

9. What function do gears perform?
10. What are the component parts of the mechanical constants of a motor's transfer function?
11. The motor's transfer function relates armature displacement to armature voltage. How can the transfer function that relates load displacement and armature voltage be determined?
12. Summarize the steps taken to linearize a nonlinear system.

Problems

1. Derive the Laplace transform for the following time functions: [Section: 2.2]
 - a. $u(t)$
 - b. $t u(t)$
 - c. $\sin \omega t u(t)$
 - d. $\cos \omega t u(t)$
2. Using the Laplace transform pairs of Table 2.1 and the Laplace transform theorems of Table 2.2, derive the Laplace transforms for the following time functions: [Section: 2.2]
 - a. $e^{-at} \sin \omega t u(t)$
 - b. $e^{-at} \cos \omega t u(t)$
 - c. $t^3 u(t)$
3. Repeat Problem 18 in Chapter 1, using Laplace transforms. Assume that the forcing functions are zero prior to $t = 0-$. [Section: 2.2]
4. Repeat Problem 19 in Chapter 1, using Laplace transforms. Use the following initial conditions for each part as follows: (a) $x(0) = 4$, $x'(0) = -4$; (b) $x(0) = 4$, $x'(0) = 1$; (c) $x(0) = 2$, $x'(0) = 3$, where $x'(0) = \frac{dx}{dt}(0)$. Assume that the forcing functions are zero prior to $t = 0-$. [Section: 2.2]
5. Use MATLAB and the Symbolic Math Toolbox to find the Laplace transform of the following time functions: [Section: 2.2]

Symbolic Math
SM

 - a. $f(t) = 8t^2 \cos(3t + 45^\circ)$
 - b. $f(t) = 3te^{-2t} \sin(4t + 60^\circ)$
6. Use MATLAB and the Symbolic Math Toolbox to find the inverse

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Laplace transform of the following frequency functions: [Section: 2.2]

- a. $G(s) = \frac{(s^2 + 3s + 10)(s + 5)}{(s + 3)(s + 4)(s^2 + 2s + 100)}$
- b. $G(s) = \frac{s^3 + 4s^2 + 2s + 6}{(s + 8)(s^2 + 8s + 3)(s^2 + 5s + 7)}$

7. A system is described by the following differential equation:

$$\frac{d^3y}{dt^3} + 3 \frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4 \frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 8x$$

Find the expression for the transfer function of the system, $Y(s)/X(s)$. [Section: 2.3]

8. For each of the following transfer functions, write the corresponding differential equation. [Section: 2.3]

- a. $\frac{X(s)}{F(s)} = \frac{7}{s^2 + 5s + 10}$
- b. $\frac{X(s)}{F(s)} = \frac{15}{(s + 10)(s + 11)}$
- c. $\frac{X(s)}{F(s)} = \frac{s + 3}{s^3 + 11s^2 + 12s + 18}$

9. Write the differential equation for the system shown in Figure P2.1. [Section: 2.3]

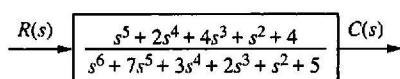


FIGURE P2.1

10. Write the differential equation that is mathematically equivalent to the

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block diagram shown in Figure P2.2. Assume that $r(t) = 3t^3$. [Section: 2.3]

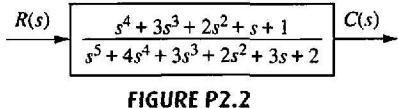


FIGURE P2.2

11. A system is described by the following differential equation: [Section 2.3]

$$\frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 3x = 1$$

with the initial conditions $x(0) = 1$, $\dot{x}(0) = -1$. Show a block diagram of the system, giving its transfer function and all pertinent inputs and outputs. (Hint: the initial conditions will show up as added inputs to an effective system with zero initial conditions.)

12. Use MATLAB to generate the transfer function: [Section: 2.3] MATLAB
ML

$$G(s) = \frac{5(s+15)(s+26)(s+72)}{s(s+55)(s^2+5s+30)(s+56)(s^2+27s+52)}$$

in the following ways:

- a. the ratio of factors;
- b. the ratio of polynomials.

13. Repeat Problem 12 for the following transfer function: [Section: 2.3] MATLAB
ML

$$G(s) = \frac{s^4 + 25s^3 + 20s^2 + 15s + 42}{s^5 + 13s^4 + 9s^3 + 37s^2 + 35s + 50}$$

14. Use MATLAB to generate the partial-fraction expansion of the following function: [Section: 2.3]

$$F(s) = \frac{10^4(s+5)(s+70)}{s(s+45)(s+55)(s^2+7s+110)(s^2+6s+95)}$$

15. Use MATLAB and the Symbolic Math Toolbox to input and form LTI objects in polynomial and factored form for the following frequency functions: [Section: 2.3] Symbolic Math
SM

a. $G(s) = \frac{45(s^2 + 37s + 74)(s^3 + 28s^2 + 32s + 16)}{(s+39)(s+47)(s^2 + 2s + 100)(s^3 + 27s^2 + 18s + 15)}$

b. $G(s) = \frac{56(s+14)(s^3 + 49s^2 + 62s + 53)}{(s^2 + 81s^2 + 76s + 65)(s^2 + 88s + 33)(s^2 + 56s + 77)}$

16. Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for each network shown in Figure P2.3. [Section: 2.4]

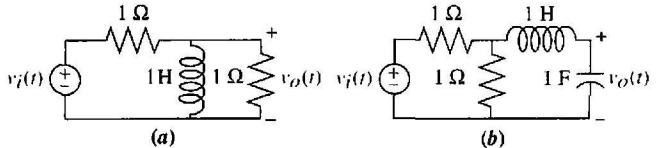


FIGURE P2.3

17. Find the transfer function, $G(s) = V_L(s)/V(s)$, for each network shown in Figure P2.4. [Section: 2.4]

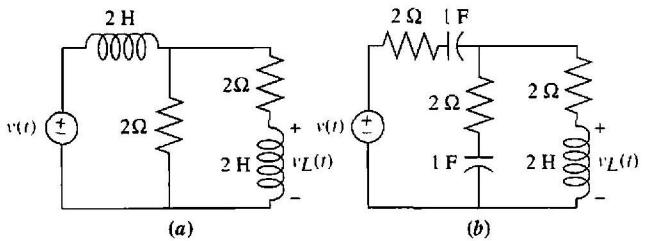


FIGURE P2.4

18. Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for each network shown in Figure P2.5. Solve the problem using mesh analysis. [Section: 2.4]

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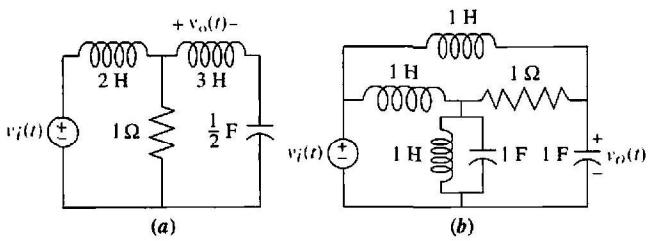


FIGURE P2.5

19. Repeat Problem 18 using nodal equations. [Section: 2.4]

20. a. Write, but do not solve, the mesh and nodal equations for the network of Figure P2.6. [Section: 2.4]
- b. Use MATLAB, the Symbolic Math Toolbox, and the equations found in part a to solve for the transfer function, $G(s) = V_o(s)/V(s)$. Use both the

mesh and nodal equations and show that either set yields the same transfer function. [Section: 2.4]

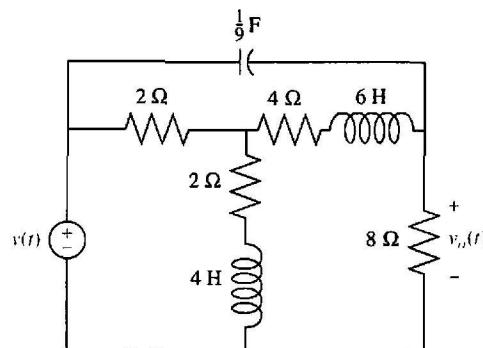
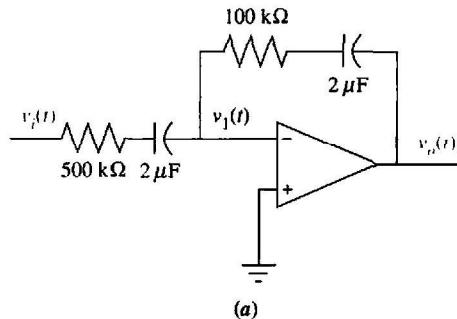


FIGURE P2.6

21. Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for each operational amplifier circuit shown in Figure P2.7. [Section: 2.4]



(a)

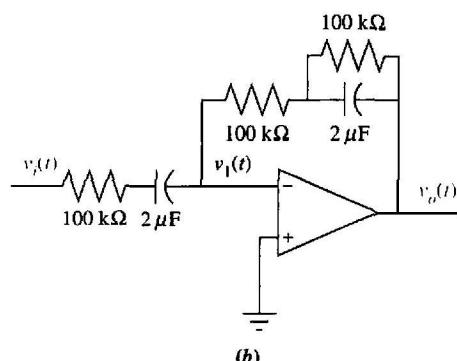
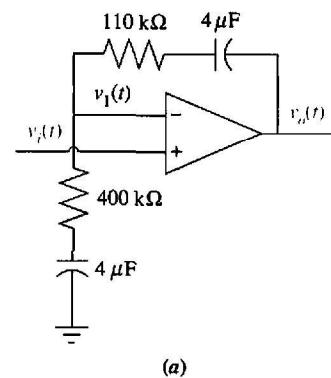


FIGURE P2.7

22. Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for each operational amplifier circuit shown in Figure P2.8. [Section: 2.4]



(a)

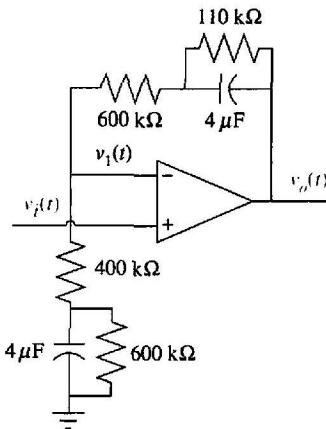


FIGURE P2.8

23. Find the transfer function, $G(s) = X_1(s)/F(s)$, for the translational mechanical system shown in Figure P2.9. [Section: 2.5]

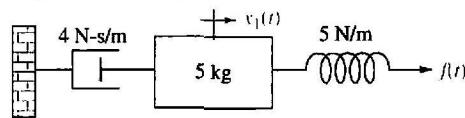


FIGURE P2.9

24. Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical network shown in Figure P2.10. [Section: 2.5]

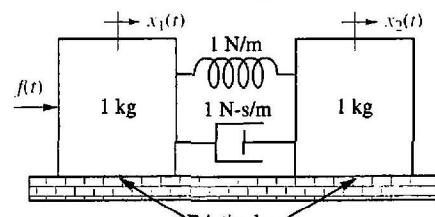


FIGURE P2.10

25. Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical system shown in Figure P2.11. (Hint: place a zero mass at $x_2(t)$.) [Section: 2.5]

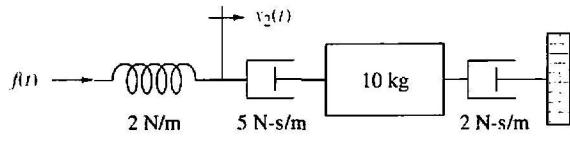


FIGURE P2.11

26. For the system of Figure P2.12 find the transfer function, $G(s) = X_1(s)/F(s)$. [Section: 2.5]

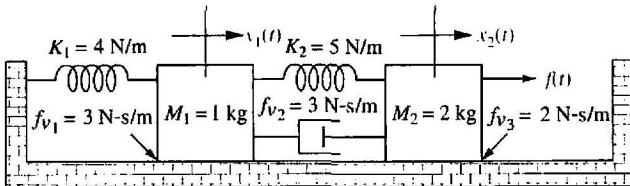


FIGURE P2.12

27. Find the transfer function, $G(s) = X_3(s)/F(s)$, for the translational mechanical system shown in Figure P2.13. [Section: 2.5]

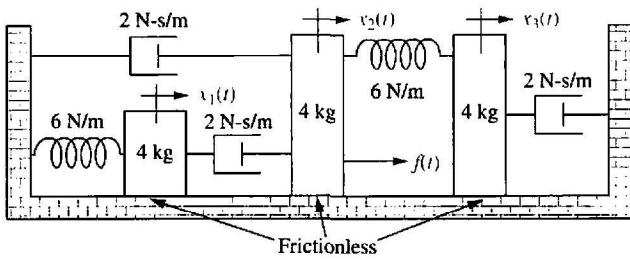
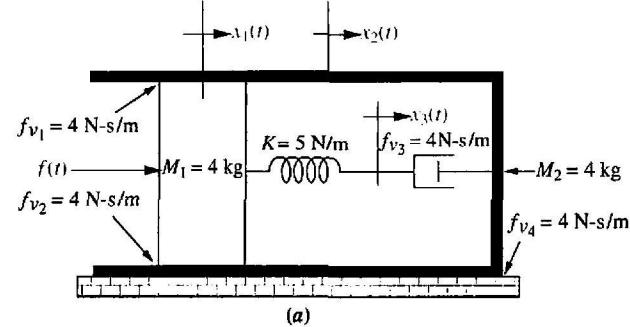


FIGURE P2.13

28. Find the transfer function, $X_3(s)/F(s)$, for each system shown in Figure P2.14. [Section: 2.5]



(a)

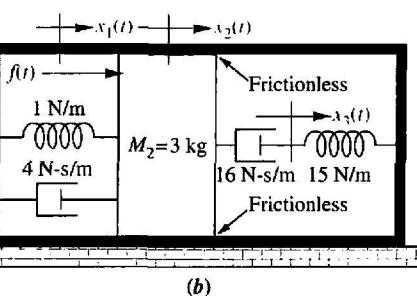


FIGURE P2.14

29. Write, but do not solve, the equations of motion for the translational mechanical system shown in Figure P2.15. [Section: 2.5]

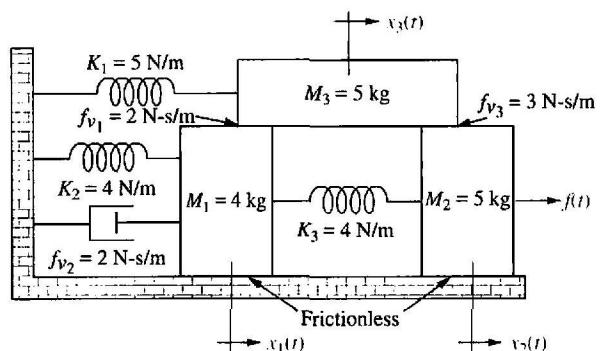
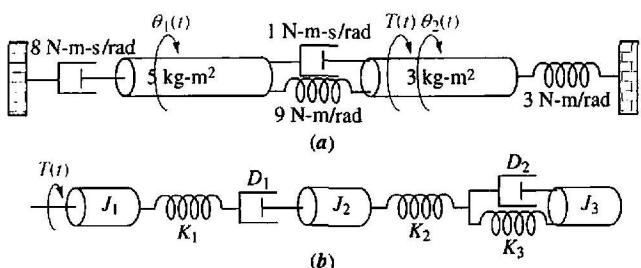
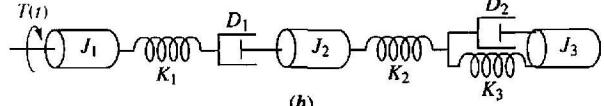


FIGURE P2.15

30. For each of the rotational mechanical systems shown in Figure P2.16, write, but do not solve, the equations of motion. [Section: 2.6]



(a)



(b)

FIGURE P2.16

31. For the rotational mechanical system shown in Figure P2.17, find the transfer function $G(s) = \theta_2(s)/T(s)$ [Section: 2.6]

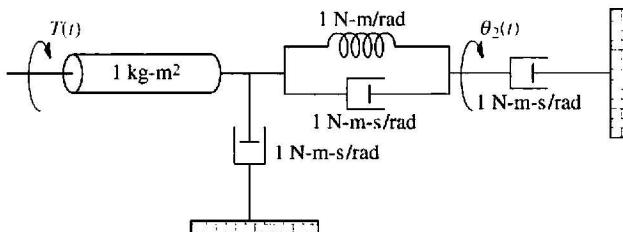


FIGURE P2.17

32. For the rotational mechanical system with gears shown in Figure P2.18, find the transfer function, $G(s) = \theta_3(s)/T(s)$. The gears have inertia and bearing friction as shown. [Section: 2.7]

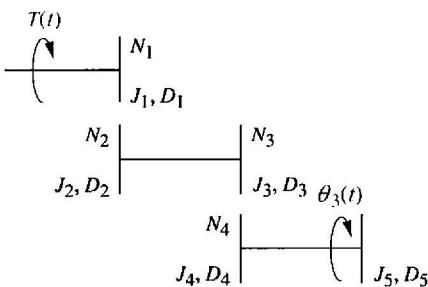


FIGURE P2.18

33. For the rotational system shown in Figure P2.19, find the transfer function, $G(s) = \theta_2(s)/T(s)$. [Section: 2.7]

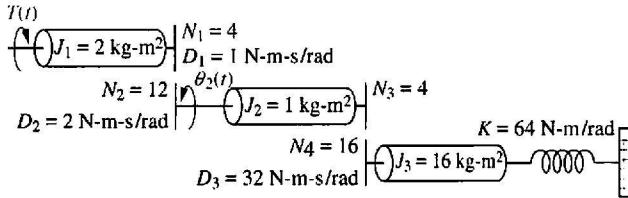


FIGURE P2.19

34. Find the transfer function, $G(s) = \theta_2(s)/T(s)$, for the rotational mechanical system shown in Figure P2.20. [Section: 2.7]

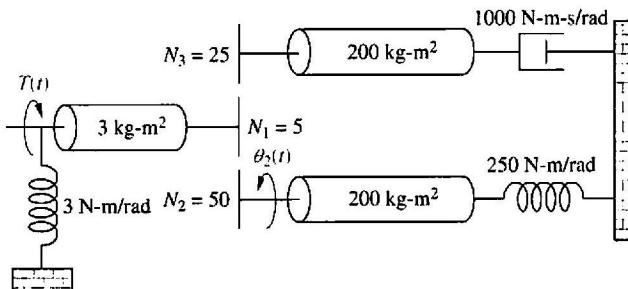


FIGURE P2.20

35. Find the transfer function, $G(s) = \theta_4(s)/T(s)$, for the rotational system shown in Figure P2.21. [Section: 2.7]

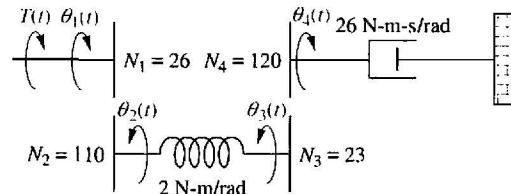


FIGURE P2.21

36. For the rotational system shown in Figure P2.22, find the transfer function, $G(s) = \theta_L(s)/T(s)$. [Section: 2.7]

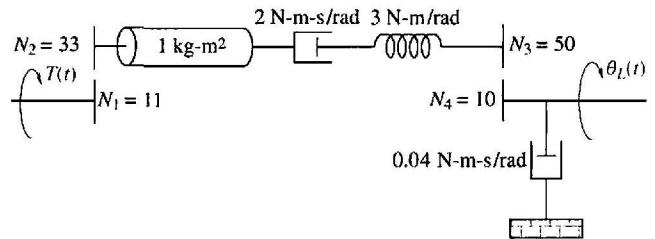


FIGURE P2.22

37. For the rotational system shown in Figure P2.23, write the equations of motion from which the transfer function, $G(s) = \theta_1(s)/T(s)$, can be found. [Section: 2.7]

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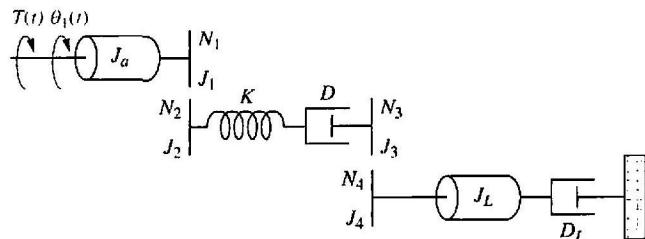


FIGURE P2.23

38. Given the rotational system shown in Figure P2.24, find the transfer function, $G(s) = \theta_6(s)/\theta_1(s)$. [Section: 2.7]

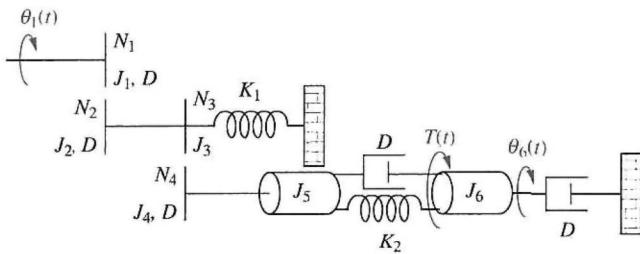


FIGURE P2.24

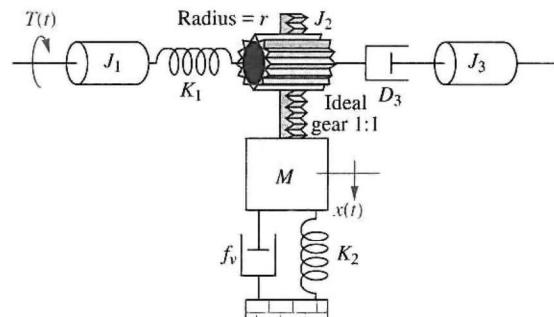


FIGURE P2.27

39. In the system shown in Figure P2.25, the inertia, J , of radius, r , is constrained to move only about the stationary axis A . A viscous damping force of translational value f_v exists between the bodies J and M . If an external force, $f(t)$, is applied to the mass, find the transfer function, $G(s) = \theta(s)/F(s)$. [Sections: 2.5; 2.6]

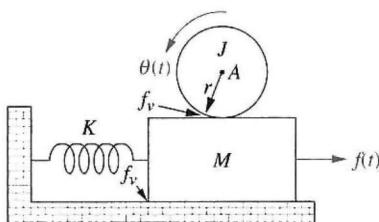


FIGURE P2.25

40. For the combined translational and rotational system shown in Figure P2.26, find the transfer function, $G(s) = X(s)/T(s)$. [Sections: 2.5; 2.6; 2.7]

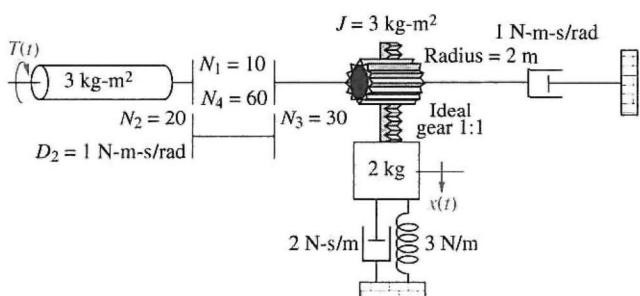


FIGURE P2.26

41. Given the combined translational and rotational system shown in Figure P2.27, find the transfer function, $G(s) = X(s)/T(s)$. [Sections: 2.5; 2.6]

42. For the motor, load, and torque-speed curve shown in Figure P2.28, find the transfer function, $G(s) = \theta_L(s)/E_a(s)$. [Section: 2.8]

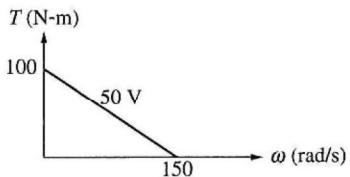
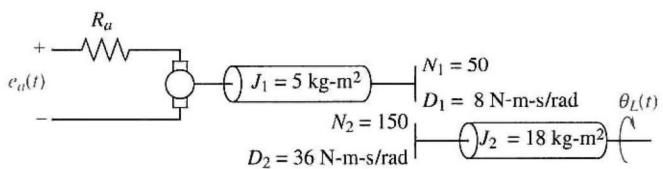


FIGURE P2.28

43. The motor whose torque-speed characteristics are shown in Figure P2.29 drives the load shown in the diagram. Some of the gears have inertia. Find the transfer function, $G(s) = \theta_2(s)/E_a(s)$. [Section: 2.8]

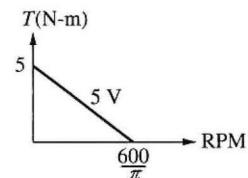
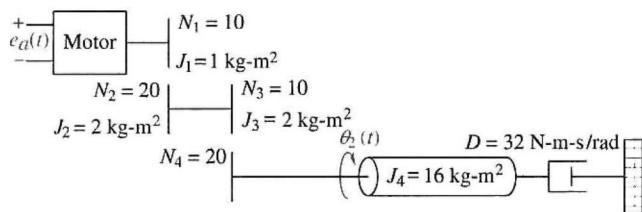


FIGURE P2.29

44. A dc motor develops 55 N-m of torque at a speed of 600 rad/s when 12 volts are applied. It stalls out at this voltage with 100 N-m of torque. If the inertia and damping of the armature are 7 kg-m² and 3 N-m-s/rad, respectively, find the transfer function, $G(s) = \theta_L(s)/E_a(s)$, of this motor if it drives an inertia load of 105 kg-m² through a gear train, as shown in Figure P2.30. [Section: 2.8]

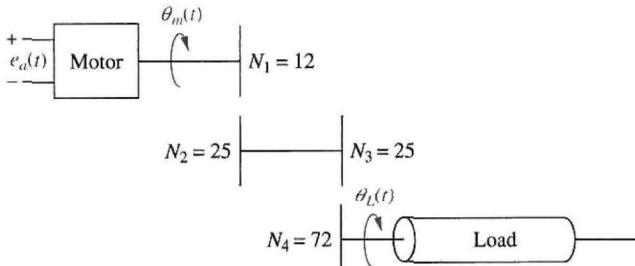


FIGURE P2.30

45. In this chapter, we derived the transfer function of a dc motor relating the angular displacement output to the armature voltage input. Often we want to control the output torque rather than the displacement. Derive the transfer function of the motor that relates output torque to input armature voltage. [Section: 2.8]

46. Find the transfer function, $G(s) = X(s)/E_a(s)$, for the system shown in Figure P2.31. [Sections: 2.5–2.8]

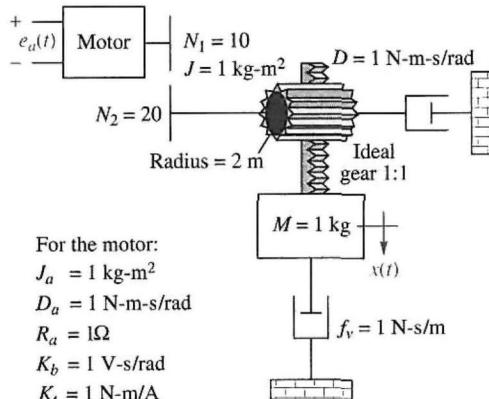


FIGURE P2.31

47. Find the series and parallel analogs for the translational mechanical system shown in Figure 2.20 in the text. [Section: 2.9]

48. Find the series and parallel analogs for the rotational mechanical systems shown in Figure P2.16(b) in the problems. [Section: 2.9]

49. A system's output, c , is related to the system's input, r , by the straight-line relationship, $c = 5r + 7$. Is the system linear? [Section: 2.10]

50. Consider the differential equation

$$\frac{d^2x}{dt^2} + 3 \frac{dx}{dt} + 2x = f(x)$$

where $f(x)$ is the input and is a function of the output, x . If $f(x) = \sin x$, linearize the differential equation for small excursions. [Section: 2.10]

a. $x = 0$

b. $x = \pi$

51. Consider the differential equation

$$\frac{d^3x}{dt^3} + 10 \frac{d^2x}{dt^2} + 31 \frac{dx}{dt} + 30x = f(x)$$



where $f(x)$ is the input and is a function of the output, x . If $f(x) = e^{-x}$, linearize the differential equation for x near 0. [Section: 2.10]

52. Many systems are *piecewise* linear. That is, over a large range of variable values, the system can be described linearly. A system with amplifier saturation is one such example. Given the differential equation

$$\frac{d^2x}{dt^2} + 17 \frac{dx}{dt} + 50x = f(x)$$

assume that $f(x)$ is as shown in Figure P2.32. Write the differential equation for each of the following ranges of x : [Section: 2.10]

a. $-\infty < x < -3$

b. $-3 < x < 3$

c. $3 < x < \infty$

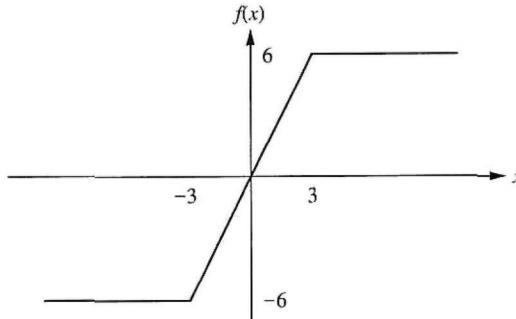


FIGURE P2.32

53. For the translational mechanical system with a nonlinear spring shown in Figure P2.33, find the transfer function, $G(s) = X(s)/F(s)$, for small excursions around $f(t) = 1$. The spring is defined by $x_s(t) = 1 - e^{-f_s(t)}$, where $x_s(t)$ is the spring displacement and $f_s(t)$ is the spring force. [Section: 2.10]

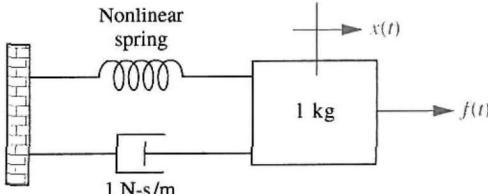


FIGURE P2.33

54. Consider the restaurant plate dispenser shown in Figure P2.34, which consists of a vertical stack of dishes supported by a compressed spring. As each plate is removed, the reduced weight on the dispenser causes the remaining plates to rise. Assume that the mass of the system minus the top plate is M , the viscous friction between the piston and the sides of the cylinder is f_v , the spring constant is K , and the weight of a single plate is W_D . Find the transfer function, $Y(s)/F(s)$, where $F(s)$ is the step reduction in force felt when the top plate is removed, and $Y(s)$ is the vertical displacement of the dispenser in an upward direction.

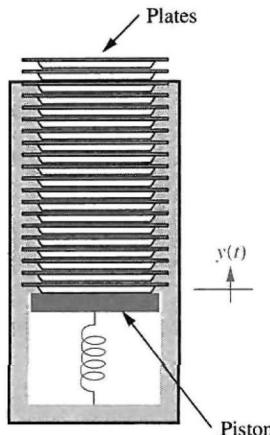


FIGURE P2.34 Plate dispenser

55. Each inner ear in a human has a set of three nearly perpendicular semicircular canals of about 0.28 mm in diameter filled with fluid. Hair-cell transducers

that deflect with skull movements and whose main purpose is to work as attitude sensors as well as help us maintain our sense of direction and equilibrium are attached to the canals. As the hair cells move, they deflect a waterproof flap called the *cupula*. It has been shown that the skull and cupula movements are related by the following equation (Milsum, 1966):

$$J\ddot{\phi} + b\dot{\phi} + k\phi = (aJ)\ddot{\psi}$$

where

J = moment of inertia of the fluid in the thin tube (constant)

b = torque per unit relative angular velocity (constant)

k = torque per unit relative angular displacement (constant)

a = constant

$\phi(t)$ = angular deflection of the cupula (output)

$\ddot{\psi}(t)$ = skull's angular acceleration (input)

Find the transfer function $\frac{\Phi(s)}{\ddot{\Psi}(s)}$.

56. Diabetes is an illness that has risen to epidemic proportions, affecting about 3% of the total world population in 2003. A differential equation model that describes the total population size of diabetics is

$$\frac{dC(t)}{dt} = -(\lambda + \mu + \delta + \gamma + \nu)C(t) + \lambda N(t)$$

$$\frac{dN(t)}{dt} = -(\nu + \delta)C(t) - \mu N(t) + I(t)$$

with the initial conditions $C(0) = C_0$ and $N(0) = N_0$ and

$I(t)$ = the system input: the number of new cases of diabetes

$C(t)$ = number of diabetics with complications

$N(t)$ = the system output: the total number of diabetics with and without complications

μ = natural mortality rate (constant)

λ = probability of developing a complication (constant)

δ = mortality rate due to complications (constant)

ν = rate at which patients with complications become severely disabled (constant)

γ = rate at which complications are cured (constant)

Assume the following values for parameters: $\nu = \delta = 0.05/\text{yr}$, $\mu = 0.02/\text{yr}$, $\gamma = 0.08/\text{yr}$, $\lambda = 0.7$, with initial conditions $C_0 = 47,000,500$ and $N_0 = 61,100,500$. Assume also that diabetic incidence is constant $I(t) = I = 6 \times 10^6$ (Boutayeb, 2004).

- Draw a block diagram of the system showing the output $N(s)$, the input $I(s)$, the transfer function, and the initial conditions.
 - Use any method to find the analytic expression for $N(t)$ for $t \geq 0$.
57. The circuit shown in Figure P2.35(a) is excited with the pulse shown in Figure P2.35(b).

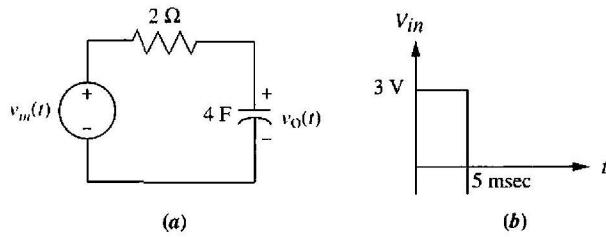


FIGURE P2.35

The Laplace transform can be used to calculate $v_o(t)$ in two different ways: The “exact” method is performed by writing $v_i(t) = 3[u(t) - u(t - 0.005)]$, from which we use the Laplace transform to obtain

$$V_{in}(s) = 3 \frac{1 - e^{-0.005s}}{s}$$

(Hint: look at Item 5 in Table 2.2, the time shift theorem.) In the second approach the pulse is approximated by an impulse input having the same area (energy) as the original input. From Figure P2.35(b): $v_{in}(t) \approx (3 \text{ V})(5 \text{ msec}) \delta(t) = 0.015\delta(t)$. In this case, $V_{in}(s) = 0.015$. This approximation can be used as long as the width of the pulse of Figure P2.35(b) is much smaller than the circuit’s smallest time constant. (Here, $\tau = RC = (2\Omega)(4 \text{ F}) = 8 \text{ sec} \gg 5 \text{ msec}$.)

- Assuming the capacitor is initially discharged, obtain an analytic expression for $v_o(t)$ using both methods.
 - Plot the results of both methods using any means available to you, and compare both outputs. Discuss the differences.
58. In a magnetic levitation experiment a metallic object is held up in the air suspended under an electromagnet. The vertical displacement of the object can

be described by the following nonlinear differential equation (Galvão, 2003):

$$m \frac{d^2 H}{dt^2} = mg - k \frac{I^2}{H^2}$$

where

m = mass of the metallic object

g = gravity acceleration constant

k = a positive constant

H = distance between the electromagnet and the metallic object (output signal)

I = electromagnet’s current (input signal)

- Show that a system’s equilibrium will be achieved when $H_0 = I_0 \sqrt{\frac{k}{mg}}$.
- Linearize the equation about the equilibrium point found in Part a and show that the resulting transfer function obtained from the linearized differential equation can be expressed as

$$\frac{\delta H(s)}{\delta I(s)} = - \frac{a}{s^2 - b^2}$$

with $a > 0$. Hint: to perform the linearization, define $\delta H = H(t) - H_0$ and $\delta I = I(t) - I_0$; substitute into the original equation. This will give

$$m \frac{d^2(H_0 + \delta H)}{dt^2} = mg - k \frac{(I_0 + \delta I)^2}{(H_0 + \delta H)^2} = \gamma$$

Now get a first-order Taylor’s series approximation on the right-hand side of the equation. Namely, calculate

$$m \frac{d^2 \delta H}{dt^2} = \frac{\partial \gamma}{\partial \delta H} \Big|_{\delta H=0, \delta I=0} \delta H + \frac{\partial \gamma}{\partial \delta I} \Big|_{\delta H=0, \delta I=0} \delta I$$

59. Figure P2.36 shows a quarter-car model commonly used for analyzing suspension systems. The car’s tire is considered to act as a spring without damping, as shown. The parameters of the model are (Lin, 1997)

M_b = car’s body mass

M_{us} = wheel’s mass

K_a = strut’s spring constant

K_t = tire’s spring constant

f_v = strut’s damping constant

r = road disturbance (input)

x_s = car’s vertical displacement

x_w = wheel’s vertical displacement

Obtain the transfer function from the road disturbance to the car's vertical displacement $\frac{X_s(s)}{R(s)}$.

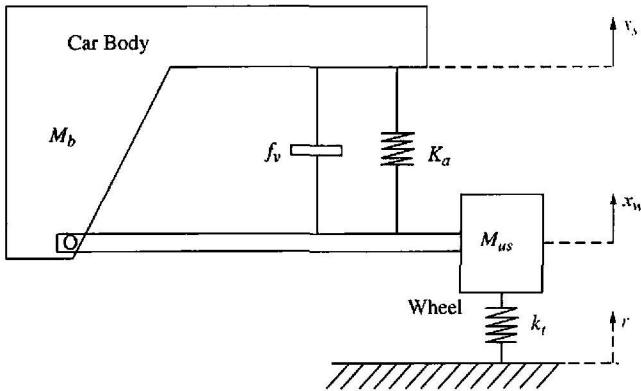
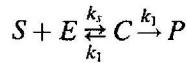


FIGURE P2.36 Quarter-car model used for suspension design. (© 1997 IEEE)

60. Enzymes are large proteins that biological systems use to increase the rate at which reactions occur. For example, food is usually composed of large molecules that are hard to digest; enzymes break down the large molecules into small nutrients as part of the digestive process. One such enzyme is amylase, contained in human saliva. It is commonly known that if you place a piece of uncooked pasta in your mouth its taste will change from paper-like to sweet as amylase breaks down the carbohydrates into sugars. Enzyme breakdown is often expressed by the following relation:



In this expression a substrate (S) interacts with an enzyme (E) to form a combined product (C) at a rate k_1 . The intermediate compound is reversible and gets disassociated at a rate k_{-1} . Simultaneously some of the compound is transformed into the final product (P) at a rate k_2 . The kinetics describing this reaction are known as the Michaelis-Menten equations and consist of four nonlinear differential equations. However, under some conditions these equations can be simplified. Let E_0 and S_0 be the initial concentrations of enzyme and substrate, respectively. It is generally accepted that under some energetic conditions or when the enzyme concentration is very big ($E_0 \gg S_0$),

the kinetics for this reaction are given by

$$\frac{dS}{dt} = k_\psi (\tilde{K}_s C - S)$$

$$\frac{dC}{dt} = k_\psi (S - \tilde{K}_M C)$$

$$\frac{dP}{dt} = k_2 C$$

where the following constant terms are used (Schnell, 2004) :

$$k_\psi = k_1 E_0$$

$$\tilde{K}_s = \frac{k-1}{k_\psi}$$

and

$$\tilde{K}_M = \tilde{K}_s + \frac{k_2}{k_\psi}$$

- Assuming the initial conditions for the reaction are $S(0) = S_0$, $E(0) = E_0$, $C(0) = P(0) = 0$, find the Laplace transform expressions for S , C , and P : $\mathcal{L}\{S\}$, $\mathcal{L}\{C\}$, and $\mathcal{L}\{P\}$, respectively.
- Use the final theorem to find $S(\infty)$, $C(\infty)$, and $P(\infty)$.
- Humans are able to stand on two legs through a complex feedback system that includes several sensory inputs—equilibrium and visual along with muscle actuation. In order to gain a better understanding of the workings of the postural feedback mechanism, an individual is asked to stand on a platform to which sensors are attached at the base. Vibration actuators are attached with straps to the individual's calves. As the vibration actuators are stimulated, the individual sways and movements are recorded. It was hypothesized that the human postural dynamics are analogous to those of a cart with a balancing standing pole attached (inverted pendulum). In that case, the dynamics can be described by the following two equations:

$$J \frac{d^2\theta}{dt^2} = mgl \sin \theta(t) + T_{\text{bal}} + T_d(t)$$

$$T_{\text{bal}}(t) = -mgl \sin \theta(t) + kJ\theta(t) - \eta J\dot{\theta}(t)$$

$$-\rho J \int_0^t \theta(t) dt$$

where m is the individual's mass; l is the height of the individual's center of gravity; g is the gravitational constant; J is the individual's equivalent moment of inertia; η , ρ , and k are constants given by the body's postural control system; $\theta(t)$ is the

individual's angle with respect to a vertical line; $T_{\text{bal}}(t)$ is the torque generated by the body muscles to maintain balance; and $T_d(t)$ is the external torque input disturbance. Find the transfer function $\frac{\Theta(s)}{T_d(s)}$ (Johansson, 1988).

62. Figure P2.37 shows a crane hoisting a load. Although the actual system's model is highly nonlinear, if the rope is considered to be stiff with a fixed length L , the system can be modeled using the following equations:

$$\begin{aligned} m_L \ddot{x}_{La} &= m_L g \phi \\ m_T \ddot{x}_T &= f_T - m_L g \phi \\ x_{La} &= x_T - x_L \\ x_L &= L \phi \end{aligned}$$

where m_L is the mass of the load, m_T is the mass of the cart, x_T and x_L are displacements as defined in the figure, ϕ is the rope angle with respect to the vertical, and f_T is the force applied to the cart (Martinen, 1990).

- a. Obtain the transfer function from cart velocity to rope angle $\frac{\Phi(s)}{V_T(s)}$.
- b. Assume that the cart is driven at a constant velocity V_0 and obtain an expression for the resulting $\phi(t)$. Show that under this condition, the load will sway with a frequency $\omega_0 = \sqrt{\frac{g}{L}}$.
- c. Find the transfer function from the applied force to the cart's position, $\frac{X_T(s)}{F_T(s)}$.
- d. Show that if a constant force is applied to the cart, its velocity will increase without bound as $t \rightarrow \infty$.

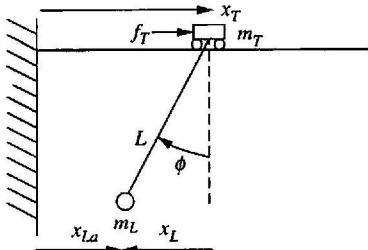


FIGURE P2.37 (© 1990 IEEE)

63. In 1978, Malthus developed a model for human growth population that is also commonly used to model bacterial growth as follows. Let $N(t)$ be the population density observed at time t . Let K be the

rate of reproduction per unit time. Neglecting population deaths, the population density at a time $t + \Delta t$ (with small Δt) is given by

$$N(t + \Delta t) \approx N(t) + KN(t)\Delta t$$

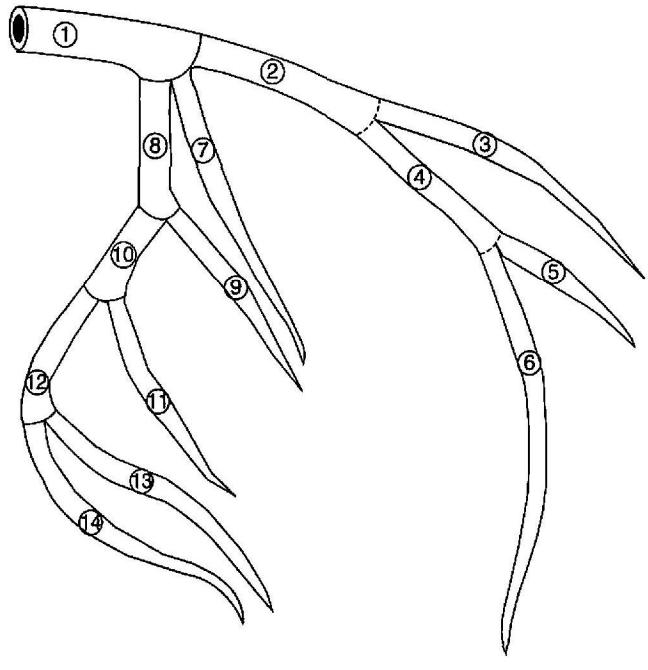
which also can be written as

$$\frac{N(t + \Delta t) - N(t)}{\Delta t} = KN(t)$$

Since $N(t)$ can be considered to be a very large number, letting $\Delta t \rightarrow 0$ gives the following differential equation (Edelstein-Keshet, 2005):

$$\frac{dN(t)}{dt} = KN(t)$$

- a. Assuming an initial population $N(0) = N_0$, solve the differential equation by finding $N(t)$.
- b. Find the time at which the population is double the initial population.
- 64. Blood vessel blockages can in some instances be diagnosed through noninvasive techniques such as the use of sensitive microphones to detect flow acoustic anomalies. In order to predict the sound properties of the left coronary artery, a model has been developed that partitions the artery into 14 segments, as shown in Figure P2.38(a).



(a)

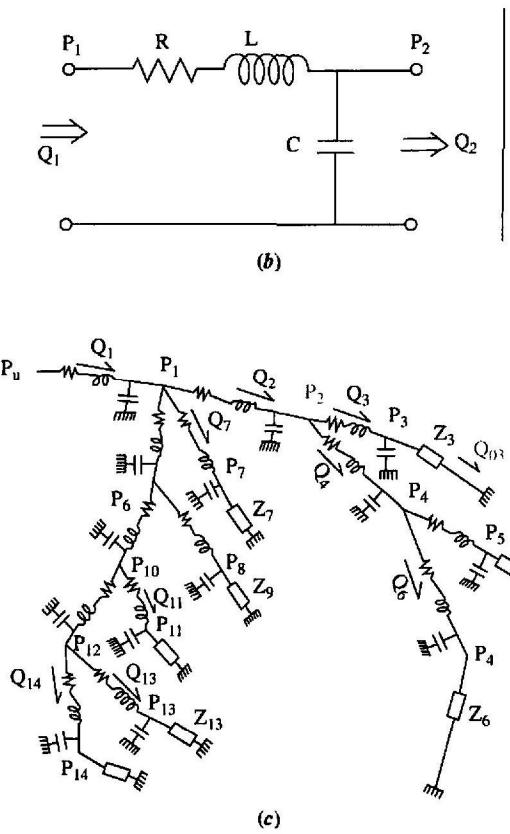


FIGURE P2.38 (© 1990 IEEE)

Each segment is then modeled through the analogous electrical circuit of Figure P2.38(b), resulting in the total model shown in Figure P2.38(c), where eight terminal resistances (Z) have been added. In the electrical model, pressure is analogous to voltage and blood flow is analogous to current. As an example, for Segment 3 it was experimentally verified that $R_3 = 4176 \Omega$, $C_3 = 0.98 \mu\text{F}$, $L_3 = 140.6 \text{ H}$, and $Z_3 = 308,163 \Omega$ (Wang, 1990).

- For Segment 3, find the transfer function from input pressure to blood flow through Z_3 , $\frac{Q_{03}(s)}{P_2(s)}$.
- It is well known in circuit analysis that if a constant input is applied to a circuit such as the one of Figure P2.38(b), the capacitor can be substituted by an open circuit and the inductor can be substituted by a short circuit as time approaches infinity. Use this fact to calculate the flow through Z_3 after a constant unit pressure pulse is applied and time approaches infinity.

- Verify the result obtained in Part b using the transfer function obtained in Part a and applying the final value theorem.

- In order to design an underwater vehicle that has the characteristics of both a long-range transit vehicle (torpedo-like) and a highly maneuverable low-speed vehicle (boxlike), researchers have developed a thruster that mimics that of squid jet locomotion (Krieg, 2008). It has been demonstrated there that the average normalized thrust due to a command

step input, $U(s) = \frac{T_{ref}}{s}$, is given by:

$$T(t) = T_{ref}(1 - e^{-\lambda t}) + a \sin(2\pi f t)$$

where T_{ref} is the reference or desired thrust, λ is the system's damping constant, a is the amplitude of the oscillation caused by the pumping action of the actuator, f is the actuator frequency, and $T(t)$ is the average resulting normalized thrust. Find the thruster's transfer function $\frac{T(s)}{U(s)}$. Show all steps.

- The Gompertz growth model is commonly used to model tumor cell growth. Let $v(t)$ be the tumor's volume, then

$$\frac{dv(t)}{dt} = \lambda e^{-\alpha t} v(t)$$

where λ and α are two appropriate constants (Edelstein-Keshet, 2005).

- Verify that the solution to this equation is given by $v(t) = v_0 e^{\lambda/\alpha(1-e^{-\alpha t})}$, where v_0 is the initial tumor volume.
- This model takes into account the fact that when nutrients and oxygen are scarce at the tumor's core, its growth is impaired. Find the final predicted tumor volume (let $t \rightarrow \infty$).
- For a specific mouse tumor, it was experimentally found that $\lambda = 2.5$ days, $\alpha = 0.1$ days with $v_0 = 50 \times 10^{-3} \text{ mm}^3$ (Chignola, 2005). Use any method available to make a plot of $v(t)$ vs. t .
- Check the result obtained in Part b with the results from the graph from Part c.

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

- High-speed rail pantograph.** Problem 21 in Chapter 1 discusses active control of a pantograph mechanism for high-speed rail systems. The diagram for the pantograph and catenary coupling is shown in Figure P2.39(a). Assume the simplified model shown in Figure P2.39(b), where the catenary is represented by the spring, K_{ave} (O'Connor, 1997).

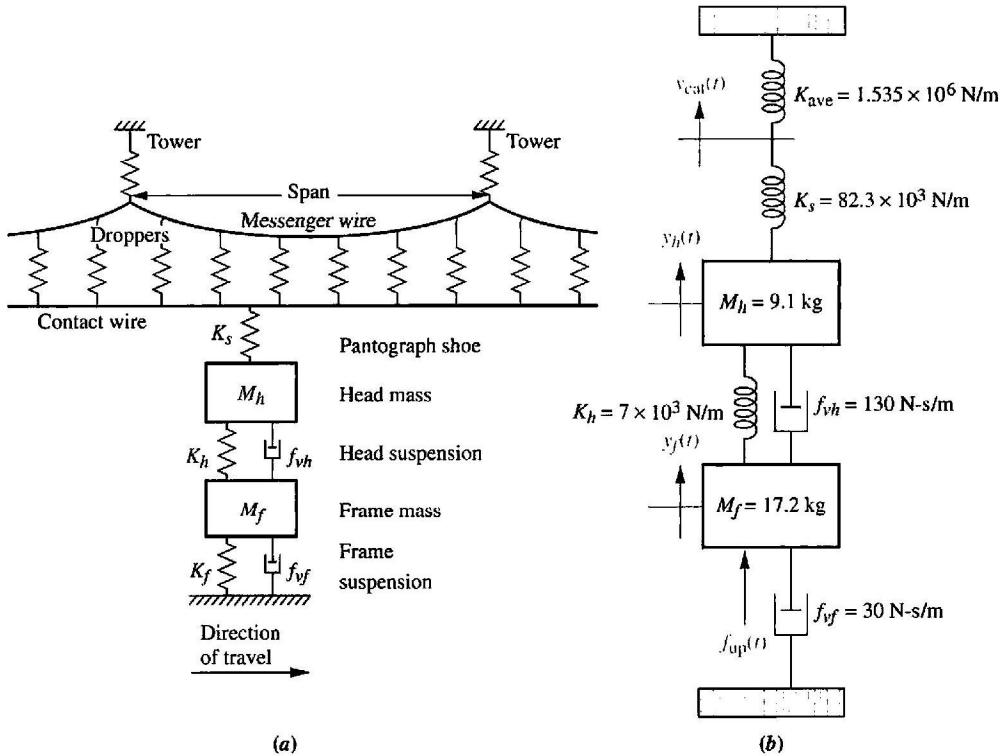


FIGURE P2.39 a. Coupling of pantograph and catenary; b. simplified representation showing the active-control force (Reprinted with permission of ASME.)

- Find the transfer function, $G_1(s) = Y_{cat}(s)/F_{up}(s)$, where $y_{cat}(t)$ is the catenary displacement and $f_{up}(t)$ is the upward force applied to the pantograph under active control.
 - Find the transfer function $G_2(s) = Y_h(s)/F_{up}(s)$, where $y_h(t)$ is the pantograph head displacement.
 - Find the transfer function, $G(s) = (Y_h(s) - Y_{cat}(s))/F_{up}(s)$.
68. **Control of HIV/AIDS.** HIV inflicts its damage by infecting healthy CD4 + T cells (a type of white blood cell) that are necessary to fight infection. As the virus embeds in a T cell and the immune system produces more of these cells to fight the infection, the virus propagates in an opportunistic fashion. As we now develop a simple HIV model, refer to Figure P2.40. Normally T cells are produced at a rate s and die at a rate d . The HIV virus is present in the bloodstream in the infected individual. These viruses in the bloodstream, called *free viruses*, infect healthy T cells at a rate β . Also, the viruses reproduce through the T cell multiplication process or otherwise at a rate k . Free viruses die at a rate c . Infected T cells die at a rate μ .

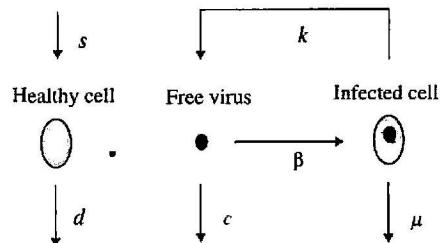


FIGURE P2.40 (© 2004 IEEE)

A simple mathematical model that illustrates these interactions is given by the following equations (Craig, 2004):

$$\frac{dT}{dt} = s - dT - \beta T v$$

$$\frac{dT^*}{dt} = \beta T v - \mu T^*$$

$$\frac{dv}{dt} = k T^* - c v$$

where

- T = number of healthy T cells
- T^* = number of infected T cells
- v = number of free viruses

- The system is nonlinear; thus linearization is necessary to find transfer functions as you will do in subsequent chapters. The nonlinear nature of this model can be seen from the above equations. Determine which of these equations are linear, which are nonlinear, and explain why.
- The system has two equilibrium points. Show that these are given by

$$(T_0, T_0^*, v_0) = \left(\frac{s}{d}, 0, 0 \right)$$

and

$$(T_0, T_0^*, v_0) = \left(\frac{c\mu}{\beta k}, \frac{s}{\mu} - \frac{cd}{\beta k}, \frac{sk}{c\mu} - \frac{d}{\beta} \right)$$

69. **Hybrid vehicle.** Problem 23 in Chapter 1 discusses the cruise control of serial, parallel, and split-power hybrid electric vehicles (HEVs). The functional block diagrams developed for these HEVs indicated that the speed of a vehicle depends upon the balance between the motive forces (developed by the gasoline engine and/or the electric motor) and running resistive forces. The resistive forces include the aerodynamic drag, rolling resistance, and climbing resistance. Figure P2.41 illustrates the running resistances for a car moving uphill (*Bosch, 2007*).

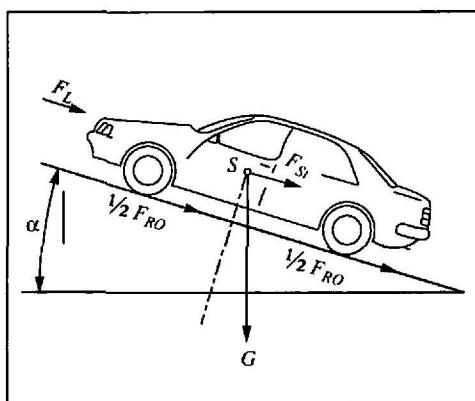


FIGURE P2.41 Running resistances

The total running resistance, F_w , is calculated as $F_w = F_{Ro} + F_L + F_{St}$, where F_{Ro} is the rolling resistance, F_L is the aerodynamic drag, and F_{St} is the climbing resistance. The aerodynamic drag is proportional to the square of the sum of car velocity, v , and the head-wind velocity, v_{hw} , or $v + v_{hw}$. The other two resistances are functions of car weight, G , and the gradient of the road (given by the gradient angle, α), as seen from the following equations:

$$F_{Ro} = fG \cos \alpha = fmg \cos \alpha$$

where

f = coefficient of rolling resistance,

m = car mass, in kg,

g = gravitational acceleration, in m/s^2 .

$$F_L = 0.5\rho C_w A(v + v_{hw})^2$$

where

ρ = air density, in kg/m^3 ,

C_w = coefficient of aerodynamic drag,

A = largest cross-section of the car, in kg/m^2 .

$$F_{St} = G \sin \alpha = mg \sin \alpha$$

The motive force, F , available at the drive wheels is:

$$F = \frac{T i_{tot}}{r} \eta_{tot} = \frac{P \eta_{tot}}{v}$$

where

T = motive torque,

P = motive power,

i_{tot} = total transmission ratio,

r = tire radius,

η_{tot} = total drive-train efficiency.

The surplus force, $F - F_w$, accelerates the vehicle (or retards it when $F_w > F$). Letting $a = \frac{F - F_w}{k_m \cdot m}$, where a is the acceleration and k_m is a coefficient that compensates for the apparent increase in vehicle mass due to rotating masses (wheels, flywheel, crankshaft, etc.):

- Show that car acceleration,¹⁶ a , may be determined from the equation:

$$F = fmg \cos \alpha + mg \sin \alpha + 0.5\rho C_w A(v + v_{hw})^2 + k_m ma$$

¹⁶ Other quantities, such as top speed, climbing ability, etc., may also be calculated by manipulation from that equation.