THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2 **ADVANCED MATHEMATICS 2**
(For Both School and Private Candidates)

Time: 3 Hours **Thursday, 10th February 2011 a.m.**

INSTRUCTIONS

1. This paper consists of **sixteen (16)** questions in sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

This paper consists of 5 printed pages.

2011/0001

SECTION A (60 marks)

Answer **all** questions in this section.

1. (a) By using mathematical tables evaluate:
$$\frac{[12\tan^{-1}(3.42)]e^{8.22}}{(0.001182)^{1/2}}.$$

(b) By using a non – programmable scientific calculator compute the following expression to eight significant figures:
$$\frac{(7\ln 13.42)(42.8425)^{4/5}}{(0.03482)^{3/2}[11\sin^{-1}(0.8543)]}.$$

(6 marks)

2. (a) Check the validity of the following argument $\sim p \rightarrow (q \leftrightarrow \sim r)$, $\sim r \rightarrow \sim p$, $q \rightarrow \sim r$ and $\sim r$.

(b) What does the proposition $[(p \wedge q) \wedge r] \rightarrow (p \wedge q)$ represent?

(6 marks)

3. (a) What is the length of the tangent to the circle $4x^2 + 4y^2 - 8x - 2y + 15 = 0$ from the point (2, 3)?

(b) Show that the circles $x^2 + y^2 - 16x - 12y + 75 = 0$ and $5x^2 + 5y^2 - 32x - 24y + 75 = 0$ touch each other.

(6 marks)

4. (a) Solve the equation $\frac{1}{2}\tan^{-1} x = \tan^{-1}\left(\frac{1-x}{1+x}\right)$.

(b) In the triangle ABC, $AB = x - y$, $BC = x$, $CA = x + y$. Show that $\cos B = \frac{x - 4y}{2(x - y)}$.

(c) Prove that $\frac{\cos 3\theta}{\cos\theta} - \frac{\cos 6\theta}{\cos 2\theta} = 2(\cos 2\theta - \cos 4\theta)$.

(6 marks)

5. (a) If α, β and γ are the roots of the equation $x^3 - 5x^2 + 5x - 2 = 0$, find the equation whose roots are $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$.

(b) Prove that the equation $(k-2)x^2 + 2x - k = 0$ has real roots for all values of k.

(c) When the expression $x^5 + 2x^2 + ax + b$ is divided by $x^2 - 4$, the remainder is $3x + 1$. Find the values of a and b.

(6 marks)

6. (a) Find the centre and equations of the asymptotes of the hyperbola $xy - x - 2y = 6$.

(b) Find the equation for the set of points $P(x, y)$ such that they are equidistant from the origin O and the line $x = 4$.

(6 marks)

7. (a) If $y = (\cosh^{-1} x)^2$, show that $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$.

(b) Show that $y = C_1 e^{2x} + C_2 e^{-2x}$ is the same as $y = b_1 \cosh 2x + b_2 \sinh 2x$.

(6 marks)

8. The table below shows the marks obtained by students in a certain test.

Marks	Number of Students
0 – 10	10
10 – 20	7
20 – 30	3
30 – 40	5
40 – 50	15
50 – 60	10
60 – 70	15
70 – 80	10
80 – 90	10
90 - 100	5

(a) Using the assumed mean A = 55, calculate the mean mark.

(b) Calculate the standard deviation of the distribution.

(6 marks)

9. There are four defective oranges in a package of 10. If two oranges are selected at random one after another without replacement, what is the probability of each of the following?

(a) One defective and one good oranges will be selected.

(b) At least one defective orange will be selected. **(6 marks)**

10. (a) Show that, when n is a positive integer:
$\left(\sqrt{3}-i\right)^n+\left(\sqrt{3}+i\right)^n=2^{n+1}\cos\left(\frac{n\pi}{6}\right)$.

(b) If $z=x+iy$, find real values of x and y which satisfies $z\bar{z}-6i=12-2iz$ where $\bar{z}$ is the conjugate of z. **(6 marks)**

SECTION B (40 marks)

Answer any **four (4)** questions from this section. Extra questions will **not** be marked.

11. (a) Use knowledge on vectors to prove the sine rule for plane triangles.

(b) A force given by $\underline{F}=3\underline{i}+2\underline{j}-4\underline{k}$ is applied at the point (1, -1, 2). Find the moment of $\underline{F}$ about the point (2, -1, 3).

(c) Show that $\underline{a}=(2\underline{i}-2\underline{j}+\underline{k})/3$, $\underline{b}=(\underline{i}+2\underline{j}+2\underline{k})/3$ and $\underline{c}=(2\underline{i}+\underline{j}-2\underline{k})/3$ are mutually orthogonal unit vectors. **(10 marks)**

12. (a) The roots of the equation $3x^2+4x-5=0$ are α and β. Find:

(i) $\frac{1}{\alpha}+\frac{1}{\beta}$,

(ii) $\alpha^2+\beta^2$,

(iii) an equation whose roots are α^3 and β^3.

(b) If $f(x)=ax^2+bx+c$ leaves remainders 1, 25 and 1 on division by $x-1$, $x+1$ and $x-2$ respectively, show that $f(x)$ is a perfect square. **(10 marks)**

13. (a) Determine the coordinates of the point at which the normal to the curve $xy = 8$ at the point (4, 2), cuts the tangent to the curve $16x^2 - y^2 = 64$ at the point $\left(\frac{5}{2}, 6\right)$.

(b) AB is a chord of the rectangular hyperbola $xy = c^2$ and M is its midpoint. If AB has a constant length, find the locus of M.

(10 marks)

14. (a) Differentiate the following hyperbolic functions with respect to x:

(i) $f(x) = \cosh^{-1}(3 - 2x)$,

(ii) $f(x) = \sinh^{-1}(\tan x)$.

(b) Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$.

(c) Show that $\tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$.

(10 marks)

15. (a) Simplify each of the following propositions by using laws of algebra of propositions:

(i) $q \rightarrow (\sim p \rightarrow \sim q)$,

(ii) $\sim(p \vee q) \vee (\sim p \wedge q)$.

(b) Use the truth table to test the validity of the following argument: "You like Geography and Advanced Mathematics or you do not like Geography and Economics".

(10 marks)

16. (a) A curve is given by the parametric equations $x = a(\theta + \sin\theta)$ and $y = a(1 + \cos\theta)$. Find the length of the curve between the points $\theta = 0$ and $\theta = \pi/2$.

(b) Integrate the following:

$$\int \frac{e^x}{\sqrt{(5 - 4e^x - e^{2x})}} dx.$$

(c) By using binomial expansion of $(1 + 2x)^{1/2}$, evaluate $(1.02)^{1/2}$ correct to five decimal places.

(10 marks)