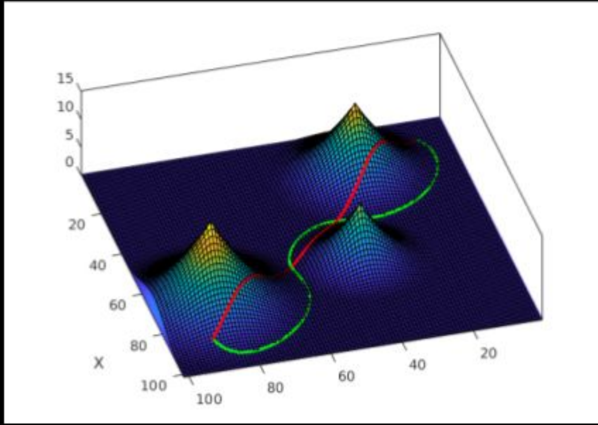
Homotopy constrained planning

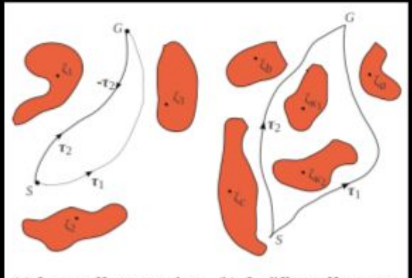
Muhammad Suhail & Ramkumar

Trajectory Optimization with Homotopy Class Constrained Planning In 2D



Trajectory optimization finds a path restricted to the homotopy class of the initial guess.

Homotopy Class of Trajectories



(a) In same Homotopy class, forming a closed contour. (b) In different Homotopy classes, enclosing obstacles.

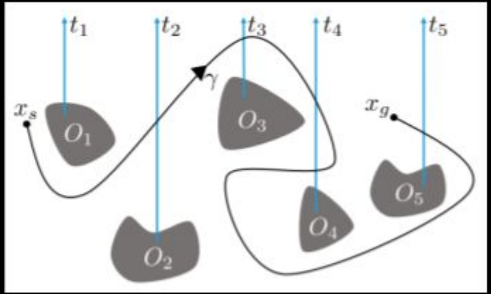
Two trajectories T_1 and T_2 connecting the same start and goal are homotopic iff one can be deformed into the other without intersecting any obstacle.

Identifying Homotopy Classes

Cauchy Integral Theorem

$$\oint_\gamma \frac{f(z)\mathrm{d}z}{z - z_0} = 2\pi i f(z_0)$$

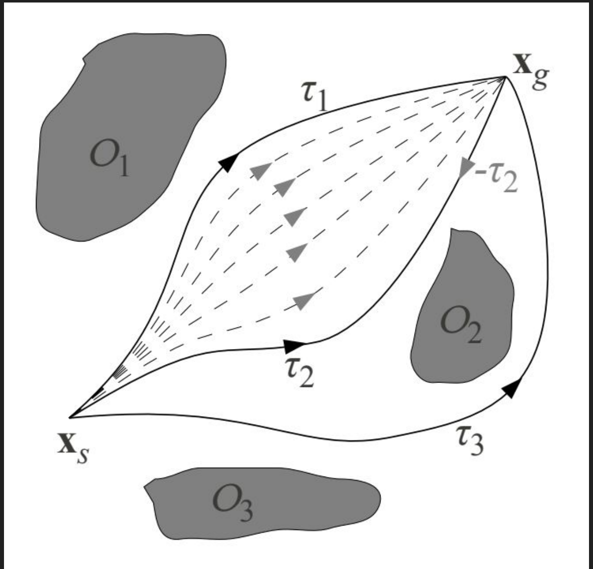
H-Signature



Problem Statement

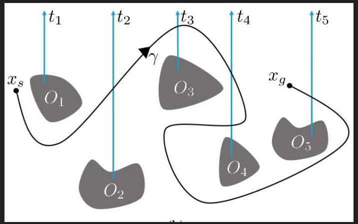
Our goal was to intelligently find paths in different homotopies and demonstrate the ability of our planner and the use of homotopy constrained planning in different application areas:

1. Point robot
2. Trajectory Optimization
3. Planar Arm



Bhattacharya et al

Math definition of the problem



Vinitha et al.

- H signatures are unique for trajectories of different homotopy classes and can be computed using L-values or reduced words.
- We plan on H Signature Augmented Graphs. Node in the graph is given by $\{v, \mathcal{H}_2(v_s, v)\}$
- Our graph is defined as follows:

$$\mathcal{G}_L(\mathcal{G}) = \{\mathcal{V}_L, \mathcal{E}_L\}$$

where,

1.

$$\mathcal{V}_L = \left\{ \{z, \Lambda\} \middle| \begin{array}{l} z \in \mathcal{V}, \text{ and,} \\ \Lambda \notin \mathcal{B} \text{ (or equivalently, } \Lambda' \in \mathcal{A}) \\ \quad \text{if } z = z_g, \text{ and,} \\ \Lambda = L(z_s \to z) \text{ for some trajectory} \\ \quad z_s \to z, \text{ from } z_s \text{ to } z \end{array} \right\}$$

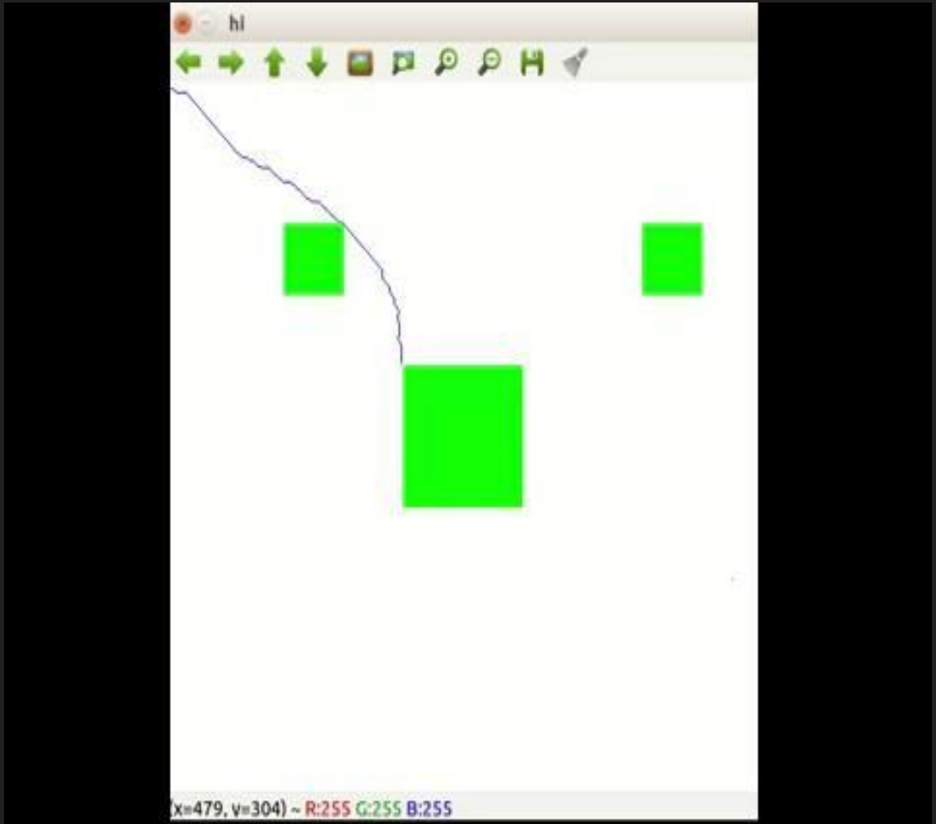
2. And, edge $\{\{z, \Lambda\} \to \{z', \Lambda'\}\}$ is in $\mathcal{E}_L$ for $\{z, \Lambda\} \in \mathcal{V}_L$ and $\{z', \Lambda'\} \in \mathcal{V}_L$, iff
   i. $\{z \to z'\} \in \mathcal{E}$, and
   ii. $\Lambda' = \Lambda + L(z \to z')$, where $L(z \to z')$ is the L-value of the straight line segment joining the adjacent nodes z and z'
3. And, the cost/weight associated with an edge $\{\{z, \Lambda\} \to \{z', \Lambda'\}\} \in \mathcal{E}_L$ is same as the cost of the edge $\{z \to z'\} \in \mathcal{E}$.

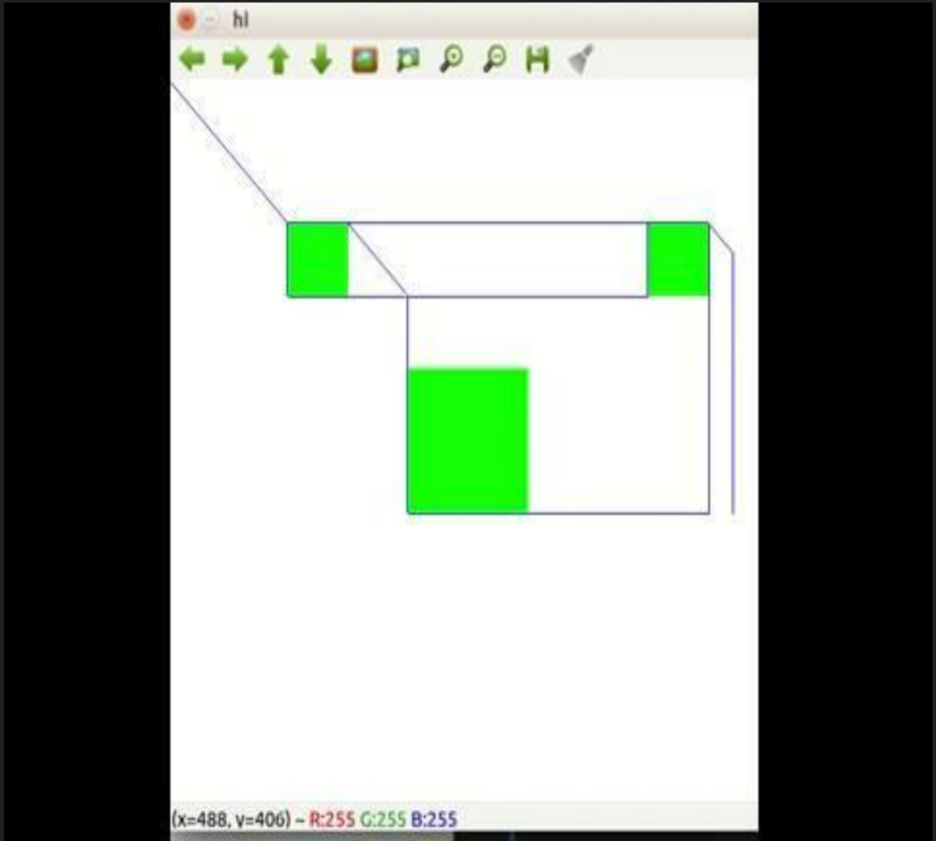
Bhattacharya et al.

Algorithms

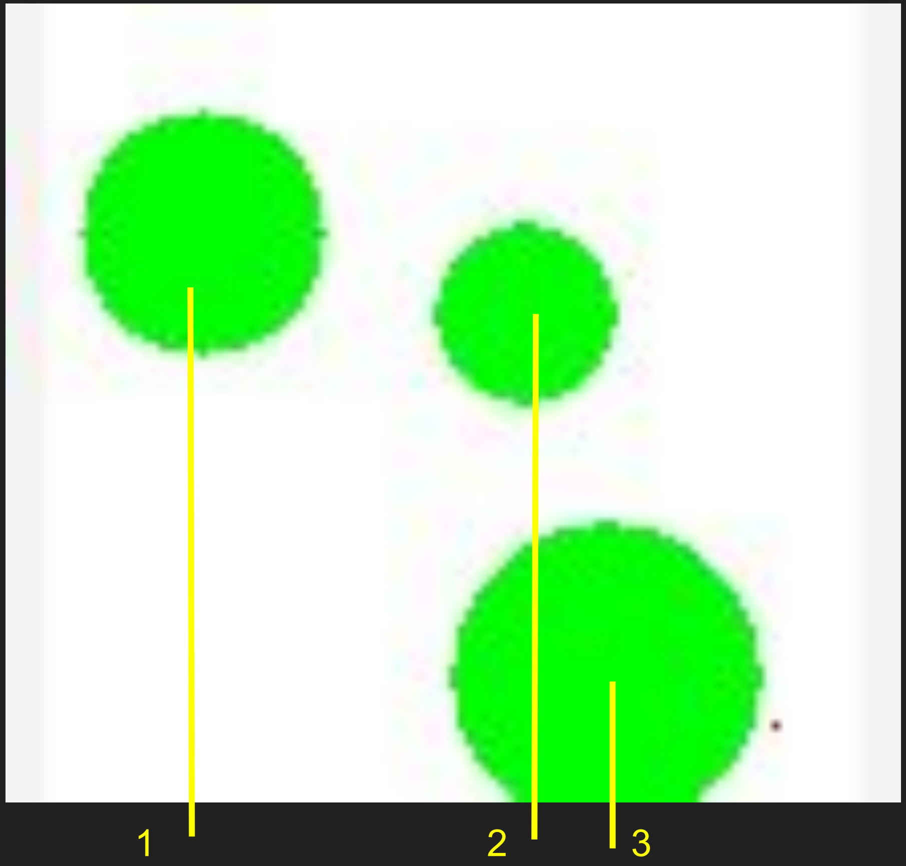
1. Point robot:
    a. A-star - Euclidean Heuristic and cost function
    b. MHA* - Euclidean Heuristic (Anchor Search) and (Euclidean distance + 100* Number of mismatched variables) (Inadmissible Heuristic).
    c. For the path seen in the movie, A-star expands 36239923 states, while MHA* expands 24560948
2. Trajectory optimization:
    a. Traj Opt will converge to a global solution, only if the initial estimate is in the same homotopy as the global optimum.
    b. By providing paths from every homotopy in the environment, we can ensure that the algorithm converges to the global optimum
3. Planar Arm:
    a. Weighted A-star
    b. The h-signature is calculated using the reduced word approach using the end effector's trajectory, but the planning is in C-space.

MHA* & A* (goal signature = 123)

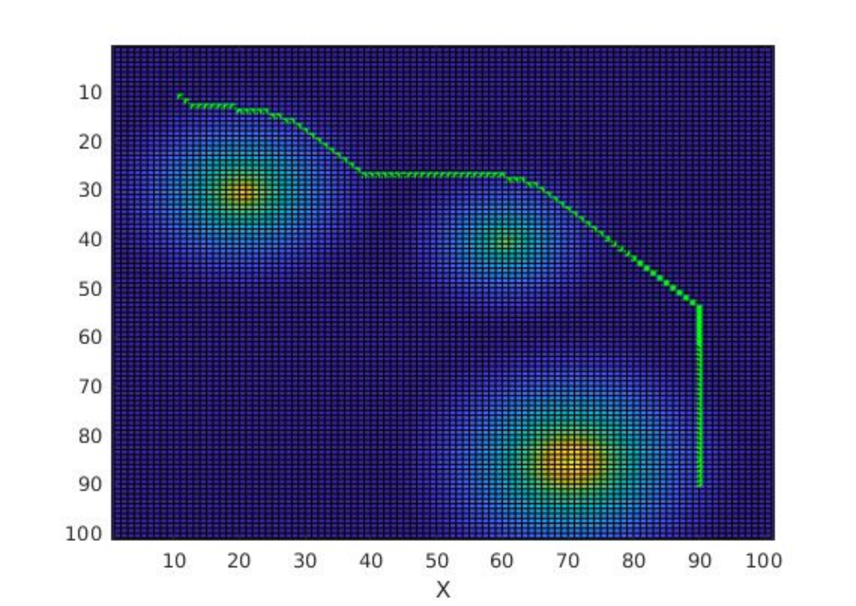


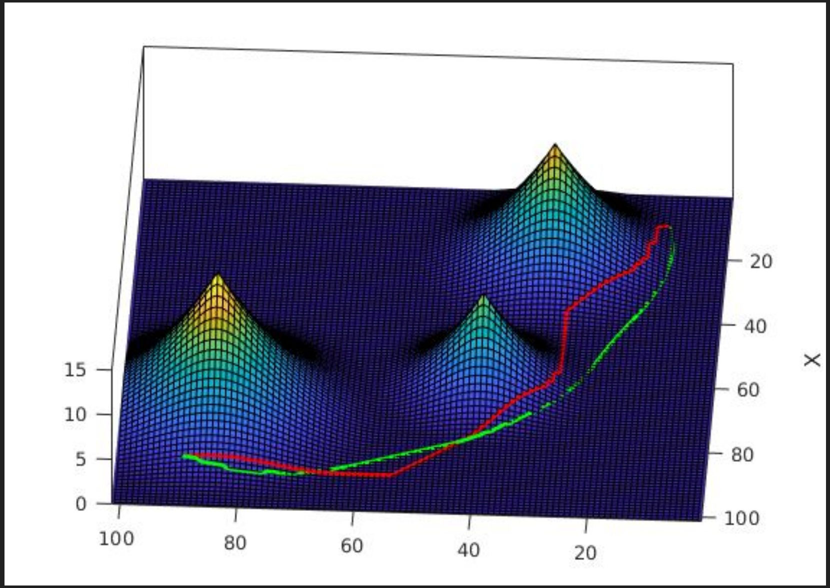


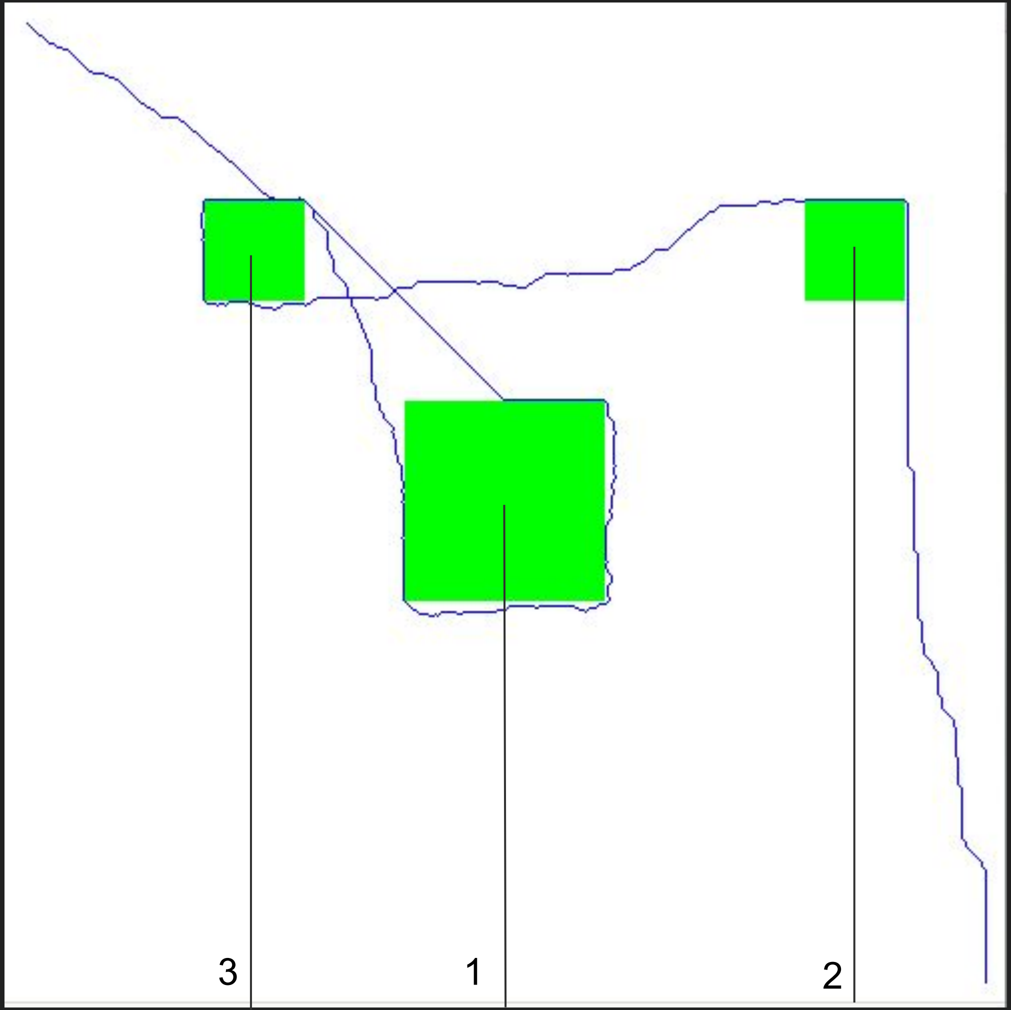
Costmap Input with H-signatures



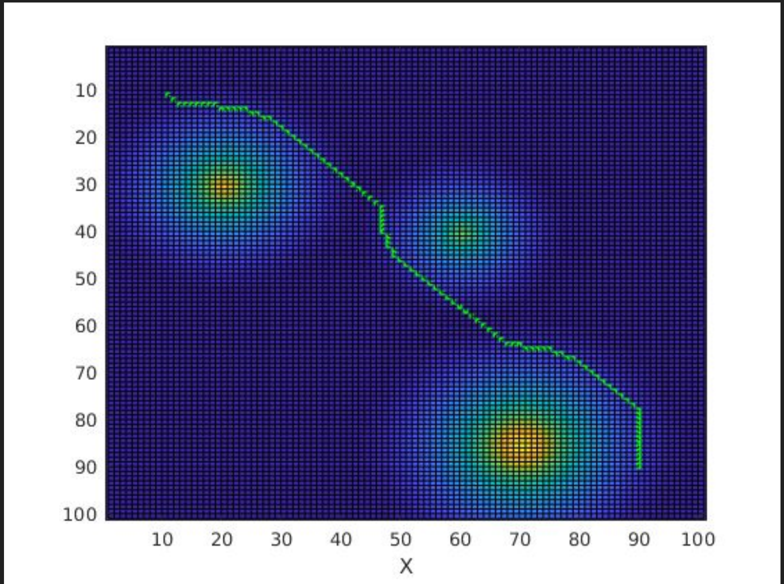
A* with Gradient Descent (goal signature = NULL)

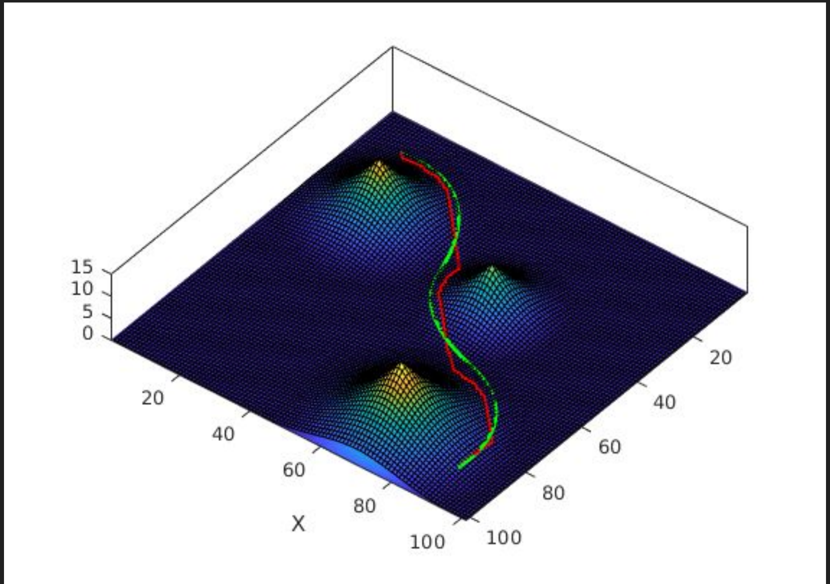




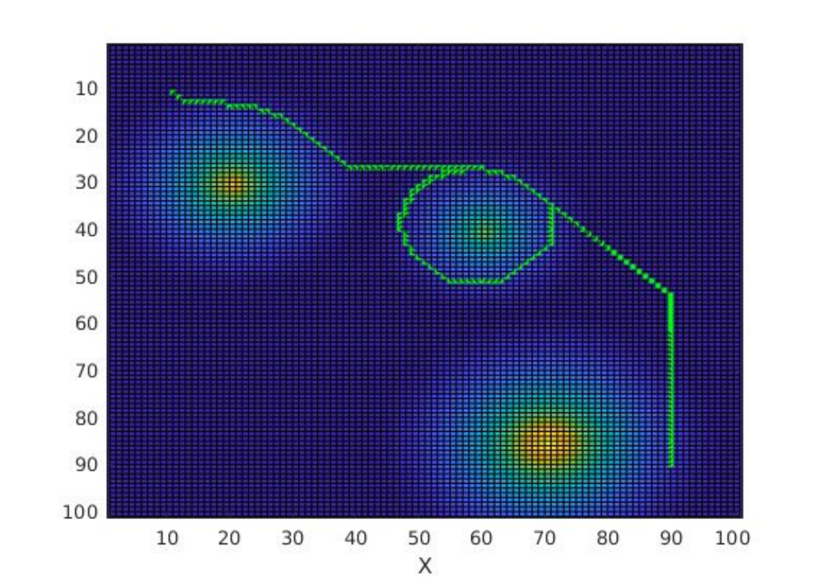

3
1
2

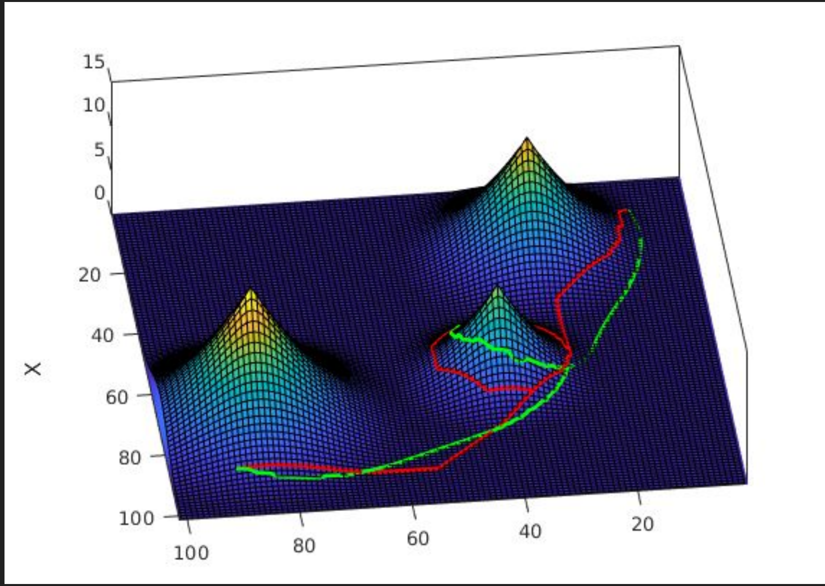
A* with Gradient Descent (goal signature = 2)

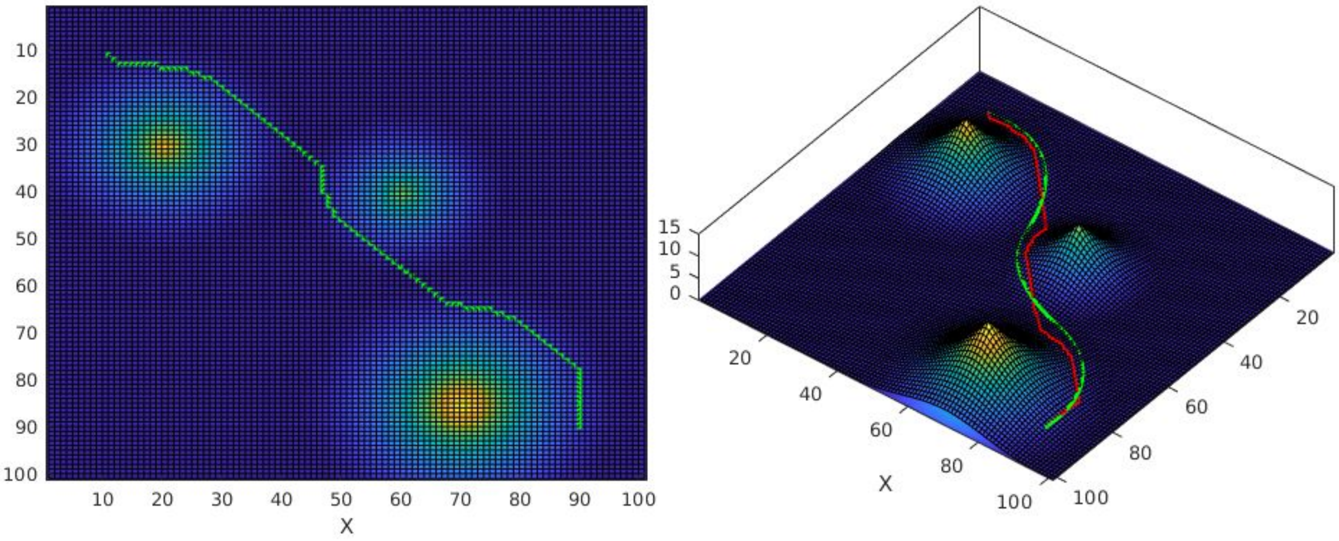


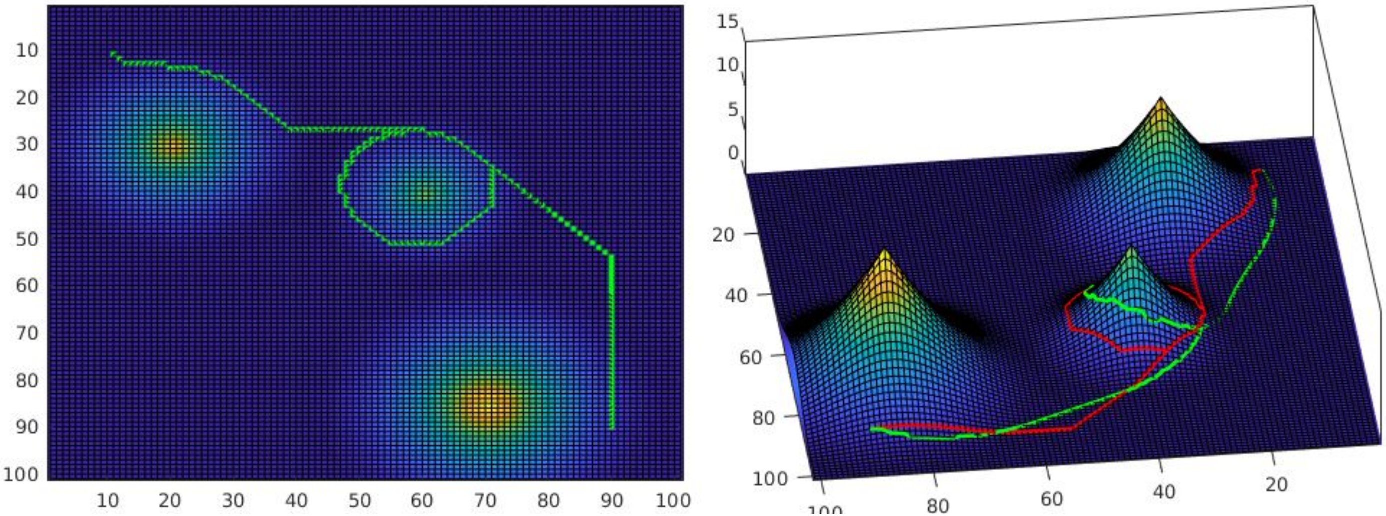


A* with Gradient Descent (goal signature = -2)






10
20
30
40
50
60
70
80
90
100
10 20 30 40 50 60 70 80 90 100
X
15
10
5
0
20
40
60
80
100
X
20
40
60
80
100



10
20
30
40
50
60
70
80
90
100
10
20
30
40
50
60
70
80
90
100
15
10
5
0
20
40
60
80
100
100
80
60
40
20

Manipulator planning with homotopy