} = \begin{pmatrix} \bar{A}_{11} & \bar{A}_{12} \\ 0 & \bar{A}_{22} \end{pmatrix}, \ \bar{B} = PB = \begin{pmatrix} \bar{B}_1 \\ 0 \end{pmatrix}$$
$$\bar{A}_{11} \in \mathbb{R}^{r\times r}, \ \bar{B}_1 \in \mathbb{R}^{r\times m}$$

Also, $(\bar{A}_{11}, \bar{B}_1)$ is controllable, and $G(s) = (sI - \bar{A}_{11})^{-1}\bar{B}_1$.

Observability

$$\dot{x} = Ax + Bu, \ y = Cx, \ x \in \mathbb{R}^n, \ y \in \mathbb{R}^p$$

Definition (Definition 6.O1)
The state-space equation is said to be observable if for any unknown initial condition, there exists a finite t_1 such that the knowledge of the input and the output over $[0, t_1]$ is suffices to determine uniquely the initial condition $x(0)$.

W.L.G., $u = 0$, (since u is completely known)

Observability

Note that

$$y(t) = Ce^{At}x(0)$$

Hence, if $N(Ce^{At}) = \emptyset$, i.e., $\dim(N(Ce^{At})) = nullity(Ce^{At}) = 0$, then the system is observable.

- $N(Ce^{At})$: unobservable subspace

Observability

Let

$$\mathcal{O} = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{pmatrix}$$

$\mathcal{O}$: Observability matrix, $\mathcal{O} \in \mathbb{R}^{pn \times n}$

Observability

Theorem: $N(Ce^{At}) = N(\mathcal{O})$
Proof: We will show that $N(Ce^{At}) \subset N(\mathcal{O})$ and $N(Ce^{At}) \supset N(\mathcal{O})$.

If $x_0 \in N(Ce^{At})$, then

$$0 = Ce^{At}x_0 \Rightarrow 0 = C\left(\frac{d}{dt}e^{At}\right)\Big|_{t=0} x_0 \Rightarrow 0 = CA^k x_0, \ \forall k \geq 0$$

Hence, $x_0 \in N(\mathcal{O})$.
If $x_0 \in N(\mathcal{O})$. then $x_0 \in N(Ce^{At})$, since by C-H Theorem, we have

$$Ce^{At} = C\beta_0(t)I + \cdots + CA^{n-1}\beta_{n-1}(t)$$

Observability

If $N(Ce^{At}) = \emptyset$, i.e., $\dim(N(Ce^{At})) = nullity(Ce^{At}) = 0$, then the system is observable.

- We need $N(Ce^{At}) = N(\mathcal{O}) = \emptyset$
- Hence, by the rank-nullity theorem, the system is observable if $rank(\mathcal{O}) = n$
- We say that the system is observable if and only if the pair (C, A) is observable
- Observability also does not depend on the time (by C-H Theorem)

Observability

Duality Theorem (Theorem 6.5 of the textbook)
The following are equivalent:

- (C, A) is observable
- (A^T, C^T) is controllable

Observability

Proof: (A^T, C^T) is controllable if and only if

$$\mathcal{O}^T = (C^T \; A^T C^T \; \cdots \; (A^T)^{n-1} C^T)$$

$$rank(\mathcal{O}^T) = n = rank(\mathcal{O})$$

$$\mathcal{O} = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{pmatrix}$$

Observability

Theorem (Theorem 6.O1)
If (A, C) is observable, and A is stable, then there exists a unique solution of

$$A^T P + PA = -C^T C,$$

where $P = \int_0^\infty e^{A^T\tau} C^T C e^{A\tau} d\tau > 0$.

Theorem (Theorem 6.O1)
(C, A) is observable if and only if

$$rank \begin{pmatrix} C \\ A - \lambda I \end{pmatrix} = n$$

Observability

Theorem (Theorem 6.O1)
(C, A) is observable if and only if the observability Gramian

$$Q_t = \int_0^t e^{A^T\tau} C^T C e^{A\tau} d\tau > 0$$

Theorem (Theorem 6.O3) Let $\bar{A} = PAP^{-1}$ and $\bar{C} = CP^{-1}$. Then (C, A) is observable if and only if $(\bar{C}, \bar{A})$ is observable.

Observability

$$\dot{x} = Ax + Bu, \;\; y = Cx, \;\; G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B$$

Kalman Decomposition Theorem (Theorem 6.O6 of the textbook):
Suppose that $rank(\mathcal{O}) = q < n$. Let

$$P = \begin{pmatrix} v_1 \\ \vdots \\ v_q \\ v_{q+1} \\ \vdots \\ v_n \end{pmatrix}, \;\; v_1, \ldots, v_q\text{: eigenvectors.}$$

Observability

Let $z = Px$. Then $\dot{z} = PAP^{-1}z + PBu,\ y(t) = CP^{-1}z$, and

$$\bar{A} = PAP^{-1} = \begin{pmatrix} \bar{A}_{11} & 0 \\ \bar{A}_{21} & \bar{A}_{22} \end{pmatrix},\ \bar{B} = PB = \begin{pmatrix} \bar{B}_1 \\ \bar{B}_2 \end{pmatrix},\ \bar{C} = \begin{pmatrix} \bar{C}_1 & 0 \end{pmatrix}$$

$$\bar{A}_{11} \in \mathbb{R}^{q\times q},\ \bar{C}_1 \in \mathbb{R}^{p\times q}$$

Also, $(\bar{C}_1, \bar{A}_{11})$ is observable, and $G(s) = \bar{C}_1(sI - A_{11})^{-1}\bar{B}_1$.

Kalman Decomposition Theorem

Theorem (Theorem 6.7 of the textbook)
We can extract the state that is controllable and observable.

Fact: The state space equation with the observable canonical form is always observable

Discrete-Time LTI System

Discrete-time LTI system

$$x(k+1) = Ax(k) + Bu(k), \;\; y(k) = Cx(k)$$

- (A, B) is controllable if and only if $rank(\mathcal{C}) = n$
- (C, A) is observable if and only if $rank(\mathcal{O}) = n$

Minimum Energy Control (page 189)

$$\dot{x} = Ax + Bu, \; x(0) = x_0, \; x(t_1) = x_f, \; (A, B) \text{ controllable}$$

$$W_{t_1} = \int_0^{t_1} e^{A\tau} B B^T e^{A^T \tau} d\tau > 0, \; \text{invertible}$$

Let

$$u^*(t) = -B^T e^{A^T(t_1 - t)} W_{t_1}^{-1} (e^{At_1} x_0 - x_f)$$

$$x(t_1) = e^{At_1} x_0 - \underbrace{\left(\int_0^{t_1} e^{A(t_1 - \tau)} B B^T e^{A^T(t_1 - \tau)} d\tau \right)}_{W_{t_1}} W_{t_1}^{-1} (e^{At_1} x_0 - x_f) = x_f$$

Minimum Energy Control (page 189)

We can show that the controller u^* is the minimum energy controller in the sense that for any controller u that transfers the state from x_0 to x_f, we have

$$\int_0^{t_1} \|u(t)\|^2 dt \geq \int_0^{t_1} \|u^*(t)\|^2 dt, \ \forall u$$

Stabilizability & Detectability

Weaker notions of controllability and observability

A system is stabilizable if and only if $\bar{A}_{22}$ is stable and $(\bar{A}_{11}, \bar{B}_1)$ is controllable

A system is detectable if and only if $\bar{A}_{22}$ is stable and $(\bar{C}_1, \bar{A}_{11})$ is observable

How about the example on pages 4-5. Is it stabilizable? Is it detectable?

Controllability & Observability: LTV system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \; y(t) = C(t)x(t)$$

$$W_t = \int_0^t \Phi(t,\tau)B(\tau)B^T(\tau)\Phi^T(t,\tau)d\tau \geq 0, \; \forall t \geq 0$$

$$Q_t = \int_0^t \Phi^T(t,\tau)C^T(\tau)C(\tau)\Phi(t,\tau)d\tau \geq 0, \; \forall t \geq 0$$

The LTV system is

- is controllable if and only if there exists $t_f > 0$ such that $W_{t_f} > 0$
- is observable if and only if there exists $t_f > 0$ such that $Q_{t_f} > 0$
- W_t: controllability Graminan
- Q_t: observability Graminan

Minimal Realizations

We have seen that the realization of the state-space equation is not unique.

$$\dot{x} = Ax + Bu, \;\; y = Cx + Du, \;\; x(0) = 0$$
$$\dot{x} = A_1 x + B_1 u, \;\; y = C_1 x + D_1 u, \;\; x(0) = 0$$
$$y(t) = C \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau = C_1 \int_0^t e^{A_1(t-\tau)} B_1 u(\tau) d\tau$$

Minimal Realizations

Lemma (not in the textbook)

- Two system realizations (A, B, C, D) and (A_1, B_1, C_1, D_1) are equivalent if and only if $D = D_1$ and

$$Ce^{At}B = C_1 e^{A_1 t} B_1, \ \forall t \geq 0$$

- Two system realizations (A, B, C, D) and (A_1, B_1, C_1, D_1) are equivalent if and only if $D = D_1$ and

$$CA^k B = C_1 A_1^k B, \ \forall k \geq 0$$

Minimal Realizations

In view of the Kalman decomposition, we have the following result:
⇒ Suppose (A, B, C, D) is a system realization. If either (C, A) is not observable or (A, B) is not controllable, then there exists a lower-order realization (A_1, B_1, C_1, D_1) for the system

Definition (page 233 of the textbook)
Realizations with the smallest possible dimension are called minimal realizations

Minimal Realizations

Theorem (Theorem 7.M2 (page 254))
(A, B, C, D) is a minimial realization of the transfer function $G(s)$ if and only if (A, B) is controllable and (C, A) is observable

If the system is not controllable or not observable (or not controllable and observable), then there are pole-zero cancellations in a transfer function.

MATLAB Commands

- controllability matrix: $ctrb(A, B)$
- observability matrix: $ctrb(A^T, C^T)$
- minimal realization: $minreal(A, B, C, D)$ $\Rightarrow$ reduce the system order that has only controllable and observable state
- Mostly, we use the balanced realization (Chapter 7.4) $\Rightarrow$ related to controllability and observability Gramians (robust control, advanced control topic)

Conclusions

In this chapter

- Controllability
- Observability
- Duality
- Kalman decomposition
- Minimal realization

Next chapter: control system design

- Pole-placement
- observer design
- Optimum system design (LQR + Kalman filter)