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GRAVITATIONAL DEPOSITION OF PARTICLES FROM LAMINAR AEROSOL FLOW THROUGH INCLINED CIRCULAR TUBES

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Abstract—Two He–Ne-laser photometers were used to measure gravitational deposition of particles carried with laminar air flows through inclined circular tubes. If the magnitude of the projection of the settling velocity of aerosol particles in the direction of the air flow is small compared to the magnitude of the maximum velocity of the laminar air flow ($rT \sin \beta/L = v \sin \beta/u_0 \ll 1$), gravitational deposition for upward and downward flow can be described by

$$\mathrm{DE} = \frac{2}{\pi}[2\kappa\sqrt{(1-\kappa^{2/3})} - \kappa^{1/3}\sqrt{(1-\kappa^{2/3})} + \arcsin \kappa^{1/3}],$$

where $\kappa = 3T \cos \beta/4$. The problem was applied to a system of randomly oriented circular tubes for which several analytical approximations were derived.

NOMENCLATURE

DE deposition
L length of a circular tube
N_0 number of particles entering a tube
N number of particles leaving a tube
PE penetration
P_N penetration of particles through the nitrogen mixer
Q volumetric aerosol flow rate
Q_N volumetric nitrogen flow rate
T sedimentation parameter, $T = vt/2r$
c_0 number concentration of particles entering a tube
c number concentration of particles leaving a tube
r radius of a circular tube
t mean residence time of aerosol particles in a tube
u_0 maximum velocity of Hagen–Poiseuille flow through a circular tube, $u_0 = 2u$
u mean velocity of Hagen–Poisseuille flow through a circular tube
v terminal settling velocity of an aerosol particle
β angle of inclination to the horizontal.

INTRODUCTION

Applying the method of the limiting trajectories and neglecting the inertia and Brownian motion of particles Pich (1972) calculated the gravitational deposition of particles from laminar aerosol flow through straight horizontal channels of different cross-section which for a circular tube yielded

$$\mathrm{DE}(T, \beta = 0) = 1 - \mathrm{PE}(T, \beta = 0)$$
$$= \frac{2}{\pi}[2\epsilon\sqrt{(1-\epsilon^{2/3})} - \epsilon^{1/3}\sqrt{(1-\epsilon^{2/3})} + \arcsin \epsilon^{1/3}], \tag{1}$$

where PE is the fraction of aerosol particles which escapes gravitational deposition in the tube (penetration) and $\epsilon = 3T/4$. $T = vt/2r$ is the dimensionless sedimentation parameter. It is defined in such a way that for $T \geq 1$ all particles will be deposited from a calm aerosol in a horizontal circular tube or from an aerosol slug flow (uniform velocity profile) through such a tube.

It was also shown by Pich (1972) that the gravitational deposition from laminar flow through a tube of elliptical cross-section is also given by equation (1) when ϵ

is replaced by (ϵm) with m being the major-to-minor semiaxis ratio of the ellipse. Equation (1) is in agreement with expressions derived by Thomas (1958) and Natanson [quoted by Fuchs (1964)]. Independently, Harris (1972) derived the same result using also the method of the limiting trajectories.

In a horizontal tube $2r$ is the maximum distance a particle can settle through air before it will be deposited. In an inclined tube this distance is $2r/\cos\beta$ so that equation (1) for inclined tubes can be extended to

$$\begin{aligned}\mathrm{DE}(T,\beta) &= 1 - \mathrm{PE}(T,\beta)\\ &= \frac{2}{\pi}[2\kappa\sqrt{(1-\kappa^{2/3})} - \kappa^{1/3}\sqrt{(1-\kappa^{2/3})} + \arcsin\kappa^{1/3}],\end{aligned} \tag{2}$$

where $\kappa = \epsilon\cos\beta = 3T\cos\beta/4$. The penetration of particles carried with a laminar air flow through circular tubes inclined at different angles is shown in Fig. 1, revealing for each curve a cut-off at $\kappa = 1$.

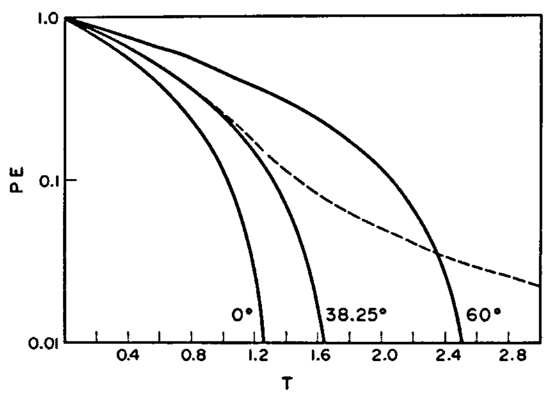

Fig. 1. Fraction of aerosol particles which escapes gravitational deposition (penetration) from laminar flow through a circular tube inclined at different angles to the horizontal as function of the sedimentation parameter. The dashed line represents the penetration through a system of randomly oriented identical circular tubes.

Based on the concepts of the particle trajectory function and the limiting trajectory, Wang (1975) extended the theory of gravitational deposition of particles from a laminar aerosol flow to deposition in inclined straight flat and circular tubes of which the inlet and outlet cross-sections are normal to the axes of the tubes. It turned out that the deposition depends not only on the sedimentation parameter but also on the direction of flow. For uphill flow the gravitational deposition is given by

$$\begin{aligned}\mathrm{DE}(\gamma) &= 1 - \mathrm{PE}(\gamma)\\ &= 1 - \frac{2}{\pi}\{(1-2\gamma)\sqrt{[\gamma(1-\gamma)]} + \arcsin\sqrt{(1-\gamma)}\},\end{aligned} \tag{3}$$

where

$$\gamma = \frac{(3T\cos\beta/4)^{2/3}}{1 - rT\sin\beta/L}.$$

As long as the magnitude of the projection of the settling velocity of the particles in the direction of the air flow is small compared to the maximum velocity of the laminar air flow ($rT\sin\beta/L = v\sin\beta/u_0 \ll 1$), equation (2) appears as special case of equation (3). For a horizontal tube equation (3) reduces to equation (1).

For downhill flow the gravitational deposition is given by

$$\begin{aligned}\mathrm{DE}(\zeta_1) &= 1 - \mathrm{PE}(\zeta_1)\\ &= 1 - \frac{2}{\pi}\left\{\frac{\sqrt{(1-\zeta_1^2)}}{1+2rT\sin\beta/L}[\zeta_1(1+rT\sin\beta/L) - 3T\cos\beta/2] + \arcsin\sqrt{(1-\zeta_1^2)}\right\},\end{aligned} \tag{4}$$

where

$$\zeta_1 = \frac{3T\cos\beta/4}{1+3rT\sin\beta/2L}.$$

Equation (4) can neither be reduced to equation (1) nor to equation (2). In Wang's theory, ζ_1 is an approximative root of the third order equation

$$\zeta^3 + 3rT\zeta\sin\beta/2L - 3T/4 = 0, \tag{5}$$

derived by the reguli-falsi method. Using, however,

$$\zeta_2 = \frac{(3T\cos\beta/4)^{1/3}}{1+3rT\sin\beta/2L}, \tag{6}$$

which for $rT\sin\beta/L = v\sin\beta/u_0 \ll 1$ is an even better approximative solution of equation (5) than ξ_1, equation (2) also appears as special case of equation (4) for $rT\sin\beta/L \ll 1$. Consequently, sedimentation losses of particles from a laminar aerosol flow through an inclined circular tube should be independent of the direction of flow provided $rT\sin\beta/L = v\sin\beta/u_0 \ll 1$.

It is the purpose of this paper to check the validity of equation (2) experimentally.

MATERIAL AND METHODS

Monodisperse droplets generated by condensation of bis (2-ethylhexyl) sebacate vapour upon sodium chloride nuclei in pure nitrogen (Stahlhofen *et al.*, 1975) were used. The diameter distributions of the aerosols were evaluated by light scattering on a great number of single droplets with the high resolution spectrometer LASI which was calibrated with polystyrene, iron oxide and bis (2-ethylhexyl) sebacate particles of known geometrical diameter (Gebhart *et al.*, 1976). Their standard deviations were about 1.08 and their modal diameters covered the range between 3 and 4 μm. The bulk density of the droplet material was 0.91 g/cm^3. The aerosol generation ensured that the droplets were uncharged.

The length of the circular tubes was 100 or 200 cm and their radius 0.8 cm. The volumetric aerosol flow rate was evaluated by measuring the pressure drop in a pneumotachograph with a capacitance manometer. It was varied between about 6 and 100 cm^3/sec. The aerosol flow through the tubes, therefore, could be characterized by Reynold numbers up to about 600.

The experimental conditions allowed the determination of the penetration as function of the sedimentation parameter for values up to $T \simeq 1.2$. The maximum angle of inclination was 30°. Hence, the conditions under which equation (2) is valid were satisfied: $rT\sin\beta/L = v\sin\beta/u_0 < 0.005 \ll 1$. For uphill flow, the fraction of particles retreated from the tubes and the fraction not able to enter the tubes (Wang, 1975) were negligible under the conditions of this study.

In general, the penetration of aerosol particles through a tube can be evaluated by counting the number of particles entering and leaving the tube

$$\mathrm{PE} = \frac{N}{N_0} = \frac{\int cQ\,\mathrm{d}t}{\int c_0 Q\,\mathrm{d}t}. \tag{7}$$

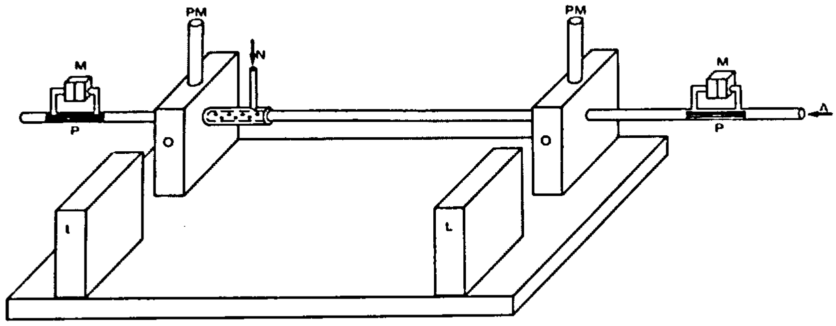

Fig. 2. Experimental set-up; (A) aerosol inlet, (L) laser, (M) capacitance manometer, (N) nitrogen supply for mixer, (P) pneumotachograph, (PM) photomultiplier.

In the case when c and c_0 remain constant over the period of measurement the penetration is simply given by PE $= c/c_0$.

In the present study, the particle number concentration of the aerosol at the inlet and outlet cross-sections of the circular tubes were simultaneously determined with two He–Ne-laser photometers, the prototype of which was already described elsewhere (Heyder *et al.*, 1973). The output currents of their photomultipliers were fed to linear amplifiers with variable amplification factors and then recorded with a multichannel u.v. light recorder. While passing particle-free air through the light scattering photometers their background current were compensated. The amplification factors were varied until both instruments exhibited the same response when exposed to the same aerosol. The experimental set-up is shown in Fig. 2. The photometers were mounted on a plain plate which could be inclined at different angles.

Only particles passing the sensing volume of a photometer contribute to its response. In this study, the sensing volumes of the photometers were cylinders of 3 mm width and 10 mm length with axes perpendicular to the aerosol flow. Therefore, only in case of uniformly distributed particles over the cross-sections of the photometers their output currents were proportional to the particle number concentrations of the aerosols.

Those conditions were, however, not satisfied in the photometer at the end of the tube due to the steadily declining upper limiting particle trajectory towards the bottom wall of the tube thus leaving an expanding particle-free zone at the top wall along the tube. Therefore, as shown in Fig. 2, a nitrogen mixer was introduced between the end of the tube and the second photometer to ensure that all particles which penetrated the tube were uniformly distributed over the cross-section of the tube.

The mixer consisted of two concentric cylinders of which the inner cylinder had the same diameter as the circular tube but its surface was covered with holes of 0.2 mm dia. By introducing compressed particle-free nitrogen into the hollow space between the cylinders at a flow rate of 2.6 cm^3/sec, nitrogen jets emerged from these holes which thoroughly mixed the particles.

Since the losses of particles in the nitrogen mixer were not negligible, the penetration of particles through the mixer had to be measured separately. For this purpose, only the mixer was placed between the two photometers. The dilution of the aerosols due to adding nitrogen to them could be taken into account by measuring the volumetric aerosol flow rates behind the second photometer also, as illustrated in Fig. 2. Thus, the penetration of particles through a circular tube was given by

$$\mathrm{PE} = \frac{c}{c_0}\left(1 + \frac{Q_N}{Q}\right)\frac{1}{P_N}. \tag{8}$$

RESULTS AND DISCUSSION

The penetration of particles through the nitrogen mixer was found to be dependent on the particle diameter and the volumetric aerosol flow rate through the circular tube.

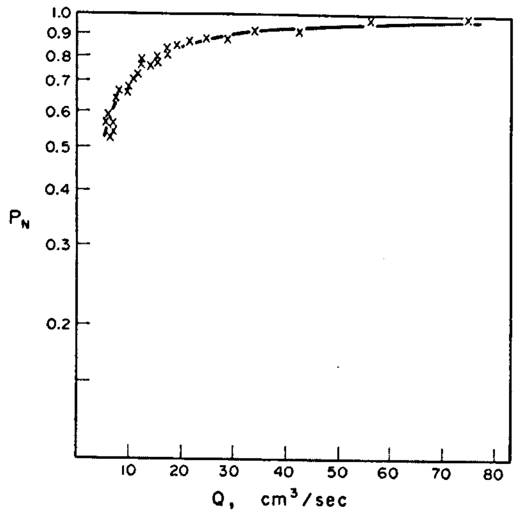

Fig. 3. Penetration of 3.5 μm particles through the nitrogen mixer as function of the volumetric aerosol flow rate through the mixer.

The penetration of 3.5 μm particles as function of the aerosol flow rate is shown in Fig. 3. For $Q \gg Q_N$, the particles had an appreciable chance to penetrate the mixer. For Q approaching Q_N, the penetration fell rapidly.

In Fig. 4, the penetration of particles carried with a laminar air flow through a horizontal and an inclined circular tube as function of the sedimentation parameter is shown. Within the experimental errors the measured penetration confirmed the predicted values. It can also be seen that in an inclined tube the penetration for uphill flow is the same as for downhill flow. Therefore, provided $rT \sin \beta/L = v \sin \beta/u_0 \ll 1$, gravitational deposition of particles from a laminar aerosol flow through inclined straight circular tubes is given by equation (2) regardless the direction of the aerosol flow.

The apparent penetration of particles through a tube, which is the penetration without nitrogen mixing of the particles at the end of the tube, is also shown in Fig. 4. As long as the upper limiting trajectory of the particles passed through the outlet cross-section of the tube above the laser beam the apparent penetration was higher than the penetration. If this trajectory, however, fell below the laser beam the apparent penetration rapidly approached zero penetration. The dependence of the apparent penetration on the sedimentation parameter as shown in Fig. 4 revealed that the particles

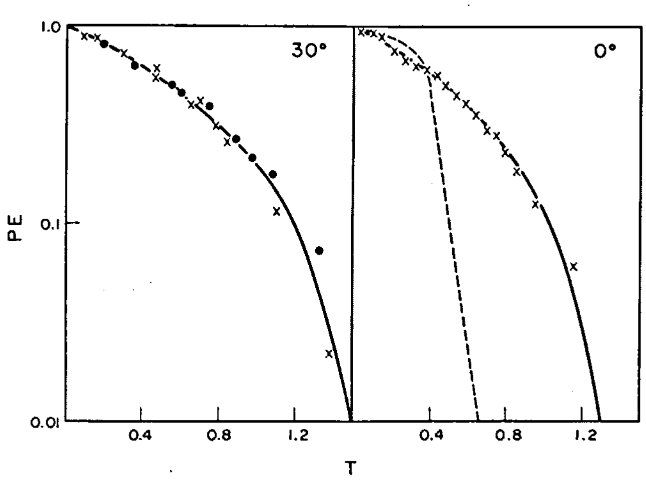

Fig. 4. Comparison of theoretically predicted penetration of particles carried with a laminar air flow through a horizontal ($\beta = 0$) and an inclined ($\beta = 30°$) circular tube with experimental values (points). The dashed line represents the apparent penetration as explained in the text. ×, Uphill flow; ●, downhill flow.

were deposited by undisturbed settling in the tube which is one of the basic assumptions in the theory of gravitational deposition from laminar flow.

APPLICATION TO PARTICLE DEPOSITION IN THE HUMAN RESPIRATORY TRACT

The small airways in the human respiratory tract can be visualized as a system of randomly oriented circular tubes. During aerosol breathing, in half the airways the aerosol flows upwards with respect to the horizontal. In the other half, it flows downwards. Since for $rT \sin \beta / L = v \sin \beta / u_0 \ll 1$, equation (2) proved valid for both directions of the aerosol flow, it can be used to calculate the gravitational deposition of particles from laminar aerosol flows through such a system of randomly oriented circular tubes for $rT/L = v/u_0 \ll 1$, according to

$$\mathrm{DE}(T) = 1 - \mathrm{PE}(T) = 1 - \int_0^{\pi/2} \mathrm{PE}(T, \beta) \cos \beta \, \mathrm{d}\beta. \tag{9}$$

PE(T) was calculated by numerical integration utilizing a Wang computer. Whenever $3T \cos \beta/4$ exceeded one, zero penetration was taken. The results are also shown in Fig. 1.

For practical applications, as the estimation of particle deposition in the human lungs, simple approximations are desirable. As can be seen from Fig. 1, up to $T = 0.5$ the penetration through the system is almost identical with the penetration through a single tube inclined at $\beta = 38.25°$ to the horizontal. For $0.5 < T < 0.8$, the penetration through this tube is less than 2% lower than the penetration through the system. For higher values of the sedimentation parameter, the penetration through the single tube is no longer representative for the penetration through the system. $\pi/4$ is the expectation of $\cos \beta$ for $0 \leq \beta \leq \pi/2$ which explains the exceptional position of $\beta = \arccos \pi/4 = 38.25°$.

Therefore, for $rT/L = v/u_0 \ll 1$ and $T < 0.5$, the penetration through a system of randomly oriented indentical circular tubes is given by

$$\mathrm{PE}(T, 38.25^0) = 1 - \frac{2}{\pi}\left[\frac{3\pi T}{8}\sqrt{\left(1 - \left(\frac{3\pi T}{16}\right)^{2/3}\right)} - \left(\frac{3\pi T}{16}\right)^{1/3}\sqrt{\left(1 - \left(\frac{3\pi T}{16}\right)^{2/3}\right)} + \arcsin\left(\frac{3\pi T}{16}\right)^{1/3}\right]. \tag{10}$$

Pich (1972) obtained $\mathrm{PE} = 1 - 4T/\pi$ for $T \ll 1$ expanding the functions of equation (1) in MacLaurin series. Based on this concept, equation (10) can be approximated by

$$\mathrm{PE} = 1 - T \tag{11}$$

which for $T \leq 0.1$ gives slightly lower penetration (less than 1%) than equation (10).

It has, however, to be considered that small values of the sedimentation parameter are not the only limitation of equations (9)–(11). To estimate the upper limit of the particle-to-air velocity for the application of these equations, equations (3), (4) and (6) were used to calculate the penetration of particles through a system of randomly oriented circular tubes assuming that in half the tubes uphill flow occurs and downhill flow in the other half, respectively. This penetration is given by

$$\mathrm{PE}(T, r/L) = \tfrac{1}{2} \int_0^{\pi/2} [\mathrm{PE}(\gamma) + \mathrm{PE}(\zeta_2)] \cos \beta \, \mathrm{d}\beta. \tag{12}$$

PE(T, 0.2) is shown, together with the penetration PE(T) [equation (9)], in Fig. 5. As can be seen the approximation PE(T) is valid for $T \leq 0.2$, i.e. for $rT/L = v/u_0 \leq 0.04$,

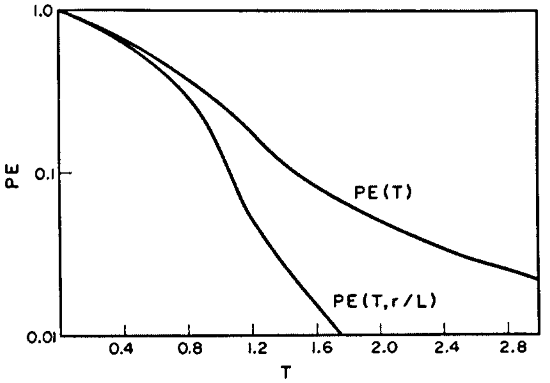

Fig. 5. Comparison of the penetration of particles carried with a laminar air flow through a system of randomly oriented identical circular tubes, PE($T,r/L \doteq 0.2$), with the approximative penetration, PE(T).

where the penetration is overestimated by less than 1%. In this range, however, as already discussed, PE(T) is well described by equation (10) or even equation (11). With increasing values of $rT/L = v/u_0$, more and more complicated functions have to be used to estimate the penetration of aerosol particles through a system of randomly oriented circular tubes, starting with equation (11), passing over, first to equation (10), then to equation (9) in order to derive, finally, at equation (12).

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