Additional Examples

1 One Dimensional non-symmetric gaussian $\hat{\rho}$

We choose $\rho_0 = 0.5$ and

$$\hat{\rho} = 0.5(1-t) + t\frac{1}{0.5604}e^{-10((y+0.2)^2)}.$$

The prefactor $\frac{1}{0.5604}$ ensures that the mass of $\hat{\rho}$ is one at all times. Choosing $n = 61$, $N = 60$, ODE Tols $= 10^{-8}$, Optimality Tols $= 10^{-4}$. For $\beta = 10^{-3}$ and $\gamma = -1$, $J_{FW} = 0.1084$ and $J_{Opt} = 0.0055$, see 1. For $\beta = 10^{-3}$ and $\gamma = 1$, $J_{FW} =$ and $J_{Opt} =$, see 2. Other choices of β and γ behave as expected.

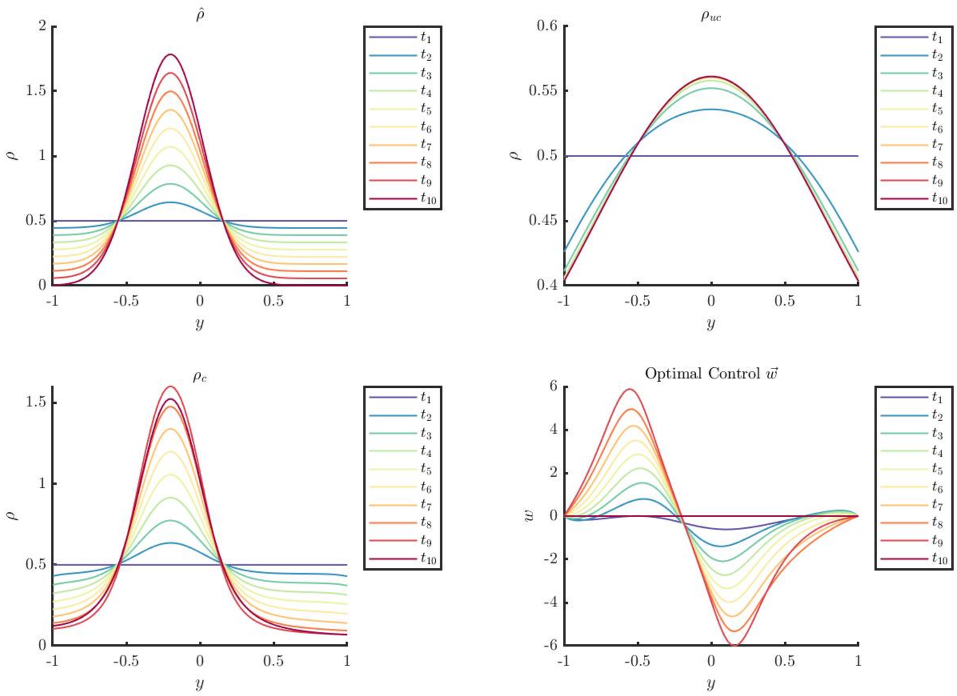

Figure 1: 1D Example with $\beta = 10^{-3}$, $\gamma = -1$

2 Two Dimensional, Example 1

++ 2D seems to work now because I fixed mass conservation, which wasn't correct the last time I ran it. ++

We choose $\rho_0 = 0.25$ and

$$\hat{\rho} = 0.25(1-t) + t * \frac{1}{4}((\cos(\pi y_1) + 1)(\cos(\pi y_2) + 1)),$$

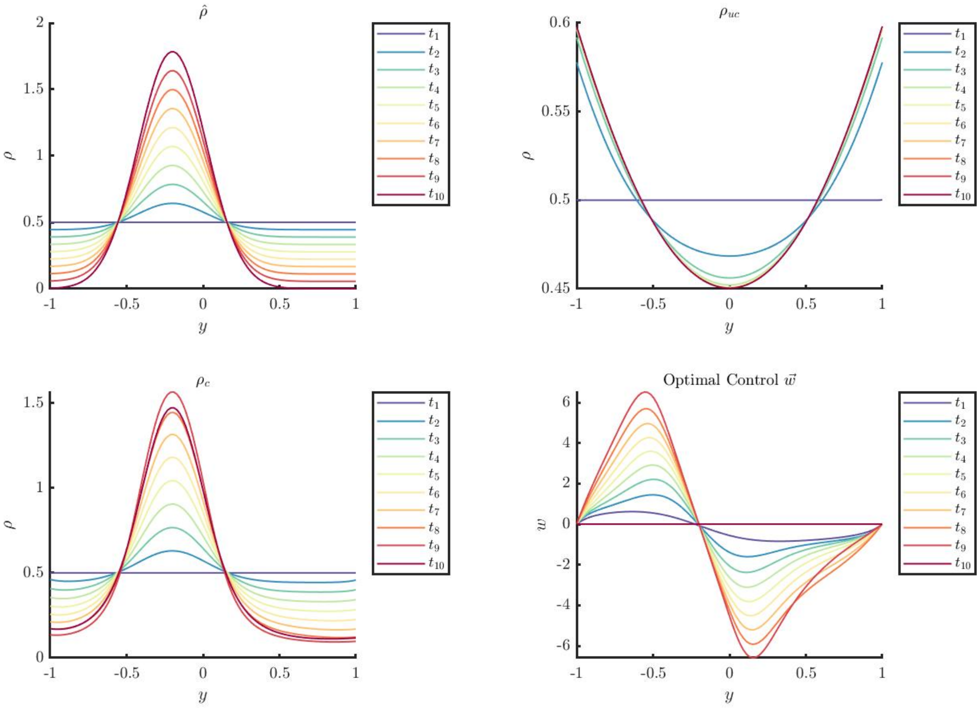

Figure 2: 1D Example with $\beta = 10^{-3}$, $\gamma = 1$

see 3 We choose $n = 20$, $N_1, N_2 = 30$. Tolerances are $10^{-8}/10^{-4}$. For $\beta = 10^{-3}$ and $\gamma = 1$, $J_{FW} = 0.0596$ and $J_{Opt} = 0.0170$, see 4, 5, 6. For $\beta = 10^{-3}$ and $\gamma = -1$, $J_{FW} = 0.0334$ and $J_{Opt} = 0.0020$, see 7, 8, 9.

3 Two Dimensional, Example 2

4 Two Dimensional, Example 3

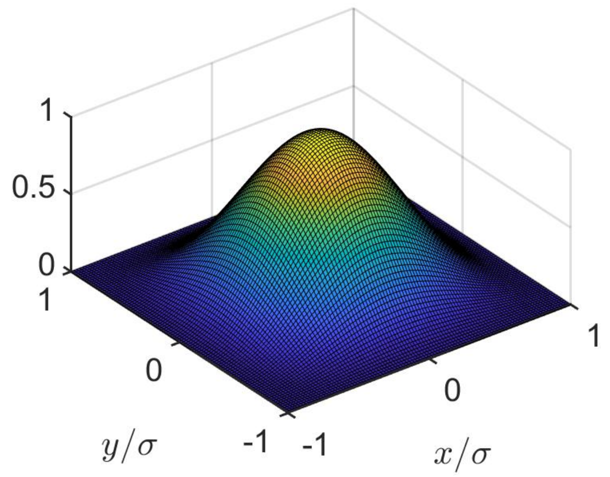

Figure 3: 2D Example 1, $\hat{\rho}$ at $t = 20$

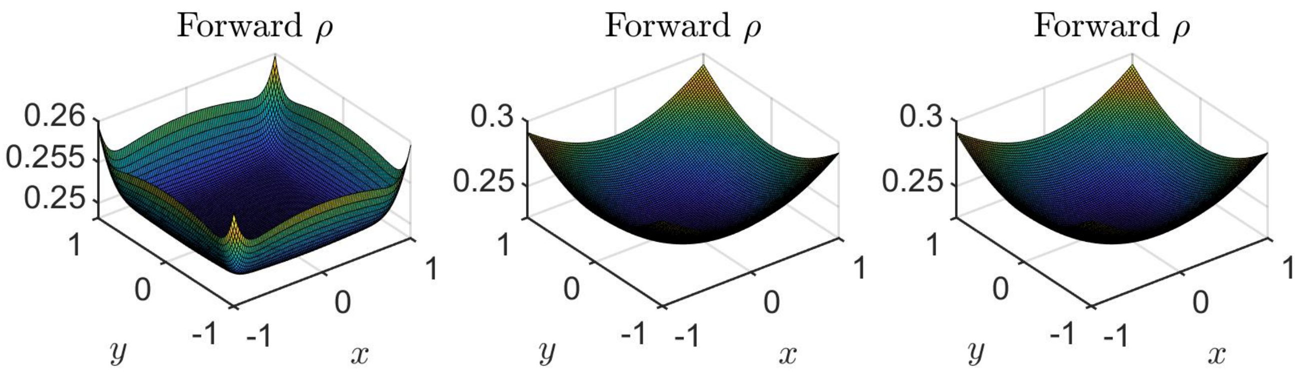

Figure 4: 2D Example 1, ρ forward, $t = 2, 10, 20$, $\beta = 10^{-3}$, $\gamma = 1$

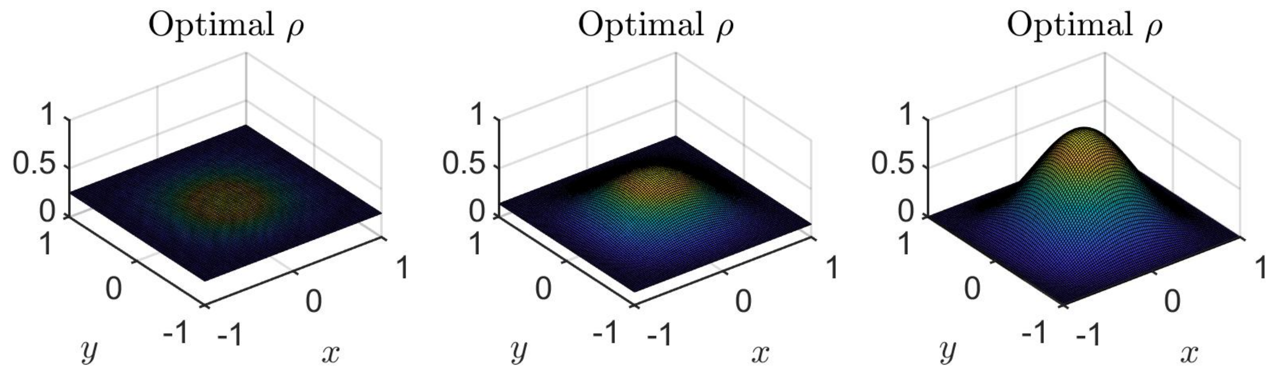

Figure 5: 2D Example 1, ρ optimal, $t = 2, 10, 20$, $\beta = 10^{-3}$, $\gamma = 1$

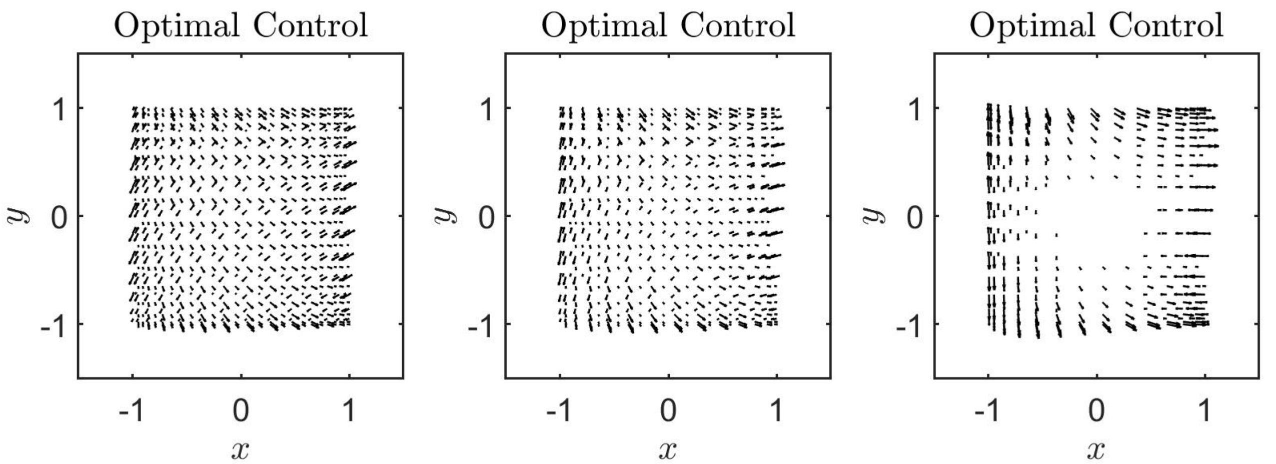

Figure 6: 2D Example 1, Optimal Control, $t = 2, 10, 19$, $\beta = 10^{-3}$, $\gamma = 1$

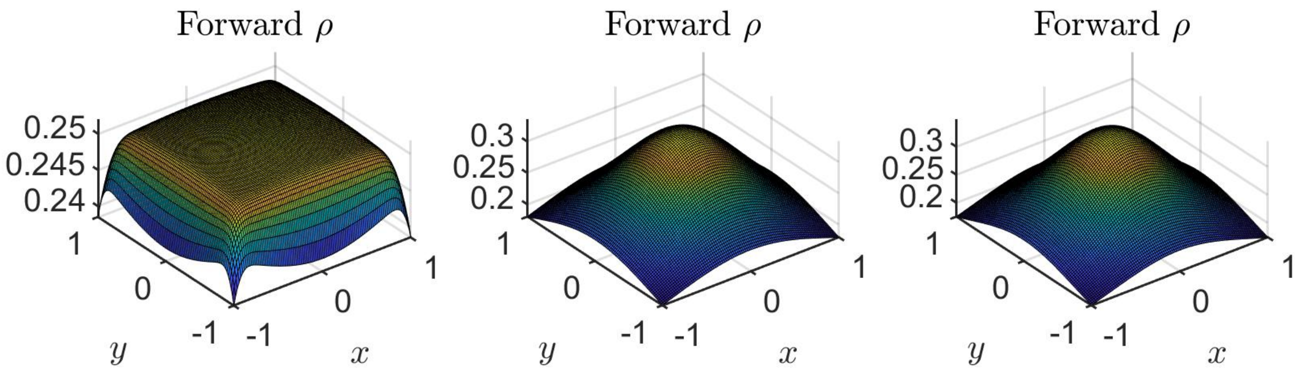

Figure 7: 2D Example 1, ρ forward, $t = 2, 10, 20$, $\beta = 10^{-3}$, $\gamma = -1$

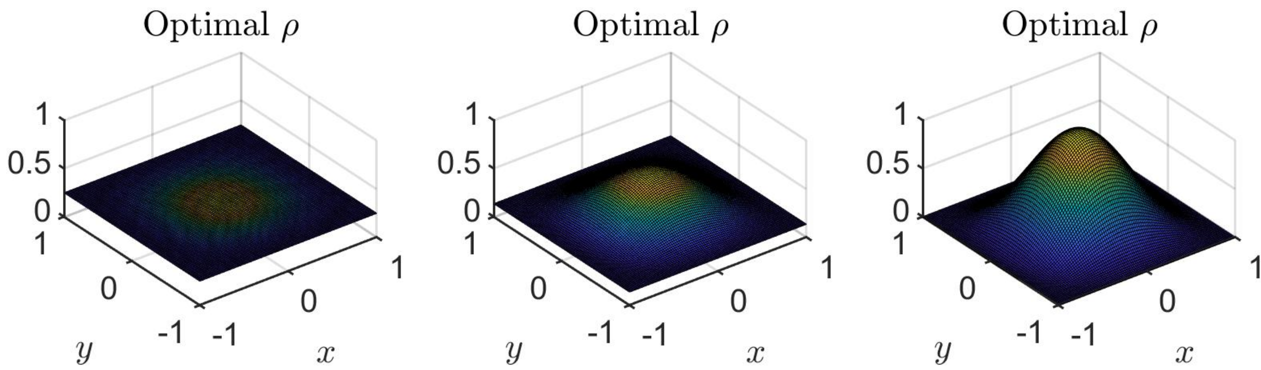

Figure 8: 2D Example 1, ρ optimal, $t = 2, 10, 20$, $\beta = 10^{-3}$, $\gamma = -1$

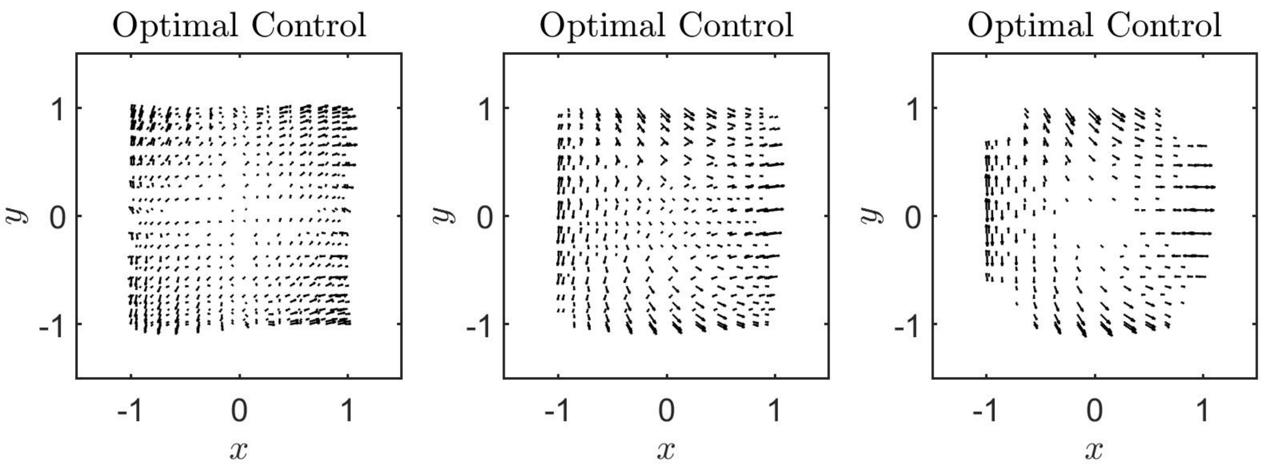

Figure 9: 2D Example 1, Optimal Control, $t = 2, 10, 19$, $\beta = 10^{-3}$, $\gamma = -1$