gamma	beta	Iters	JFW	JOpt
1×10^{-03}	0.000	671	0.04166667	0.00144668
1×10^{-03}	0.000	726	0.06690197	0.01091870

Notes/ Questions on Report:

- Paper plots: color bar from t0 to T or other way around?
- don't know what's up with this extra arrow in the vector plots.
- Comment: Figure 8 to 11 and Figure 14 to 17 are the interesting examples.

Currently running some configurations of Ex2 and Ex4 on server.

Notes/ Questions on Presentation:

- only Neumann Flow control below.
- summary slide: different models?
- how to call $\vec{w}$ in talk (I'll explain what I mean by this).
- if too long, what should go?
- figure references ok?

1 Paper plots fixed

See figures 1, 2 and 3.

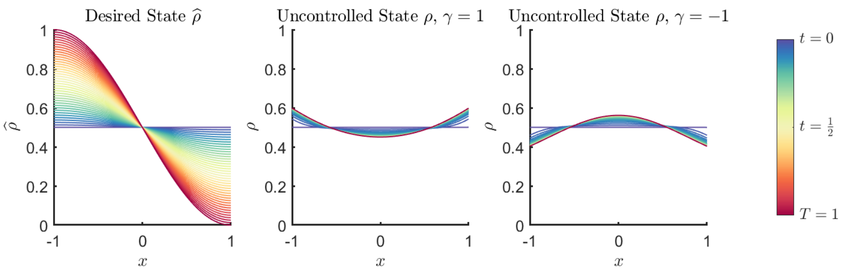

Figure 1: Example 1.

2 2D Example 1

We choose $\rho_0 = 0.25$ and

$$\hat{\rho} = 0.25(1-t) + t\frac{1}{4}((\cos(\pi y_1) + 1)(\cos(\pi y_2) + 1)),$$

as in last week's report. See figures 4 and 5.

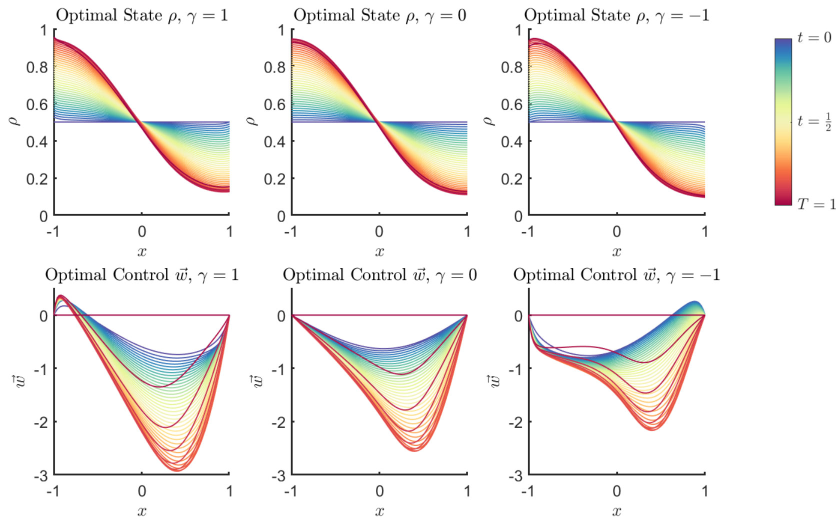

Figure 2: Example 1.

3 2D Example 2

Choose $\rho_0 = 0.25$ and target:

$$\widehat{\rho} = \frac{1}{4}(1-t) + t\left(\frac{1}{4}\sin\left(\frac{\pi}{2}(x_1-2)\right)\sin\left(\frac{\pi}{2}(x_2-2)\right) + \frac{1}{4}\right).$$

Choose $n = 10$, $N = 20$ (probably need more in the future but it's quick). For $\beta = 10^{-3}$, $\gamma = -1$, we get $J_{FW} = 0.0130$ and $J_{Opt} = 7.2994 \times 10^{-4}$, see figures 8 and 9. For $\beta = 10^{-3}$, $\gamma = 1$, we get $J_{FW} = 0.0108$ and $J_{Opt} = 0.0023$, see figures 10 and 11.

4 2D Example 3

Choose $\rho_0 = \frac{1}{4}$ and the target:

$$\widehat{\rho} = \frac{1}{4}(1-t) + t\frac{1}{0.31405}e^{-10((y_1+0.2)^2+(y_2+0.2)^2))}.$$

Note the target doesn't satisfy the boundary conditions. This converges for $\beta = 10^{-1}$ but diverges for $\beta = 10^{-3}$ for various N, n and γ. Probably due to steep $\widehat{\rho}$, so for small β we have advection dominance. Choose $\beta = 10^{-1}$, $n = 20$, $N = 30$, Tols $= 10^{-8}/10^{-4}$. Then for $\gamma = -1$, we get $J_{FW} = 0.1993$, $J_{Opt} = 0.1595$, see figures 12 and 13.

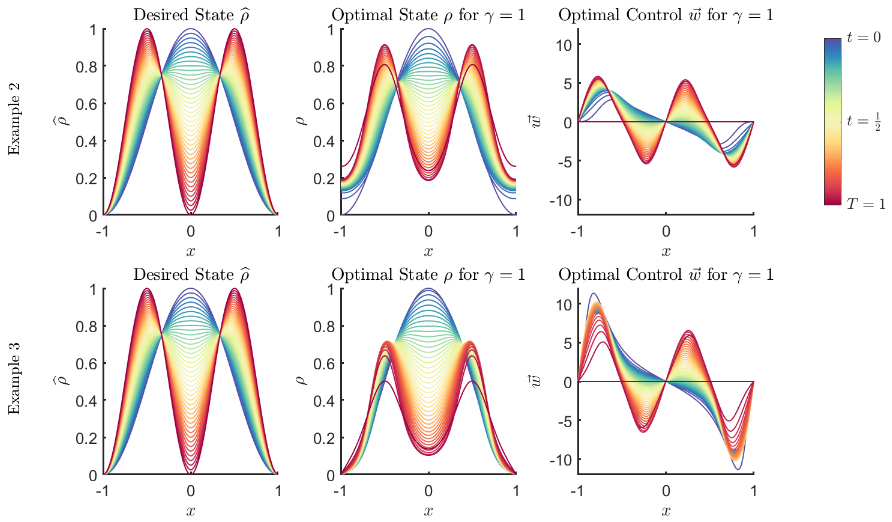

Figure 3: Example 2/ Example 3.

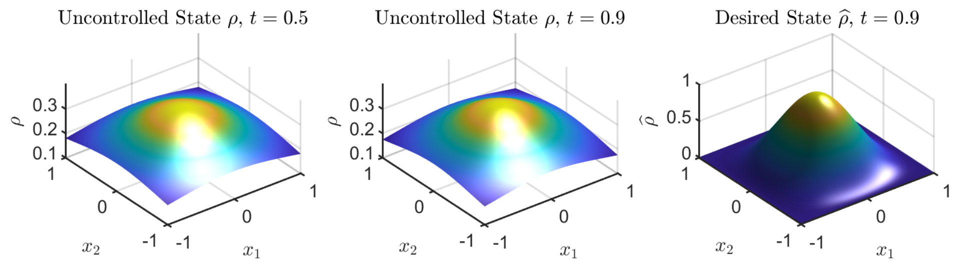

Figure 4: 2D Example 1, uncontrolled ρ and $\widehat{\rho}$, $\beta = 10^{-3}$, $\gamma = -1$.

5 2D Example 4

We consider a very similar example to Example 3, but with less steep desired state. We have $\rho_0 = 0.25$, as before, and the target:

$$\widehat{\rho} = \frac{1}{4}(1-t) + t\frac{1}{0.9921}e^{-3((y_1+0.2)^2+(y_2+0.2)^2))}.$$

Again, this does not satisfy the no-flux boundary conditions. We have $n = 10$, $N = 20$, which needs to be increased in the future.

Then for $\gamma = -1$, we get $J_{FW} = 0.0329$, $J_{Opt} = 0.0014$, see figures 14 and 15.

Then for $\gamma = 1$, we get $J_{FW} = 0.0524$, $J_{Opt} = 0.0135$, see figures 16 and 17.

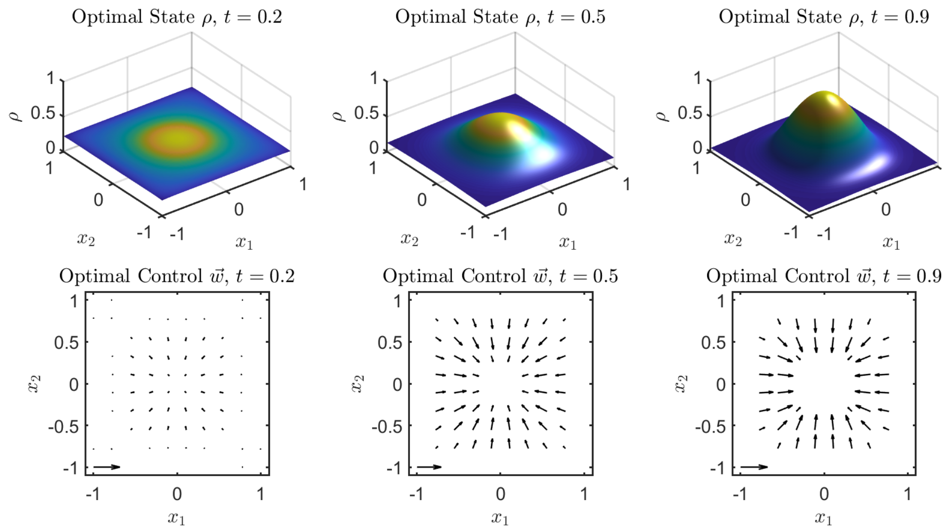

Figure 5: 2D Example 1, controlled ρ and optimal control $\vec{w}$, $\beta = 10^{-3}$, $\gamma = -1$.

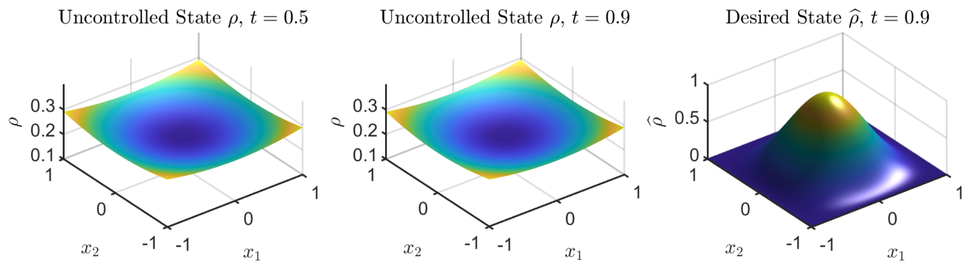

Figure 6: 2D Example 1, uncontrolled ρ and $\widehat{\rho}$, $\beta = 10^{-3}$, $\gamma = 1$.

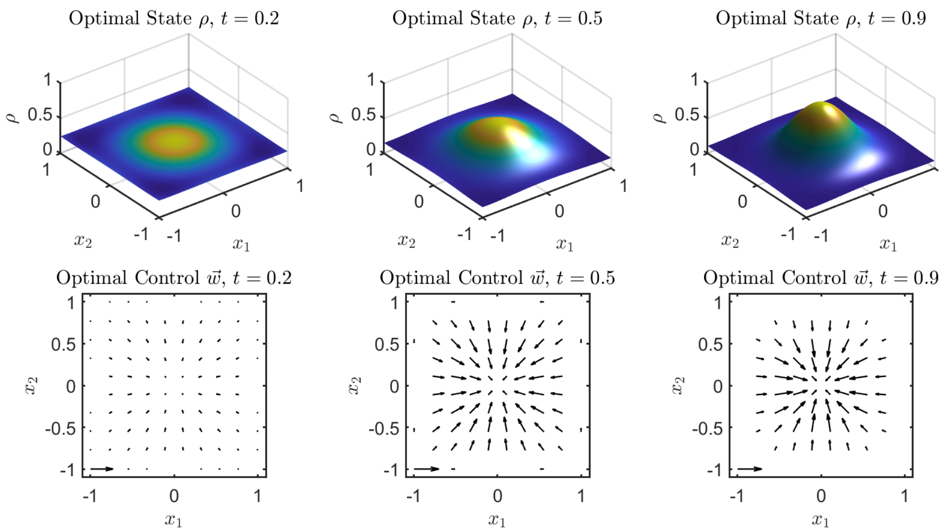

Figure 7: 2D Example 1, controlled ρ and optimal control $\vec{w}$, $\beta = 10^{-3}$, $\gamma = 1$.

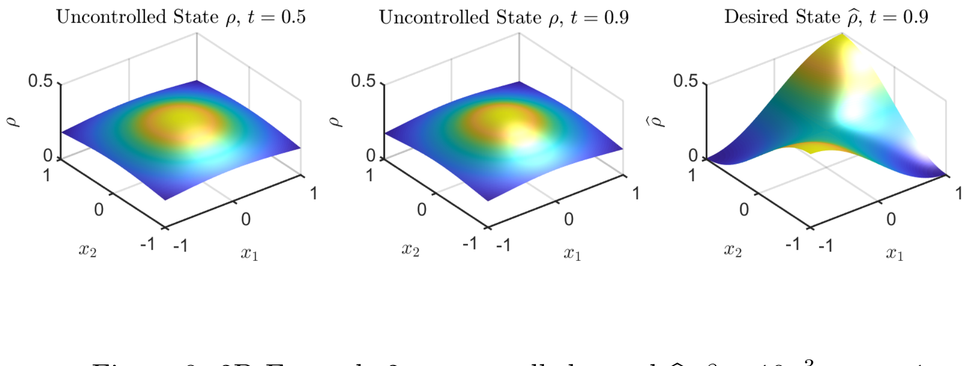

Figure 8: 2D Example 2, uncontrolled ρ and $\widehat{\rho}$, $\beta = 10^{-3}$, $\gamma = -1$.

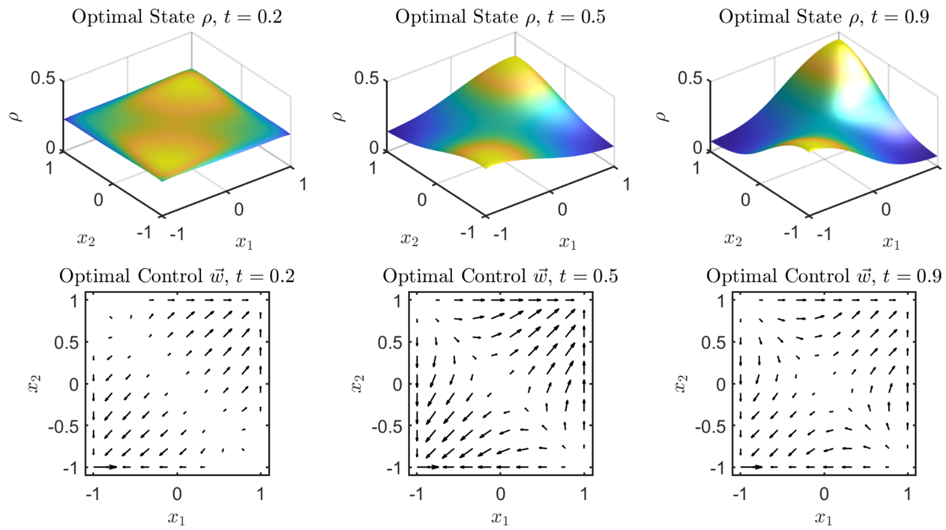

Figure 9: 2D Example 2, controlled ρ and optimal control $\vec{w}$, $\beta = 10^{-3}$, $\gamma = -1$.

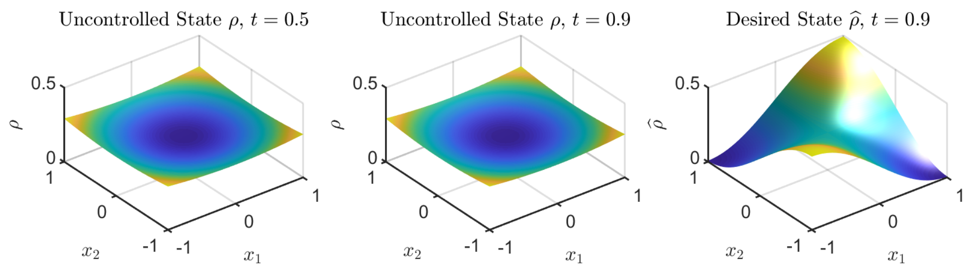

Figure 10: 2D Example 2, uncontrolled ρ and $\widehat{\rho}$, $\beta = 10^{-3}$, $\gamma = 1$.

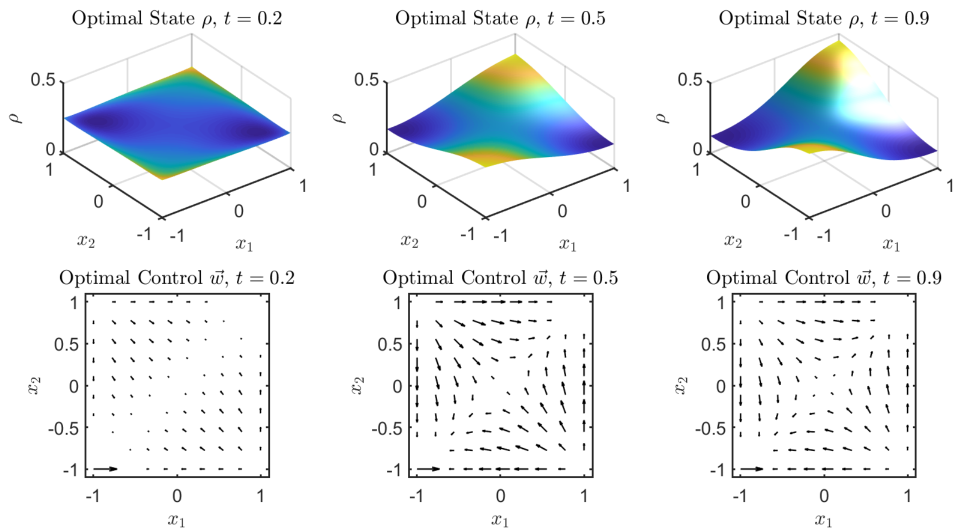

Figure 11: 2D Example 2, controlled ρ and optimal control $\vec{w}$, $\beta = 10^{-3}$, $\gamma = 1$.

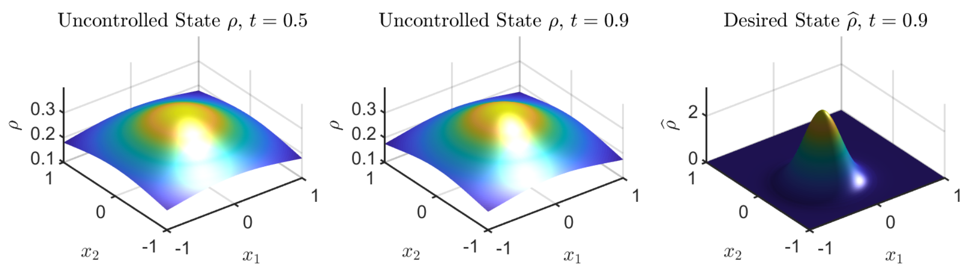

Figure 12: 2D Example 3, uncontrolled ρ and $\widehat{\rho}$, $\beta = 10^{-1}$, $\gamma = -1$.

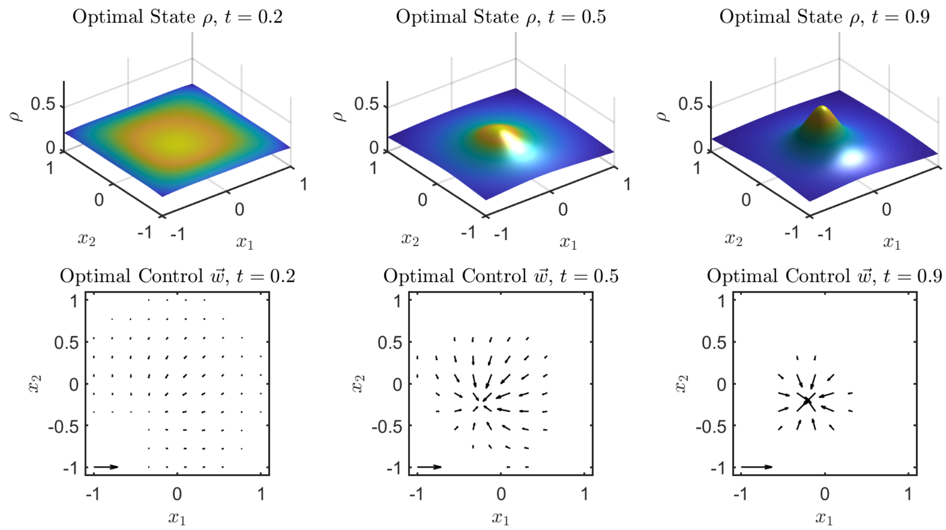

Figure 13: 2D Example 3, controlled ρ and optimal control $\vec{w}$, $\beta = 10^{-1}$, $\gamma = -1$.

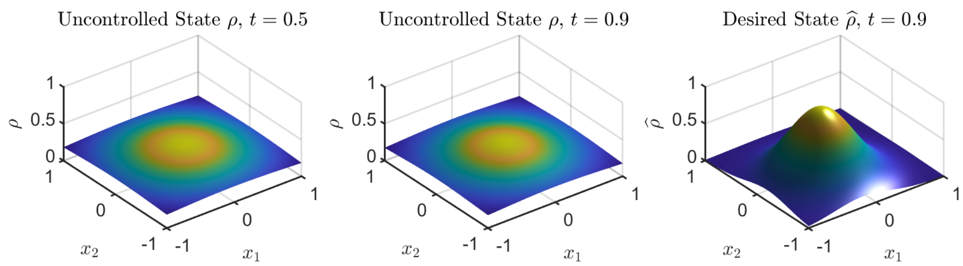

Figure 14: 2D Example 4, uncontrolled ρ and $\widehat{\rho}$, $\beta = 10^{-3}$, $\gamma = -1$.

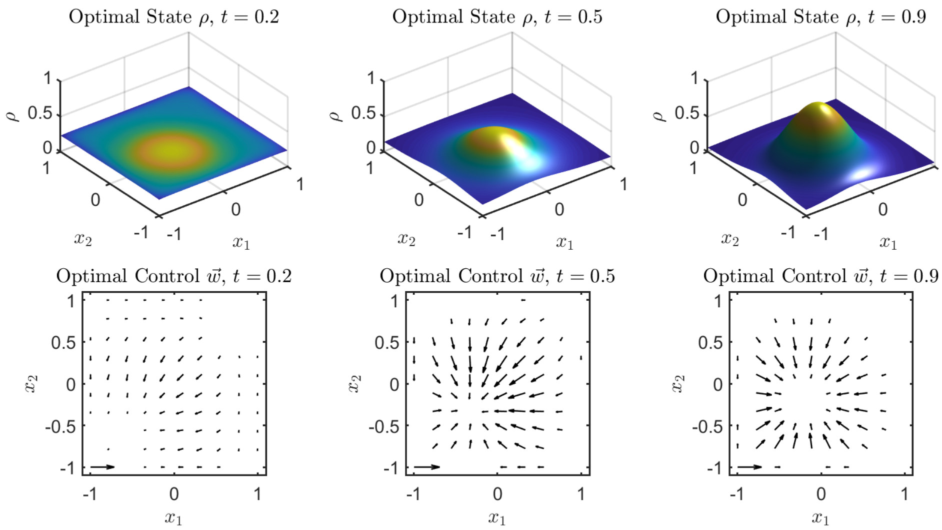

Figure 15: 2D Example 4, controlled ρ and optimal control $\vec{w}$, $\beta = 10^{-3}$, $\gamma = -1$.

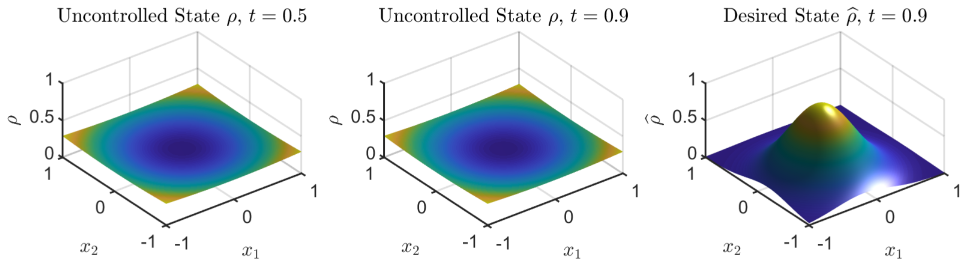

Figure 16: 2D Example 4, uncontrolled ρ and $\widehat{\rho}$, $\beta = 10^{-3}$, $\gamma = 1$.

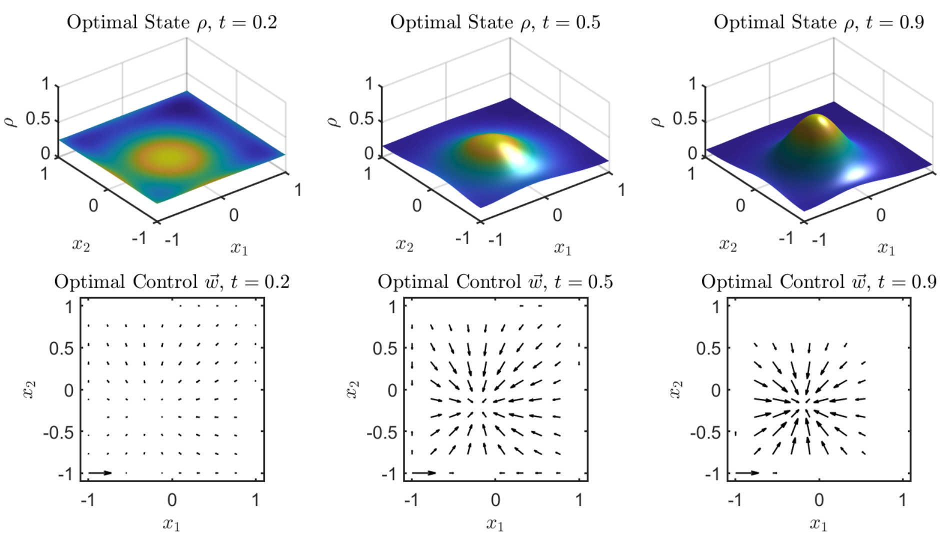

Figure 17: 2D Example 4, controlled ρ and optimal control $\vec{w}$, $\beta = 10^{-3}$, $\gamma = 1$.