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Task 1:

$$L = T - U \qquad \Bigg| \; T = \sum_i \frac{1}{2} m_i v_i^2 + \frac{1}{2} w_i^T I_i w_i$$

$$T = \frac{1}{2}\left(m_1 \sqrt{\dot{x}_{s_1}^2 + \dot{y}_{s_1}^2}^2 + m_2 \sqrt{\dot{x}_{s_2} + \dot{y}_{s_2}}^2 \right)$$

$$+ \frac{1}{2}\dot{q}_1 I_1 \dot{q}_1 + \frac{1}{2}(\dot{q}_1 + \dot{q}_2) I_2 (\dot{q}_1 + \dot{q}_2)$$

$$\dot{x}_{s_1} = \frac{d}{dt} x_{s_1} = -d_1 \cdot \sin(\tilde{q}_1) \cdot \dot{q}_1$$

$$\dot{y}_{s_1} = \frac{d}{dt} y_{s_1} = d_1 \cos(\hat{q}_1) \cdot \dot{q}_1$$

$$\dot{x}_{s_2} = \frac{d}{dt} x_{s_2} = -l_1 \sin(\tilde{q}_1)\dot{q}_1 - d_2 \sin(\hat{q}_1 + q_2)(\dot{q}_1 + \dot{q}_2)$$

$$\dot{y}_{s_2} = \frac{d}{dt} y_{s_2} = l_1 \cos(\tilde{q}_1)\dot{q}_1 + d_2 \cos(\hat{q}_1 + q_2)(\dot{q}_1 + \dot{q}_2)$$

$$\dot{x}_{s_1}^2 + \dot{y}_{s_1}^2 = d_1^2 \cdot \left(\sin^2(\hat{q}_1) + \cos^2(\tilde{q}_1) \right) \dot{q}_1^2$$

$$= d_1^2 \dot{q}_1^2$$

$$\dot{x}_{s_2}^2 + \dot{y}_{s_2}^2 = l_1^2 \sin^2(\tilde{q}_1)\dot{q}_1^2 + 2 l_1 \sin(\tilde{q}_1)\dot{q}_1 d_2 \sin(\hat{q}_1 + q_2)(\dot{q}_1 + \dot{q}_2)$$

$$+ d_2^2 \sin^2(\hat{q}_1 + q_2)(\dot{q}_1 + \dot{q}_2)^2$$

$$+ l_1^2 \cos^2(\tilde{q}_1)\dot{q}_1^2 + 2 l_1 \cos(\hat{q}_1)\dot{q}_1 d_2 \cos(\hat{q}_1 + q_2)(\tilde{q}_1 + \dot{q}_2)$$

$$+ d_2^2 \cos^2(\hat{q}_1 + q_2)(\dot{q}_1 + \dot{q}_2)^2$$

$$= l_1^2 \dot{q}_1^2 + d_2^2 (\dot{q}_1 + \dot{q}_2)^2$$

$$+ 2 l_1 \dot{q}_1 d_2 (\dot{q}_1 + \dot{q}_2) \left[\sin(\tilde{q_1}) \sin(\tilde{q_1} + q_2) + \cos(\tilde{q_1}) \cos(\hat{q_1} + q_2) \right]$$

$$= l_1^2 \dot{q}_1^2 + d_2^2 (\tilde{\dot{q_1}} + \dot{q_2})^2 + 2 l_1 \dot{q}_1 d_2 (\dot{q}_1 + \dot{q}_2) \cos(q_2)$$

$$\Rightarrow T = \frac{1}{2} m_1 \cdot d_1^2 \dot{q}_1^2$$

$$+ \frac{1}{2} m_2 \left(l_1^2 \dot{q}_1^2 + d_2^2 (\dot{q}_1 + \dot{q}_2)^2 + 2 l_1 \dot{q}_1 d_2 (\dot{q}_1 + \dot{q}_2) \cos(q_2) \right)$$

$$+ \frac{1}{2} I_1 \dot{q}_1^2 + \frac{1}{2} I_2 (\dot{q}_1 + \dot{q}_2)^2$$

$$U = m_1 \cdot g \cdot (y_{S_1}) + m_2 \cdot g \cdot (y_{S_2})$$

$$= m_1 g \cdot d_1 \sin(\tilde{q_1}) + m_2 \cdot g \cdot (l_1 \sin(\hat{q_1}) + d_2 \sin(\hat{q_1} + q_2))$$

$$L = T - U$$

$$\frac{1}{2} m_1 \cdot d_1^2 \dot{q}_1^2 + \frac{1}{2} m_2 \left(l_1^2 \dot{q}_1^2 + d_2^2 (\dot{q}_1 + \dot{q}_2)^2 + 2 l_1 \dot{q}_1 d_2 (\dot{q}_1 + \dot{q}_2) \cos(q_2) \right)$$

$$+ \frac{1}{2} I_1 \dot{q}_1^2 + \frac{1}{2} I_2 (\dot{q}_1 + \dot{q}_2)^2$$

$$- m_1 g \cdot d_1 \sin(\tilde{q_1}) - m_2 \cdot g \cdot (l_1 \sin(\hat{q_1}) + d_2 \sin(\hat{q_1} + q_2))$$

$$-\frac{\partial L}{\partial q_1} = + m_1 \cdot g \, d_1 \cos(\tilde{q_1}) + m_2 g \left(l_1 \cos(\tilde{q_1}) + d_2 \cos(\tilde{q_1} + q_2) \right)$$

$$= \cos(\tilde{q_1}) \cdot \left(+ m_1 g d_1 + m_2 g l_1 \right) + \cos(\tilde{q_1} + q_2) \left(m_2 g \, d_2 \right)$$

$$-\frac{\partial L}{\partial q_2} = m_2 l_1 \dot{q_1} d_2 (\dot{q_1} + \dot{q_2}) \sin(q_2) + m_2 g \, d_2 \cos(\tilde{q_1} + q_2)$$

$$\frac{\partial L}{\partial \dot{q_2}} = m_2 d_2^2 (\dot{q_1} + \dot{q_2}) + m_2 l_1 \dot{q_1} d_2 \cos(q_2) + I_2 (\dot{q_1} + \dot{q_2})$$

$$= \dot{q_2} \left[m_2 d_2^2 + I_2 \right)$$

$$+ \dot{q_1} \left[m_2 d_2^2 + m_2 l_1 d_2 \cos(q_2) + I_2 \right]$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q_2}} = \ddot{q_2} \left[m_2 d_2^2 + I_2 \right]$$

$$+ \dot{q_1} \dot{q_2} \left[- m_2 l_1 d_2 \sin(q_2) \right]$$

$$+ \ddot{q_1} \left[m_2 d_2^2 + m_2 l_1 d_2 \cos(q_2) + I_2 \right]$$

$$\frac{\partial L}{\partial \dot{q}_1} = m_1 d_1^2 \dot{q}_1 + \frac{1}{2} m_2 \left[2 l_1^2 \dot{q}_1 + d_2^2 \, 2 (\dot{q}_1 + \dot{q}_2) + 4 l_1 \dot{q}_1 d_2 \cos(q_2) \right.$$

$$\left. + 2 l_1 d_2 \dot{q}_2 \cos(q_2) \right] + I_1 \dot{q}_1 + I_2 (\dot{q}_1 + \dot{q}_2)$$

$$= \dot{q}_1 \left(m_1 d_1^2 + m_2 \left(l_1^2 + d_2^2 + 2 l_1 d_2 \cos(q_2) \right) + I_1 + I_2 \right)$$

$$+ \dot{q}_2 \left(m_2 \left(d_2^2 + l_1 d_2 \cos(q_2) \right) + I_2 \right)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_1} = \ddot{q}_1 \left(m_1 d_1^2 + m_2 \left(l_1^2 + d_2^2 + 2 l_1 d_2 \cos(q_2) \right) + I_1 + I_2 \right)$$

$$+ \dot{q}_1 \dot{q}_2 \left(-2 l_1 d_2 m_2 \sin(q_2) \right) - \dot{q}_2^2 m_2 l_1 d_2 \sin(q_2)$$

$$+ \ddot{q}_2 \left(m_2 \left(d_2^2 + l_1 d_2 \cos(q_2) \right) + I_2 \right)$$

$\square$ = Matrix M $\square$ = Vektor C

$$M =$$

$$
\begin{bmatrix}
m_1 d_1^2 + m_2\left(l_1^2 + d_2^2 + 2l_1 d_2 \cos(q_2)\right) + I_1 + I_2 & \left(m_2\left(d_2^2 + l_1 d_2 \cos(q_2)\right) + I_2\right) \\[2em]
\left\{m_2 d_2^2 + m_2 l_1 d_2 \cos(q_2) + I_2\right\} & \left[m_2 d_2^2 + I_2\right]
\end{bmatrix}
$$

$$C_{11} =$$

$$
+ \dot{q}_1 \dot{q}_2 \left(-2l_1 d_2 m_2 \sin(q_2)\right) - \dot{q}_2^2 m_2 l_1 d_2 \sin(q_2)
$$

$$
+ \cos(\hat{q_1}) \cdot \left(+ m_1 g d_1 + m_2 g l_1\right) - \cos(\hat{q_1} + q_2)\left(m_2 g \, d_2\right)
$$

$$
= - l_1 d_2 m_2 \sin(q_2)\left[2\dot{q}_1 \dot{q}_2 + \dot{q}_2^2\right]
$$

$$
+ \left[\cos(\hat{q_1})\left(m_1 d_1 + m_2 l_1\right) + \cos(\hat{q_1} + q_2)\left(m_2 d_2\right)\right] g
$$

$$C_{22} =$$

$$
+ \dot{q}_1 \dot{q}_2 \left[-m_2 l_1 d_2 \sin(q_2)\right]
$$

$$
+ m_2 l_1 \dot{q}_1 d_2 (\dot{q}_1 + \dot{q}_2)\sin(q_2) + m_2 g d_2 \cos(\hat{q_1} + q_2)
$$