Report 30/04/2020

All plotting times are defined by 'linspace(0,1,10)'. All examples are run with FixPt, and $\beta = 10^{-3}$ only. $\lambda = 0.01$, $N = 60$, $n = 61$, OLD Tol $= 10^{-8}$, Optimality Tol $= 10^{-4}$.

1 Neumann Flow Control - Asymmetric Example 1

For this, the initial condition for ρ is $\rho_{IC} = 0.5$, and the Flow term is zero in the forward problem. The target is:

$$\hat{\rho} = 0.5(1-t) + t(\frac{1}{2}\sin(\pi(y-2)/2) + \frac{1}{2}).$$

We consider $\beta = 10^{-3}$ and $\gamma = 0$, $\gamma = 1$ and $\gamma = -1$. All of these examples converge in about 700 iterations, using FixPt. $\lambda = 0.01$, $N = 60$, $n = 61$, OLD Tol $= 10^{-8}$, Optimality Tol $= 10^{-4}$. When $\gamma = 0$, $J_{FW} = 0.0417$ and $J_{Opt} = 0.0014$, see Figure 1. When $\gamma = -1$,

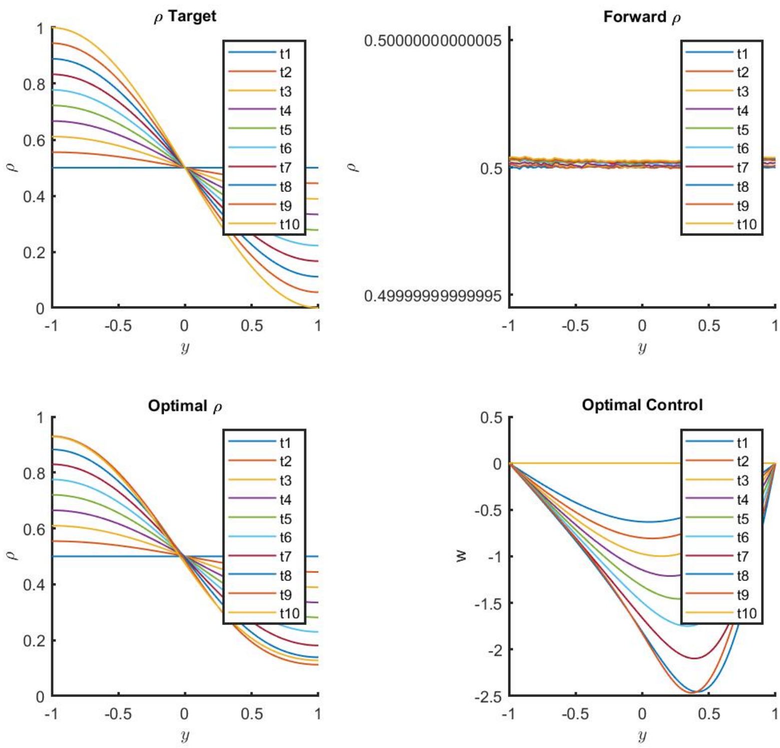

Figure 1: Results for Neumann Flow, Asymmetric Example 1, $\gamma = 0$.

$J_{FW} = 0.0438$ and $J_{Opt} = 0.0011$, see Figure 2. When $\gamma = 1$, $J_{FW} = 0.0434$ and $J_{Opt} = 0.0020$, see Figure 3.

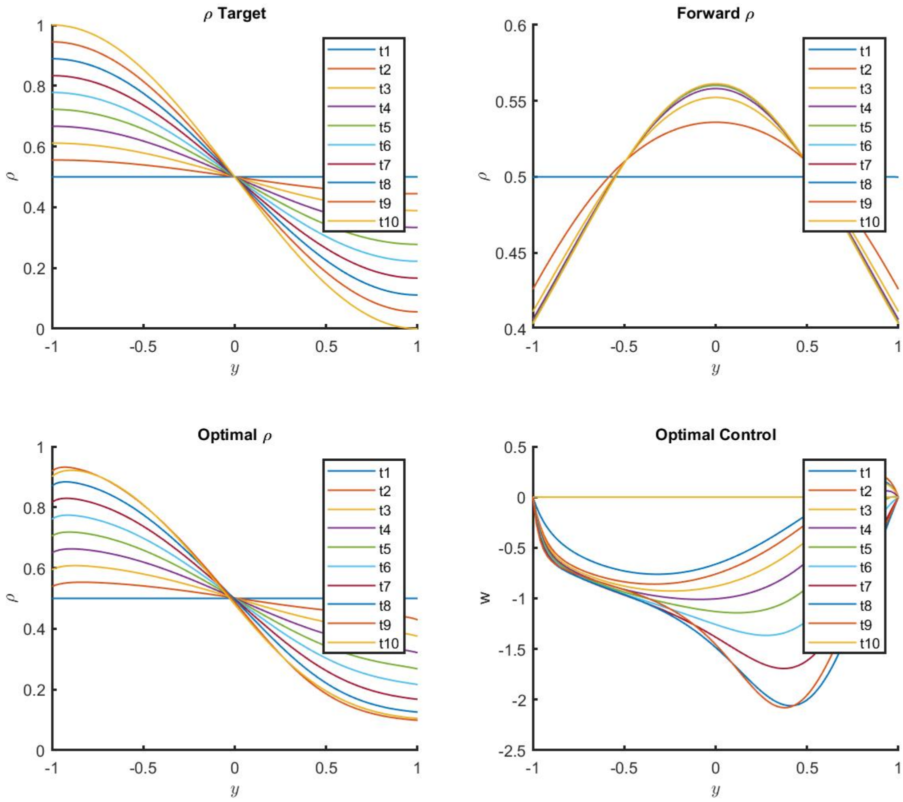

Figure 2: Results for Neumann Flow, Asymmetric Example 1, $\gamma = -1$.

2 Neumann Flow Control - Asymmetric Example 2

For this, the initial condition for ρ is:

$$\rho_{IC} = (-\frac{1}{2}\sin(\pi(y-2)/2) + \frac{1}{2}),$$

and the Flow term is zero in the forward problem. The target is:

$$\hat{\rho} = (1-t)(-\frac{1}{2}\sin(\pi(y-2)/2) + \frac{1}{2}) + t(\frac{1}{2}\sin(\pi(y-2)/2) + \frac{1}{2}),$$

which is similar to the target of the example above but with the new ρ_{CIC} incorporated.
We consider $\beta = 10^{-3}$ and $\gamma = 0$, $\gamma = 1$ and $\gamma = -1$. All of these examples converge in about 700 iterations and take about 15 min, using FixPt. $\lambda = 0.01$, $N = 60$, $n = 61$, OLD Tol $= 10^{-8}$, Optimality Tol $= 10^{-4}$. When $\gamma = 0$, $J_{FW} = 0.0321$ and $J_{Opt} = 0.0013$, see Figure 4. When $\gamma = -1$, $J_{FW} = 0.0384$ and $J_{Opt} = 0.0012$, see Figure 5. When $\gamma = 1$, $J_{FW} = 0.0307$ and $J_{Opt} = 0.0017$, see Figure 6.

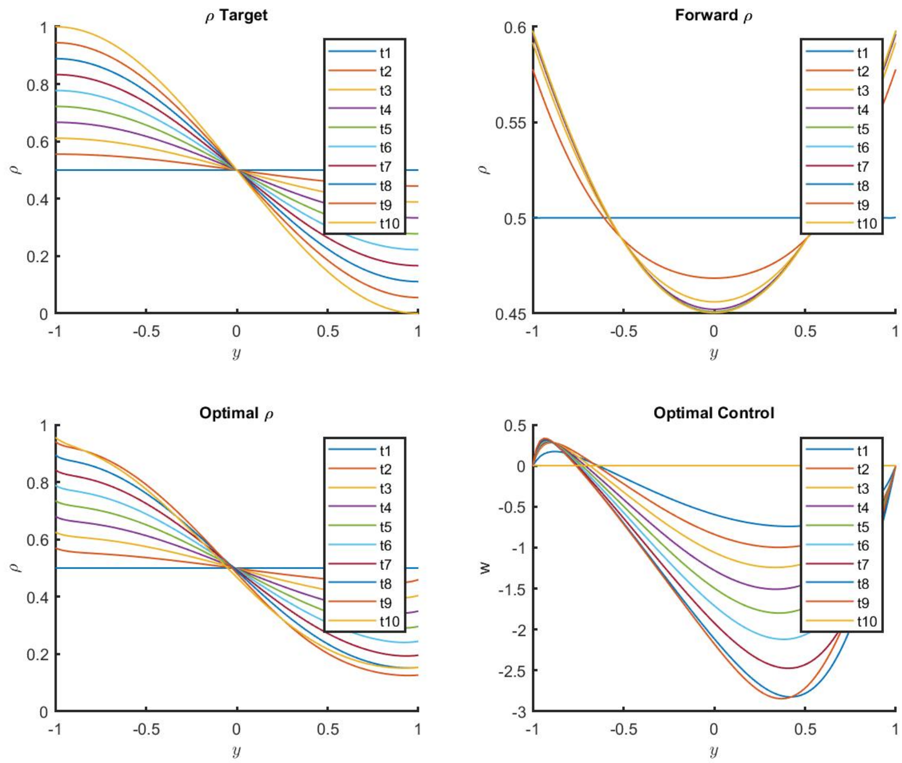

Figure 3: Results for Neumann Flow, Asymmetric Example 1, $\gamma = 1$.

3 Dirichlet Flow Control - Symmetric Example

We take the Dirichlet Boundary condition to be $\rho_{\partial\Omega} = 0.5$. Then the initial condition for ρ is $\rho_{IC} = 0.5$, the forward Flow term is zero. The target is:

$$\hat{\rho} = 0.5(1-t) + t(-\frac{1}{4}\cos(\pi y) + \frac{1}{4}).$$

We consider $\beta = 10^{-3}$ and $\gamma = 0$, $\gamma = 1$ and $\gamma = -1$. All of these examples converge in under 5 minutes and about 700 iterations, using FixPt. $\lambda = 0.01$, $N = 60$, $n = 61$, OLD Tol $= 10^{-8}$, Optimality Tol $= 10^{-4}$. When $\gamma = 0$, $J_{FW} = 0.0313$ and $J_{Opt} = 0.0018$, see Figure 7. When $\gamma = -1$, $J_{FW} = 0.0741$ and $J_{Opt} = 0.0022$, see Figure 8. When $\gamma = 1$, $J_{FW} = 0.0148$ and $J_{Opt} = 0.0015$, see Figure 9. The results for the three cases are slightly but not very visibly different.

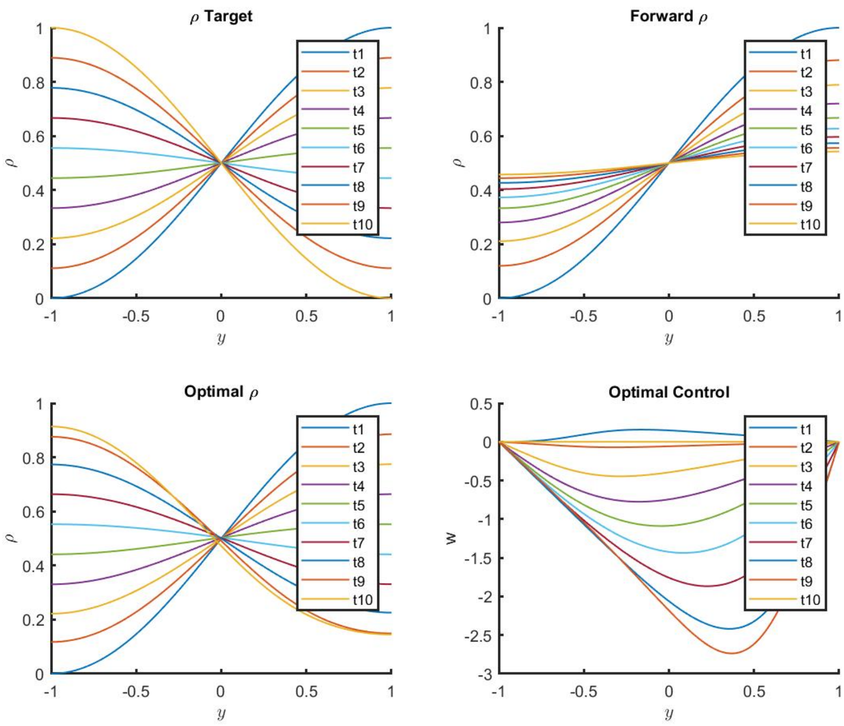

Figure 4: Results for Neumann Flow, Asymmetric Example 2, $\gamma = 0$.

4 Dirichlet Flow Control - Asymmetric Example

We take the Dirichlet Boundary condition to be $\rho_{\partial\Omega} = 0.5$. Then the initial condition for ρ is $\rho_{IC} = 0.5$, the forward Flow term is zero. The target is:

$$\hat{\rho} = 0.5(1-t) + t(\frac{1}{2}(\sin(\pi y) + 1)).$$

We consider $\beta = 10^{-3}$ and $\gamma = 0$, $\gamma = 1$ and $\gamma = -1$. All of these examples converge in under 5 minutes and about 700 iterations, using FixPt. $\lambda = 0.01$, $N = 60$, $n = 61$, OLD Tol $= 10^{-8}$, Optimality Tol $= 10^{-4}$. When $\gamma = 0$, $J_{FW} = 0.0417$ and $J_{Opt} = 0.0027$, see Figure 10.
When $\gamma = -1$, $J_{FW} = 0.0510$ and $J_{Opt} = 0.0026$, see Figure 11.
When $\gamma = 1$, $J_{FW} = 0.0452$ and $J_{Opt} = 0.0030$, see Figure 12.
An interesting observation is that even though J_{FW} is smaller for $\gamma = 1$, J_{Opt} is smaller for $\gamma = -1$. Overall, the particle interactions don't seem to have a large impact on the results.

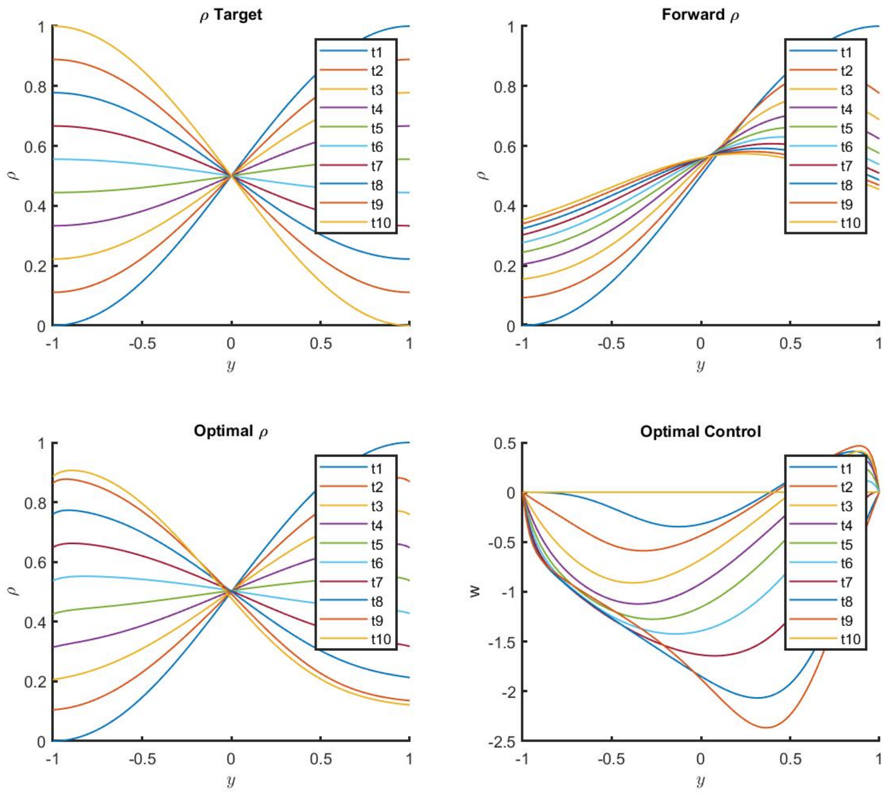

Figure 5: Results for Neumann Flow, Asymmetric Example 2, $\gamma = -1$.

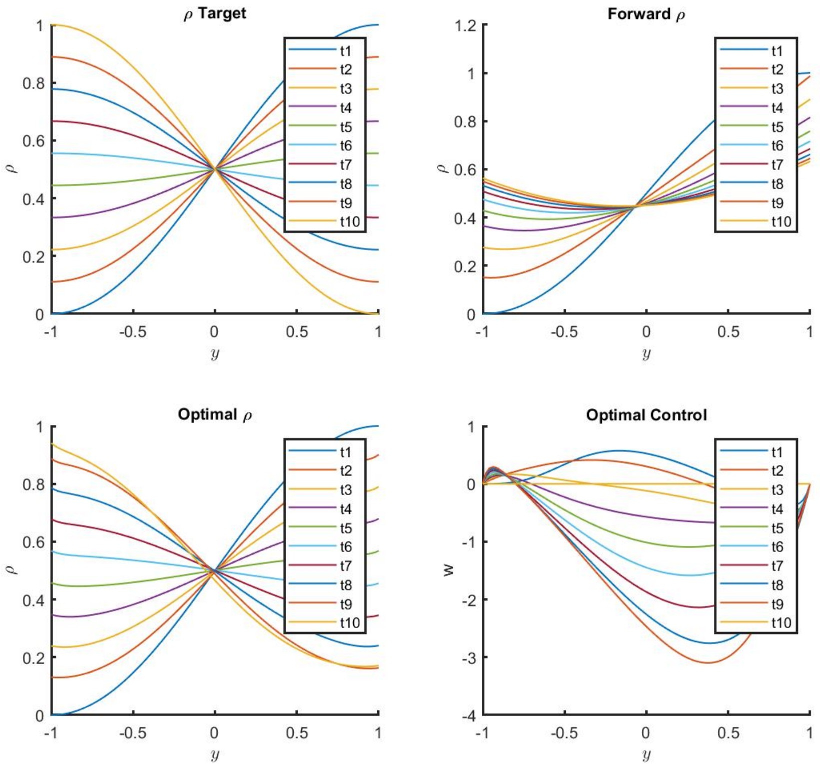

Figure 6: Results for Neumann Flow, Asymmetric Example 2, $\gamma = 1$.

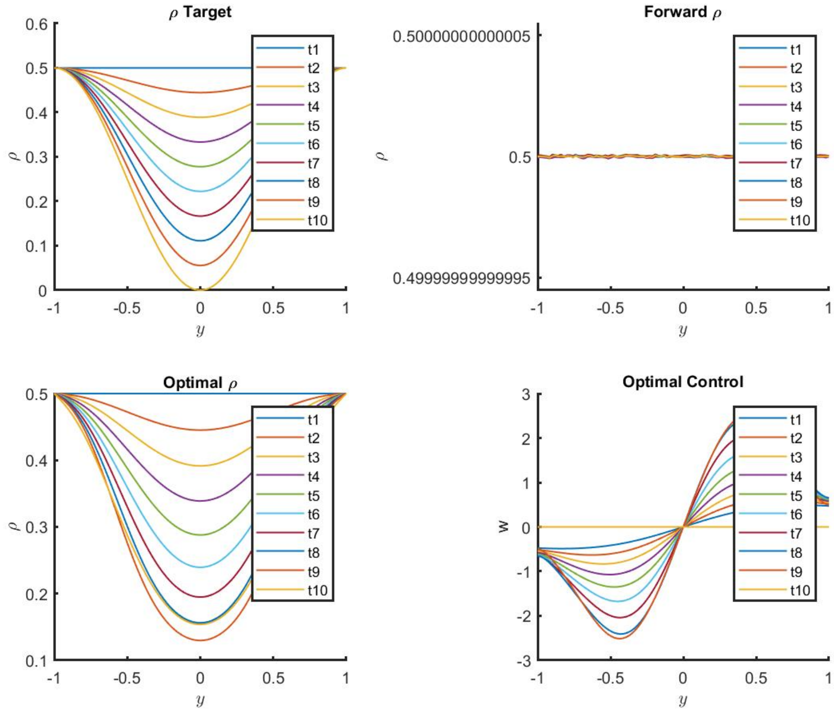

Figure 7: Results for Dirichlet Flow, Symmetric Example, $\gamma = 0$.

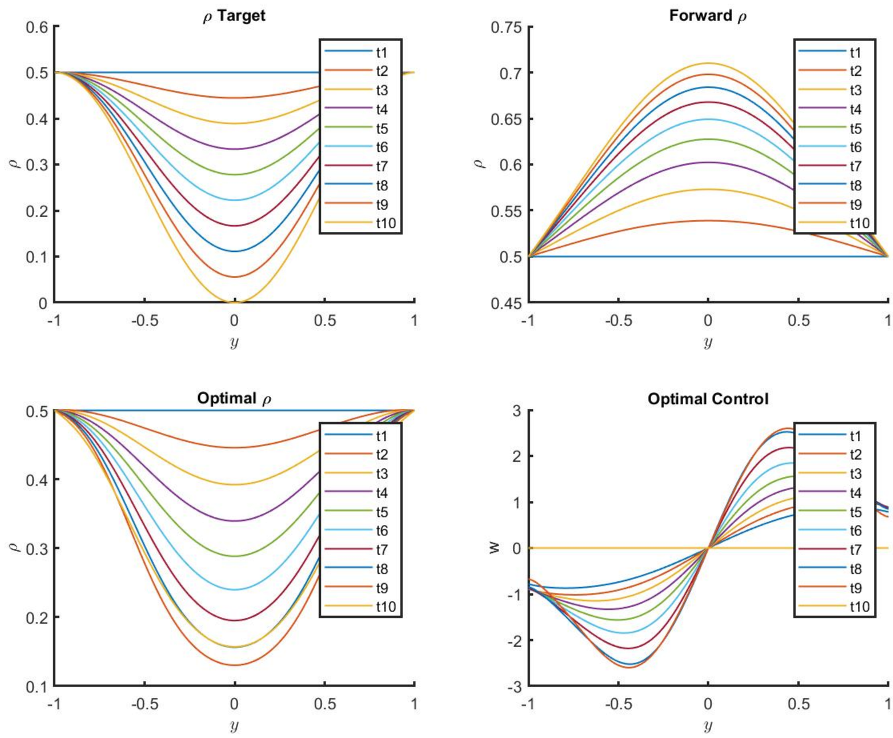

Figure 8: Results for Dirichlet Flow, Symmetric Example, $\gamma = -1$.

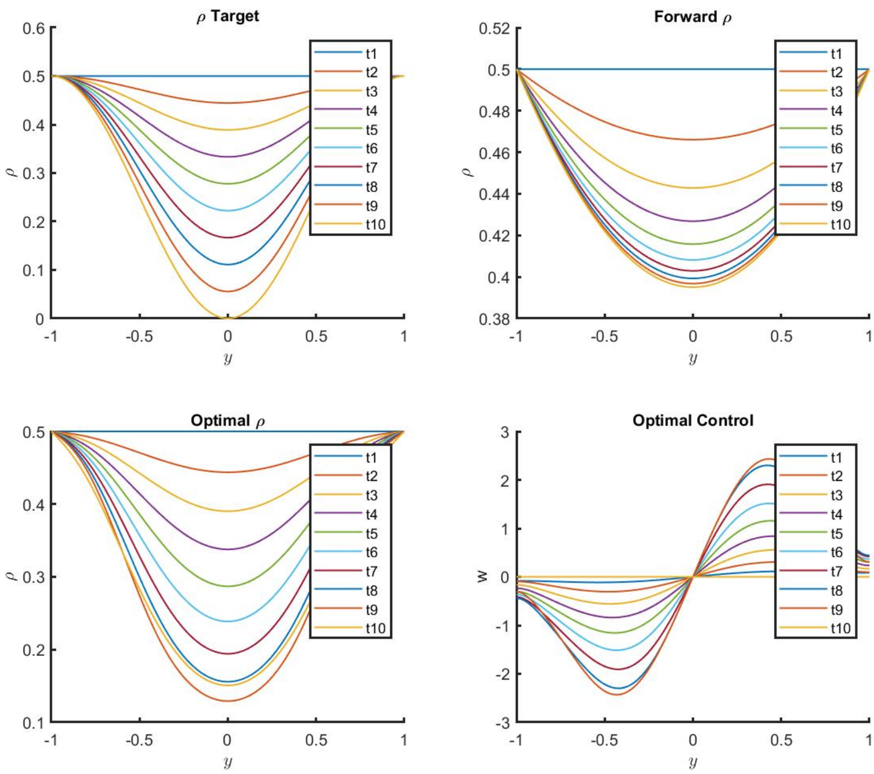

Figure 9: Results for Dirichlet Flow, Symmetric Example, $\gamma = 1$.

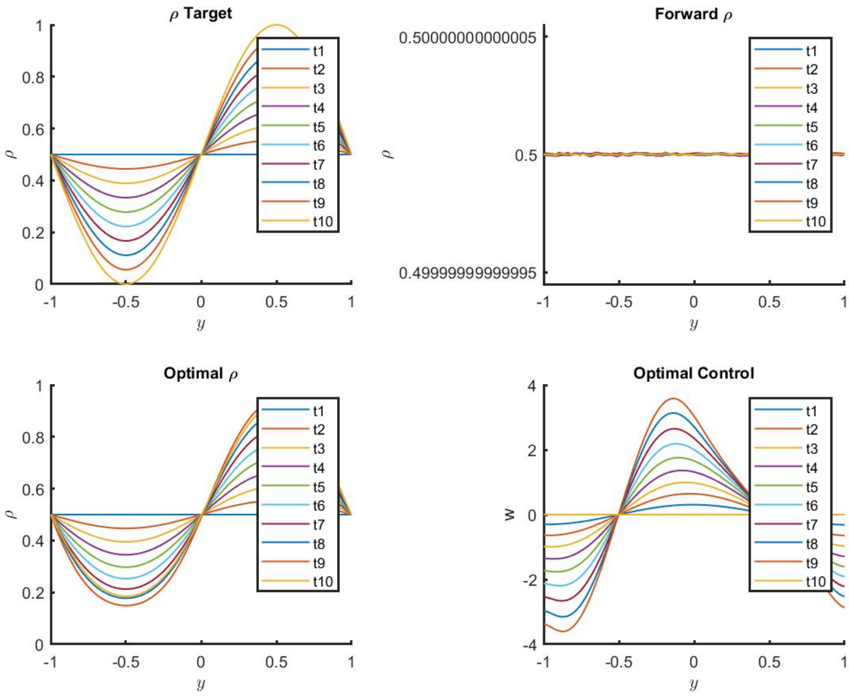

Figure 10: Results for Dirichlet Flow, Asymmetric Example, $\gamma = 0$.

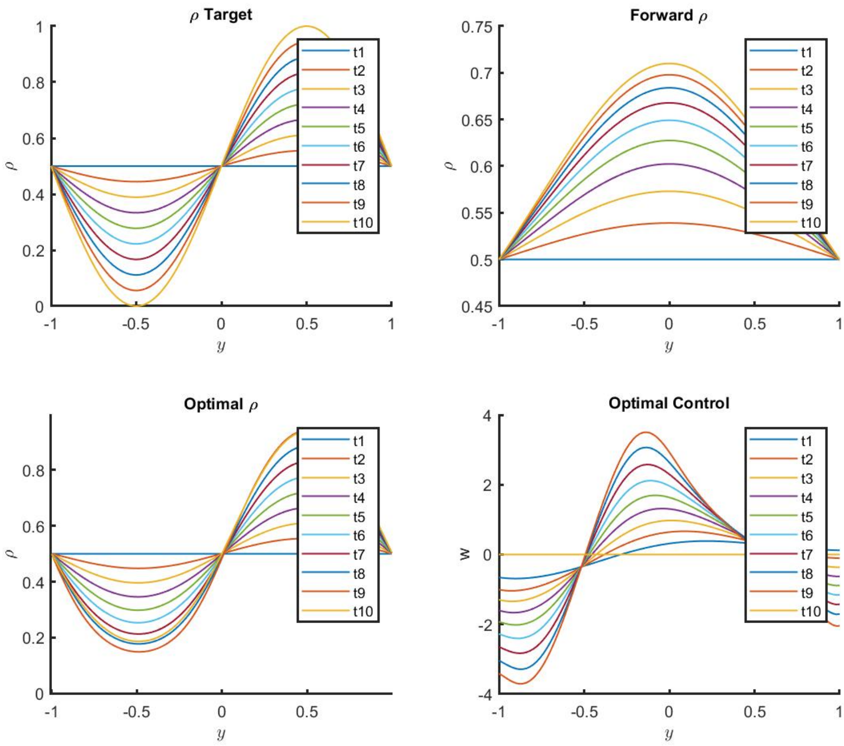

Figure 11: Results for Dirichlet Flow, Asymmetric Example, $\gamma = -1$.

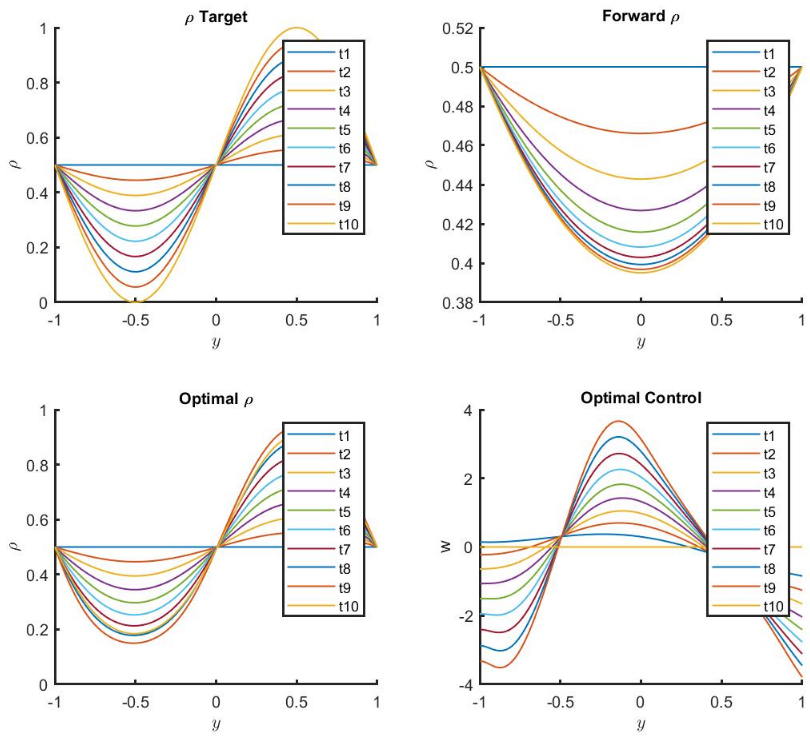

Figure 12: Results for Dirichlet Flow, Asymmetric Example, $\gamma = -1$.