++ Typo in Volume?: Interp 2D ++ ++ go through Control Plotting Function ++ streamlines not working ++

Paper Example 3D

We choose

$$\begin{aligned}
\rho_0 &= 0.125 \\
V_{ext} &= ((x_1+0.3)^2-1)((x_1-0.4)^2-0.5) \\
&\quad ((x_2+0.3)^2-1)((x_2-0.4)^2-0.5)((x_3+0.3)^2-1)((x_3-0.4)^2-0.5) \\
\widehat{\rho} &= 0.125(1-t)+t\left(\frac{\pi}{4}\right)^3\cos\left(\frac{\pi x_1}{2}\right)\cos\left(\frac{\pi x_2}{2}\right)\cos\left(\frac{\pi x_3}{2}\right)
\end{aligned}$$

The external potential is shown in Figure 1.

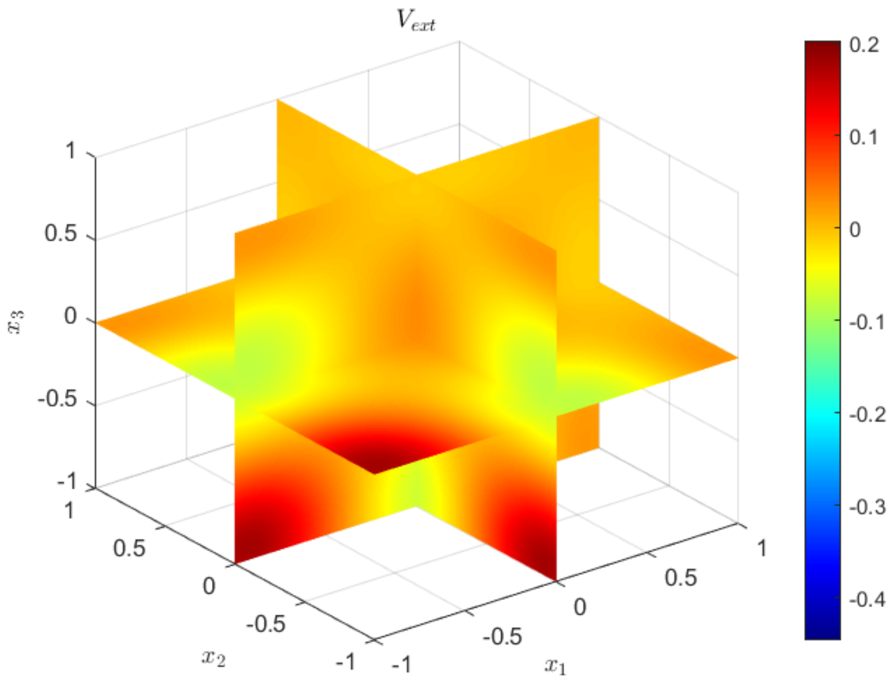

Figure 1: External Potential V_{ext}

For $N = 20$ and $n = 11$, with $\beta = 10^{-3}$ we get for $\kappa = 0$, $\mathcal{J}_c = 0.0078$, with $\mathcal{J}_1 = 0.0071$ and $\mathcal{J}_2 = 8.5034$. This can be compared to $\mathcal{J}_{uc} = 0.0195$ from the computed forward problem with $\mathbf{w} = \vec{0}$.

For $\kappa = 1$, we get that $\mathcal{J}_c = 0.0102$, with $\mathcal{J}_1 = 0.0097$, $\mathcal{J}_2 = 10.7306$. Compare to $\mathcal{J}_{uc} = 0.0232$. For $\kappa = -1$ we have $\mathcal{J}_c = 0.0059$, $\mathcal{J}_1 = 0.0054$, $\mathcal{J}_2 = 6.4039$. Compare to $\mathcal{J}_{uc} = 0.0477$.

While the forward problem takes around 12 minutes to solve, the optimal control problem with Newton-Krylov takes about 35 hours for 10 outer iterations, which is enough for convergence. Mass is conserved to 10^{-4}. The results can be seen in Figures 2, 3 and 4.

The controls are plotted in Figures 5, 6 and 7. They are all normalized to the maximum over all three controls and scaled by a factor of 2 for visibility. The figures are still not good though. I am not sure how the scaling works – $\kappa = 0$ should have way smaller arrows by that logic.

The norm of the control (in Euclidean norm) can be seen in Figures 8, 9 and 10. Between these there is not much of a qualitative difference. However, the control for $\kappa = 0$ is of order 10^{-4}, while the control for $\kappa = 1$ and $\kappa = -1$ is of order 1.

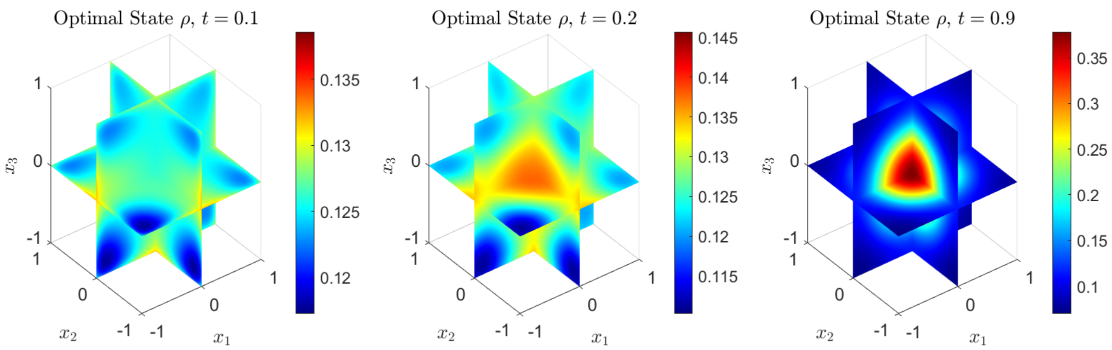

Figure 2: Optimal state ρ for $\kappa = 1$.

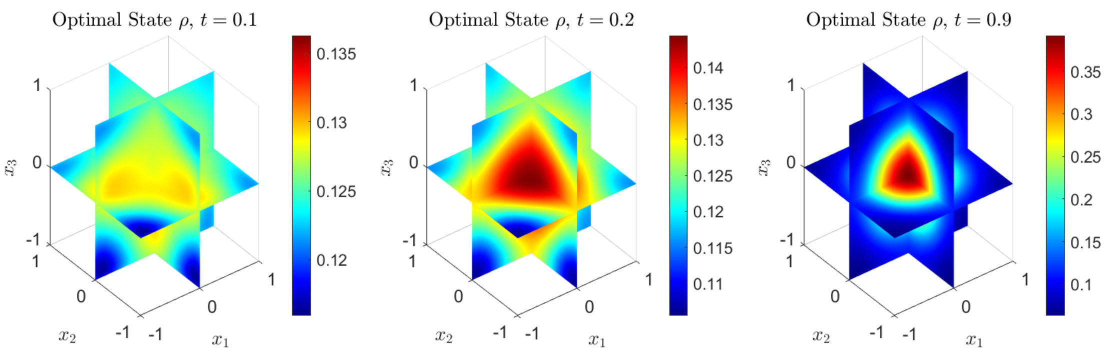

Figure 3: Optimal state ρ for $\kappa = 0$.

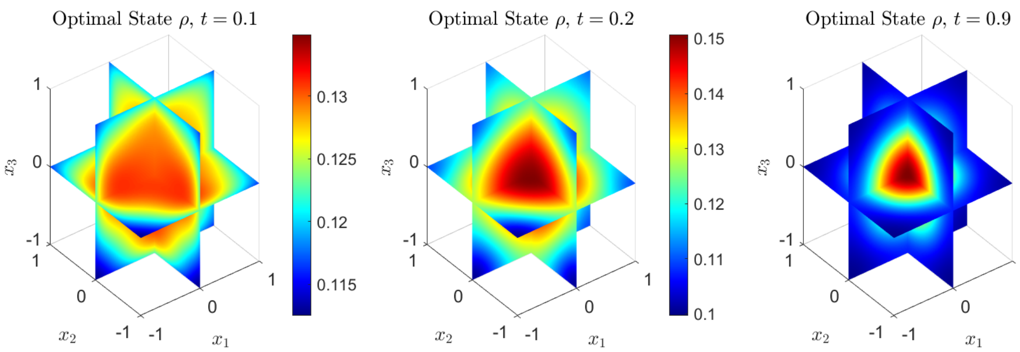

Figure 4: Optimal state ρ for $\kappa = -1$.

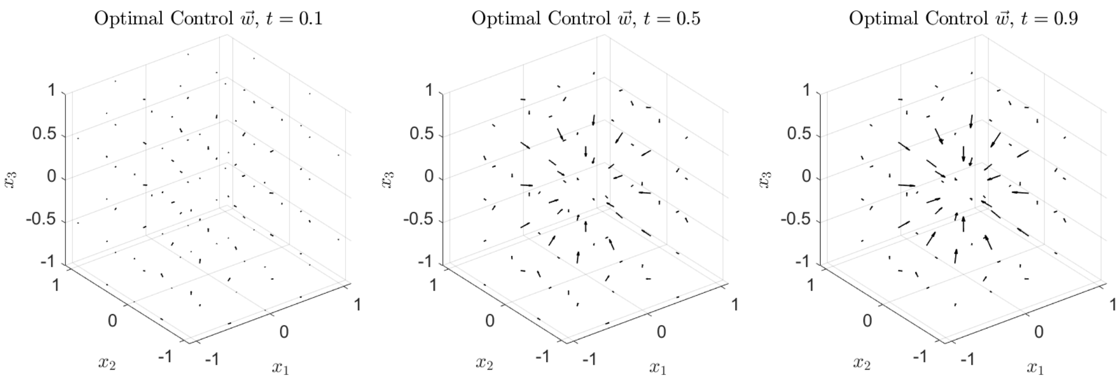

Figure 5: Optimal control $\mathbf{w}$ for $\kappa = 1$.

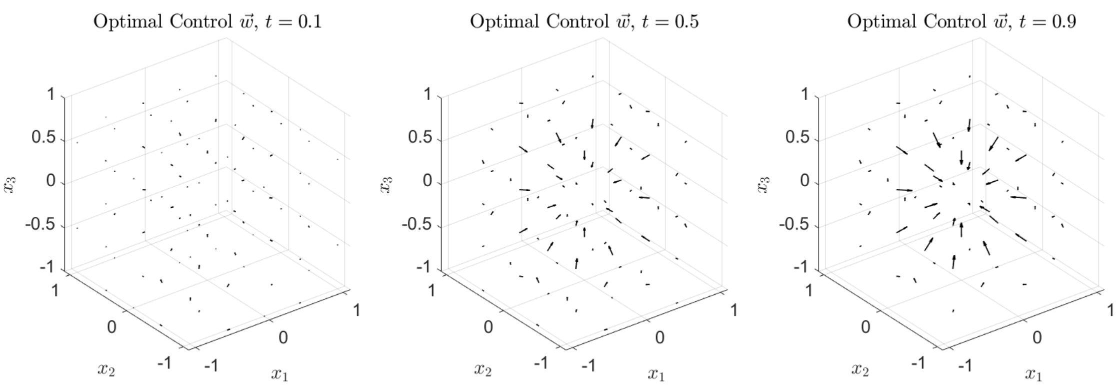

Figure 6: Optimal control $\mathbf{w}$ for $\kappa = 0$.

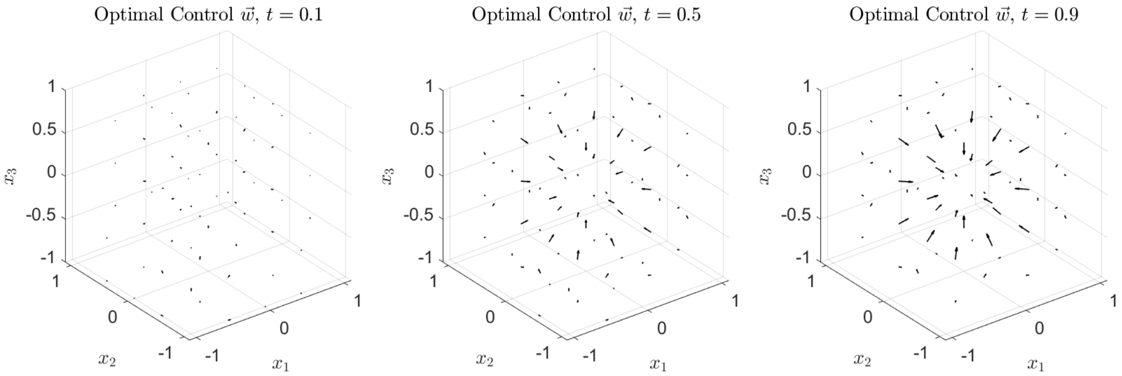

Figure 7: Optimal control $\mathbf{w}$ for $\kappa = -1$.

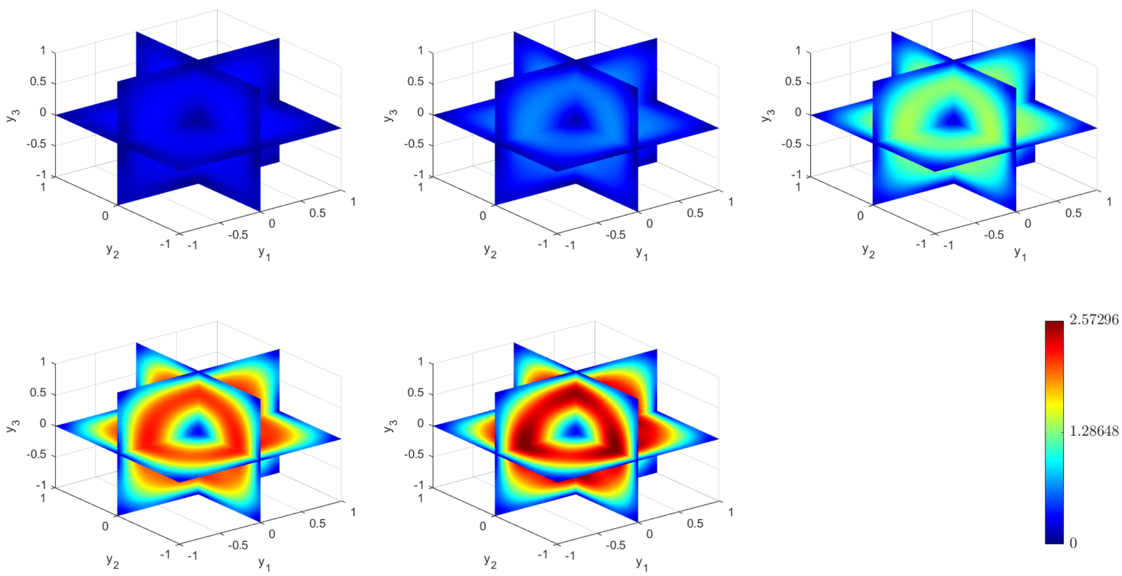

Figure 8: Norm of the optimal control $\mathbf{w}$, at times 1, 3, 5, 7 and 9 for $\kappa = 1$.

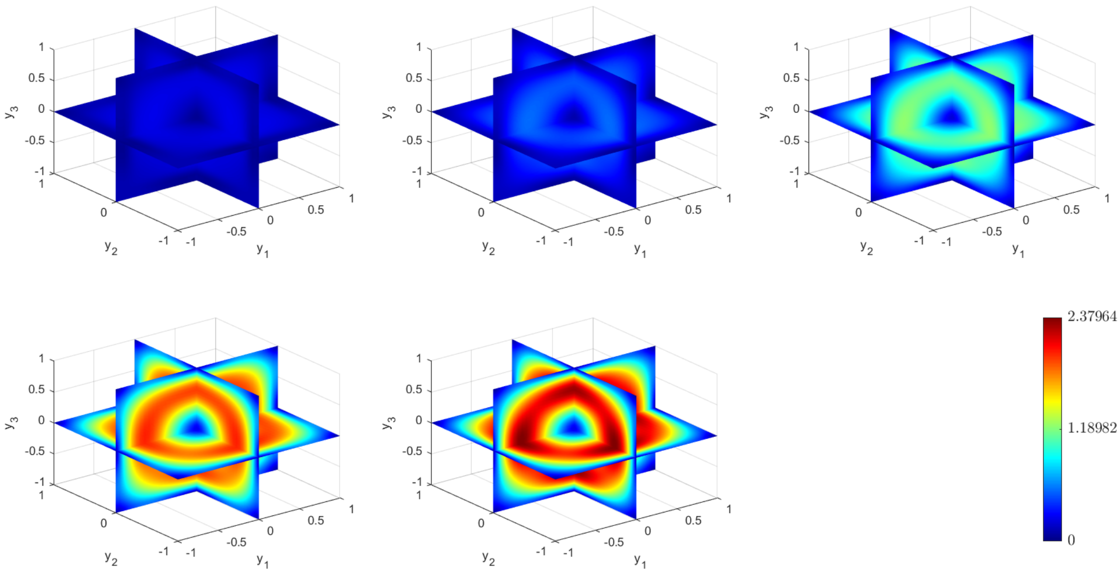

Figure 9: Norm of the optimal control $\mathbf{w}$, at times 1, 3, 5, 7 and 9 for $\kappa = 0$.

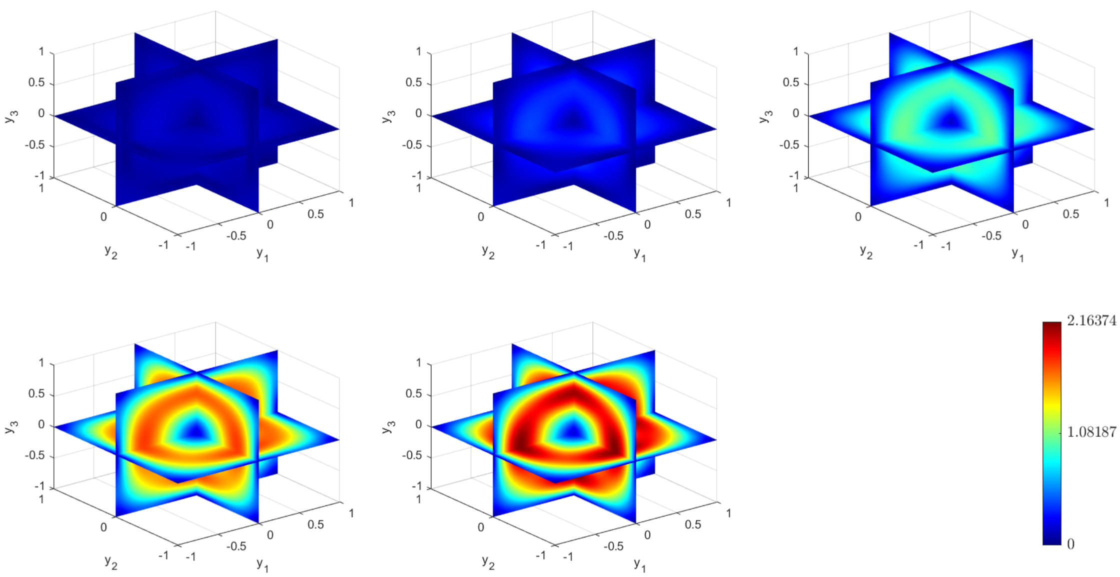

Figure 10: Norm of the optimal control $\mathbf{w}$, at times 1, 3, 5, 7 and 9 for $\kappa = -1$.