1 Sedimentation Optimization

We have $N = 40$ and $n = 30$ for each shape. We choose the ODE tolerance to be 10^{-7} and the optimization tolerance is 10^{-3}. I fixed the problems with the implementation of the sedimentation optimization code. First, I tested whether setting $\widehat{\rho} = \rho_{FW}$ would converge within one iteration. This happened. Then I set up a test problem which sets $\widehat{\rho}$ to be the forward solution for $V_{ext} = ay$, where $a = 0.1$, as in Archer's paper. Then I set up the optimization forward problem to be such that $a = 0.01$ and $\mathbf{w} = \mathbf{0}$. We expect the control to act downward, since the strength of gravity a is decreased. We also expect that the cost $\mathcal{J}$ is decreasing from the baseline J_{FW} when optimizing. For $\beta = 10^{-3}$ and $\beta = 10^{-1}$ this works well. When $\beta = 10^{-3}$ we get $J_{FW} = 0.4955$ and $J_{Opt} = 0.0556$. The results can be seen in Figures 1, 2 and 3.

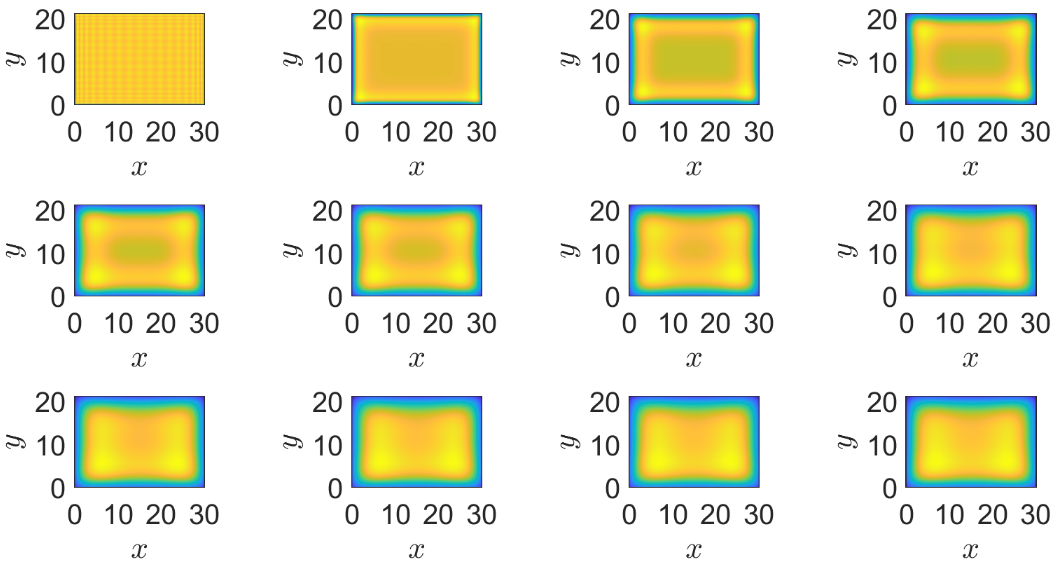

Figure 1: Forward ρ for $a = 0.01$

1.1 Multishape

Just to test whether the multishape OCP works now (since there have been issues in the past which have been fixed when fixing another bug). We have $N = 20$ and $n = 30$ for each shape. We choose the ODE tolerance to be 10^{-7} and the optimization tolerance is 10^{-3}. We choose the target with $a = 0.1$ and the forward problem with 0.099, so that we get quick convergence, since we just want to see whether the method is working. We get $J_{FW} = 4.4833 \times 10^{-5}$ and $J_{Opt} = 2.6884 \times 10^{-6}$. The results can be seen in Figures 4 and 5.

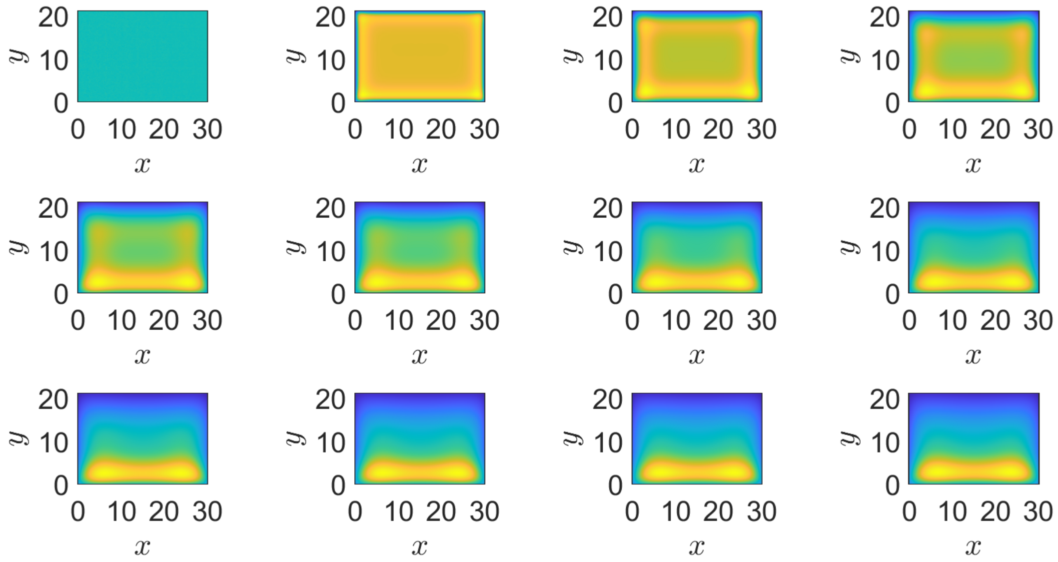

Figure 2: Optimal ρ for $a = 0.01$

2 Time-independent control

We now use the example with the same configurations as in the first section. The difference will be that the gradient equation is:

$$\mathbf{w} = -\frac{1}{\beta}\int_0^T \rho \nabla q dt.$$

This means, we get a $\mathbf{w}$ which is averaged over the time horizon and therefore time independent. This seems to work well. $J_{FW} = 0.4855$ and $J_{Opt} = 0.0733$. The results can be seen in Figures 6, 7 and 8.

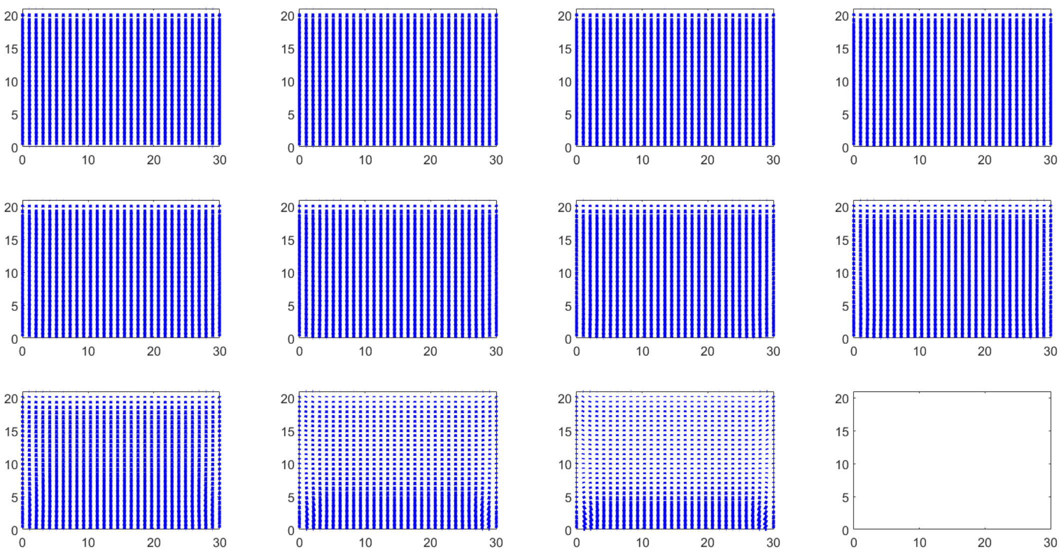

Figure 3: Optimal Control for $a = 0.01$

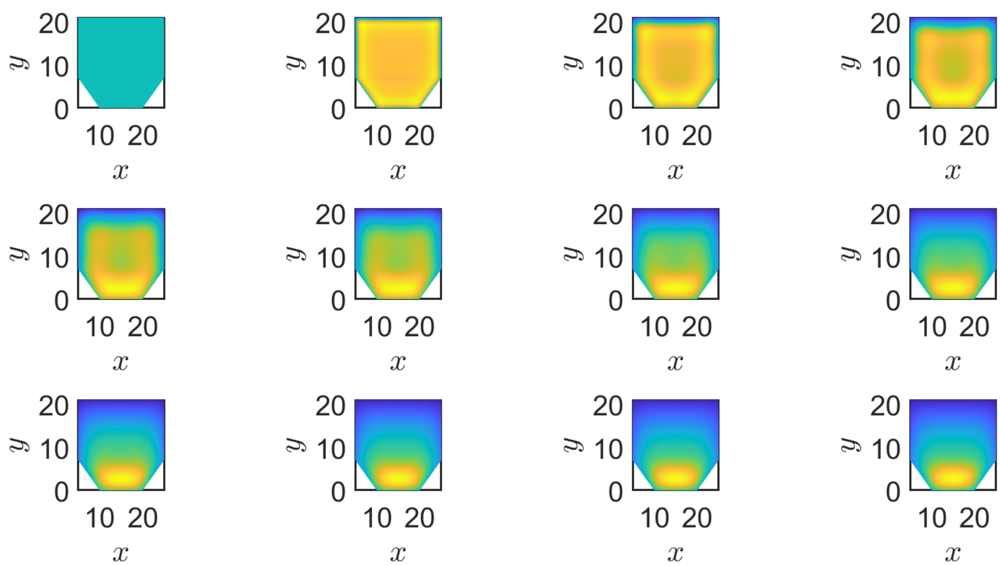

Figure 4: Optimal ρ for $a = 0.099$

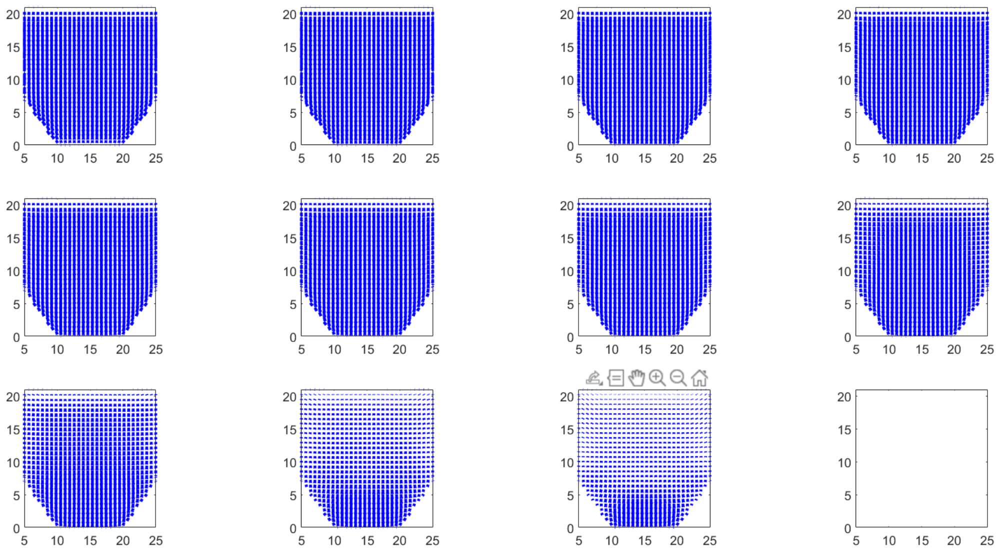

Figure 5: Optimal Control for $a = 0.099$

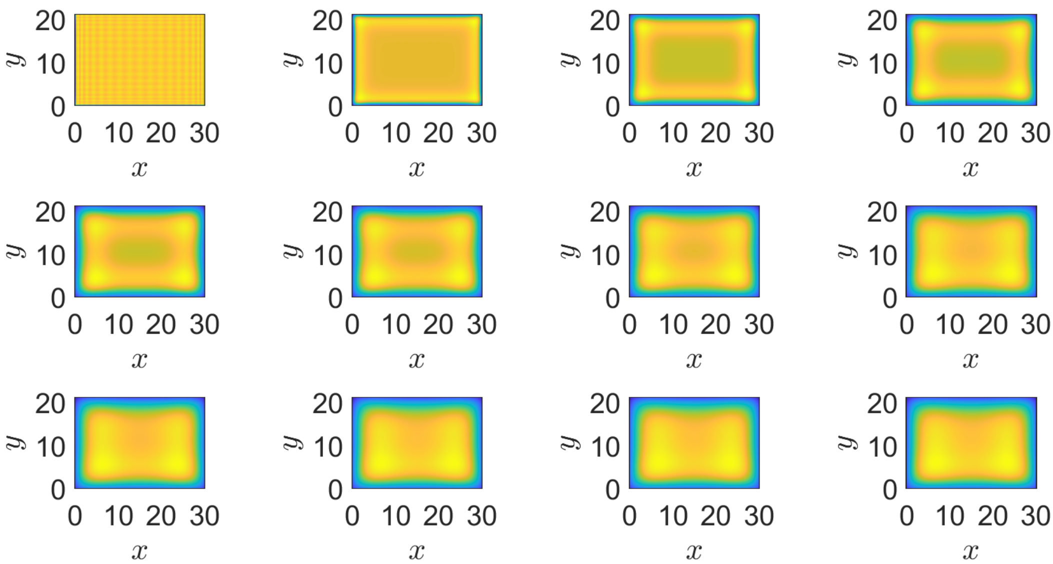

Figure 6: Time-independent; Forward ρ for $a = 0.01$

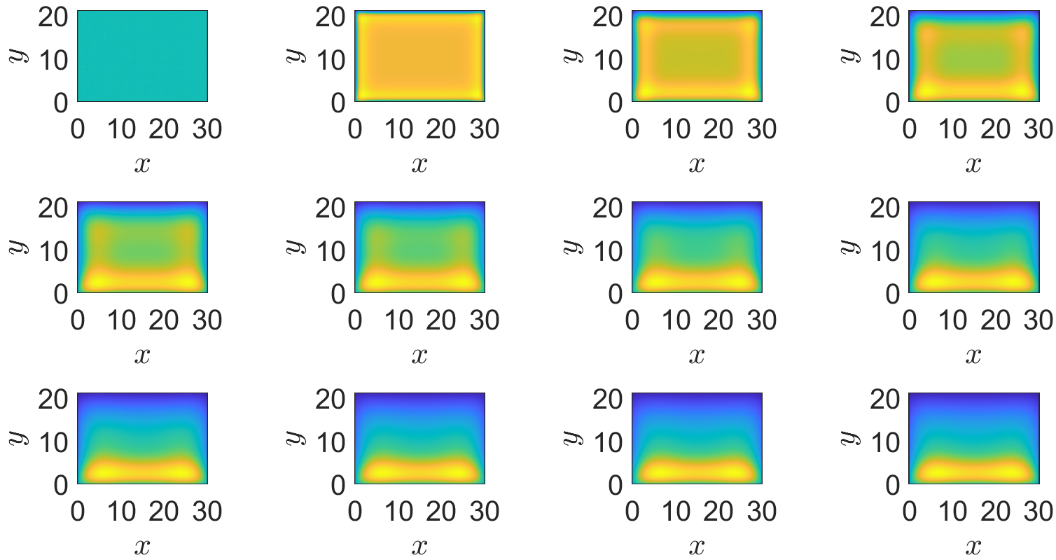

Figure 7: Time-independent; Optimal ρ for $a = 0.01$

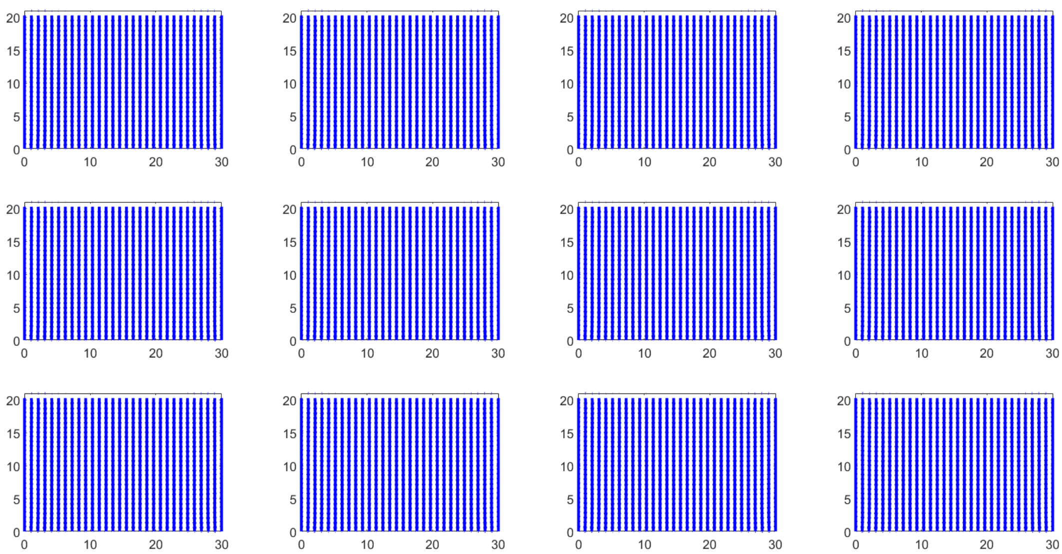

Figure 8: Time-independent; Optimal Control for $a = 0.01$