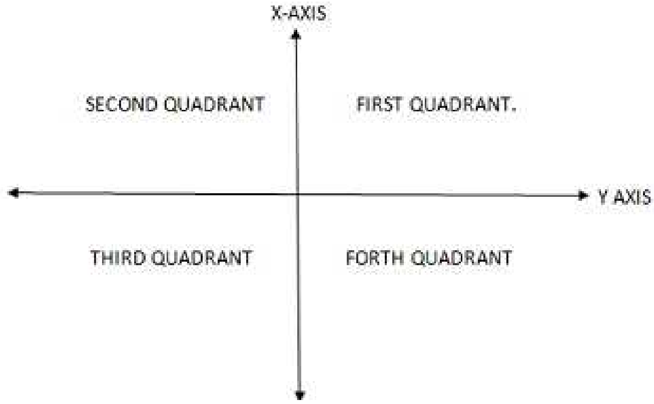
Basic mathematics notes Form Four

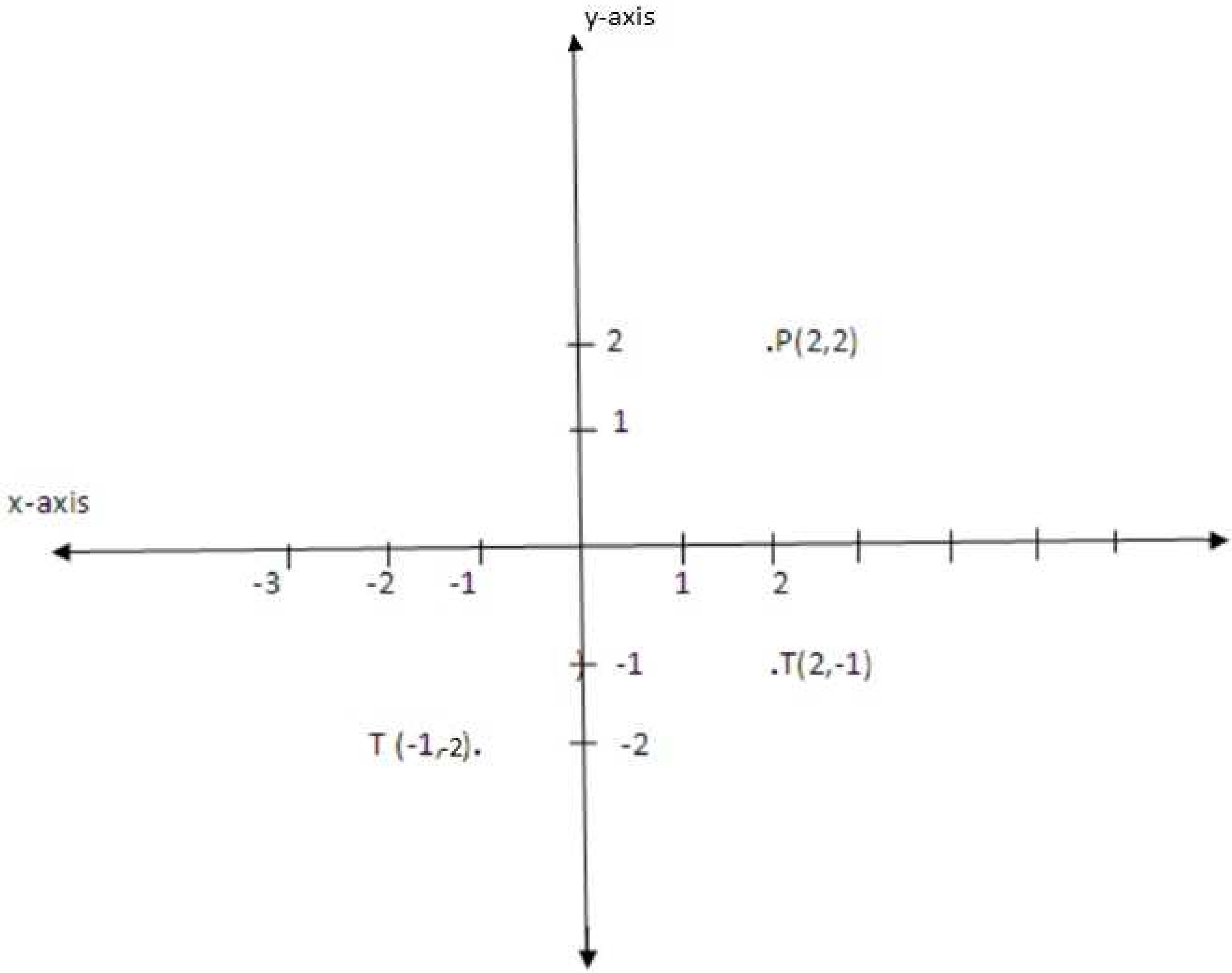
COORDINATE GEOMETRY



X –absissae.
Y- Ordinate. coordinate

<u>Exercise</u>

1. Plot the following point. P (2,2), T (-1, -2), L (2, -1)



2. In which quadrants is the?
   a. Abscissa positive? I
   b. Ordinate negative III
   c. Abscissa negative II
   d. Ordinate positive I
   e. Abscissa negative and ordinate negative? III

EQUATIONS IN A STRAIGHT LINE

Gradient / slope

Slope = $\frac{\text{change in y coordinates}}{\text{Change in x coordinates}}$

Equation

A (3,2) N (x,y) m=1

Gradient = $\frac{\text{Change in } y}{\text{Change in } X}$

$1 = \frac{y-2}{X-3}$

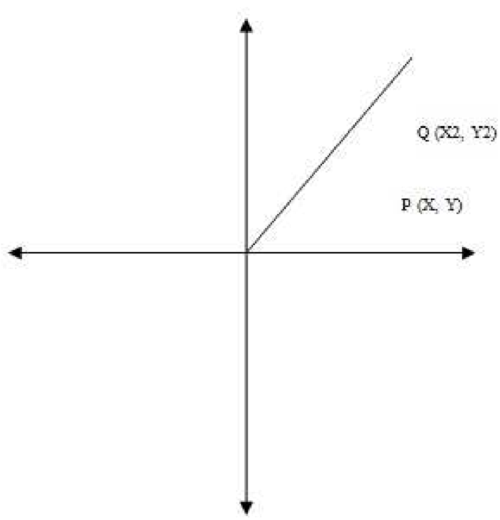
y - 2 = x - 3

y =x – 3 + 2

y=x - 1

Consider two points P (x, y) and (X_2, Y2) are given and lie on the same line.

If there exists point N (x, y) which lies on PQ, where $X_1 \neq X_2$ the N lies on the same line If and Only if the slope of PN if the same as the slope of PQ.



Slope PQ = $\frac{\text{Change in } y}{\text{Change in } X}$

P(x ,y) and Q (X_2, Y_2)

Slope of PQ (M) = $\frac{Y2-Y1}{X2-X1}$

Slope at PN = $\frac{\text{Change in } y}{\text{Change in } X}$

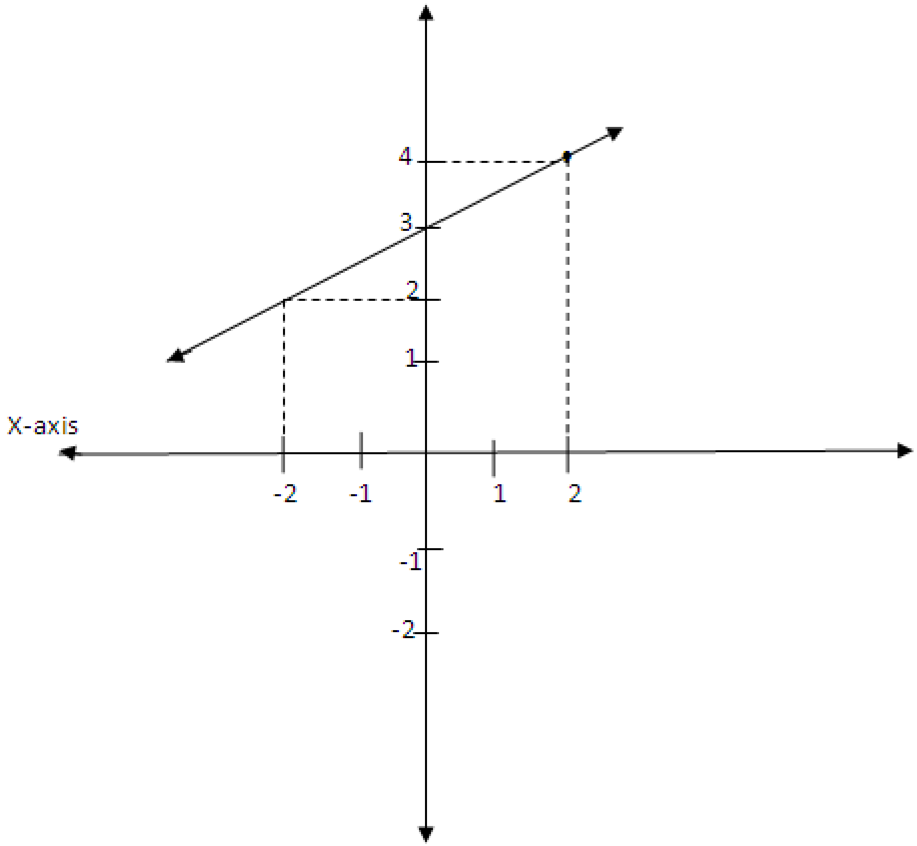
P(X ,Y) and N (X, Y)

M = $\frac{Y-Y1}{X-X1}$

Exercise

1. A straight line is drawn through (2, 4) and (-2, 2) . Draw a graph to find where it intersects.
    a. The y- axis
    b. The x-axis

    Solution:



(a) (0 , 3)

(b) (-6, 0)

X_2 y_2 x, y

(2, 4)   (-2, 2)

$M = \frac{\Delta y}{\Delta x}$

M = Slope

Δy = Change in y

Δx = Change in x

$M = \frac{4-2}{2 - {}^-2}$

$M = \frac{2}{2+2} = \frac{\cancel{2}}{\cancel{4}} = \frac{1}{2}$

$M = \frac{1}{2}$

Equation of line

Choose the points (2, 4)

$M = \frac{1}{2}$

$y = mx + c$

$\frac{1}{2} = \frac{y-()}{x-()}$

$2(y-4) = x-2$

$2y - 8 = x-2$

$2y-8+8 = x-2+8$

$\frac{2y}{2} = \frac{x+6}{2}$

$y = \frac{x+6}{2}$

y intercept the value of x will be 0

$y = \frac{0+6}{2} = \frac{\cancel{6}}{\cancel{2}}\ 3$

(0, 3)

x- intercept value of y is equal to 0

$y = \frac{x+6}{2}$

$0 = \frac{x+6}{2}$

$0 \times 2 = x + 6$

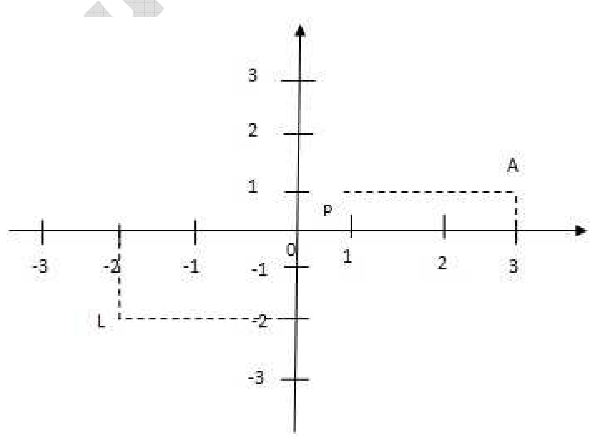
$0 = x + 6$

$X = -6$

(-6,0)

Will intersect in point (-6, 3)

2. In figure below, find the coordinates of the following points; A,P and L



A (3, 1), P (0,0) , L(-2,-2)

3. Find the gradient of the straight line joining each of the following pairs of points.

a. (1,6) and (5,7)

b. (3,2) and (7,-3)

c. (-3,4) and (8,1)

Solution:

(a) $M = \frac{\Delta y}{\Delta x} = \frac{7-6}{5-1} = \frac{1}{4}$

(b) $M = \frac{-3-2}{7-3} = \frac{-5}{4}$

$= \frac{-5}{4}$

(c) $M = \frac{1-4}{8-3} = \frac{-3}{11}$

4. Find the equation of the line of 2 which passes through the point (3,5)

Solution;

M = 2

$M = \frac{Y2-Y1}{X2-X1}$

$2 = \frac{Y-5}{X-3}$

2X - 6 = Y - 5

2X – 6 + 5 = Y

Y=2X - 1

5. For each of the following conditions, find the equations of the line.

a. Passing through points (4,7) having gradient of 3.

b. Passing through point (4,7) and (3,4)

c. Passing through A (4,-3) whose slope is 2/5 of the slope of the line joining A (4,-3) to B (9,7)

Solution

a). $3 = \frac{Y-7}{X-4}$

3X-12= Y-7

Y=3X-5

b). $M= \frac{7-4}{4-3}$

M=3

$3= \frac{Y-4}{X-3}$

3X – 9 = Y - 4

Y =3X - 5

c). $M= \frac{7+3}{9-4} = 2$

$^{2}/_{5} \times 2 = {}^{4}/_{5}$

$^{4}/_{5} = \frac{Y- -3}{X-4}$

4X – 16 – 15 = 5Y

$Y = \frac{4X-31}{5}$

6. Verify that the points (-2,2) and (-6,0) lie on the line joining points A (-4,1) and B (2,4).

Solution

$M = \frac{Y2-Y1}{X2-X1} = \frac{0-2}{-6+2}$

$M = {}^{1}/_{2}$

Also

$M = \frac{4-1}{2+4}$

$M = {}^{1}/_{2}$

Slope = $\frac{5}{2}$ Point (3 -4)

$\frac{5}{2} = \frac{y - {}^{-}4}{x-3}$

$\frac{5}{2} = \frac{y+4}{x-3}$

7. Find the equations of the following straight lines in the form of ax + by + c = 0

a. The line joining the points (2,4) and (-3,1)

$M = \frac{4-1}{2--3} = {}^{3}/_{5}$

${}^{3}/_{5} = \frac{y-4}{x-2}$

$5y - 20 = 3x - 6$

$5y = 3x + 14$

$3x - 5y + 14 = 0$

b. The line through (3,1) with gradient $-{}^{3}/_{5}$

Solution:

$M = -{}^{3}/_{5}$

$M = \frac{y2 - y1}{x2 - x1}$

$-{}^{3}/_{5} = \frac{y-1}{x-3}$

$-3x + 9 - 5y + 5 = 0$

$-3x - 5y + 14 = 0$

(c) = The line through (3,-4) and which has the same slope as the line 5x-2y =3

$5x - 2y = 3$

$5x - 3 = +2y$

$\frac{5x-3}{2} = \frac{\cancel{2}y}{\cancel{2}}$

$Y = \frac{5x-3}{2}$

Slope = $\frac{5}{2}$ Point (3 -4)

$\frac{5}{2} = \frac{y - {}^{-}4}{x-3}$

$\frac{5}{2} = \frac{y+4}{x-3}$

$2y + 8 = 5x + -15 - 8$

$2y + 8 - 8 = 5x - 15 - 8$

$2y = 5x - 23$

$0 = 5x - 2y - 23$

$\therefore 5x - 2y - 23 = 0$

8. Determine the value of K in order the line whose equation is Kx – y + 5 passes through that point (3,5)

   Solution:

   $Kx - y + 5 = 0$

   $Kx - 5 + 5 = 0$

   $3K = 0$

   $K = 0$

9. What must be the value of T to allow the line represented by the equation 3X-Ty=16 to pass through the point (5,-4)

   Solution

   $3x - Ty = 16$

   $3(5) - Tx - 4 = 16$

   $15 + 4T = 16$

   $4T = 1$

   $T = {}^{1}/_{4}$

10. Find the equation of a line with a slope ${}^{2}/_{3}$ having the same Y-intercept as the line

    $2x - 5y + 20 = 0$

Solution:

$y = mx + c$

$5y = 2x + 20$

$y = \frac{2x}{5} + 4$

y – intercept x = 0

$y = 4$

points (0 , 4)

$y = m(x - x1) + y1$

$y = \frac{2}{3}(x - 0) + 4$

$y = \frac{2x}{3} + 4$

11. Determine the value of m and c so that the line Y = mx + c will pass through the points (-1, 4) and (3, 5).

Solution:

$M = \frac{5-4}{3+1} = \frac{1}{4}$

$M = \frac{1}{4}$

$\frac{1}{4} = \frac{y-5}{x-3}$

$x - 3 = 4y - 20$

$x + 17 = 4y$

$y = \frac{x}{4} + \frac{17}{4}$

$c = \frac{17}{4}$

Therefore gradient (M) = $\frac{1}{4}$ and c = $\frac{17}{4}$

EQUATION OF A STRAIGHT LINE.

Slope of PQ (M) = $\frac{Y-Y1}{X-X1}$

$Y-Y_1 = M(X-X_1)$

$Y = MX - MX_1 + Y_1$

$Y = MX + C$

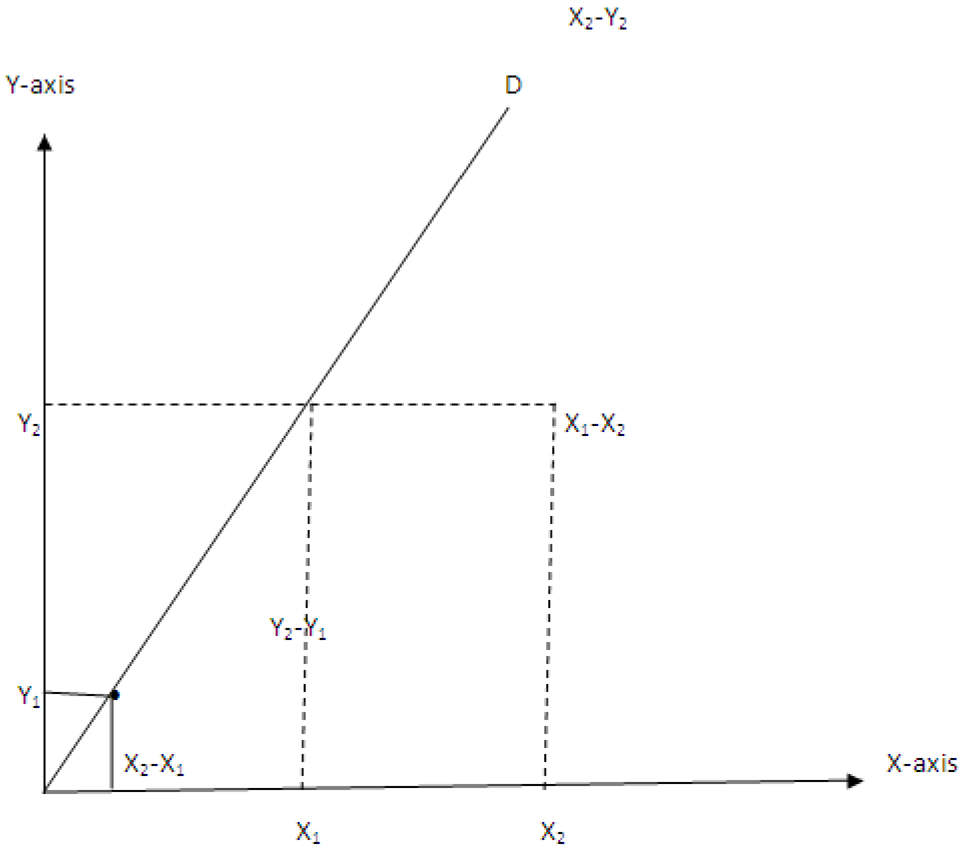
Example

(3, 5) slope = 2

$Y – 5 = 2(X-3)$

$Y = 2x – 6 + 5$

Mid point of a straight line



Similarities;

$\triangle PCP \sim \triangle QPR$

$\frac{PC}{QD} = \frac{CQ}{DR} = \frac{PQ}{QR}$

$\frac{PQ}{QR} = 1$

Take;

$\frac{PC}{QD} = \frac{PQ}{QR}$

$\frac{X - X1}{X2 - X} = 1$

$X-X_1 = X_2 - X$

$2X = X_1 + X_2$

$X = \frac{X1 + X2}{2}$

$\frac{QC}{DR} = \frac{PQ}{QR}$

QR

$\frac{Y - Y1}{Y2 - Y} = 1$

$2Y = Y_2 + Y_1$

$Y = \frac{Y2 + Y1}{2}$

Mid point (x, y) = $\left(\frac{X1+X2}{2}, \frac{X1+X2}{2}\right)$

EXERCISE

1. Find the coordinates of the mid points joining each of the following pairs.

   a. (7,1) and (3,5)

   Midpoint = $\left(\frac{X1+X2}{2}, \frac{X1+X2}{2}\right)$

   $= \left(\frac{7+3}{2}, \frac{1+5}{2}\right)$

   = (5, 3)

   b. (0,0) and (12, 3)

Mid point = $\left(\frac{0+12}{2}, \frac{0+3}{2}\right)$

= (6, 1.5)

DISTANCE BETWEEN TWO POINTS

$PQ^2 = PC^2 + BC^2$

$PQ^2 = (X-X_1)^2 + (Y-Y_1)^2$

$\overline{PQ} = \sqrt{(X-X1)^2 + (Y-Y1)^2}$

EXERCISE

1. If the line from $(-4, Y_1)$ to $(X_2, -3)$ is bisected at (1,-1) .find the values of Y_1 and X_2

   Solution

   $1 = \frac{(-4 + X_2)/2}{2}$

   $2 = -4 + X_2$

   $X_2 = 6$

   $-1 = \frac{Y - 3}{2}$

   $-2 = Y_1 + -3$

   $Y_1 = 1$

2. The mid point of a line segment is (-2,5) and one end point is (1,7) . Find the other end point.

   Solution:

Mid point = $\left(\frac{X1+X2}{2}, \frac{X1+X2}{2}\right)$

= $\left(\frac{1+{X2}/{2}}{2}, \frac{7+{Y2}/{2}}{2}\right)$

$-2 = \frac{1+x2}{2}$

$-4 = 1 + X_2$

$X_2 = -5$

$5 = \frac{7+Y2}{2}$

$10 = 7 + Y_2$

$Y_2 = 3$

The other points is (-5 , 3)

3. The mid points of the sides of a triangle are (2 , 0) and (4, -3 ½) and (6 , ½) .Find the vertices of the triangle if one of them is (4,3) .

Solution

i. Mid point = (2,0)

$2 = \frac{4 + X_2}{2}$

$4 = 4 + X_2$

$X_2 = 0$

$0 = \underline{3 + Y_2}$

2

$Y_2 = -3$

ii. $4 = \frac{4 + X_2}{2}$

$X_2 = 8 - 4$

$X_2 = 4$

$-3.5 = \frac{3 + Y_2}{2}$

$Y_2 = -7 - 3$

$Y_2 = -10.$

iii. $6 = \frac{4 + X_2}{2}$

$12 = 4 + x_2$

$X_2 = 8$

$0.5 = \frac{3 + Y_2}{2}$

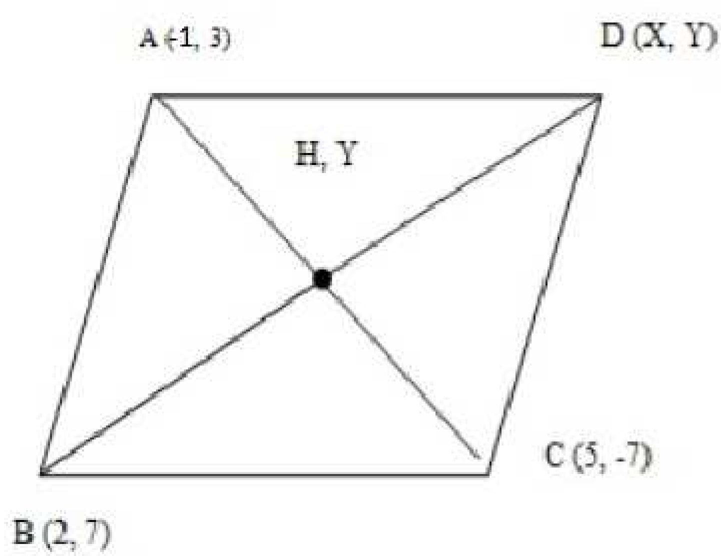
$1 = 3 + y_2$

$Y_2 = -2$

:. The vertices of the triangle are (0 , -3), (4 , 10) and (8 , -2)

4. Three vertices of a parallelogram ABCD are A (-1,3) ,B(2,7) and C (5,-7). Find the coordinates of vertex D using the principle that the diagonals dissect each other.

Solution:



Mid point H (x, y) = $\left(\frac{5-1}{2}, \frac{-7+3}{2}\right)$

= (2,-2)

(2,-2) = $\frac{2+X}{2}, \frac{7+Y}{2}$

4 = 2+ X

X = 2

-4 = 7 + Y

Y= -11

D= (2,-11)

EXERCISE

1.Find the distance between the line segments joining each of the following pairs of points.

a. (1,3) and (4,7)

Solution

D = $\sqrt{(x2-x1)^2+(y2-y1)^2}$

D = $\sqrt{(4-1)^2+(7-3)^2}$

D = $\sqrt{(3)^2+(4)^2}$

D = $\sqrt{9+16}$

D= $\sqrt{25}$

D = 5

b. (1,2) and (5,2)

Solution;

D = $\sqrt{(x2-x1)^2+(y2-y1)^2}$

D = $\sqrt{(5-1)^2+(2-2)^2}$

D = $\sqrt{(4)^2+(0)^2}$

D = $\sqrt{16}$

D = 4

2.Find the distance of the following point from the origin.

(-15, 8) (0, 0)

Solution

D = $\sqrt{(x2-x1)^2+(y2-y1)^2}$

D = $\sqrt{(0--15)^2+(0-8)^2}$

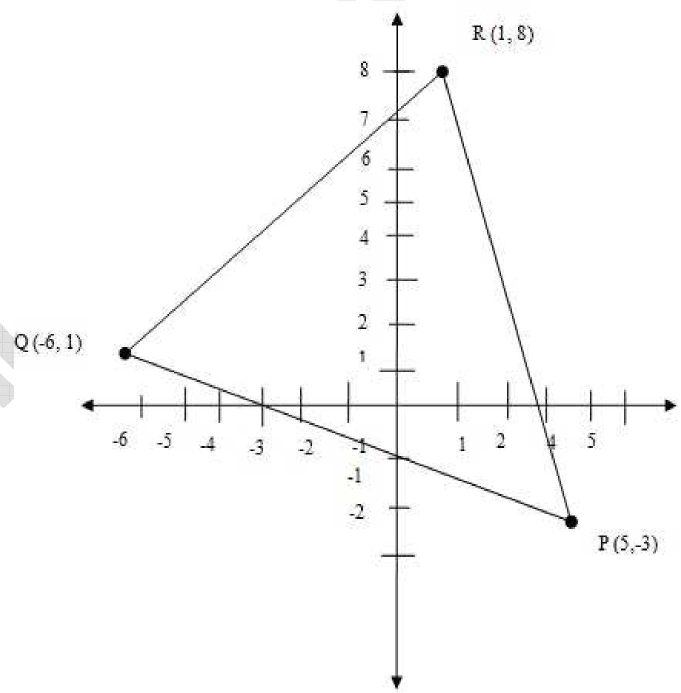
D = $\sqrt{(15)^2+(-8)^2}$

D = $\sqrt{225\ +\ 64}$

D= $\sqrt{289}$

D = 17

3. P, Q, R are the points (5,-3) (-6,1) (1,8) respectively . Show that triangle PQR is isosceles



QP = $\sqrt{(x2-x1)^2+(y2-y1)^2}$

QP = $\sqrt{(5-(-6))^2+(-3-1)^2}$

QP = $\sqrt{(11)^2+(-4)^2}$

QP = $\sqrt{121 + 16}$

QP= $\sqrt{137}$

PR = $\sqrt{(x2-x1)^2+(y2-y1)^2}$

PR = $\sqrt{(5-1)^2+(-3-8)^2}$

PR = $\sqrt{(4)^2+(-11)^2}$

PR = $\sqrt{16+121}$

PR= $\sqrt{137}$

Therefore triangle PQR is isosceles

PARALLEL LINES

Two lines are parallel if they have the same slope.

Example

1. Find whether AB is parallel to PQ in the following case.
   a. A(4,3) , B (8,4) P (7,1) Q (6,5)

Solution

Slope of AB = $\frac{\text{Change in Y}}{\text{Change in X}}$

$= \frac{4-3}{8-4} = {}^1\!/_4$

Slope of PQ= $\frac{5-1}{7-6}$ = -4

Therefore AB and PQ are not parallel line

2. Find the equation of the line through the point (6,2) and parallel to the line

   X +3Y - 13=0

Solution

X+3Y -13 =0

3Y = -X+13

Y= -X/3 + 13/3

Slope = -1/3

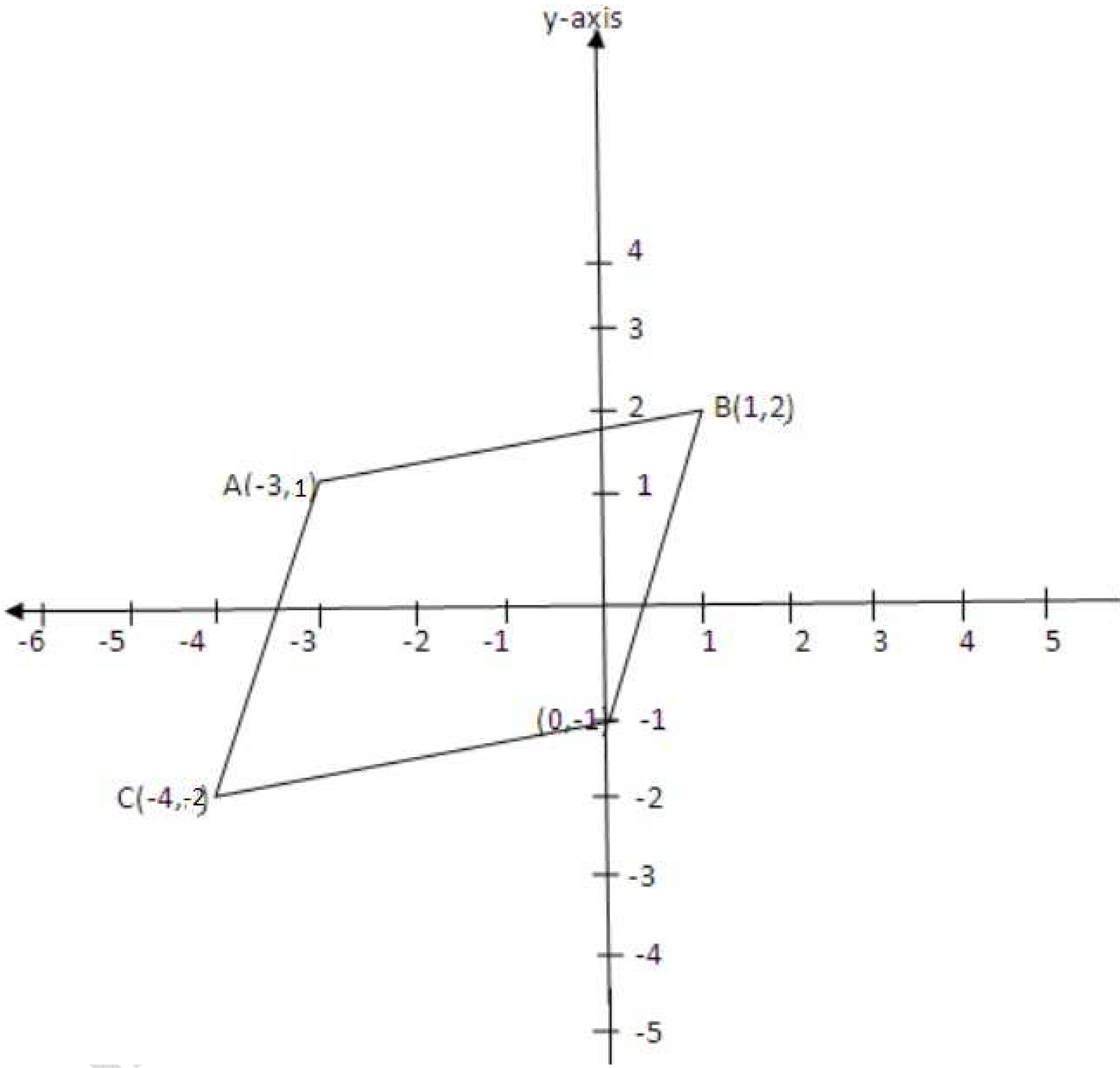
Equation of a straight line

$Y - Y_1 = M (X - X_1)$

Y - 2 = -1/3 (x-6)

Y = -x/3 + 4

3. Show that A (-3, 1), B (1,2) , C(0,-1) and D (-4,-2) are vertices of a parallelograms.

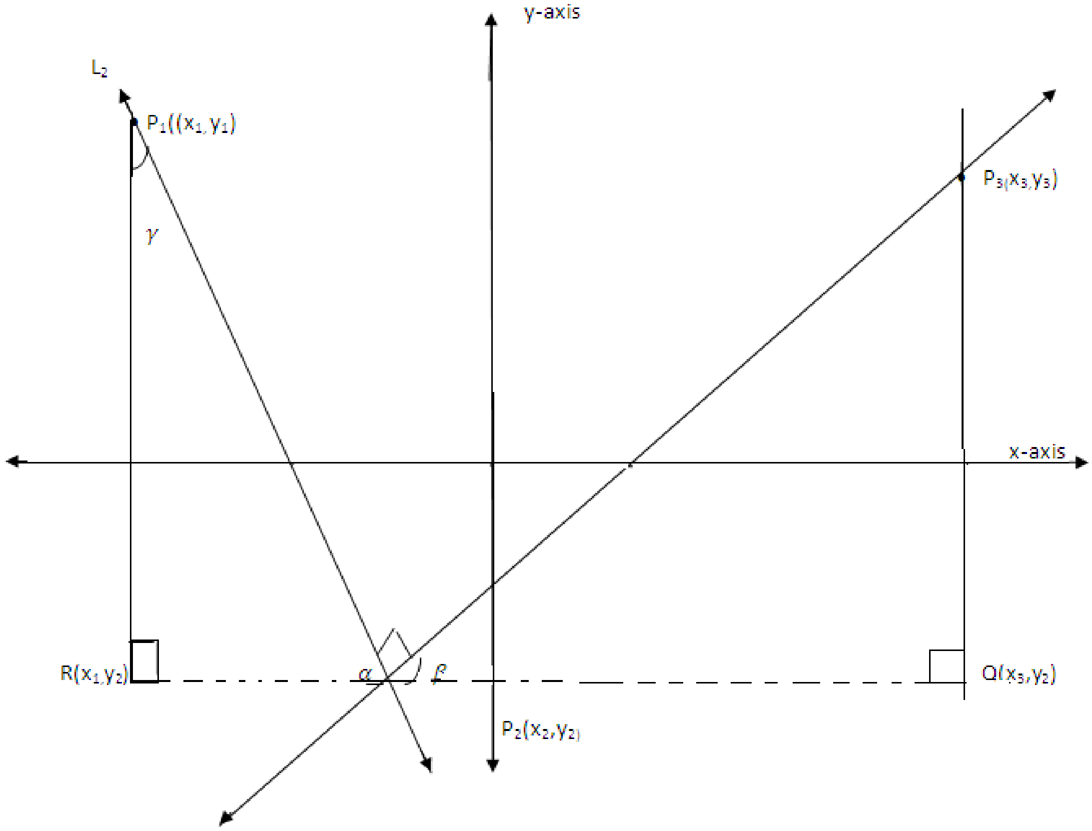


Slope AB = $\frac{2-1}{1--3} = {}^{1}/_{4}$

Slope CD = $\frac{-2--1}{0--4} = {}^{-1}/_{4}$

PERPENDICULAR LINES

Two lines are perpendicular if they intersect at right angle. Suppose that two lines L_1 and L_2 are perpendicular with slopes M_1 and M_2 as shown below.



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Choose Point $P(x_1,y_1)$, $P_2(x_2, y_2)$ $P_3(x_3, y_3)$, R and Q

Also $\propto$, β and γ are the Greek letters Alpha, beta and gamma respectively representing the degree measures of the triangles as indicated. Then

$\alpha + \beta = 90^0$(complementary angles)

$\alpha + \gamma = 90^0$(complementary angles)

$\beta = \gamma$

$\therefore \Delta P_2QP_3 \sim \Delta P_1RP_2$

$\frac{P_2q}{QP_3} = \frac{P_1R}{RP_2}$

$\frac{x_3 - x_2}{y_3 - y_2} = \frac{y_2 - y_1}{x_2 - x_1}$

But the slope of $L_1 = M_1 = \frac{y_3 - y_2}{x_3 - x_1}$

and slope of $L_2 = M_1 = M_1 = = \frac{y_2 - y_1}{x_2 - x_1}$

Therefore $\frac{1}{M_1} = - M_2$ 0r $M_1 M_2 = -1$

If two non-vectorlines are perpendicular with slopes M_1 and M_2,then

$$\boxed{M_1M_2 = -1}$$

Two lines are perpendicular if they intersect at right angles.

If two non vertical lines are perpendicular with slopes M_1 and M_2, then

$M_1 \times M_2 = 1$

Example

1. Find the equation of the line through P (-2 , 5) and perpendicular to the line

   $6X - 7Y = 4$

Solution

y = mx + c

From the equation we get

$Y = (\,^{6}/_{7}\,)X - \,^{4}/_{7}$

$M_1 = \,^{6}/_{7}$

$M_1 \times M_2 = -1$

$(\,^{6}/_{7}) M_2 = -1$

$M_2 = -\,^{7}/_{6}$

Equation $M = -\,^{7}/_{6}$ (-2, 5)

Slope $M_2 = \frac{-7}{6}$

$\frac{y-5}{x+2} \times \frac{-7}{6}$

6y- 30 = -7x-14

6y + 7x = 16

Or 7x + 6y- 16 =0

2. Find the equation of the line through the point (6,2) and perpendicular to the line joining P (3,-1) and Q (-2 ,1)

Solution:

Slope of P and Q = $\frac{1 - -1}{-2-3} = -\,^{2}/_{5}$

$M_1 \times M_2 = -1$

M_2= -1 x -5/2 = 5/2

Equation M = 5/2 (6,2)

$\frac{5}{2} = \frac{y-2}{x-6}$

2(y-2)= 5(x-6)

2y-4 = 5x-30

2y-4 +30-5x =0

2y-5x+26=0

3. Find the equation of a line perpendicular to the equation 3X- 11Y -4 = 0

And passing through (- 3, 8)

Solution:

3X – 11Y – 4 =0

Y =mx + c

-11y = -3X +4

Y= 3/11 X – 4/11

M = 3/11

M_2 = - 11/3

Equation M = - $^{11}/_{3}$ (-3,8)

$$\text{Slope } \frac{-11}{3} = \frac{y-8}{x- -3}$$

$$= -11x-33 = 3y-24$$

$$= 3y-24 +33 +11x$$

$$= 3y+11x +9$$

$$.\ : 3y+ 11x +9 =0$$

4. Show that A (-3 , 2) , B (5 , 6) and C (7 , 2) are vertices of a right angled triangle.

Solution

$$\frac{6-2}{5-3} = \frac{\cancel{4}}{\cancel{8}} \quad = \frac{1}{2}$$

$$\frac{2-6}{7-5} = \frac{-4}{2} \ = -2$$

$$\frac{2-2}{7-3} = \frac{0}{10} = 0$$

Slope of AB x slop of BC = -1

Hence AB is perpendicular to BC

5. Determine which two sides of the following triangles ABC contain a right angle. A(3,2) , B (5,-4) , C (1, -2)

Solution

$$\text{Slope AB} = \frac{-4-2}{5-3} = \frac{-6}{2} = -3$$

Slope BC = $\underline{-2 + 4}$ = $\underline{2}$ = - $\underline{1}$ = $\underline{\ 1\ }$

1-5   -4   2   2

Slope AC = $\frac{-2-2}{1-3} = \frac{-4}{-2} = 2$

Slope of AB x slope of AC = -1

-(1/2) x 2 = -1

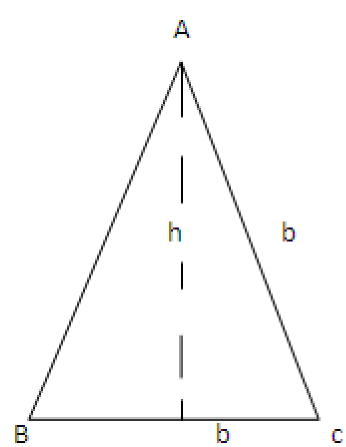
Therefore AB is perpendicular to AC

AREAS AND VOLUMES

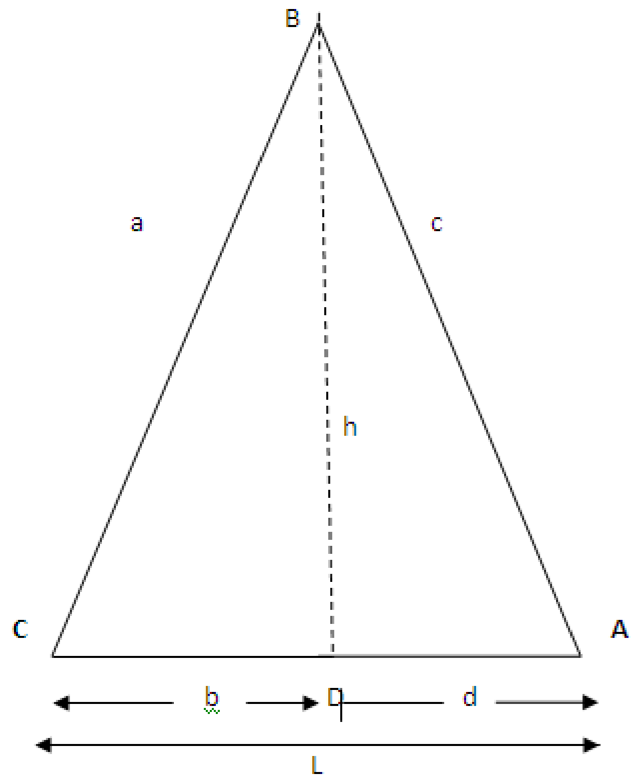
AREAS

CASE

1.Right angled triangle Area = ½ b x h



2. Triangle with altitude that lies within the triangle

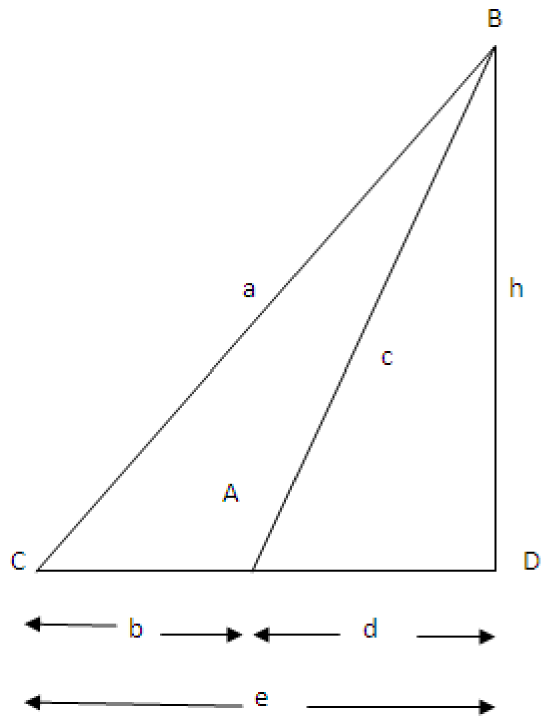


Area 0f Δ ABC = ½ bh + ½ d h

= ½ h(b + d)

= ½ hL

3. A triangle where the altitude of triangle lies outside of the triangle.



Area of ABC = area of triangle BCD – area of triangle ABD

$= \frac{1}{2} h (b+d) - \frac{1}{2} h d$

$= \frac{1}{2} h e - \frac{1}{2} h d$

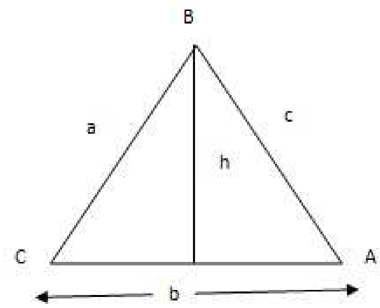
$= \frac{1}{2} h (e-d)$

In all the triangle the formula is the same.

Thus if you were given a triangle with a base b and its corresponding height (altitude) h, its area is equal to $\frac{1}{2}$ b h

CASE II

We can also use the knowledge of trigonometrical ratios.



Area of triangle ABC = ½ b h

= ½ b a sin c

Sin A = h/c

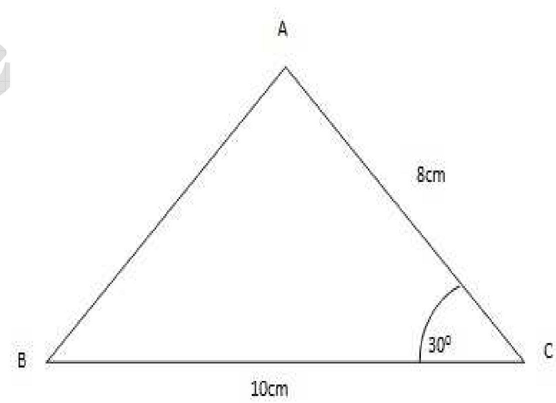
h= c sin A

the area of triangle ABC = ½ b c sin A

<u>Example</u>

1. The length of two sides of a triangle are 8cm and 10 cm. find he area of the triangle, if the included angle is 30 ^{0}.

<u>Solution;</u>

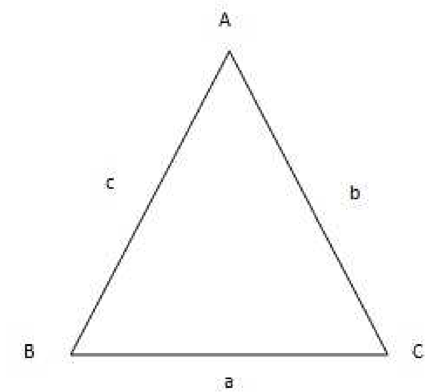


Area = ½ x 10 x 8 x sin 30^0

= 40 x ½Cm^2

=$20cm^2$

2. The area of triangle ABC with sides a,b,c.



Area of triangle ABC

= ½ c b sin A

= ½ a c sin B

= ½ a b sin C

Example

The base of triangle PQR is 17 cm long. If corresponding height is 20cm, find the area of the triangle.

Solution;

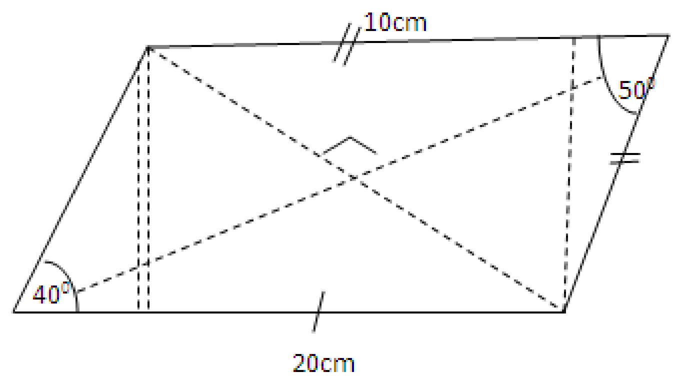
Area of triangle PQR = ½ b h

= ½ x 17 x 20

= 170 cm^2

Qn. 9

what is the area of the paper required to make the kite shown in the figure



Solution

(i) = ½ x 20 x 20 x sin 40º

= 200 x 0.6428

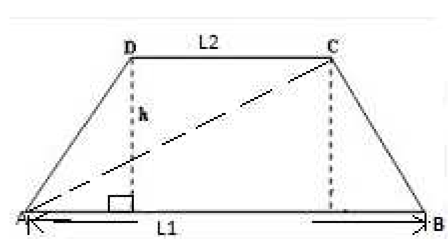
$=128.56cm^2$

(ii)= ½ x 10 x 10 x sin 50^0

= 50 x 0.7660

$= 38.3\ cm^2$

AREA OF TRAPEZIUM



Area of trapezium ABCD = area of triangle ABC + area required of triangle ADC

$= {}^{1}/_{2} L_1 h + {}^{1}/_{2} L_2 h$

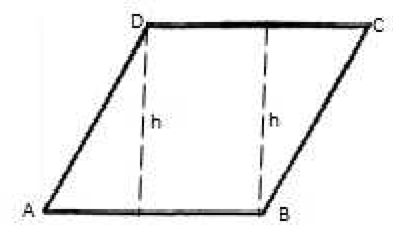
$= {}^{1}/_{2} h(L_1 + L_2)$

Examples.

1. Calculate the height of trapezium with area 84 square units and bases 16 units and 8 units as shown ;

Area = ½ h (b_1+ b_2)
84 = ½ h (16+ 8)
84 = 12 h
h = 7 units.

AREA OF PARALLELOGRAM



Area of parallelogram ABCD = area of ΔABD + ΔBCD

$= \frac{1}{2} AB\,h + \frac{1}{2} CD\,h$

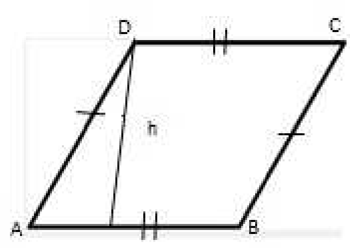
$= \frac{1}{2} h \left(\overline{AB} + \overline{DC} \right)$

$= \frac{1}{2} h \left(2\,\overline{AB} \right)$

$= h \times \overline{AB}$

Area of parallelogram = bh

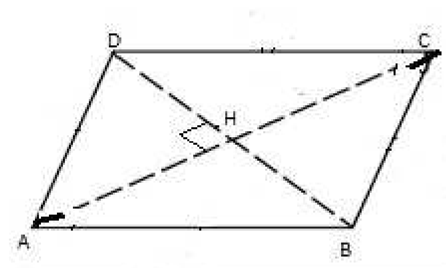
AREA OF RHOMBUS



A rhombus is also a parallelogram.

Area = bh

We can also find the area of the rhombus by considering the diagonals of a rhombus.



AC and DB are the diagonals.

Area of triangle ABC = area of triangle ADC

Area of rhombus ABCD= 2 (area of triangle ABC)

OR = 2 (area of triangle ADC)

Area of triangle ABCD= 2 (area of ΔABH or 2x area BCH)

Area of triangle ABC = $\frac{1}{4}\overline{DB}\times\overline{AC}$

Area of rhombus ABCD = 2(area of triangle BC)

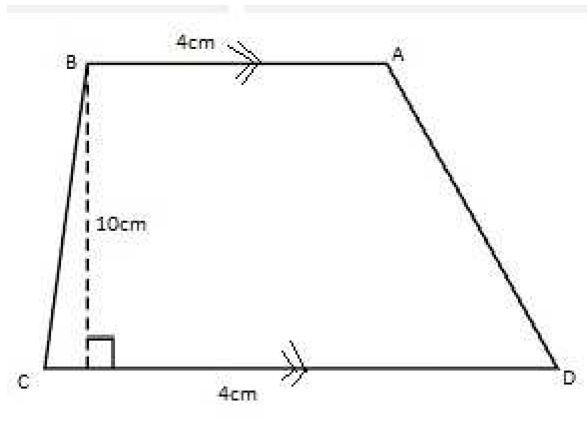
$= 2 \times \frac{1}{4}\overline{BD} \times \overline{AC}$

$= \frac{1}{2} \times \overline{DB} \times \overline{AC}$

EXERCISE

1. 1. Calculate the area of a rhombus whose diagonals are 12dm and 10 dm.

2. 2. Calculate the area of the trapezium ABCD shown in the figure below



Solution

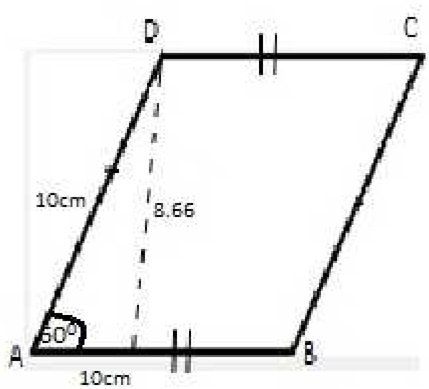
$$Area = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2}x\ 10(6+4)$$

$$\frac{1}{2} \times 10\ \times 10$$

$$= 50\ cm^2$$

3. ABCD is a parallelogram with A= 10cm, BAD = 60^0. Calculate the area of the parallelogram.



Solution

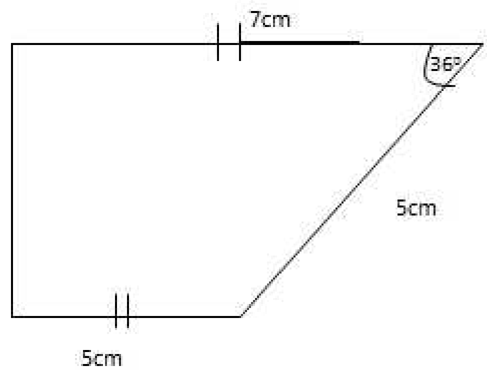
$$\sin 60^0 = opp/10$$

$$\sin 60^0 \times 10 = X$$

$$X = 0.8660 \times 10$$

$$X = 8.66\ cm$$

4. 4. Find the area of trapezium ABCD shown in the figure below;



Sin 36^0 = h/ s

Solution4:

h = 0.5878x 5 cm

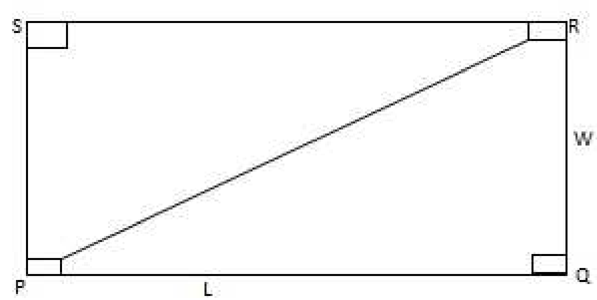
h= 2.939 cm

area = ½ x 2.939 (7+5)

= ½ x 2.939(12)

= 17.634cm^2

AREA OF A RECTANGLE



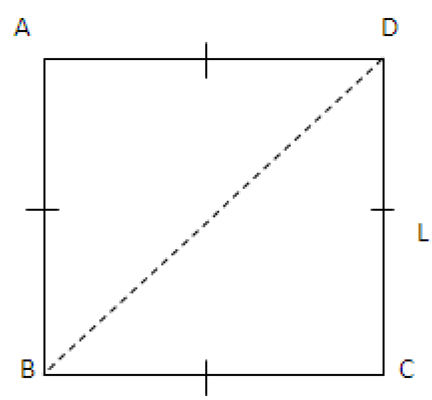
area of rectangle PQRS = area of triangle PQR+ area of PSR

$= \frac{1}{2}\,\overline{PQ} \times \overline{RQ} + \frac{1}{2}\,\overline{SR} \times \overline{PS}$

$= \frac{1}{2} \times L \times W + \frac{1}{2} \times L \times W$

$= LW$

AREA OF SQUARE



A square is a rectangle with equal sides.
Area of triangle ABC = area of triangle ADC

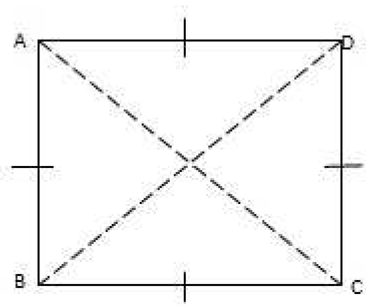
A square is a rectangle with equal sides.
Area of triangle ABC = area of triangle ADC

$= \frac{1}{2} \times \overline{AB} \times \overline{BC} + \frac{1}{2} \times \overline{BC} \times \overline{DC}$

$= \frac{1}{2} L \times L + \frac{1}{2} \times L \times L$

$= L^2$

Also we can find the area of a square by considering the diagonals.



$\Delta ABC = \Delta ADC$

Area of triangle ABC = Area of triangle ADC

Area of square ABCD = 2 (area of ABC or ADC)

Area of triangle ABC $= 2\times(\frac{1}{2}\times\overline{AC}\times\frac{1}{2}\times\overline{DB})$

Area of ABC $= \overline{AC}\times\overline{DB}\times\frac{1}{4}$

Area of square ABCD $= 2\times\frac{1}{4}\times\overline{AC}\times\overline{DB}$

Where $\overline{AC}=\overline{DB}$

Area $=\frac{1}{2}\overline{DB^2}$

Example

1. Find the area of square in which diagonals have length of $12.5cm^2$

Solution

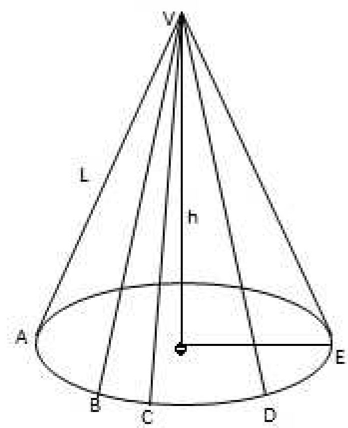
Area = ½ (length of diagonals)^2

= ½ $(12.5)^2$

= $78.125cm^2$

TOTAL SURFACE AREA OF A RIGHT CIRCULAR CONE

Right circular cone Is the one whose vertex is vertically above the center of the base of the cone.



Total surface area of a cone = area of curved surface + base area.

BUT;

area of curved surface (lateral surface) = area of small triangles.

If we consider our cone , AB, BC , CD and DC are approximated line segments, hence we have small triangles VAB,VBC , VCD and VDE.

Hence area of curved surface

= ½ AB x VA + ½ BC x VC + ½ CD x VC + ½ DEx VD

But VA= VB = VC = VD = VE

Area of curved surface = ½ AxBxL+ ½ BxCxL+ ½ xCxDxL+ ½ DxExL

= ½ L (AB+BC+CD+DE)

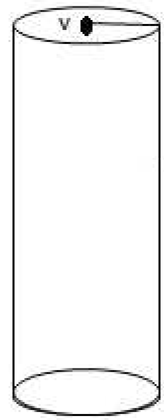
= ½ L ($2\pi R$)

= πRL

Total surface area = $\pi R^2 + \pi RL$

= $\pi R(R+L)$

<u>TOTAL SURFACE AREA OF A RIGHT CYLINDER</u>



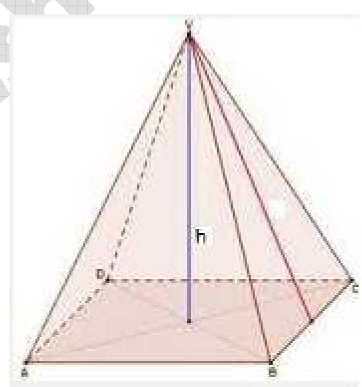
Total surface area of a right cylinder

= area of curved surface+ bases area.

$= 2\pi Rh + \pi R^2$

TOTAL SURFACE AREA OF A RIGHT PYRAMID

A right pyramid is the one which the slant edges joining the vertex to the corner of the base are equal.



Total surface area of a right pyramid

=area of triangle VAB+ VBC+VDC+VDA + area of the base.

= lateral surface + area of the base.

BUT

As VAB, VDC, VBC and VDA are isosceles triangles. Then VA,VB,VC and VD are slant height.

Example

Consider the data below of a right pyramid. Find the total surface area of the pyramid.

Total surface area of a pyramid

= area of laterals + base area

=area of $\Delta VAB + \Delta VBC + \Delta VDC + \Delta VDA +$ base area

$(VA^2) = (AK^2) + (KV^2)$
$(12^2) = (6^2) + (KV^2)$
$(KV^2) = 144 - 36$
$KV = 6\sqrt{8}$

$(VC^2) = (KC^2) + (VK^2)$
$12^2 = 4^2 + (VK^2)$
$VK = 8\sqrt{2}$

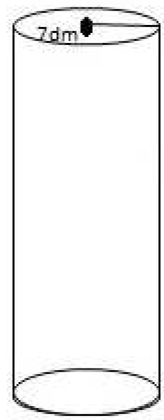
$h_1 = \sqrt{108} = 6\sqrt{3}$
$h_2 = \sqrt{128} = 8\sqrt{2}$

area of $\triangle VAB = \frac{1}{2} \times 12 \times 6\sqrt{3}$
$= 72\sqrt{3}$
Area of $\triangle BCV = \frac{1}{2} \times 8 \times 8\sqrt{2}$
$= 64\sqrt{2}$
Base area $= 12 \times 8 = 96 cm^2$

EXERCISE

1. The radius of a base of right circular cylinder is 7dm and height is 10 dm. find;
(a). The total surface area.

Solution:

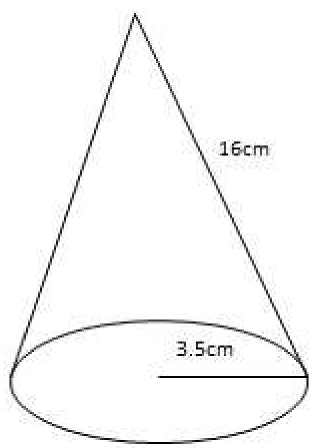


The total surface area = $2\pi R$ (h+r)

= 2 x 3.14 x 7 dm (10+7)

= 74.732 dm^2

2. Calculate the lateral surface area of the right cone shown below.



= $\pi r(r+L)$

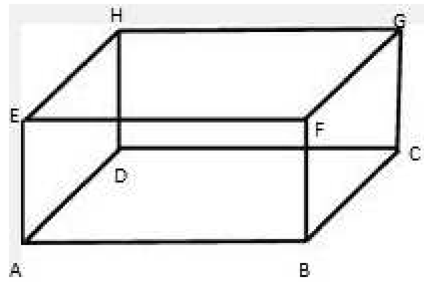
= 3.14x3.5x(3.5+1.6)

= 3.14x3.5x19.5

= 214.305cm^2

<u>THE TOTAL SURFACE AREA OF A RIGHT PRISM</u>

A right prism is a prism in which each of the vertical edges is perpendicular to the plane of the base. an example of right prism is shown in the figure below where EABF, FBSG, HDCG and EADH are faces made up the lateral surface. and ABCD and EFGH are bases.



A right prism is a prism in which each of the vertical edges is perpendicular to the plane of the base. an example of right prism is shown in the figure below where EABF, FBSG, HDCG and EADH are faces made up the lateral surface. and EFGH are bases.

The total surface area of a prism ABCDEFG

= Area of lateral surface + base area

$$(\overline{AB} \times \overline{BF}) + (\overline{BC} \times \overline{CG}) + (\overline{DC} \times \overline{DH}) + (\overline{AD} \times \overline{AE})$$

But $\overline{BF} = \overline{CG} = \overline{DH} = \overline{AE}$ -these are heights of our prisms.

Total surface area $= BF(\overline{AB}+\overline{BC}+\overline{DC}+\overline{AD})$

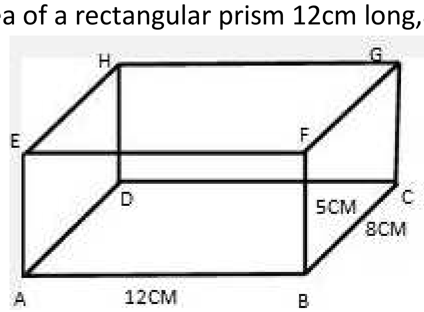
$= BF(\text{perimeter})$

Bases area $= 2(\overline{AB} \text{ x } BC)$ or $2(\overline{EF} \text{ X } \overline{FG})$

Total surface area $= \overline{BF}$ (perimeter) + 2(area of one base)

Example

Find the total surface area of a rectangular prism 12cm long,8 cm wide, and 5 cm high.



soln

Surface Area = BF(ABxBC)2

=5(12 x8) 2

= 5cm x 192cm

=960 cm^2

Base area = 12x8 x2

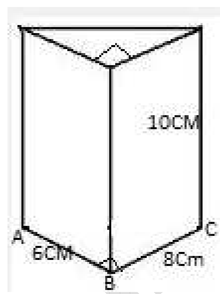
= 192 cm^2 . : Total surface area = 240 cm^2 +192 cm^2 = 432 cm^2

Exercise

1. The altitude of a rectangular prism is 4cm and the width and length of its base are 12cm and 3 cm respectively. Calculate the total surface area of the prism.

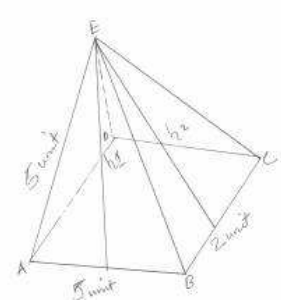
2. One side of a cube is 4dm. calculate
   a. The lateral surface area.
   b. Total surface area.

3. Figure below shows a right triangular prism whose base is a right angles triangle. Calculate its total surface area.



4. The altitude of a square pyramid is 5units long and a side of the base is 5 units long. Find the area of a horizontal cross-section at distance 2 units above the base.

Solution



Answers

Solution1(a)

= 4(2+3) 2 +2(2x3)

= 4x10 + 12

$= 40 + 12$

$= 53cm^2$

Solution2.

(a) Lateral area $= 2\ (4+4+4+4)$

$=2x\ 16$

$=32dm^2$

(b)Total surface area $= 32 + 2(4+4)$

$= 48\ dm^2$

Solution3.

$=(AB^2) + (BC^2) = (\ AC^2)$

$=6^2 + 8^2 = AC^2$

$AC=10$

$= 8x10cm^2= 80\ cm^2$

Area of triangle = ½ b h

= ½ x 6 x 8 x 2

$= 48cm^2$

Area of rectangle b = 10cmx10cm

$=100cm^2$

Total surface area = $100cm^2 + 48\ cm^2 + 80cm^2$

$=228cm^2$

Solution4.

$a^2+b^2=c^2$

$2.5^2 + b^2 = 5^2$

$b^2 = 25-6.25$

$b = 4.33 = h$

$a^2+b^2=c^2$

$a^2=251$

$a = \sqrt{24}$

$A_1 = (½ \times 5 \times 4.33) \times 2$

$=21.65cm^2$

$A_2 = \ \times 2 \times 4.89 \times 2$

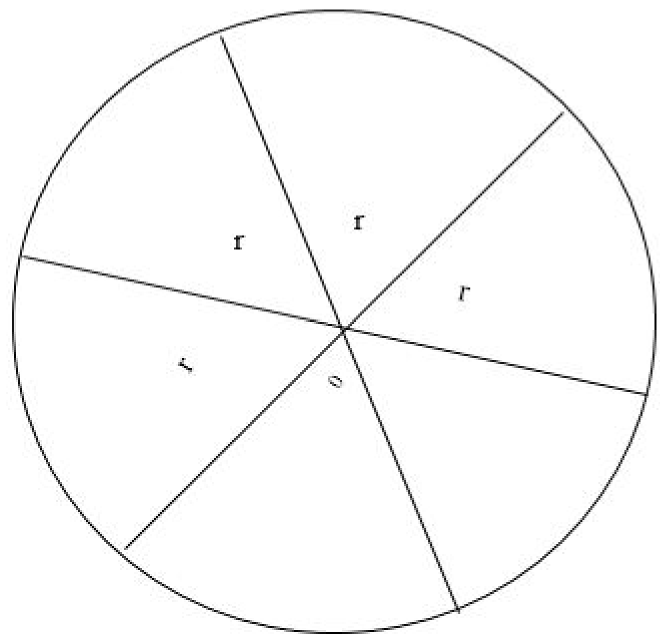
$A_2 = 9.798$

Area = $9.798+21.65\ cm^2$

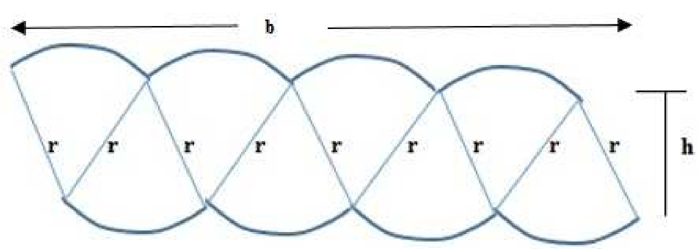
$=31.448cm^2$

<u>AREA OF A CIRCLE</u>

Consider a circle with several radii (r).



Re-arrange those pieces from a circle to form a parallelogram.



$$A = bh$$

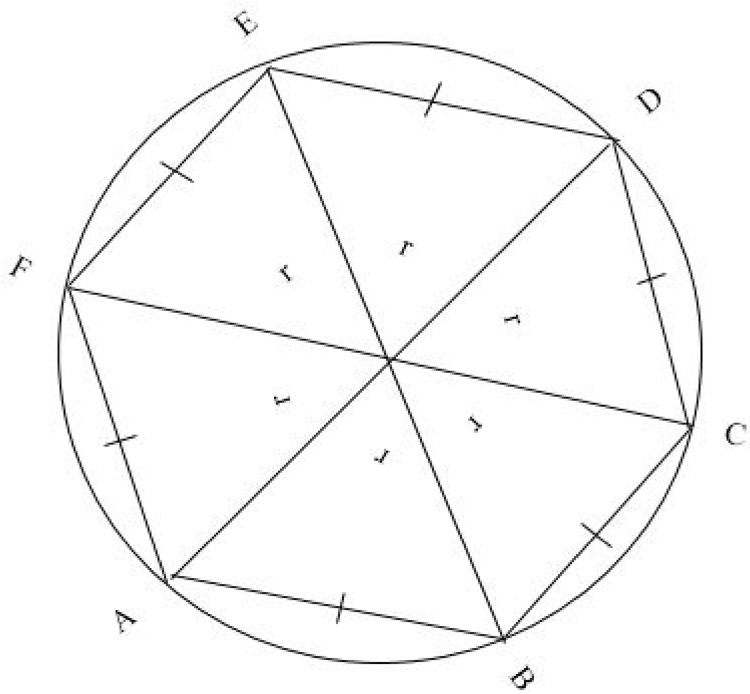
$$= \tfrac{1}{2}\, c\, r$$

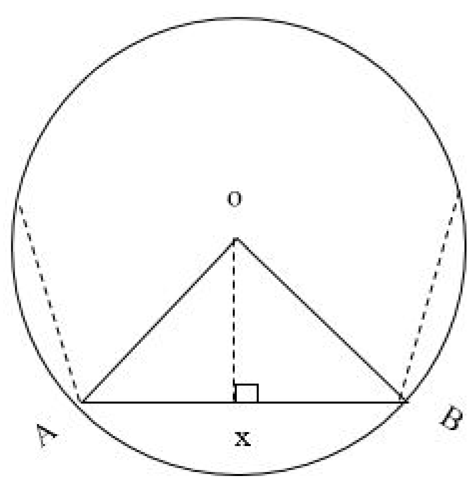
$$\tfrac{1}{2}\, c\, r = 2\pi r$$

Area of a circle = πr^2

Area of a sphere = $4\,\pi r^2$

<u>LENGTH AND PERIMETER OF A RECTANGULAR POLYGON INSCRIBED IN A CIRCLE.</u>





Consider triangle $\hat{AOB}$

AO is perpendicular to AB

But AOB = 360/n

$A\hat{O}X = B\hat{O}X$

$A\hat{O}X + B\hat{O}X = A\hat{O}B$

Hence $A\hat{O}X = \frac{1}{2} A\hat{O}B = \frac{1}{2}(360^0/n) = 180^0/n$

$\overline{AB}$ is one of the sides of our regular polygons inscribed in a circle.

$\overline{AB} = S$

$\overline{AX} = \frac{1}{2}\overline{AB} = \frac{1}{2}S$

Consider AOX

$\sin A\hat{O}X = \frac{\overline{AX}}{\overline{AO}}$

$\sin 180^0/n = \frac{(1/2)S}{r}$

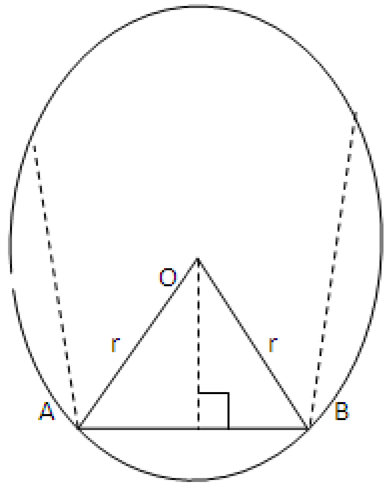
$s = \frac{2r\sin 180}{n}$

$s = \frac{d \sin 180^\circ}{n}$

p=ns

p= n(dsin180Ëš)

AREA OF A RECTANGULAR POLYGON INSCRIBED IN A CIRCLE



Area of a $\Delta AOB = \frac{1}{2}\ \overline{AOX}\ \overline{BOX}\ \sin Y$

$AOB = \frac{1}{2} r \times r \frac{\sin 360}{n}$

Area of regular polygon inscribed in a circle.= $n\left(\frac{1}{2} r^2 \frac{\sin 360}{n}\right)$

Exercise

1. Find the length of one side of a regular nine –sided polygon inscribed in a circle of radius 10 cm^2.

2. Find the radius of a circle which inscribes an equilateral triangle with perimeter 24 cm.

3. Find the area of a 9-sided polygon inscribed in a circle with radius 5 cm.

4. Find the area between two concentric circles.

Answers

Solution 1.

$$S = d \sin \frac{180^0}{n}$$

$$= 20 \frac{\sin 180^0}{9}$$

$$= 20 \sin 20^0$$

$$= 20 \times 0.3420$$

$$= 6.84cm$$

Solution2.

$$P = nd \sin \frac{180^0}{n}$$

$$24cm = 3d \frac{\sin 180^0}{3}$$

$$24 \text{ cm} = 3d \sin 60^0$$

$$24 = 3d\ 0.8660$$

$$D = 9.237cm$$

$$R = 4.\ 6185cm$$

Solution3.

$$\text{Area} = {}^1\!/_2\, n\, r^2 \sin \frac{360}{n}$$

$= ½ \times 9 \times 25 \times \sin 40^0$

$= 11.25 \times 0.6428$

$= 72.315cm^2$

Solution4.

Area = πr^2

$= 3.14 \times 6 \times 6$

$= 113.04\ cm^2$

Area = πr^2

$= 3.14 \times 4 \times 4$

$= 50.24cm^2$

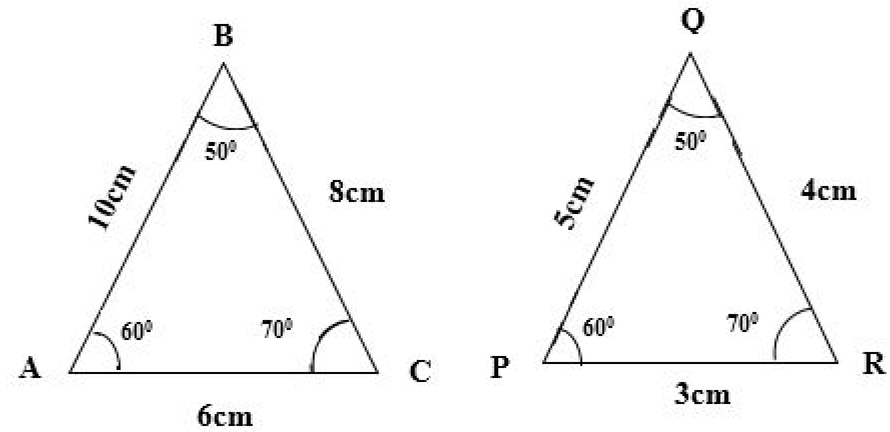
Area between circles = $113.04cm^2 - 50.24cm^2$

$= 62.80cm^2$

AREAS OF SIMILAR FIGURES

Similarity

Two polygons are similar when their corresponding angles are equal and corresponding sides are proportional.



Similarity of polygons

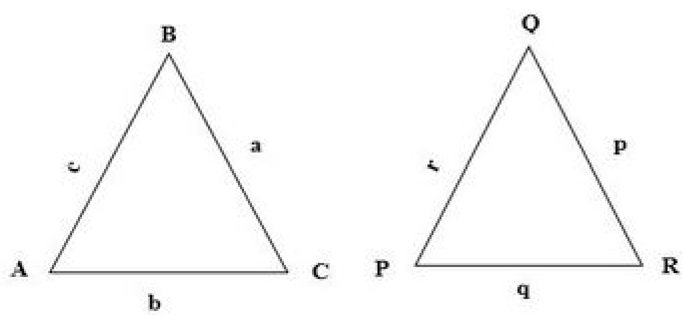
If corresponding angles are equal, also if the corresponding side are proportional.

$$A\hat{B}C = P\hat{Q}R = 50^0$$

$$B\hat{A}C = Q\hat{P}R = 60^0$$

$$A\hat{C}B = P\hat{R}Q = 70^0$$

$$\frac{\overline{AB}}{\overline{PQ}} = \frac{10cm}{5cm} = 2$$



Area of triangle ABC = ½x a x c sin B

Area of triangle PQR = ½ r p sin Q

$$\frac{\text{Area of triangle ABC}}{\text{Area of triangle PQR}} = \frac{\frac{1}{2}\, ac \sin B}{\frac{1}{2}\, r p \sin Q}$$

But Sin B = sin Q

$$\frac{\text{Area of triangle ABC}}{\text{Area of triangle PQR}} = \frac{\overline{AC}}{\overline{PR}} = K^2$$

Exercise.

1. Two triangles are similar, A side is 6cm long. The corresponding side to the other is 20cm. if the area of the first is 90 cm^2 .what is the area of the second?

2. The ratios of the areas of two circles is 50: 72. If the radius of the smaller circle is 15 cm, find the radius of the larger circle

3. Two triangles are similar. A side of one is 2 units long .the corresponding side of the other is 5 units long. What is the ratio of their areas?

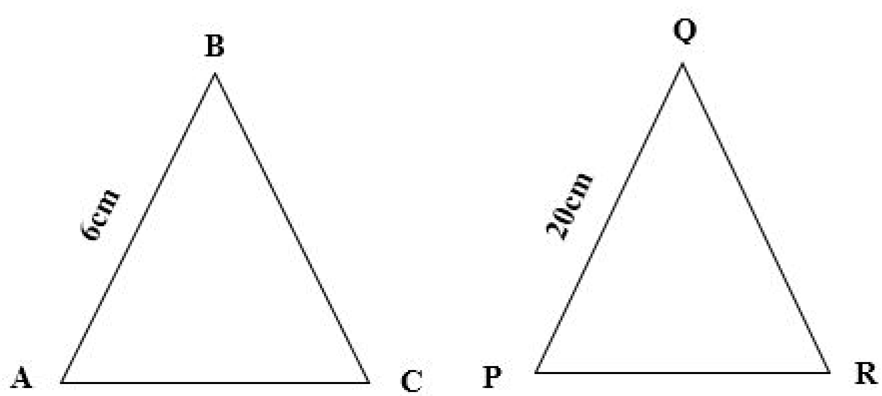
4. Two triangles are similar. The ratio of their areas is $^{25}/_{9}$. What is the ratio of their corresponding sides?

5. Two polygons are similar. A side of one is 8 cm long .the corresponding side of the other is18 cm . the area of the first is $16cm^2$. Find the area of the second.

6. The ratio of the area of two circles is 50: 72. If the radius of the smaller circle is 15 cm, find the radius of the larger circle.

Answers

Solution1.



Area of triangle ABC = K^2

Area of triangle PQR

Where K = 6/20

$$\frac{90cm}{x} = \frac{3^2}{10}$$

$X=100cm^2$

$$\frac{90cm}{X} = \frac{3}{10}^2$$

Solution 2

$$\frac{\text{Area of small circle}}{\text{Area of large circle}} = K^2$$

Where K = 15/X

$$\frac{50}{72} = \frac{15^2}{X}$$

$X = \sqrt{324}$

X = 18cm

Solution 3.

$$\frac{\text{Area of triangle 1}}{\text{Area of triangle 2}} = K^2$$

$= (^2/_5)^2$

$= ^4/_{25}$

Areas = 4:25

Solution4.

$^{25}/_9 = K^2$

$K = ^5/_3$

The ratio of corresponding sides = 5:4.

Solution5.

$$\frac{\text{Area of triangle 1}}{\text{Area of triangle 2}} = K^2$$

$$\frac{16cm^2}{x} = \left[\frac{8}{18}\right]^2$$

$$x = 81\ cm^2$$

Solution6.

$\frac{\text{Area of small circle}}{\text{Area of big circle}} = K^2$

$$\frac{50}{72} = \left[\frac{15}{R}\right]^2$$

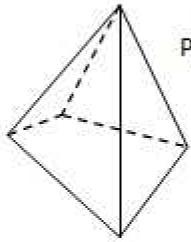
$$R = 18cm.$$

THREE-DEMENSIONAL FIGURES

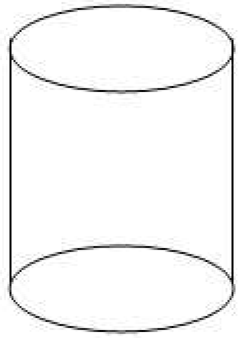
INTRODUCTION

A three dimensional figure is a solid figure having three measures.

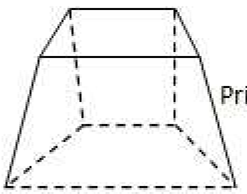
Some examples are pyramids, cylinders, prisms, cubes and cuboids.



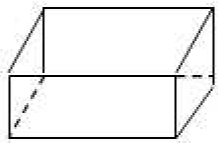
Pyramid



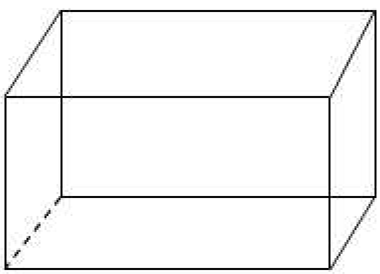
Cylinder



Prism



cuboid



Cube

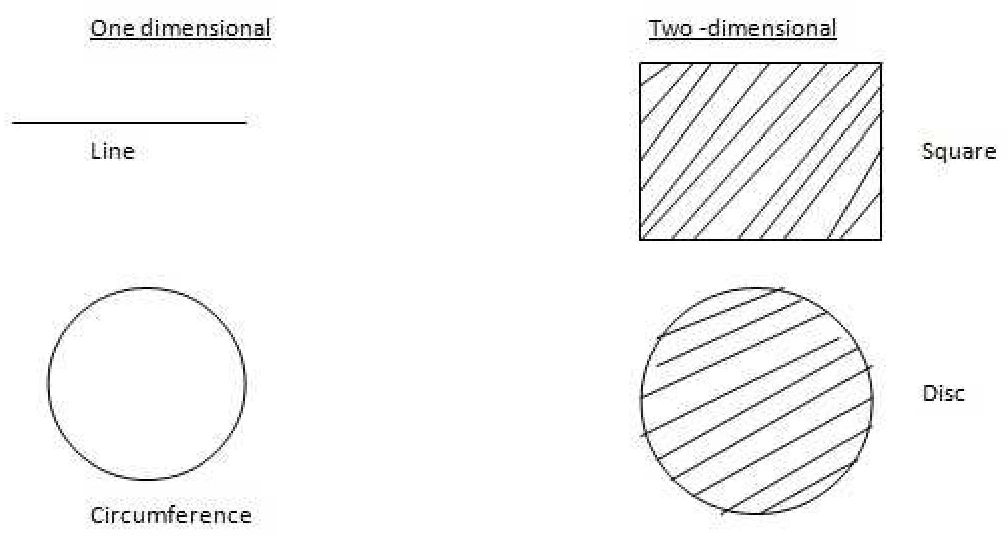
- Apart from three dimensional figures, there are also one and two dimensional figures.

Examples

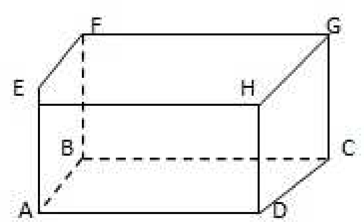
1. A line is one – dimensional. There is one direction to move along it. (up or down it).

2. A flat shape is two- dimensional. There are two directions to move across it (up or down, and left or right).

The diagram below show objects which are one or two –dimensional.



-The –dimensional space contains line and planes. Consider a diagram of a cube:



AB—is a line and

ABCD is a plane

Note:

When we refer to a cube or cuboid ABCDEFGH, then ABCD and EFGH are apposite faces, with, E opposite A, F opposite B and so on , as in the diagram above

- Line and planes can be parallel or perpendicular. In the diagram above:

  - The lines AD and EH are parallel.
  - The lines AD and AE are perpendicular.
  - The planes ABCD and EFGH are parallel
  - The planes ABCD and AEHD are perpendicular
  - The lines AD and CE are neither parallel nor perpendicular and the planes ABCD and EHCB are neither parallel nor perpendicular

EXERCISE 3.1A

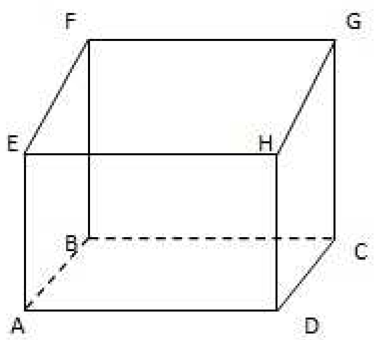
The diagram show a cube ABCDEFGH. The face ABCD is horizontal. Give

a) (a) Another horizontal face

b) (b)A vertical face

c) (c)A horizontal line

d) (d)A vertical line



2. Refer to the diagram for question. Write down

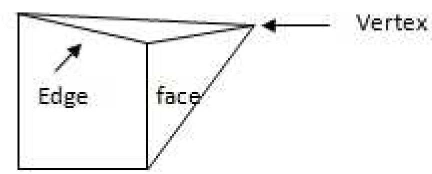
a) Two faces which meet at GC

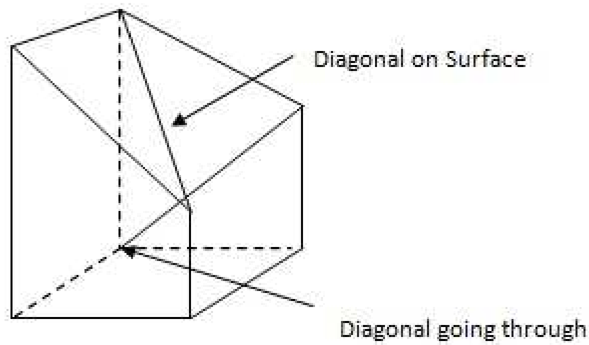
b) Three edge which meet at F

<u>CLASSIFYING THREE-DIMENSIONAL FIGURES</u>

<u>Definitions</u>:

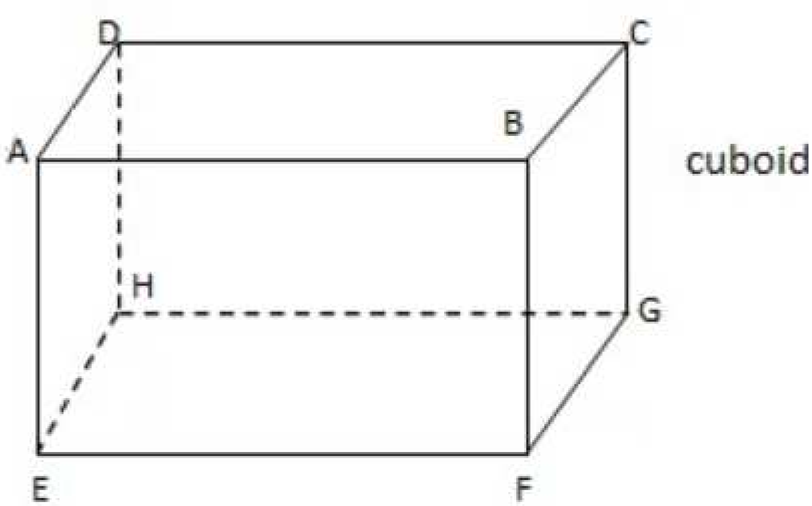
1. 1) A polyhedron is a solid whose surface consists of flat polygons.
2. 2)A face is a flat surface of a solid.
3. 3)An edge is a line where two faces meet.
4. 4)A vertex is a point where three or more faces meet.
5. 5)A diagonal is a line joining two vertices which is not an edge. A diagonal may go through the solid or be an its surface.





CUBOID

A cuboid is a box shape with faces that are rectangles. The diagram show a cuboid ABCDEFGH. All are rectangles that are parallel or perpendicular to each other.



Faces: 6

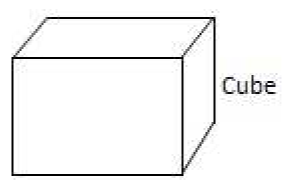
Edge: 12

Vertices: 8

-AG and BG are diagonals. AG goes through the solid, and GB lies on its surface.

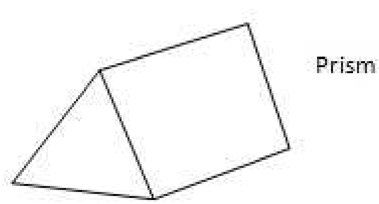
CUBE

-A cube is a cuboid with all edges the same length. All the faces are squares.



PRISM

-A prism is a solid which has the same cross- section through. A triangular prism has a triangular cross-section



Faces: 5

Edges: 9

Vertices: 6

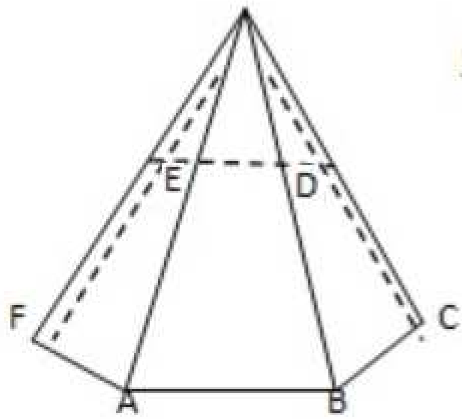
PYRAMID

A pyramid has a flat base in the shape of a polygons and all other edges come to a point called the vertex

TYPES OF PYRAMID

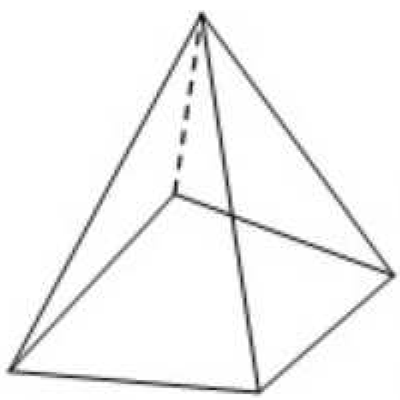
1.

1. A hexagonal base pyramid



A hexagonal base pyramid ABCDEF and vertex V.

2. A square based pyramid

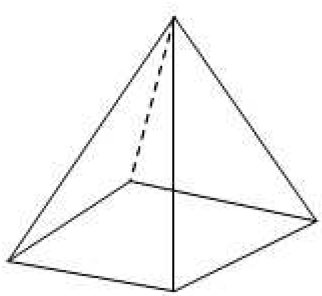


Faces: 5

Edges: 8

Vertices: 5

-If the vertex of the pyramid is above the centre of the base it is a right pyramid.



EXERCISE 3.1B

1. 1) Find everyday objects in the shape of

(a) a cuboid (b) a cube (c) a prism

2. 2)What are the mathematical names for the shapes of these objects?

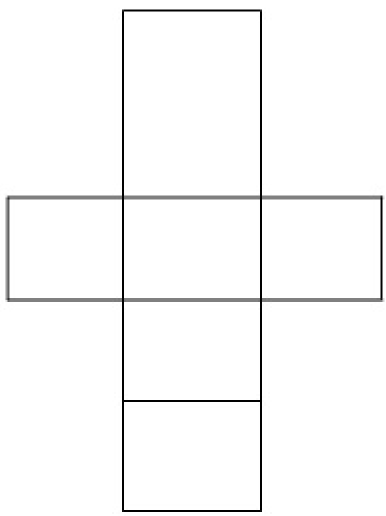
(a) a match box (b) a football (c) an unsharpened pencil (d) the tip of a sharpened pencil.

CONSTRUCTION OF THREE-DIMENSIONAL FIGURES

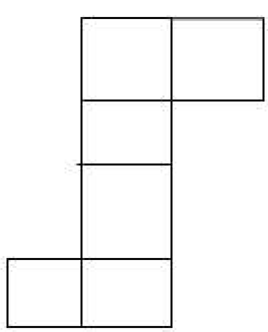
-You can make a model of a solid out of paper. The shape that you cut out is a net. The diagram show a net for a cube.

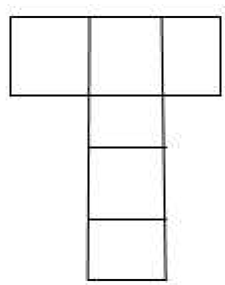
Notice that:

There are six squares for the six faces of the cube



-There are many possible different nets for the same object. The nets below will also make a cube.

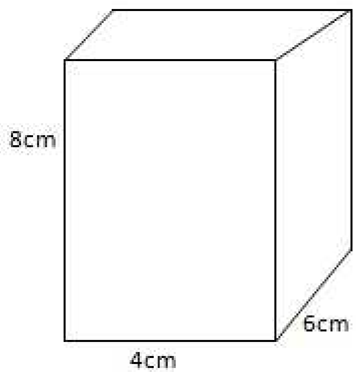




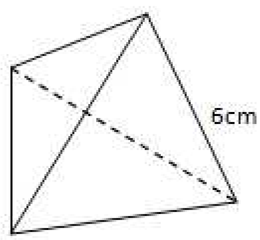
EXERCISE 3.2.A

Make nets for solids and construct the shapes

a) A cuboid which is 4cm by 6cm 8cm

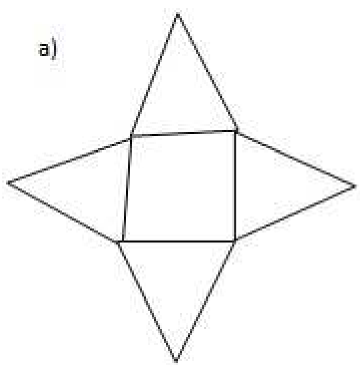


b) A tetrahedron with all sides 6cm

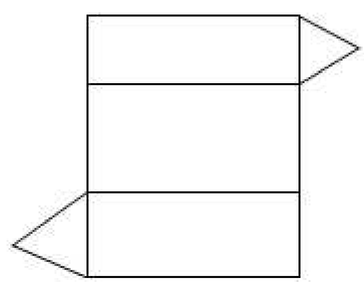


2)What sort of solid will be made from each of these net?

a)



b)

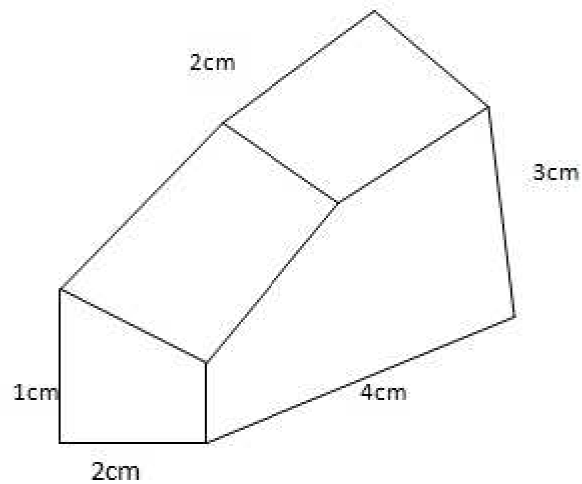


<u>SKETCHING THREE-DIMENSIONAL FIGURES</u>

-We can represent a three-dimensional object in two dimensional object in two dimensional.

<u>I. By the use of oblique projection.</u>

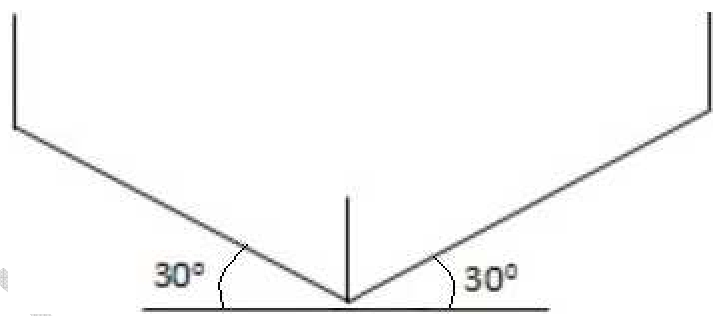
Look at the solid shown below:



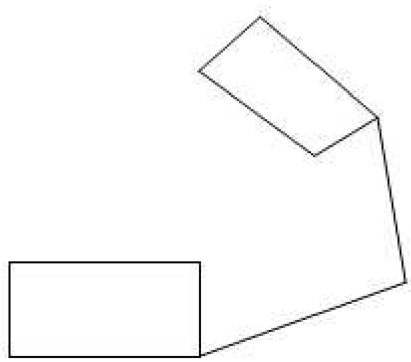
-It is not drawn to scale.

-The following steps will give an accurate representation

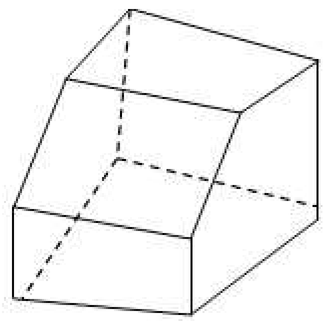
i. Draw the base lines first at 30^0 to the horizontal. Then add the vertical lines. There are all parallel.



ii. Now add any top edges, noting that all edges that are parallel will still be parallel in the oblique projection.

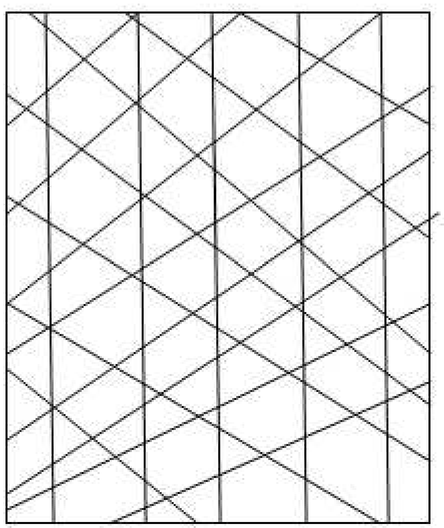


iii. You can now draw the sloping edges, together with any hidden edges that are drawn as dotted lines. The find diagram is now shown.

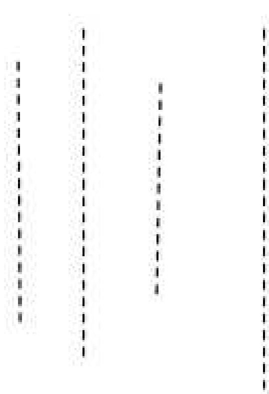


II. By the use of Isometric paper

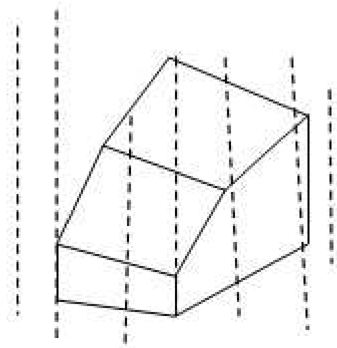
-There is special paper to make drawing solids easier. This is Isometric paper. On this paper there are lines at 60^0 to each other these lines to draw solids.



-Sometimes this paper has dots instead of lines

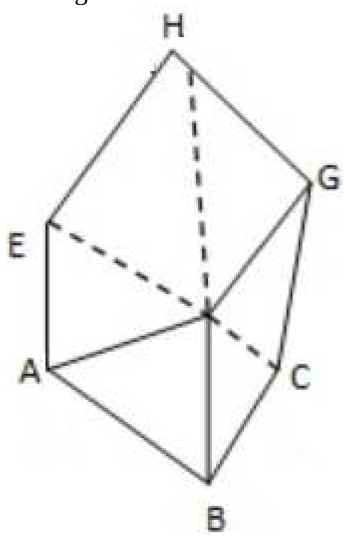


- - On a special grid the shape above looks like the diagram below



Exercise 3.3A

1. 1) The diagram show a cube ABCDEFGH drawn in oblique projection



Write down

a) a)Two faces.

b) b)Two edges.

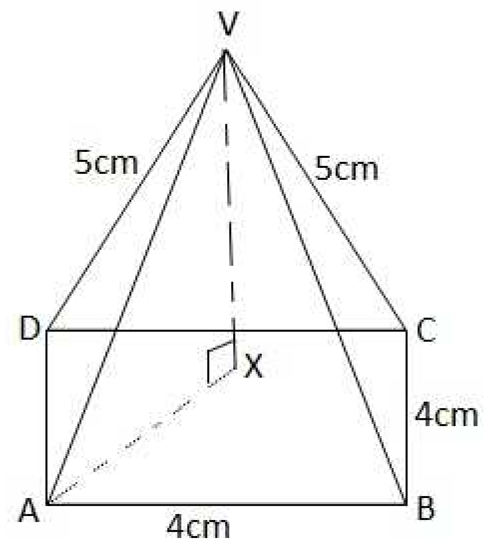
c) c) Two diagonals that pass through the shape.

d) d)Two diagonals that lie on the surface of the shape.

ANGLE BETWEEN LINE AND PLANE

Example

VABCD is a pyramid, with VA= VB=VC=VD=5cm and ABCD a square of side 4cm. Find the angle between VA and ABCD.



<u>Solution</u>

-Drop a perpendicular from V to ABCD. This meets ABCD at x, the centre of the square. So the projection of VA on ABCD is AX . The angle we want is <VAX.

By Pythagoras theorem in ΔABC

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{4^2 + 4^2} = \sqrt{32}$$

$$AX = \frac{1}{2}AC = \frac{1}{2}\sqrt{32}$$

The cosine of <VAX = $\frac{AX}{VA}$

$$= \frac{\frac{1}{2}\sqrt{32}}{5}$$

= 0.5657

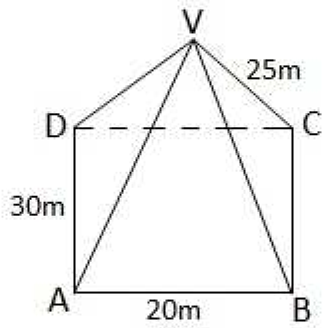
Therefore <VAX s 55.6º

The angle between $\overline{VA}$ and ABCD is 55.6º

<u>EXERCISE 3.3B</u>

1. 1) ABCDEFGH is a cube with a side of 4cm. Find the angle between the line AG and the face ABCD

2. 2) VABCD is a pyramid with a rectangular base ABCD. AB=20M, AD=30M and VA=VB=VC=VD=25M. Find the angle between VA and ABCD



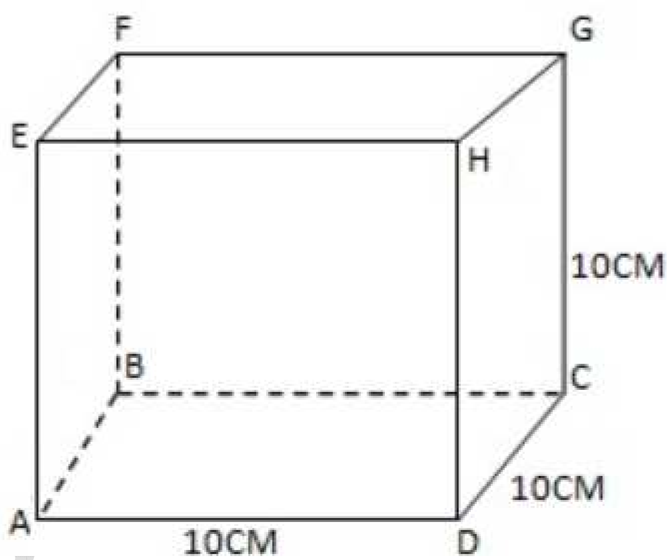
ANGLE BETWEEN TWO PLANES

Example

The diagram show a cube ABCDEFGH of side 10cm. Find the angles between

a) a) ABCD and ABGH F G

b) b) FHA and FHDB



Solution

A) a)The planes meet in the line AB.

-AD is a line in ABCD which is perpendicular to AB.

-AH is a line in ABGH which is perpendicular to AB. So the angle we want is the angle between AD and AH Which is < DAH.

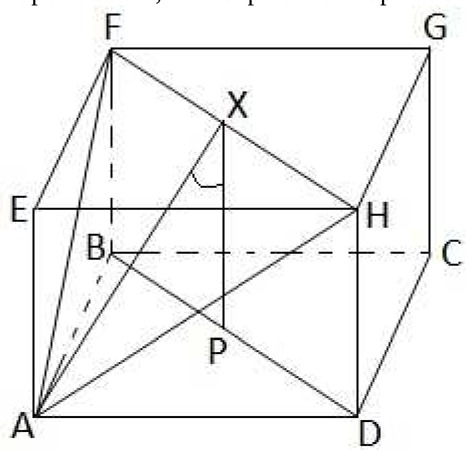
$\text{Tan} < DAH = \frac{HD}{AD} = \frac{10}{10} = 1$

$< DAH = \tan^{-1} 1 = 45^0$

$\therefore$The angle between ABCD and ABGH is 45^0

b). The planes meet in the line FH.

Let x be the midpoint of FH, and let p be the midpoint of BD



-Then XA is a line in FHA which is perpendicular to FH.XP is a line FHDB which is perpendicular to FH

-So the angle we want is the angle between XA and XP, which is <AXP.

-XP is the height of the cube: which is 10cm.

-AP is half the diagonal of ABCD ie

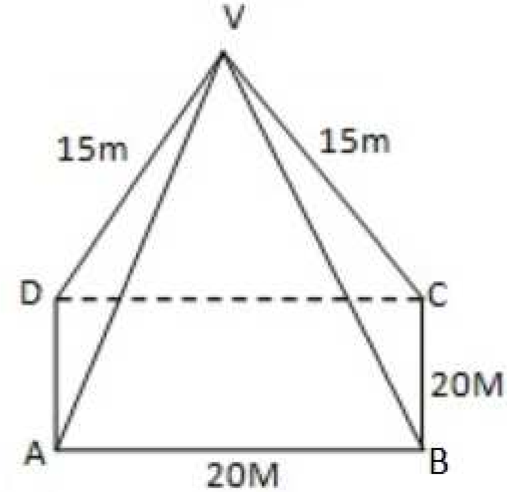
$$\frac{1}{2}\sqrt{10^2+10^2} = \frac{\frac{1}{2}\sqrt{200}}{10} = 0.7071$$

$< AXP = {}^{-1} 0.7071 = 35.3^0$

The angle between FHA and FHDB is 35.3^0

EXERCISE 3.3C

1. The diagram show a pyramid VABCD in which ABCD is a square of side 20m, and VA= VB=VC=VD=15m



Find the angles between the planes

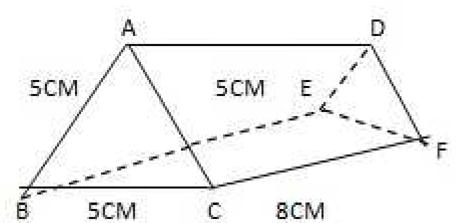
a) VAB and ABCD

b) VAB and VCD

2. 2) A prism has length 8cm, and its cross- section is an equilateral triangle of side 5cm. Find angles between the planes.

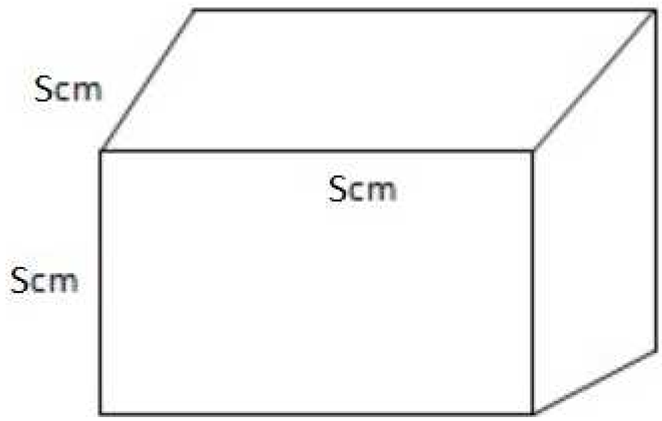
   a) ADEB and BEFC

   b) AEF ad BEFC



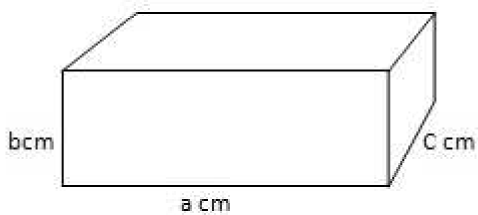
SURFACE AREA OF THREE-DIMENSIONAL FIGURES

Consider the diagram which shows a cube of 5cm . It has 6 faces, which are squares of side 5cm. So the total surface area is



$= Scm \times Scm \times Scm = 6S^2cm^2$

2. Consider the diagram which show a cuboid which is a cm by b cm by c cm. It has 6 faces which are rectangles: two are a cm by b cm, two are b cm by c cm and two are c cm by a cm. so its total surface area is



=(a cm × b cm)×2+ (b cm × c cm) x 2 + (a cm× c cm)×2

=2ab cm2 + 2bc cm2+2ac cm2

$=(2ab+2bc+2ac)cm^2$

Example 1:

Find the surface area of a cuboid which is 12cm by 10cm by 8cm.

Solution:

Given:

a = 12cm, b = 10cm and c = 8cm

Formula = $(2ab+2bc)cm^2$

=(2×12×10)+(2×10×8)+(2×12×8)

= 240 + 160 = 192

= 592

The surface area is $592cm^2$

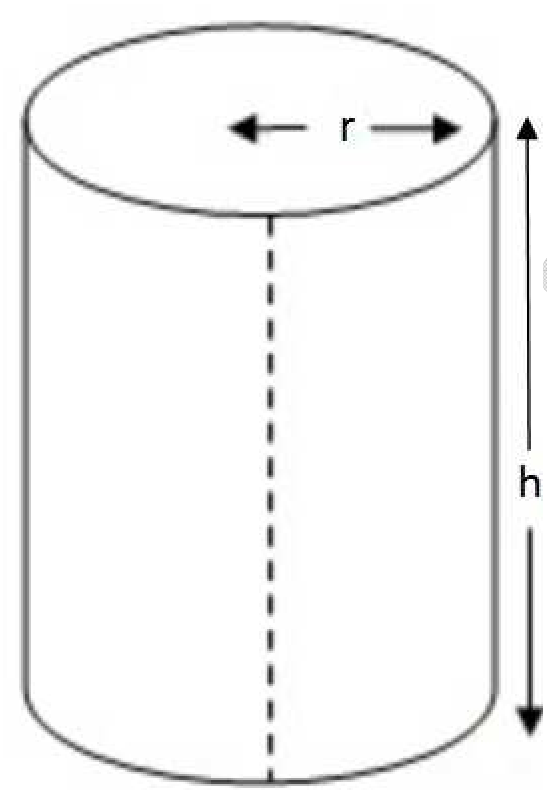
EXERCISE 3.4A

1. 1. A tea crate has a square base of side 0.8m, and its height is 1.1m. Find the surface area of the crate.

2. 2. A room is 5.2m long, 2.5m high and 4.5m wide. Find the surface area of the wall and the wiling.

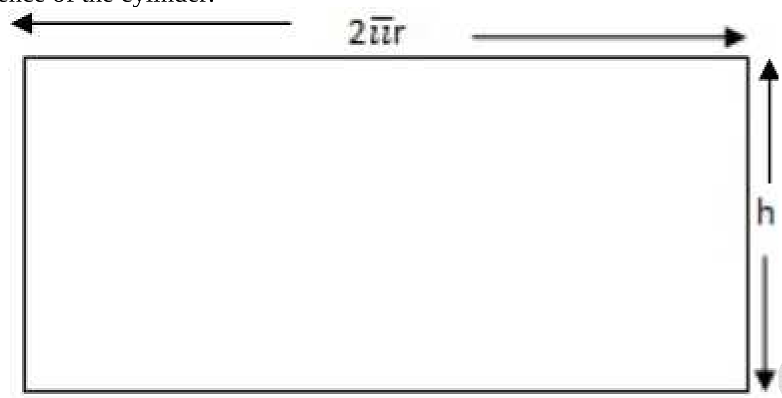
CYLINDER

The diagram show a cylinder, which has height him and top radius r cm.



- - The surface of a cylinder consists of a circular top and bottom, and curved side.

- -Imagine cutting the cylinder down the side, and unfolding it. The curved side becomes a rectangle, with height h and width the circumference of the cylinder.



Area of top and bottom = $2\pi r^2$

Area of curved side = $h \times 2\pi r$

Total area = $2\pi r^2 + 2\pi rh$

$= 2\pi(r+h)$

Example

Find the curved surface area of a cylinder which is 6cm high and with radius 4cm.

Solution

Given

$r = 4cm$, $h = 6cm$

Formula $= 2\pi r(r+h)$

$= 2\pi \times 4 \times (4+6)$

$= \pi \times 8 \times 10$

$= 80 \times \pi$ Since $\pi = 3.14$

$= 80 \times 3.14$

$= 251.20$

$\therefore$ The surface area is $25\ 1cm^2$

EXERCISE 3.4 B

1. A tin of shoe polish is 2cm high and 4cm in radius. Find its surface area.

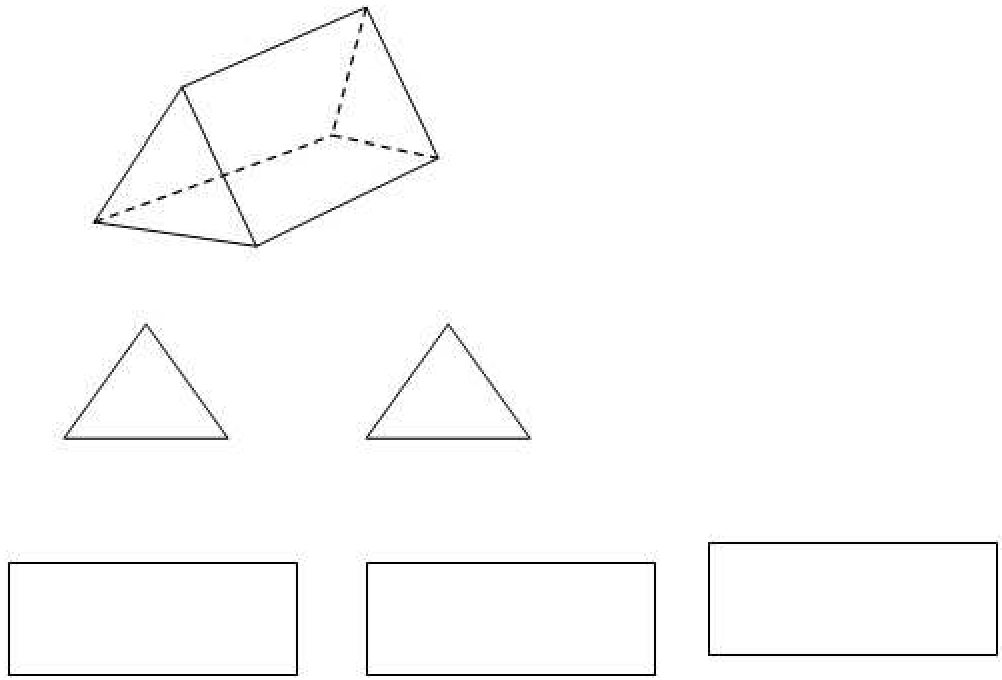
2. A paint roller is a cylinder which is 15cm long and with radius 3cm. Find the area of wall it can cover in one revolution

PRISMS

- Recall that a prism is a solid with a constant cross section

- In many cases the cross-section is a triangle.

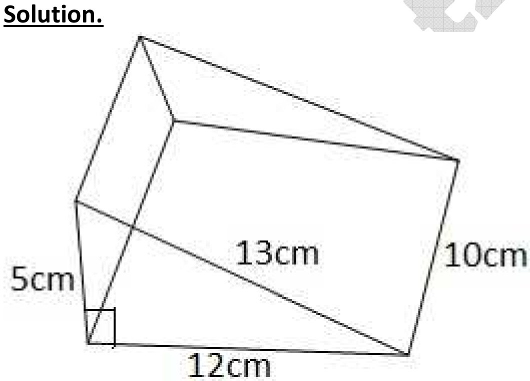
- The surface of a prism consists of the two cross-sections and the sides. In particular, the surface of a triangular and the sides. In particular, the surface of a triangular prism consists of two triangles and three rectangles.



Example

A prism has a cross-section which is a triangle with sides 5cm, 12cm and and 13cm. Its length is 10cm. Find Its surface area

Solution.



-The surface consists of:

- . Two triangles of side 5,12and 13
- . Three rectangles: 5, by 10, 12 by 10 and 13 by 10

Note:

The two triangles are right angled.

- The area of each triangle is

The area of each triangle is

Formula= $\frac{1}{2}bh$

$=\frac{1}{2}\times5\times12=30cm^2$

The areas of the rectangle are: Formula=l ×w

=(50×10)+ (12×10) =(13×10)

=50 +120+130

$=300cm^2$

-Total area =(2×30)+300

=60+300

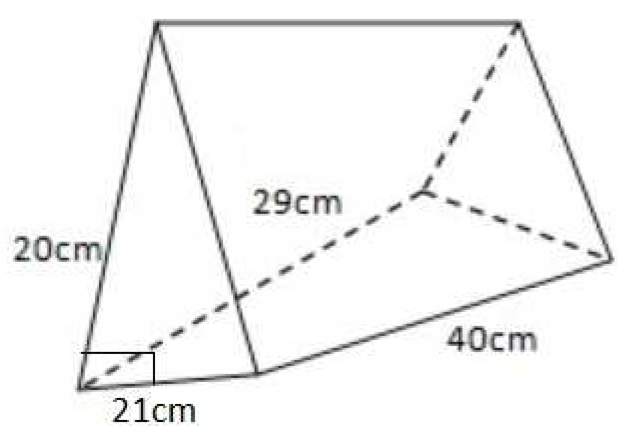
$=360cm^2$

The surface area is $360cm^2$

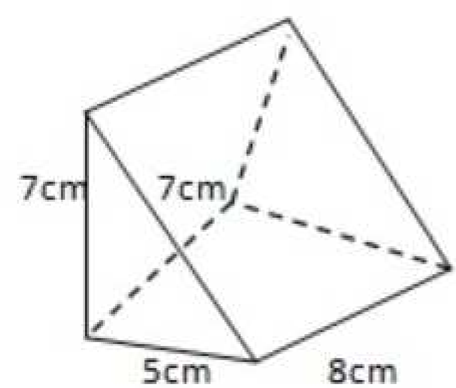
EXERCISE 3.4C

Find the surface area of the prisms shown below

a)



b)



2.The cross-section of a prism is a regular pentagon of side 8cm. The prism is 30cm long. Find the surface area of the prism.

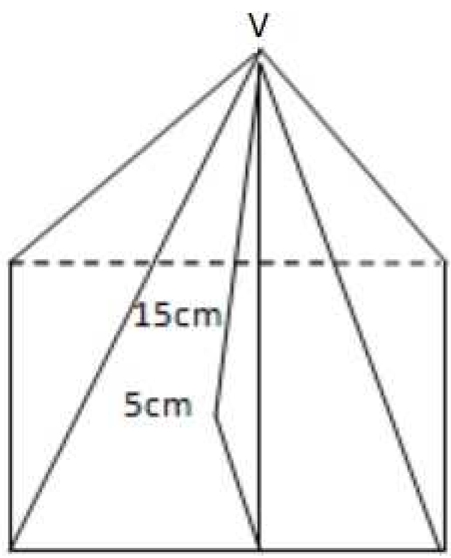
PYRAMIDS

The surface area of a pyramid is the sum of the base area and the area of the triangular sides. Sometimes you need to use Pythagoras' theorem to find the area of the sides. The following example shows the method.

Example

A pyramid has a square base of side 10cm and height 15cm.

Solution:



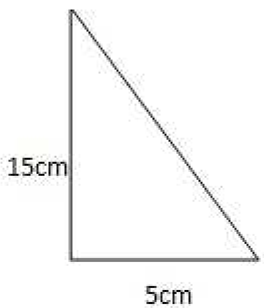
-The area of the base is

Formula = $S \times S = S^2$

$= 10 \times 10$

$= 100cm^2$

- Take a line from the vertex V to the middle of one of the sides of the base. The vertical rise of this line is 15cm.
- The horizontal run of this line is $\frac{1}{2} \times 10 = 5cm$
- Hence the length of the line is



By Pythagoras theorem

$\sqrt{15^2 + 5} = \sqrt{225 + 25} = \sqrt{250cm}$

-So the area of each triangle is

Formula $= \frac{1}{2}bh$

$= \frac{1}{2} \times 10 \times \sqrt{250}$

$= 5\sqrt{250}\ cm^2$

The total are is

$100 + 4 \times 5\sqrt{250}$

$100 + 20\sqrt{250}$

100+316.2

=416.2

The surface area is $416cm^2$

Exercise 3.4 D

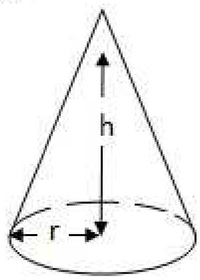
1. A pyramid has a square base of side 10cm and height 12cm. Find the surface area of its triangular faces.
2. A pyramid has a rectangular base which is 40cm by 60cm. Its vertex is 20cm above the centre of the base. Find the total surface area of the pyramid.

CONES

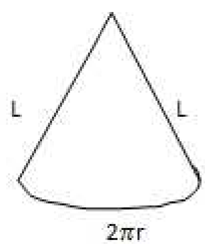
-The height h, of a cone is its perpendicular height. It is not the length of the slanting edge.

-The length of the slanting edge, l is given by Pythagoras’ theorem.

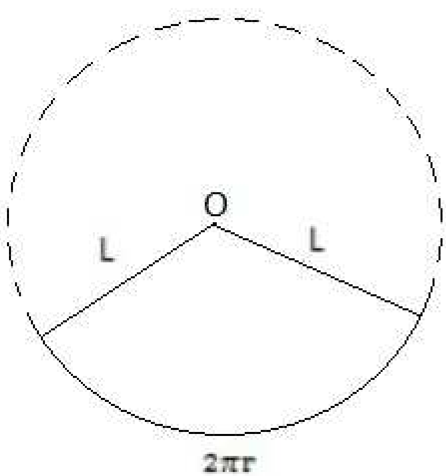
$L=\sqrt{r^2+h^2}$



-Imagine cutting along the side of the cone unfolding. You would get a sector of a circle. The radius of this circle is l and the arc length is the circumference of the cone base, 2πr. Hence the area of the curved side, which is the area of this sector, is the area of the circle, πl^2



πL^2 Is reduce in the ratio of the circumference of the cone base and the complete circle



$$\frac{2\pi r}{2\pi r}=\frac{r}{L}$$

- Area of the curved side = $\pi L^2 \times \frac{r}{l} = \pi rl$

∴ The total area of the cone is

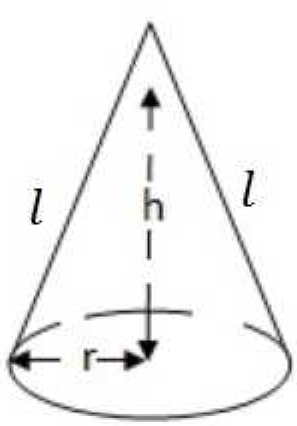
$\pi r^2 + \pi rl = \pi r(r+L)$

Example

A cone has base radius 4cm and height 3cm . Find its curved surface area.

Solution

The slant height l is l is given by



$$l = \sqrt{r^2 + h^2}$$

$$l = \sqrt{4^2 + 3^2}$$

$$l = \sqrt{16 + 9}$$

$$l = \sqrt{25}$$

$$l = 5$$

Formula, Area= π r (r+L)

= π ×4×(4=5)

= π × 36

= 113.1

There curved area of the cone is 113cm^2

EXERCISE 3.4E

Find the surface area of these cones.

a) a) With base radius 5cm and height 12cm.

b) b) With base radius 7m and height24m

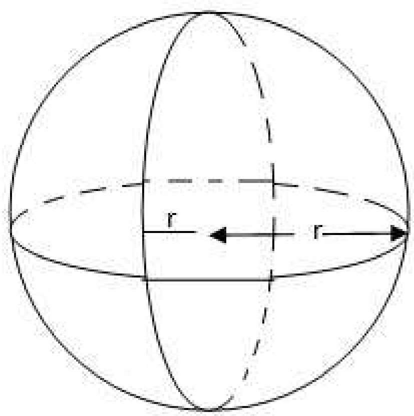
2.The base radius of a cone is c cm and its surface area is 160 cm2. Find its slant height and hence find its height.

SPHERES

-A sphere is a round solid, like a ball.

If the sphere has radius r, then its surface area A is

$A = 4\pi^2$



Example

Find the surface area of sphere of radius 0.46m

Solution

Given,

Radius r = 0.46

Area = $4\pi^2$

$= 4\times \pi(46)^2$

$= 2.660$

∴The surface area is 2.66m2

EXERCISE 3.4F

1. Find the radio of the spheres with area

a) a) $64\pi cm$ b) $0.44m^2$

2. A sphere has surface area 48cm2. Find its radius

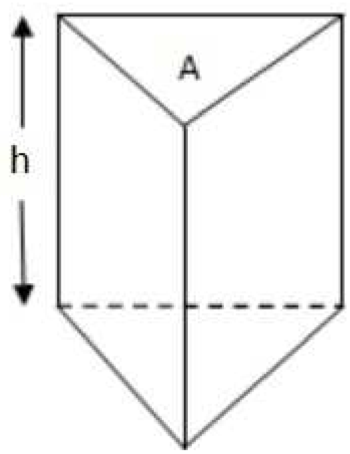
<u>VOLUMES OF THREE – DIMENSIONAL FIGURES</u>

<u>PRISMS</u>

Recall that a prism is a solid with a constant cross-section.

If the area of cross –section is A and the height is h, then the volume is

V=Ah



- The cuboid and the cylinder are examples of prisms. The volume of a cuboid is given by

  V = lbh (The cross –section is a circle)

- The volume of a cylinder is given by

  $V = \pi^r h$ (The cross –section is a circle)

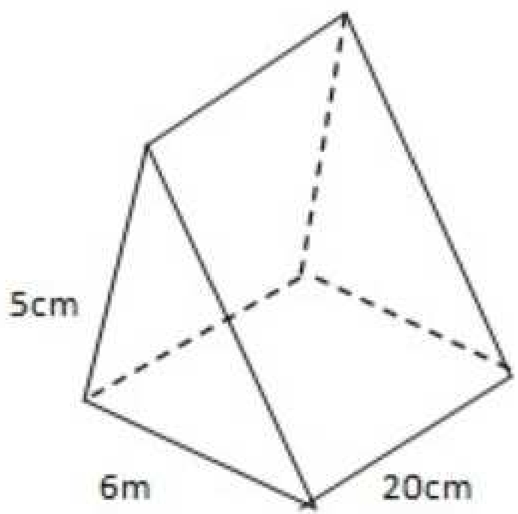
- For both of these solids, the volume is given by

  V= Ah, where A is the area of cross-section.

Example

The cross-section of a right angled prism is a triangle for which the shorter are 5cm and 6cm . The prism is 20cm long . Find its volume

Solution:



- The area of cross-section
- $A = \frac{1}{2}ab$

  $= \frac{1}{2} \times 5 \times 6$

  $= 15$

- Now multiply by the height

Volume = 20×15

= 300

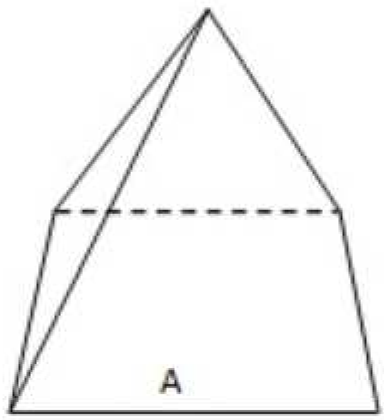
∴ There volume is $300cm^3$

EXERCISE 3.5.A

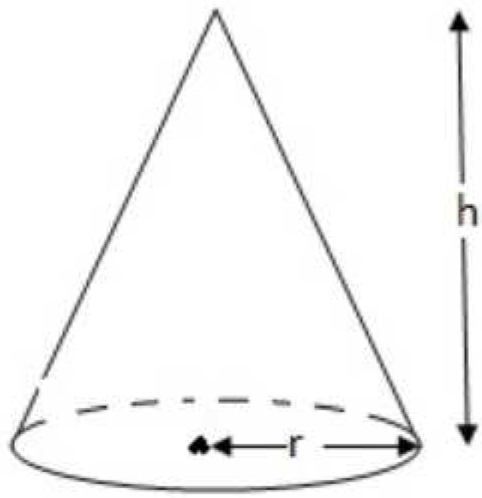
1. 1. The cross-section of a prism is a regular hexagen with side 0.8m. The prime is 1.6 long. Find the volume of the prism.

2. 2. The cross-section of a prism is a right angled triangle with sides 12cm, 9cm and 15cm, and with height 25cm. Find the volume of the prism.

PYRAMIDS AND CONES



a pyramid



a cone

- The pyramid tapers to a point from a base, which is usually a rectangle.

- The cone tapers to a point from a base, which is a circle.

- For both these solids, the volume is a third that of the corresponding prism with the same base. The volume of a pyramid is a third of the corresponding cuboid

$$V=\frac{1}{3}\pi Ah$$

∴ The volume of a cone is a third that of the corresponding cylinder

$$V=\frac{1}{3}\pi r^2h$$

<u>Example</u>

A pyramid has a square base. Its height is 7cm and its volume is 56cm^3. Find the side of the base.

Solution

- Suppose the side of the base is x2 cm^2

- Formula for the volume of a pyramid

-Formula for the volume of a pyramid

$V=\frac{1}{3}Ah$

$56=\frac{1}{3}\times x^2\times 7$

Hence $x^2=\frac{56\times 3}{7}$

$X^2=24$

$X=\sqrt{24}$

$X=4.9$

∴The side of the base is 4.9cm

EXERCISE 3.5.B

1. 1. A pyramid has a square base of side 10cm and volume $500cm^3$. Find its height.
2. 2. A cone has height 12cm and volume $50cm^3$. Find its base radius.

SPHERES

If a sphere has radius r, then its volume is

$$V=\frac{4}{3}\pi r^3$$

Example 1:

A sphere has radius 8.7cm. Find its volume.

Solution

Apply the formula

$V=\frac{4}{3}\times\pi\times(8.7)^3$

$V=2760$

∴The volume is $2760cm^3$

Example 2.

A sphere has volume $100cm^3$. Find its radius

Solution

Let the radius Le r. Then $V = \frac{4}{3}\pi^3$

$100 = \frac{4}{3}\pi^3$

$r3 = \frac{3}{4} \times 100) \div \pi = 23.87$

$3\sqrt{r^3} = 3\sqrt{23.87} = 2.88$

∴ The radius is 2.88m

EXERCISE 3.5C

1. 1. A sphere has volume 1.6m3

   a) Find its radius.

   b) Find its surface area.

2. 2. A sphere has surface area 56cm2

   a) Find its radius

   b) Find its volume

SOLUTIONS

EXERCISE 3.1.A

1. a) EFGH b) HDCG c) AD d)EA
2. a) GCDH and GCBF b)FG,FE and FB

EXERCISE 3.1 B

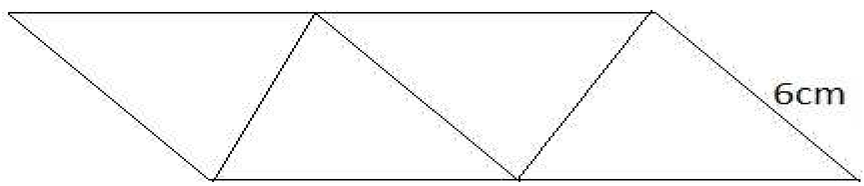
1. a) A Match box b) A die

2. a)Cuboid b) Sphere c) Cylinder d)Cone

EXERCISE 3.2A

1. 1. a)

b)



2. 2. a) A pyramid
b) A prism

EXERCISE 3.3A

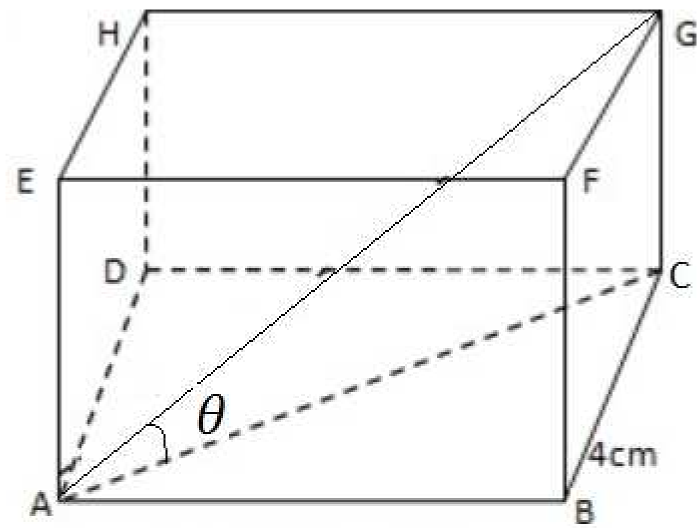
1. a) ABCD and BCGF

b) AB and DC

c) EC and AG

d) EB and FC

EXERCISE 3.3.B



$$AC = \sqrt{AB^2 + BC^2}$$

$$AC = \sqrt{4^2 + 4^2}$$

$$AC = \sqrt{16 + 16}$$

$$AC = \sqrt{32}$$

$$\tan\theta = \frac{4}{\sqrt{32}}$$

$$\tan\theta = 0.7071$$

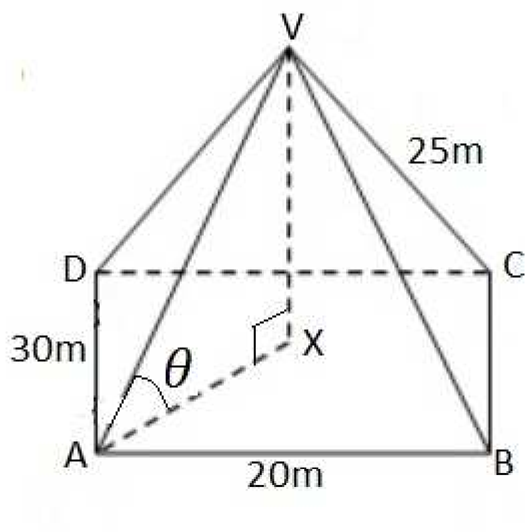
$$\theta = \tan^{-1}0.7071$$

$$\theta = 35.3^0$$

∴The angle between the line AG and the face ABCD is 35.3^0

2. 2. **Given**

AB=20M, AD=30m and VA =VB=VC=VD=25m



$AC = \sqrt{AB^2 + BC^2}$

$AC = \sqrt{20^2 + 30^2}$

$AC = \sqrt{400 + 900}$

$AC = \sqrt{1300}$

$AX = \frac{1}{2} AC$

$= \frac{1}{2}\sqrt{1300}$

$\text{Cos } \theta = \frac{AX}{AV}$

$= \frac{1}{2}\sqrt{1300}$

25

$= 43.9^0$

$\therefore$ The angle between V A and ABCD is 43.9^0

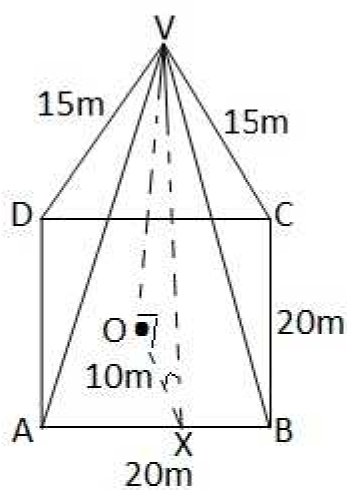
EXERCISE 3.3C

Given

- A square base of side 20m

- VA = =VC=VD =15M

- Angle between the planes VAB and ABCD = ?



$$\text{Cos } V\hat{X}O = \frac{OX}{VX}$$

$$\text{COS } V\widehat{XO} = \frac{10}{15}$$

$$\text{COS } V\hat{X}O = \frac{2}{3}$$

$$V\hat{X}O = \text{CS}^{-1}\frac{2}{3}$$

$$48.19^0$$

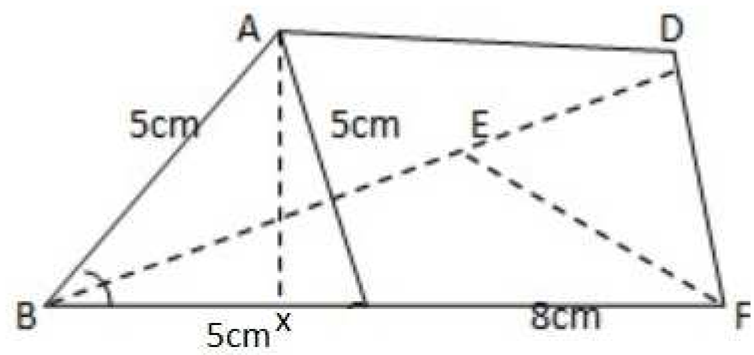
∴ The angle between the planes VAB and ABCD is 48.19^0

2. **Given**

- Length of the prism 8cm

- Side of the triangle 5cm

- Angle between the planes ADEB and BEFC =?



$BX = \frac{1}{2}BC$

$BX = \frac{1}{2} X\, 5 = 2.5cm$

$COS\ A\hat{B}X = \frac{BX}{AB}$

$COS\ A\hat{B}X = \frac{2.5}{5} = \frac{1}{2} = 2.5$

$A\hat{B}X = COS^{-1}\ 0.5$

$A\hat{B}X = 60^0$

$\therefore$ The angle between the planes ADEB and BEFC is 60^0

EXERCISE 3.4A

1. Given

1. A square base of side 0.8m and height 1.1m

Formula = Base area = (0.8×0.8)×2

= 0.64 ×2

= 1.28

Then other areas

$= (0.8 \times 1.1) \times 4 = 3.52$

- The total area = $1.28 + 3.52 = 4.8$

∴The surface area of the crate $4.8m^2$

2. Given

- A room with length = 5.2m, height=2.5m and width= 4.5m

-The area of the wall = length × width

$= 5.2 \times 4.5$

$= 23.4\ m^2$

- The area of the wall =(height × length) ×2 + (height ×width)×2

$= (2.5\times5.2)x2 + (2.5\times4.5)\times2$

$= 26+22.5$

$= 48.5m^2$

- The total area = $23.4m^2 + 48.5m$

$= 71.9m^2$

∴ The surface are of the wall and the wiling is $71.9m^2$

<u>**EXERCISE 3.4B**</u>

1. 1. **Given**

- Height = 2cm
- Radius = 4cm
- Formula = $2\ \pi r\ (r+h)$

$= 2\ \pi\times4(4+2)$

$= 8\pi\times6$

$= 48\pi$

$=150.72$

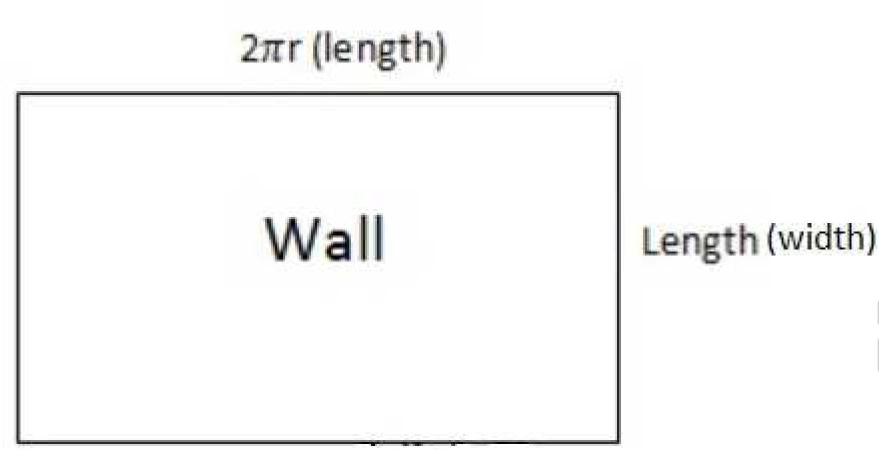
∴The surface area is $151cm^2$

2. **Given**

Length =15cm

Radius=3cm

Area of the wall = length × width



Area of the wall = $2\pi r \times L$

$=2\times\pi\times3\times15$

$= 90\ \pi$

$= 282.6cm^2$

EXERCISE 3.4C

1. a)surface area of the prisms=

- Base area = Rectangle=L×W

-The area of rectangle are

=(21×40) + (29×40)

=840 ×1,160 +800

$=2,800cm^2$

-The area of a triangle $= \frac{1}{2}bh$

$= \frac{1}{2} \times 21 \times 20$

$= 210\ cm^2$

But the prism have 2 triangles

$210 \times 2 = 420cm^2$

-The total area is 2,8000+420=3,220

The surface area of the prism is $3,220cm^2$

Given

Side = 8cm

Length=30cm

-A regular pentagon has 5 sides

-Area of a regular pentagon has 5 rectangles which are equal.

- The area of rectangles are

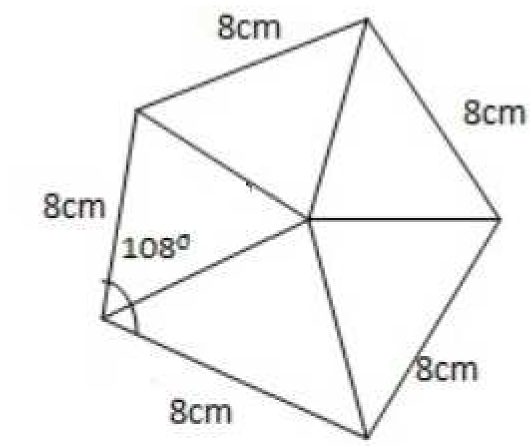
Formula = L×n

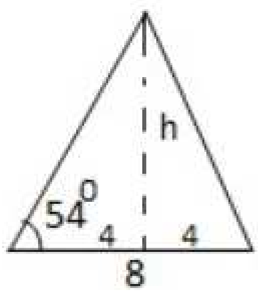
$= (30 \times 8) \times 5$

$= 240 \times 5$

$= 1,200cm^2$

Base area





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- Sum of interior angle = $(n-2)180^0$
  $= (5-2)180^0$
  $= 3\times180^0$
  $= 540^0$

Interior angle = $\frac{540}{5} = 180^0$

-tan $54^0 = \frac{h}{4}$

-tan $54^0 \times 4 = h$

$h = 1.3764\times4$

$h = 5.5056$.cm

-Area of a triangle $= \frac{1}{2}bh$

$= \frac{1}{2}\times8\times5.5056$

$= 22.0224$

- For base areas, that is top and bottom area there are 10 triangles.

- Therefore 22.0224×10=220.224 = 220

Hence

The total area = 1,200 + 220 = 1,420cm^2

∴ The surface area of the prism is 1420cm^2

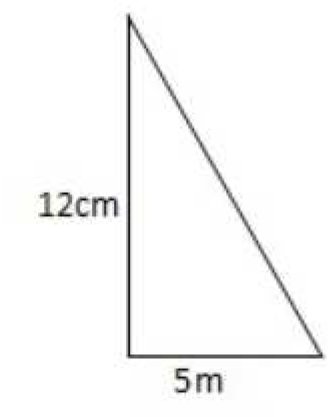
<u>EXERCISE 3.4 D</u>

Given

-A square base of side = 10cm and height is 12cm. the base area =5×5=5^0

=10×10

=100^2



$=\sqrt{12^2+5^2}$

$=\sqrt{144+25}$

$=\sqrt{169}$

=13cm

-The area of a triangle $=\frac{1}{2}$bh

$=\frac{1}{2}\times10\times13$

=65cm^2

- Total are of all triangles 65cm^2 × 4 = 260cm^2

∴The surface area of triangular faces is 260cm^2

EXERCISE 3.4

a) Formula for

PROBABILITY

Defn:

Probability is a branch of mathematics which deals with and shows how to measure these uncertainties of events in every day life. It provides a quantitative occurrences and situations. In other words. It is a measure of chances.

Probability set and Event

Suppose that an experiment of tossing a fair coin is done once. The outcomes expected from this experiment are that the coin will show up a head (H) or a tail (T). The results of the experiment are called outcome. All expected outcome from the possibility set or sample space, which is denoted by S.A

Specified outcome is called an even and is denoted by E. For example in this experiment the outcome that a head shows up is an event and it is a subset of the possibility set. Thus S ={H,T} and E ={H}

An event may not occur. For example, If the event is that a head occurs in tossing a coin once but a tail occurs instead. Then the event did not occur and is denoted by E': E'. Is the complement of E. In this experiment the space S = {H,T}. If the outcome is a head showing up, the E = {H}. So the event that is a head does not show up is E' = {T}

Formula of Probability of Event

$$P(E) = \frac{\text{Number of times the event has occured}}{\text{Total number of times the experiment has been done}}$$

$$P(E) = \frac{n(E)}{n(S)}$$

Keys

Outcomes;

Sample space (S)

Event (E)

The probability of an event is the number P where $0 \le P \le 1$ that measures the likelihood that the event will occur when the experiment is performed.

Example of probability experiment

1. Tossing a coin with outcomes; head or tail.
2. Throwing a die with outcomes 1, 2, 3,4,5,6.
3. Aiming at a target with outcomes of Success or failure
4. Testing an electrical component, outcomes defective or non- defective.

Event.

An event A is a subset of the sample space ($A \subseteq S$) or event is a specific outcome.

$$\frac{\textit{Sample space or possibility set}}{\text{Are all expected outcomes performed in an experiment}}$$

Example.

1. Three coins are tossed once each.

$S = \{HHH, HHT, HTH, HHT, THH, THT, TTH, TTT\}$

$E = \{HHH\}$ is an event where exactly 3 heads appear.

$E = \{HTH, HHT, THH\}$ is an event where exactly one tail shows up.

2. A die is tossed once and the results are recorded.

   a. The sample space
   b. The event that an even number occurs.
   c. The event that an even number does not occur.

Answer

a. $S = \{1,2,3,4,5,6\}$

b. $E = \{2,4,6\}$

c. E = $\{1,3,5\}$

3. Give the possibility set of the experiment of selecting even number less than 20.

$$S = \{2,4,6,8,10,12,14,16,18\}$$

4. Give the possibility set of not selecting an even number from a set of numbers less than 9.

$$S = \{1,3,5,7\}$$

DISCRETE RANDOM VARIABLE.

Is the one that assumes only a countable number if values .

If "S" has "n" number of elements and if by symmetry each of these elements has the same probability to occur , then this probability is in 1/n . if A has "m" elements,

then $p(A) = \frac{m}{N}$

Example.

1.If we toss two fair coins, let A be the event that at least one head shows up . find P (A).

Solution:

S = $\{HH, HT, TT, TH\}$

n(s) =4

E(n) = $\{HT, HH, TH\}$

At least greater or equal

nE(A) = 3

P(A) = $\frac{ne(A)}{n(S)}$

= 3/4 = 0.75

2.find the probability that an odd number appears in a single toss of a fair die.

Solution:

S = $\{1,2,3,4,5,6\}$

n(S) = 6

E(n) = $\{1,3,6\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{\cancel{3}}{\cancel{6}} \quad \frac{1}{3}$$

$$.:\ P(E) = \underline{\underline{\frac{1}{3}}}$$

Some properties of probability.

1. **0≤P(A)≤1**
2. **P(S)=1**
3. **P(Q)=0**
4. **P(A) = 1-P(A)**

Exercise 1.1

1. If the probability that it rains at Dar-ES-Salaam on April 1st in any year is 0.4, what is the probability that it will not rain at Dar-Es-Salaam on April first next year?

Solution:

P(A) = 1- P(A)

= 1- o.4

= 0.6

2. A box contains 6 volley balls and 8 footballs of the same size. If one ball is picked at random, what is the probability that it will be a football?

Solution:

n(S)= 14

n(f)= 8

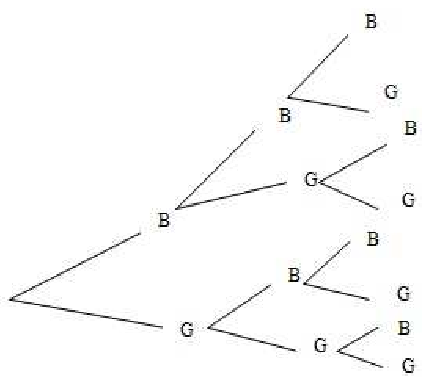
p(f) = 8/14

p(f) = 4/7

COMBINED EVENT AND TREE DIAGRAM

Example

In a family of three children, write down an event where all the children are boys.



S= $\{BBB, BBG, BGB, GBB, GBG, GGB, GGG, GBG\}$

E= $\{BBB\}$

E= $\{GGG\}$

P(GGG) = $\frac{n(E)}{n(S)}$

= 1/8

The probability at least two are boys;

E = $\{GBB, BGB, BBG, BBB\}$

n(E) = 4

$P(E) = {}^{4}/_{8}$

$= {}^{1}/_{2}$

Exercise 1.2

1. 1. If a two digit numeral is written choosing the 10 digit from the set (1,2,3,4) and unit's digit from (5,6) . find the probability that a number greater than 20 will appear.

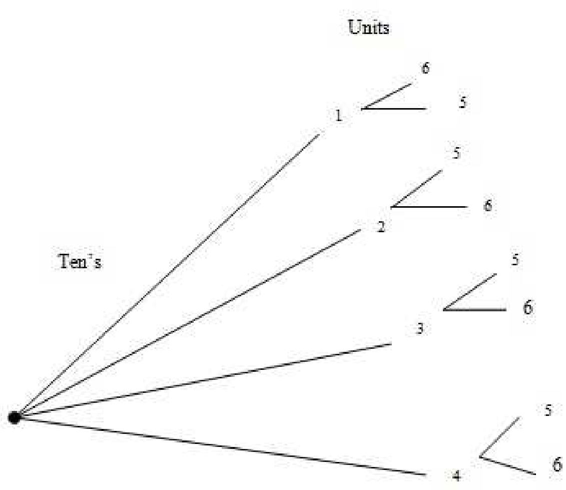
2.A pair of dice is tossed . find the probability that the sum of the two numbers obtained is;

a. At least 8
b. At most 10

3.Maria has two blouses, one is green and the other is yellow. She also has a blue , white and black shirt . what is the probability that she will put on a yellow blouse and a blue shirt ?

Answers

Solution1.



S= $\{16 ,15, 25, 26, 35, 36, 45, 46\}$

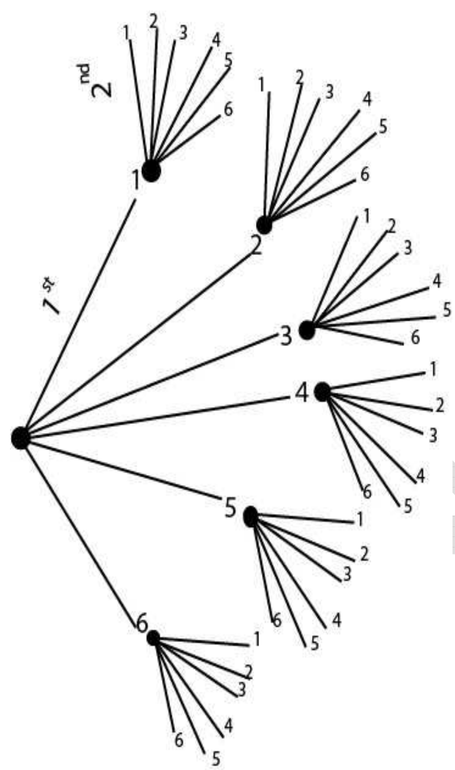
E = $\{25, 26 ,35, 36, 45, 46\}$

$$P(E) = \frac{n(E)}{n(s)}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

Solution 2.



$$S=\begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6)(4,1)(4,2)(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5)(6,6) \end{Bmatrix}$$

(a).

n(s) = 36

n(E) = 15

P(E) = $^{15}/_{36}$

(b).

n(E) = 33

p(E) = $^{33}/_{36}$ = $^{11}/_{13}$

Solution 3.

	Shirts			
blouse		blue	White	Black
	Green	BG	WG	BG
	yellow	BY	WY	BY

$$S = \{BG, WG, GB, BY, WY, BY\}$$

n(S) = 6

E= $\{BG\}$

n(E)= 1

P(E)= $^{1}/_{6}$

INTERSECTION

The intersection of two events A and B denoted by A$\cap$B is the event that occurs if A and B are on a single performance of the experiment.

UNION

The union of two events A and B , denoted by A$\cup$B is the event that occurs if either A or B or both occur on using performance of the experiments.

Example.

1. Consider a die toss experiment , events are defines as;

E(A)= $\{toss\ an\ even\ number\ shows\ up\}$

E(B)= $\{toss\ a\ number\ than\ or\ equal\ to\ 3\ shows\ up\}$

Find

(a). P(A $\cup$ B)

(b). P(A $\cap$ B)

Solution:

S= $\{1,2,3,4,5,6\}$

E(A) = $\{2,4,6\}$

E(B)= $\{1,2,3\}$

a. A $\cup$ B= $\{1,2,3,4,6\}$

P(A $\cup$ B) = $^5/_6$

b. A $\cap$ B = $\{2\}$

P(A $\cap$ B) = $^1/_6$

1. The probability that a man watches TV in any evening is 0.6, the probability that he listens the radio is 0.3 and the probability the does both is 0.15. what is the probability that he does neither?

Solution:

P(T) = 0.6

P(R)= 0.3

P(T $\cap$ R) = 0.15

P(A $\cup$ B) = P(A) + P(B) – P(AnB)

$P(T \cup R) = 0.6 + 0.3 - 0.15$

$= 0.75$

$P(TUR)^1 = 1 - P(TUR)$

$= 1 - 0.75$

$= 0.25$

2. At a second hand show room, 20% of the cars have no engine, 40% of the cars have bad tyres, and 15% have no engine and have tyres. what is the probability that a car chosen at random has good tyres and an engine?

Solution:

$P(A) = 20\%$

$P(B) = 40\%$

$P(A \cap B) = 15\%$

$P(A \cup B) = {}^{20}/_{100} + {}^{40}/_{100} - {}^{15}/_{100}$

$= {}^{45}/_{100}$

$P(A\hat{I}„ \cap B\hat{I}„) = 100 - P(A \cup B)$

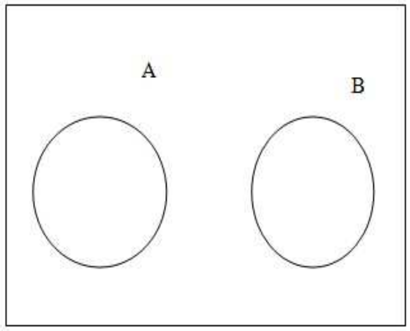
$= (100\% - 45\%)$

$P(A\hat{I}„ \cap B\hat{I}„) = 55\%$

MUTUALLY EXCLUSIVE EVENTS

If one and only one event among two or more events can take place at a time, then the events are called mutually exclusive.

-



Event A and B are mutually exclusive if $A \cap B = 0$ and therefore

$P(A \cup B) = P(A) + P(B)$

Example

1. Find the probability that an even number or an odd number greater than one occurs when a die is tossed once.

Solution

S = $\{1,2,3,4,5,6\}$

E(A) = $\{2,4,6\}$

E(D) = $\{3,5\}$

$P(A \cup D) = P(A) + P(B)$

$= {}^{3}/_{6} + {}^{2}/_{6}$

$= {}^{5}/_{6}$

2.

In a family of four children, what is the probability that ;

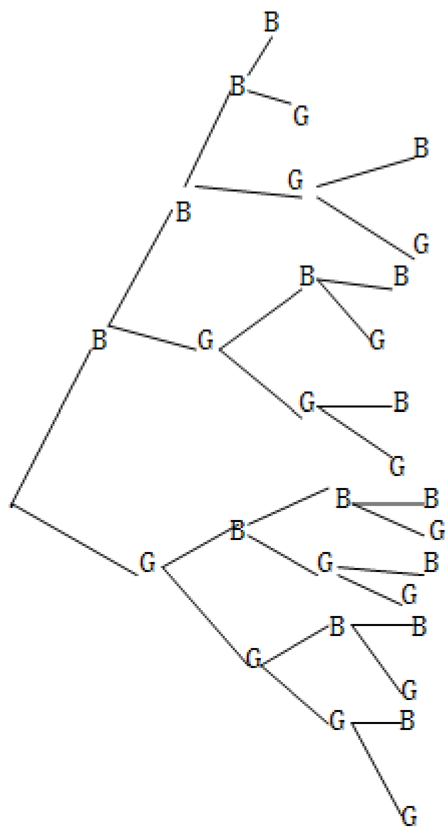
a) Two are boys and two are girls.

b) All are boys

c) Exactly 4 are girls

d) At least 4 are boys

Solution



$$\left\{\begin{array}{l} \text{BBBB, BBBG, BBGB, BBGG, BGBB, BGBG} \\ \text{BGGB, BGGG, GBBB, GBBG, GBGB} \\ \text{GBGG, GGBB, GGBG, GGGB, GGGG} \end{array}\right\}$$

(a) All are boys and two are girls

$$\frac{1}{\underset{8}{\cancel{16}}} \times \frac{\overset{3}{\cancel{6}}}{16} = \frac{3}{128}$$

(b) All are boys

$\frac{1}{16}$

(c) Exactly four are girls

$\frac{1}{16}$

(d) At least four are boys

$\frac{1}{16}$

<u>INDEPENDENT EVENTS</u>

Two events are independent if the occurrence of one event or non occurrence of the event does not affect the probability of the other event.

$P(E_1 \text{ and } E_2) = P(E_1) - P(E_2)$

Or $P(E_1 \cap E_2)$

<u>Exercise</u>

1. If the probability that A will be alive in 20 years is 0.7 and the probability that B will be alive in 20 years is 0.5, then the probability that they will both be alive in 20 years is ?

Solution

$P(A \cap B) = P(A) \times P(B)$

$= 0.7 \times 0.5$

$= 0.35$

2. The probabilities that Halima and perpetuate will be selected for further studies are respectively 0.4 and 0.7. Calculate the probability that both of them will be selected for further studies.

Solution

$P(A \cap B) = P(A) \times P(B)$

$=0.4 \times 0.7$

$=0.28$

3. One bag contains 4 white balls and 2 black balls , another contains 3 white balls ad 5 black balls. If a ball is drawn from each bag , find the probability that

a. Both are white

b. Both are black

c. One is white and one is black.

Solution:

a. $P(w_1w_2) = P(w_1) \times P(w_2)$

$= {}^{4}/_{6} \times {}^{3}/_{8}$

$= {}^{12}/_{48}$

$= {}^{1}/_{4}$

b. $P(B_{1+}B_2)= P(B_1)\ P(B_2)$

$\frac{2}{6} \times \frac{5}{8} = \frac{\cancel{10}}{\cancel{48}} = \frac{5}{24}$

c. $(w_1 B_2 \text{ or } B_1w_2)= P\ (w_1) \times P(\ B_2) + P(B_1) \times P(w_2)$

$(\frac{4}{6} + \frac{3}{8}) \times (\frac{2}{6} + \frac{5}{8})$

$\frac{16+12}{24}\ \frac{8+15}{24}$

$\frac{28}{24} \times \frac{23}{24} = \frac{644}{576} = \underline{\underline{1.12}}$

$\frac{\cancel{3}}{\cancel{30}} = \frac{1}{10}$

DEPENDENT EVENTS

Events are dependent when one event affects the probability of the occurrence of the other.

$P\ (E_1/E_2) = P(E_1) \times P(E_1/E_2)$

Not replaced $P(RWB) = P(R) \times P(W) \times P\ (B)$

$= {}^{6}/_{15} \times {}^{4}/_{14} \times {}^{5}/_{13}$

Exercise

1. The probability that a man and his wife will be alive for fifty years are 3/10 and 1/3 respectively. Find the probability that

   a. both will be alive,

   b. At least one will be alive.

Solution:

(a) $P(A \cap B) = P(A) \times (PB)$

$$\frac{3}{10} \times \frac{1}{3} = \frac{\cancel{3}}{\cancel{30}} = \frac{1}{10}$$

(b)

$$\left(\frac{3}{10} + \frac{1}{3}\right) + \left(\frac{\cancel{3}}{10} \times \frac{1}{\cancel{3}}\right)$$

$$\underline{\underline{0.733}}$$

2. A fair die is tossed once and the number showing up is recorded. find the probability of an even number greater than two showing up.

Solution:

$S = \{1,2,3,4,5,6\}$

$E = \{2,4,6\}$

$P(E) = \frac{2}{6}$

$= \frac{1}{3}$

3. A box contain 4 white balls and 5 black balls. two balls are drawn at random from the box. Find the probability that both the balls are

   a. white

b. black

solution:

a. $P(A_1/A_2) = P(A)$

$$\frac{\cancel{4}^{1}}{9} \times \frac{3}{\cancel{8}_{2}} = \frac{3}{18} \times \frac{3}{8} = \frac{\cancel{3}}{\cancel{18}} \quad \frac{1}{6}$$

$$\therefore \frac{1}{6}$$

b. $\frac{5}{9} \times \frac{\cancel{4}^{1}}{\cancel{8}_{2}} = \frac{5}{18}$

THE MULTIPLICATIVE RULE

Permutations;

A permutation of 'n' different objects taken 'r' at time is an arrangement of 'r' out of 'n' objects with attention given to the order of arrangement. the number of permutations of 'n' objects taken 'r' at a time is denoted by;

$$^{n}P_{r} \text{ or } nPr = \frac{n!}{(n-r)!}$$

Example

1. The number of permutations of the letters a, b, c taken two at a time is;

$$\frac{n!}{(n-r)!} = {}^{3}P_{2} = \frac{3!}{(3-2)!} = \frac{3 \times 2 \times 1}{1}$$

These are ab, ba, ac, cabc, cb.

2. In how many ways can 10 people be seated on a bench if only 4 seats are available?

Solution

$$^{10}P_4 = \frac{10!}{(10-4)!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

= 5040way

3. <u>Evaluate</u>

$$^{8}P_3 = \frac{8!}{(8-3)!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

= 336ways

4. How many four digits numbers can be formed with 10 digits 0,1,2,3,4,5,6,7,8,9

a. Repetition is allowed.

b. Repetitions are not allowed.

Solution

a. P_1 P_2 P_3 P_4

10 10 10 10

= 10 x10x10x10

= 10000 ways

b. P_1 P_2 P_3 P_4

10 9 8 7

= 10x9x8x7 = 5040ways

NOTE : the number of permutations of 'r' objects consisting of groups of which n_1 are alike and n_2 are alike is

$$\frac{n!}{n1!n2!}$$ where $n = n_1 + n_2$

COMBINATIONS

Combinations of n different objects taken 'r' at a time is a selection of 'r' out of the 'n' objects .take 'r' at a time denoted by;

$$^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

Example

1. The number of combination of letters a,b,c taken two at a time.

Solution

$$^{3}C_{2} = \frac{3!}{(3-2)!\,2!}$$

$$= \frac{3 \times 2 \times 1}{1 \times 2 \times 1}$$

$$=3$$

These are ab, bc and ac

NOTE ab, ba are the same.

2. How many arrangements are there of the letters of the word "solopaga"?

Solution

n =8

n_1 (0) = 2

N2 (A)= 2

No arrangement = $\frac{n!}{n1!n2!}$

$$= \frac{8!}{2!\,2!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1}$$

$= 10080$

3. It is known that $^{n}C_{r} = {}^{n}C_{n-r}$

   Find x given that $^{20}C_{18} = {}^{20}C_{x}$

Solution

$^{20}C_{x} = {}^{20}C_{20-18}$

$= {}^{20}C_{2}$

$x = 2$

4. A box contains 8 red,3 white and 9 blue balls. If 3 balls are drawn at random , determine the probability that;

   a. All 3 are red
   b. All 3 are white
   c. 2 red and one white
   d. 1 of each color is drawn
   e. At least one is white.

Solution

a. $P(R_1R_2R_3) = \frac{8C3}{20C3}$

$8C3 = \frac{8!}{(8-3)!3!} = \frac{8 \times 7 \times 6 \times 5!}{5! \times 3 \times 2 \times 1} = 56$

$$20C3 = \frac{20!}{(20-3)!3!} = \frac{20x19x18x17!}{17!x3x2x1} = 1140$$

$$P(R_1R_2R_3) = \frac{56}{1140}$$

b. $P(W_1W_2W_3) = \frac{3C3}{20C3}$

$$3C3 = \frac{3!}{(3-3)!3!} = \frac{3!}{0!x\,3\,!} = 1$$

Note: 20C3=1140

$$P(W_1W_2W_3) = \frac{1}{1140}$$

c. $P(R_1R_2) = \frac{2}{20} \times \frac{1}{19} = \frac{2}{380} = \frac{1}{190}$

$$P(1W) = \frac{1}{20}$$

$P(R_1R_2)$ and $P(1W) =$ $P(R_1R_2) \times P(1W) = \frac{1}{190} x \frac{1}{20} = \frac{1}{3800}$

d. $P(RWB) = \frac{8C1\ 3C19C1}{20\,C_3}$

$$= \frac{7!x1! \quad 2!x\ 1! \quad 8! \ \ 8x7!\ x\ 3x2!\ x\ 9x8!}{1140}$$

$$= \frac{216}{1140}$$

e. $P(\ W_1\ W_2\ W_{\ 3}) + P(W_1\ W_2W_3) + P(W_1\ W_2W_3)$

$$= \frac{3C1\ 17C2\ + 3C2\ 17C1\ +\ 3C3}{1140} = {}^{23}/_{57}$$

$$= {}^{81}/_{1140}$$

5. A jar contains 3 white peas, 2 red peas and 2 yellow peas. Three peas are drawn at random ,find the probability that;

a. At least 2 peas are white

b. Exactly 2 peas are white.

Solution

a. $P(W_1W_2W_3) + P(W_1W_2W_3)$

$= \frac{3C2\ 4C1}{35} + \frac{3C3}{35} = \frac{13}{35}$

b. $P(W_1\ W_2W_3)$

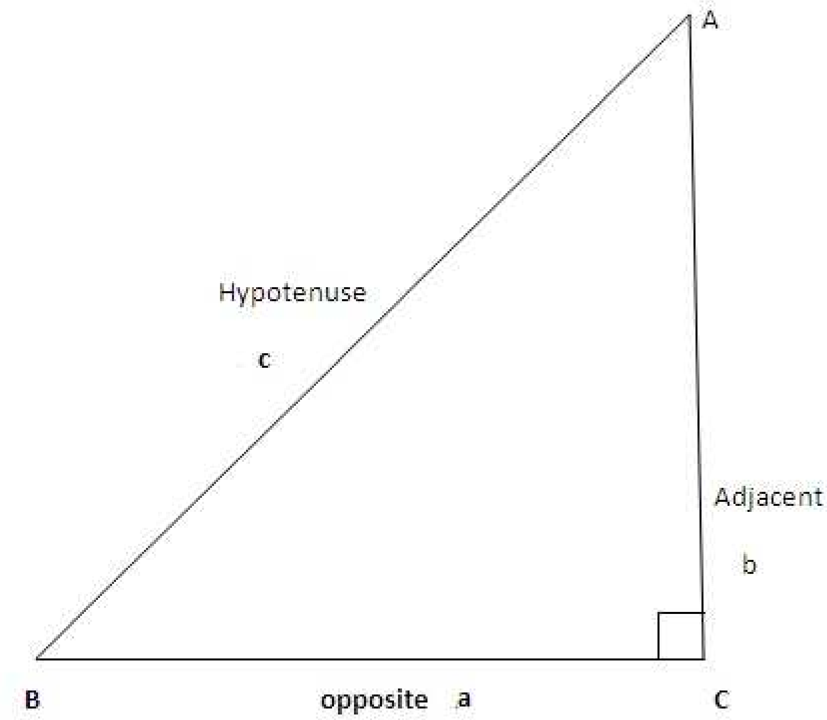
$= \frac{3C2\ 4C1}{35} = \frac{12}{35}$

TRIGONOMETRY

<u>Trigonometrical ratios in a unit circle</u>

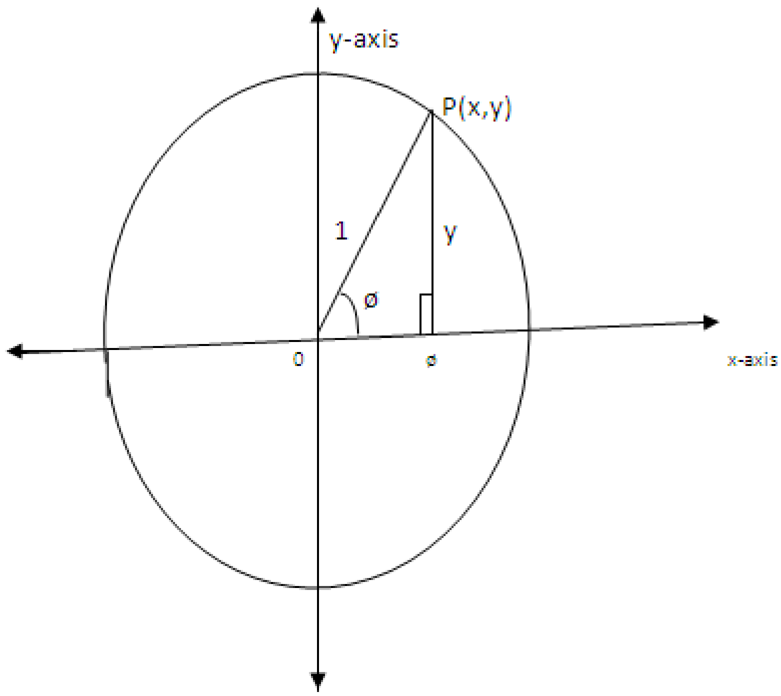
The three trigonometrical ratios of sine, cosine and tangent have been defined earlier, using the sides of a right-angled triangle as follows

If A is an angle as shown



Consider a circle of units subdivided into four congruent sectors of the coordinate axes whose origin is at the center of the circle as shown below.

Let ø be any acute angle ($0 < ø < 90^0$) and let P with coordinates (x, y) be the point where OP intersect with the circle then



$\textbf{Sin } \emptyset = \frac{PQ}{OP} = \frac{Y}{1} = y$

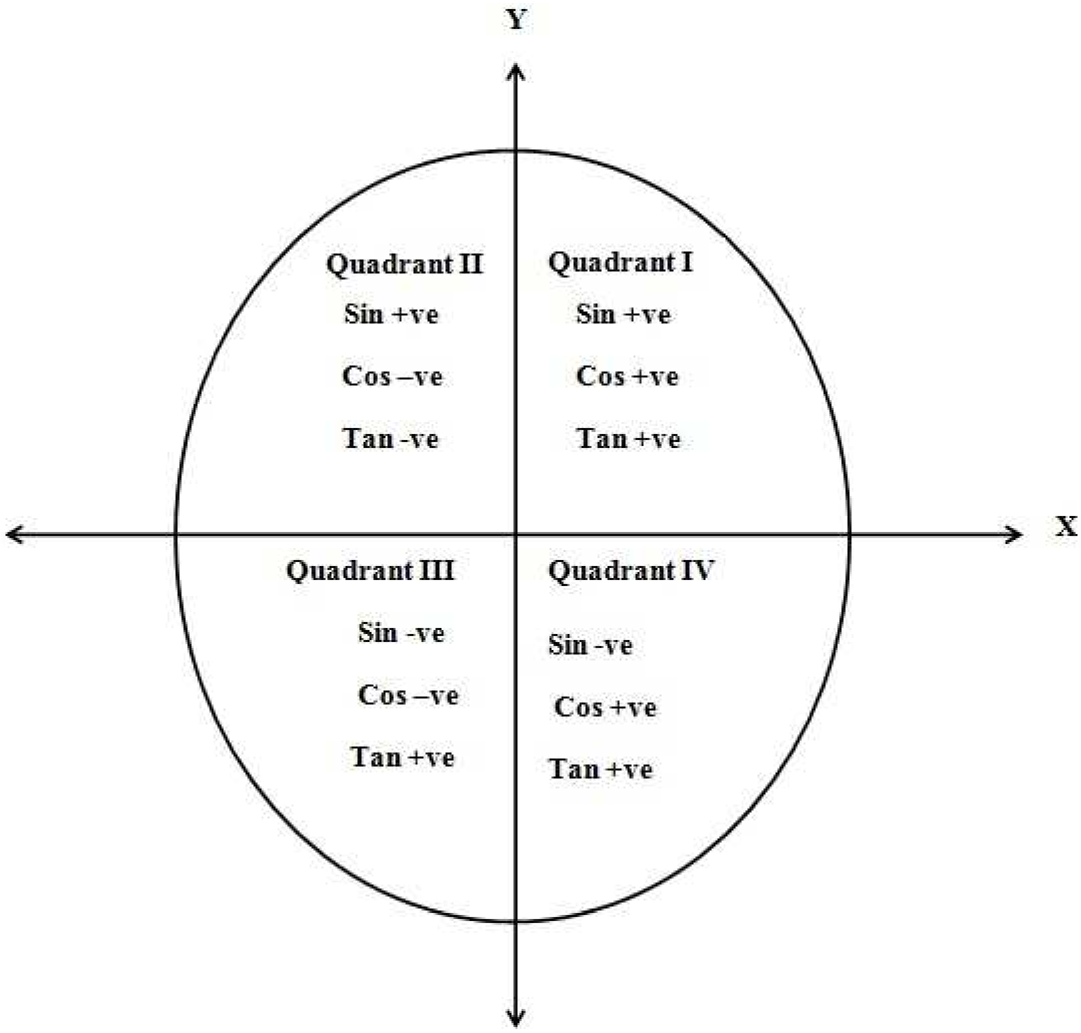
$\textbf{Cos } \emptyset = \frac{OQ}{OP} = \frac{x}{1} = x$

$\textbf{tan } \emptyset = \frac{PQ}{OQ} = \frac{Y}{X}$

SIGNS OF THE TRIGONOMETRICAL RATIOS

The trigonometrical ratios can be positive or negative depending on the size of the angle and the quadrant in which the angle is found.

The results obtained are illustrated below. There results will be a help in determining whether sine, cosine and tangent of an angle is positive or negative.

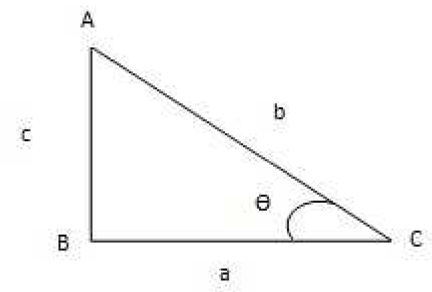


Obtuse angle $90^0 < \theta < 180^0$

$\text{Sin } (180-\theta)^0 = y$

$\text{Cos } (180-\theta)^0 = -x$

$\text{Tan } (180-\theta)^0 = -{}^{y}/_{x}$



Sin θ = $\frac{\text{opposite}}{\text{Hypotenuse}} = \frac{c}{b}$

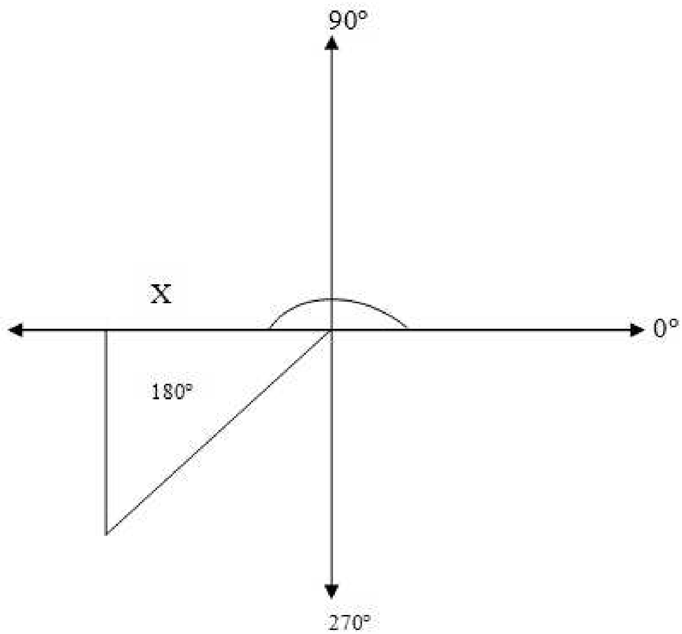
Cos θ = $\frac{\text{adjacent}}{\text{Hypotenuse}} = \frac{a}{b}$

Tan θ = $\frac{\text{opposite}}{\text{Adjacent}} = {}^{c}/_{a}$

Tan θ = $\frac{\text{Sin}\theta}{\text{Cos}\,\theta}$

Tan θ = $\frac{c/b}{a/b}$

Tan θ = ${}^{c}/_{a}$



EXERCISE

1. Write the signs of each of the following

   a. Cos 160^0 = negative

   b. Cos 310^0 = positive

   c. Cos 75^0 = positive

   d. Sin 220^0 = negative

   e. Cos 335^0 = positive

   f. Tan 190^0 = positive

2. Express the following in terms of sine, cosine or tangent of an acute angle

a). Cos 308^0
$=360^0 - 308^0$
$= 52^0$
$=\text{Cos } 52^0$

b). Sine 217^0
$= (-217 - 180)^0$
$= -(37^0)$
$= \text{Sin } -37^0$

c). Tan 175^0

= $-(180^0 - 175^0)$

= -5^0

Tan -5^0

d). Tan 333^0

= $-(360 - 333)^0$

= -27^0

= $\sin -27^0$

e). Cos 103^0

= $-(180 - 103)$

= -77^0

= $\cos -77^0$

3. Express the following in terms if sine

a). Sin 130^0

= $(180 - 130)^0$

= $\underline{\sin\ 50^0}$

b). Sin 230^0

= $-(230 - 180)^0$

= $\underline{-\sin 50^0}$

c). Sin 310^0

= $-(360 – 310)^0$

= $\underline{-\sin 50^0}$

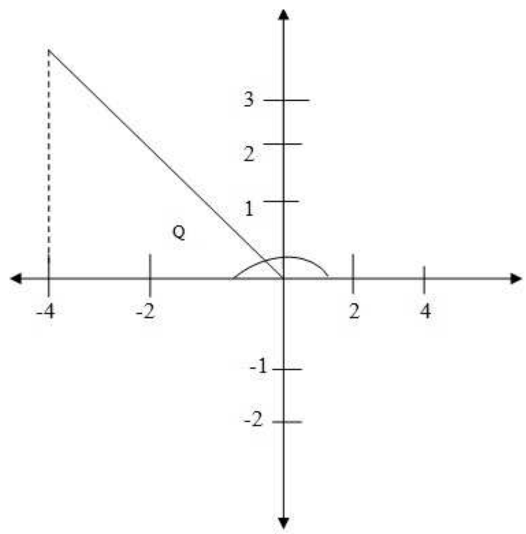
<u>Examples</u>

1. Let θ be any angle and P with coordinates (-4, 3) be a point in the terminal point of op, see the figure below. Find

a. sinθ

b. cosθ

c. tanθ



$(OP)^2 = (-4)^2 + (3)^2$

OP= 5

a. Sin θ = sin (180- θ) = 3/5

b. Cos θ = cos(180 – θ) = -4/5

c. Tan θ = tan (180-θ) = - ¾

EXERCISE

1. Find the cos θ and tan if θ is the angle made by the positive x- axis, from the line from the origin to each of the following points.

   a. (2,6)

Solution

$\text{Hyp} = \sqrt{2^2 + 6^2}$

$\sqrt{40}$

$\text{Cos } \emptyset = \frac{adj}{hyp}$

Cos θ = $\frac{2}{\sqrt{40}}$

Sin θ = $\frac{6}{\sqrt{40}}$

$\text{Tan}\phi = \frac{\cancel{6}}{\cancel{2}}\ 3.$ $\underline{\underline{\text{Tan}\phi = 3}}$ $\text{Sin} = \frac{\text{opp}}{\text{Hypotenus}}$

b. (-12 , 5)

$\sqrt{-12^2 + 5^2}$

$\sqrt{144^2 + 25}$

$\sqrt{169} = 13$

Cos θ = $-\frac{12}{13}$

Tan θ = $-\frac{5}{12}$

Sin θ = $\frac{5}{13}$

c. (-4,-3)

$\sqrt{(-4)^2 + (-3)^2}$

$= \sqrt{16 + 9} = \sqrt{25} = 5$

Cos θ = $-\frac{4}{5}$

Tan θ = $-\frac{3}{4}$

Sin θ = $-\frac{3}{5}$

POSITIVE AND NEGATIVE ANGLES

NOTE;

1. If θ is positive, the negative angle corresponding to θ is $(-360^0 + \theta)$
2. If θ is negative, the positive angle corresponding to θ is $(360^0 + \theta)$

Example

1. Find the positive or negative angles corresponding to each of the following angles.
   a. $304^0 = (-360^0 + \theta) = (-360^0 + 304) = -56^0$
   b. $-115^0 = (360^0 + \theta) = 360^0 + -115^0 = 245^0$

2. Find the sine, cosine and tangent of each of the following angles.
   a. 144^0

Solution

$144^0 = 180^0 - 144^0 = 36^0$

$= \sin 36^0 = 0.5878$

$= \text{cosine } 36^0 = -0.8090$

$= \tan 36^0 = -0.7265$

b). -231^0

$= 360^0 + \theta = 360^0 + -231$

$= 129^0$

$= 180^0 - 129^0$

$= 51^0$

$\sin 51^0 = 0.7771$

$\text{Cosine } 51^0 = 0.6293$

$\tan 51^0 = -1.2349$

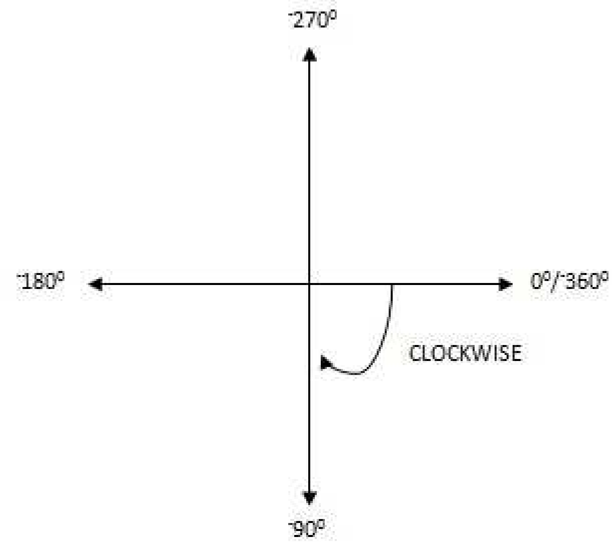
c). 310^0

$= 360^0 - 310^0 = 50^0$

$\sin 310^0 = \sin 50^0 = 0.7660$

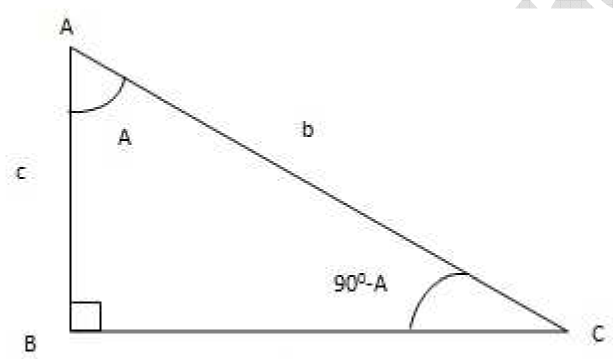
$\text{Cosine } 310^0 = \text{cosine } 50^0 = 0.6428$

$\tan 310^0 = \tan 50^0 = 1.1918$



RELATIONSHIP BETWEEN TRIGONOMETRIC RATIOS

Consider a triangle A, B, C in which angles A and C are complimentary angles. ie $A+C = 90^0$



Sin A = a/b
Cos C = a/b
Sin A = Cos C = a/b
C = 90-A
Sin A = Cos $(90-A)^0$ = Cos C = a/b
Cos A = c/b
$(\text{Sin A})^2 = \sin^2 A$

$\text{Sin}^2 A + \text{Cos}^2 A = \frac{b^2}{b^2} = 1$

Exercise

1. Given that Sin θ = 4/9 .find Cos θ

Solution

$Sin^2\theta + cos^2\theta = 1$

$(4/9)^2 + cos^2\theta = 1$

$Cos^2\theta = 1 - 16/81$

$Cos\ \theta = \frac{\sqrt{65}}{9}$

2. If Sin θ = 0.9397 and Cos θ = 0.3420. Find without using tables tan θ.

Solution

$Tan\ \theta = \frac{Sin\ \theta}{Cos\ \theta} = \frac{0.9397}{0.3420}$ $\underline{= 2.748}$

3. If $Sin\ \alpha = \frac{\sqrt{2}}{5}$ find sin(90-α)

Solution

$\alpha + \beta = 90^0$

$\beta = 90^0 - \alpha$

$Sin\ \alpha = Cos\ (90^0 - \alpha) = Cos\ \beta = \frac{\sqrt{2}}{5}$

$Sin^2\beta + Cos^2\beta = 1$

$Sin^2\beta + \left(\frac{\sqrt{2}}{5}\right)^2$

$Sin^2\beta = \frac{25-2}{25}$

$\text{Sin}^2\beta = \frac{23}{25}$

$\text{Sin } \beta = \text{Sin } (90-\alpha) = \frac{\sqrt{23}}{25}$

4. If Sin A = 0.9744 and Cos A = 0.225

Find without using tables Tan A?

Solution

$\text{Tan A} = \frac{\text{Sin A}}{\text{Cos A}}$

$= \frac{0.9744}{0.225}$

Tan A = 4.3307

5. Find without using tables sin α if cos α = 0.9272 and tan α=0.404

Solution

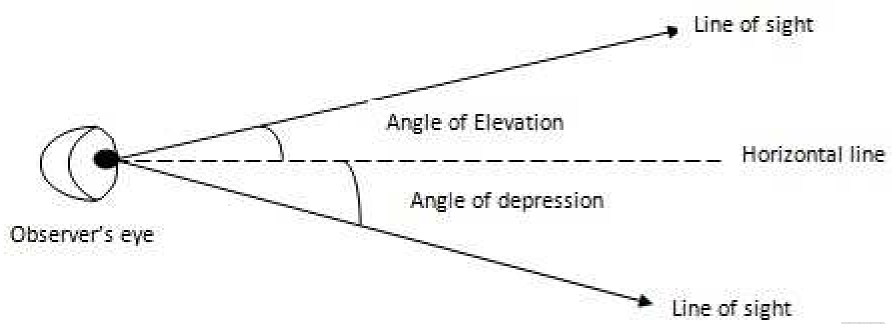
$\text{Tan } \alpha = \frac{\sin \alpha}{\text{Cos } \alpha}$

$0.404 = \frac{sin\alpha}{0.9272}$

Sin α = 0.3746

APPLICATIONS OF TRIGONOMETRICAL RATIOS

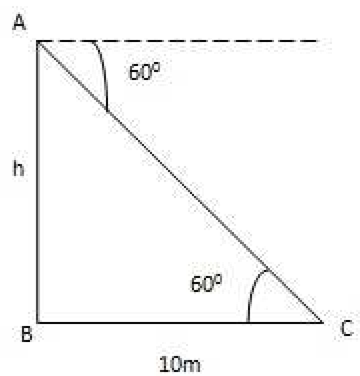
Angles of depression and elevation



Example and exercise

1. From the top of a tower , the angle of depression of a point on the ground 1M away from the base of the tower is 60^0. How high is the tower?

Solution



Tan60^0= $\frac{\text{opposite}}{\text{Adjacent}}$

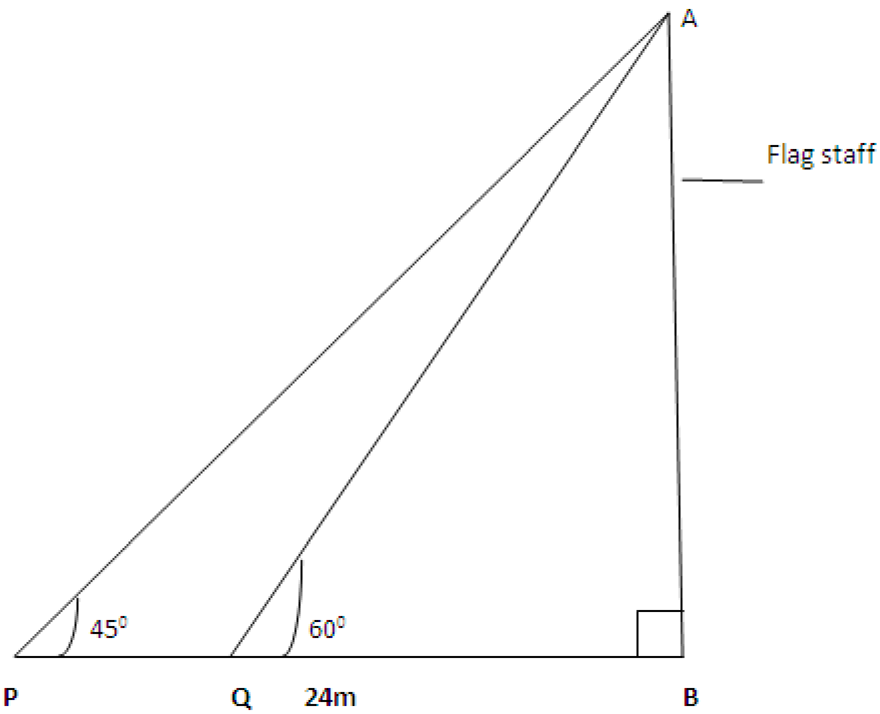
Tan60^0= $^{BA}/_{10m}$

BA = tan 60^0 x 10 m

BA = 17.321m = height of a tower

2. P and O are two pegs on level ground and both lie due west of a flag staff. The angle of elevation of the top of the flagstaff from P is 45^0 and from Q is 60^0. Find distance PQ.

Solution



$\text{Tan}45^0 = \frac{AB}{24m}$

$AB = 24m \times \tan 45^0$

$AB = 24m$

$\text{Tan } 60^0 = {}^{AB}/_{QB}$

$QB = \frac{24m}{\text{Tan } 60^0}$

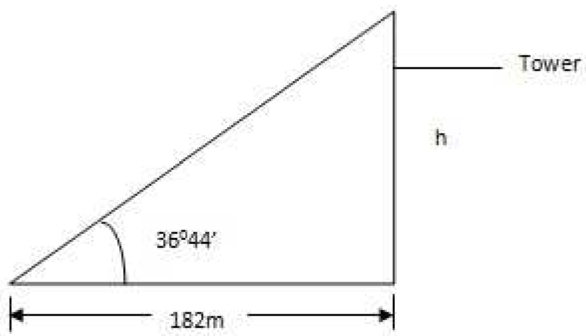
$QB = 13.85m$

$PQ = 24m – 13.85m$

$PQ = 10.15m$

3. At a point 182 m from the foot of a tower on a level road, the angle of elevation of the top of the tower is $36^0 44^1$. Find the height of the tower.

Solution



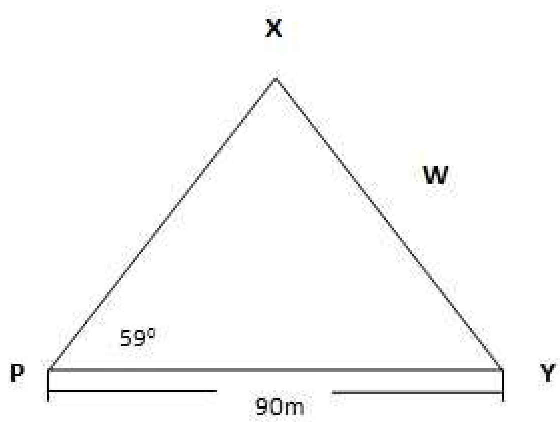
$\text{Tan } 36^0 44' = \frac{h}{182m}$

h = 0.7463 x 182 m

h = 135.8266m

4. x and y are two points on opposite banks of a river(figure below) . If PY measures 90m and XPY = 59^0. Find the width of the river.

Solution



$$\tan 59^0 = \frac{w}{90m}$$

$W = \tan 59^0 \times 90m$

$= 1.6643 \times 90m$

$= 149.787m$

TRIGONOMETRIC SPECIAL ANGLES

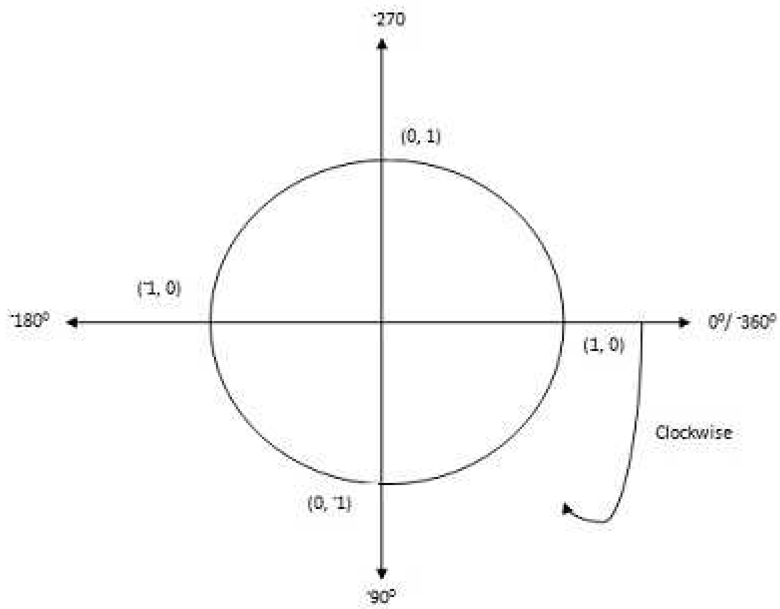
angle	0^0	90^0	180^0	270^0	360^0
Sine	0	1	0	-1	0
Cosine	1	0	-1	0	1
Tangent	0	∞	0	∞	0

Cos θ= $^{x}/_{1}$

Cos θ = x

Sin θ = $^{y}/_{1}$

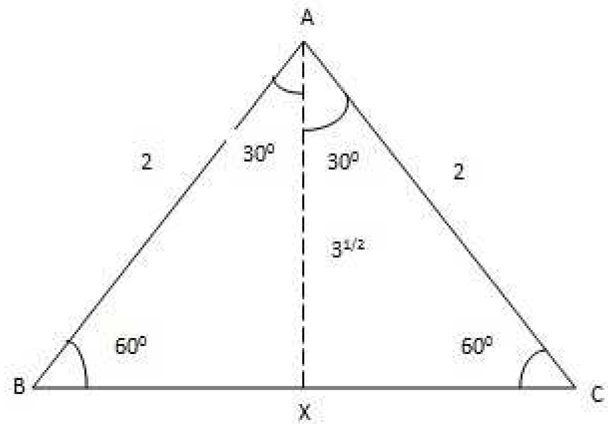
Sin θ = y



<u>OTHER SPECIAL ANGLES:</u>

Consider an equilateral triangle ABC

Let each side to have two (2) units



Using Pythagoras theorem

$(AB)^2 = (BX)^2 + (AX)^2$
$2^2 = 1^2 + (AX)^2$
$AX = \sqrt{3}$

$\text{Sin } 60^0 = \frac{\sqrt{3}}{2}$

$\text{Cos } 60^0 = \frac{1}{2}$

$\text{Tan } \varnothing = \frac{\sqrt{3}}{1}$

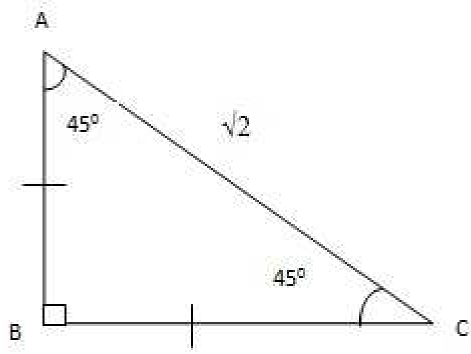
$\text{Tan } 60^0 = \sqrt{3}$

$\text{Tan } 60^0 = \frac{3}{\sqrt{3}}$

$\text{Sin } 30^0 = \frac{1}{2}$
$\text{Cos } 30^0 = \frac{\sqrt{3}}{2}$

$\text{Tan } 30^0 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

For 45^0
Consider an isosceles triangle ABC



Sin $45^0 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Cos $45^0 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Tan $45^0 = 1$

EXAMPLES

1. Find the sine, cosine and tangent of each of the following angles.

(a) -135^0

$= 360^0 + -135^0$
$= 360^0 - 135^0$
$= 225^0$
$= 225^0 - 180^0$

Sin $45^0 = \frac{\sqrt{2}}{2}$

Cos $45^0 = \frac{\sqrt{2}}{2}$

Tan $(-135^0) = \tan 45^0 = 1$

b. 330^0

$= 360^0 - 330^0$
$= 30^0$

$= \sin 330^0 = -\sin 30^0 = -\frac{1}{2}$

$\text{Cos } 330^0 = \cos 30^0 = \frac{\sqrt{3}}{2}$

$\tan 330^0 = -\tan 30^0 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

2. Find the values of the following without using tables.

$$\frac{\text{Tan } 30^\circ \ \sin 60^\circ}{\text{Cos } 45^\circ}$$

$$= \frac{1/(\sqrt{3}) \ x \ (\sqrt{3})/2}{\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{\sqrt{2}} \text{ x } \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

3. $\frac{\tan(-300)\cos 600}{\text{Sin }(-45)}$

$$= \frac{-\frac{1}{\sqrt{3}} \text{ x } \frac{1}{2}}{\frac{\sqrt{2}}{2}}$$

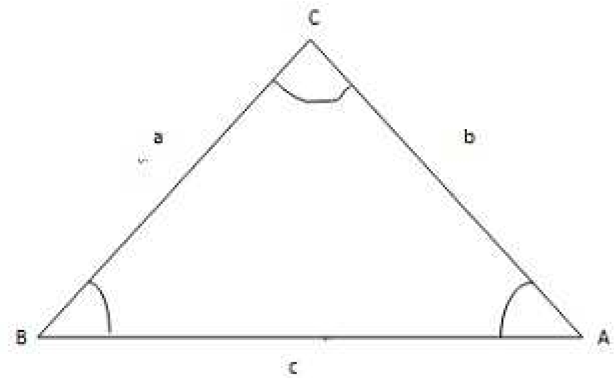
$$= \frac{\sqrt{3}}{2\sqrt{3}} \text{ x } \frac{2}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

SINE RULE

Let us consider a triangle ABC, with its including angle C

Let us find the area of the triangle using its including angle and two sides.



Area of triangle ABC = $\frac{1}{2}$ x a x b x sin C

Area of triangle ABC = $\frac{1}{2}$ x a x c x sin B

Area of triangle ABC = $\frac{1}{2}$ x b x c x sin A

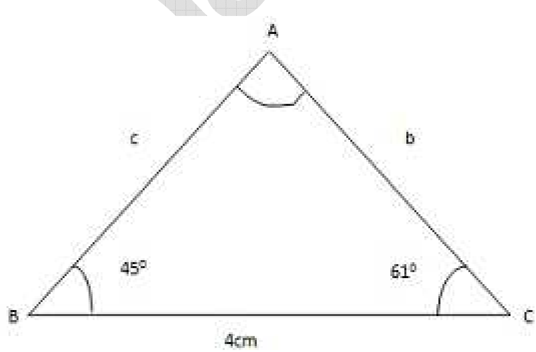
$\frac{1}{2}$ x a x b x sin c = $\frac{1}{2}$ x a x c x sin B = $\frac{1}{2}$ x b x c x sin A

Dive each by $\frac{1}{2}$ x a x c

$$\frac{\text{Sin C}}{c} = \frac{\text{Sin B}}{b} = \frac{\text{Sin A}}{a}$$

<u>Examples</u>

1. Find the unknown sides and angle sin each of the following triangles.



Solution

$$\frac{\text{Sin B}}{b} = \frac{\text{Sin A}}{a}$$

$$\frac{\sin 45}{b} = \frac{\sin A}{4\text{ cm}}$$

but A = $180^0 - (61+43)^0$

$$\frac{\sin 43}{b} = \frac{\sin 76}{4\text{cm}}$$

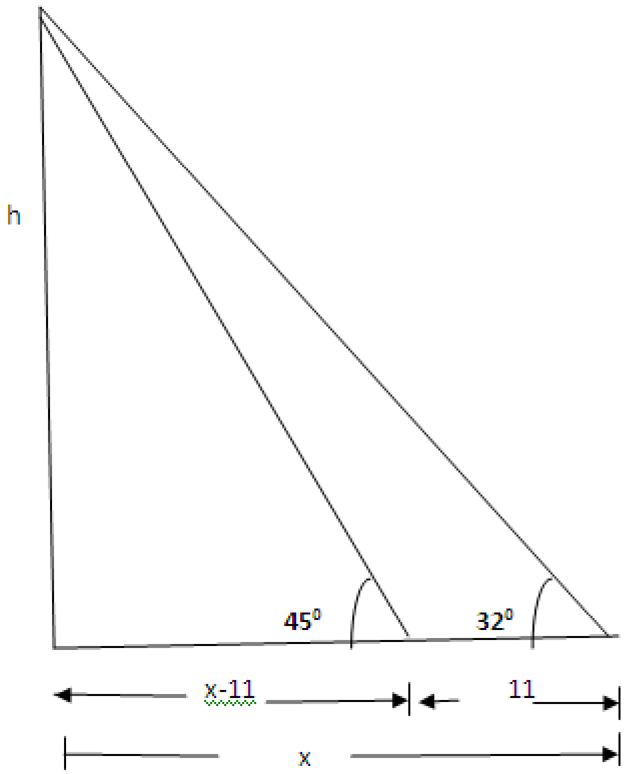
$$\frac{0.682}{b} = \frac{0.9703}{4}$$

$$b = \frac{0.682 \times 4}{0.9703}$$

No.	Logarithm
6.820×10^{-1}	$\bar{1}.8338$
4×10^{0}	$+ \underline{0.6021}$
	0.4359
9.703×10^{-10}	$\underline{\bar{1}.9869}$
2.812×10^{0}	$\underline{0.4490}$
= 2.81cm	

2. Juma notices that the angle of elevation of a coconut tree is 32^0. Walking 11 m in the direction towards the tree he notices the angle of elevation to be 45^0. Find the height of the tree.

Solution



$\text{Tan}45^0 = \frac{h}{x-11}$

$h = \tan45^0(x - 11)$

$h = x - 11$.....(i)

$\text{Tan } 32^0 = \frac{h}{x}$

$h = \tan 32^0 \times x$

$h = 0.6249x$............ii)

compare i) and ii) eqns

$x-11=0.6249x$

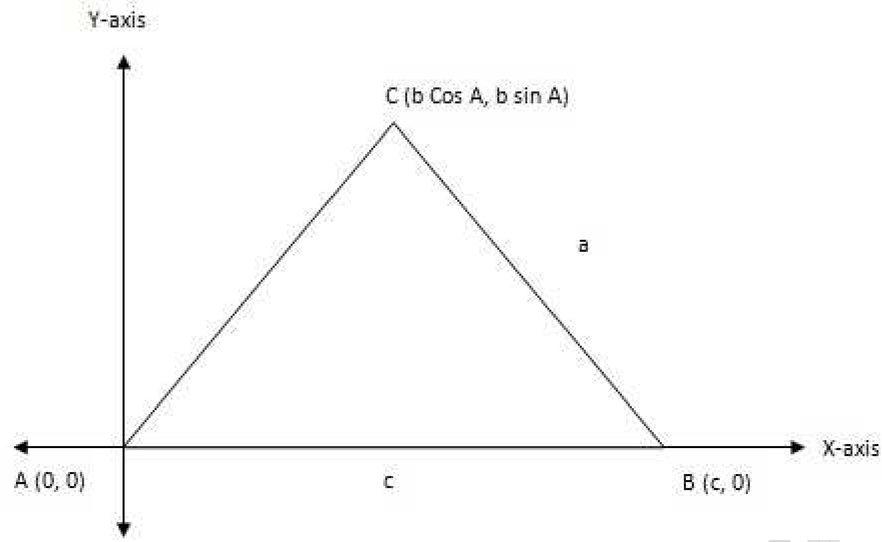
$x-0.6249x=11$

$0.3751x= 11$

$x=0.0341m$

COSINE RULE

Consider a triangle ABC whose coordinates are A (0,0) , B (c, 0) and C (b cos A , b sin A)



Cos A = $^{x}/_{b}$

Sin A = $^{y}/_{b}$

X = b cos A

Y = b sin A

$a^2 = (b \cos A - c)^2 + (b \sin A - 0)^2$

$a^2 = b^2\cos^2 A - 2bc\cos A + c^2 + b^2 \sin^2 A$

$a^2 = b^2 \sin^2 A + b^2\cos^2 A - 2bc\cos A + c^2$

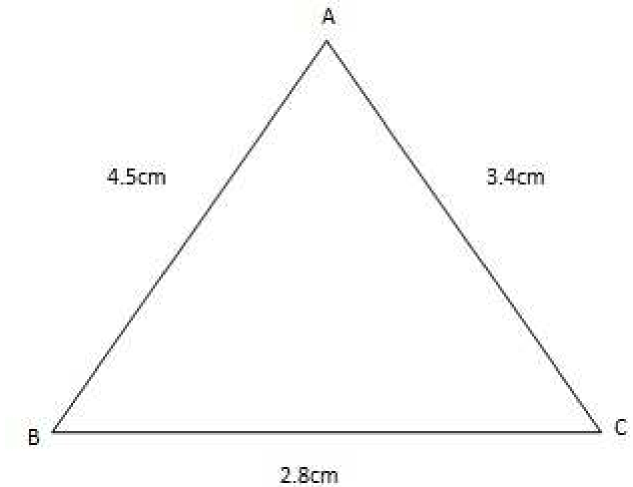
$a^2 = (\sin^2 A + \cos^2 A)b^2 + c^2 - 2bc\cos A$

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$ cosine rule

$c^2 = b^2 + a^2 - 2ab \cos c$

<u>Example</u>



Find the value of angle A.

$a^2 = b^2 + c^2 - 2bc\cos A$
$(2.8)^2 = (3.4)^2 + (4.5)^2 - 2 \times 3.4 \times 4.5 \times \cos A$
$3^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos A$
$\cos A = 5/6 = 0.8333$
$A = \cos^{-1} 0.8333$

No.	logarithm
0.5×10^{1}	1.6990
0.6×10^{1}	<u>- 1.7782</u>
8.33×10^{-1}	<u>-1.9208</u>
= 83.33	
$30^0 34'$	

$A = 33^0 30'$

<u>COSINE OF THE SUM AND DIFFERENCE OF TWO ANGLES (A and B)</u>

Cosine $(A + B) = \cos A \cos B - \sin A \sin B$
Cosine $(A-B) = \cos A \cos B + \sin A \sin B$

Verify that

Cos (90 – 60) = cos 90 cos 60 + sin 90 sin 60

Cos 30= cos 90^0cos 60^0 + sin 90^0 sin 60^0

$\frac{\sqrt{3}}{2} = 0 \times {}^1/_2 + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

<u>Questions</u>

1. find the cosine of 75^0 without using mathematical tables.

solution

Cos 75^0 = cosine (30 + 45)

= cosine 45 cosine 30 – sin45^0 sin 30^0

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times {}^1/_2$

$= \frac{2.448 - 1.414}{4}$

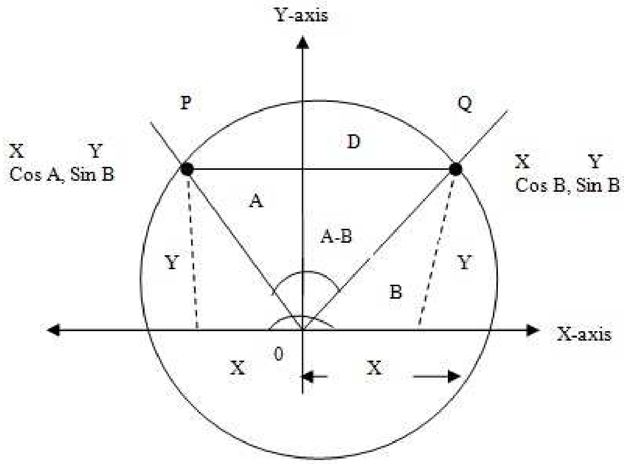
$= \frac{1.035}{4}$

= 0.2588

We use the knowledge of coordinate geometry to find the distance and cosine rule.

Consider a unit circle of radius 1 with points P and Q and angles A and B shown in the figure.

Let the distance from P to Q be d.



By distance formula

$d^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$

$d^2 = \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B$
$d^2 = \cos^2 A + \sin^2 A + \cos^2 B + \sin^2 B - 2\sin A \sin B + \sin^2 B$
$d^2 = 1+1 - 2(\cos A \cos B + \sin A \sin B)$

$d^2 = 2 - 2(\sin A \sin B + \cos A \cos B)$ ……. (i)

by the cosine rule

$d^2 = 1^2 + 1^2 - 1\cos(A-B)$

$d^2 = 2 - 2\cos(A-B)$ ……(ii)

equate equation (i) and (ii)

$2 - 2\cos(A-B) = 2-2(\cos A \cos B + \sin A \sin B)$
$-2\cos(A-B) = -2(\cos A \cos B + \sin A \sin B)$

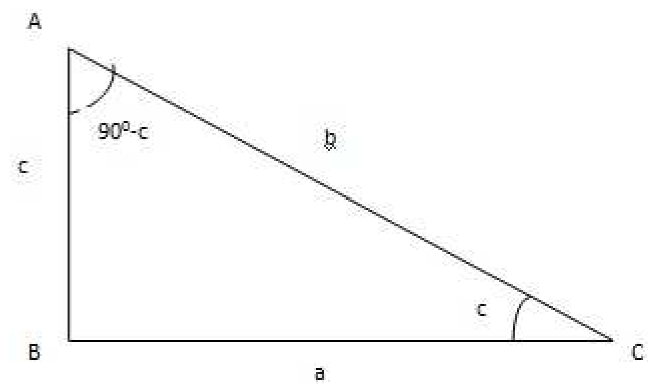
$\cos(A - B) = \cos A \cos B + \sin A \sin B$ …. (iii)

also

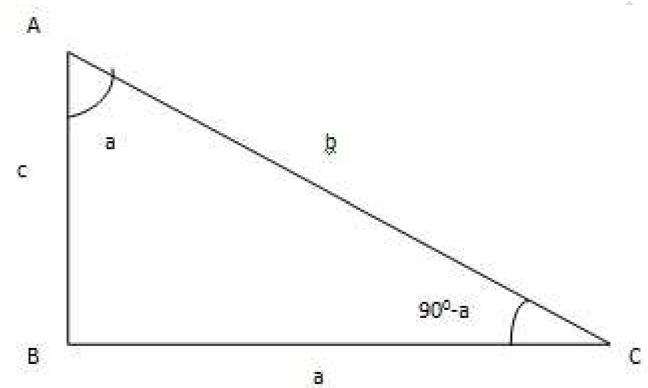
$\cos(A+B) = \cos(A - -B) = \cos A \cos -B + \sin A \sin -B$
$= \cos A \cos B - \sin A \sin B$ ……(iv)

THE SINE OF THE SUM AND DIFFERENCE OF ANY TWO ANGLES

Consider a triangle ABC and c as a acute angle.

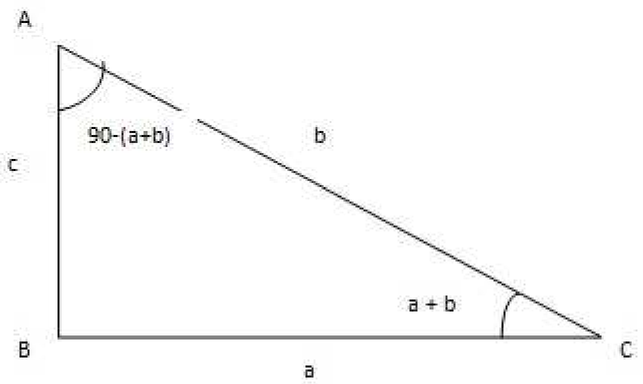


Sin C = Cos (90 – C) = $^c/_b$........ (i)
Now let C = 90 – A



Sin (90 – A) = Cos A = c/b .....(ii)

Also let c be A + B



Sin (A+ B) = cos (90 – (A+B))
Sin (A+ B) = cos((90 – A) –B)
Sin (A+B) = cos (90- A) cos B + sin (90- A) Sin B
Sin (A+B) = sin A cos B + sin B (cos A)
Sine of the difference of any two angles A and B
Refer to the above expression
Sin (A-B) = sin (A + - B) = sin A cos –B + sin –B cos A

Sin A cos B – sin B cos A __ Difference of any two angles.

<u>Exercise</u>

1. (a) Find the truth set of

   Sin θ = - $^{1}/_{2}$ in the domain $0 \le \theta \le 360^0$

   (b) verify that for any small angle A^0
   cos (90-A) = sin A

Solution

a. sin θ = - $^{1}/_{2}$
sin negative is in the 3^{rd} and 4^{th} quadrant
3^{rd}

$\Theta - 180^0 = 30^0$
$\Theta = 180 + 30^{\ 0}$
$\Theta = 210^{\ 0}$

4^{th}

$360 - \theta = 30^0$
$- \theta = 30^0 - 360^0$

$-\theta = -330^0$
$\theta = 330^0$

The truth set of $\sin\theta = {}^{-1}/_{2}$ = $0^0 \le \theta \le 360^0$

b. cos (90-A) = sin A
cos 90 cos A + sin 90 sin A =sin A
0 x cos A + 1 x sin A = sin A
Sin A = sin A

2. Use sin(S + -t) to help find a formula for sin (s $-$ t)

Solution

S $-$ t = s + (-t)
= sin S cos –t + sin (-t) cos S
= sin S cos t – sin t cos S

3 . Verify that $\sin\left({}^{2\pi}/_{3} + {}^{5\pi}/_{3}\right) = \sin {}^{2\pi}/_{3} \cos {}^{5\pi}/_{3} + \sin {}^{2\pi}/_{3} \sin {}^{5\pi}/_{3}$

$\theta = \frac{1800\ S}{\pi}$

$= \frac{1800\ x\ 2/3\ \pi}{\pi}$

$= 1800\ x\ {}^{5}/_{3}$
$= 60^0 \times 5$
$= 300^0$
$= 360^0 - 300^0$
Sin (120 + 300) = 120 cos 300 + cos 120 sin 300
Sin 420^0= sin 60 cos 60 + -cos 60 –sin 60

Sin 60 = ${}^{\sqrt{3}}/_{2}$ x ${}^{1}/_{2}$ + - ${}^{1}/_{2}$ x ${}^{\sqrt{3}}/_{2}$
${}^{\sqrt{3}}/_{2} = {}^{\sqrt{3}}/_{4} + {}^{\sqrt{3}}/_{2}$
${}^{\sqrt{3}}/_{2} = {}^{\sqrt{3}}/_{2}$

VECTORS

Displacement and Position Vectors.

Displacement

The distance moved by an object from one point A to another point B in the direction from A to B is called displacement $\overline{AB}$

Examples of vector quantities are displacement, Velocity, acceleration, force, momentum, electric field and magnetic field.

Quantities which have magnitude only scalars, for example distance, speed, Pleasure, time and temperature.

Vector are named by either two capital letters with an arrow above, like vector $\underline{\overrightarrow{OA}}$ or by a single capital or small letter in bold print like $\underline{a}$

Sometimes a single. Small letter with a bar below like a or a like $\underline{a}$

These are physical quantities which have magnitude and direction

vector quantities – have both magnitude and direction ie velocity , acceleration etc.

Scalar quantities – are quantities which only have magnitude but nit direction. ie size, mass , time....etc

Naming vectors

(i). capital letters OA

$O \longrightarrow A$

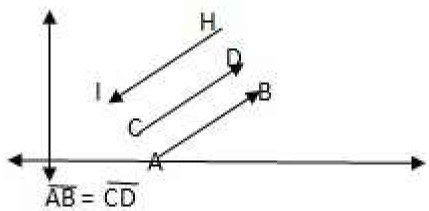
(ii). small letter a

(iii). small letter with bar

a , b

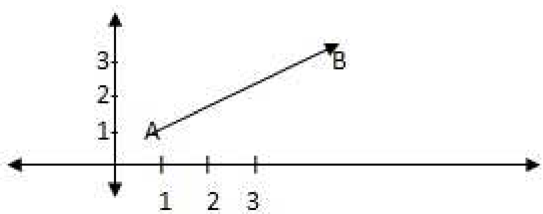
1. Equivalent vectors

Two vectors are equivalent if they have the same magnitude and direction.

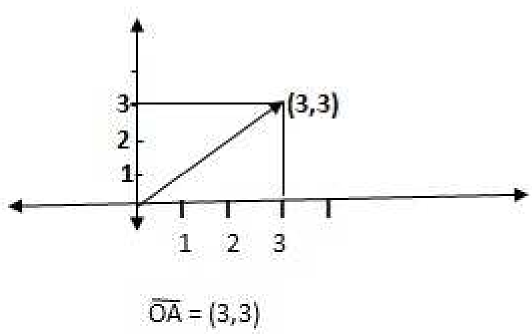


___HI is not equivalent to AB and CD

2. displacement vector



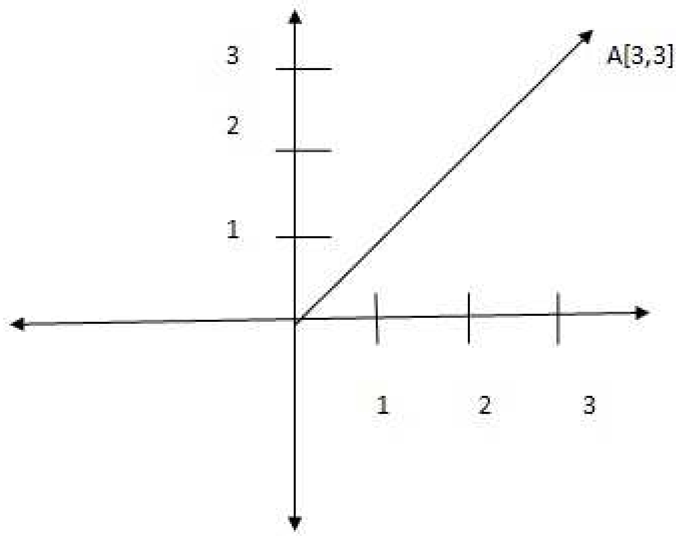
1. position vector
In the xy plane all vectors having their initial Points at the origin and their end Points elsewhere are defined as position vectors. Position vectors are named by the coordinates of their end points



OA = (3,3)

4. unit vector.
In the xy plane the position vector of unit length in the positive x-axis direction is named i and the

position vector of unit length in the positive y-axis direction is named j.
Both i and j are Unit Vectors.



$\overline{OA} = 3i + 3i$

-

<u>Questions</u>

1. write the following vectors as position vectors.

a. <u>a</u> (-3,-4) = (-3, 0) + (0,-4)

= -3(1,0) + -4 (0,1)

= -3i -4j

b. <u>b</u>(-5, 5) = (-5, 0) + (0,5)

= -5 (1,0) + 5(0,1)

= -5i + 5j

c. <u>c</u> = (-1, 6)

= (-1,0) + (0,6)

= -1 (1,0) + 6(0,1)

= -1i + 6j

Alternative = -i + 6j

2. write the following vectors as position vectors

a. S = - 8 i

= -8i + 0j

= -8 (1,0) + 0(0,1)

= (-8,0)

b. u= 7j

u = 0i +7 j

= 0(1,0) + 7 (0,1)

u = (0,7)

Exercise

1. express the following vectors in form of I and j vectors

(a)

P (-3,6)

= (-3, 0) + (0,-6)

= -3 (1,0) + 6 (0,1)

= -3i + 6j

(b)

q = (5 , -2)

= (5,0) + (0,-2)

= 5(1,0) + - 2(0,1)

= 5i - 2j

(c)

r = (-4 , 2)

r = (4,0) + (0,2)

= -4(1,0) + 2 (0,1)

= -4i + 2j

3. express each of the following in a position vector

(a). a = 3i + 2j

= 3(1,0) + 2(0,1)

= (3,0) + (0,2)

= (3,2)

(b). b = 6i – 7j

= 6(1,0) – 7 (0,1)

= (6,0) + (0,-7)

= (6,-7)

(c). e = -3i

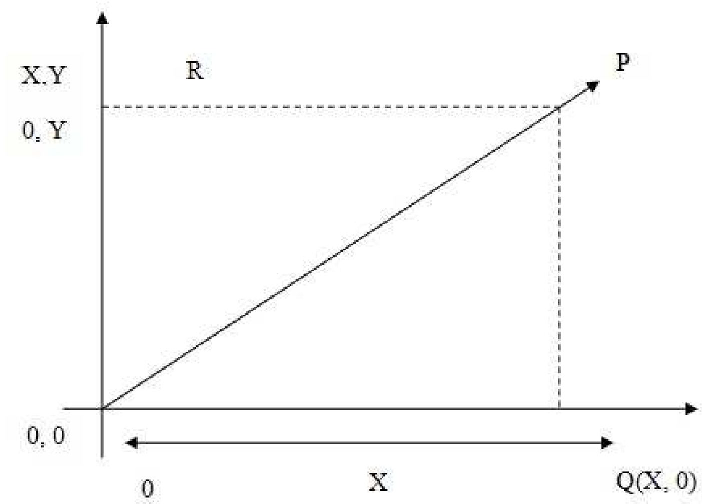
= 0 (1,0) + (0,-3)

= (0,-3)

MAGNITUDE (MODULUS) OF A VECTOR

The magnitude of vectors is used to define the size of a vector. The other name for magnitude is modulus. The magnitude or modulus of a vector is a scalar quantity.

Consider the position vector $\underline{r}$ = (x, y)



Vector OP = $\underline{r}$ = (x,y)

OP is perpendicular to the x-axis

PR is perpendicular to the y-axis

Triangle OPQ is right angled at Q

By Pythagoras theorem

$PQ^2 = OQ^2 + QP^2$

$OP^2 = x^2 + y^2$

OP = $\sqrt{x^2 + y^2}$

If $\underline{r}$ = (x,y) then

$|r| = \sqrt{x^2 + y^2}$

this is the formula for finding the magnitude

Example;

Calculate the magnitude of the position vector

v = (-3, 4)

$|v| = \sqrt{(-3)^2 + 4^2}$

= 5

Unit vector

If U is any vector, the unit vector in the direction of u is given by

$\frac{U}{|U|}$ and is denoted as $\hat{u}$

Example

Find the unit vector in the direction of the vector u= (12,5)

$|U| = \sqrt{12^2 + 5^2} = 13$

Let u be the unit vector.

$$u = \left(\frac{12}{13}, \frac{5}{13}\right) = \underline{\underline{\frac{12i}{13} + \frac{5}{13}j}}$$

Exercise

1. define the magnitude of a vector hence calculate the magnitude of

a= -12i – 5j

solution

$a = -12i - 5j$

$= -12(1,0) - 5(0,1)$

$= (-12,0) + (0,-5)$

$= (-12,-5)$

$|a| = \sqrt{-12^2 + -5^2}$

$= \sqrt{169}$

$= 13$

2. a. find $i + j$

$= 1,1$

$= \sqrt{1^2 + 1^2}$

$= \sqrt{2}$

SCALAR MULTIPLICATION

Example;

1. if $\underline{a} = 3i + 3j$ and $\underline{b} = 5i + 4J$ find, $-5\underline{a} + 3\underline{b}$

$= -5\underline{a} + 3\underline{b}$

$= -5(3i + 3j) + 3(5i + 4J)$

$= -15i - 15j + 15i + 12j$

$= -3j$

2. given $\underline{p} = (8,6)$ and $\underline{q} = (7,9)$

Find $9\underline{p} - 8\underline{q}$

$= 9(8,6) - 8(7,9)$

$= (72,54) - (56, 72)$

$= (16,-18)$

3. given vector $a = -\underline{i} + 3\underline{j}$, $b = 5\underline{i} - 3\underline{j}$ and $c = 4\underline{a} + 3\underline{b}$

(a) find the magnitude of vector c

(b) find the unit vector in the direction of vector d where $d = 2\underline{a} + -3\underline{b} + \underline{c}$

<u>solution;</u>

(a) $\underline{c} = 4\underline{a} + 3\underline{b}$

$= (-\underline{i} + 3\underline{j}) + 3(5\underline{i} - 3\underline{j})$

$= -4\underline{i} + 12\underline{j} + 15\underline{i} - 9\underline{j}$

$= 11\underline{i} + 3\underline{j} \quad (11,3)$

$|c| = \sqrt{11^2 + 3}$

$= \sqrt{121 + 9}$

$= \sqrt{130}$

b. $d = 2\underline{a} - 3\underline{b} + c$

$= 2(-i + 3\underline{j}) - 3(5\underline{i} + -3\underline{j}) + 11i + 3j$

$= -2\underline{i} + 6\underline{j} - 15\underline{i} + 9\underline{j} + 11\underline{i} + 3\underline{j}$

$= -2\underline{i} - 15\underline{i} + 11\underline{i} + 6\underline{j} + 3\underline{j} + 9\underline{j}$

$= -6\underline{i} + 18\underline{j}$

$(-6, 18)$

$|d| = \sqrt{6^2 + 18^2}$

$\hat{d} = \left(\frac{-6}{3\sqrt{40}} , \frac{18}{3\sqrt{40}}\right)$

ADDITION OF VECTORS

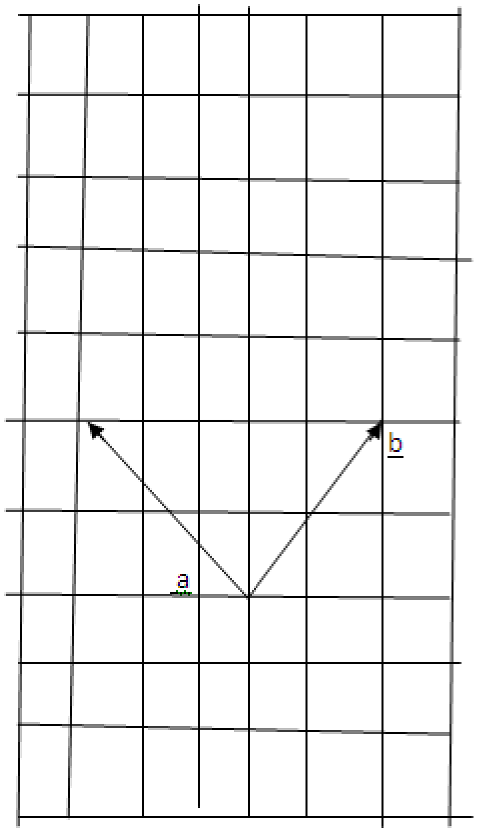
The sum of any two or more vectors is called the resultant of the given vectors**.**

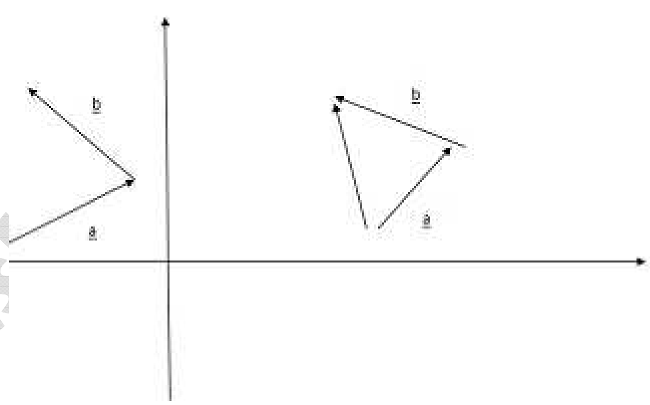
here we have 3 laws which govern us.

Resultant is the one which represents the sum of vectors.

1. **THE TRIANGLE LAW**

The initial point of the second vector is joined to the end points. The first vector, resultant is joined by completing the triangle whose initial point is the initial point of the first vector and its end point concedes with that of the second vector.





8^2

= 10

Cos α = $^6/_{10}$

Cos B = $^8/_{10}$

Hence 0.6 and 0.8 are direction cosine of <u>a</u>

Cos α = 0.6

α= $53^0 16'$

<u>Exercise:</u>

(1) Evaluate the magnitude and direction of b= -8i + 6j

Solution

$|b| = \sqrt{-8^2 + 6^2}$

$= 10$

Cos α= $^{-8}/_{10}$ = 0.8

Cos B = $^6/_{10}$

Cos B = 0.6

B = $53^0 3'$

2. Calculate the direction cosines of c = 3i + 4j , hence show that the sum of the squares of these direction cosines is one.

Solution

$|c| = \sqrt{3^2 + 4^2}$

$= 5$

Cos α = $^3/_5$= 0.6

Cos B = $^4/_5$ = 0.8

$0.6^2 + 0.8^2 = 1$

1.00 = 1

1 = 1

3. Find the direction cosine of

(a) a = i+ j

$$|a| = \sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \underline{\underline{\sqrt{2}}}$$

Cos α = $1/\sqrt{2} \times \frac{\sqrt{2}/2}{\sqrt{2}/\sqrt{2}} = \sqrt{2}/2$

Cos B = $1/\sqrt{2} \times \frac{\sqrt{2}/2}{\sqrt{2}/\sqrt{2}} = \sqrt{2}/2$

The direction of cosine a is

4. given a = (1,2) b = (-2,-1) and c = (3,7)

Calculate 2a + 3b + 4c

Solution

= 2(1,2) + 3(-2,-1) + 4(3,7)

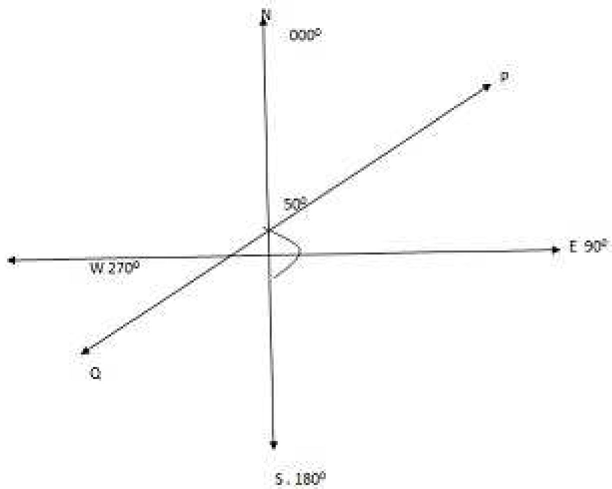
= (2,4) + (-6,-3) + (12,28)

= (8,29)

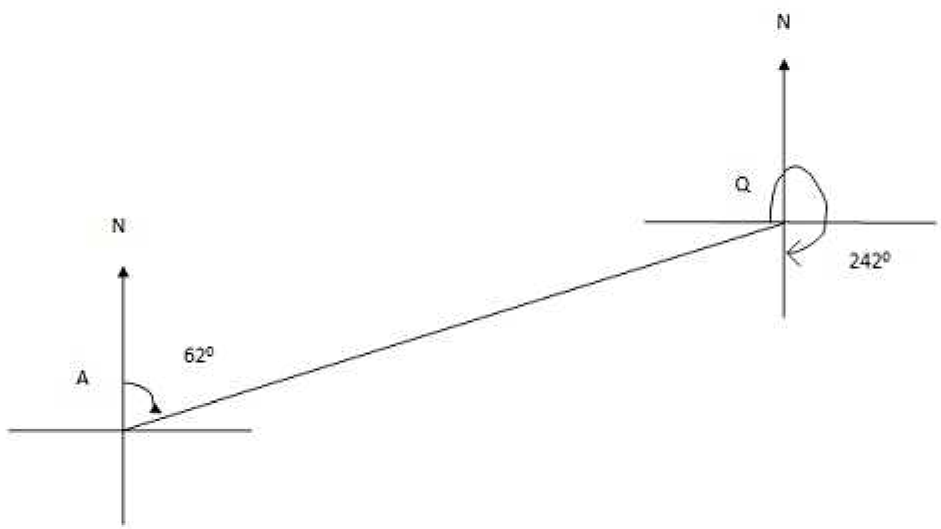
BEARING

1. Two methods used in reading bearing of all angles are measured with reference from the north direction only.

North is taken as 000^0 , east as 090^0, south as 180^0, and west as 270^0.



Point p located at a bearing of 050^0. Point Q is located at a bearing of 200^0.



Bearing of point B from point A is measured from north direction at point A to the line joining AB (bearing of point B from A is 062^0)

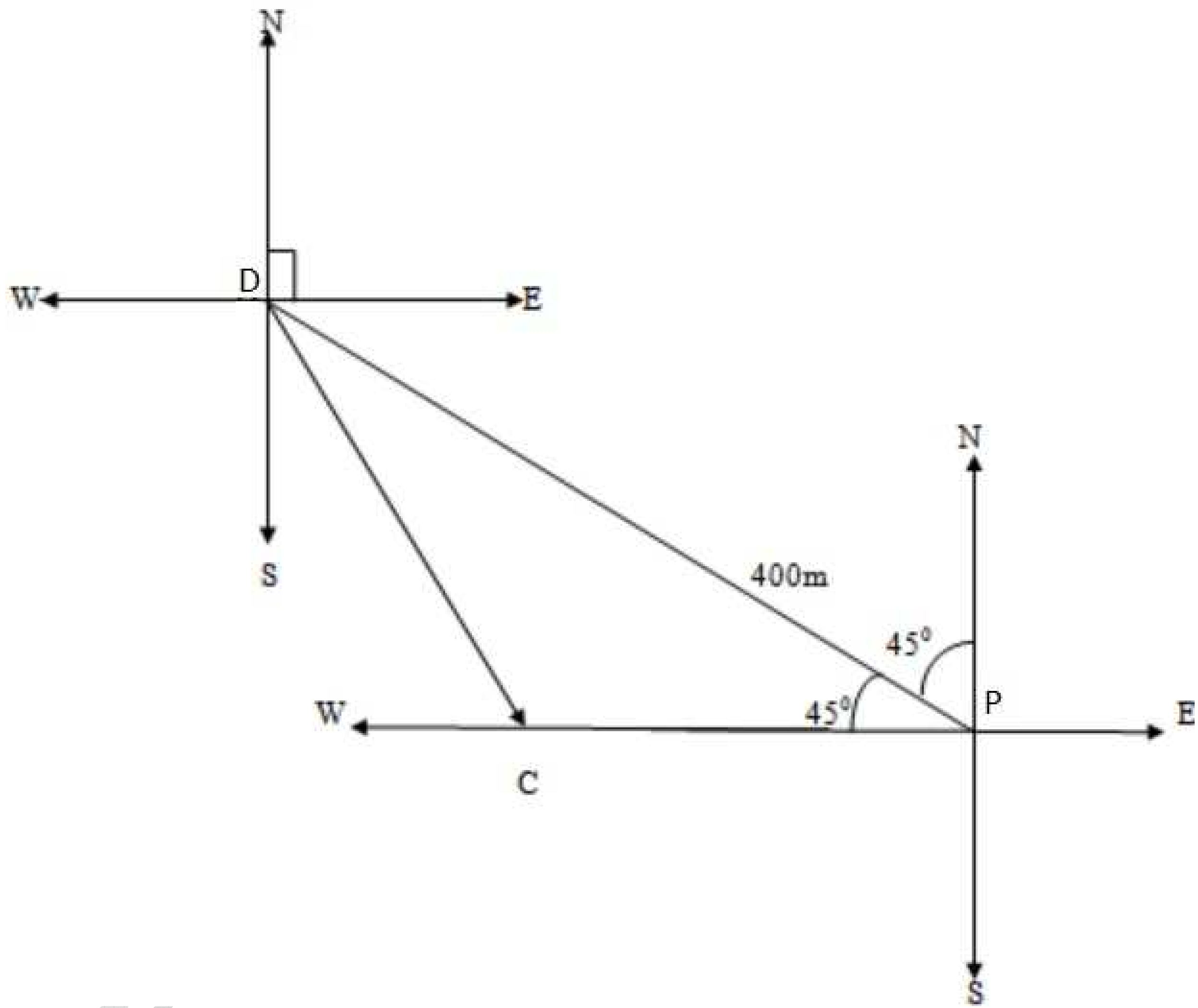
Bearing of A from B is measured from the north direction at B to the line joining BA.

(Bearing of A from B is 242^0)

Application of vectors

Questions:

1. a student walks 400m in the direction of S45^0E from the dormitory to the parade ground and then he walks 100m due west to his classroom. Find his displacement from the dormitory to the classroom.



From figure above the resultant is $\overrightarrow{DC}$. By the cosine rule.

$\overline{DC}^2 = 400^2 + 100^2 - 2 \times 400 \times 100 \cos 45^0$

$= 160000 + 10000 - 80000 \times \sqrt{2}/2$

= 170000 $-$ 56568.5

=113440

From figure above the resultant is $\overrightarrow{DC}$ By the cosine rule.

$$\overrightarrow{DC}^2 = 400^2 + 100^2 + 2\text{Cos } 45^0$$
$$= 160{,}000 + 10{,}000 - 80{,}000$$
$$= 17{,}000 - 56{,}560$$
$$= 113{,}440$$

$$\overrightarrow{DC}^2 = \sqrt{113440} = 336.8 \text{ metres}$$

Let $C\hat{D}P = \emptyset$

Then by the sine rule

$$\frac{\text{Sin } \emptyset}{100} = \frac{\sin 45^0}{336.8}$$

$$\text{Sin } \emptyset = \frac{0.707 \times 100}{336.8}$$

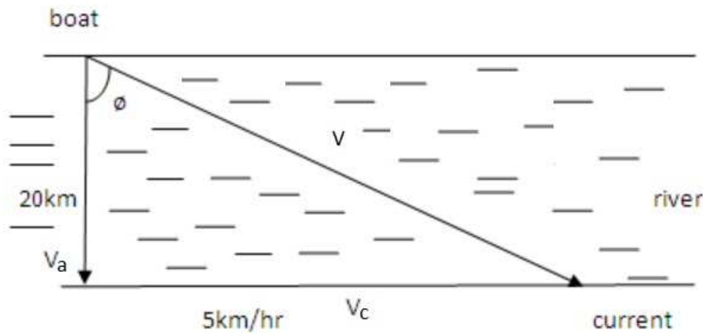
$$\emptyset = 12^0$$

Bearing = $S(45^0 - 12^0)E = S33^0E$.

The boy displacement from the dormitory to the classroom is 336.8metres at a bearing of $S33^0E$

2. A boat crosses a river at velocity of 20km/ hr southwards. The river has a current of 5km/hr due east. Calculate the resultant velocity of the boat.

Solution



<u>Solution</u>

Let the velocity of the boat be v that of the current y and the resultant velocity v

Let ø be the angles between v and v_b

$$Rr = 5i - 20j$$

$$|\mathbf{Rr}| = \sqrt{52 - 202}$$

$$= \sqrt{25 + 400}$$

$$= \sqrt{425}$$

Let θ be the angle between v and v_b.

Then, $\tan ø = \frac{5}{20} = 0.25$

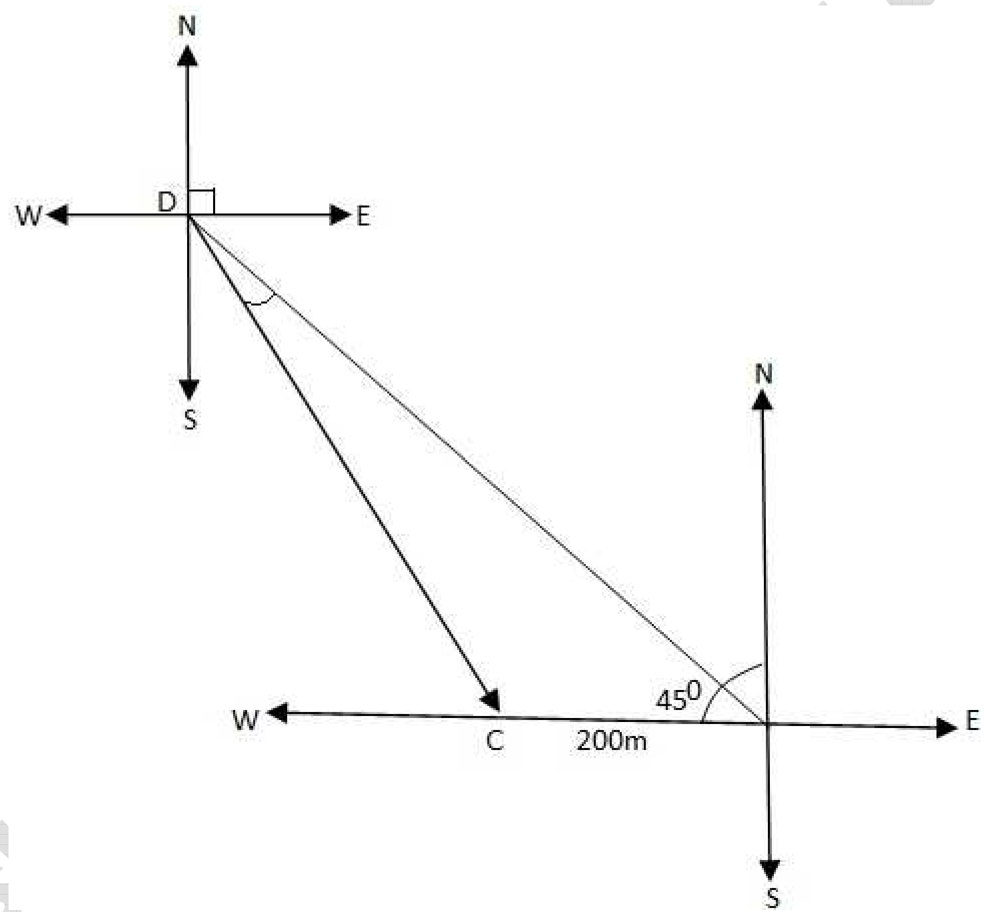
$$ø = 14^0\ 2$$

No.	logarithm
$(4.25 \times 10^2)^{1/2}$ 2.002×10^1	<u>2.6284</u> 2 1.3142

Hence the resultant velocity of the boat is 20.62km/hr at bearing of S 14

3. A student walks 500m in the direction S 45^0E from the classroom to the basketball ground and then she walks 200m due west to the dormitory. What Is her displacement from the classroom.

Solution



= 650000 – 200,000x 1.414

= 290,000- 282800

= $\sqrt{367200}$

No.	Logarithm
$(3.672 \times 10^5)^{1/2}$ 63639	$\underline{5.5649}$ 2 2.782445

= 636

4. An aeroplane flies with the speed of 100km/ hr and the wind is blowing from south with speed 40km/hr.

(a). find the time used by an aeroplane to fly due north the distance 70km.

(b). in what bearing must the pilot set his plane in order to fly due east.

(c). find the resultant speed of an aeroplane to fly east in the nearest km/hr and time taken to fly t he distance 296 km due east.

Solution

(a)

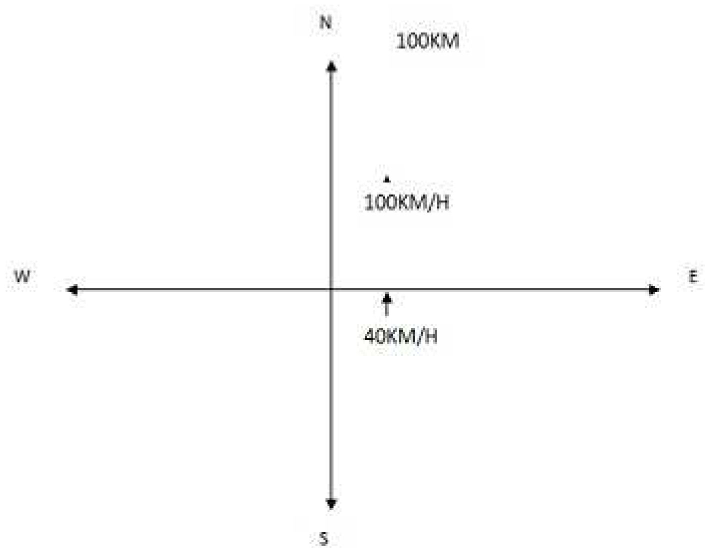
v = 100km/hr + 40km/hr = 140km/hr

Time = d/v

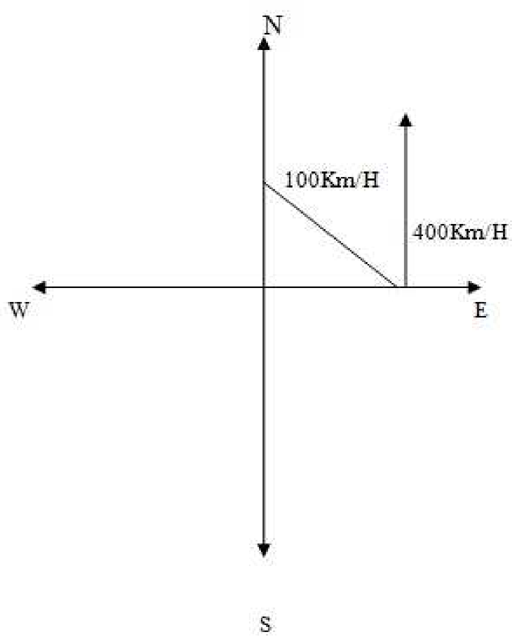
$= \frac{70\text{km}}{140\text{km/hr}}$

$= {}^{1}/_{2}$ hr

100KM



(b). cos θ= $\frac{40km}{100\ km/hr}$



$100^2 = 40^2 + X^2$

$10000 - 16000 = X^2$

X = $\sqrt{8400}$

No.	Logarithm
$(8.4 \times 10^3)^{½}$	$\frac{3.9243}{2}$
9.164×10^1	1.9621

Resultant (s) = 91.64 $\approx$ 92

T = D / V

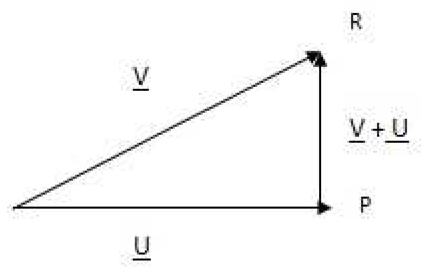
= $\frac{296}{92}$

= 3.217

2. THE PARALLELOGRAM LAW

Two vectors sharing a common initial point P to get the resultant we should complete a parallelogram.

Example;



PR = U + V = V + U

3. THE POLYGON LAW OF VECTOR ADDITION.

Here the resultant is obtained by joining endpoint to initial point of the vector one after another.

The resultant is the vector joining the initial point of the first vector to the end point of the second vector.

Exercise 5.3

1. given the vectors p, q, r, s and t in the figure.

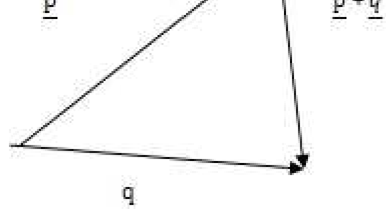
Find (a). P+Q

(b). P+Q+R

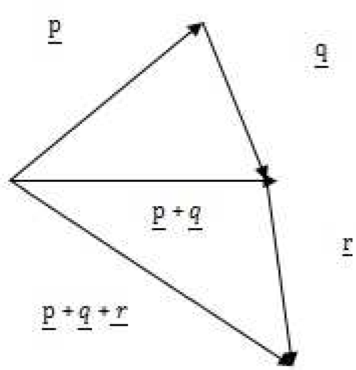
(c). P+Q+R+S

Solution

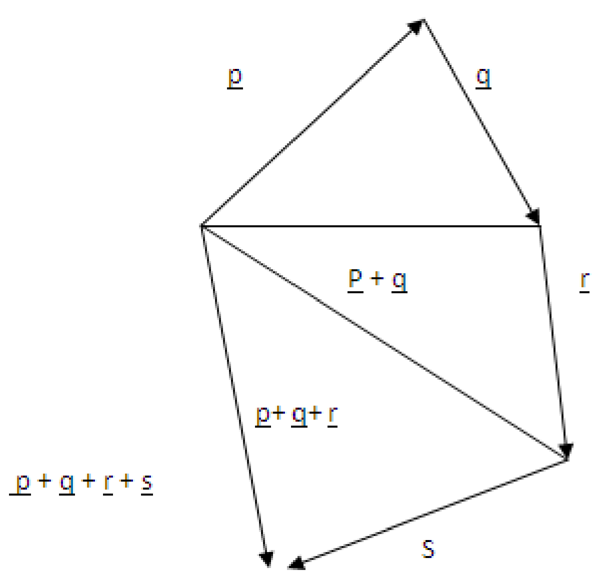
a.



b.



c.

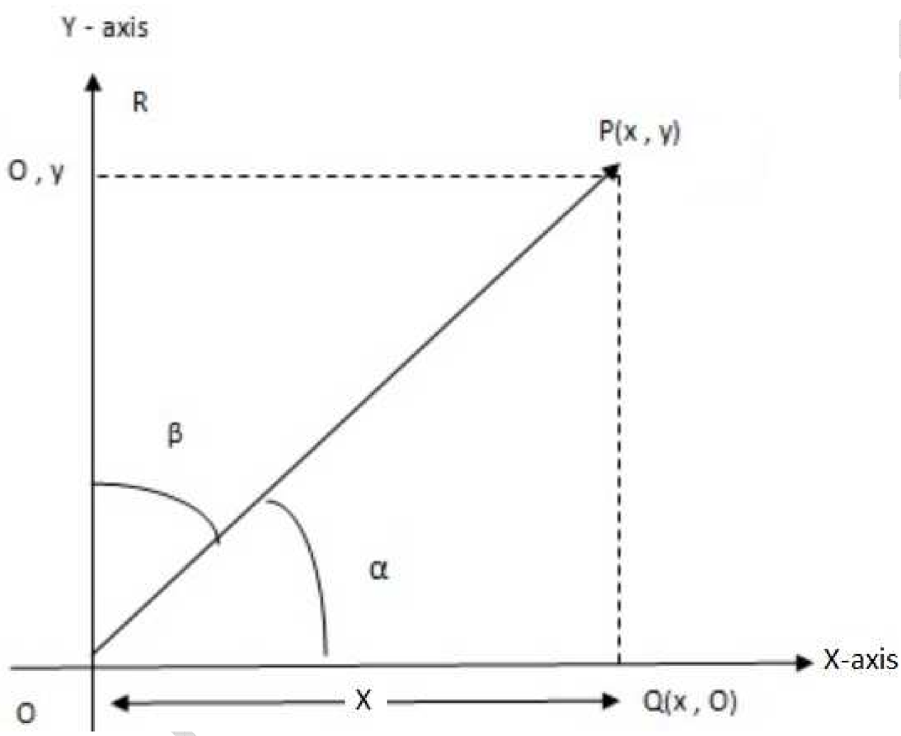


DIRECTION OF A VECTOR

1. **DIRECTION COSINE**

Consider a vector $\overrightarrow{OP}$ = (X, Y)

OP makes angle α with the positive direction along the direction of the y-axis



Triangle OQP is right angled at Q

P$\hat{O}$R = O$\hat{P}$Q (alternate interior angle)

OPQ = β

Cos α = $\frac{X}{|\overline{OP}|}$

Cos B = $\frac{y}{|\overline{OP}|}$

The values of cos α and cos B are the direction cosines of vector $\overrightarrow{OP}$

Example

1. If a = 6i + 8j

Find the direction cosines of a , hence

Find the angle that a makes with the positive axis.

$|a| = \sqrt{6^2 + 8^2}$

MATRICES

Operations on matrices

A matrix represents another way of writing information. Here the information is written as rectangular array. For example two students Juma and Anna sit a math Exam and an English Exam. Juma scores 92% and 85%, while Anna scores 66% and 86%. This can be written as.

$$\begin{pmatrix} a11\ a12 \ldots\ldots .an \\ a21\ \ a22 \ldots ..a2n \\ am\ \ \ am2 \ldots ..amn \end{pmatrix}$$

A size of a matrix is known as its order and is denoted by the number of rows times the number of columns. Therefore the order of above matrix is $\begin{pmatrix} 36 & 38 & 40 & 41 \\ 38 & 41 & 29 & 30 \\ 29 & 50 & 35 & 42 \end{pmatrix}$ each of the numbers in the matrix is called an element.

Types of matrix

(i) **Row matrix.** This is a matrix having only one row. Thus $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ is a row matrix.

(ii) **Column matrix**. This is a matrix having only one column. Thus $\begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$ is a column matrix.

(iii) **Null matrix**. This is a matrix with all its elements zero. Thus $\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$ is a null matrix or zero matrix.

(iv) **Square matrix**. This is a matrix having the same number of rows and column. Thus $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is a square matrix.

(v) **Diagonal matrix**. This is a square matrix in which all the elements are zero except the diagonal elements. Thus $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is a diagonal matrix.

Note that: The diagonal in a matrix always runs from up left to lower right.

(vi) **Unit matrix of identity matrix.** This is a diagonal or square matrix in which the diagonal elements equal to 1. An identity matrix is usually denoted by the symbol I. Thus

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(vii) **Equal matrix**. Two matrices are said to be equal if they are of the some order, responding elements are equal.

$$\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 6 & 7 \\ 9 & 10 \end{pmatrix}$$

An m x n matrix (E.g. matrix A) is a rectangular array of m x n real (or complex numbers) arranged in M horizontal rows and n vertical columns.

$$A = \begin{pmatrix} a11\,a12 \ldots\ldots .an \\ a21 \quad a22 \ldots\ldots a2n \\ am \quad am2 \ldots\ldots amn \end{pmatrix}$$

<u>Example</u>

1. The table below represents number of students in each stream in each form. Now write that information in matrix.

Form	I	II	III	IV

Stream J	36	38	30	41
Stream K	38	41	29	30
Stream L	29	50	35	42

J = $\begin{pmatrix} 36 & 38 & 40 & 41 \\ 38 & 41 & 29 & 30 \\ 29 & 50 & 35 & 42 \end{pmatrix}$

2. Give the order of the following matrices.

i.A = $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

has order 2x2

ii.B = $\begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$

has order 2x3

iii. C = (P Q) has order 1x2

iv. D = $\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$

has order 3x1

SPECIAL MATRICES

Is the matrix having all elements zero (zero matrix)

Z = $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

IDENTITY MATRIX: Is the square matrix whose elements is the leading diagonal are everywhere 1 and 0 elsewhere.

I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Identity matrix

ADDITION OF MATRICES

Matrix addition is performed by adding corresponding elements.
for example

If $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ and $B = \begin{pmatrix} e & g \\ f & h \end{pmatrix}$ then

$$A + B = \begin{pmatrix} a+e & c+g \\ b+f & d+h \end{pmatrix}$$

Example

1. Given matrices A = (1 2 3) and B = (4 5 6)

Find

i. A + B
ii. B + A

Solution

i. A + B = (1 2 3) + (4 5 6)
= (1+4 2+5 3+6)
= (5 7 9)

ii. B + A = (4 5 6) + (1 2 3)
= (4+1 5+2 6+3)
= (5 7 9)

2. If $A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 7 \\ 9 & 10 \end{pmatrix}$

Then, find A + B

$$A + B = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix} + \begin{pmatrix} 6 & 7 \\ 9 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 9 \\ 13 & 15 \end{pmatrix}$$

ADDITIVE IDENTITY MATRIX

Consider any 2 x 2 matrix $N = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ N = , where a, b, c, and dare any real number. If N + Y = Y + N = N, then N

is the addictive identity matrix.

The 2 x 2 addictive identity matrix is $Y = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Let matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

And $Z = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Then Z is an additive identity matrix.
i.e. A+Z = A and Z +A = A

Example, $A + Z = \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$= \begin{pmatrix} a+0 & b+0 \\ c+0 & d+0 \end{pmatrix}$$

$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

ADDITIVE INVERSE MATRIX

Consider any two matrices of the same order P and Q.

If P + Q = Q + P = R, then Q is called the additive inverse of P or P is called the additive inverse of Q.

i.e. Q = -P or P = -Q

Suppose,

$$P = \begin{pmatrix} a & c \\ b & d \end{pmatrix}, \quad Q = \begin{pmatrix} e & g \\ f & h \end{pmatrix} \text{ and } R = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

If P + Q = Z then,

$$Q = R - P = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$= \begin{pmatrix} 0-a & 0-c \\ 0-b & 0-d \end{pmatrix}$$

$$= \begin{pmatrix} -a & -c \\ -b & -d \end{pmatrix}$$

$$= (-1)\begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$= (-1)P$$

$$= -P$$

Therefore the additive inverse of P is -P

Example

1. Find the additive inverse of

(a) $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ (b) $C = \begin{pmatrix} -3 & 2 \\ 1 & 1 \end{pmatrix}$

Solution

a) The addictive of B = -B

$$= -\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

The addictive of B = $\begin{pmatrix} -2 & -3 \\ -4 & -5 \end{pmatrix}$

b) The addictive inverse of C = -C

$$C = -\begin{pmatrix} -3 & 2 \\ 1 & 1 \end{pmatrix}$$

The addictive inverse of C = $\begin{pmatrix} 3 & -2 \\ -1 & -1 \end{pmatrix}$

SUBTRACTION OF MATRICES

The process of subtracting a real number "f" from another real number g is the same as adding g to the additive inverse of f.

Thus f-g = f + (-g).

NOTE

When matrix P is subtracted from another matrix Q the result is the same as adding P to the additive inverse of Q.

i.e P- Q = P + (-Q).

<u>Example</u>

Given $P = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, $Q \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

Find P-Q

$$P-Q = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3-1 & 2-(-1) \\ 1-(-1) & 4-1 \end{pmatrix}$$

$$.:\ P-Q = \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix}$$

SCALAR MULTIPLICATION OF MATRICES

If $A = \begin{pmatrix} p & r \\ q & s \end{pmatrix}$

Is any real numbers, where p, q, r, s are any real numbers and x are any non zero real r

Then, $xA = x\begin{pmatrix} p & r \\ q & s \end{pmatrix}$

$Xa = \begin{pmatrix} xp & xr \\ xq & xs \end{pmatrix}$

Example If $B = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}$

i. Find 2B

Solution

$2\begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} = \begin{pmatrix} 8 & 10 \\ 12 & 14 \end{pmatrix}$

ii. Find $^1/_2$ B

Solution

$^1/_2\begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} = \begin{pmatrix} 2 & 2.5 \\ 3 & 3.5 \end{pmatrix}$

Questions

1. Given;

$A = \begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 5 & 7 \\ 6 & -8 \end{pmatrix}$

<u>FIND.</u>

(a) 3A + 2B

Solution

$3A = 3\begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$

3A = $\begin{pmatrix} 9 & 6 \\ 3 & 15 \end{pmatrix}$

2B = 2 $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

2B = $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$

3A+ 2B = $\begin{pmatrix} 9+2 & 6-2 \\ 3-2 & 15+2 \end{pmatrix}$

= $\begin{pmatrix} 11 & 4 \\ 0 & 17 \end{pmatrix}$

(b) 5 (A + B)

A+ B = $\begin{pmatrix} 3+1 & 2+-14 \\ 1-1 & 5+1 \end{pmatrix}$ = $\begin{pmatrix} 4 & 1 \\ 0 & 6 \end{pmatrix}$

5(A+B) = 5 $\begin{pmatrix} 4 & 1 \\ 1 & 6 \end{pmatrix}$

= $\begin{pmatrix} 20 & 5 \\ 0 & 30 \end{pmatrix}$

2. Using the matrices;

A = $\begin{pmatrix} 9 & 7 \\ 8 & 6 \end{pmatrix}$, B = $\begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}$ and C = $\begin{pmatrix} 2 & 4 \\ 0 & 5 \end{pmatrix}$

a) Find A (BC)

BC = $\begin{pmatrix} 5x2 + 3x0 & 3x2 + 2x4 \\ 4x2 + 2x0 & 0x3 + 2x5 \end{pmatrix}$

BC = $\begin{pmatrix} 10 & 35 \\ 8 & 26 \end{pmatrix}$

A (BC) = $\begin{pmatrix} 9 & 7 \\ 8 & 6 \end{pmatrix}\begin{pmatrix} 10 & 35 \\ 8 & 26 \end{pmatrix}$

= $\begin{pmatrix} 9X10 + 7X8 & 9X35 + 7x\,26 \\ 8X10 + 6X8 & 8X35 + 6X26 \end{pmatrix}$

= $\begin{pmatrix} 146 & 497 \\ 128 & 436 \end{pmatrix}$

b) (AB) C

AB = $\begin{pmatrix} 9X5 + 7X4 & 9X3 + 7X2 \\ 8X5 + 6X4 & 8x3 + 6X2 \end{pmatrix}$

AB = $\begin{pmatrix} 73 & 41 \\ 64 & 36 \end{pmatrix}$

C = $\begin{pmatrix} 2 & 4 \\ 0 & 5 \end{pmatrix}$

(AB)C = $\begin{pmatrix} 73X2 + 41X0 & 73X4 + 41X5 \\ 63X2 + 36X0 & 64X4 + 36X5 \end{pmatrix}$

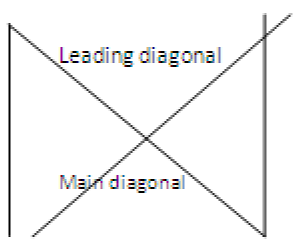
(AB)C = $\begin{pmatrix} 146 & 497 \\ 128 & 434 \end{pmatrix}$

DETERMINANT OF A MATRIX

If A is a 2×2 matrix that A = $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

Then the determinant of matrix A = $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$ is obtained by adding the product of the elements in the leading diagonal to the negative of the product of the elements of the main diagonal to the negative of the product of elements of the main diagonal and is denoted by $[A]$

Thus in 2×2 matrix



Therefore $[A] = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$= ad - bc$

NOTE

Determinants exist for square materials only.

Calculate the determinant of a matrix and tell whether the matrix is singular or non singular.

1. A = $\begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$

Solution

[A]
= (-1 x4) – (2x3)
= -4 -6
= -10

A is non singular matrix.

Inverse of matrices

The inverse of a matrix say P is another matrix denoted by P^{-1}

Inverse of matrices

The inverse of a matrix say P is another matrix denoted by P^{-1}

Such that $AA^{-1} = A^{-1}A = 1$

Where 1= $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the multiplicative identity matrix.

Consider the 2 x 2 matrix A = $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ where a, b, c, and d are any real numbers.

Let the inverse of A = A^{-1} such that $A^{-1} = \begin{pmatrix} p & r \\ q & s \end{pmatrix}$ where p, q, r and s are any real nu

Since $AA^{-1} = A^{-1}A = 1$ Then, $\begin{pmatrix} a & c \\ b & d \end{pmatrix} \times \begin{pmatrix} p & r \\ q & s \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Thus $ap + cq = 1$........... (i)

$bp + dq = 0$...........(ii)

(i) xb $abp + bcq = b$

(iii) xa $abp + adq = 0$

$bcq - adq = b$

$(bc - ad)\, q = b$

$q = \frac{-d}{ad-bc} =$

And $ar + cs = 0$........(iii)

$br + ds = 1$..........(iv)

(ii) xb $abr + bcs = a$

(iv) xa $abr + ads - a$

$bcs - ads = a$

$- (ad - bc)s = -a$

$S = \frac{a}{ad+bc}$

Thus $ap + cq = 1$........... (i)

$bp + dq = 0$...........(ii)

(i) xb $abp + bcq = b$

(iii) xa $abp + adq = 0$

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(iv) xa $abr + ads - a$

$bcs - ads = a$

$- (ad - bc)s = -a$

$S = \frac{a}{ad+bc}$

xd adq + cdq = d (ii) xd adr + cds = 0

xc + bcp = 0 (iv) xc cbr + cds = c

adp – bcp = d adr – cbr = -c

(ad – bc)p = d (ad – cb)r = -c

$P = \frac{d}{ad-bc}$ $r = \frac{-c}{ad-cb}$

Hence $A^{-1} = \begin{pmatrix} \frac{d}{ad-bc} & \frac{-c}{ad-bc} \\ \frac{-b}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$

$$\frac{1}{ad-bc}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$$

$$\frac{1}{|A|}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$$

NOTE

The inverse of a matrix say $A=\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

Can be found by interchanging the elements of the leading diagonal so that d takes place of a and a takes place of d. Change the sign of the elements in the main diagonal so that b and c becomes –b and –c respectively.

Divide each element by the determinant of A

2. Inverse exist for non singular matrix.

3. Singular matrix has no inverse because they have zero determinant.

Example

Determine the inverse of the gives matrix and indicate if it is singular or non singular.

1. A = $\begin{pmatrix} 4 & 4 \\ -4 & 4 \end{pmatrix}$

Solution

Determinant; (A) = (4x4) – (-4x4)

= 16+16

= 32

$A^{-1} = {}^{1}/_{32}\begin{pmatrix} 4 & -4 \\ 4 & 4 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 1/8 & -1/8 \\ 1/8 & 1/8 \end{pmatrix}$

A is non singular matrix.

2. B = $\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$

Solution

Determinant (B) = (-1x1) – (-1x-1)

= -1-1

= -2

$B^{-1} = -1/2 \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} {}^{1}/_{2} & {}^{-1}/_{2} \\ {}^{-1}/_{2} & {}^{-1}/_{2} \end{pmatrix}$

<u>MATRICES ON SOLVING SIMULTANEOUS EQUATIONS</u>

<u>Questions</u>

1.5X + 6Y = 1

7X + 8Y = 15

$\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \end{pmatrix}$

Let

$\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$ be A

$|A|$ = (5x8) – (7x6)

= 40 – 42

= -2

$A^{-1} = -1/2 \begin{pmatrix} 8 & 6 \\ 7 & 5 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} -4 & 3 \\ 3.5 & -2.5 \end{pmatrix}$

$= \begin{pmatrix} -4 & 3 \\ 3.5 & -2.5 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 3.5 & -2.5 \end{pmatrix}$

$= \begin{pmatrix} -4x5 + 3x7 & -4x6 + 3x8 \\ 3.5x5 + 2.5x7 & 3.5x5 + -2.5x8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4x11 + 3x15 \\ 3.5x11 + -2.5x15 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

x = 1

y = 1

2.Solve the following simultaneous equation by matrix.

4X + 2Y= 40

X + 3Y = 35

$\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 40 \\ 35 \end{pmatrix}$

Let $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ be B

$|B|$ = (4x3) – (2x1)

= 12 – 2

= 10

$B^{-1} = 1/10 \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{pmatrix}$

$\begin{pmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.3x40 + -0.2x35 \\ -0.1x40 + 0.4x35 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

X = 5
Y = 10

CRAMMERS RULE FOR SOLVING SIMULTANEOUS EQUATIONS

Is a rule used to solve the simultaneous equations

Consider the following examples

Solve the following system of simultaneous equation

1. 5X+6Y = 11
   7X + 8Y = 15

Solution

$$\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \end{pmatrix}$$

Let A = $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

$|A|$ = (5x8) – (7x6)
= 40-42
= -2

Let B = $\begin{pmatrix} 11 & 6 \\ 15 & 8 \end{pmatrix}$

$|B|$ = (11x8) – (15x6)
= 88- 90
= -2

X= $\left|\frac{B}{A}\right| = \frac{-2}{-2}$= 1

X= 1

Let C = $\begin{pmatrix} 5 & 11 \\ 7 & 15 \end{pmatrix}$

$|C|$ = (5x15) – (7x11)

$= 75 \,\overline{\ }\, 77$

$= -2$

$Y = \frac{C}{A} = \frac{-2}{-2}$

2. $4X-6= -3Y$

$4+5Y = -2X$

Solution

$4X+ 3Y = 6$

$-2X-5Y = 4$

$$\begin{pmatrix} 4 & 3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

Let $A = \begin{pmatrix} 4 & 3 \\ -2 & 5 \end{pmatrix}$

$|A| = (4X-5) – (-2X3)$

$= -20 + 6$

$= 14$

Let $B = \begin{pmatrix} 6 & 3 \\ 4 & -5 \end{pmatrix}$

$A = (4X-5) – (-2X3)$

Let $B = \begin{pmatrix} 4 & 3 \\ -2 & 5 \end{pmatrix}$

$|B| =(6X-5) – (4X3)$

$= -30 -12$

$= -42$

$X= \left|\frac{B}{A}\right| = \frac{-42}{-14} = 3$

$X= 3$

Let $C = \begin{pmatrix} 4 & 6 \\ -2 & 4 \end{pmatrix}$

$|C| = (4X4) – (-2X6)$

$= 16- -12$

$= 28$

$Y = \left|\frac{C}{A}\right|$

$Y = \frac{28}{-14}$

Y = -2

TRANSFORMATION IN THE PLANE

Transformation in the plane is a mapping which shifts an object from one position to another within the same plane.

Examples of transformations in the xy plane are

i. Reflection

ii. Rotation

iii. Enlargement

iv. Translation

REFLECTION

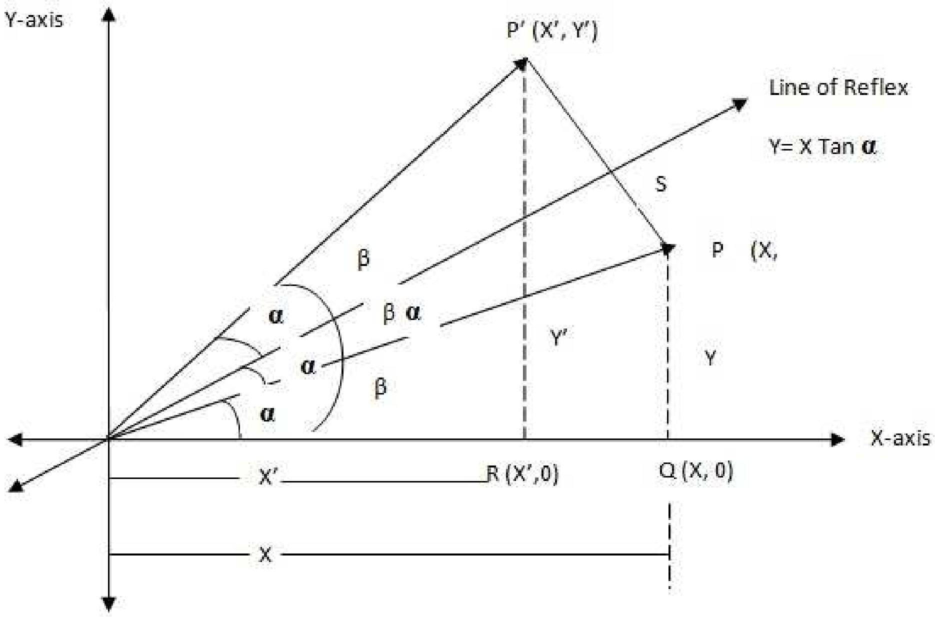
-The action or process of sending back light, heat or sound from a surface.

ISOMETRIC MAPPING

-Is a transformation which the object size is maintained.

-Reflection is an example of isometric mapping.

Reflection in the line included an angle (α) passing through the origin.



$\overrightarrow{OP}$ inclined at B with the coordinates being (X, Y)

$\overrightarrow{OP}$ is the image of $\overrightarrow{OP}$ under reflection is $\overrightarrow{OS}$

PP is perpendicular to **OS** ($\overrightarrow{OS}$ is the line of reflection)

POS= α ¯ β

Δ **OPQ** is right angled at Q

Hence X = OP cos B……(i)

Y = OP sin B

$\overrightarrow{QR}$ is perpendicular to the axis of X at R

Coordinates of R are (X^1,0)

OR= X^1

$RP^1=Y^1$

Δ OP^1R is right angled at R

$P^1OS = POS = \alpha - \beta$

Angle $P^1OR = \alpha - \beta + \alpha - \beta + \beta = 2\alpha - \beta$

$\text{Cos}(2\alpha-\beta) = \frac{X^1}{OP^1}$

$X^1 = OP^1 \cos(2\alpha-\beta)$….. (iii)

$\frac{Y^1}{OP^1} = \sin(2\alpha-\beta)$

$Y^1 = OP^1 \sin(2\alpha-\beta)$……(iv)

Cos (A+B) = cos A cos B – sin A sin B

Sin(A+B) = sin A cos B+ sin B cos A

X^1 = OP^1cos 2α cosβ + OP sin 2α cos β.... (iii)

Y^1= OP sin 2α cos β – OP^1 sin B cos 2α

X^1= OP cos 2α cos β+ OP sin 2α sin β

$Y^{1=}$ OPsin2αcos β – Op sinβ cos 2α

X^1 = OP cos β cos2α + OP sin β sin 2α

Y^1= OP cos βsin 2α – OP sin β cos 2α

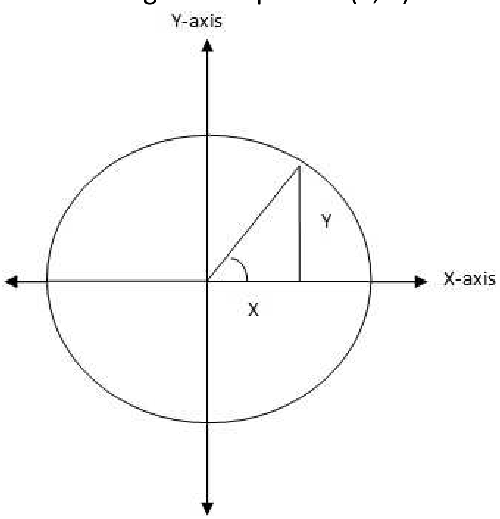
X^1 = X cos 2α + Y sin 2α .....(i)

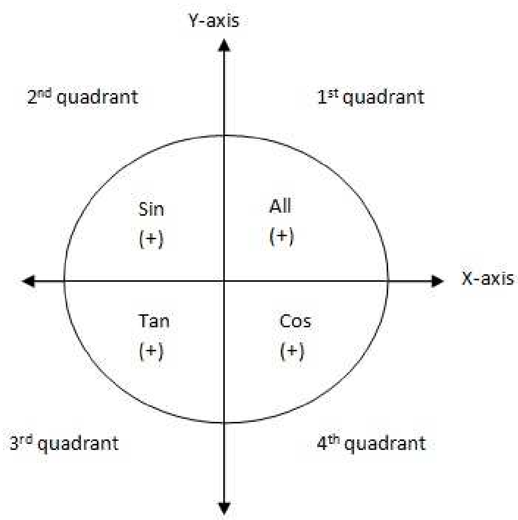
Y^1 = X sin 2α – Y cos 2 α .....(ii)

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

Exercise

1. Find the image of the point A (1, 2) after a reflection in the Y= X plane.





Solution;

Y= X

Y/X = 1

Tan α = Y/X = 1

α= 90^0

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos 90 & \sin 90 \\ \sin 90 & -\cos 90 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

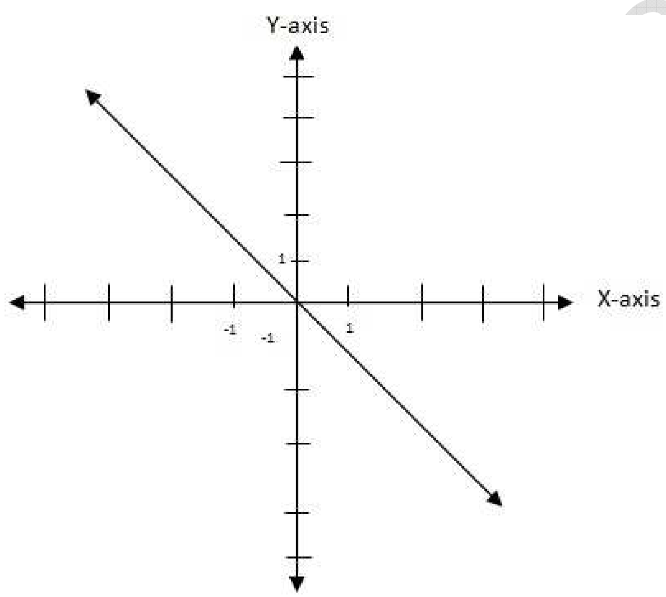
$(X^1, Y^1) = (2, 1)$

2. Find the image of B (3,4) after a reflection in the line Y= -X followed by another reflection in the line Y= 0

$M_y = -X$
$M_y = 0$

Y= -X

X	-3	-2	-1	0	1	2
Y	3	2	1	0	-1	-2



y = ˉx
y = ˉ1
x
$\alpha = -45^0$ for clockwise movement
Or
$\alpha = 135^0$ anticlockwise movement

$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos 2 \text{ x } 135 & \sin 2 \text{ x } 135 \\ \sin 2 \text{ x } 135 & -\cos 2 \text{ x } 135 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$

$(X^1, Y^1) = (-4, -3)$

Followed reflection at Y = 0

Tan α = 0

α = 0

$\begin{pmatrix} X^{11} \\ Y^{11} \end{pmatrix} = \begin{pmatrix} \cos 0 & \sin 0 \\ \sin 0 & -\cos 0 \end{pmatrix} \begin{pmatrix} -4 \\ -3 \end{pmatrix}$

$\begin{pmatrix} X^{11} \\ Y^{11} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -4 \\ -3 \end{pmatrix}$

$\begin{pmatrix} X^{11} \\ Y^{11} \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$

$(X^{11}, Y^{11}) = (-4, 3)$

In questions 3 to 6, write the matrix of reflection in a given line.

3. Y= 0 (the X axis)

Y= 0

Tan α= 0

α= 0

$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos 0 & \sin 0 \\ \sin 0 & \cos 0 \end{pmatrix}$

$M_x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

4. Y = X

$Y/_X = 1$

Tan α= 1

A= 90^0

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos 90 & \sin 90 \\ \sin 90 & \cos 90 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

5. X = 0

Tan α = 0

α= 0

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos 0 & \sin 0 \\ \sin 0 & \cos 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

6. Find the image of the point (1, 2) after a reflection in the line Y= X followed by another reflection in the line Y= -X.

$Y/X = 1$

Tan α = 90^0

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos 90 & \sin 90 \\ \sin 90 & \cos 90 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0x1 & + & 1x2 \\ 1x1 & + & 0x2 \end{pmatrix}$$

= (2, 1)

ROTATION

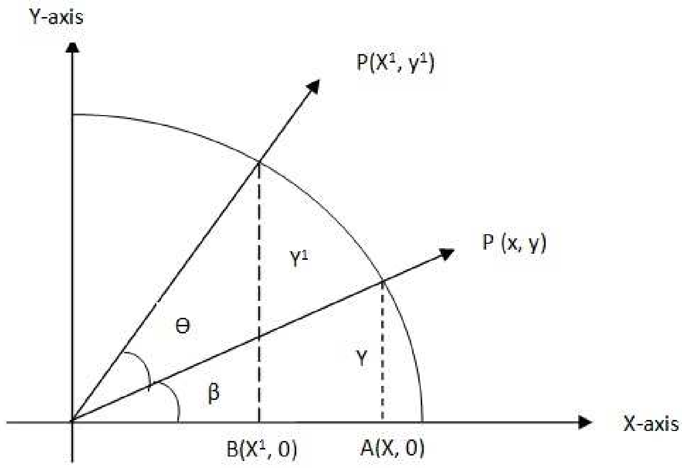
Find the image of the point B (1,2) after a rotation by 90^0 about the origin.

Solution:

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$R_\theta = \begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix}$$

IMAGE OF A POINT ROTATED AT INCLINED LINE AT ANGLE B



Let $\overline{OP}$ be inclined at an angle B

$\overrightarrow{OP} = \overrightarrow{OP'}$

PA is perpendicular to the X – axis at A

$\overline{OA} = X$, $\overline{AP} = Y$

ΔOAP is right angled at A with POA = B

$\text{Cos B} = \frac{X}{OP}$

X = OP Cos B …….(i)

$\text{Sin B} = \frac{Y}{OP}$

Y = OP sin B ….. (ii)

$\overline{PB}$ is perpendicular to the X – axis at B

$\text{Cos}(B+\theta) = \frac{X^1}{OP}$

$X^1 = \overline{OP}\cos(B+\theta)$

$X^1 = \overline{OP}\cos B\cos\theta - OP\sin B\sin\theta$

$X^1 = X\cos\theta - Y\sin\theta$ ……(iii)

$\text{Sin}(B+\theta) = \frac{Y^1}{OP}$

$Y^1 = \overline{OP}\sin(B+\theta)$

$Y^1 = \overline{OP}\sin B\cos\theta + \overline{OP}\sin\theta\cos B$

$Y^1 = Y\cos\theta + X\sin\theta$ …..(iv)

In matrix form

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

<u>Question</u>

<u>The matrix of rotation</u>

1. 90^0 about the origin.

$$R = \begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

2. find the image of (1,2) after a rotation of 90^0 followed by another rotation of 270^0 about the origin.

Solution

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} X^{11} \\ Y^{11} \end{pmatrix} = \begin{pmatrix} \cos 270 & -\sin 270 \\ \sin 270 & \cos 270 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

3. Find the image of 3X + 4Y + 6 = 0 under a rotation of 90^0 about the origin.

Solution:

3X + 4Y = -6

X intercept, Y = 0

3X = -6

X = -2

Y intercept, X = 0

4Y= -6

Y = $^{-3}/_{2}$

(-2, $^{-3}/_{2}$)

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix} \begin{pmatrix} -2 \\ -3/2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ -3/2 \end{pmatrix}$$

$$\begin{pmatrix} X^1 \\ Y^1 \end{pmatrix} = \begin{pmatrix} -3/2 \\ -2 \end{pmatrix}$$

TRANSLATION

Exercise

1. A translation takes every point a distance 1 unit to the left and 2 units downwards. Find where it takes.

a. (0,0)
b. (1,1)
c. (3,7)

Solution

a. (0,0)

(a,b) = (-1,-2)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

b. (1,1)

(a,b) = (-1,-2)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

C. (3,7)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

2. If translation takes the origin to (8,7). Given

U= (-12, 12) , v= (6,-16)

find T(u+v)

= (u + v)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -12 \\ 12 \end{pmatrix} + \begin{pmatrix} 6 \\ -16 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$$

$$T = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \end{pmatrix} + \begin{pmatrix} 8 \\ 7 \end{pmatrix}$$

$$T(u+v) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

3. Find the image if the line 3x + 4Y + 6 = 0 under a translation by the vector (-6,-1)

$3x + 4Y - 6 = 0$

Solution

$Y = mx + c$

$4Y = -3/4\ x - 3/2$
X intercept, y=0
$3x + 0 + 6 = 0$
$3X = -6$
$X = -2$
$(-2,0)$

Y intercept, X= 0
$0 + 4Y + 6 = 0$
$4Y = -6$
$Y = {}^{-3}/_{2}$
$(0, {}^{-3}/_{2})$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix} + \begin{pmatrix} 8 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{-3}{2} \end{pmatrix} + \begin{pmatrix} -6 \\ -1 \end{pmatrix} = \begin{pmatrix} -6 \\ \frac{-5}{2} \end{pmatrix}$$

$(-8,-1)$ and $(-6, {}^{-5}/_{2})$

$Y - Y_1 = M(X - X_1)$
$Y - -1 = M(X - -8)$
$Y + 1 = {}^{-3}/_{4}\ (X+8)$
$Y+1 = {}^{-3}/_{4}\ X + {}^{-24}/_{4}$
$Y = {}^{-3}/_{4}X - 7$

4. Find the image of the line Y = X under a translation by the vector (5,4)

Solution

Y = X

X	-2	1
Y	-2	1

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

(3,2) and (6,5)

Slope $= \frac{Y2-Y1}{X2-X1}$

$= \frac{5-2}{6-3}$

$= {}^{3}/_{3}$

M =1 (3 ,2)

$Y-Y_1 = M(X-X_1)$

Y-2 = 1(X - 3)

Y-2 = X - 3

Y = X – 3 + 2

Y = X -1

LINEAR TRANSFORMATIONS

Consider transformation T,
Let u and v be two vectors
Let t be the real number
The t is a linear transformation if it obeys the following properties.
i. T (tu) = t T (u)
ii. T(u+v) = T (u) + T(v)

ENLARGEMENT

The transformation which magnifies an object such that its image is proportionally increased or decreased in size by some factor.

General matrix of enlargement is $\begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$

where k is non zero or real number (Linear scale factor)

EXERCISE

1. Find the image of (1,2) under the enlargement by T= $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Solution

$$\begin{bmatrix} X^1 \\ Y^1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad 1 \quad = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$(X^1, Y^1) = (5, 10)$

2. Find the image of (-1/2 , -1/3) under the enlargement by

T= $\begin{bmatrix} -12 & 0 \\ 0 & -12 \end{bmatrix}$

Solution

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -12 & 0 \\ 0 & -12 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

$(X', Y') = (6, 4)$

3. Find the enlargement matrix which maps the point (3,-4) into (18, -24).

$$\begin{pmatrix}18\\-24\end{pmatrix} = \begin{pmatrix}k & 0\\0 & k\end{pmatrix}\begin{pmatrix}-3\\4\end{pmatrix}$$

$$\begin{pmatrix}18\\-24\end{pmatrix} = \begin{pmatrix}-3k\\4k\end{pmatrix}$$

-24 = 4k

18 = -3k

K= -6

-24 = 4K

K = -6

(-6,-6)

LINEAR PROGRAMMING

SIMULTANEOUS EQUATIONS

Solving simultaneous equations graphically, the solution is given by the point of intersection of the two lines.

Examples

1. Solve graphically the following simultaneous equations.

   2x - y = 1

   3x+3y =6

   Equation 1
   2x - y = 1

   X intercept, y= 0

   2x - y = 1

   2x - 0 = 1

   x = $^1/_2$

$(^1/_2, 0)$

Y intercept, x = o

2x - y = 1

y = 1

(0, -1)

Equation 2.

3x + 3y = 6

X intercept,Y=0

3x-0 = 6

X = 2

(2,0)

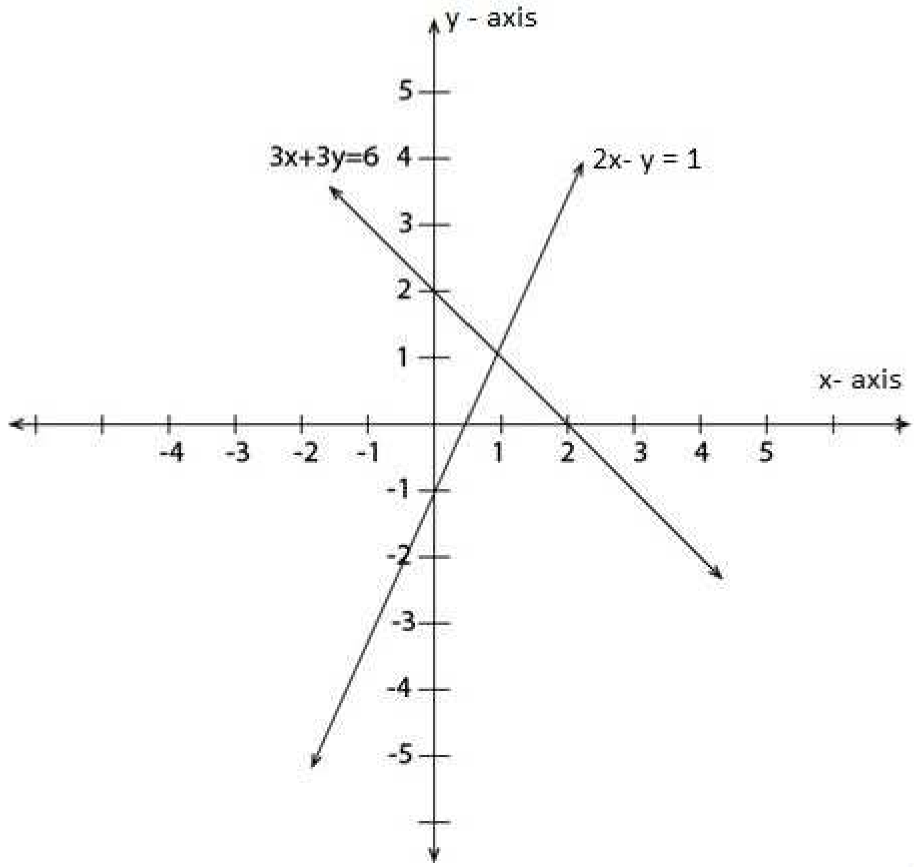
Y intercept,X=0

0 + 3y = 6

3Y = 6

y = 2

(0,2)



Solution (1,1) i.e x = 1 and y = 1.

2. Ali paid 34 shillings for 10 oranges and 35 mangoes. Moshi went to the same fruit market and paid 24 shillings for 16 oranges and 18 mangoes .what was the price of a mango and for an orange.

Solution:

Let X be a price of an orange

Let y be the price of mangoes

$10X + 35Y = 34$

$16X + 18Y = 24$

Solution:

$$\begin{cases} \_16 \quad 10X + 35\ Y = 34 \\ 10 \quad 16XC + 18\ Y = 24 \end{cases}$$

$$\begin{cases} 160\ X + 560\ Y = 544 \\ -\ 160\ X + 180\ Y = 240 \end{cases}$$

380 y = 314

Y = 0.8

10X + 35y = 34

10X + 35 x 0.8 = 34

10X + 28 = 34

X= 0.6

X intercept,Y=0

10 X + 35 Y = 34

10X + 0 = 34

10 X = 34

X = 3.4 (3.4,0)

Y intercept , X=0

0 + 35 Y = 34

35Y = 34

Y = 1 (0,1)

X intercept Y =0

16X + 18 Y = 24

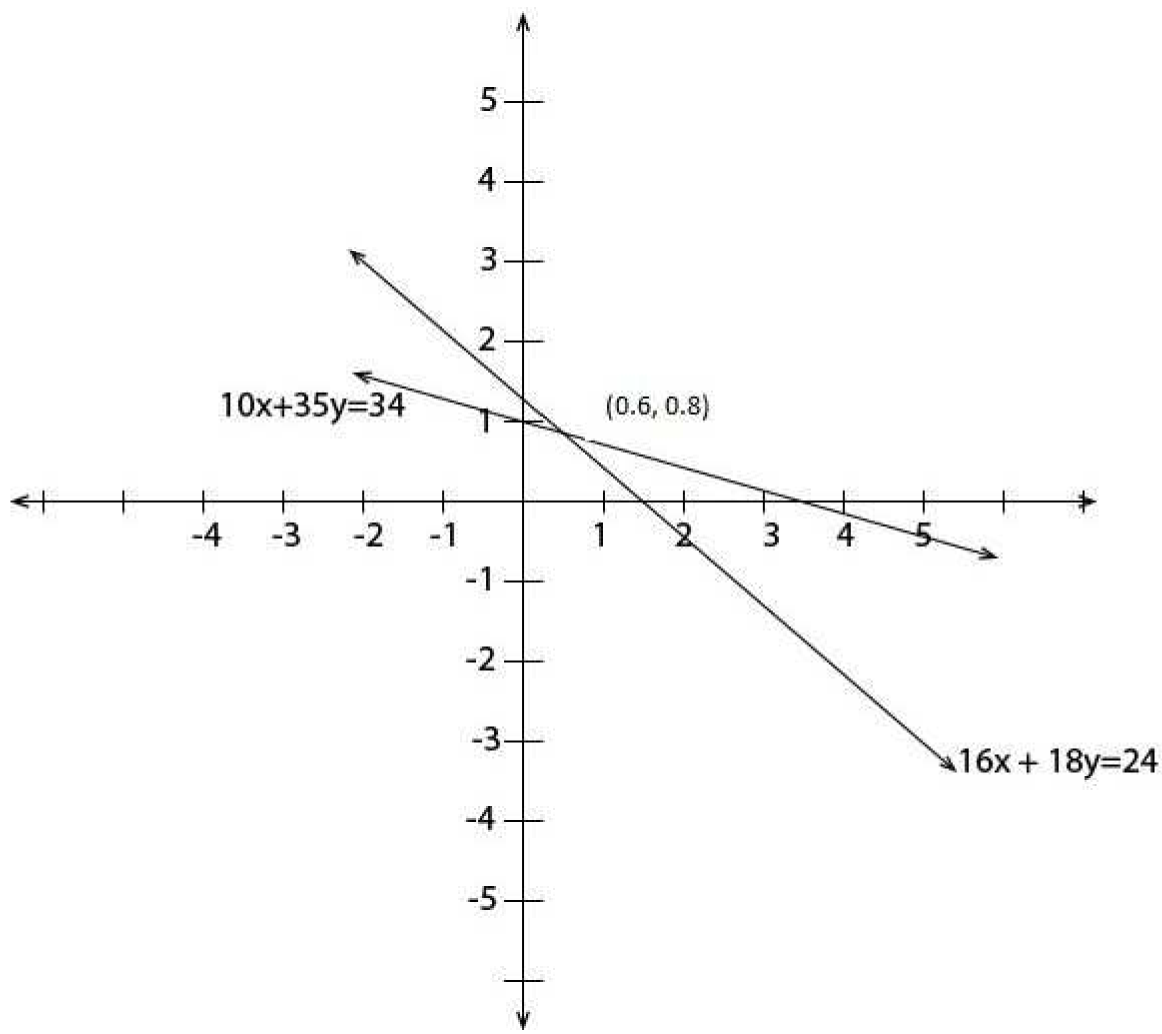
16X = 24

X = 1.5 (1.5, 0)

Y intercept X= 0

18Y = 24

$Y = 1.3^{*}$ (0 ,1.3)



x = 0.6

y = 0.8

∴Price of an orange is 0.6sh and price of a mango is 0.8sh.

3. $7X + 3Y + 12 = 0$

$5X - 2Y + 2 = 0$

Solution:

$7X + 3Y + 12 = 0$

X intercept y=0

$7X + 0 = -12$

X = $-17/2$

Y intercept X=0

$0 + 3Y = -12$

$3Y = -12$

$Y = 4$

$5X - 2Y = -2$

X intercept y=0

$5X - 0 = -2$

X= $-2/5$

Y intercept X= 0

$0 - 2y = -2$

$Y = 1$

4. $2c + 2d = c - d$

$2d + 2 = c + 1$

Solution:

2c + 2d = c - d

2c –c + 2d + d = 0

c + 3d =0…….(1)

2d + 2 = c + 1

2d + 2- c-1= 0

2d – c + 1= 0 ……..(2)

c + 3d = 0

c intercept, d = 0

c = 0

d intercept , c = 0

3d = 0

d = 0

2d – c = -1

C intercept, d= 0

-c = -1

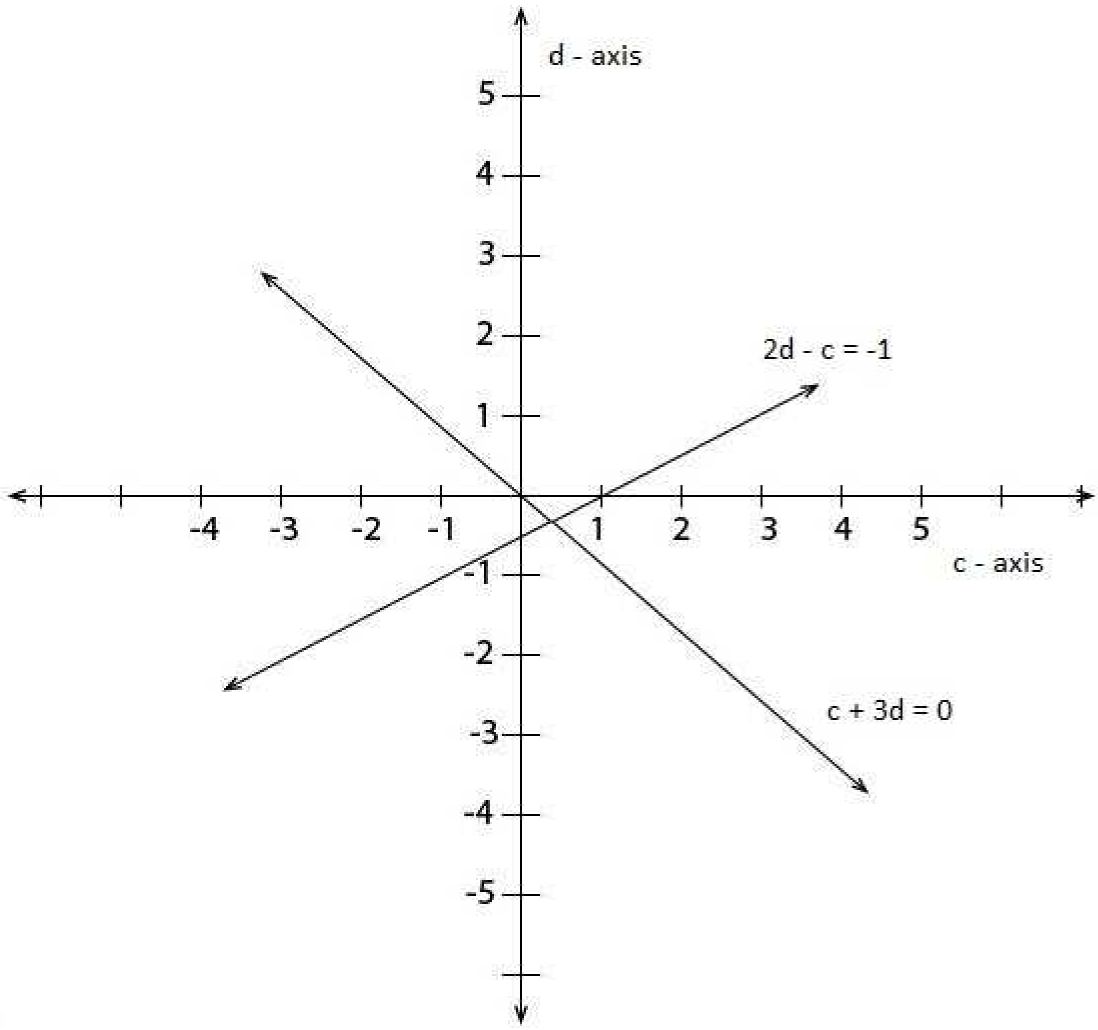
c = 1

d intercept , c= 0

0 - 2d = 1

d = - $^{1}/_{2}$

d = (0, - 0.5)



Solution (0.6, -0.2)

By substitution method

c + 3d = 0 -------(i)

c – 2d = 1--------(ii)

equation;

c = -3d

c -2d = 1

c – 2(-0.2) = 1

c + 0.4 = 1 – 0.4

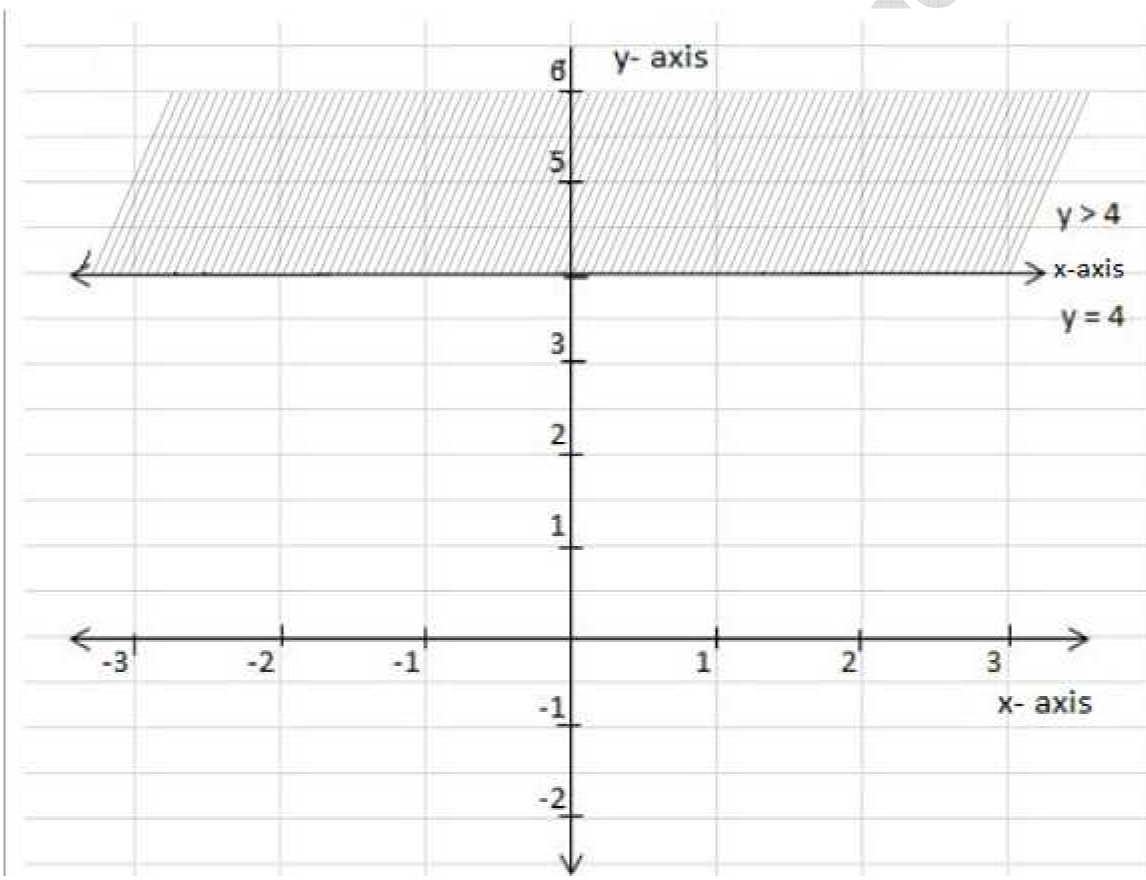
c = 0.6

:. c = 0.6 , d = -0.2

LINEAR INEQUALITIES

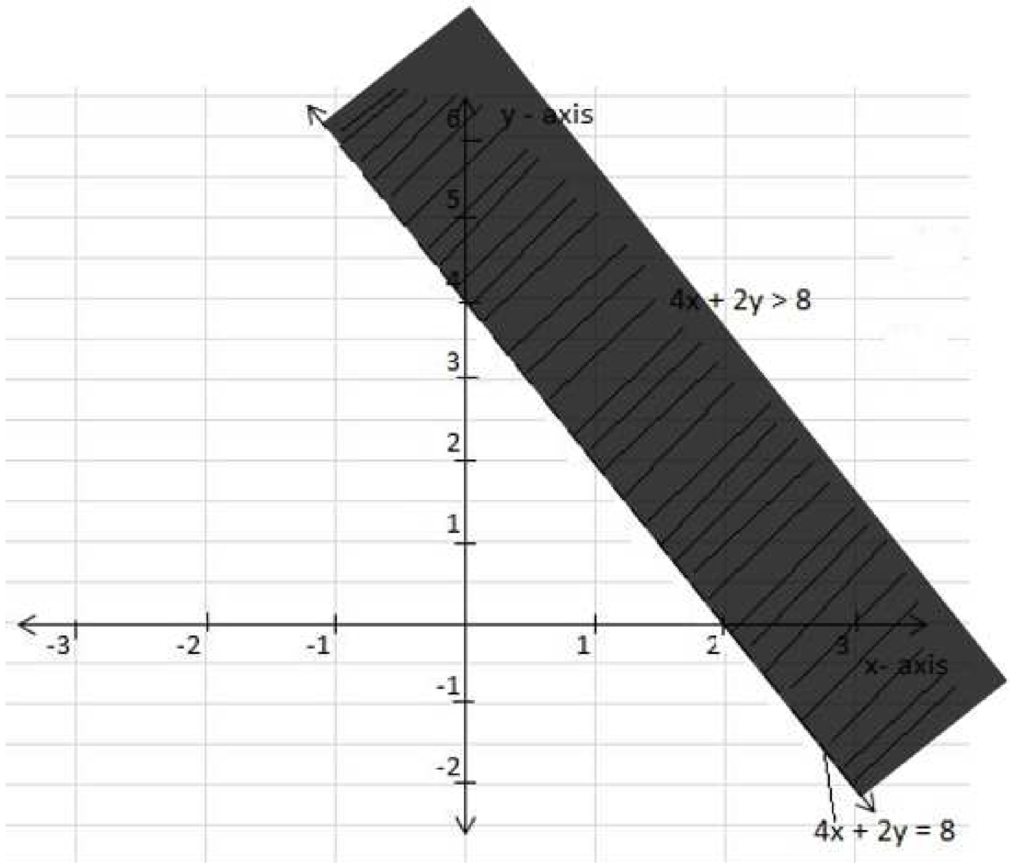
When a linear equation of the form ax + by + c = 0 is represented on a coordinate plane, it separates the plane into two disjoint sets.
E.g. y = 4 separate the plane into two disjoint sets
The points above the line y= 4 satisfy the relation y>4, while those in the lower half plane satisfy the relation y < 4.

SHADING OF REGIONS
When drawing the inequality $y \geq 4$, first draw the line of separation to separate the plane y>4, and the plane y< 4 which is y = 0, Then shade the **unwanted** region.



Note: The shaded region together with the line y = 4 represents $y \geq 4$

Boundaries of half- plane
Equations are used to describe the boundaries of half- plane. The boundary lines are continuous to include points on the line when the inequality is written using the sign $\geq$ or $\leq$where as the boundary lines are dotted if the inequality is written in the of form < or >.
E.g. Draw and show the half- plane representing the inequality 4x + 2y > 8



Feasible Region

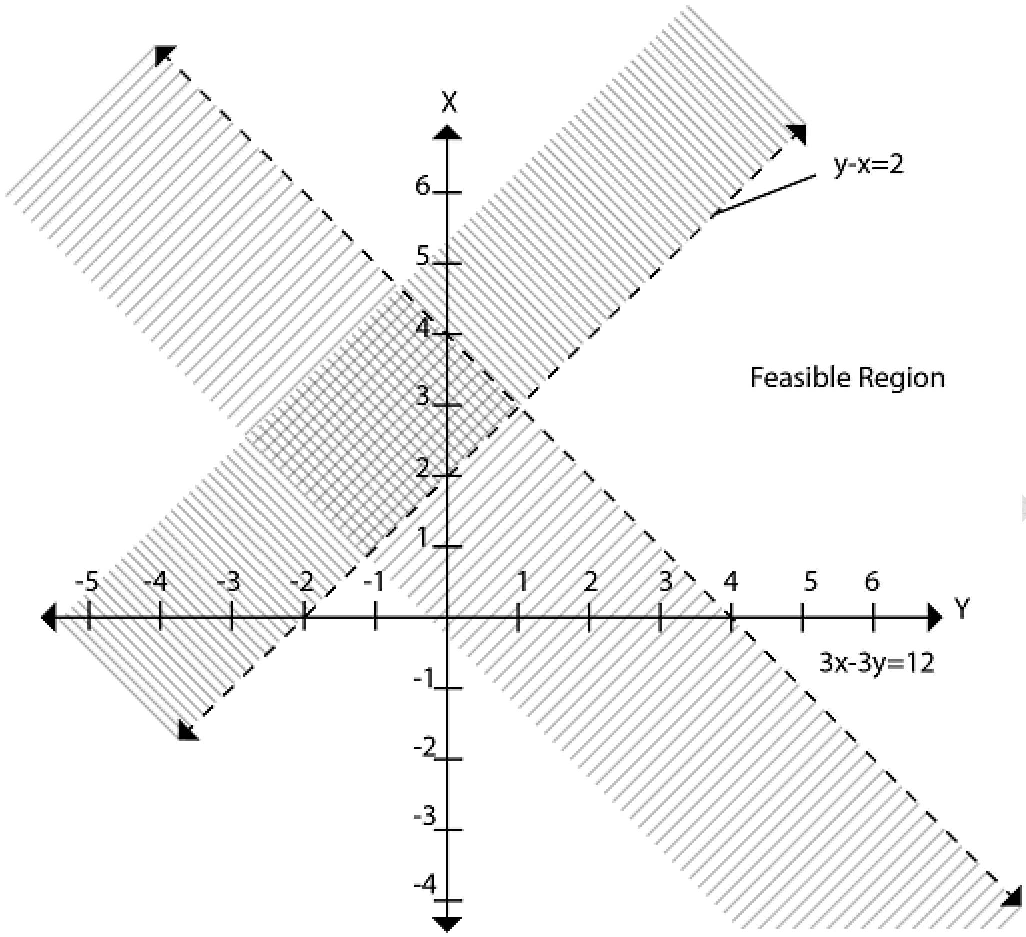
Feasible region is the region of intersection of all the inequality given in a problem. In order the feasible region to be seen clearly the unwanted region of the inequalities should be shaded leaving the required region clear.

Example 1.

Show the feasible region which satisfy the following inequalities $3x + 3y \geq 12$ and $y - x \leq 2$

Solution

$3x + 3y \geq 12$,	(0, 4) (4, 0)
$y - x \leq 2$,	(0, 2) (-2, 0)



From $3x + 3y \geq 12$, after drawing the line of separation i.e $3x + 3y = 12$, The point above or below the line is used to test which region satisfy the inequality.
e.g. (0, 0) the origin

$3(0) + 3(0) \geq 12$

$0 \geq 12$ Not true

$\therefore$ The region below the line does not satisfy the inequality instead. The opposite region satisfies.The region below is shaded to live the wanted region clear.

For the case of $y - x \leq 2$
testing using the origin (0, 0)

$0 - 0 \leq 2$

$0 \leq 2$ (it is true)

The region below is the wanted region, therefore we shade the region above.

Example 2
A bread dealer can buy up to 150 loaves ifbread .premium bread costs 200/= per loaf and royal bread costs 280/= per loaf. The dealer can spend no more than 36000/= in the business. Premium bread sells at a profit of 40/= per loaf while royal bread sells at a profit of 50% per loaf .how many loaves of bread of each type should the dealer buy in order to generate maximum profit?

Solution:

Let X be the no. of premium bread.

Let Y be the no of Royal bread

$200X + 250Y \leq 36000$

$X + Y \leq 150$

$X \geq 0$

$Y \geq 0$

X intercept, Y=0

$200X + 250Y = 36,000$

$200X = 36000$

$X = 180$

$(180,0)$

X intercept, Y = 0

$X + Y = 150$

$X = 150$

$(150, 0)$

Y intercept, X = 0

$0 + 250Y = 36,000$

$250Y = 36,000$

$Y = 144$

$(0, 144)$

Y intercept, X = 0

$0 + Y = 150$

$Y = 150$

objective function

$40X + 50/100 \times 250Y =$ Maximum profit

$40X + 125Y =$ Maximum profit

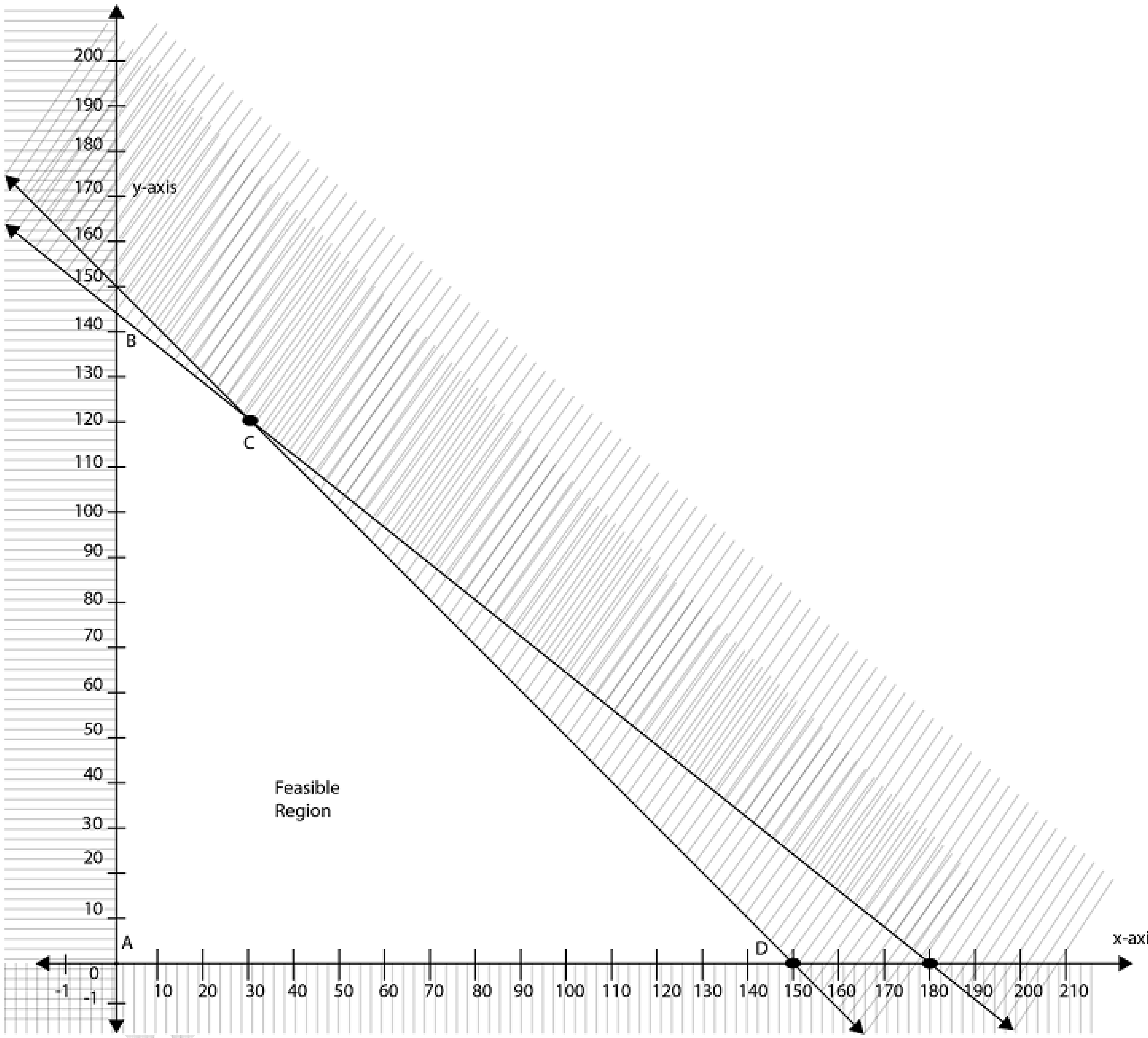
A (6, 0)

B (144, 0)

C (0,144)

$40X + 125Y =$ Maximum profit

$40 + 125X144 = 18,040$



The Corner points

A (0, 0) C (30, 120)

B (0, 144) D (150, 0)

$f(x, y) = 40x + 125y$ maximize

$f(0, 0) \quad = 40x + 125y = 0$

$f(0, 144) = 40x + 125y = 18000$
$f(30, 144) = 40x + 125y = 15120$
$f(150, 0) = 40x + 125y = 6000$

optimal point is (0, 144)
∴ In order for the bread dealer to get maximum profit, he should prepare 144 royal breads only.

Example 3.
Draw a graph and show the feasible region which is satisfied by the inequalities ;

$X \geq 0$, $2x + y \leq 4$, $2x + 3y \geq 8$, $y \geq 0$, $y + 3x = 9$, $x + y \leq 10$

Which If any of the inequalities can be omitted without affecting the answer.

SOLUTION

$2x + y = 4$

X intercept, Y = 0

$2x + 0 = 4$

$x = 2$

(2, 0)

Y intercept

$0 + y = 4$

$y = 4$

(0, 4)

$2x + 3y = 8$

X intercept, Y =0

$2x + 0 = 8$

$2x = 8$

$x = 4$

(4, 0)

y intercept, x =0

0 + 3Y = 8

y =8/3

(0 , 2.7)

x + 3y = 9

x intercept, y =0

x + 0 = 9

x = 9

y intercept, y = 0

0 + 3Y = 9

y = 3

(0, 3)

x + y = 10

x intercept, y =0

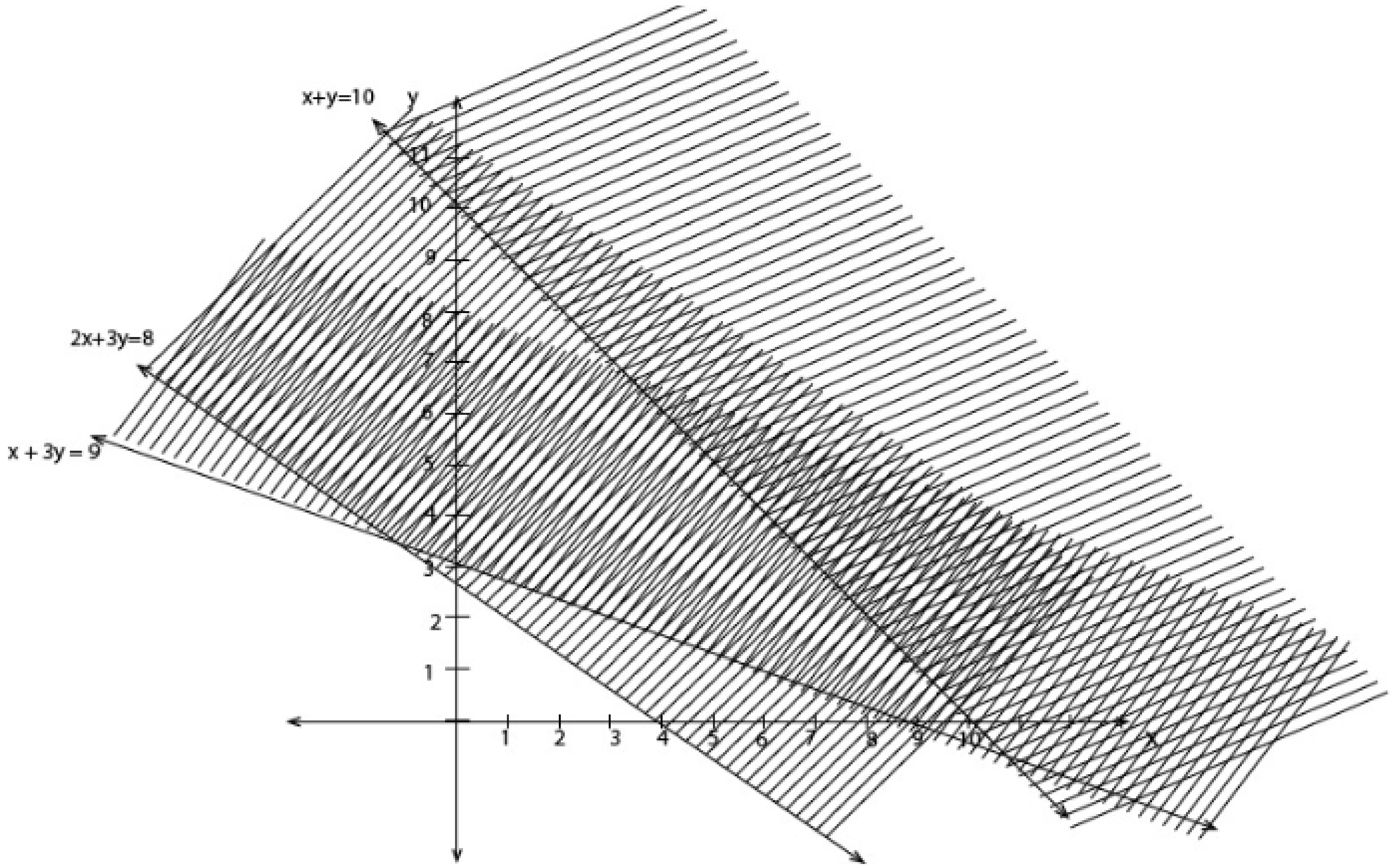
x =10

(10, 0)

y intercept, x =0

y = 10

(0, 10)



Example 4.

George buys X pencils 10 shillings each,(X+8) exercise books at 10 shillings each. If he wishes to have some change as town bus fare from a 200shs note, form an inequality in X and solve it to find the range of X.

Solution:

X no. of pencils

0 X + 10(X+8) ≤200

10X + 10X + 80 = 200

20 X = 120

X = 6

X < 6

The range of x is = 1, 2,3,4,5.

Exercise 8.1

1. Show by shading out the unwanted regions, the half planes representing the following simultaneous inequalities.

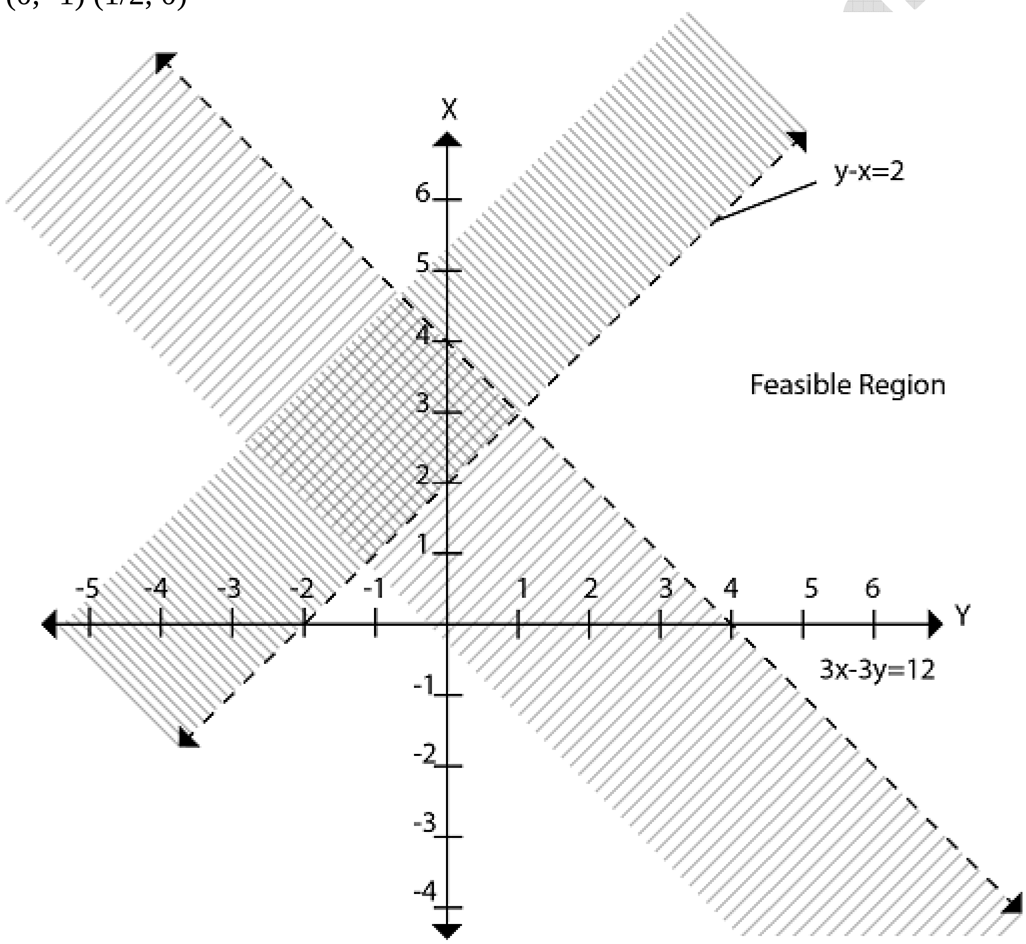
a) $y < 2x - 1$, $y > 3 - x$

b) $-3 \leq x - y < 2$

c) $y < 2x$, $y > 3 - x$, $y > -1$

Solution

a) $y < 2x - 1$	$y > 3 - x$
$1 < 2x - y$	$y + x > 3$
$2x - y > 1$	$y + x = 3$
$2x - y = 1$	(0, 3) (3,0)
(0, -1) (1/2, 0)	

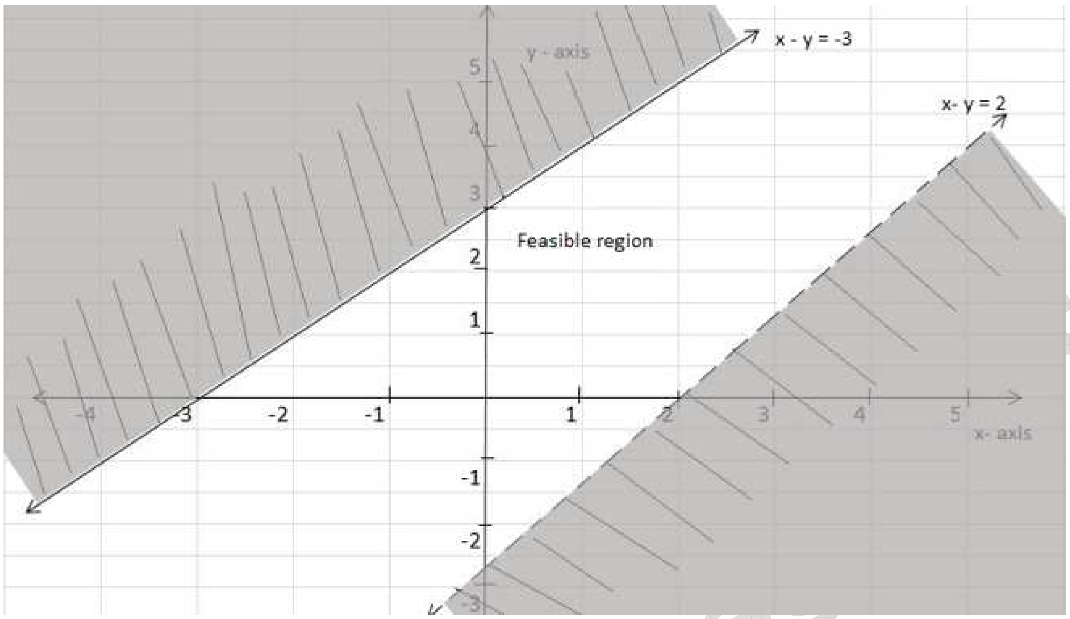


(b) $-3 \leq x - y < 2$

$-3 \leq x - y$	$x - y < 2$
$x - y \geq -3$	$x - y = 2$

$x - y = -3$ $(0, -2)$ $(2, 0)$

$(0, 3)$ $(-3, 0)$



(c) $y > 2x - 1$

$y - 2x > -1$

$y - 2x = -1$

$(0, -1)$ $(1/2, 0)$

$y > -1$

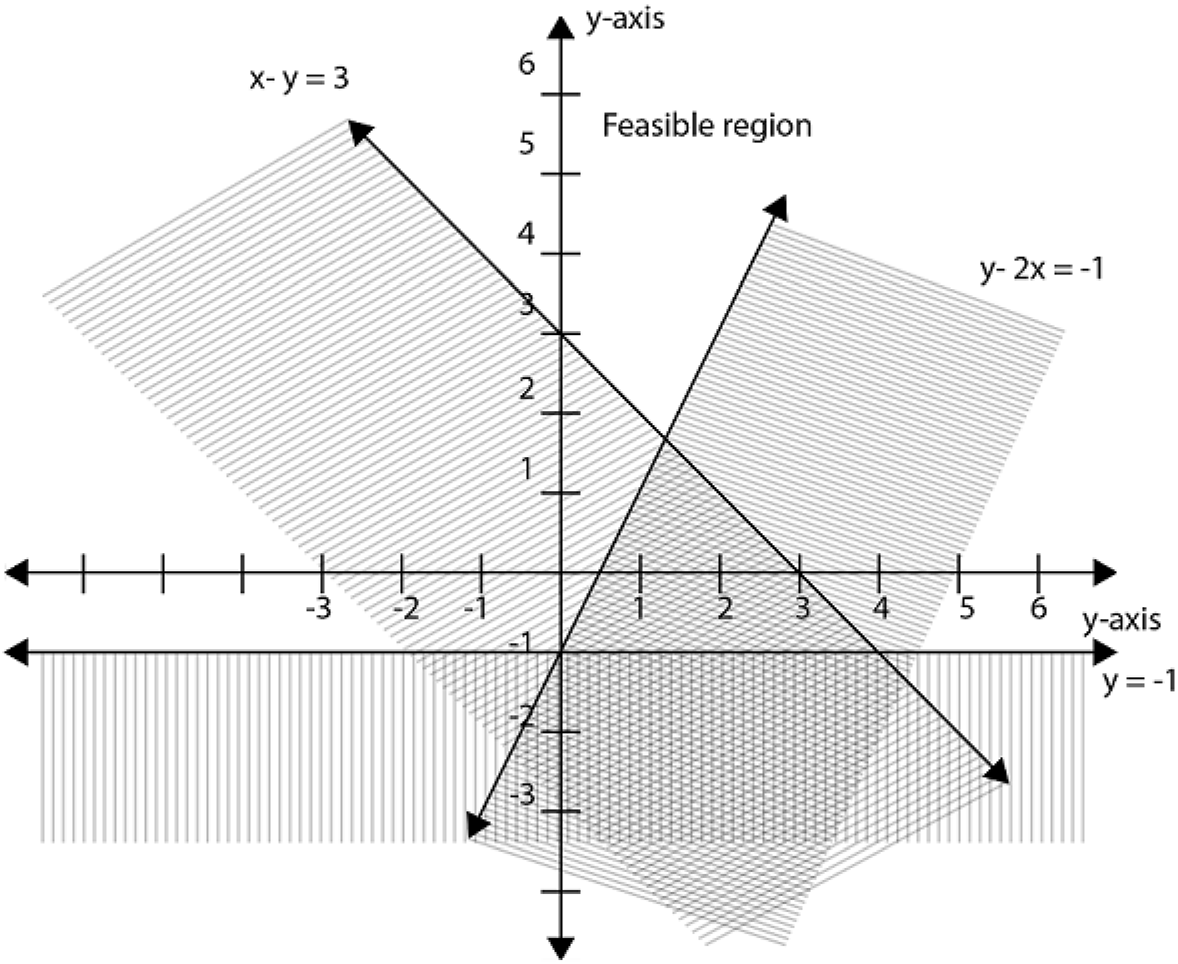
$y = -1$

$y > 3 - x$

$y + x > 3$

$y + x = 3$

$(0, 3)$ $(3, 0)$



Linear Programming

Linear programming is a branch of mathematics which enables to solve problems which one wants to get the greatest or least value of a quantity.

Solving linear programming problems

1.A farm is to be planted with wheat and maize while observing the following.

	Wheat	Maize	Maximum Total
Days of labour per hectare	2	1	10
Labour cost per hectare(Shs)	700	600	4200
Cost of fertilizers per hectare(Shs)	300	400	2400

If wheat yields a profit of 800 Sh. per hectare while maiza yields 600sh.per hectare.How should the area be planted in order to get maximum profit?

Solution

Step1:

Express the information provided in mathematical form

Let x represents number of hectares of wheat planted

y represents number of hectares of maize to be planted

Step2: The objective function

This is the function to be optimized. The objective of the problem is to maximize the profit.

Therefore

800x - profit yielded by wheat

600y - profit yielded by maize

The objective function $f(x,y) = 800x + 600y$ is to be maximized.

Step3.Availability.

$2x + y \leq 10$ (Available days of labor)

$700x + 600y \leq 4200$ (Available money for labors cost)

$300x + 400y \leq 2400$ (Available money for fertilizer)

$x \geq 0$, $y \geq 0$ hectares of wheat and maize can only be positive. (non - negative)

$\therefore$ The problem is to maximize $f(x,y) = 800x + 600y$ subject to

$2x + y \leq 10$

$700x + 600y \leq 4200$ i.e $7x + 6y \leq 42$

$300x + 400y \leq 2400$ i.e $3x + 4y \leq 24$

$x \geq 0$,

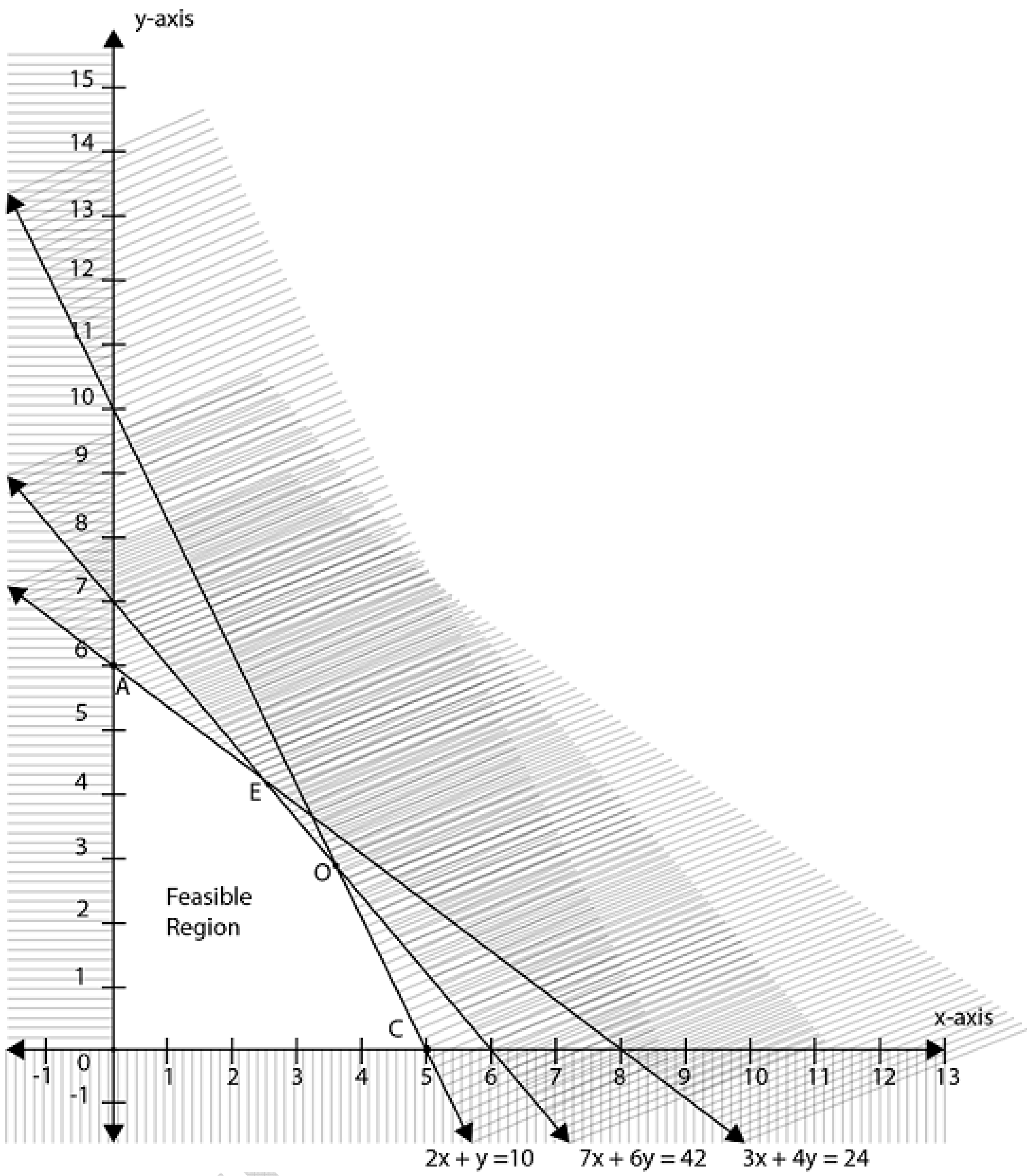
$y \geq 0$

Step4.Plot the constraints on the graph

$2x + y \leq 10$ (0,10) (5,0)

$7x + 6y \leq 42$ (0,7) (6,0)

$3x + 4y \leq 24$ (0,6) (8, 0)



step 5.The Identify the corner points.
A(0, 6), B(0,0), C(5,0), D(3.3, 2.4), E(2.4, 4.2)

Step6.Substitute the corner points to the objects function.
f(x, y) = 800x + 600y
f(0, 6) = 800x + 600y = 3600
f(0, 0) = 800x + 600y = 0
f(5, 0) = 800x + 600y = 40000
f(3.8, 2.4) = 800x + 600y = 4480
f(2.4, 4.2) = 800x + 600y = 4500

Note: The point gives the best solution is called optimal point and the best solution is called the optimal solution

∴ f(2.4, 4.2) is the optimal point and 4500 is the optimal solution.
2.4 hectares should be planted wheat and while 4.2 hectares should be planted maize in order to get maximum profit

EXERCISE 8.2

1. A doctor prescribes that in order obtain an adequate supply of Vitamins A and C, his patient shall have portions of food 1 and 2. The number of the units of vitamin A and C are given in a table below,

	A	C
Food 1	3	2
Food 2	1	7

The doctor prescribes a minimum of 14 units of vitamin A and 21 units of vitamin C. What are the least number of portion of food 1 and 2 that will fit the doctors prescription?

Solution
Let x represents number of portion of food 1 and y represent number of portion of food 2.

	A	C
Food 1(x)	3	2
Food 2(y)	1	7
Minimum Units	14	21

$$3x + y \geq 14$$
$$2x + 7y \geq 21$$
$$x \geq 0$$
$$y \geq 0$$

Objective function
To minimize $f(x,y) = x + y$

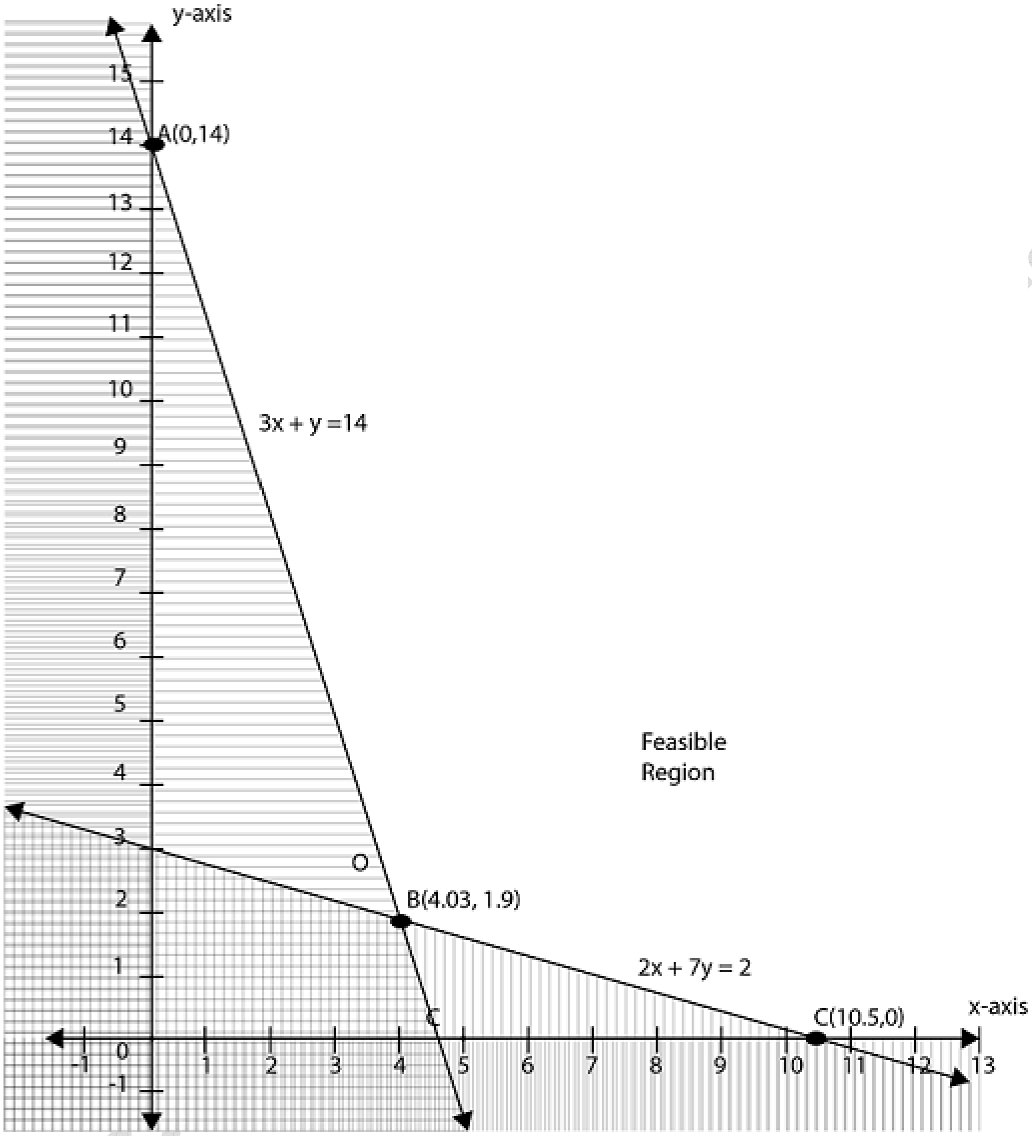
To represents the constraint on the x-y plane

$3x + y = 14$ (0, 14) (4.667, 0)

$2x + 7y = 21$ (0, 3) (10.5, 0)

$x = 0$

$y = 0$



Corner points

A(0, 14) B(4.03, 1.9) C(10.5, 0)

$f(x,y) = x + y$

$f(0, 14) = 0 + 14$

$f(4.03, 1.9) = 4.03 + 1.9$

$f(10.5, 0) = 10.5 + 0$

Best solution(optimal solution) = 5.93 and the optimal point is (4.03, 1.9).

$\therefore$ 4.03 is the portion of the food 1 and 1.9 is the portion of food 2 that should be taken to fit the doctors prescription.