Chaos synchronization in three coupled SQUIDs

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I. INTRODUCTION

II.

For three coupled SQUID devices, the set of dimensionless equations is:

$$\begin{bmatrix} \ddot{\Phi}_1 + \gamma\dot{\Phi}_1 + \beta\sin(2\pi\Phi_1) \\ \ddot{\Phi}_2 + \gamma\dot{\Phi}_2 + \beta\sin(2\pi\Phi_2) \\ \ddot{\Phi}_3 + \gamma\dot{\Phi}_3 + \beta\sin(2\pi\Phi_3) \end{bmatrix} = \frac{1}{\det(\Lambda)} \begin{bmatrix} 1-\lambda^2 & -\lambda+\frac{\lambda^2}{8} & \lambda^2-\frac{\lambda}{8} \\ -\lambda+\frac{\lambda^2}{8} & 1-\frac{\lambda^2}{64} & -\lambda+\frac{\lambda^2}{8} \\ \lambda^2-\frac{\lambda}{8} & -\lambda+\frac{\lambda^2}{8} & 1-\lambda^2 \end{bmatrix} \begin{bmatrix} \Phi_{ext}-\Phi_1 \\ \Phi_{ext}-\Phi_2 \\ \Phi_{ext}-\Phi_3 \end{bmatrix} \tag{1}$$

,

where:

$$\det(\Lambda) = 1 - \frac{129\lambda^2}{64} + \frac{\lambda^3}{4} \tag{2}$$

ϕ_{ac} is the amplitude of an alternating (ac) flux, Ω its relative frequency rescaling by the inductive-capacitive SQUID frequency and γ is the loss coefficient. $\beta = I_c L/2\pi$ where L is the self-inductance of the SQUID ring and I_c is the critical current which characterizes a Josephson junction. Finally, ϕ is the magnetic flux rescaling by the flux quantum and λ is the coupling coefficient between nearest neighboring SQUIDs.

In order to quantify the synchronization between $\phi_1(t)$ and $\phi_3(t)$ we calculate the η measurement [1]:

$$\eta(t) = \sqrt{(\phi_1(t)-\phi_3(t))^2 + (\dot{\phi_1}(t)-\dot{\phi_3}(t))^2}, \tag{3}$$

where for the mean value of η in time close to zero ($\langle\eta\rangle_t \simeq 0$) we have almost perfect synchronization, for $0 < \langle\eta\rangle_t \leqslant 0.3$ we have intermittent synchronization and finally for $\langle\eta\rangle_t > 0.3$ the two time series $\phi_1(t)$ and $\phi_3(t)$ are unsynchronized.

The phenomena of chaos synchronization has also been observed in a system of three coupled lasers [2].

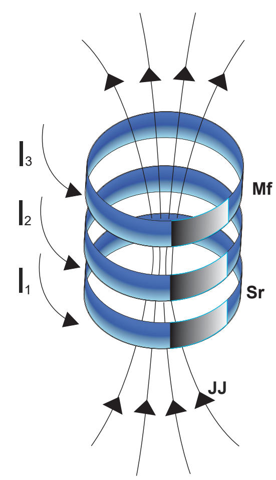

FIG. 1. Schematic diagram of a SQUID trimer with positive magnetic coupling strength, in a magnetic field where (Mf) is the Magnetic field, (Sr) is the Superconducting ring, (JJ) is the Josephson Junction, and (I_1), (I_2) and (I_3) are the induced currents.

All the numerical analysis have been obtained using Julia programming language and the DynamicalSystems package [3]. All the code for this paper can be found in `https://github.com/Joniald/Squid_Trimer`.

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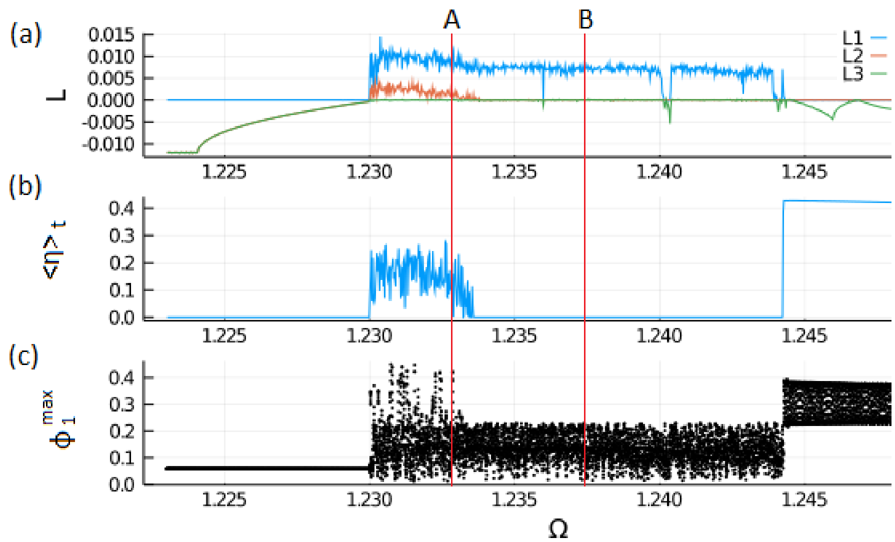

FIG. 2. (a) The three maxima Lyapunov exponents (the rest are negative), (b) the mean value over time of η and (c) maximum value of the magnetic flux ϕ_1 as a function of the driving frequency. Red line A corresponds to $\Omega = 1.233$ and B to $\Omega = 1.2375$. We observe two main regions: The first one is between $(\Omega = 1.23, \Omega = 1.234)$ corresponds to meta-stable hyper chaos synchronization where ($\langle\eta\rangle_t \neq 0$, $L_1 > 0$, $L_2 > 0$) and the second one lying between $(\Omega = 1.234, \Omega = 1.244)$ corresponds to chaos synchronization where ($\langle\eta\rangle_t = 0$, $L_1 > 0$). Parameters: $\lambda = 0.1075$, $\phi_{ac} = 0.02$, $\gamma = 0.024$, $\phi_{dc} = 0$ and $\beta = 0.1369$.

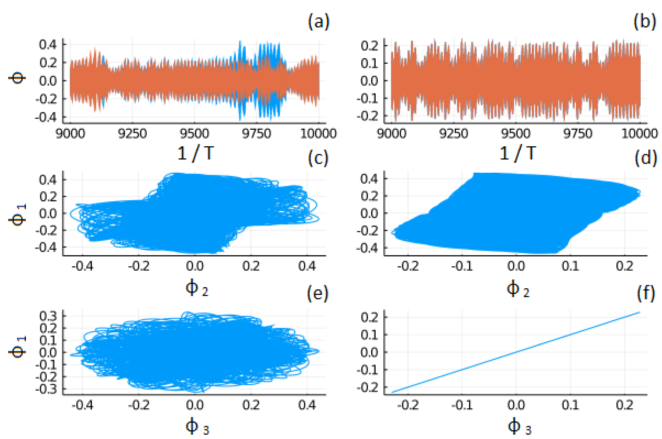

FIG. 3. Unsynchronized chaos for a set of parameters as in Fig. 2 (A red line) where $\Omega = 1.233$. (a) Time series for ϕ_1 and ϕ_3. (c) Projection of the flow onto ϕ_1 - ϕ_2 plane. (e) Projection into the ϕ_1 - ϕ_3 plane. (b), (d) and (f) the same for synchronized chaos where the set of parameters as in Fig. 2 (B red line) where $\Omega = 1.2375$.

III. CONCLUSIONS

IV. ACKNOWLEDGEMENTS

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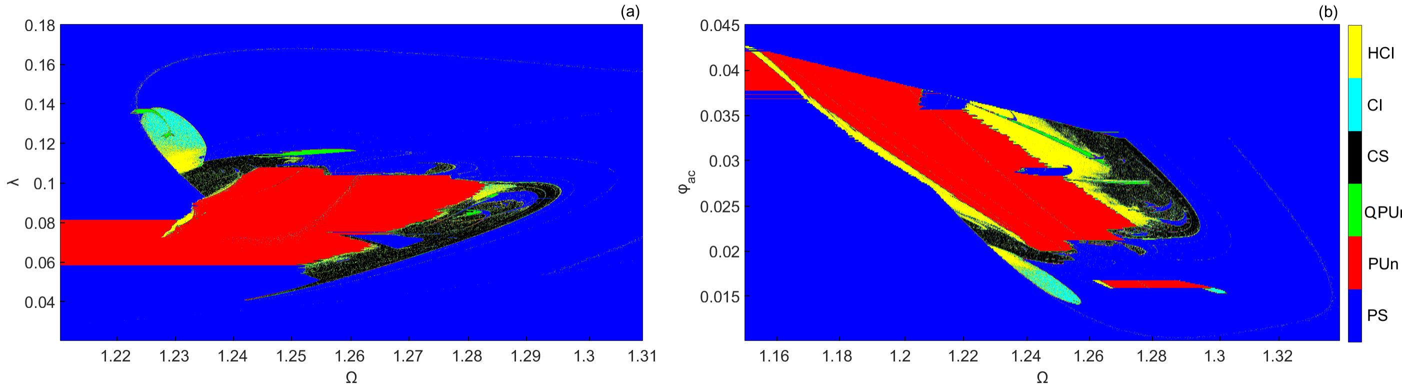

FIG. 4. Map of different dynamical regions in the (a) (λ,Ω) parameter space where $\phi_{ac} = 0.02$ and (b) in the (ϕ_{ac},Ω) parameter space where $\lambda = 0.02$. Depending on three maxima Lyapunov exponents ($L_1 > L_2 > L_3$) and $\langle\eta\rangle_t$ measurement, we observe six different areas. Periodic synchronization (PS) where $L_1 = 0$ and $\langle\eta\rangle_t < 0.01$, Periodic unsynchronized solution (PUn) where $L_1 = 0$ and $0.3 < \langle\eta\rangle_t$, Quasiperiodic unsynchronized solution (QPUn) where $L_1 = L_2 = 0$ and $\langle\eta\rangle_t > 0.3$, Chaos synchronization (CS) where $L_1 > 0, L_2 = 0$ and $\langle\eta\rangle_t < 0.01$, Chaos intermittent synchronization (CI) where $L_1 > 0, L_2 = 0$ and $0.01 < \langle\eta\rangle_t < 0.3$ and finally Hyperchaos intermittent synchronization (HCI) where $L_1 > 0, L_2 > 0, L_3 = 0$ and $0.01 < \langle\eta\rangle_t < 0.3$.

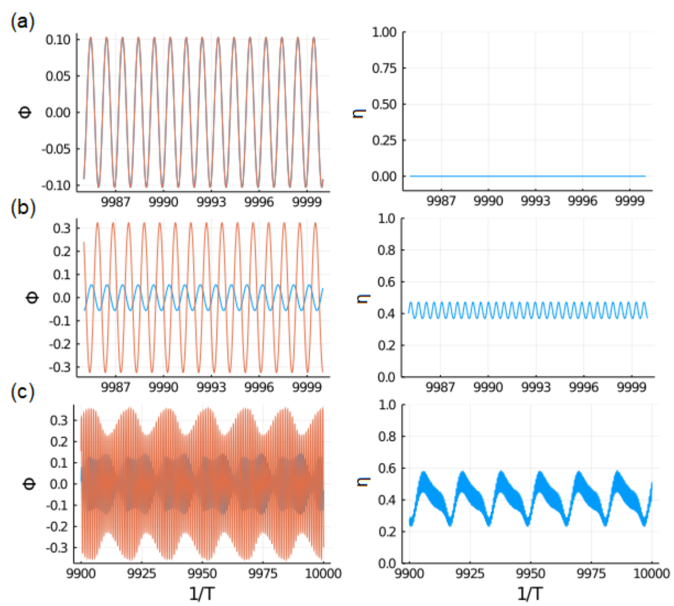

FIG. 5. (a) Periodic synchronization (PS) where $\Omega = 1.21$ and $\lambda = 0.16$. (b) Periodic unsynchronized (PUn) solution where $\Omega = 1.24$ and $\lambda = 0.07$ (c) Quasiperiodic unsynchronized (QPUn) solution where $\Omega = 1.255$ and $\lambda = 0.118$. In first column the time series of the magnetic flux for the first SQUID, (red line), and the third SQUID, (blue line). In second column η over time. Other parameters: $\phi_{ac} = 0.02$, $\gamma = 0.024$, $\phi_{dc} = 0$ and $\beta = 0.1369$.

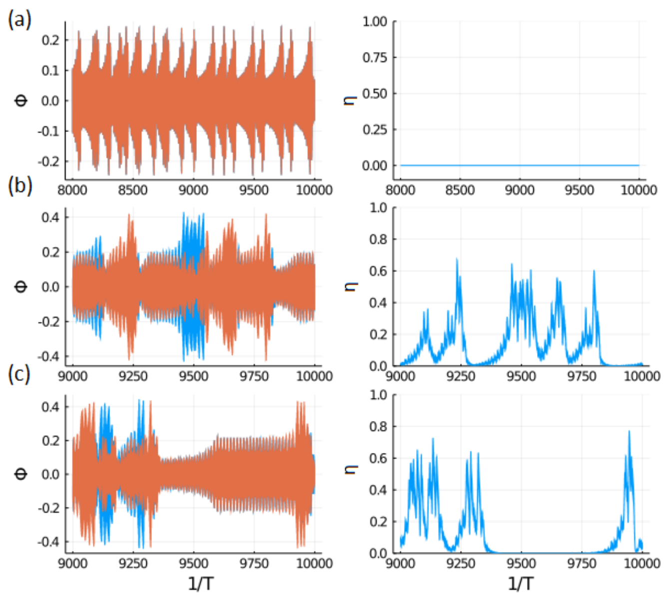

FIG. 6. (a) Chaos synchronization (CS) where $\Omega = 1.235$ and $\lambda = 0.1$. (b) Chaos intermittent synchronization (CI) where $\Omega = 1.23$ and $\lambda = 0.125$. (c) Hyperchaos intermittent synchronization (HCI) where $\Omega = 1.23$ and $\lambda = 0.11$. First and second column as in Fig.5. Other parameters: $\phi_{ac} = 0.02$, $\gamma = 0.024$, $\phi_{dc} = 0$ and $\beta = 0.1369$.