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Visual Computing Institute | RWTH AACHEN UNIVERSITY

# Basic Techniques in Computer Graphics

## Assignment 11

Date Published: January 23rd 2018, Date Due: January 30th 2018

- All assignments (programming and text) have to be done in teams of 3–4 students. Teams with less than 3 or more than 4 students will receive no points.
- Hand in **one solution per team per assignment**.
- Every team must work independently. Teams with identical solutions will receive no points.
- Solutions are due 18:00 on January 30th 2018. Late submissions will receive zero points. No exceptions!
- Instructions for **programming assignments**:
  - Download the solution template (a zip archive) through the L$^2$P course room.
  - Unzip the archive and populate the `assignmentXX/MEMBERS.txt` file. Any team member not listed in this file will not receive any points! (Also see the instructions in the file.)
  - Complete the solution.
  - Prepare a new zip archive containing your solution. It must contain exactly those files that you changed. **Only change those files you are explicitly asked to change in the task description.** The directory layout must be the same as in the archive you downloaded. (At the very least it must contain the `assignmentXX/MEMBERS.txt`.)
  - Upload your zip archive through the L$^2$P before the deadline.
  - Your solution must compile and run correctly **on our lab computers** using the exact same `Makefile` provided to you. If it does not, you will receive no points.
- Instructions for **text assignments**:
  - Prepare your solutions on paper.
  - If you write your solution by hand, write neatly! Anything we cannot decipher will receive zero points. No exceptions!
  - If you hand in more than one sheet, staple your sheets together. (No paper clips!)
  - Put the names and student ID numbers of all team members onto every sheet.
  - Unless explicitly asked otherwise, always justify your answer.
  - Be concise!
  - Put your solution into the designated drop box at our chair before the deadline. (1st floor, E3 building.)

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## Exercise 1 Bernstein Polynomials and Bézier Curves [6 Points]

### (a) Derivatives of Bernstein Polynomials [2 Points]

Show: The first derivative of a Bernstein polynomial of degree $n$ can be expressed as a difference of two Bernstein polynomials of degree $n-1$:

$$\frac{\mathrm{d}}{\mathrm{d}t} B_i^n(t) = n \left( B_{i-1}^{n-1}(t) - B_i^{n-1}(t) \right).$$

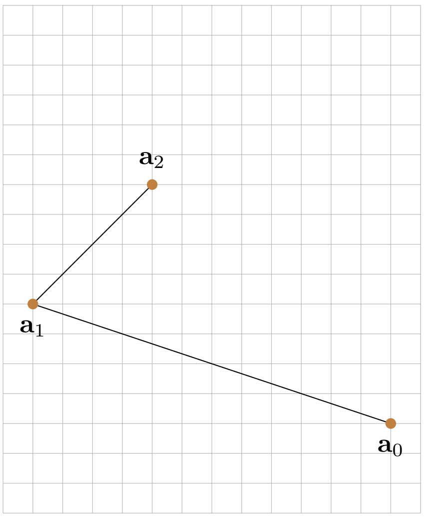
### (b) Derivatives of Bézier Curves [2 Points]

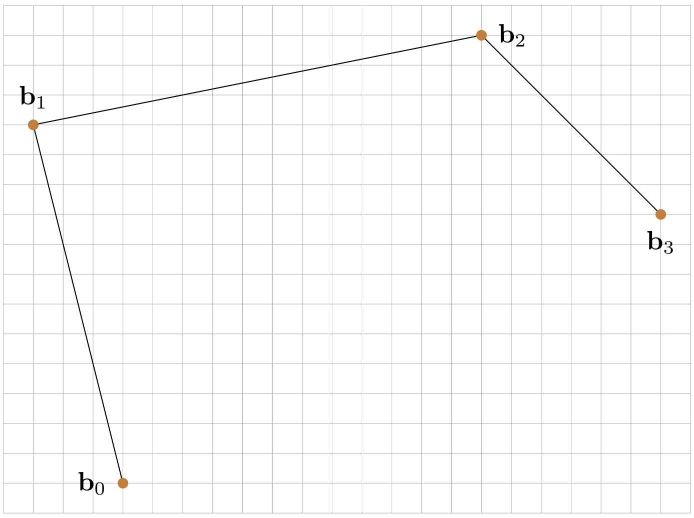
Given a Bézier curve $\mathbf{b}(t)$ of degree $n$ with control points $\mathbf{p}_0, \ldots, \mathbf{p}_n$, show that the derivative $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{b}(t)$ is again a Bézier curve $\mathbf{b}'(t)$ (of degree $n-1$). What are the control points of the derivative curve?

*Hint: Use the identity from task (a).*

### (c) De Casteljau Algorithm [2 Points]

Use the de Casteljau algorithm to evaluate the two Bézier curves defined by their control polygons below. Evaluate the left curve ($\mathbf{a}(t)$) at $t = \frac{1}{2}$ and the right curve ($\mathbf{b}(t)$) at $t = \frac{1}{3}$. Perform the evaluation graphically. You do not need to perform any computations.



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## Exercise 2 Splines **[4 Points]**

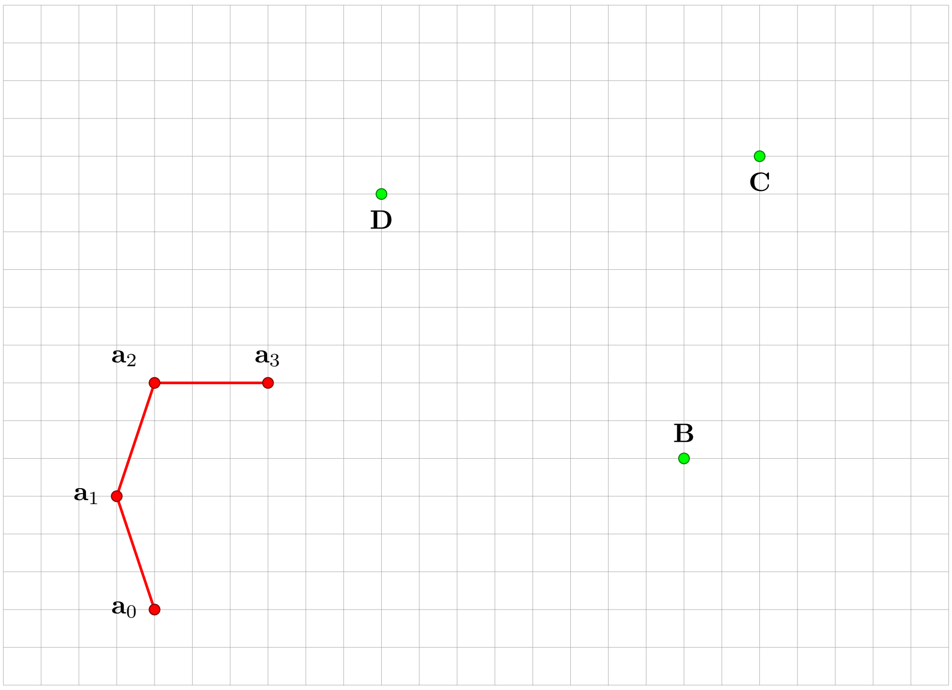
### (a) Interpolating Bézier Spline Construction **[2 Point]**

Starting from an initial curve segment, incrementally construct a cubic (degree 3) Bézier spline with $C^2$ continuity that interpolates the points $\mathbf{B}, \mathbf{C}, \mathbf{D}$! The initial cubic Bézier curve $\mathbf{a}(t)$ is given by the control points $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ (indicated by the red control polygon).

1. First, construct the control polygon $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ of a cubic Bézier curve $\mathbf{b}(t)$ such that curves $\mathbf{a}$ and $\mathbf{b}$ join with $C^2$ continuity and its endpoint interpolates point $\mathbf{B}$ (i. e. $\mathbf{b}(1) = \mathbf{B}$).
2. Then, construct the control polygon $\mathbf{c}_0, \mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3$ of a cubic Bézier curve $\mathbf{c}(t)$ such that curves $\mathbf{b}$ and $\mathbf{c}$ join with $C^2$ continuity and its endpoint interpolates point $\mathbf{C}$ (i. e. $\mathbf{c}(1) = \mathbf{C}$).
3. Finally, construct the control polygon $\mathbf{d}_0, \mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3$ of a cubic Bézier curve $\mathbf{d}(t)$ such that curves $\mathbf{c}$ and $\mathbf{d}$ join with $C^2$ continuity and its endpoint interpolates point $\mathbf{D}$ (i. e. $\mathbf{d}(1) = \mathbf{D}$).

Include all auxiliary constructions (such as A-frames) in your solution. Also, don't forget to label all Bézier control points.

*Note: You do not need to perform any computations. Do all constructions graphically using the methods presented in the lecture!*



## (b) B-Spline to Bézier Spline Conversion **[2 Point]**

In the figure below, you are given the control polygon $\mathbf{A}, \ldots, \mathbf{G}$ specifying a cubic B-spline curve. Using the A-frame construction method introduced in the lecture, graphically construct the control points for the Bézier spline describing the same curve. Your Bézier spline should consist of 4 segments, each of them a cubic (degree 3) Bézier curve:

1. $\mathbf{b}(t)$, close to the line segment $\overline{\mathbf{BC}}$, defined by control points $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$
2. $\mathbf{c}(t)$, close to the line segment $\overline{\mathbf{CD}}$, defined by control points $\mathbf{c}_0, \mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3$
3. $\mathbf{d}(t)$, close to the line segment $\overline{\mathbf{DE}}$, defined by control points $\mathbf{d}_0, \mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3$
4. $\mathbf{e}(t)$, close to the line segment $\overline{\mathbf{EF}}$, defined by control points $\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$

Include all auxiliary constructions in your solution. Also, don't forget to label all Bézier control points.

*Note: You do not need to perform any computations. Do all constructions graphically using the methods presented in the lecture!*