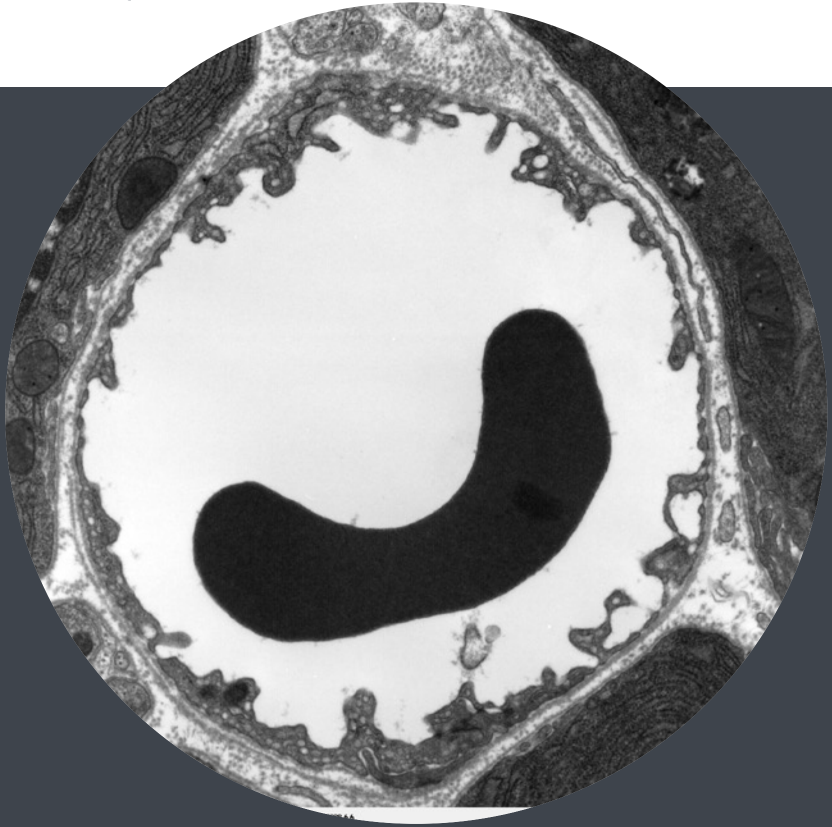
University of Stuttgart
Institute for Modelling Hydraulic and Environmental Systems

Chapter 1

Physiology and basics



Data-integrated Simulation Science A – Part D Biofluidmechnik

1.3 Dimensionless numbers characterizing blood flow

Dimensionless numbers in the context of blood flow

Navier Stokes equations

$$\overbrace{\rho\frac{\partial \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v})}^{\text{inertia}} - \overbrace{\nabla \cdot \boldsymbol{\sigma}}^{\text{viscous stress}} - \overbrace{\rho \boldsymbol{b}}^{\text{body forces}} = \boldsymbol{0},$$

$$\nabla \cdot (\rho \boldsymbol{v}) = 0,$$

Rescale:

$$\boldsymbol{v}^\star = \boldsymbol{v}/V$$
$$\nabla^\star = L\nabla$$
$$t^\star = tV/L$$
$$p^\star = pL/(V\mu)$$

Nondimensionalized Navier Stokes equations (momentum balance, no body forces)

$$\mathrm{Re}\left(\frac{\partial \boldsymbol{v}^\star}{\partial t^\star} + \nabla^\star \cdot (\boldsymbol{v}^\star \otimes \boldsymbol{v}^\star)\right) - \nabla^\star \cdot (2\boldsymbol{D}^\star(\boldsymbol{u}^\star) - p^\star \boldsymbol{I}) = \boldsymbol{0},$$

(see lecture notes for details)

$$\boxed{\mathrm{Re} := \frac{\rho V L}{\mu}}$$

- Reducing the parameter space dimension for experiments
- Good to get a feeling for the importance of certain effects when modelling new systems

Dimensionless numbers in the context of blood flow

Reynolds number

- Ratio of intertial forces to viscous forces
- Characterises flow type (laminar/turbulent)
- Tube flow stays turbulent above Re>2050
- In blood flow turbulence may occur at smaller Reynolds numbers (ca. 300-1000) depending on vessel geometry (see [1] and references therein), e.g. in aneurysms

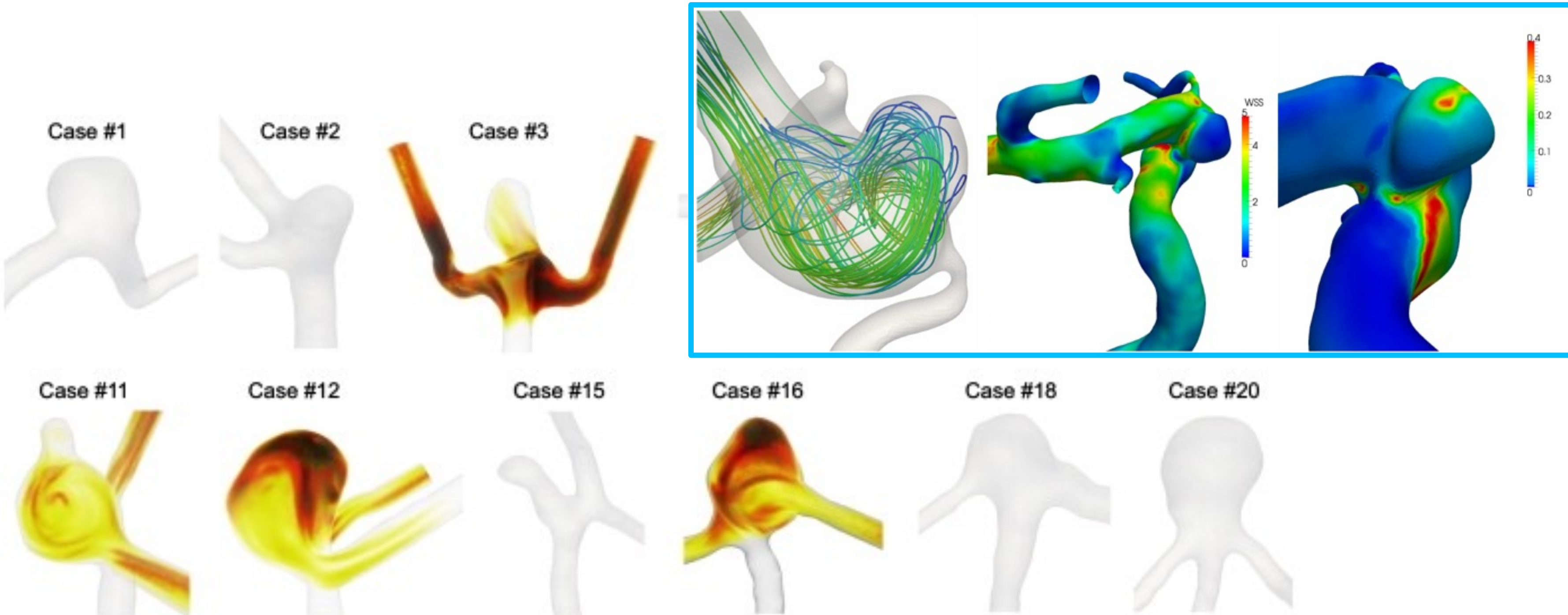
$$\boxed{\mathrm{Re} := \frac{\rho V L}{\mu}}$$

ρ: fluid density (kg/m^3); V: characteristic fluid velocity (m/s); L: characteristic length (e.g. vessel diameter); μ: dynamic viscosity (Pa·s)

[1] Saqr, K.M., Tupin, S., Rashad, S. *et al.* **Physiologic blood flow is turbulent.** *Sci Rep* **10,** 15492 (2020). https://doi.org/10.1038/s41598-020-72309-8

Dimensionless numbers in the context of blood flow

Example Aneurysms



Kristian Valen-Sendstad, Kent-André Mardal, David A. Steinman, (2013) **High-resolution CFD detects high-frequency velocity fluctuations in bifurcation, but not sidewall, aneurysms**, Journal of Biomechanics, https://doi.org/10.1016/j.jbiomech.2012.10.042.

Dimensionless numbers in the context of blood flow

Womersley number [1,2] (“Wo“, also “α“)

- Ratio of (pulsatile) inertia forces to viscous forces
- Characterises influence of pulsatility
- Can be used to define microcirculation (Wo < 1)

(Wo << 1): friction-dominated

(Wo >> 1): inertia-dominated

$$\text{Wo} := \frac{L}{2}\left(\frac{\rho\omega}{\mu}\right)^{1/2}$$

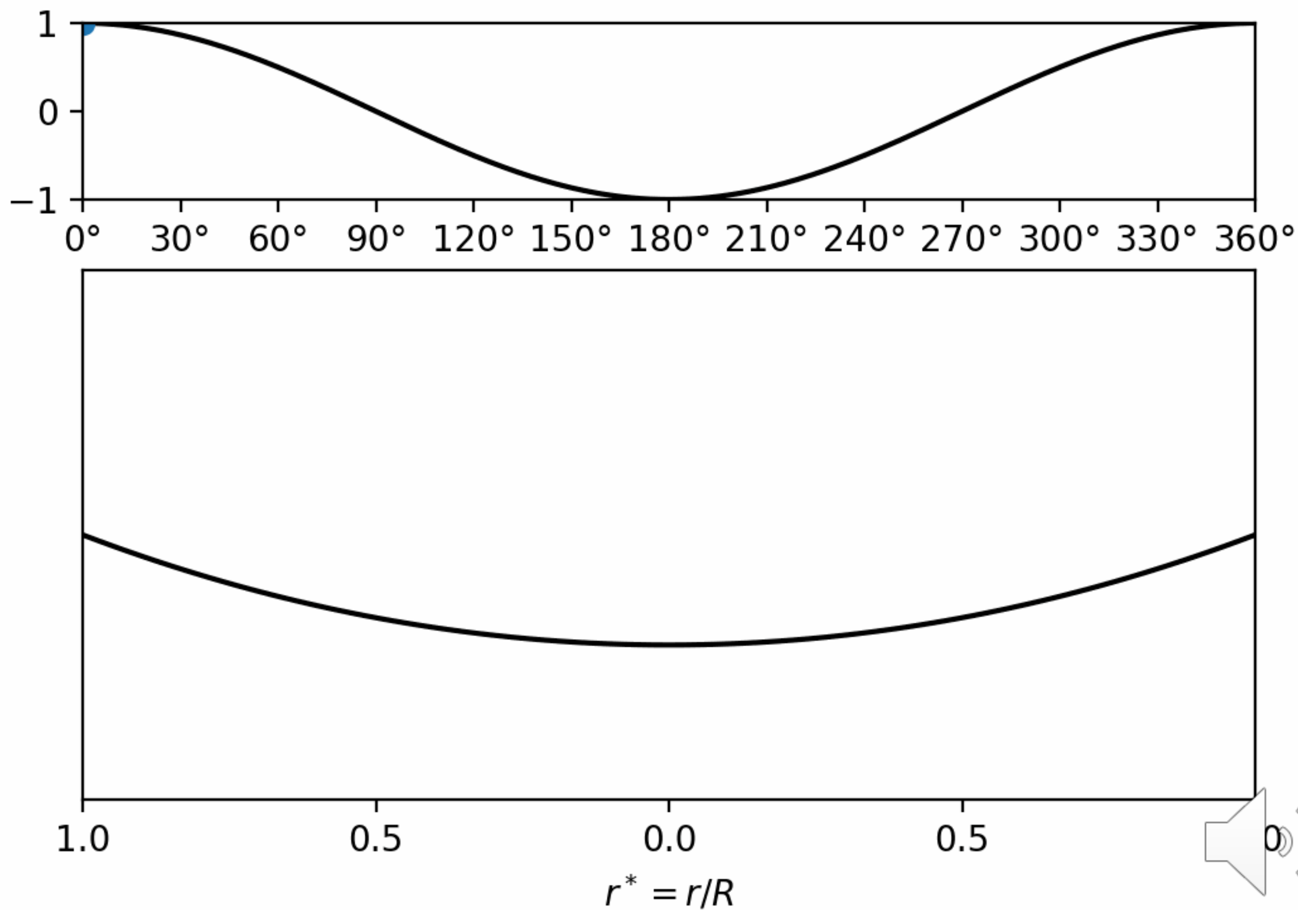
ρ: fluid density (kg/m^3); L: Characteristic length (m; e.g. vessel diameter); μ: dynamic viscosity (Pa·s); ω: Angular frequency (s^{-1}, e.g. 2π · heart beat frequency)

[1] Womersley, J. R. (1955). **Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known**. *The Journal of physiology*, *127*(3), 553-563. https://doi.org/10.1113/jphysiol.1955.sp005276

[2] Womersley, J. R. (1957). **Oscillatory Flow in Arteries: the Constrained Elastic Tube as a Model of Arterial Flow and Pulse Transmission**. *Phys. Med. Biol.* **2** 178 https://doi.org/10.1088/0031-9155/2/2/305

Womersley number

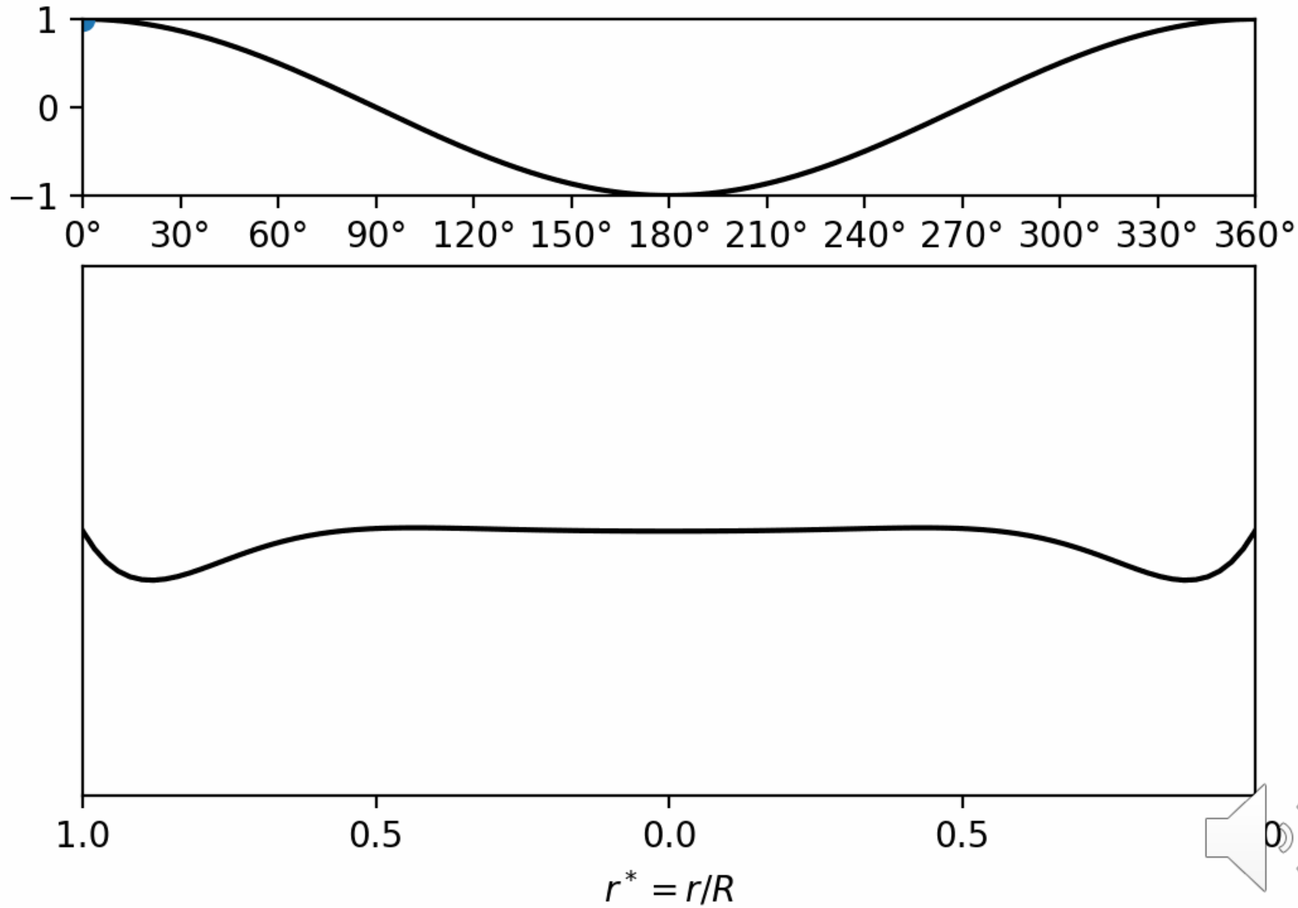
Poiseuille flow, no boundary layer effects, velocity and pressure gradient in sync



Wo = 1

Womersley number

Some boundary layer effects

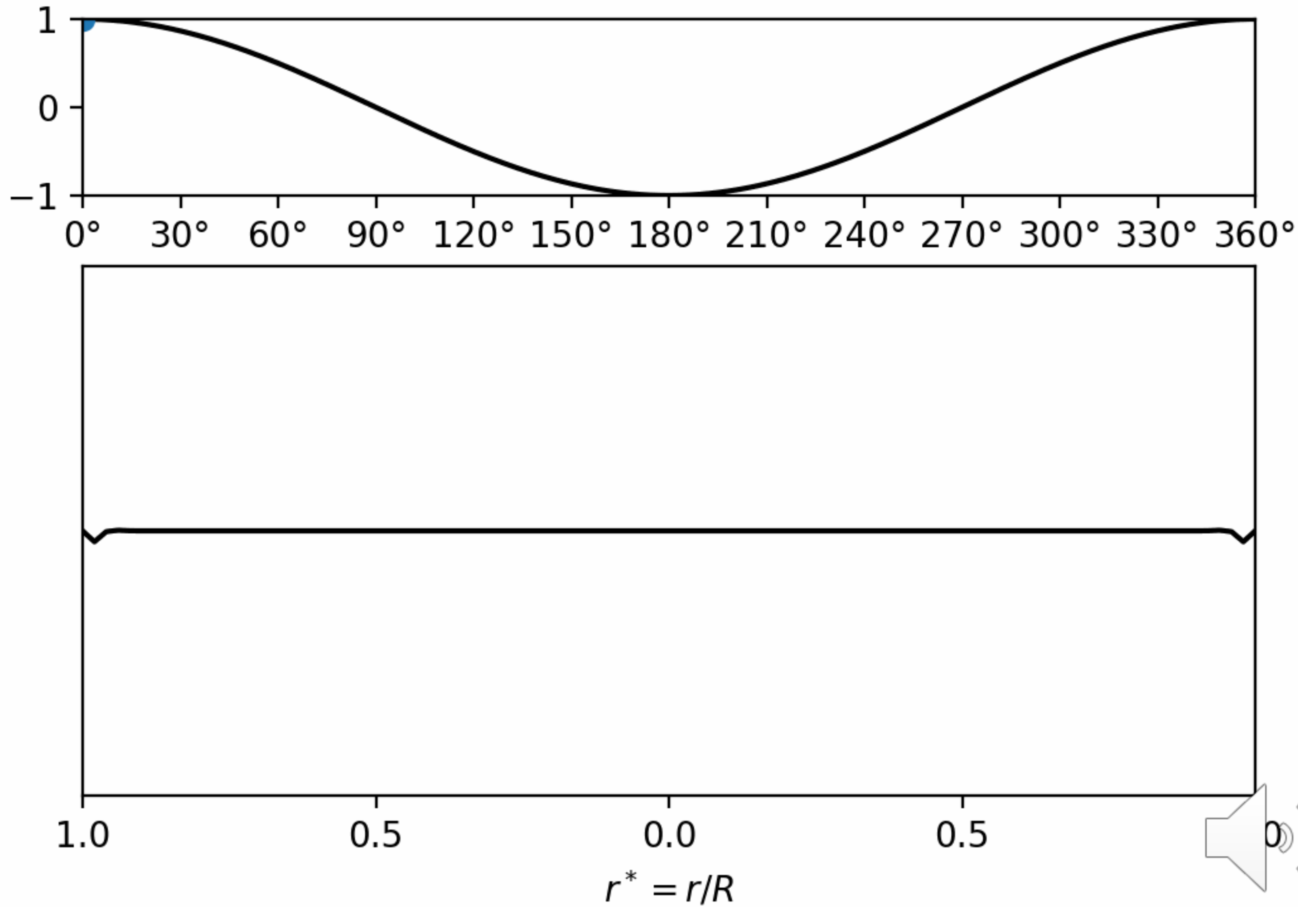


Wo = 10

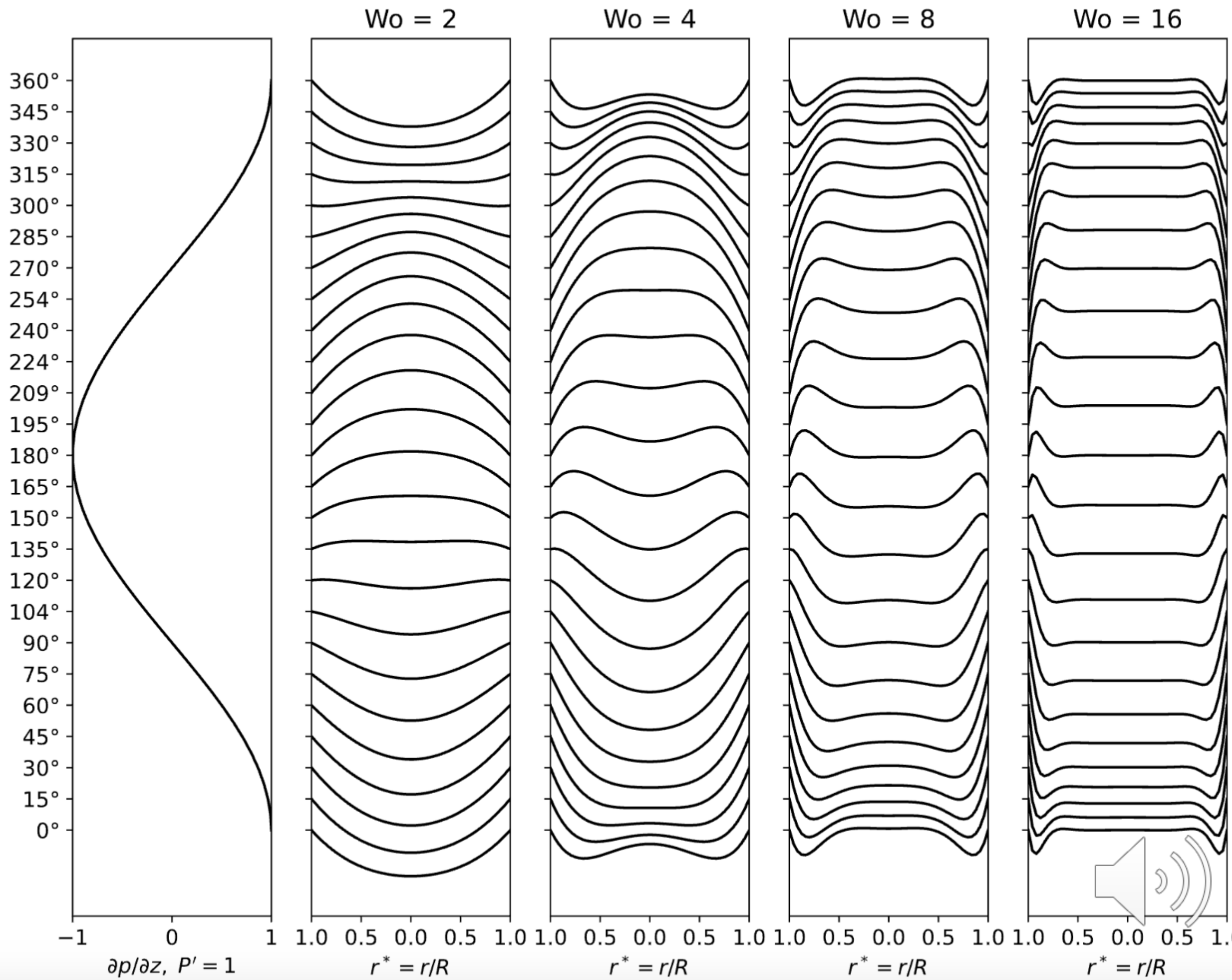
Womersley number

Strong boundary layer effects
Velocity and pressure gradient peak in 90° phase shift

Wo = 100



Womersley number



Dimensionless numbers in the context of blood flow

Womersley and Reynolds numbers (estimates / overview)

Vessel type	Wall thickness in mm	Blood volume (%)	Re	Wo
Aorta	2.0	2.0	4000.0	10.5
Large arteries	1.0	5.0	2500.0	7.7
Small arteries	1.0	5.0	100.0	1.4
Arterioles	0.03	5.0	0.5	0.014
Capillaries	0.001	5.0	0.003	0.007
Venules	0.003	25.0	0.5	0.014
Large veins	0.5	50.0	1600.0	5.6

this course

Dimensionless numbers in the context of blood flow

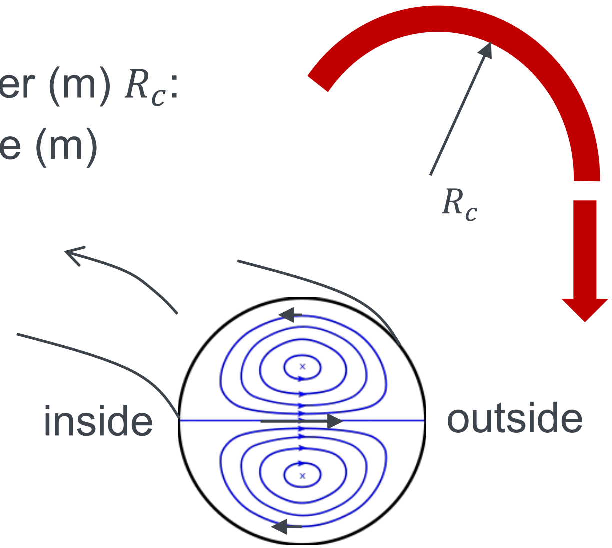
Dean number [1,2] (curved pipes)

- Ratio of inertial and centripetal forces to viscous forces

$$\boxed{\mathrm{De} := \mathrm{Re}\sqrt{\frac{L}{2R_c}}}$$

L: Channel diameter (m) R_c: Radius of curvature (m)

- De < ≈ 60: unidirectional flow (no vortices)
- De ≈ 60-80: two stable Dean vortices
- De > ≈ 400: turbulence



[1] Dean, W. R. (1927). **Note on the motion of fluid in a curved pipe**. Phil. Mag. 4 (20): 208–223. doi:10.1080/14786440708564324.
[2] Dean, W. R. (1928). **The streamline motion of fluid in a curved pipe**. Phil. Mag. Series 7. 5 (30): 673–695. doi:10.1080/14786440408564513.

Dimensionless numbers in the context of blood flow

Péclet number (mass transport)

- Ratio of advective to diffusive flow/transport
- Characterises dominant transport mechanism (>> 1: advection-dominated, < 1: diffusion-dominated)

$$\boxed{\mathrm{Pe} \coloneqq \frac{LV}{D}}$$

L: Characteristic length (m; here: *axial* distance);
V: Characteristic fluid velocity (m/s);
D: Diffusion coefficient (m^2/s)

Example: Oxygen (D ≈ 2e-5 cm^2/s) in artery (L=1cm, v=10cm/s), capillary (L=1mm, v=0.5mm/s), extra-vascular tissue (L=50μm, v=0.01μm/s):

Artery: Pe ≈ 5e5 → strongly advection-dominated; **Capillary**: Pe ≈ 250 → slightly advection-dominated; **extra-vascular tissue**: Pe ≈ 0.00025 → diffusion-dominated

Vascular system (Summary)

Complexity:

- (Hierarchical) structure of the network
- Large number of vessels

Diversity:

- Different types of flow
- Different properties of blood vessels
- Amount/position/size of features such as vessels may vary significantly among individuals (even in the same species)
- Modelling and simulation is demanding (large data quantity, more general models to account for patient-specific data instead of simulations in simplified geometries)

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Thank you!

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