CSE 530: Algorithms & Complexity
Assignment 2

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This homework assignment is due on Monday 9 April, 14:30 at the beginning of the lecture. **Please include your name and student ID.** Each question or subquestion is worth 10 marks. So the total is 70 marks. You should follow the academic integrity rules that are described at the end of the slides of Lecture 0.

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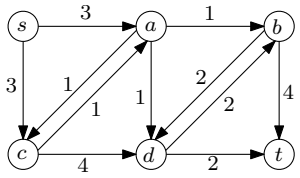
1. Find a minimum $s - t$ cut in the graph below. Justify your answer.



2. A factory builds three types of radios A, B and C. Each radio is produced by the work of three specialists Pierre, Paul and Jacques. Pierre works at most 24 hours per week. Paul works at most 45 hours per week. Jacques works at most 30 hours per week. The resources necessary to build each type of radio and their selling prices are given in the following table:

	Radio A	Radio B	Radio C
Pierre	1h	2h	2h
Paul	2h	1h	2h
Jacques	1h	3h	1h
Selling price	\$15	\$10	\$12

We assume that the factory has no problem selling all the radios it produces. The goal is to maximize the revenue over one week.

(a) Formulate this problem as a linear program.

(b) Solve this linear program numerically using a linear program solver.

3. Consider the following problem:

$$\begin{array}{ll} \text{maximize} & x_1 + 2x_2 \\ \text{subject to} & |x_1| + |x_2| \leqslant 1. \end{array}$$

(a) Show that it is equivalent to a linear program.

(b) Solve this linear program by hand.

4. We consider a generalized version of the maximum flow problem, where, in addition to the skew symmetry, flow conservation, and capacity constraint for each edge, we add a capacity constraint for each vertex (except s and t): The total amount of flow entering a vertex v is at most $c(v)$, where $c(v)$ is a given capacity. So a flow obeys the following constraints:

- $\forall u, v \in V,\ f(u,v) \leqslant c(u,v).$ (*Edge capacity constraint*)
- $\forall v \in V \setminus \{s,t\},\ \sum_{u \in V} f(u,v) \leqslant c(v).$ (*Vertex capacity constraint*)
- $\forall u, v \in V,\ f(u,v) = -f(v,u).$ (*Skew symmetry*)
- $\forall u \in V \setminus \{s,t\},\ \sum_{v \in V} f(u,v) = 0.$ (*Flow conservation*)

Give an $O(|V|.|E|^2)$-time algorithm for the problem of computing a maximum flow in a network $G(V,E)$ of this type.

5. You are given a rectilinear polygon whose vertices have integer coordinates. (See Figure 1.) Give a polynomial-time algorithm to decide whether this polygon can be partitioned into 2×1 and 1×2 rectangles. When the answer is positive, your algorithm should also return one such partition.

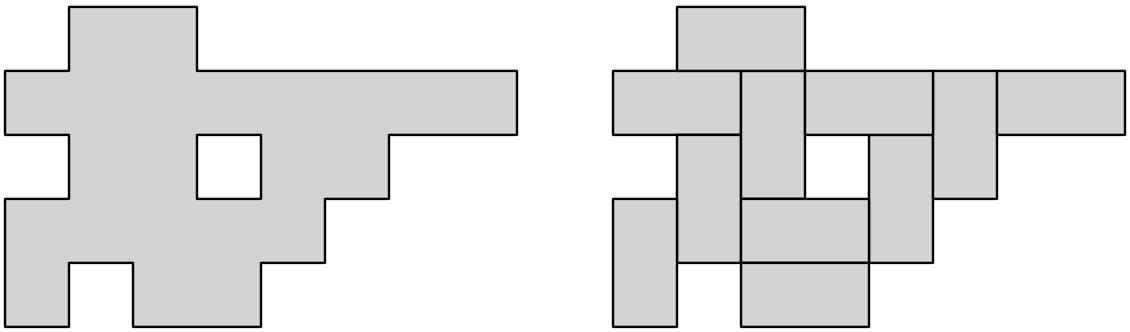


Figure 1: The input polygon (left) and its partition into 2×1 and 1×2 rectangles (right).