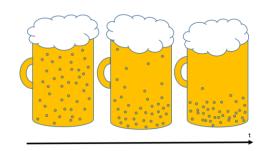$$\min_{\rho, \vec{w}} \quad \frac{1}{2}\|\rho - \widehat{\rho}\|^2_{L_2(\Sigma)} + \frac{\beta}{2}\|\vec{w}\|^2_{L_2(\Sigma)}$$

$$\partial_t \rho = \nabla^2 \rho - \nabla \cdot (\rho \vec{w}) + \nabla \cdot (\rho \nabla V_{\text{ext}}) + \nabla \cdot \int_\Omega \rho(\vec{x})\rho(\vec{x}')\nabla V_2(|\vec{x} - \vec{x}'|)d\vec{x}'$$