Report 16/04/2020

1   Code Questions

ODE times vs OutTimes

Dirichlet BCs

Interpolated of variable ρ, p in optimization

fsolve error measure

J error measure.

2   Investigating why 'large w' causes problems

Looking at the time interpolation

Two versions of w were considered:

1. w_{Cheb}, which is the exact solution evaluated at the Chebyshev Points.

2. w_{Equi}, which is the exact solution evaluated on an equispaced grid.

Then, perturb w_{Cheb} by $0.1\tilde{g}(t)$ and interpolate this onto the equispaced grid.

Furthermore, also perturb w_{Equi} by $0.1\tilde{g}(t)$ on the equispaced grid. Compare the two:

$$InterpErr = Interp * (w_{Cheb}(1 + 0.1\tilde{g}(t_{Cheb}))) - w_{Equi}(1 + 0.1\tilde{g}(t_{Equi})).$$

Furthermore, the forward solution for ρ is computed using the perturbed w_{Cheb} and interpolated on an equispaced grid. This is compared with rho_{Ex} evaluated on the equispaced grid. The following figures show the InterpErr errors in time, when the scaling parameter in $\tilde{g}$ is $a = 2$. For each space point one line. In 1 it shows a non-converging situation with 1ρ, $1p$, while in Figure 2 it shows a converging situation with 0.4ρ, $0.4p$. The errors in ρ for the two cases look very different, while the interpolation error in w is vaguely the same. The perturbation in w scales with the size of w, but is of the same shape.

When the scaling parameter in $\tilde{g}$ is $a = 0.5$ we have the following results. For 1ρ and $1p$ scaling, again this does not converge. See Figure 3. Again, for 0.4ρ, $0.4p$ the problem converges, but the error in ρ looks very different. See Figures 3 and 4. Conclusion: there doesn't seem to be a problem with time interpolation. The interpolation error for w is within the norm. However, it does seem to correlate with the steepness of the perturbation function (?!). The bigger ρ and p are, the 'weirder' are the errors in ρ (Opti vs Exact).

2.1   Checking how varying $\tilde{g}$ parameter a changes ρ

In Figure 5, there are the three considered perturbations applied to w. In Figure 6, the effect of these on 2ρ is displayed and in Figure 7 the effect on 0.5ρ. The location of the 'fold' does
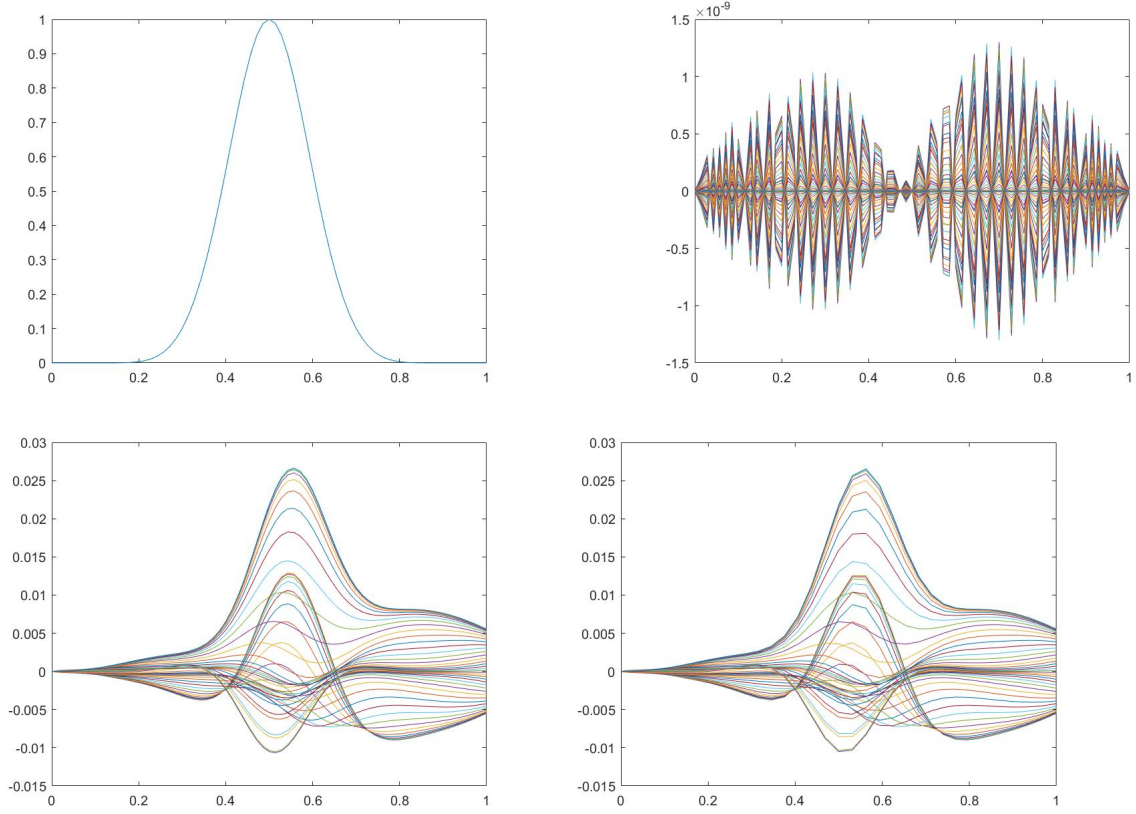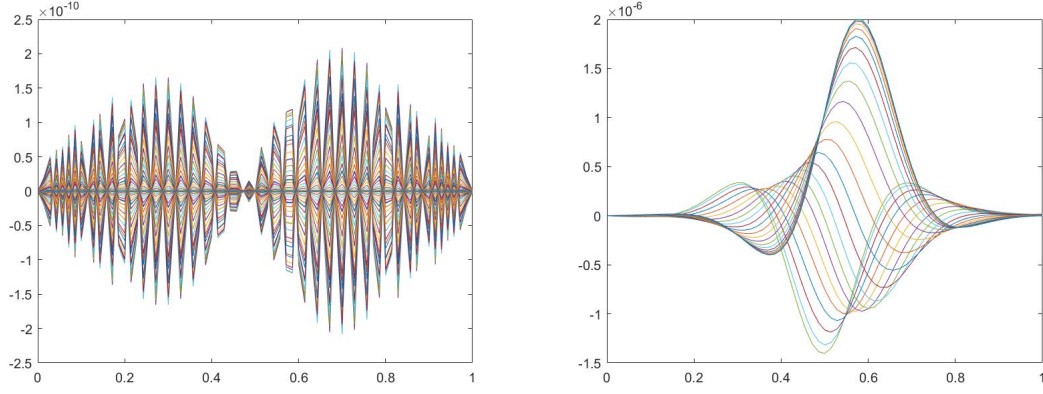
Figure 1: Interpolation Error in w due to the perturbation and resulting error in rho_{Opti} vs rho_{Ex} (left equi, right Cheb). Note, ρ did not converge! In L2Linf relative norm it is:$2.5362e{-}10$.

not seem to be correlated with the shape of $\tilde{g}(t)$.

2.2 Investigating $\rho = e^h$

Rewriting the forward equation in terms of $\rho = e^h$:

$$\partial_t \rho = \Delta\rho - \nabla \cdot (w\rho) + f$$
$$\partial_t h = \Delta h + \nabla h \cdot \nabla h - \nabla \cdot w + \nabla h \cdot w + fe^{-h}.$$

Checked code on Diffusion case first. Works. (FW solution = exact solution). Forward Case in AD works too.

3 Quick investigation on Interacting problems

First tried the same exact solutions as the Neumann flow control ones, but β fully on p, and ρ, w independent of it. However, the PDE is the Flow control PDE without Force term and
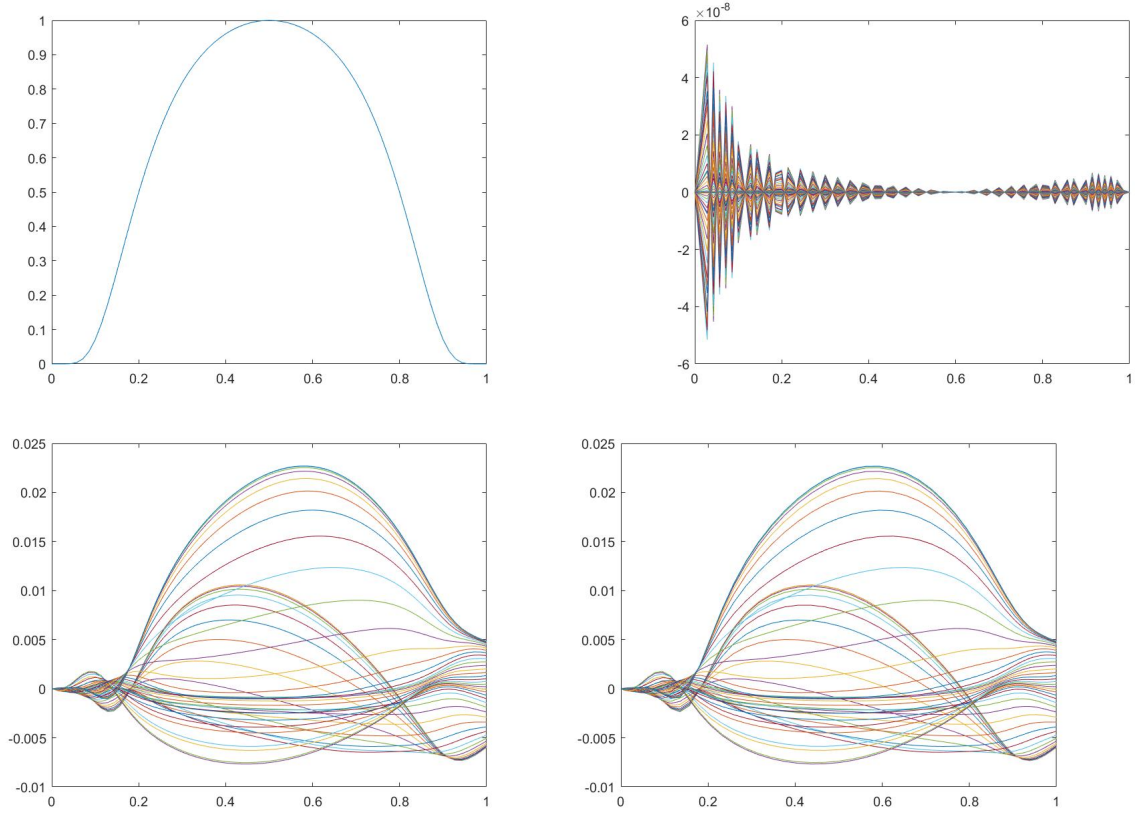
Figure 2: Interpolation Error in w due to the perturbation and resulting error in rho_{Opti} vs rho_{Ex} (left equi, right Cheb). ρ converged. In L2Linf relative norm it is:$1.4398e - 10$.

with included particle interaction term. If we scale ρ and p such that w is order 1, this works well. Tested for $\gamma = 1$ and $\gamma = -1$. Some problems have been tested with Picard, some with PixPt. Different values of β have been investigated. All of these problems converge, however it is noticeable that the cost functionals are very close together. The optimal solution does better, but only marginally so. Furthermore, it seems that the Forward solution doesn't change so much with γ, see Figure 8. For 1ρ the impact on gamma is stronger. Already for 0.4ρ and $0.2p$ the problem diverges for $\beta = 10^{-3}$ (but converges for $\beta = 10^{-1}$) This is equivalent to w of order 2.

The next idea was that $\hat{\rho}$ should be independent of β as well. Chose

$$\hat{\rho} = \cos(\pi y + \pi/2) + 2.$$

This is a shift in the shape but not a change in mass. Using the scaling that w is of order 1, this converges for $\beta = 10^{-1}$, but not for $\beta = 10^{-3}$. Changed $\hat{\rho}$ to $\hat{\rho} = \cos(\pi y + \pi/4) + 2$ in case it is easier but it didn't improve the situation.

Need to think of a better problem but also looks like a similar issue to the one with large w.

Figure 3: Interpolation Error in w due to the perturbation and resulting error in rho_{Opti} vs rho_{Ex} (left equi, right Cheb). Note, ρ did not converge! In L2Linf relative norm it is:$4.3039e-09$.
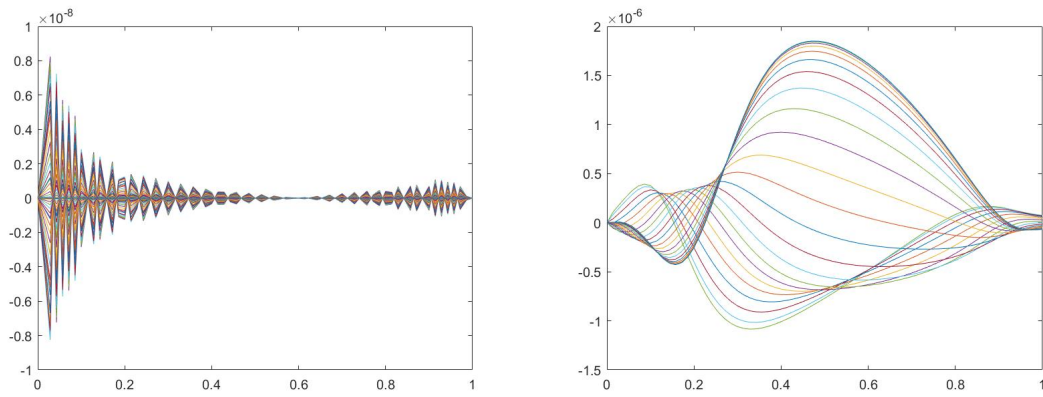

Figure 4: Interpolation Error in w due to the perturbation and resulting error in rho_{Opti} vs rho_{Ex} (left equi, right Cheb). ρ converged. In L2Linf relative norm it is:$4.3039e-09$.
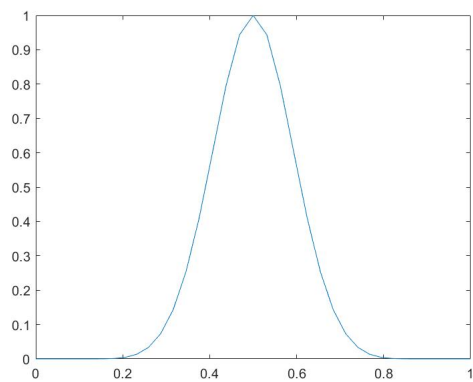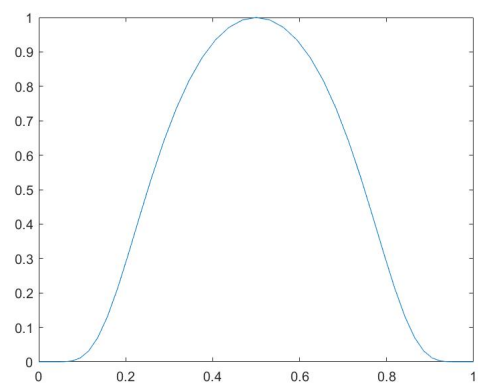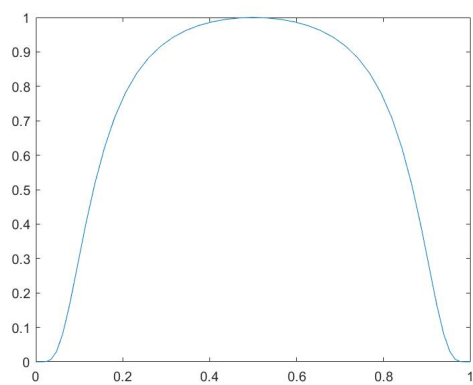
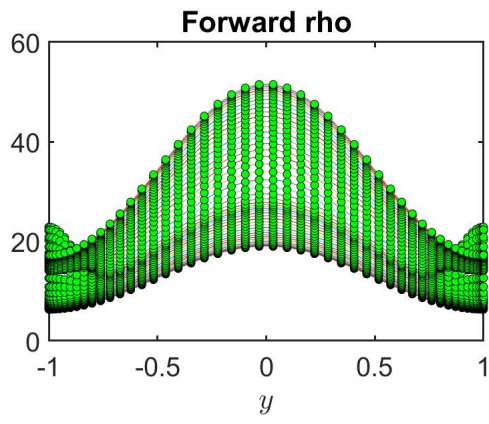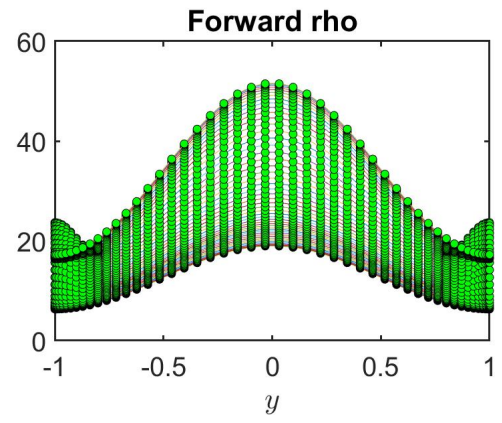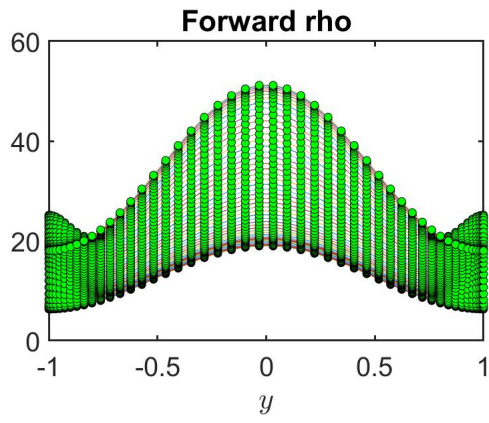Figure 5: $\tilde{g}(t)$ with $a = 0.3$, $a = 0.7$ and $a = 2$.

Figure 6: 2ρ forward solution for w perturbed by $0.1\tilde{g}(t)$, with $a = 0.3$, $a = 0.7$ and $a = 2$.
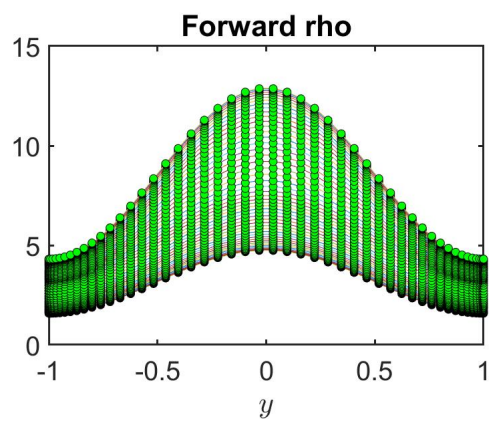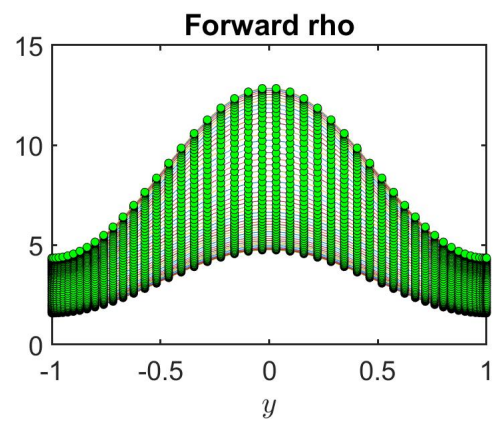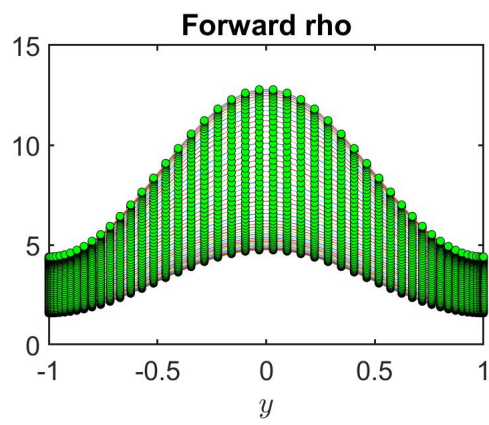
Figure 7: 0.5ρ forward solution for w perturbed by $0.1\tilde{g}(t)$, with $a = 0.3$, $a = 0.7$ and $a = 2$.
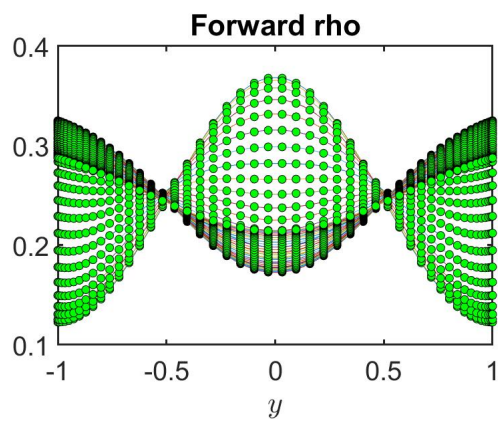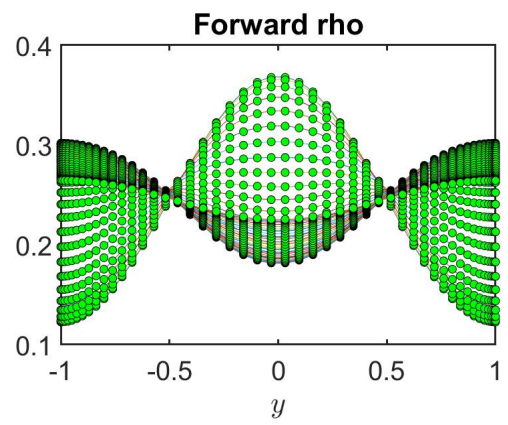
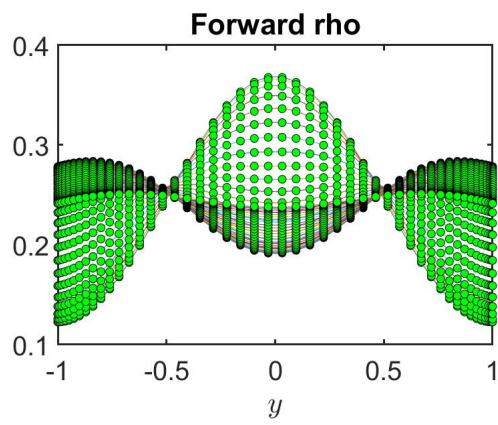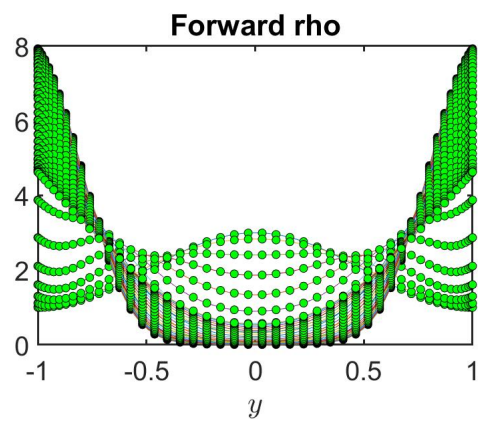Figure 8: $1/8.1441\rho$ forward solution with $\gamma = -1, 0, 1$
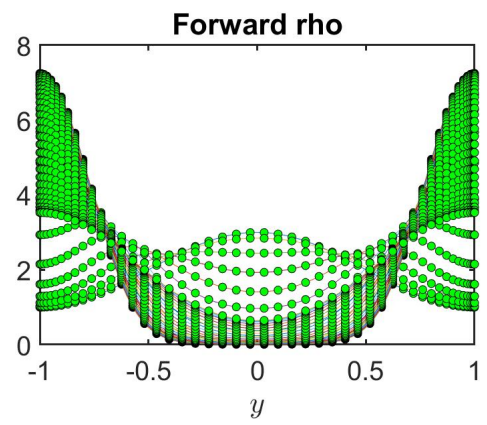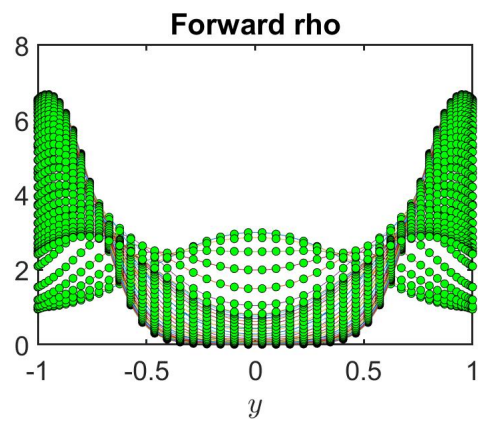
Figure 9: 1ρ forward solution with $\gamma = -1, 0, 1$