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Basic Techniques in Computer Graphics

Assignment 3

Date Published: November 14th 2017, Date Due: November 21st 2017

- All assignments (programming and text) have to be done in teams of 3–4 students. Teams with less than 3 or more than 4 students will receive no points.
- Hand in **one solution per team per assignment**.
- Every team must work independently. Teams with identical solutions will receive no points.
- Solutions are due 18:00 on November 21st 2017. Late submissions will receive zero points. No exceptions!
- Instructions for **programming assignments**:
  - Download the solution template (a zip archive) through the L^2P course room.
  - Unzip the archive and populate the `assignmentXX/MEMBERS.txt` file. Any team member not listed in this file will not receive any points! (Also see the instructions in the file.)
  - Complete the solution.
  - Prepare a new zip archive containing your solution. It must contain exactly those files that you changed. **Only change those files you are explicitly asked to change in the task description.** The directory layout must be the same as in the archive you downloaded. (At the very least it must contain the `assignmentXX/MEMBERS.txt`.)
  - Upload your zip archive through the L^2P before the deadline.
  - Your solution must compile and run correctly **on our lab computers** using the exact same `Makefile` provided to you. If it does not, you will receive no points.
- Instructions for **text assignments**:
  - Prepare your solutions on paper.
  - If you write your solution by hand, write neatly! Anything we cannot decipher will receive zero points. No exceptions!
  - If you hand in more than one sheet, staple your sheets together. (No paper clips!)
  - Put the names and matriculation numbers of all team members onto every sheet.
  - Unless explicitly asked otherwise, always justify your answer.
  - Be concise!
  - Put your solution into the designated drop box at our chair before the deadline. (1st floor, E3 building.)

Exercise 1 Extended & Homogeneous Coordinates **[1 Point]**

Please explain *in your own words* and *very concisely*:

(a) Extended Coordinates **[0.5 Points]**

How do *extended coordinates* represent points and vectors. What kind of transformations can be represented in extended coordinates using matrices?

(b) Homogeneous Coordinates **[0.5 Points]**

How do *homogeneous coordinates* represent points and vectors. What kind of transformations can be represented in homogeneous coordinates using matrices?

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Exercise 2 Transformations **[2 Points]**

In this exercise you are to consider the transformation of some orthonormal coordinate system into another orthonormal coordinate system.

Let $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2 \in \mathbb{R}^3$ be three vectors that form an orthonormal coordinate system, i.e. $\mathbf{p}_0^\mathsf{T}\mathbf{p}_1 = 0$, $\mathbf{p}_1^\mathsf{T}\mathbf{p}_2 = 0$, $\mathbf{p}_2^\mathsf{T}\mathbf{p}_0 = 0$, and $\|\mathbf{p}_0\| = \|\mathbf{p}_1\| = \|\mathbf{p}_2\| = 1$. Similarly, let $\mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2 \in \mathbb{R}^3$ also be three vectors that form an orthonormal coordinate system.

(a) Inverse of an Orthonormal Matrix **[0.5 Points]**

Show that $P^\mathsf{T} = P^{-1}$, where $P = \begin{pmatrix} \mathbf{p}_0 & \mathbf{p}_1 & \mathbf{p}_2 \end{pmatrix}$, i.e. the colums of P are formed by the vectors $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$.

(b) Rigid Transformations **[1 Point]**

Derive the linear transformation matrix $M \in \mathbb{R}^{3\times 3}$ that maps $\mathbf{p}_i$ to $\mathbf{q}_i$ for all $i \in \{0, 1, 2\}$.

(c) Inverse of Rigid Transformations **[0.5 Points]**

Show that $M^\mathsf{T} = M^{-1}$. (You may use the result from (a).)

Exercise 3 Vanishing Points and Geometric Objects **[2 Points]**

Due to perspective foreshortening, the projections of parallel lines (1D geometric objects) meet in their vanishing point (a 0D geometric object). Higher dimensional geometric objects in turn also have higher dimensional vanishing objects. In this exercise, you should investigate the vanishing object of planes (2D geometric objects), given that *the used projection is the standard projection*.

(a) Vanishing Points of Lines **[1 Point]**

Given the three points $\mathbf{p}_0 = \begin{pmatrix} -1 & 3 & -5 \end{pmatrix}^\mathsf{T}$, $\mathbf{p}_1 = \begin{pmatrix} 2 & 0 & -7 \end{pmatrix}^\mathsf{T}$, $\mathbf{p}_2 = \begin{pmatrix} 3.5 & 1 & -5 \end{pmatrix}^\mathsf{T}$, determine the vanishing points of the edges of the triangle $\Delta(\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2)$.

(b) Vanishing Points of Planes **[1 Point]**

Given a plane $\{\mathbf{p} \mid \mathbf{n}^\mathsf{T}\mathbf{p} = a\}$ with normal $\mathbf{n}$ *not parallel* to the z-axis and perpendicular distance a to the origin. Specify the set of vanishing points of all lines within the plane. Explain what kind of geometric object is formed by the vanishing points.