UNIST
Department of Mechanical Engineering

MEN 573: Advanced Control Systems I		**Spring, 2016**
Homework #9	Assigned: Friday, May 27, 2016	
Solution	Due: Wednesday, June 8, 2016 (in class)	

Problem 1.

(a) $P = \begin{bmatrix} 1 & 0 & -100 \\ 0 & 0 & 100 \\ 0 & 1 & 0 \end{bmatrix}$ is rank $= 3$. Controllable

(b) the characteristic eqn is $\Delta(s) = s^3 + 200s \Rightarrow$ $a_0 = 0$, $a_1 = 200$, $a_2 = 0$

$$q_3 = B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$q_2 = Aq_3 + a_2 B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$q_1 = Aq_2 + a_1 B = \begin{bmatrix} 100 \\ 100 \\ 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 100 & 0 & 1 \\ 100 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \bar{A} = Q^{-1} A Q \qquad \bar{B} = Q^{-1} B$$

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -200 & 0 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(c) for the poles $\lambda = -20 \pm j20, -40$, the closed loop char. eqn. is

$$\Delta_c(s) = s^3 + 80s^2 + 2400s + 32000$$

then

$$\begin{bmatrix} \bar{k}_0 \\ \bar{k}_1 \\ \bar{k}_2 \end{bmatrix}^T = \begin{bmatrix} a_{c0} - a_0 \\ a_{c1} - a_1 \\ a_{c2} - a_2 \end{bmatrix}^T = \begin{bmatrix} 32000 - 0 \\ 2400 + 200 \\ 80 - 0 \end{bmatrix}^T = \begin{bmatrix} 32000 \\ 2600 \\ 80 \end{bmatrix}^T$$

$$k = [32000 \quad 2600 \quad 80]\, Q^{-1} = \boxed{[80 \quad 240 \quad 2600] = K}$$

Let's check the above result with Matlab.

```
clc;

A = [0 0 -100;0 0 100;1 -1 0];
B=[1; 0; 0];

% check controllability
P=[B A*B A^2*B]
rank(P)

% find transformation
Q(:,3)=B;
Q(:,2)=A*Q(:,3)+0*B;
Q(:,1)=A*Q(:,2)+200*B

% compute transformation
Ab = Q^-1*A*Q
Bb = Q^-1*B

% compute closed loop char. eqn.
syms s;
Phi = expand((s+20+20*i)*(s+20-20*i)*(s+40))

[32000 2600 80]*Q^-1
```

```
P =

     1     0  -100
     0     0   100
     0     1     0


ans =

     3
```

```
Q =

   100     0     1
   100     0     0
     0     1     0


Ab =

     0     1     0
     0     0     1
     0  -200     0


Bb =

     0
     0
     1


Phi =

s^3+80*s^2+2400*s+32000


ans =   80     240     2600
```

Problem 2.

(a) we want $\lambda_c = 0, 0, 0$. See MATLAB code.

$$K = \begin{bmatrix} -17 & -3.2 & 0.4 \end{bmatrix}$$

(b) we want to compute F s.t. $y_{ss} = v$. $\Rightarrow$ FVT

$$y_{ss} = \lim_{k \to \infty} y(k) = \lim_{z \to 1} (z-1) Y(z)$$

note: $Y(z)$ is the output of the (new) CL system:

$$x(k+1) = Ax(k) + B\left(-Kx(k) + Fv(k)\right)$$

$$= (A - BK)x(k) + BFv(k)$$

also note: v is constant (step response)

$$Y(z) = C\left(zI - A_c\right)^{-1} BF\,V(z) = C\left(zI - A_c\right)^{-1} BF \frac{vz}{z-1}$$

then $$y_{ss} = \lim_{z \to 1} (z-1)\, C\left(zI - A_c\right)^{-1} BF \frac{vz}{z-1}$$

$$= \underbrace{C\left(I - A_c\right)^{-1} B}_{= \frac{2z^2 + 3z + 1}{z^4} = 6} Fv = v$$

then $6Fv = v \Rightarrow \underline{\underline{F = \frac{1}{6}}}$

note: could have used >> dcgain(sys) as well. Just make sure TF is DT.

(c) we are given the a-priori state estimator:

$$\hat{x}(k+1) = (A-LC)\,\hat{x}(k) + Bu(k) + Ly(k)$$

we want $A_e = A-LC$ to have $\lambda = 0,0,0$. Use acker command:

$$L = \begin{bmatrix} -2.6 \\ -0.4 \\ -0.2 \end{bmatrix}.$$

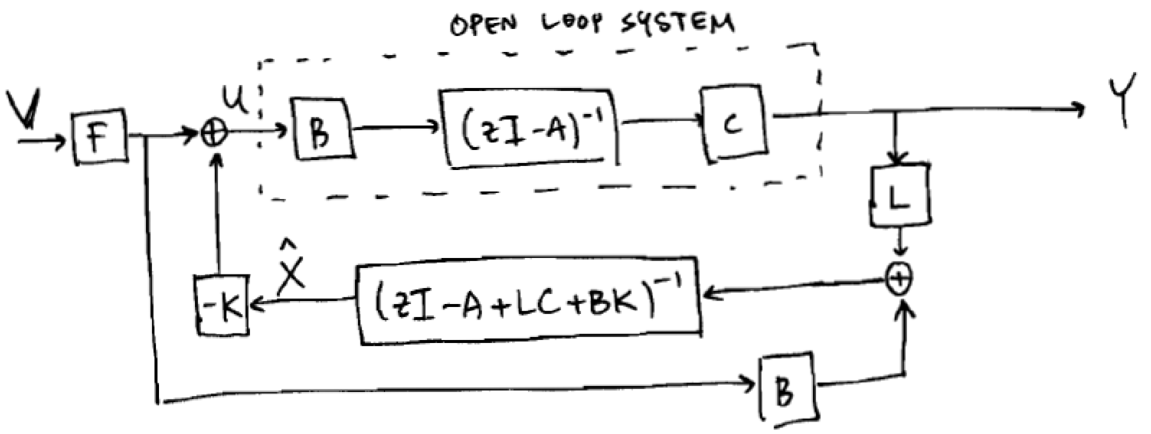
(d) simulate state observer feedback control.

we are given:

$$\hat{x}(k+1) = [A - LC - BK]\,\hat{x}(k) + BFv(k) + Ly(k)$$

$$u(k) = -K\hat{x}(k) + Fv$$

this looks like:



we only need to simulate 3 eqns: $\hat{x}(k+1)$, $x(k+1)$, $y(k)$.

(e) now we are going to design an a-posteriori observer by taking advantage of the most recent $y(k)$.

Sequence of equations:

1. estimate the step: $\hat{x}^{o}(k) =$ a-priori
2. compute the output $y(k) = C\,x(k)$
3. correct the estimate $\hat{x}(k) =$ a- posteriori

from the a- priori observer eqn

$$\hat{x}^{o}(k+1) = A\hat{x}(k) + Bu(k)$$

and the a-posterori eqn:

$$\hat{x}(k+1) = \hat{x}^{o}(k+1) + L\left(y(k+1) - C\hat{x}^{o}(k+1)\right)$$

we get

$$\hat{x}(k+1) = A\hat{x}(k)+Bu(k) + L\left(y(k+1) - CA\hat{x}(k) - CBu(k)\right)$$

$$= (A-LCA)\,\hat{x}(k) + (B-LCB)u(k) + Ly(k+1)$$

we want $A_e = (I-LC)A$ to have $\leq$values at the origin.

$\Rightarrow$ transform to observable canonical form.

Note: the form of our C.L. system is $A - L\underbrace{CA}_{\hat{C}}$

$$\det(zI-A) = \underset{\uparrow}{z^3} + \underset{\substack{\uparrow\\ a_2}}{3z^2} + \underset{\substack{\uparrow\\ a_1}}{5z} + \underset{\substack{\uparrow\\ a_0}}{5}$$

so,

$$p_1 = CA$$
$$p_2 = p_1 A + a_2 CA$$
$$p_3 = p_2 A + a_1 CA$$

then $Q = P^{-1} = \begin{bmatrix} -p_1- \\ -p_2- \\ -p_3- \end{bmatrix}^{-1}$

and $\bar{A} = Q^{-1} A Q$

$\bar{C} = CQ$

$$\bar{A} = \begin{bmatrix} -3 & 1 & 0 \\ -5 & 0 & 1 \\ -5 & 0 & 0 \end{bmatrix} \qquad \bar{C} = \begin{bmatrix} 0 & 0 & -0.2 \end{bmatrix}$$

$$\left.\begin{aligned} l_1 &= a_{e_1} - 3 = -3 \\ l_2 &= a_{e_2} - 5 = 0 - 5 = -5 \\ l_3 &= a_{e_3} - 5 = -5 \end{aligned}\right\} \quad \bar{L} = \begin{bmatrix} -3 \\ -5 \\ -5 \end{bmatrix} \Rightarrow \underline{L = Q\bar{L} = \begin{bmatrix} 0.2 \\ 0.8 \\ 0.4 \end{bmatrix}}$$

(f) we are now going to use this a-posteriori observer as state information into our feedback control law:

$$u(k) = -K\hat{x}(k) + Fv.$$

this the required sequence:

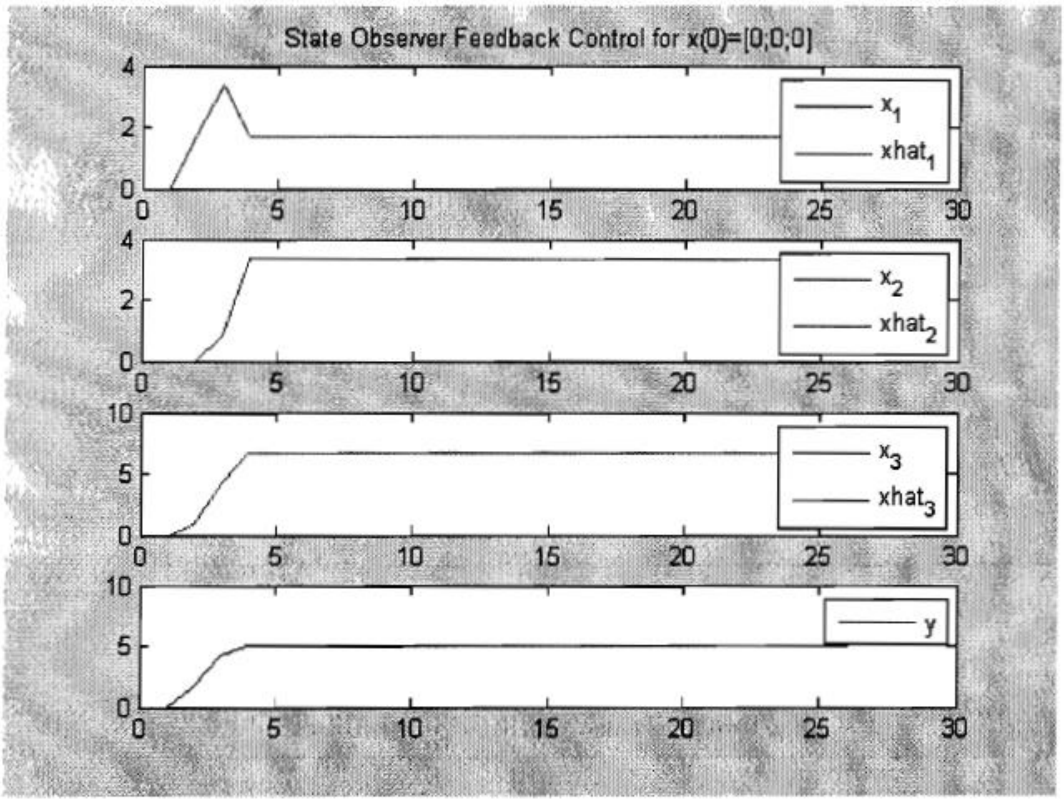
$$\begin{cases} \hat{x}^{\circ}(k+1) = A\hat{x}(k) + Bu(k) \\ x(k+1) = Ax(k) + Bu(k) \\ y(k+1) = Cx(k) \\ \hat{x}(k+1) = \hat{x}^{\circ}(k+1) + L\left(y(k+1) - C\hat{x}^{\circ}(k+1)\right) \\ u(k+1) = -K\hat{x}(k+1) + Fv. \end{cases}$$

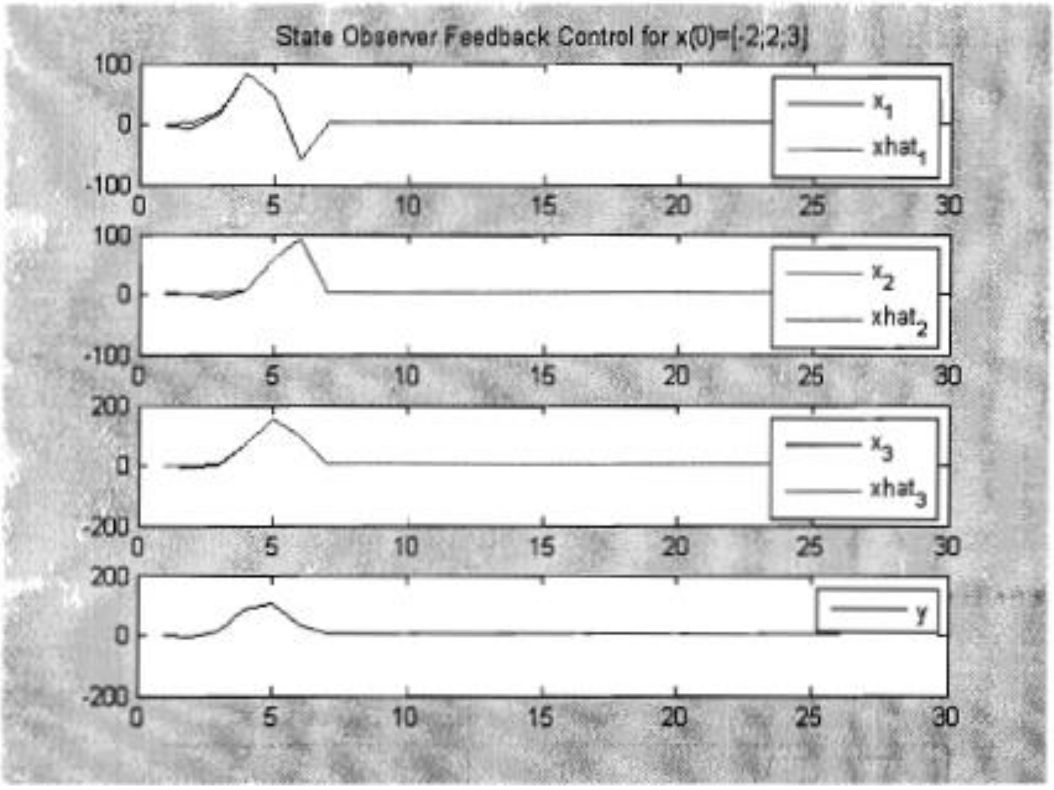
```
clc
clear all
% State Space information
A=[-1 -2 -2;0 -1 1;1 0 -1];
B=[2;0;1];
C=[1 1 0];

% (a) Find state feedback gains
K=acker(A,B,[0 0 0])
% (b) Find F
F=1/6;
% (c) Find state observer gains
L=acker(A',C',[0 0 0])'
eig(A-L*C)

% (d) computer state observer feedback for x(0) = [0;0;0]
x(:,1)=[0;0;0];
xhat(:,1)=[0;0;0];
v=5;
y(1)=C*x(:,1);
for k=1:25
    xhat(:,k+1)=(A-L*C-B*K)*xhat(:,k) +B*F*v+L*y(:,k);
    x(:,k+1)=A*x(:,k)-B*K*xhat(:,k) + B*F*v;
    y(:,k+1) = C*x(:,k+1);
end

figure(1)

subplot(411)
plot(x(1,:))
hold on;
plot(xhat(1,:),'r')
legend('x_1','xhat_1')
title('State Observer Feedback Control for x(0)=[0;0;0]')
subplot(412)
plot(x(2,:))
hold on;
plot(xhat(2,:),'r')
legend('x_2','xhat_2')
subplot(413)
plot(x(3,:))
hold on;
plot(xhat(3,:),'r')
legend('x_3','xhat_3')

subplot(414)
plot(y)
legend('y')

% (d) computer state observer feedback for x(0) = [-2;2;3];
x(:,1)=[-2;2;3];
xhat(:,1)=[0;0;0];
v=5;
y(1)=C*x(:,1);
for k=1:25
    xhat(:,k+1)=(A-L*C-B*K)*xhat(:,k) +B*F*v+L*y(:,k);
    x(:,k+1)=A*x(:,k)-B*K*xhat(:,k) + B*F*v;
    y(:,k+1) = C*x(:,k+1);
end

figure(2)

subplot(411)
plot(x(1,:))
```

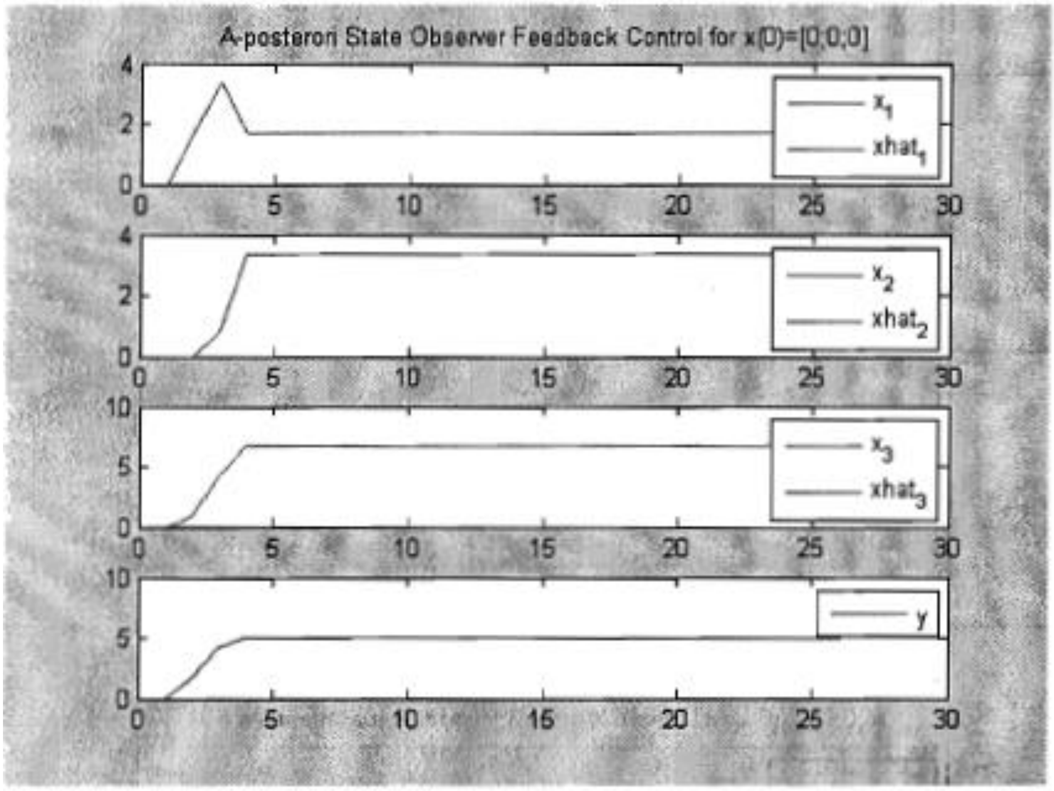
```
hold on;
plot(xhat(1,:),'r')
legend('x_1','xhat_1')
title('State Observer Feedback Control for x(0)=[-2;2;3]')
subplot(412)
plot(x(2,:))
hold on;
plot(xhat(2,:),'r')
legend('x_2','xhat_2')
subplot(413)
plot(x(3,:))
hold on;
plot(xhat(3,:),'r')
legend('x_3','xhat_3')
subplot(414)
plot(y)
legend('y')


% (e) compute a-posterori observer gains

P(1,:)=C*A;
P(2,:)=P(1,:)*A+3*C*A;
P(3,:)=P(2,:)*A+5*C*A;
Q=P^-1

Abar=Q^-1*A*Q
Cbar=C*Q

% by paper computation,
Lbar=[-3;-5;-5];
L=Q*Lbar
%check evals:
eig(A-L*C*A)


% (f) a-posterori state observer feedback
x(:,1)=[0;0;0];
xhato(:,1)=[0;0;0];
xhat(:,1) = xhato(:,1);
v=5;
u(1)=-K*xhat(:,1)+F*v;
for k=1:25
    xhato(:,k+1)=A*xhat(:,k) +B*u(k);
    x(:,k+1)=A*x(:,k)+B*u(:,k);
    y(k+1) = C*x(:,k+1);
    xhat(:,k+1) = xhato(:,k+1) + L*(y(k+1)-C*xhato(:,k+1));
    u(k+1)=-K*xhat(:,k+1) + F*v;
end


figure(3)

subplot(411)
plot(x(1,:))
hold on;
plot(xhat(1,:),'r')
legend('x_1','xhat_1')
title('A-posterori State Observer Feedback Control for x(0)=[0;0;0]')
subplot(412)
plot(x(2,:))
hold on;
plot(xhat(2,:),'r')
legend('x_2','xhat_2')
subplot(413)
plot(x(3,:))
hold on;
```

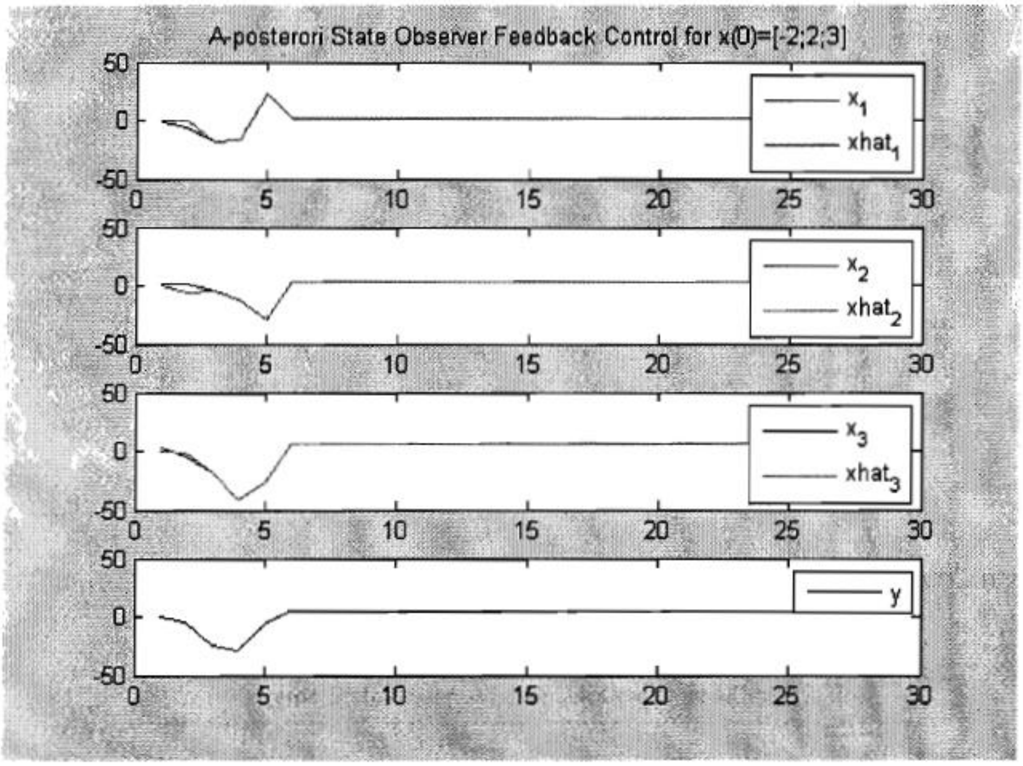
```
plot(xhat(3,:),'r')
legend('x_3','xhat_3')
subplot(414)
plot(y)
legend('y')


x(:,1)=[-2; 2; 3];
xhato(:,1)=[0;0;0];
xhat(:,1) = xhato(:,1);
v=5;
u(1)=-K*xhat(:,1)+F*v;
for k=1:25
    xhato(:,k+1)=A*xhat(:,k) +B*u(k);
    x(:,k+1)=A*x(:,k)+B*u(:,k);
    y(k+1) = C*x(:,k+1);
    xhat(:,k+1) = xhato(:,k+1) + L*(y(k+1)-C*xhato(:,k+1));
    u(k+1)=-K*xhat(:,k+1) + F*v;
end


figure(4)

subplot(411)
plot(x(1,:))
hold on;
plot(xhat(1,:),'r')
legend('x_1','xhat_1')
title('A-posterori State Observer Feedback Control for x(0)=[-2;2;3]')
subplot(412)
plot(x(2,:))
hold on;
plot(xhat(2,:),'r')
legend('x_2','xhat_2')
subplot(413)
plot(x(3,:))
hold on;
plot(xhat(3,:),'r')
legend('x_3','xhat_3')
subplot(414)
plot(y)
legend('y')
```

```
K =

   -1.7000   -3.2000    0.4000


L =

   -2.6000
   -0.4000
   -0.2000


ans =

  1.0e-005 *

   0.7601
  -0.3800 + 0.6582i
  -0.3800 - 0.6582i


Q =

    0.6000   -0.2000   -0.2000
   -0.6000    0.2000   -0.0000
    0.2000   -0.4000    0.2000


Abar =

   -3.0000    1.0000   -0.0000
   -5.0000   -0.0000    1.0000
   -5.0000         0    0.0000


Cbar =

         0         0   -0.2000


L =

    0.2000
    0.8000
    0.4000

ans =

  1.0e-005 *

   0.4551 + 0.7882i
   0.4551 - 0.7882i
  -0.9102
```


State Observer Feedback Control for x(0)=[0;0;0]
x1
xhat1
x2
xhat2
x3
xhat3
y


State Observer Feedback Control for x(0)=[-2;2;3]
x1
xhat1
x2
xhat2
x3
xhat3
y


A-posterori State Observer Feedback Control for x(0)=[0;0;0]
x1
xhat1
x2
xhat2
x3
xhat3
y


A-posterori State Observer Feedback Control for x(0)=[-2;2;3]
x1
xhat1
x2
xhat2
x3
xhat3
y