THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 **BASIC APPLIED MATHEMATICS**
(For Both School and Private Candidates)

Time: 3 Hours **Monday, 07th February 2011 a.m.**

INSTRUCTIONS

1. This paper consists of **sixteen (16)** questions in sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

This paper consists of 5 printed pages.

SECTION A (60 marks)

Answer **all** questions in this section.

1. (a) Show that the distance between (4, 1) and (10, 9) is equal to 10 units.

   (b) Find the equation of a line, in the form of $ax+by+c=0$, through the point $(1,-2)$ which is perpendicular to $2y=4x+8$. **(6 marks)**

2. (a) A quadratic equation has positive roots α and β such that $\alpha-\beta=2$ and $\alpha\beta=15$. Determine its equation, and hence obtain the quadratic equation, whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

   (b) Given the functions $f(x)=2x-5$ and $g(x)=\frac{4}{x}+7$, verify that $(f\circ g)^{-1}(x)=g^{-1}\circ f^{-1}(x)$. **(6 marks)**

3. (a) Solve the simultaneous equations $3x-y=-2$ and $x^2+4xy+7=28$.

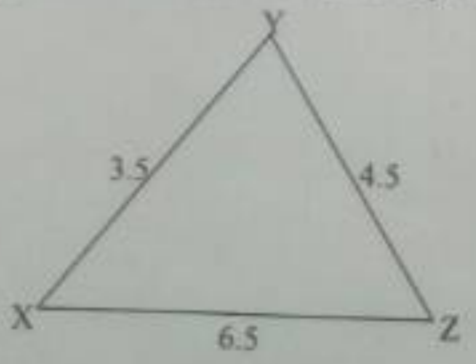
   (b) The first term of an Arithmetic Progression (A.P) is -12, and the last term is 40. If the sum of the progression is 196, find the number of terms and the common difference. **(6 marks)**

4. (a) The length (l) of a simple pendulum varies as the square of the period (T), the time to swing to and fro. A pendulum 0.994 m long has a period of approximately 2 seconds. Find:
   (i) the length of a pendulum whose period is 3 seconds,
   (ii) an equation connecting l and T.

   (b) A traveler in Uganda changed Tshs.2,000,000/= into Uganda shillings (U) at a rate of Tshs.1 = Ushs.2. He spent Ushs. 2,500,000/= and then he changed the rest back into Tshs. at the rate of Tshs.1 = Ushs.2.5. How much Tanzanian shillings did he receive? **(6 marks)**

5. (a) Prove that $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$.

   (b) In the triangle below calculate the size of angle Y.



(6 marks)

6. (a) Solve each of the following equations: -

(i) $\log x + \log 2 - \log 7 = 1$,

(ii) $\log(x+1) - \log(x-2) = 2$.

(b) Using scientific notation, evaluate $\frac{34000 \times 0.00538}{0.027 \times 430000}$ retaining up to three decimal places. **(6 marks)**

7. (a) Differentiate $\frac{(x-6)^3}{(x+5)^2}$.

(b) A container in the shape of a right circular cone of height 20 cm and base radius 2 cm is catching the drips from a tap leaking at the rate of $0.3\,cm^3s^{-1}$. Find the rate at which the surface area of water is increasing when the water is half way up the cone. **(6 marks)**

8. (a) Find $\int \cos x \sin^4 x \, dx$.

(b) Evaluate $\int_0^1 x^3 \sqrt{(x^4+3)}\, dx$, leaving your answer in surd form. **(6 marks)**

9. (a) Given that $\underline{a} = 4\underline{i} + 3\underline{j} + 12\underline{k}$ and $\underline{b} = 8\underline{i} - 6\underline{j}$, find $\underline{a}^2, \underline{b}^2$ and hence determine the angle between the vectors $\underline{a}$ and $\underline{b}$.

(b) If A and B are points (1, 1, 1) and (13, 4, 5) respectively, find, the displacement vector $\overline{AB}$ and hence the unit vector parallel to $\overline{AB}$. **(6 marks)**

10. (a) Calculate the standard deviation of the numbers 9, 3, 8, 8, 9, 8, 9, 18.

(b) Find the range of the numbers 51.6, 48.7, 50.3, 49.5, and 48.9.

(c) Calculate the mean of the distribution of marks given below:

Marks	Frequency
0 -9	0
10 - 19	3
20 – 29	7
30 - 39	12
40 - 49	18
50 - 59	22
60 – 69	17
70 -79	14
80 - 89	9
90- 99	5

(6 marks)

SECTION B (40 marks)

Answer **four (4)** questions from this section. Extra questions will **not** be marked.

11. (a) A fair die is thrown once. List the possible outcomes and hence evaluate the probability of scoring a multiple of 2.

(b) The events A and B are such that P(A) = 0.43, P(B) = 0.48 and $P(A \cup B) = 0.78$. Show that the events A and B are not independent.

(c) In how many different ways can eight cards be dealt from a pack of fifty-two playing card?

(10 marks)

12. (a) Find the product AB when

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 4 & -1 & 3 \\ 2 & 2 & 2 \\ 3 & 7 & 1 \end{pmatrix}.$$

(b) If $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & -1 \\ -1 & 2 & -1 \end{pmatrix}$, find a matrix X such that $AX + B = A$.

(c) Solve the equations $2x + 3y = 8$ and $5x - 2y = 1$ by using the inverse matrix method.

(10 marks)

13. Solve the linear programming problem:

Maximize $x + \frac{3}{2}y$ subject to the constrains: $\begin{cases} 2x + 4y \le 12 \\ 3x + 2y \le 10 \\ x, y \ge 0 \end{cases}$ **(10 marks)**

14. (a) Differentiate $f(x) = \frac{1}{x}$ from first principle.

(b) Determine $\frac{dy}{dx}$ given that $y^3 + x^4 + \cos(x + y^3) = 0$.

(c) Solve for the stationary values of the function $x^3 - 2x^2 + 11 = 0$. **(10 marks)**

15. (a) Calculate the area enclosed between the curve $y = x(x-1)(x-2)$ and the x – axis.

(b) Evaluate the integral of $\int 3^{\sqrt{2x+1}}\,dx$.

(c) What is the volume generated when the area enclosed by the curve $y = x$, the x – axis and the line $x = 2$ is rotated about the x – axis? **(10 marks)**

16. (a) Write down the unit vector which is perpendicular to the plane $4x + 3y + 2z = 12$.

(b) Find the equation of a plane through the point (2, 4, 5) and perpendicular to the vector $2\underset{\sim}{i} + 7\underset{\sim}{j} + 8\underset{\sim}{k}$.

(c) Compute the perpendicular distance of the point P(0, 14, 10) from the line whose equation is $\underset{\sim}{r} = (\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}) + \lambda(3\underset{\sim}{i} + 4\underset{\sim}{k})$. **(10 marks)**

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