UNIST
Department of Mechanical Engineering

MEN 573: Advanced Control Systems I **Spring, 2016**

Homework #6 Assigned: Wednesday, April 27, 2016
Due: Monday, May 9, 2016 (in class)

Problem 1.

Determine the range of K so that the characteristic equation,

$$s^4 + 2.9s^3 + 2.7s^2 + 0.7s + (K - 0.1) = 0$$

does not possess any root in the closed right-half plane, $\mathrm{Re}\{s\} \geq 0$.

Problem 2.

Determine the stability of the discrete time system

$$x(k+1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.008 & 0.008 & -0.79 & -0.8 \end{bmatrix} x(k)$$

by applying the bilinear transformation and Routh's criterion.

Problem 3.

Show in the z-plane how the imaginary axis of the s-plane is mapped by a bilinear transformation,

$$z = \frac{r(1+s)}{1-s} \qquad s = \frac{z-r}{z+r}$$

Problem 4.

Utilize the mapping relation in Problem 3 and the Routh criterion to determine conditions so that the all the closed loop poles of the following feedback system are inside of a circle with radius 0.5 centered at the origin of z-plane ($k > 0$), where

$$C(z) = k \qquad G(z) = \frac{1}{z(z-0.8)}$$

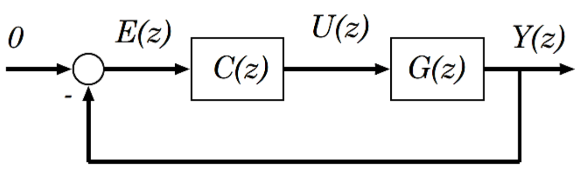

Figure 1: Feedback System

Problem 5.

Consider the following phase-lock loop dynamics

$$\ddot{y} + [a + b\cos(y)]\,\dot{y} + c\sin(y) = 0$$

where the coefficients $a \geq$, $b \geq$ and $c > 0$.

(a) Obtain a state space realization using $x_1 = y$ and $x_2 = \dot{y}$.

(b) Use the Lyapunov function candidate

$$V(x) = c(1 - \cos(x_1)) + \frac{1}{2}x_2^2$$

to show that the origin is stable in the sense of Lyapunov if $a \geq b \geq 0$.

(c) Use the same Lyapunov function and L'Salle's theorem to show that the origin is an asymptotically stable system if $a > b \geq 0$.

Problem 6.

A LTI continuous time system is given by

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & 1 \\ b & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Using the Lyapunov equation, show that the system is asymptotically stable iff $a < 0$ and $b < 0$.

Problem 7.

Given $P = P^T \in \mathcal{R}^{nxn}$ and $P > 0$ and two vectors u, $v \in \mathcal{R}^n$

(a) Show that

$$|u^T\,P\,v| \leq \lambda_{max}(P)\,\|u\|_2\,\|v\|_2,$$

where $\lambda_{max}(P) > 0$ is the largest eigenvalue of P and $\|u\|_2 = \sqrt{u^T\,u}$.

(b) Show with a counter example that

$$|u^T\,P\,v| \geq \lambda_{min}(P)\,\|u\|_2\,\|v\|_2,$$

where $\lambda_{min}(P) > 0$ is the smallest eigenvalue of P, **is not true**.