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Aspiration Efficiency: Unified Model for All Forward Sampling Angles

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■ We have developed a unified aspiration efficiency model for sampling with round inlets at 0-90° forward angles from horizontal aerosol flows. The aspiration efficiency is represented by a single equation as a function of the sampling angle, the ratio of wind to inlet velocity, and different inertial parameters for 0-60° and 45-90° sampling. The equation for 45-90° sampling is an extension of the Laktionov equation for 90°. The new model fits the experimental data within experimental accuracy.

Introduction

When airborne particles are sampled in ambient and industrial environments through an inlet, three processes may change the original aerosol concentration and size distribution: (1) aspiration to the face of the inlet, (2) bounce from the front edge of the inlet, (3) transmission losses in the inlet. The changes that occur during sampling must be assessed quantitatively so that sampling errors can be compensated for.

The overall sampling efficiency, E_s, can thus be represented by the product of three distinct efficiencies:

$$E_s = E_a E_r E_t \tag{1}$$

where E_a is the aspiration efficiency, which is the ratio of particle concentration at the inlet face to the particle concentration in the undisturbed environment. E_r is the entry efficiency, which is the ratio of particle concentration passing the inlet face to the particle concentration incident to the inlet face. E_t is the transmission efficiency, which is the ratio of the particle concentration exiting the inlet to the particle concentration just past the inlet.

Whenever possible, inlets are designed with sharp edges to that particle bounce is negligible. The sharp-edge inlets are designed so as to meet the design criteria of Belyaev and Levin (*1*), which specify that for a sharp-edged inlet (a) the ratio of outer to inner inlet diameter is ≤1.1 and (b) if the ratio of outer to inner inlet diameter is ≥1.1, the ratio of inlet thickness to inlet inner diameter is ≤0.5 and the angle of taper is ≤15°. Thus,

$$E_s = E_a E_t \tag{2}$$

The overall sampling efficiency and transmission efficiency have been measured in our wind tunnel for a wide range of wind and inlet velocities, sampling angles, and round sharp-edged inlet sizes (*2-7*). The aspiration efficiency values, calculated from these data through eq 2, have been used for the development of our unified model for aspiration efficiency.

The wind condition in the ambient and in wind tunnel systems is turbulent. The effect of turbulence intensity and scale on aspiration efficiency of sharp-edge inlets was found to be negligible at isokinetic (*8*) and anisokinetic conditions (*9*). In our studies on the effect of turbulence (*7*), we found that the turbulence intensity and scale influenced particle deposition inside the inlet because turbulent motion is superimposed on the convective flow. Our data indicated that turbulence affected the transmission efficiency for small inlets but had negligible effect on the overall sampling efficiency, from which we also conclude that the effect of turbulence on aspiration efficiency is negligible. In our wind tunnel, the turbulence intensity varied from 1.4 to 7.5% and the scale ranged from 0.5 to 10 cm, which are close to the range of values used in the other studies on the effect of turbulence (*8, 9*). The effect of turbulence has been ignored in the studies on aspiration (*1, 10-16*).

Aspiration for round sharp-edged inlets has been studied extensively (*1, 10-16*) and several equations have been developed to describe it. Comparison of these equations with our wind tunnel data has shown that some of these equations agree with our data over specific ranges of sampling angle, but none of these equations agree over the entire range of 0-90° (*4-6*), where the sampling angle is the angle at which the inlet is oriented with respect to the horizontal wind direction (*2-7*).

Available Aspiration Equations

Studies on aspiration (*1, 10-16*) have shown that the aspiration efficiency depends on the inertial behavior of the particles, the flow conditions, and the sampling angle, θ. Therefore,

$$E_a = f(\mathrm{St}, R, \theta) \tag{3}$$

where Stokes number, St, and velocity ratio, R, are

$$\mathrm{St} = \tau U_w / D_i \tag{4}$$

$$R = U_w / U_i \tag{5}$$

Particle relaxation time, τ, is

$$\tau = \rho_p d_p^2 / 18\mu \tag{6}$$

where ρ_p is the particle density, μ is the air viscosity, D_i is the inlet diameter, U_w is the wind velocity, and U_i is the inlet velocity.

Belyaev and Levin (*1*) developed the following equation for isoaxial sampling from experimental data obtained by flash illumination photography:

$$E_a = 1 + (R - 1)[1 - 1/[1 + (2 + 0.617/R)\mathrm{St}]] \tag{7}$$

valid for $0.16 < R < 5.6$, $0.18 < \mathrm{St} < 2.03$.

Davies and Subari (*10*) developed aspiration efficiency equations for sampling at 0 and 90° based on experimental data at $R < 1$. The Davies and Subari equation at 0° is

$$E_a = 1 - (1 - R) \frac{\mathrm{St}(R)^{3/2}(4R + 0.62)}{0.1\mathrm{St}(1 - R) + (R)^{3/2}[1 + \mathrm{St}(4R + 0.62)]} \tag{8}$$

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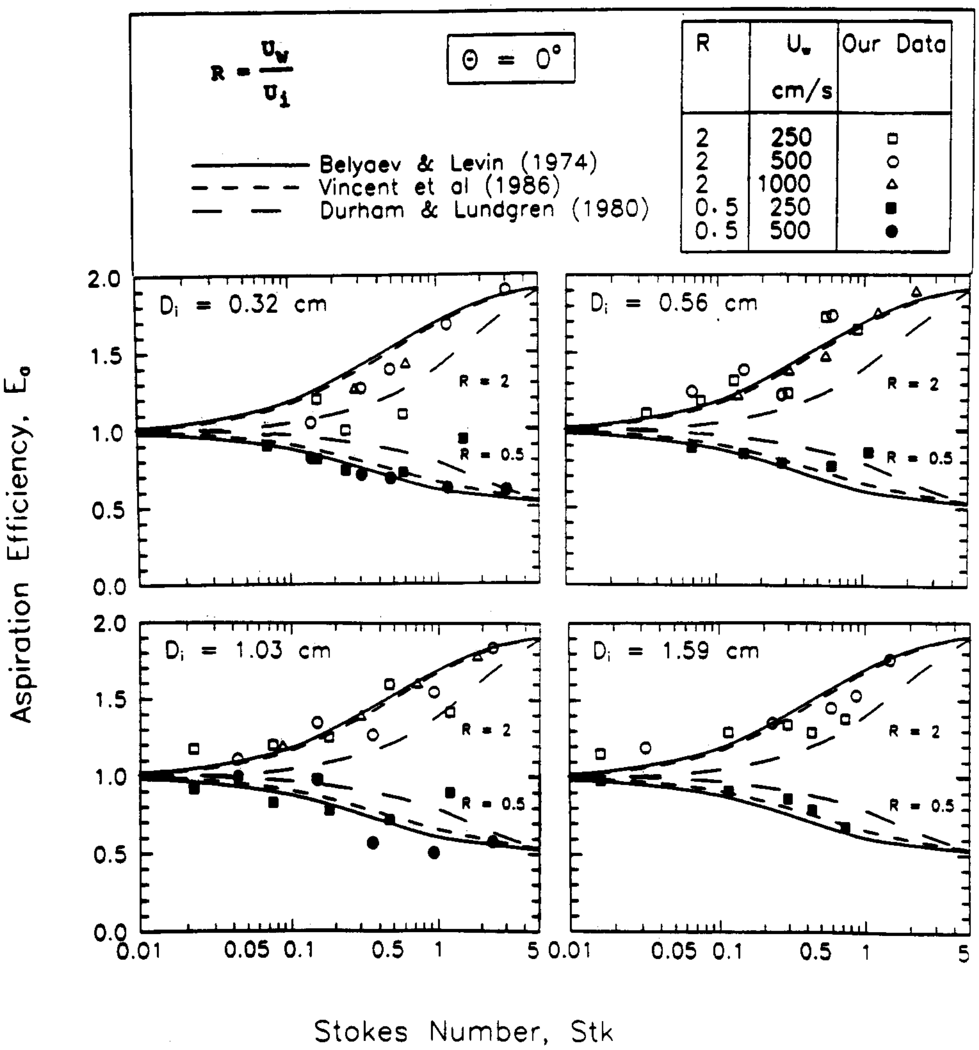

Figure 1. Aspiration efficiency data and predictions for isoaxial sampling.

valid for $0.055 \leq St \leq 6.89$.

The Davies and Subari equation for 90° is

$$E_a = 0.85 \exp[-3.8St(R)^{3/2}] \tag{9}$$

valid for $0.075 \leq St \leq 3.6$.

Laktionov (*11*) developed the following equation for 90° based on experimental data:

$$E_a = 1 - 3St^{R^{-0.5}} \tag{10}$$

valid for $1.25 \leq R \leq 6.25$; $0.003 \leq St \leq 0.2$.

Durham and Lundgren (*12*) were the first to develop an empirical equation for the entire range of 0–90°. Their equation as a function of sampling angle θ is as follows:

$$E_a = 1 + (R \cos \theta - 1)\frac{\beta(St', R)}{\beta(St', R = 1)}\beta(St') \tag{11}$$

where

$$St' = St \exp(0.022\,\theta) \tag{12}$$

$$\beta(St', R) = 1 - 1/[1 + (2 + 0.617/R)St'] \tag{13}$$

$$\beta(St') = 1 - 1/[1 + 0.55St' \exp(0.25St')] \tag{14}$$

valid for $0.01 \leq St \leq 6$.

Durham and Lundgren found that their data did not agree with their equation for 90° sampling at $R \neq 1$, but did agree with the Laktionov equation. Vincent et al. (*13*) developed the following equation for sampling at 0–90° based on their experimental data:

$$E_a = 1 + (R \cos \theta - 1)\left(1 - \frac{1}{1 + 2.1St(\cos \theta + 4R^{1/2} \sin^{1/2} \theta)}\right) \tag{15}$$

valid for $0.67 \leq R \leq 2.0$; $0.1 \leq St \leq 14.5$.

Unified Model

The development of our model has as a basis a large set of sampling efficiency data that has been obtained in our wind tunnel facility (*2–7*). Overall sampling efficiency, E_s, was determined from particle count measurements at the exit of the inlet relative to the aerosol concentration in the surrounding aerosol flow and the transmission efficiency, E_t, was determined by washoff techniques. In the work done by Okazaki and Willeke (*6*) and Okazaki et al. (*4, 5*), for 0 and 90° sampling angles, division of overall sampling efficiency by transmission efficiency gave aspiration efficiency data that matched the Belyaev and Levin equation at 0° and the Laktionov equation at 90°.

Curves representing eqs 7, 11, and 15 for aspiration efficiency at 0° sampling angle and our data for four inlet diameters are shown in Figure 1. As seen, predictions by the aspiration equations are within 10% of most of the values from our wind tunnel experiments. Therefore, the equations by Belyaev and Levin, Durham and Lundgren,

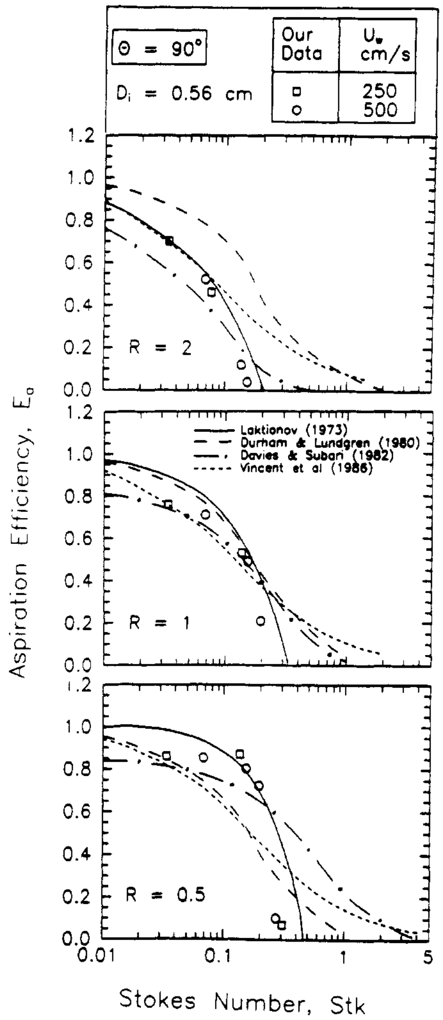

Figure 2. Data and predictions for 90° sampling.

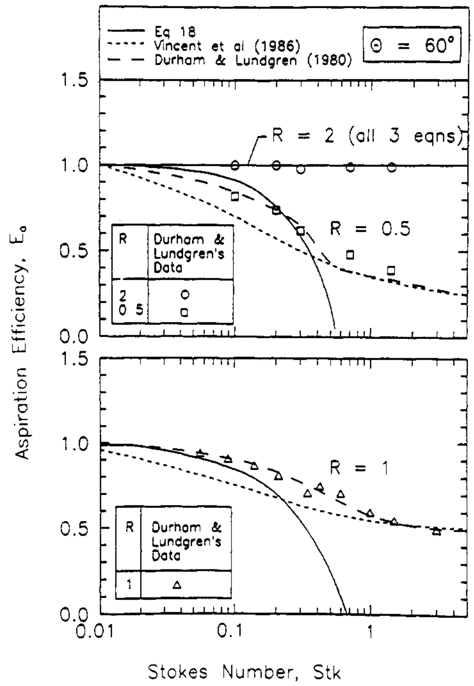

Figure 3. Data and predictions for 60° sampling.

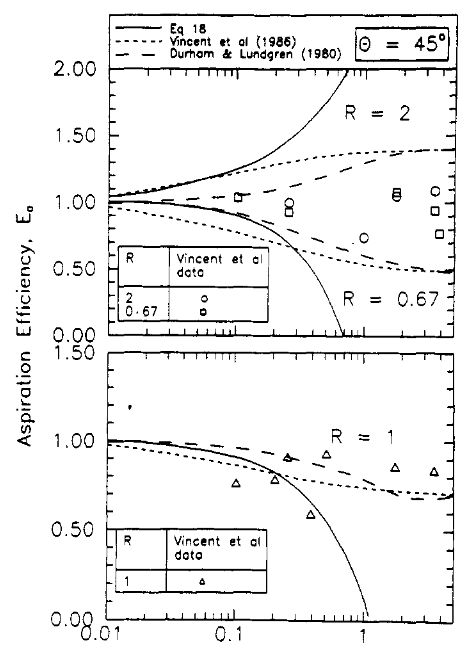

Figure 4. Data and predictions for 45° sampling.

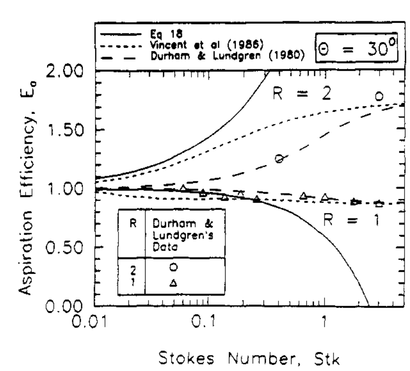

Figure 5. Data and predictions for 30° sampling.

and Vincent et al. appear to predict the aspiration efficiency within the range of experimental error.

For 90° sampling angle, our data and curves representing aspiration eqs 9, 10, 11, and 15 are shown in Figure 2. As seen, our aspiration efficiency data agree best with Laktionov, eq 10. At very small Stokes numbers, our aspiration efficiency values approach 0.85, as predicted by Davies and Subari (eq 9). For $R = 1$, the prediction by Laktionov (eq 10) agrees with Durham and Lundgren (eq 11) within the Laktionov experimental range $0.003 \leq \mathrm{St} \leq 0.2$.

Comparison of all the aspiration models (*1*, *10–16*) reveals that most of the aspiration equations (eqs 7, 10, 11, and 15) may be expressed in the following general form:

$$E_a = 1 + (R \cos \theta - 1)f \tag{16}$$

where f is an inertial parameter. This parameter is a function of Stokes number and velocity ratio and has been determined empirically for each of these equations based on experimental data.

Since our experimental aspiration efficiency data for 90° sampling agree with Laktionov, eq 10, we have extended the Laktionov equation in the form of eq 16 to include angles less than 90°. The inertial parameter is, by fitting eq 10 to eq 16,

$$f = 3\mathrm{St}^{R^{-0.5}} \tag{17}$$

Therefore, from eqs 10, 16, and 17, for angles 90° and below,

$$E_a = 1 + (R \cos \theta - 1)(3\mathrm{St}^{R^{-0.5}}) \tag{18}$$

At 90°, eq 18 reduces to eq 10.

For sampling at 60, 45, and 30° comparisons between experimental data and predictive eqs 11, 15, and 18 are shown in Figures 3–5. The experimental data for 60 and 30° sampling are from Durham and Lundgren (*12*) and for 45° sampling from Vincent et al. (*13*). The prediction by eq 18 agrees with the experimental data up to $St \approx 0.2$ for 60° at all R values, and for 45 and 30° at $R = 1$. The prediction by Durham and Lundgren (eq 11) agrees with experimental data for 60 and 30° at all R values and for 45° at $R = 1$. The prediction by Vincent et al. (eq 15) agrees with experimental data at 60° at all R values, for 45 and 30° at $R = 1$. At 45° (Figure 4) the data are widely scattered. At $R = 2$ they are lower than the predictions by all three equations. At $R = 0.67$ they are higher than the predictions by all three equations. At 30° and $R = 2$ (Figure 5) the prediction by Vincent et al. (eq 15) is higher than the experimental result of Durham and Lundgren at $St = 0.4$. The experimental result at high Stokes number agrees with the predictions by Durham and Lundgren (eq 11) and Vincent et al. (eq 15). Prediction by eq 18 also gives higher values than the experimental data at 30° and $R = 2$. There are no experimental data available for $R < 1$ at 30°.

The above analysis shows that the following equations agree the most with the available experimental data: (a) Belyaev and Levin's eq 7 at 0°, (b) Durham and Lundgren's eq 11 from 0 to ~60°, (c) eq 18 from ~45 to 90° within the Laktionov range $0.003 \leq St \leq 0.2$. The use of eqs 7, 11, and 18 results in a unified model for aspiration efficiency.

Conclusions

Through analysis of available aspiration models and the underlying experimental data base we have developed a unified model through the following general expression for aspiration efficiency:

$$E_a = 1 + (R \cos \theta - 1)f \qquad (16)$$

where f is the inertial parameter.

We recommend the use of the following parameters. These parameters are based on the best information available today. More extensive experiments may very well refine these parameters in the future.

(1) For $\theta = 0°$, use Belyaev and Levin eq 7 for which

$$f = 1 - 1/[1 + (2 + 0.617/R)St] \qquad (19)$$

valid for $0.16 < R < 5.6$; $0.18 < St < 2.03$.

(2) For $0° \leq \theta \leq 60°$, use Durham and Lundgren eq 11 for which

$$f = [1 - 1/[1 + (2 + 0.617/R)St']][1 - 1/[1 + 0.55St' \exp(0.25St')]]/[1 - 1/[1 + (2.617)St']] \qquad (20)$$

$$St' = St \exp(0.022\theta) \qquad (12)$$

valid for $0.01 \leq St \leq 6$.

(3) For approximately $45° \leq \theta \leq 90°$, use our new model for which

$$f = (3St^{R^{-0.5}}) \qquad (17)$$

valid for $1.25 \leq R \leq 6.25$; $0.003 \leq St \leq 0.2$.

Glossary

d_p	particle diameter
D_i	inlet diameter
f	inertial parameter
E_a	aspiration efficiency
E_r	entry efficiency
E_t	transmission efficiency
St	Stokes number (eq 4)
St′	modified Stokes number (eq 12)
U_w	wind velocity
U_i	inlet velocity
ρ_p	particle density
μ	air viscosity
θ	sampling angle
τ	particle relaxation time (eq 6)
β	Durham and Lundgren parameter (eq 11)

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