HW4: Linear System Theory (ECE532)

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Due Date: May 2 (Wed) at the beginning of the class.

Reading Assignment: Read Chapter 5

Note: HW must be written by LaTeX.

Problem 1:
Consider the LTI system

$$\dot{x} = Ax,\ A = \begin{pmatrix} \alpha & -1 \\ 1 & -3 \end{pmatrix}$$

where α is a scalar parameter. For what values of α of the system

- asymptotically stable?
- stable in the sense of Lyapunov?

Problem 2:
Consider the LTI system

$$\dot{x} = Ax,\ A = \begin{pmatrix} -2 & \alpha & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

where α and β are scalar parameters. For what values of α and β of the system

- asymptotically stable?
- stable in the sense of Lyapunov?

Problem 3:
Show that if all eigenvalues of a matrix A have real parts strictly less than some $-\mu < 0$, then for every $Q = Q^T > 0$, the equation $A^T P + PA + 2\mu P = -Q$ has a unique solution $P = P > 0$. (The number μ is called a stability margin). Note that problem is similar to Problem 5.26 of the textbook.

Problem 4:

You are given the LTI system

$$\dot{x} = Ax,\ A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$$

- Is $V(x) = x^T x$ a Lyapunov function for this system?
- Solve the Lyapunov equation $A^T P + PA = -Q$ with $Q = I$, and discuss what you can conclude from this solution (for P) regarding the stability of the LTI system.
- Consider now the LTI system

$$\dot{x} = Ax + b,\ A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix},\ b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

  Use the result in (ii) above to construct a Lyapunov function for this system.

- Solve Lyapunov equation $A^T P + PA = -Q$ with $Q = I$ by using MATLAB lyap

Problem 5: Problem 5.18 of the textbook (note that $A' = A^T$)

Problem 6: Problem 5.27 of the textbook

Problem 7: Problem 5.28 of the textbook

Problem 8

Consider a SISO LTI system with the transfer function $g(s) = \frac{1}{s^2+1}$. Is this system BIBO stable? Justify your answer.

Problem 9 Consider the following continuous-time system

$$\dot{x} = Ax,\ A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$$

1. Compute Lyapunov equation using MATLAB. Is the system stable?
2. Obtain the discrete-time system by using c2d in MATLAB with sampling rate $T = 0.1$
3. Solve the Lyapunov equation of the discrete-time system 2 by using MATLAB dlyap. Is the system stable?
4. Obtain the discrete-time system by using c2d in MATLAB with sampling rate $T = 0.01$
5. Solve the Lyapunov equation of the discrete-time system 4 by using MATLAB dlyap. Is the system stable?