Question: Implementation of the Forward Interaction Term

We want to implement (ignoring for now that we take the divergence of the whole thing later, because we only compute 'rhoflux' at first):

$$\begin{aligned}&\int_\Omega \rho(r)\rho(r')\nabla_r V_2(|r-r'|)dr'\\ =&\rho(r)\int_\Omega \rho(r')\nabla_r V_2(|r-r'|)dr'\end{aligned}$$

If the convolution is computed using $\nabla_r V_2(|r-r'|)$ (instead of $V_2(|r-r'|)$), then we compute:

$$rho. * (Conv * rho).$$

However, if we want to compute the convolution with $V_2(|r-r'|)$, then we need to consider:

$$\begin{aligned}&\int_\Omega \rho(r)\rho(r')\nabla_r V_2(|r-r'|)dr'\\ =&\rho(r)\nabla_r\int_\Omega V_2(|r-r'|)\rho(r')dr'\end{aligned}$$

In my logic right now, this needs to be (ignoring how the dimensions of the gradient etc fit – I think we'd need to stack two copies of rho to make it work):

$$rho. * (Grad * (Conv * rho)).$$

However, in the 2D code, this is

$$Grad * (rho. * (Conv * rho)).$$

But to me this implies that we actually solve:

$$\nabla_r\rho(r)\int_\Omega V_2(|r-r'|)\rho(r')dr',$$

so that the gradient is also applied to $\rho(r)$. And that would be wrong, right? I think in the adjoint PDE I am happy with the way it is applied. But I am not sure about the forward problem.