Counting arithmetic operations

The estimation of operation counts (multiplication, addition) is requested in Design Task 5 Problem 2. In the following, some discussion of operation counting is provided, which hopefully helps in the analysis.

1. Structures may contain mixtures of multiplications and additions of pairs of real-real, real-complex, and complex-complex values. In every case, the operation counts must be reported in terms of *real* multiplications and additions: in the case of complex values, we have multiple real operations. In the following, shorthands MPLY and ADD for operations will be used.

The rules are:

- real-real addition corresponds to one ADD.
- real-real multiplication corresponds to one MPLY.
- real-complex addition corresponds to one ADD (denoted by red $+$) as
$$a + (c + jd) = (a+c) + j(d). \tag{1}$$
- real-complex multiplication corresponds to two MPLYs (denoted by red $\times$) as
$$a \times (c + jd) = (a\times c) + j(a\times d). \tag{2}$$
- complex-complex addition corresponds to two ADDs as
$$(a + jb) + (c + jd) = (a+c) + j(b+d). \tag{3}$$
- complex-complex multiplication corresponds to four MPLYs and two real ADDs as
$$(a + jb) \times (c + jd) = (a\times c-b\times d) + j(a\times d+b\times c)) \tag{4}$$
and we treat the subtraction operation as an addition.

However, it is possible to solve the last case using only three MPLYs. By computing
$$e = c\times(a-b) \tag{5}$$
at first, we have
$$(a + jb) + (c + jd) = ((c-d)\times b+e) + j((c+d)\times a-e). \tag{6}$$
There are five ADDs here, but if $c+jd$ is a fixed coefficient value, we can compute $(c - d)$ and $(c + d)$ beforehand, which reduces the number of ADDs to three.

2. Rules for a filter, which has N coefficients:

- both coefficients and input real: N MPLYs and $N - 1$ ADDs.
- coefficients complex and input real (or vice versa): $2N$ MPLYs and $2N - 2$ ADDs are needed as real and imaginary parts are processed separately.
- both coefficients and input complex: $4N$ MPLYs and $4N - 2$ ADDs are needed, if the computation is based on the Eq. 4.

The operation counts for the last case come from the notion that complex-complex multiplications require $4N$ MPLYs and $2N$ real ADDs, and subsequent additions require $2N - 2$ ADDs.

3. If the filter is *symmetric* and both the coefficients and input are real and N is even, we can decrease the number of MPLYs to $N/2$. For odd N, this count is $(N + 1)/2$. The reduction is similar in real-complex and complex-complex cases.

4. Finally, if the output of a filter is *downsampled* or the input is *upsampled*, we can apply the polyphase decomposition and noble identities to decrease the number of multiplications and additions (recall Figure 4 and Figure 6 in **intro4a.pdf**)