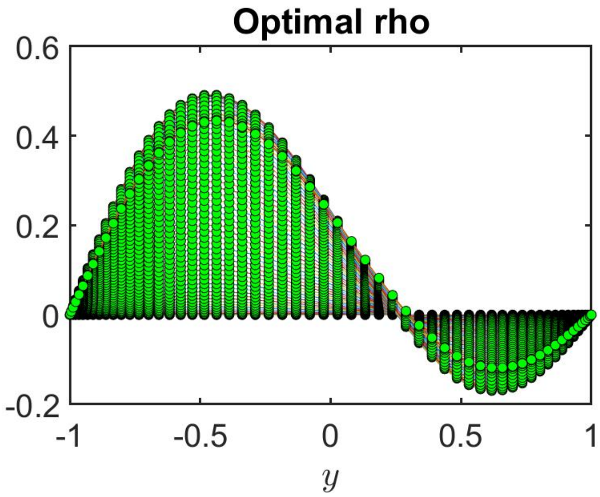
Report 23/04/2020 (Part 3)

For all problems we choose ODE tols = 10^{-8}, consistency tols = 10^{-4}, $n = 61$, $N = 60$ and force or flow are zero in the forward problems. We choose FixPt solver throughout. Results still to be verified with Picard.

1 Force Control - Dirichlet

The aim is to show a force control problem with particle interactions. Problem: generally the three choices of $\gamma = -1, 0, 1$ give almost identical results for a range of initial conditions and targets. $\gamma = 2, -2$ and $D_0 = 0.5$ shows some difference in the forward problem.
Staying with $\gamma = -1, 0, 1$ and $D_0 = 1$, we get the following results. With $\gamma = 0$ and $\beta = 10^{-3}$, the algorithm converges. $J_{FW} = 0.0365$, $J_{Opt} = 0.0032$, see Figure 1. With $\gamma = -1$ and $\beta = 10^{-3}$, the algorithm converges. $J_{FW} = 0.0365$, $J_{Opt} = 0.0031$, see Figure 2. With $\gamma = 1$ and $\beta = 10^{-3}$, the algorithm converges. $J_{FW} = 0.0365$, $J_{Opt} = 0.0033$, see Figure 3. We can see the lack of difference in pictures and in cost functional values.



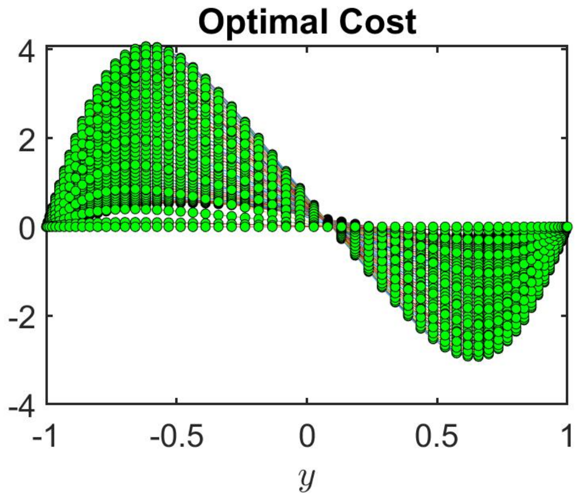


Figure 1: Solution ρ_{Opt} and w_{Opt}, with $\gamma = 0$ and $D_0 = 1$.

2 Force Control - Neumann

We choose the same configurations as in the Flow control case (moving target 1):

$$\hat{\rho} = (1-t)0.5 + t\frac{1}{4}(\cos(\pi y + \pi) + 2),$$

and the initial condition for ρ is:

$$\rho_{IC} = 0.5.$$

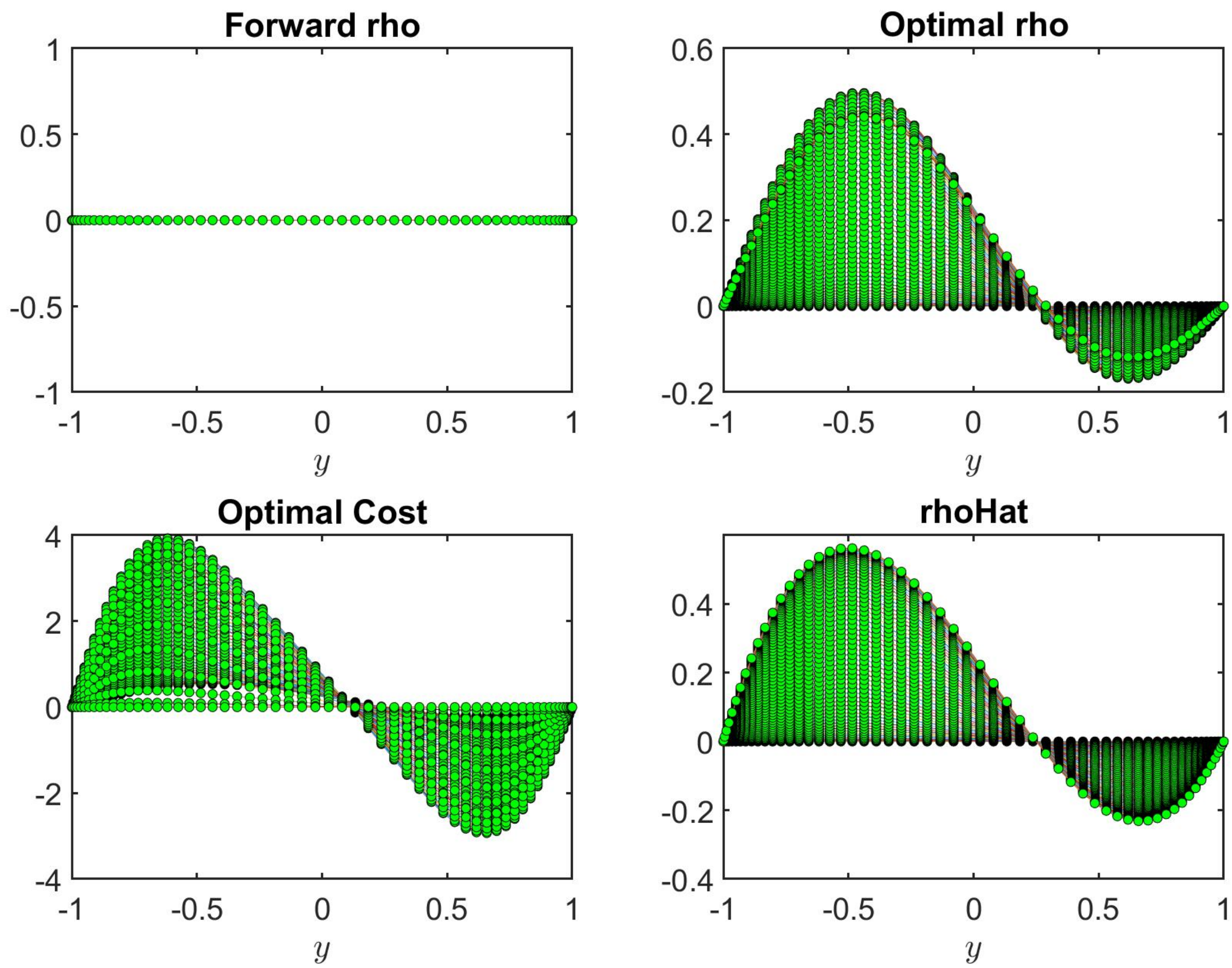


Figure 2: Solutions ρ_{FW} and ρ_{Opt}, $\hat{\rho}$ and w_{Opt}, with $\gamma = -1$ and $D_0 = 1$.

For $\gamma = 0$, $\beta = 10^{-3}$ it converges, but only if $\lambda = 0.001$. Then $J_{FW} = 0.0104$, $J_{Opt} = 0.0011$, see Figure 4.
Try $\gamma = -1$. This converges in 6506 iterations (similar to $\gamma = 0$ 25 min). $J_{FW} = 0.0041$, $J_{Opt} = 4.7534 \times 10^{-4}$, see Figure 5. Comparing the two figures, there is not much difference in the solutions of $\gamma = 0$ and $\gamma = -1$, however, the values of the cost functionals are very different, since for $\gamma = -1$ the forward solution is already closer to $\hat{\rho}$, while the forward solution for $\gamma = 0$ is constant.
Choosing $\gamma = 1$ converges as well in a similar time frame. $J_{FW} = 0.0195$, $J_{Opt} = 0.0023$, see Figure 6. The optimal solution does differ from the previous two, since the repulsive interactions result in a forward solution far from $\hat{\rho}$.

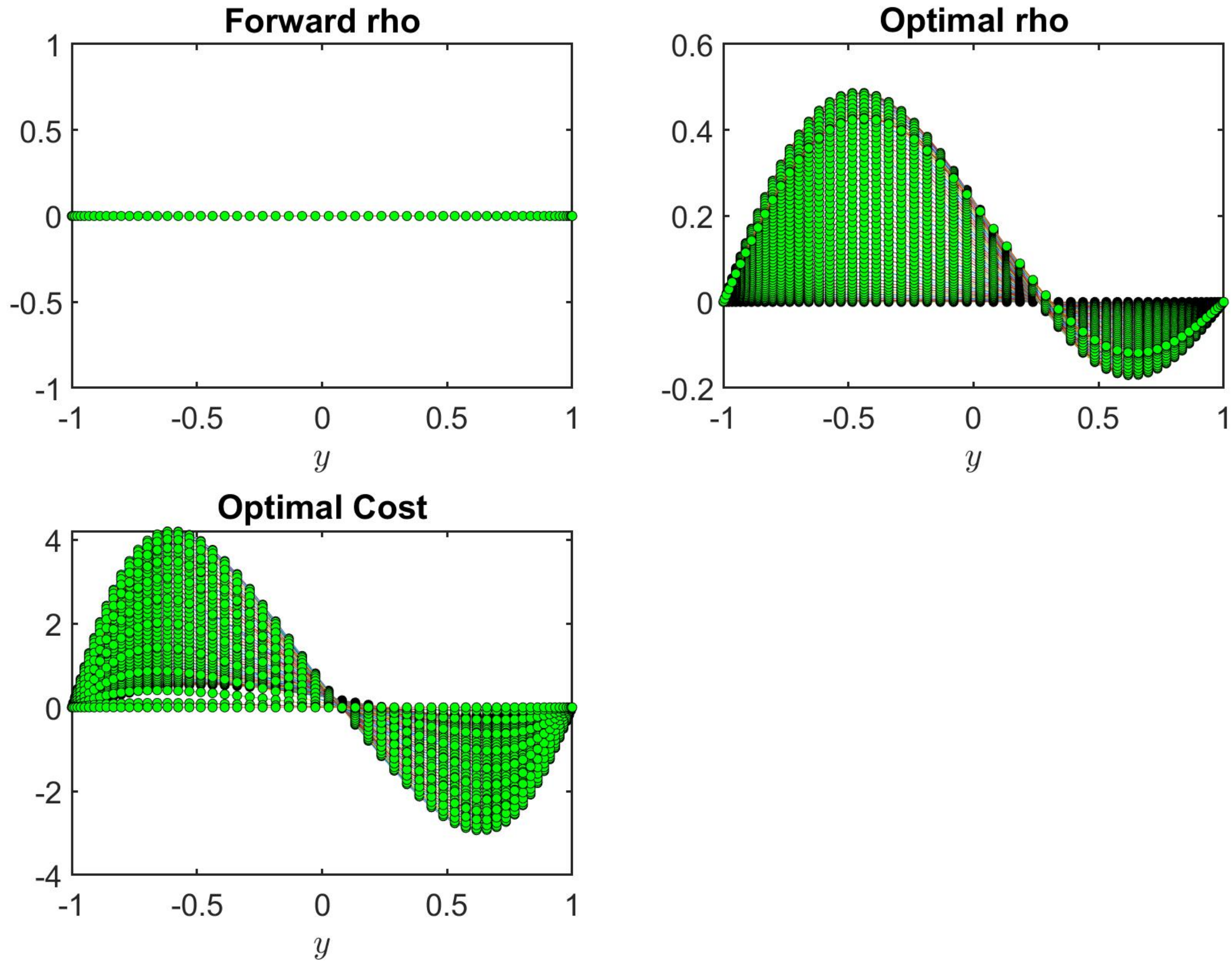


Figure 3: Solutions ρ_{FW} and ρ_{Opt}, and w_{Opt}, with $\gamma = 1$ and $D_0 = 1$.

3 Flow Control - Dirichlet

When using the same approach as in the Neumann case, the solution is a constant zero function (as the only uniform initial condition is the zero function). We therefore need another approach. The target is a moving target from the initial condition of ρ to a different particle distribution:

$$\hat{\rho} = (1-t)(-y^3 - (1/4)y^2 + y + 0.25) + t(y^3 - (1/4)y^2 - y + 0.25).$$

The initial condition for ρ is:

$$\rho_{IC} = -y^3 - (1/4)y^2 + y + 0.25.$$

At first we try the case $\gamma = 0$, $\beta = 10^{-3}$, $\lambda = 0.01$. This converges in 1013 iterations (12 min, very slow). $J_{FW} = 0.0324$, $J_{Opt} = 0.0180$, see Figure 7. For $\gamma = -1$ this converges in 1008

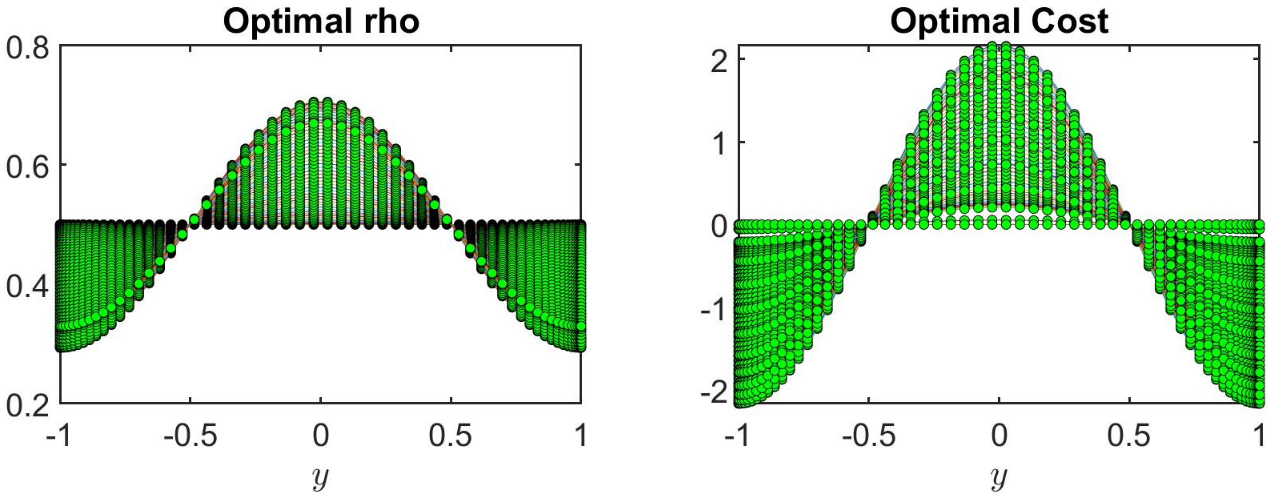


Figure 4: Solution ρ_{Opt} and w_{Opt}, with $\gamma = 0$ and $D_0 = 1$.

iterations (11 min), $J_{FW} = 0.0319$, $J_{Opt} = 0.0175$, see Figure 8. Overall, these two problems are not very different, so the particle interaction doesn't seem to have a big impact on the results. This needs to be improved. With $\gamma = 1$, this converges as well, $J_{FW} = 0.0328$, $J_{Opt} = 0.0185$, see Figure 9. Since the interactions are repulsive, they are slightly acting against $\hat{\rho}$ direction, so the cost functionals are slightly higher than for the other two values of γ. However, there is no great difference in the pictures or in the values of the cost functionals.

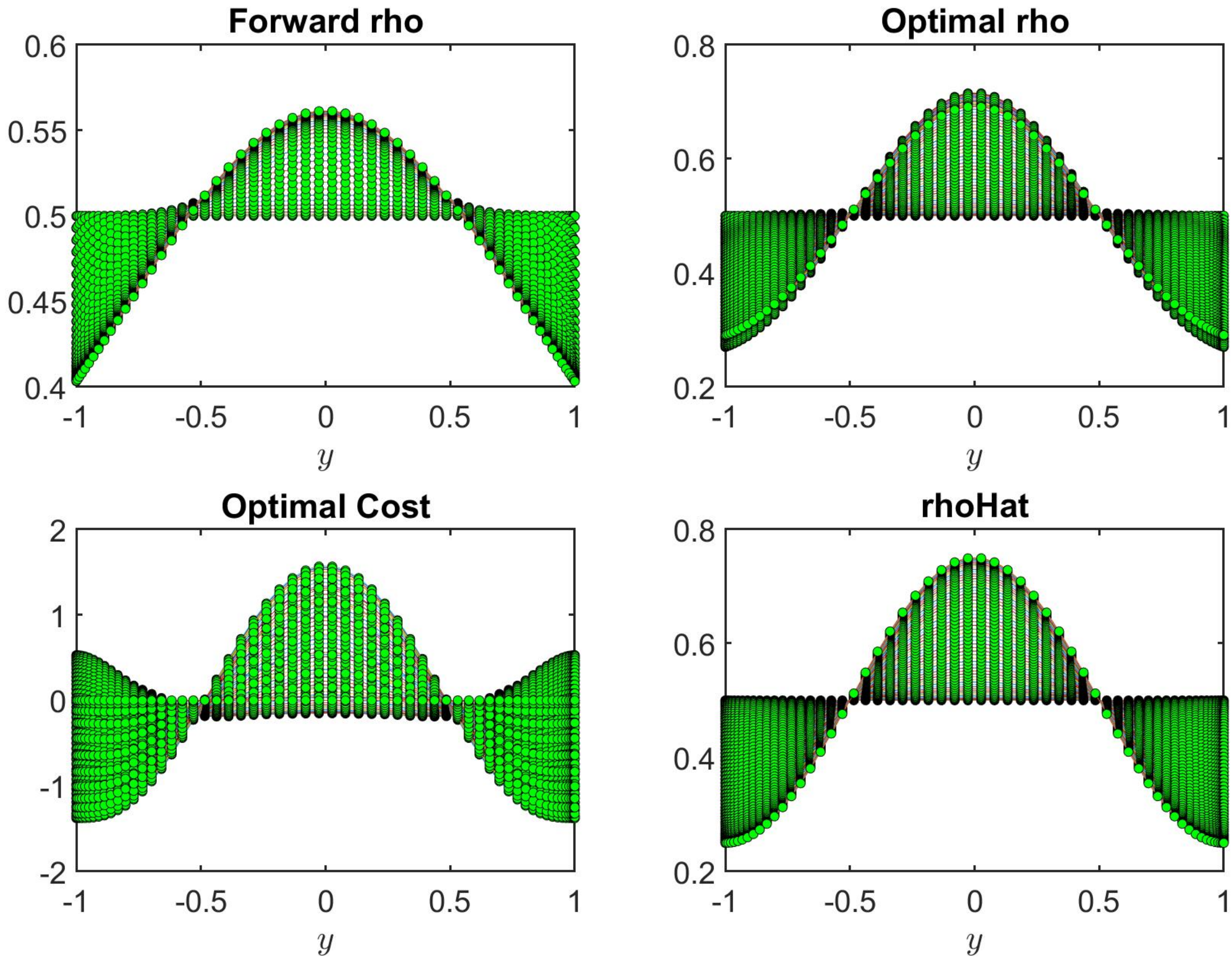


Figure 5: Solutions ρ_{FW} and ρ_{Opt}, $\hat{\rho}$ and w_{Opt}, with $\gamma = -1$ and $D_0 = 1$.

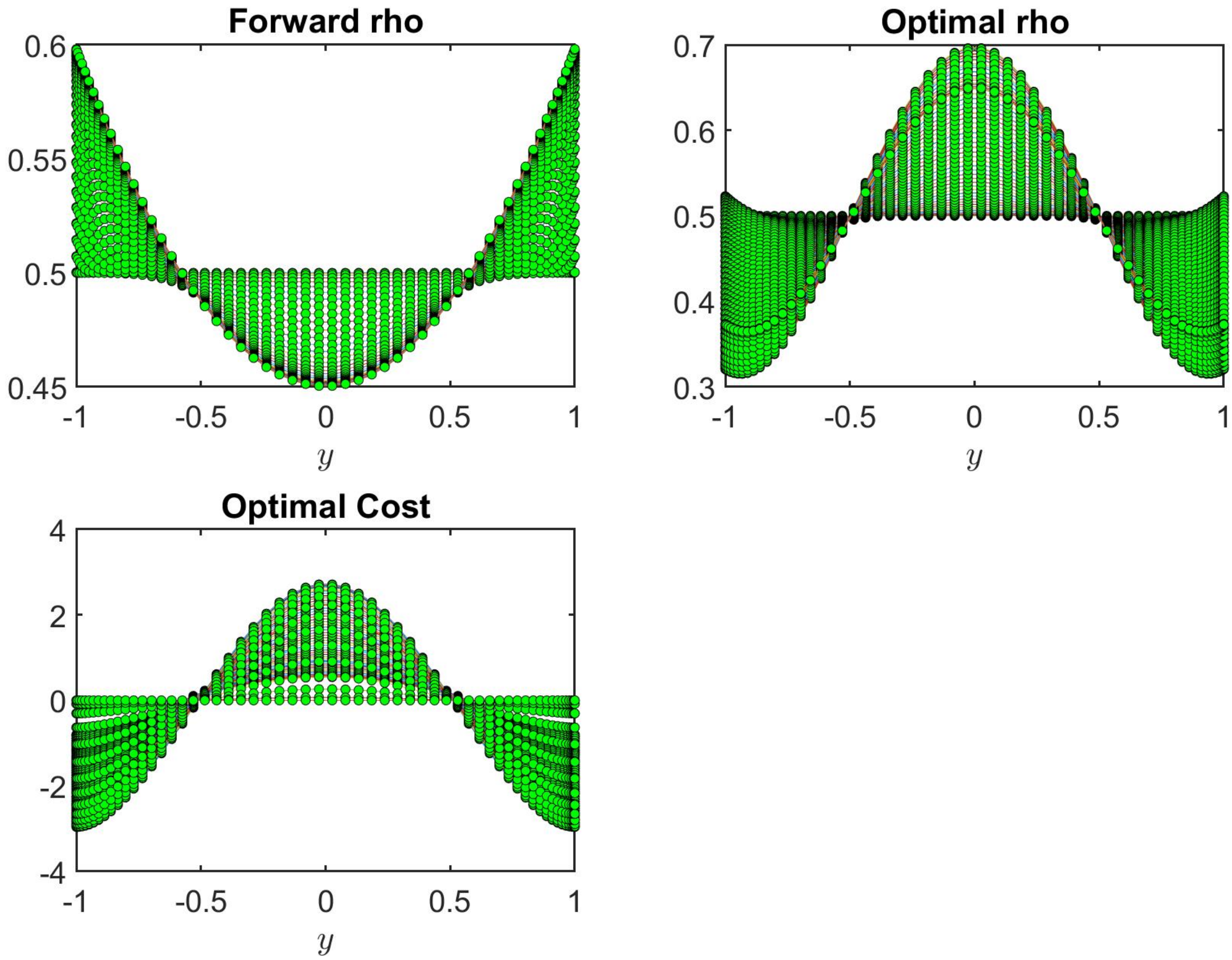


Figure 6: Solutions ρ_{FW} and ρ_{Opt}, and w_{Opt}, with $\gamma = 1$ and $D_0 = 1$.

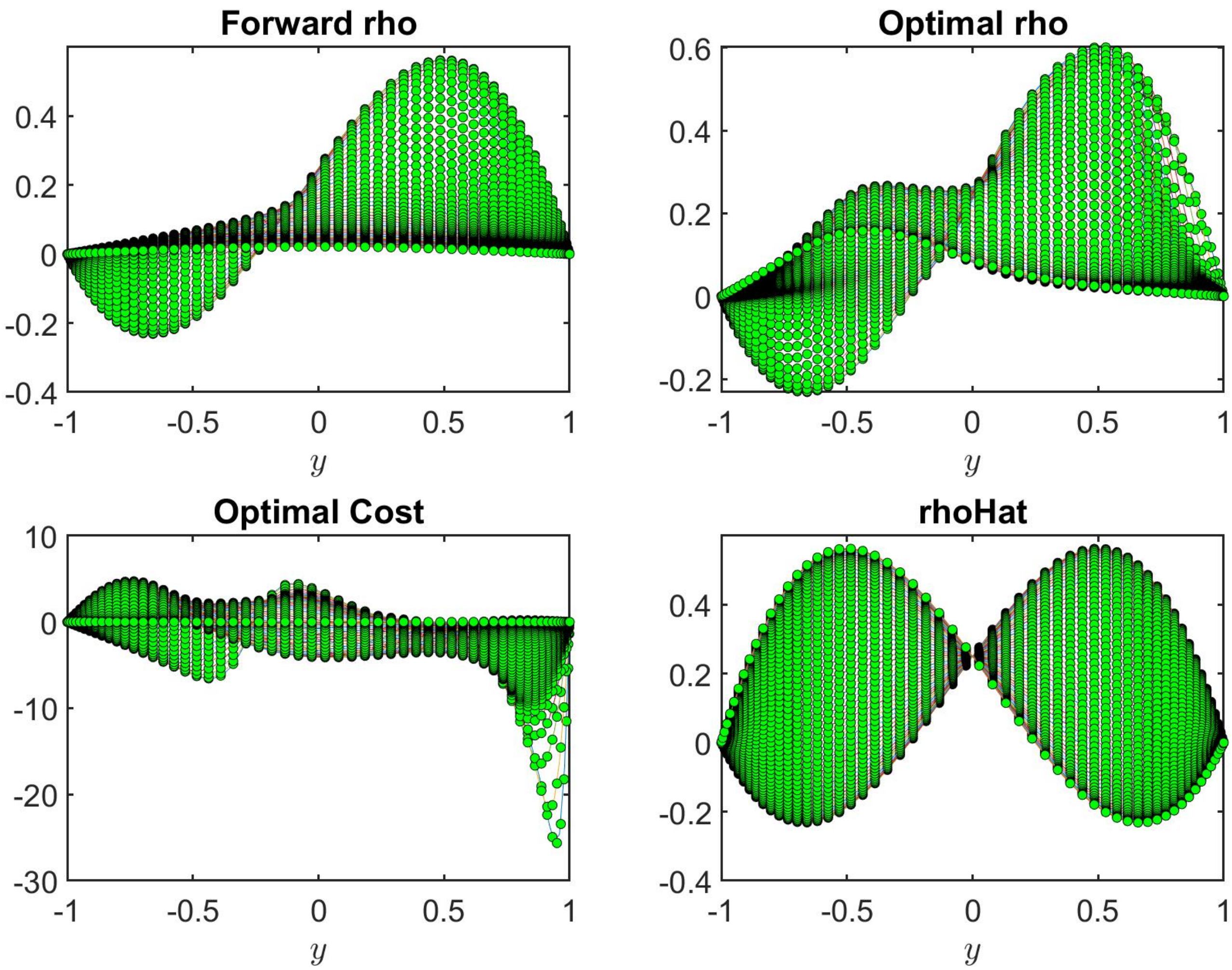


Figure 7: Solutions ρ_{FW} and ρ_{Opt}, $\hat{\rho}$ and w_{Opt}, with $\gamma = 0$ and $D_0 = 1$.

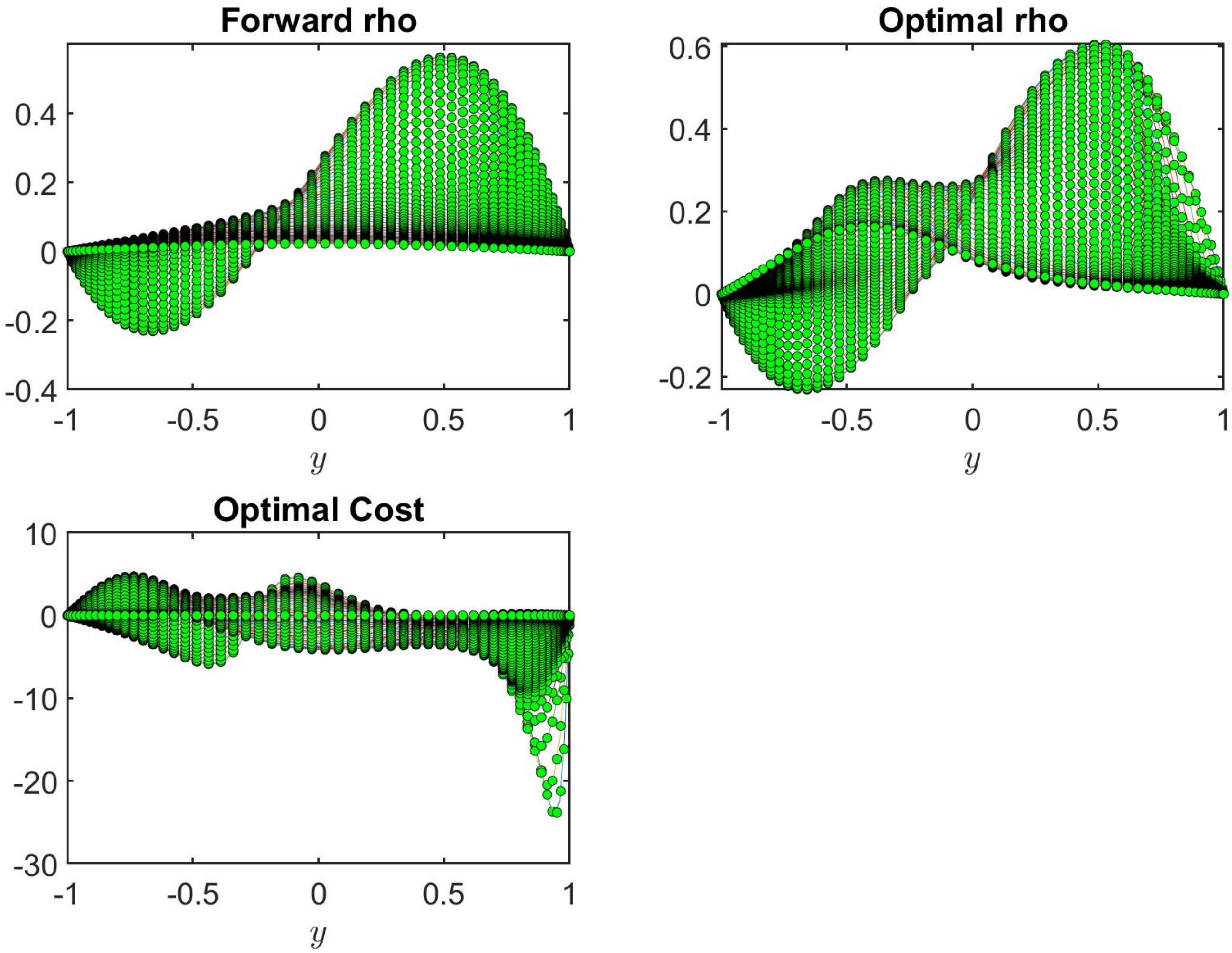


Figure 8: Solutions ρ_{FW} and ρ_{Opt}, and w_{Opt}, with $\gamma = -1$ and $D_0 = 1$.

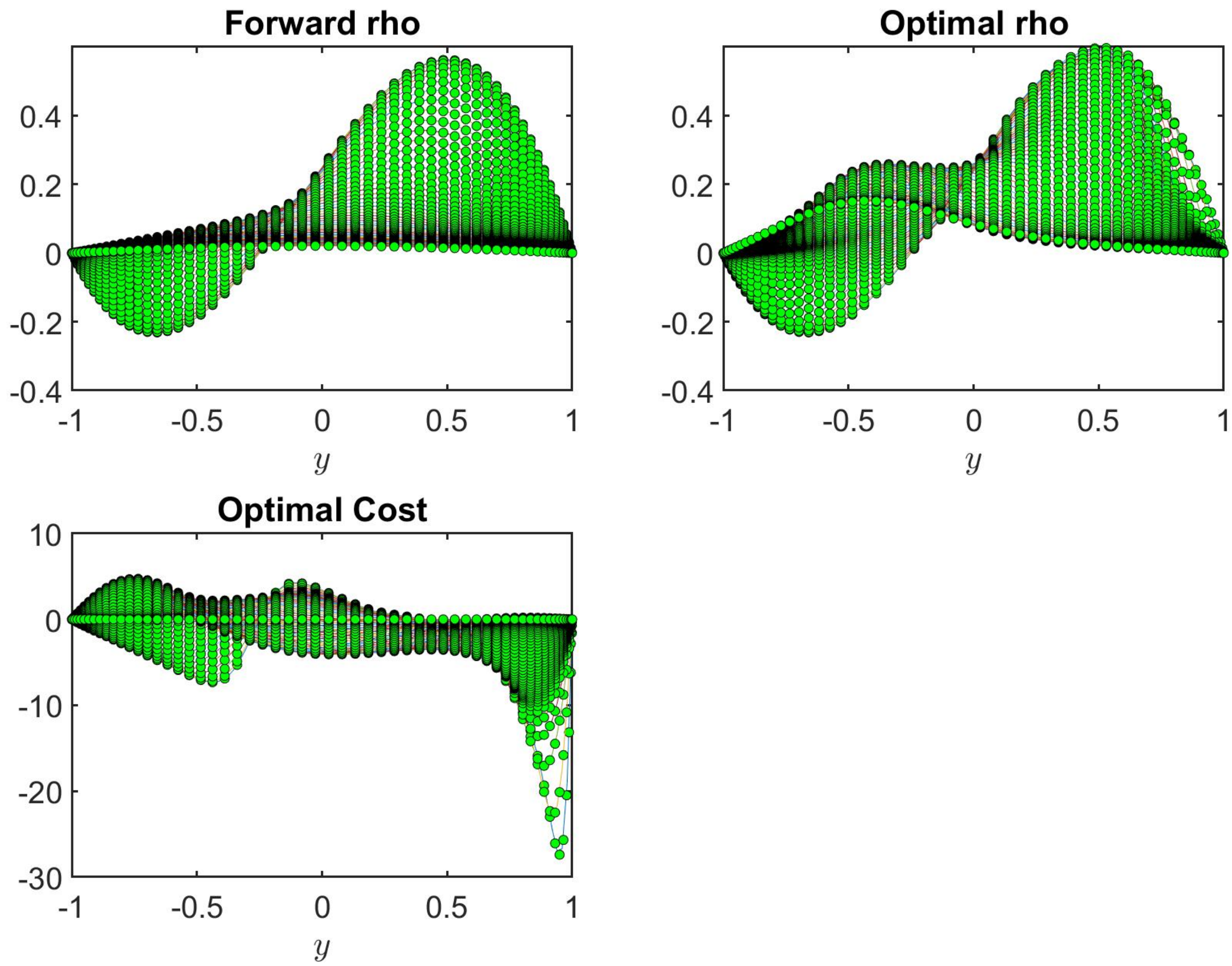


Figure 9: Solutions ρ_{FW} and ρ_{Opt}, and w_{Opt}, with $\gamma = 1$ and $D_0 = 1$.