UNIST
Department of Mechanical Engineering

MEN 573: Advanced Control Systems I **Spring, 2016**

Homework #4 Assigned: Saturday, April 2, 2016
Due: Monday, April 11, 2016 (in class)

Problem 1.

Find the solution matrix e^{At} for

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -3 \end{bmatrix}$$

using similarity transformation.

Problem 2.

Consider a third order system described by

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Find the set of all initial conditions $x(0)$, such that the free response (i.e. $u(t) = 0$) of this system will not exhibit any oscillation.

Problem 3.

Consider a second order linear time invariant (LTI) continuous time system of the form

$$\frac{d}{dt}x(t) = A\,x(t) + B\,u(t) \qquad x(0) = x_0 \tag{1}$$

where $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \in \mathcal{R}^2$ and $u(t) = \mathcal{R}$.

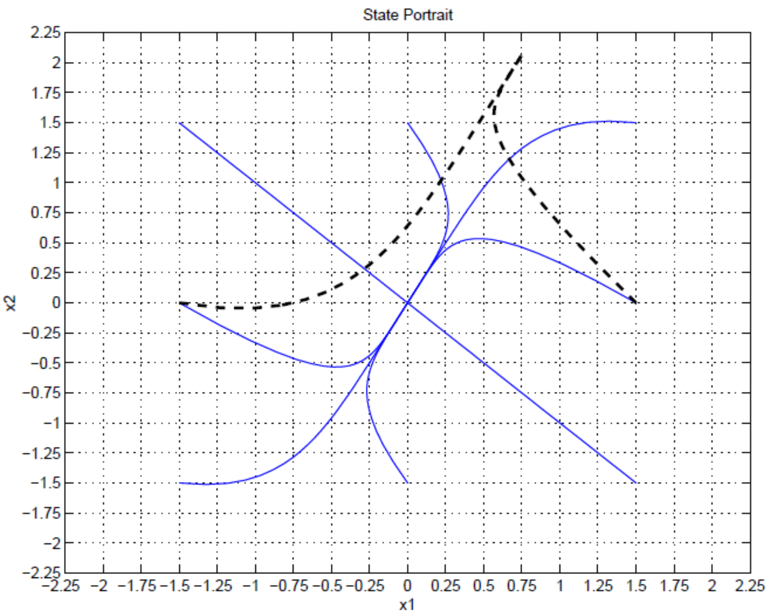

Figure 1: State Portrait of System in Eq. (1)

Fig. 1 shows the state portrait [1] of the system for several different state initial conditions and forcing inputs.

- The solid lines in Fig. 1 represent the free responses of the system (i.e. $u(t) = 0$) for several initial conditions.
- The dashed lines in Fig. 1 are the responses of the system for the following two conditions:

  (i) $x(0) = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$, $u(t) = 1$

  (ii) $x(0) = \begin{bmatrix} -1.5 \\ 0 \end{bmatrix}$, $u(t) = 1$

  Notice that in both of these cases, $\lim_{t\to\infty} x(t) = \begin{bmatrix} 0.75 & 2 \end{bmatrix}^T$.

If the matrix A satisfies

$$\mathrm{Det}[\lambda\, I - A] = \lambda^2 + 6\,\lambda + 5,$$

(a) Determine the matrix A.

(b) Determine the vector B.

Problem 4.

Consider a matrix $A \in \mathcal{R}^{2\times 2}$ such that

$$\det(\lambda I - A) = (\lambda - \sigma)^2 + \omega^2$$

Let the first eigenvalue be $\lambda_1 = \sigma + j\omega$ and its associate eigenvector $t_1 = t_R + jt_I$.

(a) Show that the eigenvector associated with the second eigenvalue, $\lambda_2 = \sigma - j\omega$ must be $t_2 = t_R - jt_I$.

(b) Show that the matrix $T_o = \begin{bmatrix} t_R & t_I \end{bmatrix} \in \mathcal{R}^{2\times 2}$ satisfies

$$A\,T_o = T_o \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

Sketch the state space trajectory (i.e. the plot of $x_2(t)$ v.s. $x_1(t)$) for LTI systems of the form $\dot{x} = Ax$, starting from the initial condition

$$x(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$

for the following three cases:

$$\text{(a)}\ A = \begin{bmatrix} -3 & 30 \\ -30 & -3 \end{bmatrix} \quad \text{(b)}\ A = \begin{bmatrix} -3 & -30 \\ 30 & -3 \end{bmatrix} \quad \text{(c)}\ A = \begin{bmatrix} -25.5 & -75 \\ 18.75 & 19.5 \end{bmatrix}$$

using the *isocline method* outlined as follows:

(i) Define $r = \frac{x_2}{x_1}$ and draw radial lines in the state plane, of constant r. For example, the line for $r = 0$ coincides with the horizontal axis, while the line $r = \infty$ coincides with the vertical axis.

(ii) For each line of constant r, you can determine the slope: $\frac{dx_2}{dx_1}$ of all state space trajectories that cross that line. For example, for case (a), the slope $\frac{dx_2}{dx_1}(r)$ is given by

$$\frac{dx_2}{dx_1}(r) = \frac{-30 - 3\,r}{-3 + 30\,r}.$$

Thus, for case (a) and along the horizontal axis (r =0), $\frac{dx_2}{dx_1}(0) = 10$ and any state space trajectory that crosses or starts from the horizontal axis will have that slope.

(iii) Sketching slopes $\frac{dx_2}{dx_1}$ along several radial lines of constant r will give you enough information on how to sketch the state space trajectory.

(d) Verify your sketches using `matlab`. For example, for case (a) you can use the following `matlab` code:

```
A = [ -3 30 ; -30 -3]
b = [1 ; 0]
c=[1,0]
d=0
sys1=ss(A,b,c,d)
x0 = b;
[y,t,x] = initial(sys,x0);
figure(1),plot(x(:,1),x(:,2))
```