

## 1 Questions, etc

- V2 Gaussian
- Exponential Accuracy
- FEM and FDM
- Modelling particle of different sizes/ anisotropic/...

- weird thing in save2png
- error when trying save2pdf

## 2 Comparing the fixed point algorithm with fsolve

In order to compute the computational time taken of the fixed point algorithm and the inbuilt Matlab function `fsolve`, Example 1 in Section ++ is considered. Note that the comparison is slightly impacted by the fact that convergence is measured differently in these two numerical methods. However, a general comparison can be made on the efficiency of the two approaches. We choose  $n = 20$ ,  $N = 30$ , the ODE solver tolerance is set to be  $10^{-8}$  and the optimality tolerance is  $10^{-4}$  and  $\beta = 10^{-3}$ . As can be seen in Table 1, the running time of the fixed point algorithm is considerably faster than for `fsolve`, while the resulting cost functionals remain the same. This can be confirmed by comparing the number of function evaluations computed with each method. The differences in  $\rho$  and  $q$  are broadly in line with the optimality tolerance set, however the control differs more (why?). (+ Note: `fsolve` says: 'Equations solved, inaccuracies possible' - it never actually reached the optimality tolerance ++)

|                     | $\gamma$ | Time taken (s)       | F.Evals | $J_{FW}$ | $J_{Opt}$ | $\rho_{Diff}$           | $q_{Diff}$              | $\vec{w}_{Diff}$ |
|---------------------|----------|----------------------|---------|----------|-----------|-------------------------|-------------------------|------------------|
| Fixed Point         | -1       | 106.4930             | 667     | 0.0438   | 0.0011    | $3.3515 \times 10^{-4}$ | $1.0922 \times 10^{-5}$ | 0.0076           |
| <code>fsolve</code> |          | $6.3670 \times 10^4$ | 35384   | 0.0438   | 0.0011    |                         |                         |                  |
| Difference          |          | fix                  |         |          |           |                         |                         |                  |
| Fixed Point         | 1        | 101.4840             | 656     | 0.0434   | 0.0020    | $6.7721 \times 10^{-4}$ | $3.8226 \times 10^{-5}$ | 0.0204           |
| <code>fsolve</code> |          | $3.3481 \times 10^4$ | 31957   | 0.0434   | 0.0020    |                         |                         |                  |
| Difference          |          |                      |         |          |           |                         |                         |                  |

Table 1: Update this table eventually.

| gamma | beta               | Iters | JFW                     | JOpt                    |
|-------|--------------------|-------|-------------------------|-------------------------|
| 0     | $1 \times 10^{-3}$ | 671   | $4.1667 \times 10^{-2}$ | $1.4467 \times 10^{-3}$ |
| 0     | $1 \times 10^{-1}$ | 656   | $4.1667 \times 10^{-2}$ | $2.8272 \times 10^{-2}$ |
| -1    | $1 \times 10^{-3}$ | 667   | $4.3751 \times 10^{-2}$ | $1.0897 \times 10^{-3}$ |
| -1    | $1 \times 10^{-1}$ | 649   | $4.3751 \times 10^{-2}$ | $2.7033 \times 10^{-2}$ |
| 0     | $1 \times 10^{-3}$ | 726   | $6.6902 \times 10^{-2}$ | $1.0919 \times 10^{-2}$ |
| 0     | $1 \times 10^{-1}$ | 770   | $6.6902 \times 10^{-2}$ | $6.0339 \times 10^{-2}$ |
| -1    | $1 \times 10^{-3}$ | 724   | $5.3559 \times 10^{-2}$ | $9.6531 \times 10^{-3}$ |
| -1    | $1 \times 10^{-1}$ | 769   | $5.3559 \times 10^{-2}$ | $4.9268 \times 10^{-2}$ |

Table 2: Test for print to matlab - ignore.