ECH 267 Nonlinear Control Theory
Final Project Report DRAFT

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Github Repo Hosted at:

https://github.com/JonnyD1117/ECH-267-Adv.-Proc.-Control

Dynamics Model of RRR-Robot Manipulator

Lagrange-Euler Equations of Motion

SymPy-Symbolic Math Library

EOM Derivation

Equation 1

$$
\begin{aligned}
&1.0 I_1 \frac{d^2}{dt^2} \mathrm{q}_1(t) + 0.25 L_2^2 m_2 \frac{d^2}{dt^2} \mathrm{q}_1(t) + 1.0 L_2^2 m_3 \frac{d^2}{dt^2} \mathrm{q}_1(t) \\
&\quad + 0.5 L_2 L_3 m_3 \sin\left(\mathrm{q}_1(t) - \mathrm{q}_2(t)\right) \left(\frac{d}{dt} \mathrm{q}_2(t)\right)^2 \\
&\quad + 0.5 L_2 L_3 m_3 \cos\left(\mathrm{q}_1(t) - \mathrm{q}_2(t)\right) \frac{d^2}{dt^2} \mathrm{q}_2(t) \\
&\quad + 0.5 L_2 g m_2 \cos\left(\mathrm{q}_1(t)\right) + 1.0 L_2 g m_3 \cos\left(\mathrm{q}_1(t)\right)
\end{aligned} \tag{1}
$$

Equation 2

$$
\begin{aligned}
&1.0 I_2 \frac{d^2}{dt^2} \mathrm{q}_2(t) - 0.5 L_2 L_3 m_3 \sin\left(\mathrm{q}_1(t) - \mathrm{q}_2(t)\right) \left(\frac{d}{dt} \mathrm{q}_1(t)\right)^2 \\
&\quad + 0.5 L_2 L_3 m_3 \cos\left(\mathrm{q}_1(t) - \mathrm{q}_2(t)\right) \frac{d^2}{dt^2} \mathrm{q}_1(t) \\
&\quad + 0.25 L_3^2 m_3 \frac{d^2}{dt^2} \mathrm{q}_2(t) + 0.5 L_3 g m_3 \cos\left(\mathrm{q}_2(t)\right)
\end{aligned} \tag{2}
$$

Equation 3

$$
T_3 = 1.0 I_3 \frac{d^2}{dt^2} \mathrm{q}_3(t) \tag{3}
$$

RRR Forward Dynamics Matrices

$$
\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + K(\theta)
$$

$$
M(\theta) = \begin{bmatrix} I_1 + 0.25 L_2^2 m_2 + L_2^2 m_3 & 0.5 L_2 L_3 m_3 \cos\left(\mathrm{q}_1(t) - \mathrm{q}_2(t)\right) & 0 \\ 0.5 L_2 L_3 m_3 \cos\left(\mathrm{q}_1(t) - \mathrm{q}_2(t)\right) & I_2 + 0.25 L_3^2 m_3 & 0 \\ 0 & 0 & I_3 \end{bmatrix}
$$

$$
V(\theta, \dot{\theta}) = \begin{bmatrix} 0.5 L_2 L_3 m_3 \sin\left(\mathrm{q}_1(t) - \mathrm{q}_2(t)\right) \left(\frac{d}{dt} \mathrm{q}_2(t)\right)^2 \\ -0.5 L_2 L_3 m_3 \sin\left(\mathrm{q}_1(t) - \mathrm{q}_2(t)\right) \left(\frac{d}{dt} \mathrm{q}_1(t)\right)^2 \\ 0 \end{bmatrix}
$$

$$K(\theta) = \begin{bmatrix} 0.5L_2 g m_2 \cos\left(\mathrm{q}_1\left(t\right)\right) + L_2 g m_3 \cos\left(\mathrm{q}_1\left(t\right)\right) \\ 0.5L_3 m_3 \cos\left(\mathrm{q}_2\left(t\right)\right) \\ 0 \end{bmatrix}$$

Inverse Dynamics

$$\ddot{\theta} = M^{-1}(\theta)[\tau - V(\theta, \dot{\theta}) - K(\theta)]$$