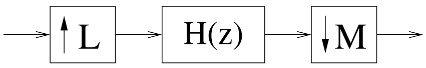
Signal Processing Systems (521279S), Fall 2025
Part 4 : Multirate processing
Design tasks, deadline for return Thu 27.11.2025 23:59

T1. (1p) Background for this task is provided in **intro4a.pdf**. Consider the sample-rate converter illustrated below. The FIR filter $H(z)$ is a low-pass filter, which provides required anti-image and anti-alias filtering for the signal. In the Table 1, coefficients h_k of the FIR filter, upsampling factor L, downsampling factor M and the input sample rate f_{in} are given. Take the values assigned to your group and solve the following subproblems.



(a) How many multiply-accumulate (MAC) operations per second (MAC/s) does the sample-rate converter require if it is implemented directly as shown above? Note: for simplicity, consider that a coefficient equal to one (1) requires also one MAC operation.

(b) Determine the polyphase decomposition of the filter $H(z)$ for the factor M (use numerical values of the coefficients h_k in your answer).

(c) Apply the decomposition from subtask (b) to the sample-rate converter to get an improved computational structure. Draw a signal flow diagram of your solution. How many MAC operations per second (MAC/s) does it require?

(d) Reconsider the flow diagram you obtained in subtask (c) and the upsampling operation in it. Recall that nobel identities 4-6 provide some ways for dealing with such operations. Can you apply those noble identities to modify the solution obtained in (c)? Discuss.

T2. (2p) Necessary background for this task is provided in **intro4b.pdf**. In addition, use the Matlab tool named **multirateN.m**, which is provided in Optima. Design a double-stage multirate solution (see Fig. 4(c) in **intro4b.pdf**) for the lowpass filter specified for your group in Table 2, where F_o is the input/output sample rate, f_p and f_s are the cutoff frequencies of the passband and stopband, A_p is the passband ripple and A_s is the stopband attenuation.

(a) Compute an upper bound ($M_{\max}$) for the overall downsampling factor $M_1 M_2$. Then, choose some reasonable values for M_1 and M_2 and use them in the following.

(b) Find appropriate filter parameters for the solution and provide a list of them in the report.

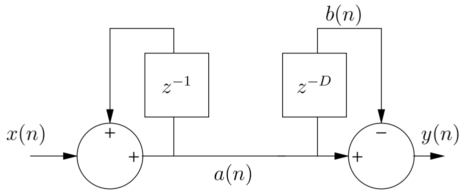
(c) Verify that your design works. Include figures of the overall frequency response and passband frequency response in the the report.

(d) Analyze how much saving in terms of MAC operations is obtained with respect to the single-rate design. Equations for this subtask are given in Sec. 3.2. of **intro4b.pdf**.

***Note.** Alternatively, you can provide a single-stage multirate solution (see Fig. 4(b) in **intro4b.pdf**) using downsampling factor $M = 5$ and answer just to problems (b), (c) and (d). However, in this case you can get only max. 1.3p from this problem.

T3. (1p) Background for the CIC filtering is provided in **intro4a.pdf (Sec. 4.2)**. The purpose of this task is to study its operation. As important background, you may also read **intro1.pdf (Sec. 2.2, Example 2)**. Parameters for the task are provided in Table 3.

(a) Consider the cascade of the integrator $I(z)$ and comb $C_D(z)$:



According to Eq. 8, the overall transfer function of this system is

$$H(z) = C_D(z)I(z) = \sum_{d=0}^{D-1} z^{-d}.$$

So, if the input $x(n)$ is a sequence of ones, $(1, 1, 1, ...)$, and the buffers associated with the delay elements are initialized by zeros, the output $y(n)$ should be a sequence $(0, 1, ..., D-1, D, D, ...)$. However, we can see that the integrator output $a(n)$ increases as $(1, 2, 3, 4, ...)$ with the sequence-of-ones input and therefore it must overflow finally. Explain how the overflow is handled in the adder in order to make the system work.

(b) Read footnote[1] below. Consider a N-stage CIC decimator with downsampling factor R and differential delay M. The input is in the fixed-point s$p.n$ format. What s$x.y$ format must be used for the decimator output and intermediate computations? Determine x and explain why $y = n$.

Test your solution using the Matlab utility **cic.m** provided in Optima. Include in the report the figures of stage inputs and outputs using a sequence of 500 constants* as an input. Last example in **help cic.m** shows, how you can plot them.

*Sequence of ones may be used for the figures, but note that testing should be done with the sequences of maximum/minimum values representable with the input s$p.n$ format.

[1] Quote from Hogenauer's (1981) article: "If the number of bits in the input stream is B_{in}, then the register growth can be used to calculate $B_{\max}$, the (number of the) most significant bit at the filter output. That is,

$$B_{\max} = \lceil N \log_2 RM + B_{\text{in}} - 1 \rceil$$

where the least significant bit of the input register is considered to be the bit number zero and where $\lceil x \rceil$ is the smallest integer not less than x." **Note**: Reading the quote carefully, it seems that B_{in} refers to the input word length, and $B_{\max} + 1$ to the output word length.

Table 1. Parameters for task T1. The difference equation of the filter $H(z)$ is

$$y(n) = \sum_{k=0}^{K-1} h_k x(n-k),$$

L is the upsampling factor, M is the downsampling factor, and f_{in} is the input sample rate. Note that the sets of coefficients h_k are far from ideal: much longer filters would be needed in practice.

Group number	Filter coefficients $h_0, h_1, \ldots, h_{K-1}$	L	M	f_{in} [MHz]
1	1,2,5,8,15,24,37,53,71,89,102,110,110,102,89,71,53,37,24,15,8,5,2,1	5	2	64.0
2	3,6,12,24,43,69,98,123,138,138,123,98,69,43,24,12,6,3	3	4	39.0
3	1,1,1,2,11,43,117,212,258,212,117,43,11,2,1,1,1	3	2	29.0
4	2,4,7,13,22,36,52,71,90,105,114,114,105,90,71,52,36,22,13,7,4,2	2	5	59.0
5	1,1,1,3,11,31,70,124,176,198,176,124,70,31,11,3,1,1,1	4	3	44.0
6	2,4,7,12,22,35,52,71,91,107,116,116,107,91,71,52,35,22,12,7,4,2	5	3	24.0
7	1,1,1,2,9,40,114,216,267,216,114,40,9,2,1,1,1	2	3	34.0
8	2,4,7,13,22,35,52,71,91,106,115,115,106,91,71,52,35,22,13,7,4,2	4	5	54.0
9	3,7,13,24,40,60,83,104,120,126,120,104,83,60,40,24,13,7,3	5	3	35.0
10	3,7,14,24,40,60,83,104,119,125,119,104,83,60,40,24,14,7,3	4	5	50.0
11	1,1,4,19,69,166,257,257,166,69,19,4,1,1	3	2	65.0
12	1,3,6,14,29,52,82,115,140,150,140,115,82,52,29,14,6,3,1	3	4	60.0
13	1,1,4,15,41,92,157,205,205,157,92,41,15,4,1,1	2	3	25.0
14	2,5,13,31,60,100,140,165,165,140,100,60,31,13,5,2	5	2	40.0
15	3,6,12,23,38,59,83,106,122,128,122,106,83,59,38,23,12,6,3	3	5	55.0
16	1,2,8,26,64,124,185,211,185,124,64,26,8,2,1	3	4	48.0
17	2,3,6,11,20,34,51,71,92,109,118,118,109,92,71,51,34,20,11,6,3,2	4	5	58.0
18	2,3,5,9,16,25,38,54,71,88,101,108,108,101,88,71,54,38,25,16,9,5,3,2	5	2	23.0
19	2,4,6,11,18,28,40,55,70,85,96,103,103,96,85,70,55,40,28,18,11,6,4,2	3	5	38.0
20	2,3,6,11,17,27,40,55,71,86,97,104,104,97,86,71,55,40,27,17,11,6,3,2	2	5	33.0
21	1,2,6,17,39,76,123,163,180,163,123,76,39,17,6,2,1	4	3	28.0
22	2,3,6,10,16,26,39,54,71,87,99,106,106,99,87,71,54,39,26,16,10,6,3,2	5	3	43.0
23	9,27,66,125,184,209,184,125,66,27,9	2	3	53.0
24	8,25,63,125,187,215,187,125,63,25,8	3	2	63.0
25	3,7,13,24,39,60,83,105,120,126,120,105,83,60,39,24,13,7,3	3	5	49.0
26	2,4,7,13,22,35,52,71,91,106,115,115,106,91,71,52,35,22,13,7,4,2	4	5	54.0
27	1,3,6,14,29,52,82,115,140,150,140,115,82,52,29,14,6,3,1	3	4	60.0
28	1,2,6,17,39,76,123,163,180,163,123,76,39,17,6,2,1	4	3	28.0
29	2,3,6,10,16,26,39,54,71,87,99,106,106,99,87,71,54,39,26,16,10,6,3,2	5	3	43.0
30	9,27,66,125,184,209,184,125,66,27,9	2	3	53.0

Table 2. Parameters for task T2. F_0 is the input/output sample rate, f_p and f_s are the filter cutoff frequencies, A_p is the passband ripple and A_s is the stopband attenuation requirement.

Group number	F_0 [kHz]	f_p [kHz]	f_s [kHz]	A_p [dB]	A_s [dB]
1	170	5.6	7.6	0.15	80
2	110	2.8	3.9	0.10	60
3	70	1.3	1.9	0.15	60
4	100	3.1	4.0	0.10	70
5	140	3.3	4.5	0.15	70
6	200	3.4	4.8	0.10	80
7	110	3.1	4.5	0.15	70
8	130	3.4	4.4	0.10	80
9	110	1.9	2.6	0.15	70
10	140	4.2	5.9	0.10	60
11	80	1.8	2.6	0.15	80
12	120	2.1	2.7	0.10	70
13	150	4.8	6.5	0.15	70
14	200	4.7	6.6	0.10	70
15	120	2.0	2.9	0.15	70
16	80	2.5	3.3	0.10	70
17	110	2.6	3.5	0.15	80
18	160	2.8	3.9	0.10	60
19	80	2.6	3.8	0.15	80
20	110	2.9	3.8	0.10	60
21	90	2.1	2.7	0.10	70
22	120	2.4	3.2	0.15	70
23	180	5.2	7.3	0.10	60
24	90	2.1	3.0	0.15	70
25	130	2.5	3.3	0.10	80
26	130	3.4	4.4	0.10	80
27	120	2.1	2.7	0.10	70
28	90	2.1	2.7	0.10	70
29	120	2.4	3.2	0.15	70
30	180	5.2	7.3	0.10	60

Table 3. Parameters for task T3.

Group number	Number of stages N	Downsampling factor R	Differential delay factor M	Input format s$p.n$
1	5	3	2	s8.5
2	4	3	2	s9.7
3	3	3	2	s8.6
4	3	3	1	s10.7
5	5	4	1	s8.6
6	5	3	2	s10.7
7	4	3	1	s9.6
8	5	3	1	s8.6
9	5	3	1	s9.6
10	5	3	1	s10.7
11	5	4	2	s8.5
12	4	3	1	s10.8
13	4	4	2	s10.8
14	4	4	1	s10.7
15	4	4	1	s8.6
16	3	4	1	s8.6
17	3	4	2	s9.6
18	4	3	2	s8.5
19	6	4	1	s9.6
20	6	3	2	s8.6
21	6	3	1	s8.5
22	5	4	1	s10.8
23	5	4	1	s9.7
24	3	4	2	s10.8
25	4	3	1	s8.5
26	6	3	1	s8.5
27	5	4	1	s10.8
28	5	4	1	s9.7
29	3	4	2	s10.8
30	4	3	1	s8.5