Report 23/04/2020 (Part 2)

Note: all of the results are done with Picard, but most have been checked with FixPt and give the same qualitative answer (i.e. which parameter configurations converge and which don't).
Note 2: Can't get the $\rho = e^h$ approach to work. It gives 'singular matrices'. Forward Result looks fine, chose $\rho > 1$ everywhere.

1 Investigating the relationship between Diffusion and Advection

We are looking at the exact flow control problem and perturb it in time by the standard bump function:

$$w_{Pert} = w_{Ex}(1 + 0.1\tilde{g}(t)).$$

Then two cases are considered:
1. Choose an example that diverges and see if changing D_0 can make it converge.
2. Choose an example that converges and see if changing D_0 breaks it.
We choose the Picard solver, $\beta = 10^{-3}$, tolerances $10^{-8}/10^{-4}$ and $\lambda = 0.1$.
For 2.:
Choosing scalerho, scalep = 1 (the coefficient in front of ρ_{IC} and p_{IC}), we have $\max \rho = 0.2576$, $\max p = 0.0543$ and $\max w = 12.7707$ (measured by 'max(max(abs))'). This converges in 246 iterations.
For 1.:
Choosing scalerho, scalep = 2, the algorithm diverges at 0.00033691 at iteration 83. The order of magnitude for the variables with this scaling are: $\max \rho = 0.5152$, $\max p = 0.1087$ and $\max w = 51.0827$. This shows that w does not grow linearly with a linear scaling of ρ and p (as expected).

1.1 Test 1: Fixing a diverging example

Trying $D_0 = 2$, the initial consistency error is 0.08751198 (with $D_0 = 1$ it is 0.10321139), so it decreased. However, the problem diverges at a consistency of 0.00038865, which is earlier than with $D_0 = 1$. For $D_0 = 3$, the initial error is 0.07665870 and it diverges at 0.00047412, which is again earlier than with lower D_0. Setting $D_0 = 10$, the initial error is 0.03677095. It diverges at 0.00043914. I am not sure why convergence doesn't improve 'linearly' as we increase the diffusion. Setting $D_0 = 20$, it diverges at 0.00023938, which is going in the right direction. Setting $D_0 = 50$, the initial error is 0.00733544 and it converges in 72 iterations. Given the difference in magnitude of ρ and w it is not so surprising that a large D_0 is needed, see Figure 2, compare to $D_0 = 1$ forward solution in Figure 1. The lowest converging problem has diffusion coefficient $D_0 = 30$. This makes perfect sense because heuristically: $\max \rho \simeq 1.5$ and

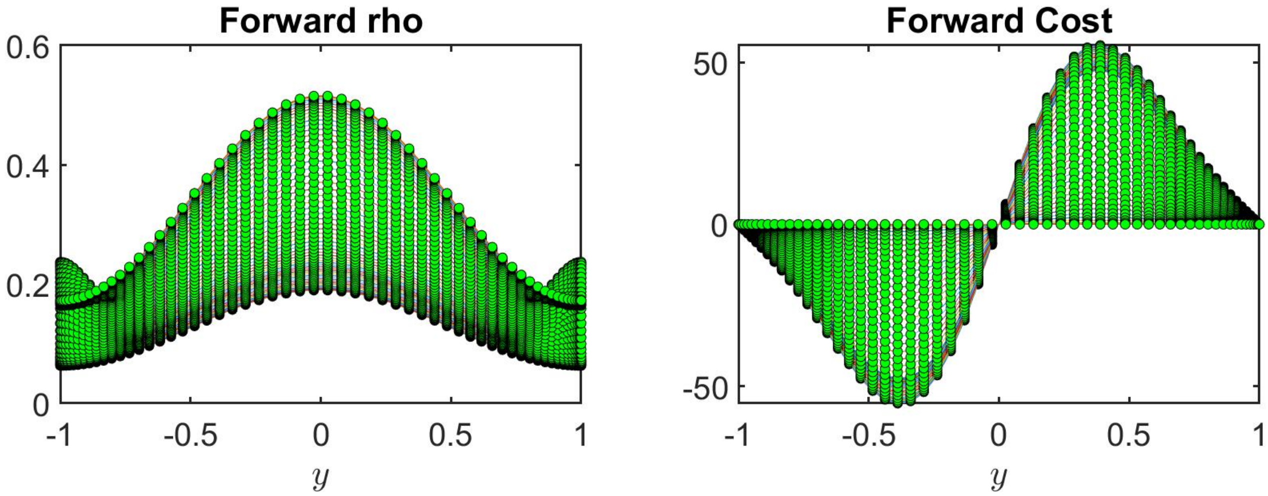

Figure 1: Solutions ρ_{FW} and w_{FW}, with $D_0 = 1$.

1.2 Test 2: Breaking a converging example

Choose $\beta = 10^{-3}$, scalerho, scalep = 1. This converges within 246 iterations, see Figure 3. When choosing $D_0 = 0.5$ instead then it diverges at 0.00029311 and with $D_0 = 0.1$ it diverges at 0.00433974. The forward solution and cost for $D_0 = 0.1$ is displayed in Figure 4. The behaviour of ρ close to the boundary is very similar to the boundary in Figure 1, which is the diverging problem in Test 1.

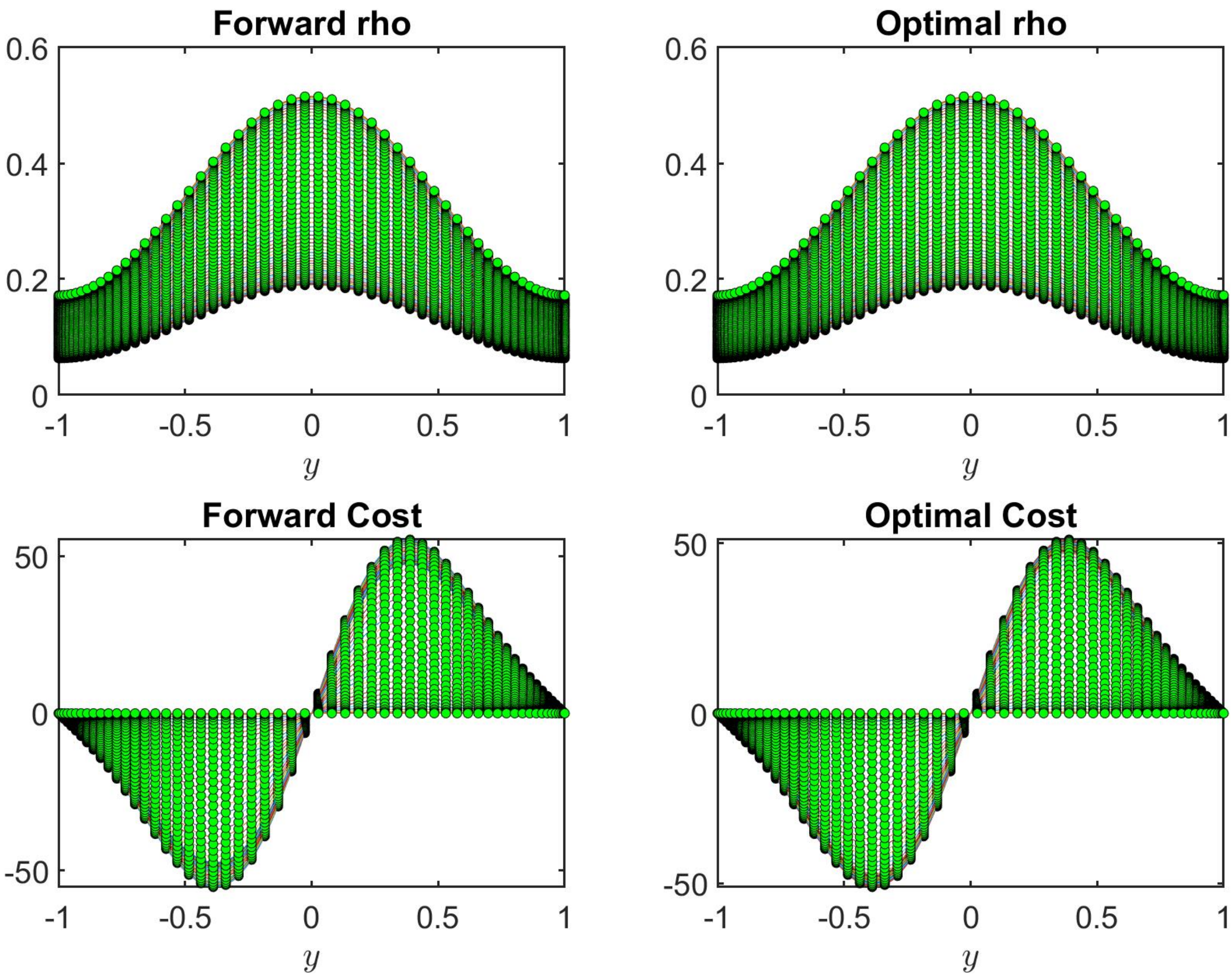

Figure 2: Solutions ρ_{FW} and ρ_{Opt}, w_{FW} and w_{Opt}, with $D_0 = 50$.

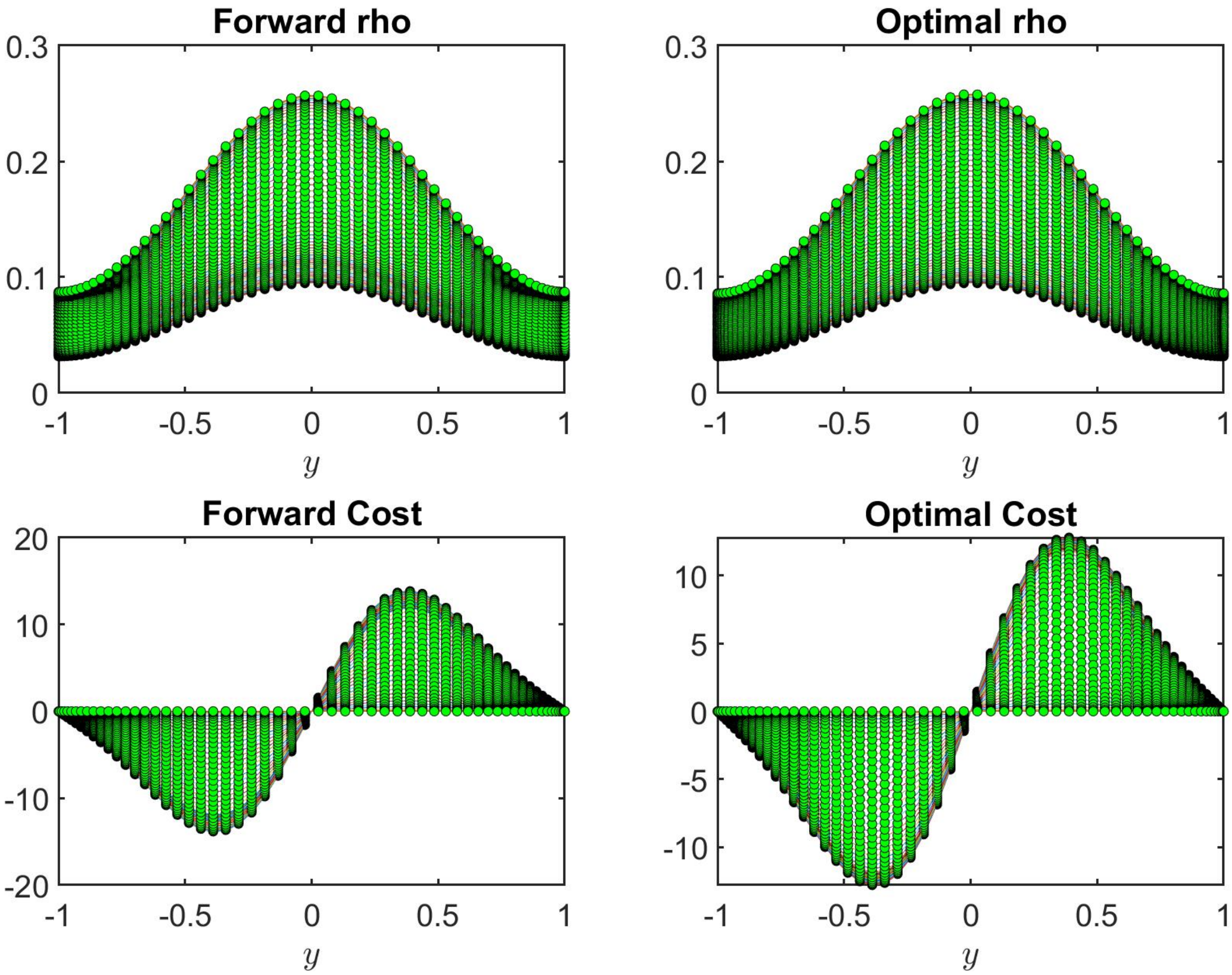

Figure 3: Solutions ρ_{FW} and ρ_{Opt}, w_{FW} and w_{Opt}, with $D_0 = 1$.

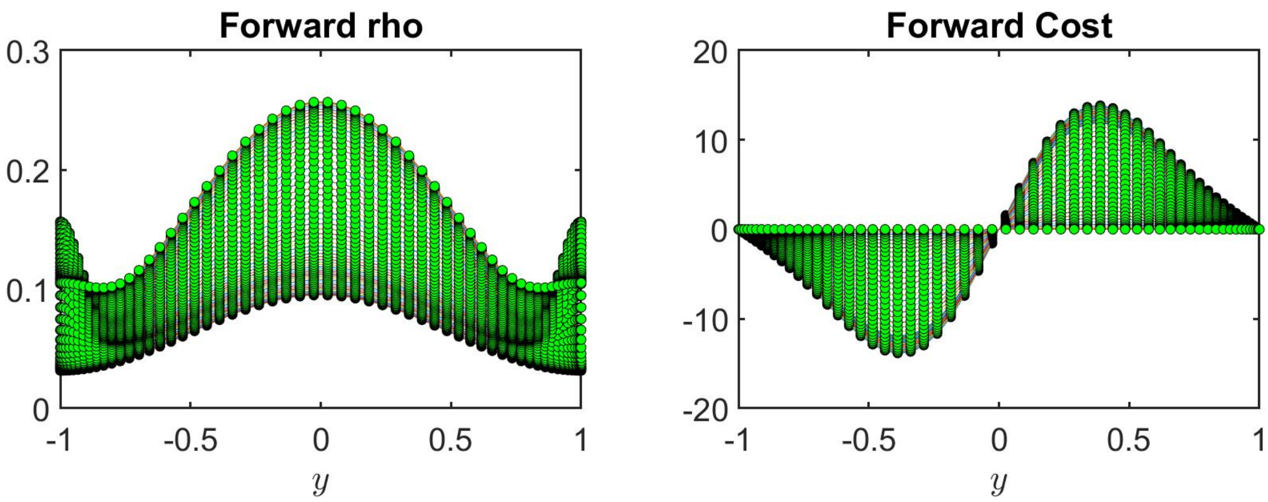

Figure 4: Solutions ρ_{FW} and w_{FW}, with $D_0 = 0.1$.