

| β / γ | 10^{-3} | 10^{-1} | 10 | 10^3 |
|------------------|-------------------|-------------------|-------------------|-------------------|
| -1 | $J_{uc} = 0.0041$ | $J_{uc} = 0.0041$ | $J_{uc} = 0.0041$ | $J_{uc} = 0.0041$ |
| | $J_c = 0.0002$ | $J_c = 0.0033$ | $J_c = 0.0040$ | $J_c = 0.0041$ |
| | Iter. = 607 | Iter. = 637 | Iter. = 311 | Iter. = 1 |
| 0 | $J_{uc} = 0.0104$ | $J_{uc} = 0.0104$ | $J_{uc} = 0.0104$ | $J_{uc} = 0.0104$ |
| | $J_c = 0.0005$ | $J_c = 0.0086$ | $J_c = 0.0104$ | $J_c = 0.0104$ |
| | Iter. = 635 | Iter. = 671 | Iter. = 340 | Iter. = 1 |
| 1 | $J_{uc} = 0.0195$ | $J_{uc} = 0.0195$ | $J_{uc} = 0.0195$ | $J_{uc} = 0.0195$ |
| | $J_c = 0.0011$ | $J_c = 0.0164$ | $J_c = 0.0195$ | $J_c = 0.0195$ |
| | Iter. = 656 | Iter. = 696 | Iter. = 356 | Iter. = 1 |

Table 1

1 Some bits and pieces that are not part of the paper.

+++ Two additional nonlinear control problems below, may be deleted? +++

2 Neumann boundary conditions, Symmetric Example 1

Consider the following symmetric setup:

$$\begin{aligned}
\hat{\rho} &= \frac{1}{2}(1-t) + t\frac{1}{4}(\cos(\pi y) + 2) \\
\rho_0 &= \frac{1}{2} \\
q_T &= 0 \\
\vec{w} &= 0 \\
f &= 0 \\
V_{ext} &= 0
\end{aligned}$$

Table 1 summarizes the results for this example. The attractive interaction term causes ρ to move towards the centre of the domain. Since $\hat{\rho}$ is also centred in the domain, J_{uc} is small for $\gamma = -1$ in comparison to the problems with $\gamma = 0$ and $\gamma = 1$. This example illustrates that the particle interaction term can have a significant impact on the optimization problem considered.

| β / γ | 10^{-3} | 10^{-1} | 10 | 10^3 |
|------------------|---|--|--|--|
| -1 | $J_{uc} = 0.0209$ $J_c = 0.0009$ Iter. = 646 | $J_{uc} = 0.0209$ $J_c = 0.0168$ Iter. = 691 | $J_{uc} = 0.0209$ $J_c = 0.0209$ Iter. = 379 | $J_{uc} = 0.0209$ $J_c = 0.0209$ Iter. = 1 |
| 0 | $J_{uc} = 0.0104$ $J_c = 0.0005$ Iter. = 635 | $J_{uc} = 0.0104$ $J_c = 0.0086$ Iter. = 671 | $J_{uc} = 0.0104$ $J_c = 0.0104$ Iter. = 340 | $J_{uc} = 0.0104$ $J_c = 0.0104$ Iter. = 1 |
| 1 | $J_{uc} = 0.0047$ $J_c = 0.00034$ Iter. = 623 | $J_{uc} = 0.0047$ $J_c = 0.0040$ Iter. = 651 | $J_{uc} = 0.0047$ $J_c = 0.0047$ Iter. = 297 | $J_{uc} = 0.0047$ $J_c = 0.0047$ Iter. = 1 |

Table 2

3 Neumann boundary conditions, Symmetric Example 2

Consider the following symmetric setup, which is the opposite of the first symmetric example:

$$\begin{aligned}
\widehat{\rho} &= \frac{1}{2}(1-t) + t\frac{1}{4}(-\cos(\pi y) + 2) \\
\rho_0 &= \frac{1}{2} \\
q_T &= 0 \\
\vec{w} &= 0 \\
f &= 0 \\
V_{ext} &= 0
\end{aligned}$$

This example can be compared to the Symmetric Example 1. Here, the desired state is having ρ clustered at both boundaries, which is similar to the effect of the repulsive interaction term $\gamma = 1$. Therefore, for this choice of interaction term, the value of the cost functional J_{uc} is smaller than the one for $\gamma = 0$ and $\gamma = -1$. This is the opposite to the observation made in the Symmetric Example 1, which is to be expected, given the two choices of desired state.