UNIST
Department of Mechanical Engineering

MEN 573: Advanced Control Systems I **Spring, 2016**

Homework #3 Assigned: Saturday, March 26, 2016
Due: Monday, April 4, 2016 (in class)

Problem 1.

A second order system is described by

$$\begin{aligned} \frac{dx_1(t)}{dt} &= x_2(t) \\ \frac{dx_2(t)}{dt} &= -4\,x_1(t) - 5\,x_2(t) + u(t) \\ y(t) &= 2\,x_1(t) + x_2(t) \\ x_1(0) &= x_{1o}\,,\ x_2(0) = x_{2o} \end{aligned} \tag{1}$$

(a) Find an expression the between $Y(s) = \mathcal{L}\{y(t)\}$ and $\{\,U(s) = \mathcal{L}\{u(t)\},\, x_{10},\, x_{20}\,\}$ in the Laplace transform domain.

(b) Determine the pole(s) and zero(s) of the input output transfer function

$$G(s) = \frac{Y(s)}{U(s)}.$$

(c) Show that when $u(t) = e^{zt}$, where z is a zero of the transfer function $G(s)$, it is possible to attain $y(t) = 0$ by properly selecting the initial conditions of the state variables (x_{1o} and x_{2o}). This implies that plant zeros block the transmission of certain input signals to the output. Find explicit expressions for such initial conditions.

(d) `Matlab exercises:`

(i) Learn how to use the `matlab` commands: `ss`, `tf` and, `zpk`.

(ii) Create a matlab state space object of the system described in Eq. (1) using the `ss` command.

(iii) Obtain the transfer function $G(s)$ in (b) using the `tf` command.

(iv) Obtain the poles and zeros of the transfer function $G(s)$ in (b) using the `zpk` command.

Problem 2.

A discrete time system is described by the following input/output transfer function

$$G(z) = \frac{z^2 + z}{z^3 - 2.6z^2 + 2.24z - 0.64} \tag{2}$$

(Note that: 1 - 2.6 + 2.24 - 0.64 = 0)

(a) Obtain the difference equation relating the input $u(k)$ and the output $y(k)$.

(b) Obtain the following state space representations of $G(z)$: (i) the controllable canonical form, (ii) the observable canonical form and (iii) the Jordan canonical form.

Problem 3.

LTI Single-Input, Single-Output (SISO) discrete time system is given by

$$x(k+1) = A\,x(k) + b\,u(k) \qquad y(k) = c\,x(k) + d\,u(k) \tag{3}$$

Defining $U(z) = \mathcal{Z}\{u(k)\}$ and $Y(z) = \mathcal{Z}\{y(k)\}$ as the Z-transforms of $u(k)$ and $y(k)$ respectively, we can define the discrete time pulse transfer function $G(z)$ by

$$G(z) = Y(z)/U(z) = c\,(zI - A)^{-1}\,b + d\,.$$

The response $y(k)$ for $k = 0, 1, 2, \cdots$ is shown in Fig. 1, when the input is given by

$$u(k) = 1 \qquad \text{for all } k \geq 0$$

and the initial condition is at the origin ($x(0) = 0$).

(a) Determine an expression for $y(k)$, for $k \geq 1$, in terms of the constants: m_0, m_1 and p shown in Fig. 1, where $0 < p < 1$.

(b) Determine an expression for the rational transfer function $G(z)$ in terms of the constants: m_0, m_1 and p.

(c) Determine the order of the system (i.e. the dimension of the state vector $x(k)$) and the value of the constant d in (3).

(d) Determine the observable canonical state space realization for this system.

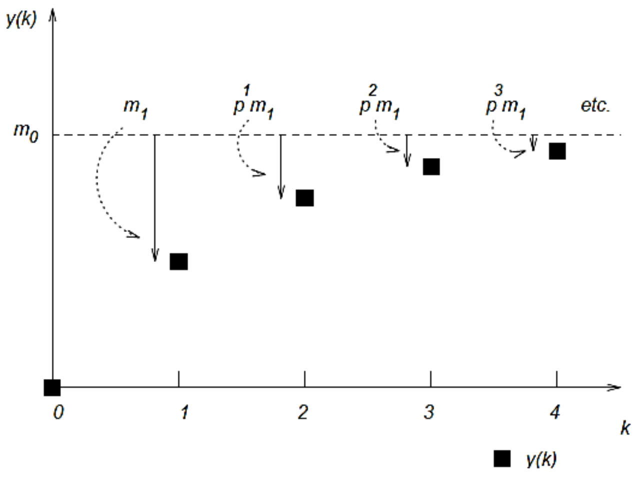

Figure 1: Unit step forced response of $G(z)$

Problem 4.

Sketched in Fig. 2 is a model of a vehicle suspension. m_b represents the mass of the vehicle body, m_t represents the lump mass of the tire and suspension, k_s and b_s are the spring constant and dashpot coefficient of the suspension, respectively, and k_t represents the tire stiffness. Differential equations to describe the motion are

$$\begin{aligned} m_b\frac{dv_b}{dt} &= f_s + b_s\left[v_t - v_b\right] \\ \frac{df_s}{dt} &= k_s\left[v_t - v_b\right] \\ m_b\frac{dv_t}{dt} &= -f_s - b_s\left[v_t - v_b\right] + f_t \\ \frac{df_t}{dt} &= k_t\left[u - v_t\right] \end{aligned} \tag{4}$$

where v_b and v_t are the vertical velocities of the body and of the unsprung mass, respectively, f_s and f_t are the forces stored in the spring and the tire, respectively, and u is the (vertical velocity) excitation from the road. Model parameter values are given as follows: $m_b = 240$ kg, $m_t = 36$ kg, $b_s = 1000$ Ns/m, $k_s = 16{,}000$ N/m and $k_t = 160{,}000$ N/m. Assume that the system input is the road vertical velocity, $u(t)$, and the output is the vehicle vertical velocity, $v_b(t)$.

`Matlab exercises:`

(a) Create a matlab state space object of the system described in Eq. (4) using the `ss` command. I will call this object `ss_sus`.

(b) Obtain the transfer function $G(s) = \frac{V_b(s)}{U(s)}$ using the `tf` command.

(c) Obtain the poles and zeros of the transfer function $G(s)$ using the `zpk` command.

(d) Plot the unit step response of $G(s)$ using the `step` command.

(e) Plot the frequency response of $G(s)$ using the `bode` command.

(f) Obtain the canonical modal state space realization using the command `canon(ss_sus,'modal')`, and understand what this commands does.

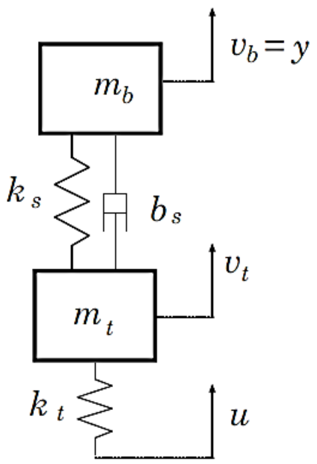

Figure 2: Tire model