UNIST
Department of Mechanical Engineering

MEN 573: Advanced Control Systems I	**Spring, 2016**
Homework #10	Assigned: Sunday, June 5, 2016 You don't have to submit this HW. The solution will be uploaded at June 12, 2016.

Problem 1.

(Axes Synchronization) Each axis of a two-axes positioning system is described by

$$X_i(s) = \frac{2}{0.02s+1} U_i(s), \quad i = 1, 2$$

The state equation for this system is

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -50 & 0 \\ 0 & -50 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} u$$

The control objective is to obtain a state feedback controller of the form

$$u(t) = -K\,x(t), \qquad K \in \mathcal{R}^{2\times 2}$$

which minimizes the cost functional

$$J = \int_0^\infty \left\{x_1^2(t) + x_2^2(t) + +s\,(x_1(2) - x_2(t))^2 + \rho\,(u_1^2(t) + u_2^2(t))\right\} dt$$

where s and ρ are weights and the third term $s\,(x_1(t) - x_2(t))^2$ penalizes the synchronization error between the two axes. I.e. the larger the value of s, the more the control action will try to make the synchronization error $(x_1(t) - x_2(t))$ small.

(a) Set $\rho = 1$ and, using the matlab function `LQR`, determine the optimal feedback gain K and the resulting closed loop poles of $A_c = A - BK$ for the following three values of s : $s = 0$, $s = 1$, $s = 100$. (You can actually solve this problem analytically if you want.)

(b) Use matlab to plot the response of the following system

$$\begin{aligned}
\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} &= \begin{bmatrix} -50 & 0 \\ 0 & -50 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}(u(t) + d(t)) \\
u(t) &= -K\,x(t)
\end{aligned}$$

where K is the feedback gain determine in part (a) and the $d(t) = \begin{bmatrix} d_1(t) & d_2(t) \end{bmatrix}^T$ is a disturbance input given by

$$\begin{aligned}
d_1(t) &= 0, \qquad \forall t \geq 0 \\
d_2(t) &= \begin{cases} 0 & 0 \leq t < 2 \\ 1 & t \geq 2 \end{cases}
\end{aligned}$$

for $x(0) = 0$, $\rho = 1$ and the following three cases: $s = 0$, $s = 1$, $s = 100$.

Problem 2.

Consider the following continuous time second-order linear time invariant system

$$\begin{aligned}\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) \\ y(t) &= x_1(t)\end{aligned}$$

Assume that control law is a state variable feedback

$$u(t) = -K\,x(t) \tag{1}$$

that minimizes the following infinite horizon cost functional

$$J = \int_0^\infty \{y^2(t) + R\,u^2(t)\}dt \tag{2}$$

(a) Determine if $\{A, C\}$ is detectable and $\{A, B\}$ is stabilizable.

(b) Draw the symmetric root locus for $R \in (0, \infty)$.

(c) Determine the asymptotic values of the eigenvalues of the close loop matrix $A - B\,K$ as $R \to \infty$.

(d) Determine the asymptotic values of the eigenvalues of the close loop matrix $A - B\,K$ as $R \to 0$.

(e) Determine:

  i. the value of R in Eq. (2),

  ii. the state feedback gain K in Eq. (1),

  when one of the eigenvalues of the close loop matrix $A - B\,K$ is $\lambda_{c1} = -2$.

Note: You do not need to find the solution of a Riccati equation in order to solve this problem.