PDE-Constrained Optimization for Multiscale Particle Dynamics

$$\min_{\boldsymbol{\rho},\mathbf{w}} \ \frac{1}{2}||\boldsymbol{\rho}-\widehat{\boldsymbol{\rho}}||_{L^2}^2 + \frac{\beta}{2}||\mathbf{w}||_{L^2}^2$$

subject to:

$$\partial_t \rho = \Delta \rho - \nabla \cdot (\rho \mathbf{w}) + \nabla \cdot \int_{\Omega} \rho(\mathbf{x}) \rho(\mathbf{x}') \nabla V_2(|\mathbf{x} - \mathbf{x}'|) d\mathbf{x} + \mathsf{BC} + \mathsf{IC}$$

