Question 1

In this question, 10K data points will be generated using the given parameters, means, covariance matrices and priors. And all of these data points will be utilized in Part A and Part B.

Part A

1.

The minimum expected risk classification rule in the form of a likelihood-ratio test is:

$$\frac{p(x|L=1)}{p(x|L=0)} = \frac{g(x|m_1, C_1)}{g(x|m_0, C_0)} > \gamma = \frac{p(L=0)}{p(L=1)} \times \frac{\lambda_{10} - \lambda_{00}}{\lambda_{01} - \lambda_{11}}$$

Where λ_{ij} is the cost associated with deciding action i when true label is j. When left value is larger than the right, the decision will be $L = 1$. Otherwise, the decision will be $L = 0$. In this problem, if we use zero-one cost/loss function, then $\lambda_{ij} = 1$ when $i \neq j$ otherwise $\lambda_{ij} = 0$. And the theoretical γ can be calculated as:

$$\gamma = \frac{0.35}{0.65} \times \frac{1-0}{1-0} = 0.5385$$

2.

The ROC curve is shown in Figure 1.

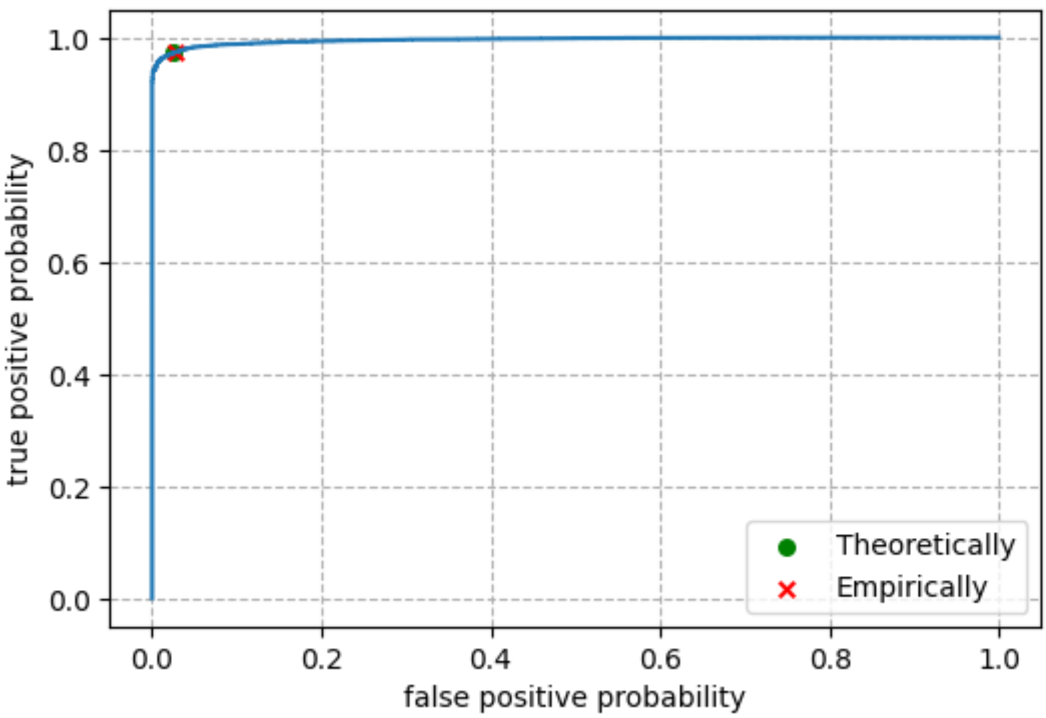


Figure 1: ROC curve for ERM classification using the knowledge of true data pdf

Through the experiments, we can find that empirically selected gamma is 0.4476 minimizes probability of error which is 0.0264. And theoretically optimal threshold is 0.5385, the probability error is 0.0266. The results of these two cases are very similar.

Type of Estimate	γ	Probability of error
Experimental	0.4476	0.0264
Theoretical	0.5385	0.0266

Part B

In this part, we obtain the Gaussian mixture distribution's parameters as below:

$$m_0 = [-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}]$$
$$m1 = [1, 1, 1, 1]$$
$$C_0 = \frac{1}{4}\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

The ROC curve is shown in Figure 2. The blue line is the ROC curve of Part A using correct knowledge and the orange line is the ROC curve of Part B using incorrect knowledge.

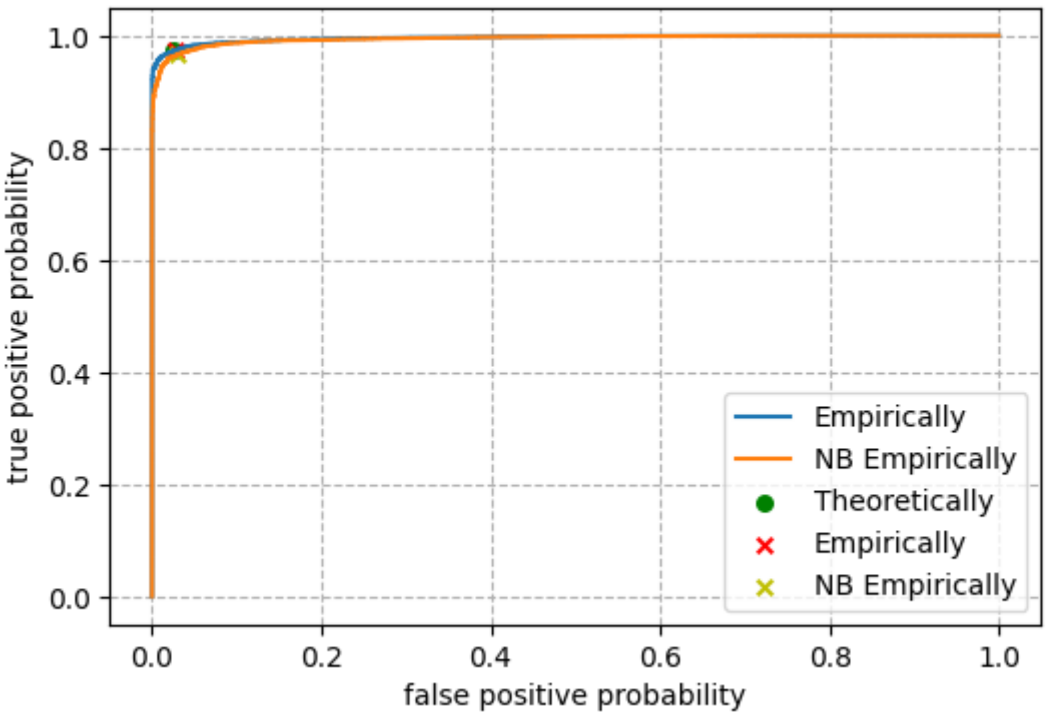


Figure 3: ROC curve for ERM classification using incorrect knowledge of data pdf

Using incorrect knowledge, empirically selected gamma 0.5092 minimizes probability of error which is 0.0321.

Type of Estimate	γ	Probability of error
Experimental	0.5092	0.0321

Type of Estimate	γ	Probability of error
Theoretical	0.5385	0.0266

From the table, it shows that using incorrect knowledge the probability of error increases comparing previous result. Therefore, this model make a little of negative effect on the minimum achievable probability of error.

Part C

The ROC curve is shown in Figure 3. The blue line is the ROC curve of Part A using correct knowledge and the orange line is the ROC curve of Part B using incorrect knowledge. And the green line of ROC is LDA.

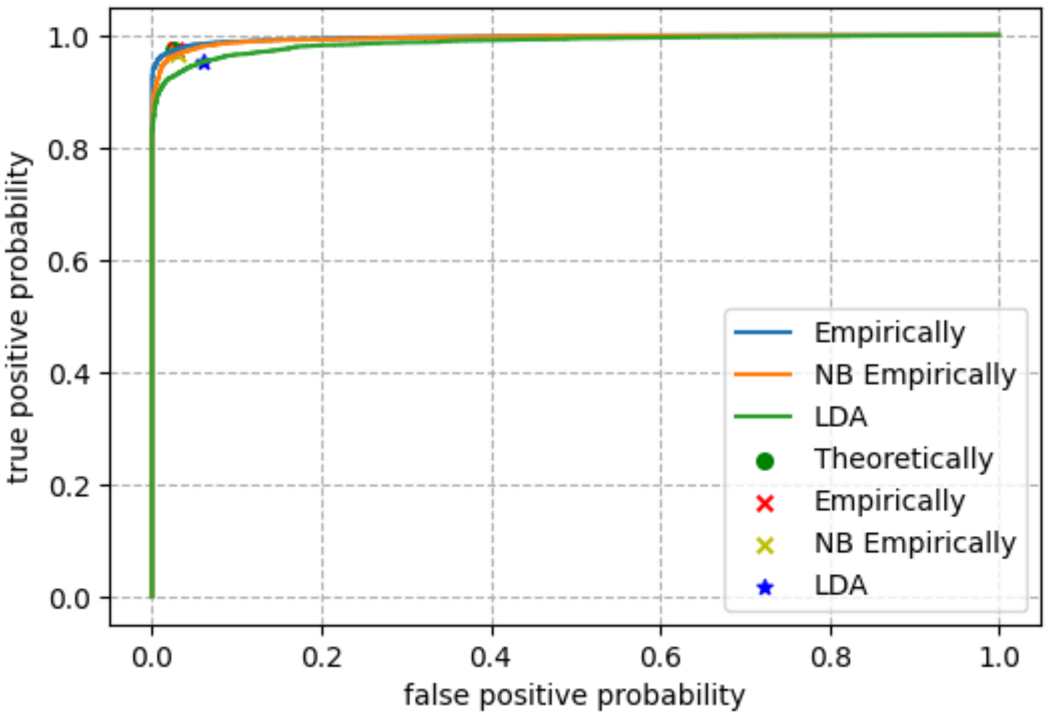


Figure 3: ROC curve for LDA classification

In experiment, $w_{LDA} = [0.59866206, 0.28086844, 0.71114533, 0.23872365]^T$. When the threshold $\tau = -0.0492$, the minimal probability of error is 0.0517. Comparing with previous two classifiers, LDA achieves largest minimal probability of error. This conclusion is consistent with ROC plot.

Question 2

Part A

I generating 10K samples from four Gaussian distributions. The data points are all three dimensional. And their scatter plot is shown in Figure 4.

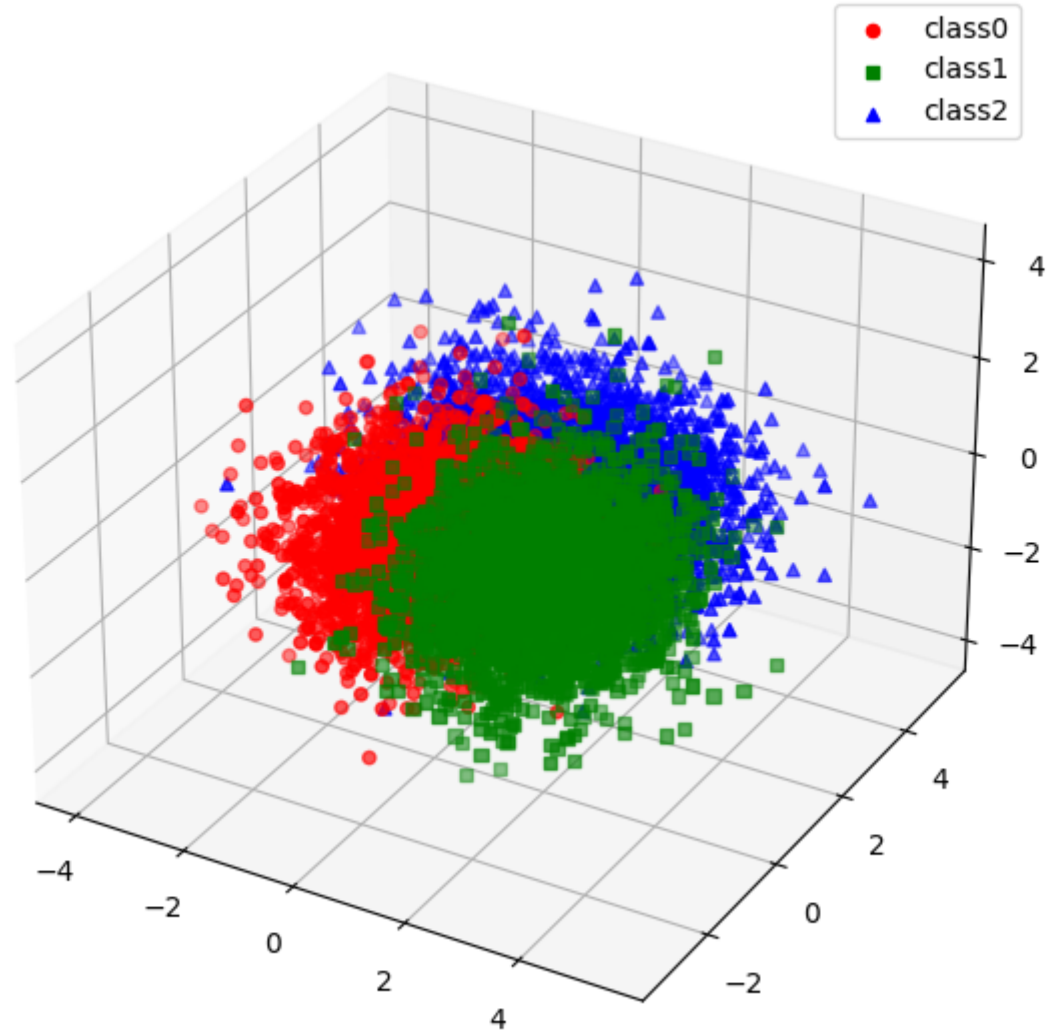


Figure 4: scatter plot of generated 10K samples

For a give data point x, the ERM decision rule is:

$$\begin{aligned} D(x) &= \arg\min_{i \in 1,2,3} R(D=i|x) \\ &= \arg\min_{i \in 1,2,3} \sum_{j=1}^{3} \lambda_{ij} p(L=j|x) \\ &= \arg\min_{i \in 1,2,3} \sum_{j=1}^{3} \lambda_{ij} p(x|L=j) p(L=j) \end{aligned}$$

Based on this rule, I complete empirical risk minimization (ERM) algorithm for data classification. And the confusion matrix is shown in Figure 5.

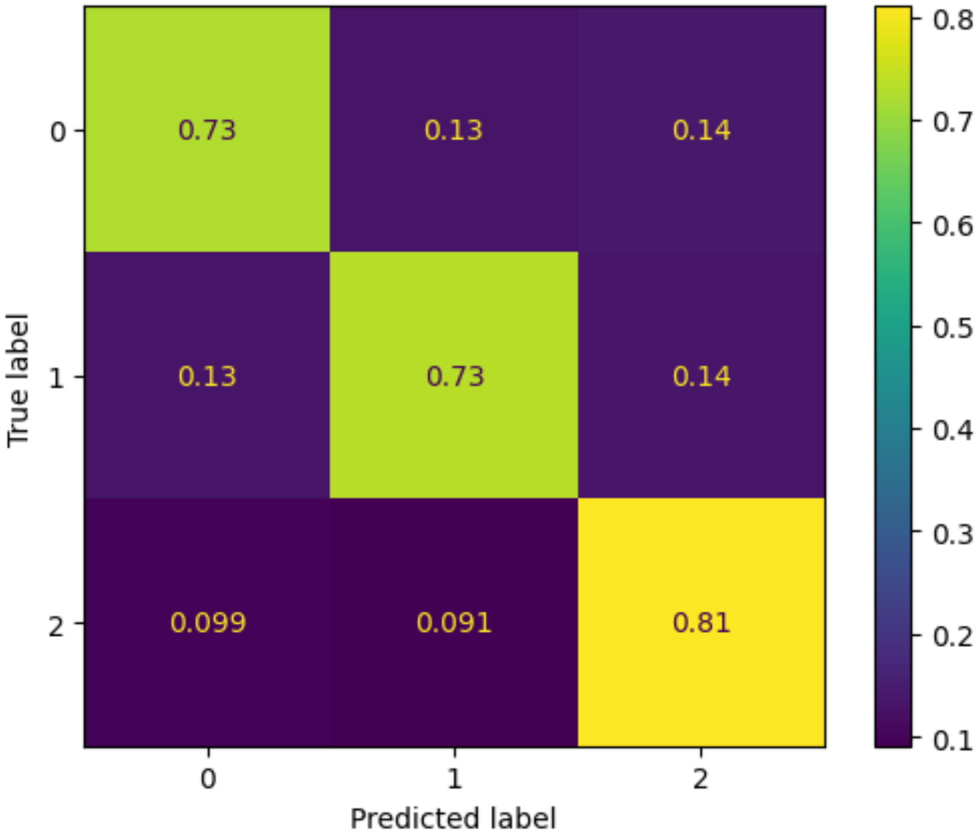


Figure 5: confusion matrix for generated 10K samples.

Furthermore, I also draw the scatter plot for each sample indicate the true class label with a different marker shape and whether it was correctly (green) or incorrectly (red) classified with a different marker color. It is shown in Figure 6.

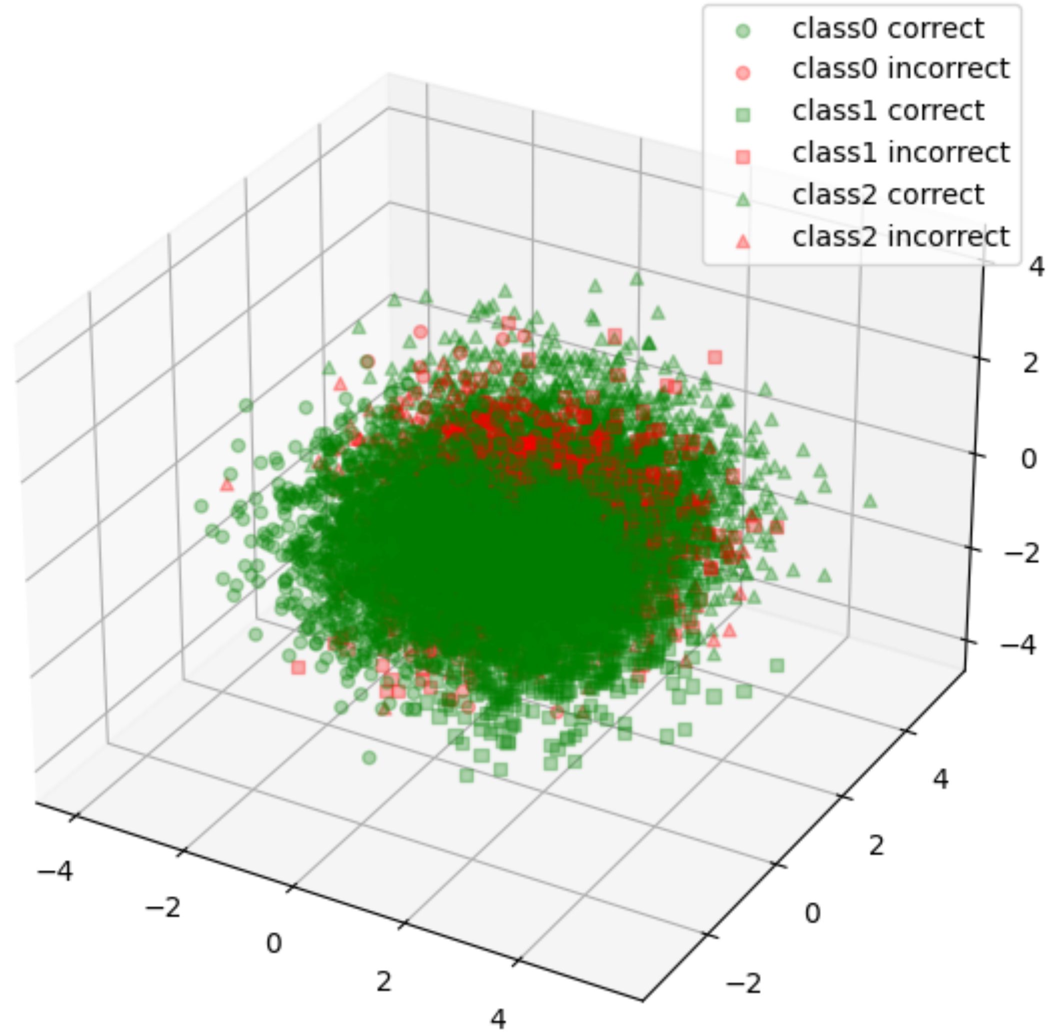


Figure 6: scatter plot for generated 10K samples with classification information.

Part B

In this part, ERM classiffcation will do twice with the following two different loss matrices which respectively care 10 times or 100 times more about not making mistakes when L = 3. For Λ_{10}, Figure 7 is the confusion matrix visualization for 10 times loss matirx. And Figure 8 is its 3-d scatter plot with data sample classification correct or incorrect information.

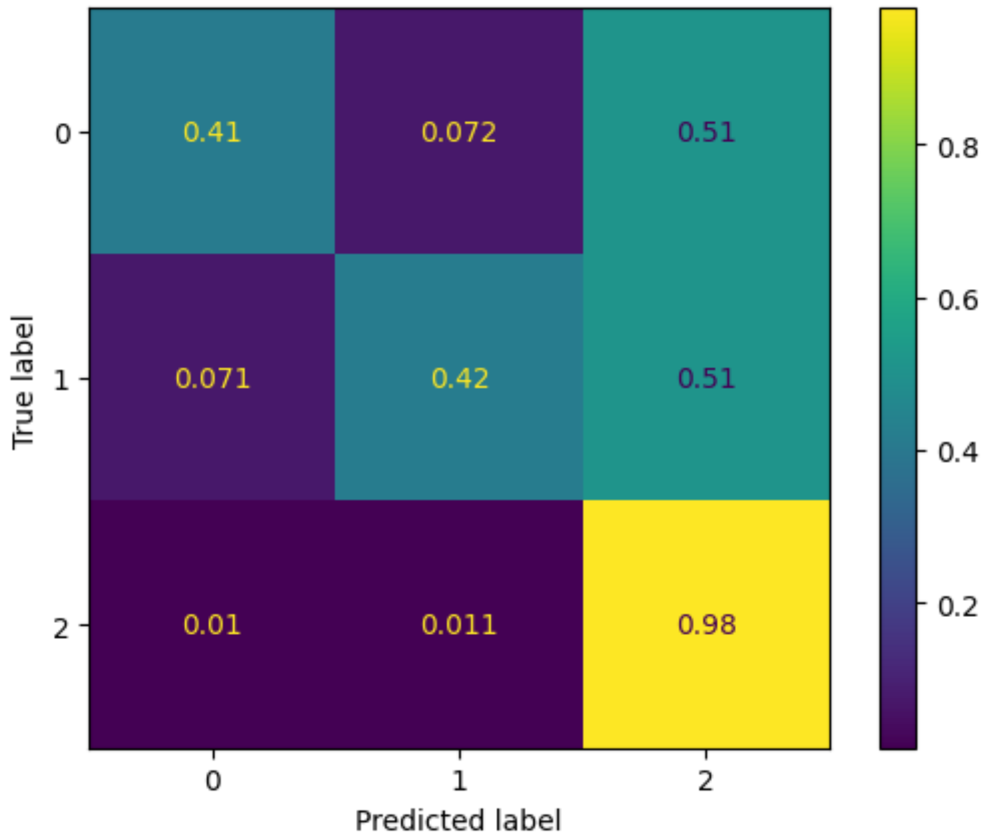


Figure 7: confusion matrix for 10 times loss matrix.

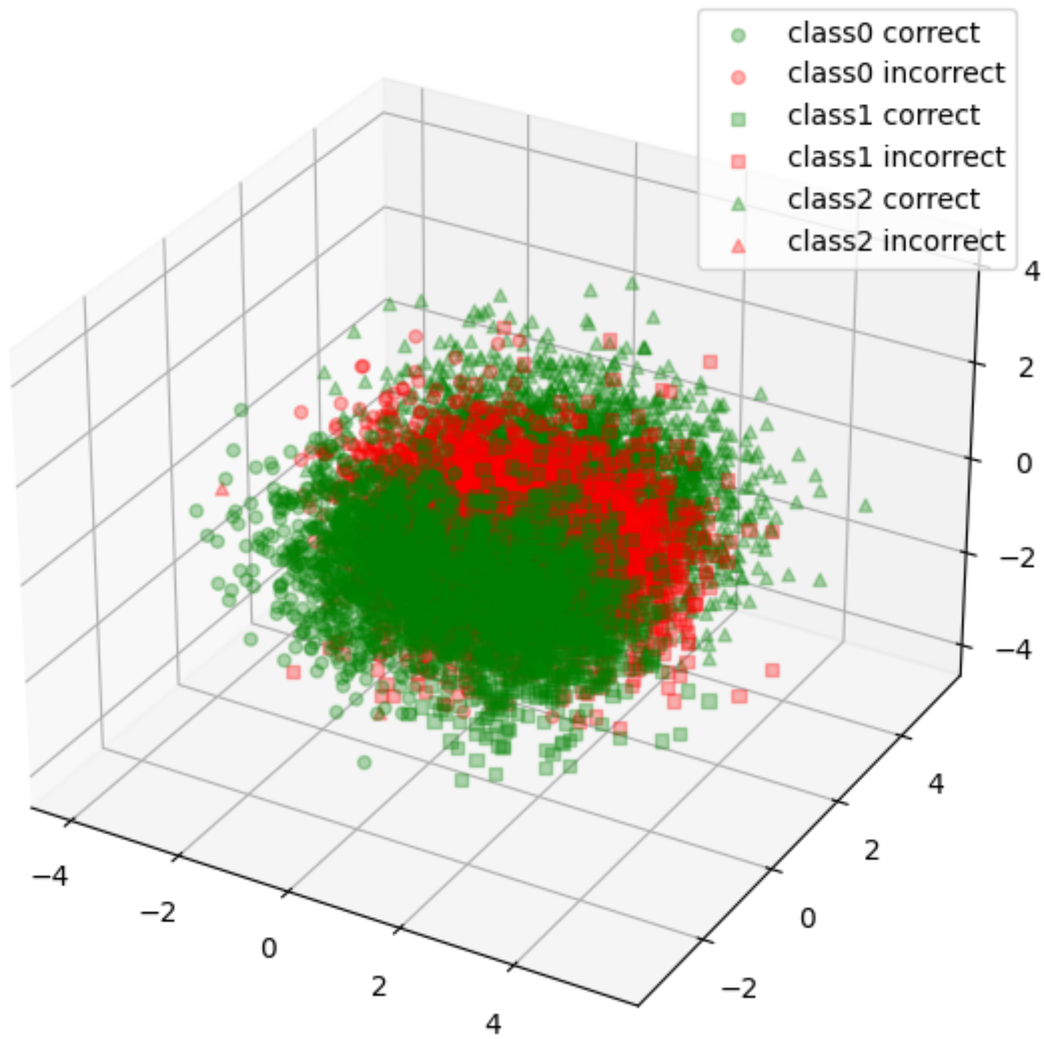


Figure 8: scatter plot with data sample classification correct or incorrect information

For Λ_{100}, Figure 9 is the confusion matrix visualization for 10 times loss matrix. And Figure 10 is its 3-d scatter plot with data sample classification correct or incorrect information.

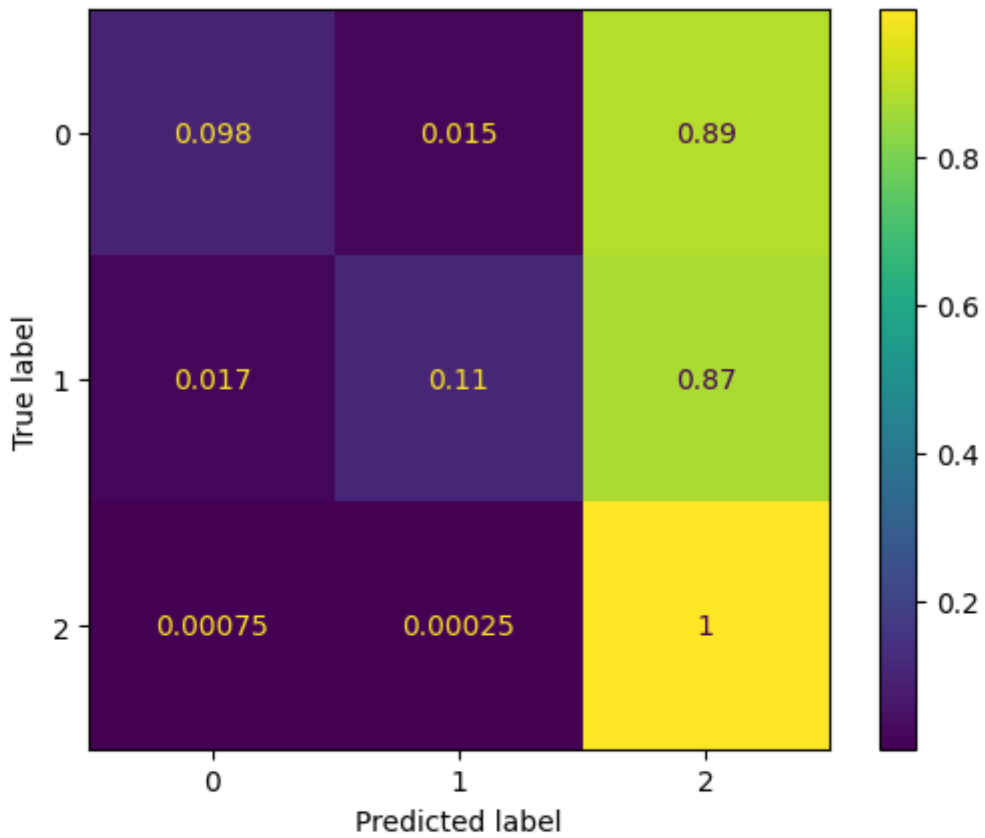


Figure 9: confusion matrix for 100 times loss matrix.

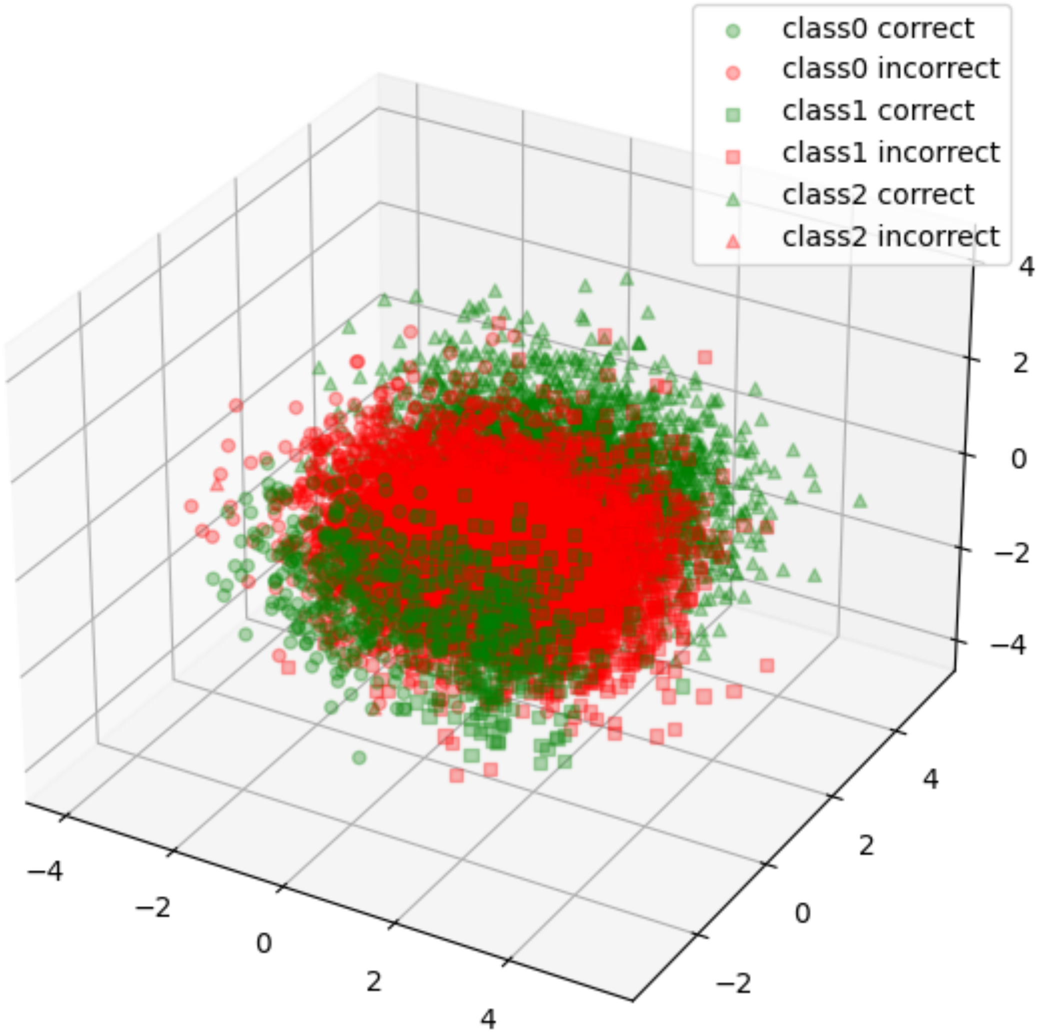


Figure 10: scatter plot with data sample classification correct or incorrect information

Interesting insights

Comparing Part A and Part B, it can be find that each sample share the same importance in classification. Therefore, its classification result which is bad or good is similar for different class. While using different loss matrix, it means the classifier will pay more attention to certain class. For Λ_{100}, it can be see that the classifier tends to classifies all data samples as label 2. In data imbalanced problem or outlier detection, using such loss matrix is useful. If we have no more information about the dataset, Part A using 0-1 loss is our first consideration.

Question 3

For Wine Quality dataset, it consists of 11 features, and class labels from 0 to 10 indicating wine quality scores. There are 4898 samples.

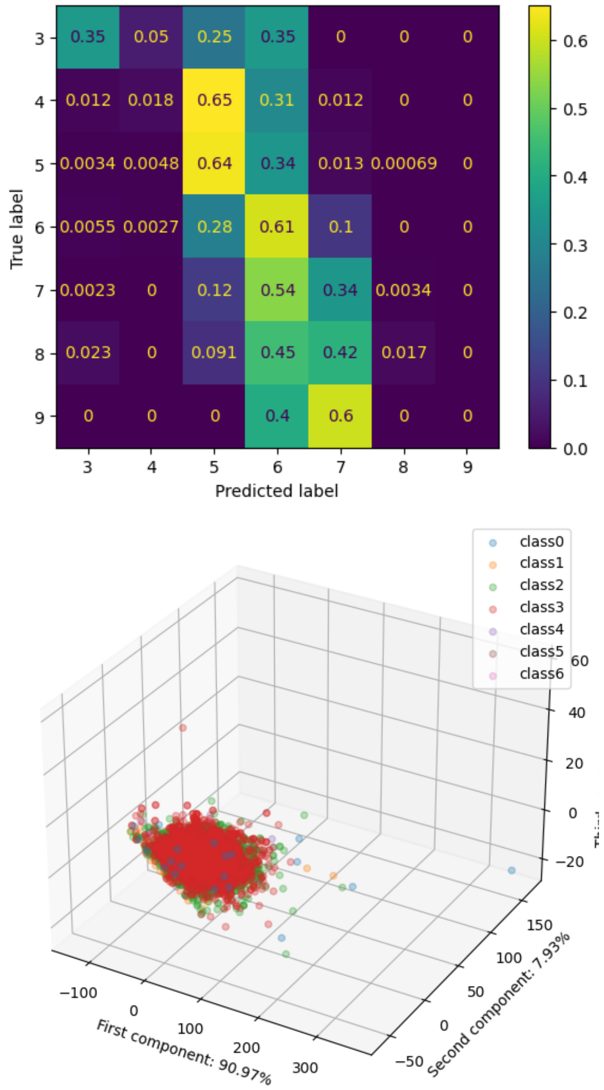


Figure 11:confusion matrix of wine quality classification and its 3d projected scatter plot

Figure 11 show the confusion matrix of the wine dataset classification result. And the plot at right is the 3-d plot using PCA for selecting features. The first three dimensions' explained variance ratio is $[0.90965734, 0.07933386, 0.01015427]$. The empirical estimated minimum probability error is 0.4726. The error is very large. And the scatter plot also show that sample of these 6 classes are overlapped heavily.

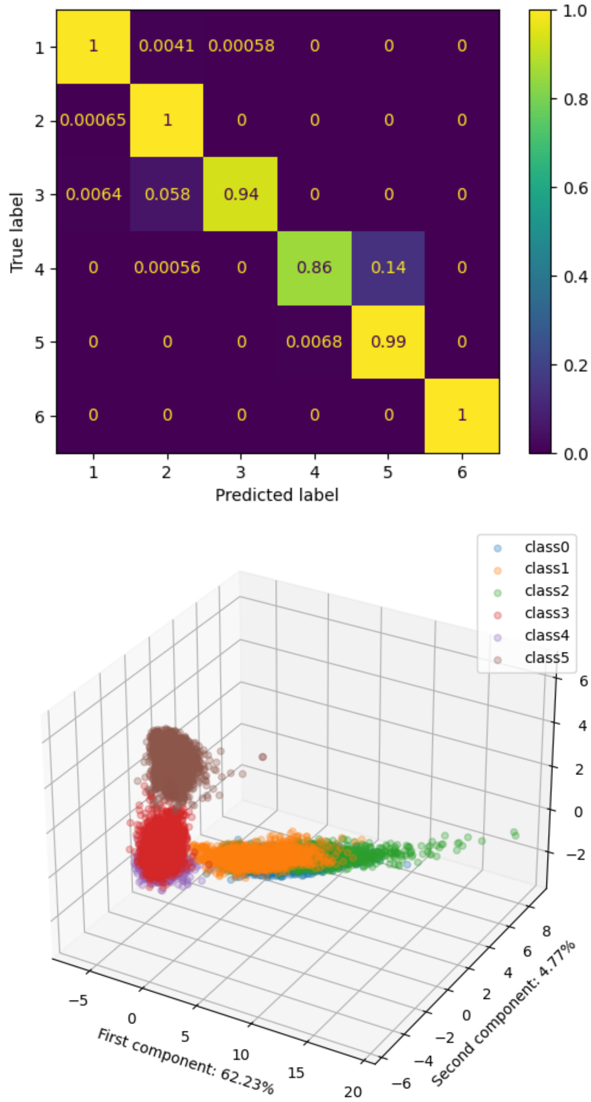


Figure 12: confusion matrix of HumanActivity classification and its 3d projected scatter plot

For Human Activity Recognition dataset, it consists of 561 features, and 6 activity labels. There are 10299 samples. Figure 12 show the confusion matrix of the HumanActivity dataset classification result. And the plot at right is the 3-d plot using PCA for selecting features. The first three dimensions' explained variance ratio is $[0.62227069, 0.04772595, 0.04018191]$. The empirical estimated minimum probability error is 0.0359. This result is very good. And the the diagonal elements of confusion matrix are or approximate 1 that means perfect classfication. And from the 3d scatter plot , it's can easily to see that these 6 classes of samples have good separation. Therefore, ERM performs better in HumanActivity dataset than Wine quality dataset.

Code Appendix

[2]:
```
import numpy as np
import pandas as pd
from scipy.stats import multivariate_normal
import matplotlib.pyplot as plt
from mpl_toolkits import mplot3d
from sklearn.metrics import confusion_matrix,ConfusionMatrixDisplay
from sklearn.preprocessing import LabelEncoder
from sklearn.decomposition import PCA
```

0.1 Question 1

0.1.1 Part A

[3]:
```
def generation(gmm_params,num=10000):
### gmm_params is a dictionary, its key are mu,Sigma,priors. It can represent a␣
↪GMM distribution.
    np.random.seed(2023)
    priors = gmm_params['prior']
    C = len(priors)
    num_list = (num * priors).astype(int)
    mu = gmm_params['mu']
    cov = gmm_params['sigma']
    data = np.concatenate([np.random.multivariate_normal(mu[i],cov[i],size=val)␣
↪for i,val in enumerate(num_list)])
    label = np.concatenate([i*np.ones(val) for i,val in enumerate(num_list)])
    return data,label
```

[4]:
```
m0 = np.array([-1./2,-1./2,-1./2,-1./2])
c0 = 1./4*np.array([[2,-0.5,0.3,0],[-0.5,1,-0.5,0],[0.3,-0.5,1,0],[0,0,0,2]])
m1 = np.array([1,1,1,1])
c1 = np.array([[1,0.3,-0.2,0],[0.3,2,0.3,0],[-0.2,0.3,1,0],[0,0,0,3]])
m = np.array([[-1./2,-1./2,-1./2,-1./2],[1,1,1,1]])
c = np.array([c0,c1])

gmm_params = {}
gmm_params['mu'] = m
gmm_params['sigma'] = c
gmm_params['prior'] = np.array([0.35,0.65])
```

```
data,label = generation(gmm_params)
```

[5]:
```
def ratioCal(data,label,gmm_params):
    mu = gmm_params['mu']
    sigma = gmm_params['sigma']
    prior = gmm_params['prior']
    ratio = []
    for i,item in enumerate(data):
        p1 = multivariate_normal.pdf(item,mu[1],sigma[1])
        p0 = multivariate_normal.pdf(item,m[0],sigma[0])
        ratio.append(p1/p0)
    return np.array(ratio)

def tpr_fpr_cal(label,ratio,val_list):
    tpr = []
    fpr = []
    for val in val_list:
        pred = np.zeros_like(label)
        pred[ratio>val] = 1
        tpr.append(np.logical_and(label==1,pred==1).sum()/(label==1).sum())
        fpr.append(np.logical_and(label==0,pred==1).sum()/(label==0).sum())
    tpr = np.array(tpr)
    fpr = np.array(fpr)
    return tpr,fpr
```

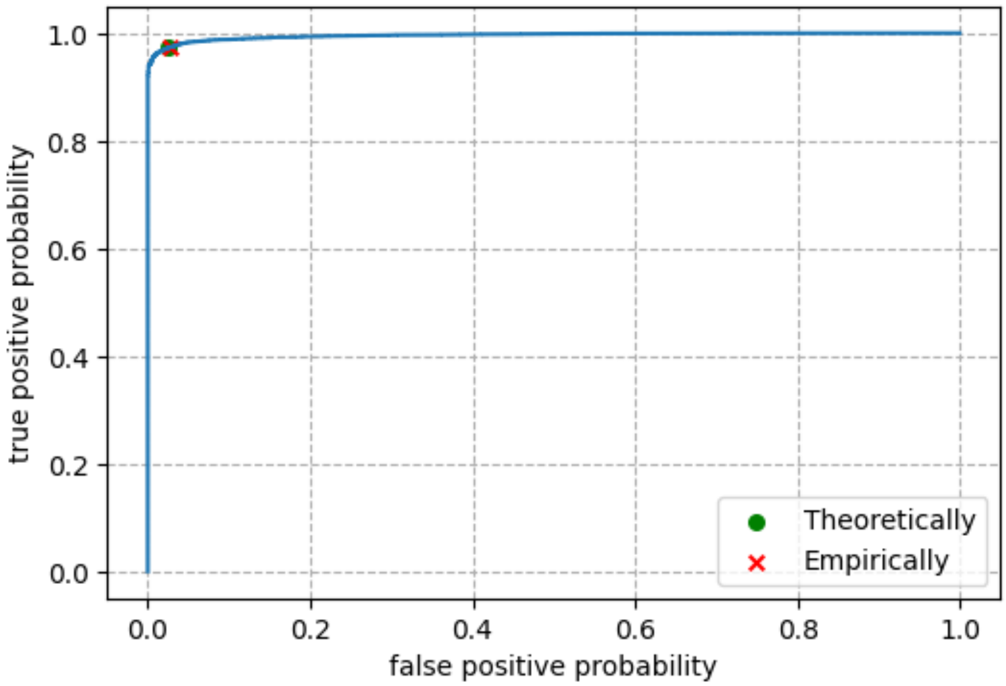
[6]:
```
ratio = ratioCal(data,label,gmm_params)
val_list = np.sort(ratio)
tpr,fpr = tpr_fpr_cal(label,ratio,val_list)
```

[7]:
```
prior = gmm_params['prior']
prob_err = fpr * prior[0] + (1 - tpr) * prior[1]
prob_err_min = min(prob_err)
threshold_exp = val_list[np.argmin(prob_err)]

optimal_gamma = prior[0]/prior[1]
pred = np.zeros_like(label)
pred[ratio>optimal_gamma] = 1
tpr_optimal = np.logical_and(label==1,pred==1).sum()/(label==1).sum()
fpr_optimal = np.logical_and(label==0,pred==1).sum()/(label==0).sum()
prob_err_opt = fpr_optimal * prior[0] + (1-tpr_optimal)*prior[1]

print(f'Empirically selected gamma {threshold_exp} minimizes P(error) which is␣
  ↪{prob_err_min}!')
print(f'Theoretially optimal theoreshold is {optimal_gamma}, the probability of␣
  ↪error is {prob_err_opt}')
```

```
Empirically selected gamma 0.44762079014733613 minimizes P(error) which is
```

```
0.02640000000000003!
Theoretially optimal theoreshold is 0.5384615384615384, the probability of error
is 0.026599999999999997
```

[8]:
```
plt.figure(figsize=(6,4))
plt.plot(fpr,tpr)
plt.scatter(fpr_optimal,tpr_optimal,marker='o',c='g',s=30,label='Theoretically')
plt.scatter(fpr[np.argmin(prob_err)],tpr[np.
 ↪argmin(prob_err)],marker='x',c='r',s=30,label='Empirically')
plt.xlabel('false positive probability')
plt.ylabel('true positive probability')
plt.grid(linestyle='--')
plt.legend()
plt.show()
```



0.1.2 Part B

[9]:
```
m0 = np.array([-1./2,-1./2,-1./2,-1./2])
c0 = 1./4*np.array([[2,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,2]])
m1 = np.array([1,1,1,1])
c1 = np.array([[1,0,0,0],[0,2,0,0],[0,0,1,0],[0,0,0,3]])
m = np.array([[-1./2,-1./2,-1./2,-1./2],[1,1,1,1]])
c = np.array([c0,c1])
```

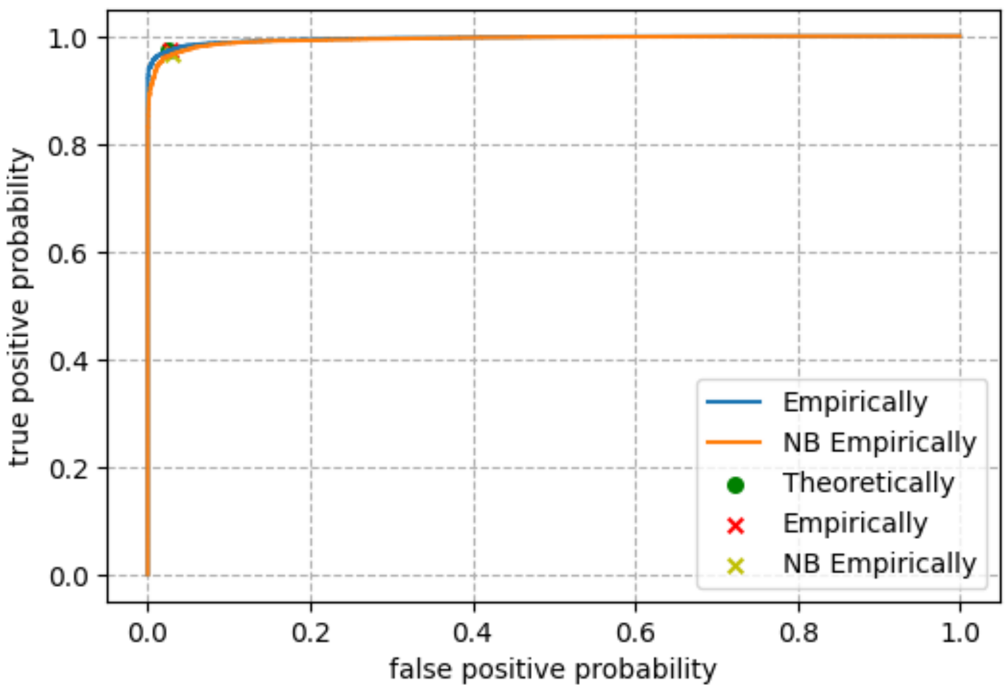
```
gmm_params = {}
gmm_params['mu'] = m
gmm_params['sigma'] = c
gmm_params['prior'] = np.array([0.35,0.65])
```

[10]:
```
ratio_1 = ratioCal(data,label,gmm_params)
val_list_1 = np.sort(ratio_1)
tpr_1,fpr_1 = tpr_fpr_cal(label,ratio_1,val_list_1)
```

[11]:
```
prob_err_1 = fpr_1 * prior[0] + (1 - tpr_1) * prior[1]
prob_err_min_1 = min(prob_err_1)
threshold_exp_1 = val_list_1[np.argmin(prob_err_1)]

print(f'Using incorrect knowledge, empirically selected gamma {threshold_exp_1}␣
 ↪minimizes P(error) which is {prob_err_min_1}!')
```

```
Using incorrect knowledge, empirically selected gamma 0.5092261718997159
minimizes P(error) which is 0.032100000000000024!
```

[12]:
```
plt.figure(figsize=(6,4))
plt.plot(fpr,tpr,label='Empirically')
plt.plot(fpr_1,tpr_1,label='NB Empirically')
plt.scatter(fpr_optimal,tpr_optimal,marker='o',c='g',s=30,label='Theoretically')
plt.scatter(fpr[np.argmin(prob_err)],tpr[np.
 ↪argmin(prob_err)],marker='x',c='r',s=30,label='Empirically')
plt.scatter(fpr_1[np.argmin(prob_err_1)],tpr_1[np.
 ↪argmin(prob_err_1)],marker='x',c='y',s=30,label='NB Empirically')

plt.xlabel('false positive probability')
plt.ylabel('true positive probability')
plt.grid(linestyle='--')
plt.legend()
plt.show()
```



0.1.3 Part C

[25]:
```
def lda(X,Y,C=2):
    mu = np.array([np.mean(X[Y==i],axis=0).reshape(-1,1) for i in range(C)])
    sigma = np.array([np.cov(X[Y==i].T) for i in range(C)])
    sb = (mu[1]-mu[0]).dot((mu[1]-mu[0]).T)
    sw = sigma[0] + sigma[1]
    eigval, eigvec = np.linalg.eig(np.linalg.inv(sw).dot(sb))
    idx = eigval.argsort()[::-1]
    eigvec = eigvec[:, idx]
    w = eigvec[:, 0]
    z = X@(w)
    return w, z
```

[16]:
```
w,z = lda(X=data,Y=label)
```

[26]:
```
w
```

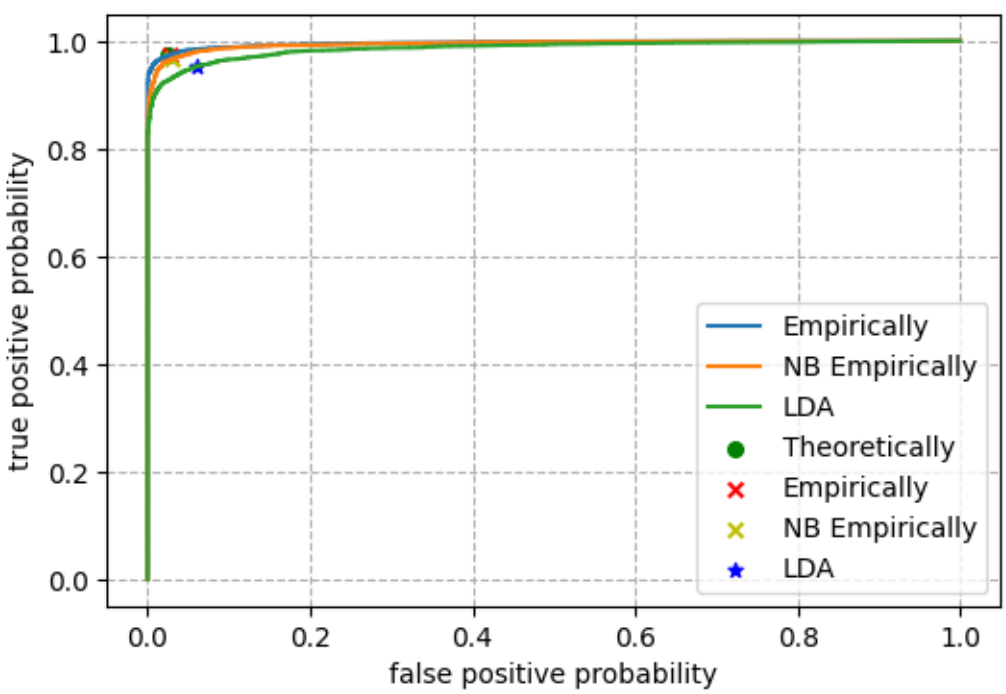
[26]:
```
array([0.59866206+0.j, 0.28086844+0.j, 0.71114533+0.j, 0.23872365+0.j])
```

[17]:
```
val_list_2 = np.sort(z)
tpr_2,fpr_2 = tpr_fpr_cal(label,z,val_list_2)
```

[24]:
```
prob_err_2 = fpr_2 * prior[0] + (1 - tpr_2) * prior[1]
prob_err_min_2 = min(prob_err_2)
threshold_exp_2 = val_list_2[np.argmin(prob_err_2)]
print(f'Using LDA, selected tao {threshold_exp_2} minimizes P(error) which is␣
↪{prob_err_min_2}!')
```

```
Using LDA, selected tao (-0.04922788135126424+0j) minimizes P(error) which is
0.05169999999999997!
```

[23]:
```
plt.figure(figsize=(6,4))
plt.plot(fpr,tpr,label='Empirically')
plt.plot(fpr_1,tpr_1,label='NB Empirically')
plt.plot(fpr_2,tpr_2,label='LDA')
plt.scatter(fpr_optimal,tpr_optimal,marker='o',c='g',s=30,label='Theoretically')
plt.scatter(fpr[np.argmin(prob_err)],tpr[np.
↪argmin(prob_err)],marker='x',c='r',s=30,label='Empirically')
plt.scatter(fpr_1[np.argmin(prob_err_1)],tpr_1[np.
↪argmin(prob_err_1)],marker='x',c='y',s=30,label='NB Empirically')
plt.scatter(fpr_2[np.argmin(prob_err_2)],tpr_2[np.
↪argmin(prob_err_2)],marker='*',c='b',s=30,label='LDA')

plt.xlabel('false positive probability')
plt.ylabel('true positive probability')
plt.grid(linestyle='--')
plt.legend()
plt.show()
```



0.2 Question 2

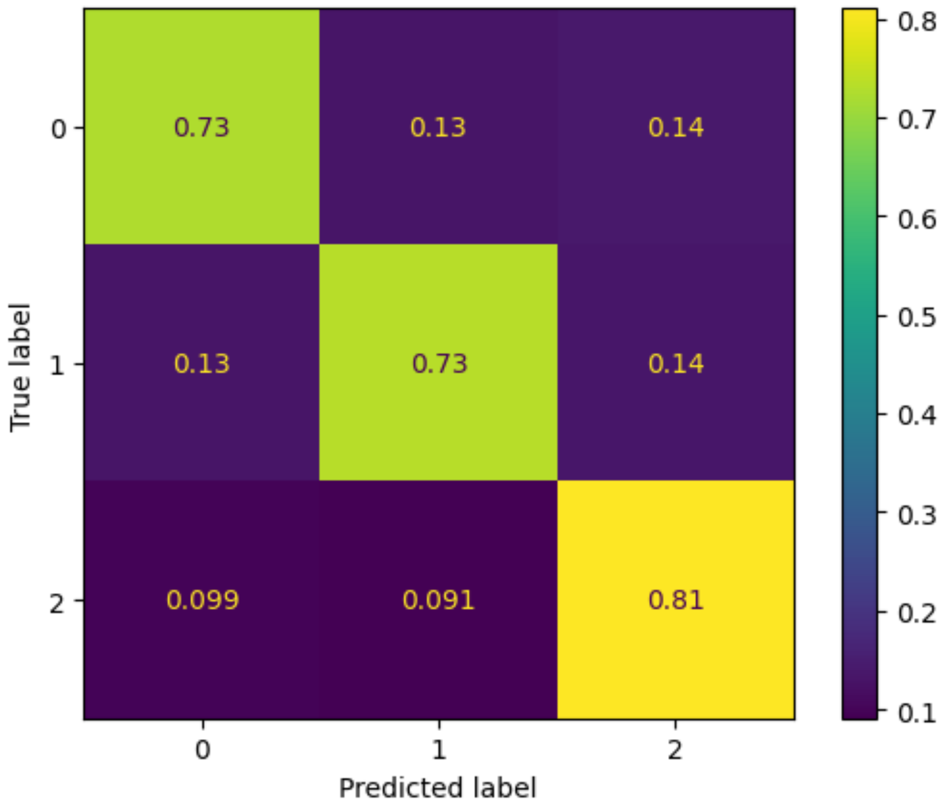
0.2.1 Part A

[63]:
```
def ERM(X, Lambda, gmm_params):
    class_cond_likelihoods = np.array([multivariate_normal.pdf(X,␣
↪gmm_params['mu'][i], gmm_params['sigma'][i]) for i in␣
↪range(len(gmm_params['prior']))])
    class_priors = np.diag(gmm_params['prior'])
    print(class_cond_likelihoods.shape,class_priors.shape)
    class_posteriors = class_priors.dot(class_cond_likelihoods)
    class_posteriors[-2,:] = class_posteriors[-2,:] + class_posteriors[-1,:]
    class_posteriors = class_posteriors[:-1,:]
    risk_mat = Lambda@class_posteriors
    return np.argmin(risk_mat, axis=0)
```

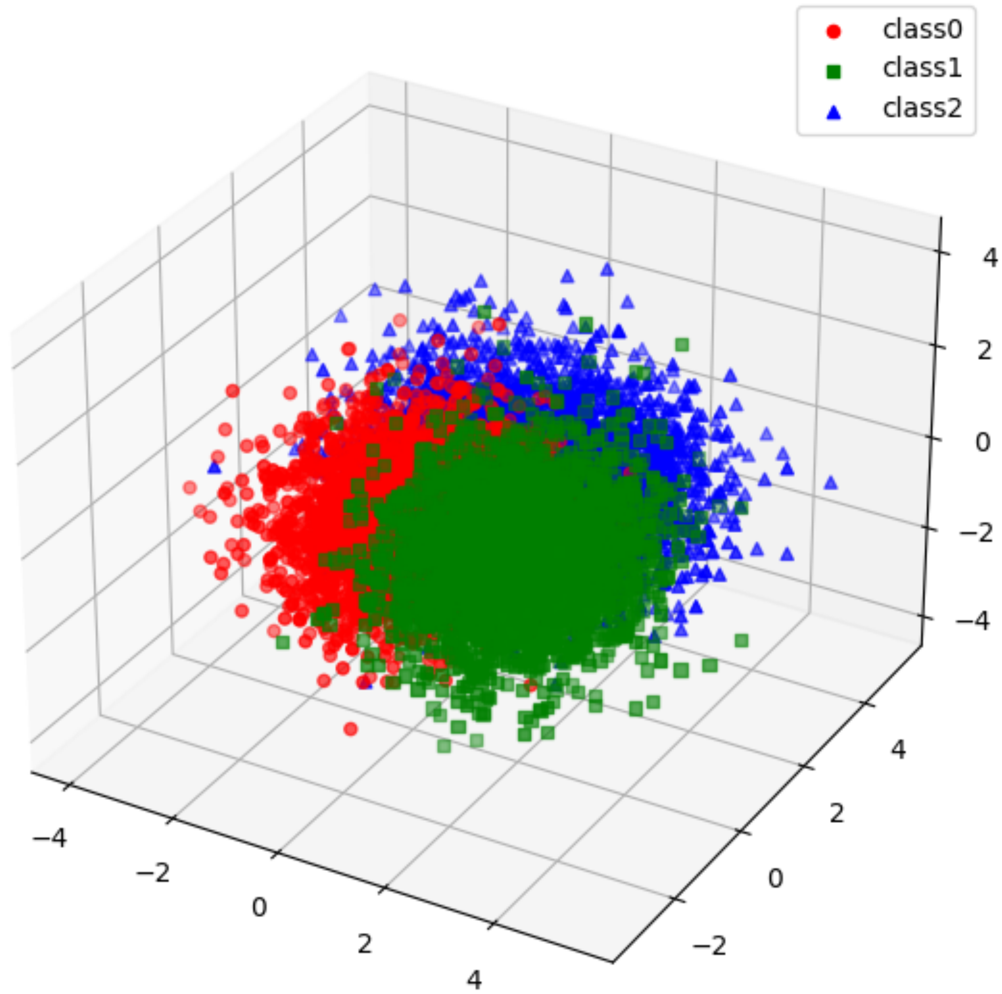
[64]:
```
gmm_params = {}
gmm_params['mu'] = np.array([[0,0,0],[2,0,0],[0,2,0],[2,2,0]])
gmm_params['sigma'] = np.array([np.eye(3),np.eye(3),np.eye(3),np.eye(3)])
gmm_params['prior'] = np.array([0.3,0.3,0.2,0.2])
x,y = generation(gmm_params,num=10000)
y[y==3] = 2
```

[68]:
```
loss = np.ones((3,3)) - np.eye(3)
pred = ERM(x,loss,gmm_params)
cm = confusion_matrix(y,pred)
prob_err = 1 - np.diag(cm).sum()/cm.sum()
print(f'The empirical estimated minimum probability error is {prob_err}')
cm_prob = cm/cm.sum(axis=1,keepdims=True)
disp = ConfusionMatrixDisplay(confusion_matrix=cm_prob,display_labels=[0,1,2])
disp.plot()
plt.show()
```

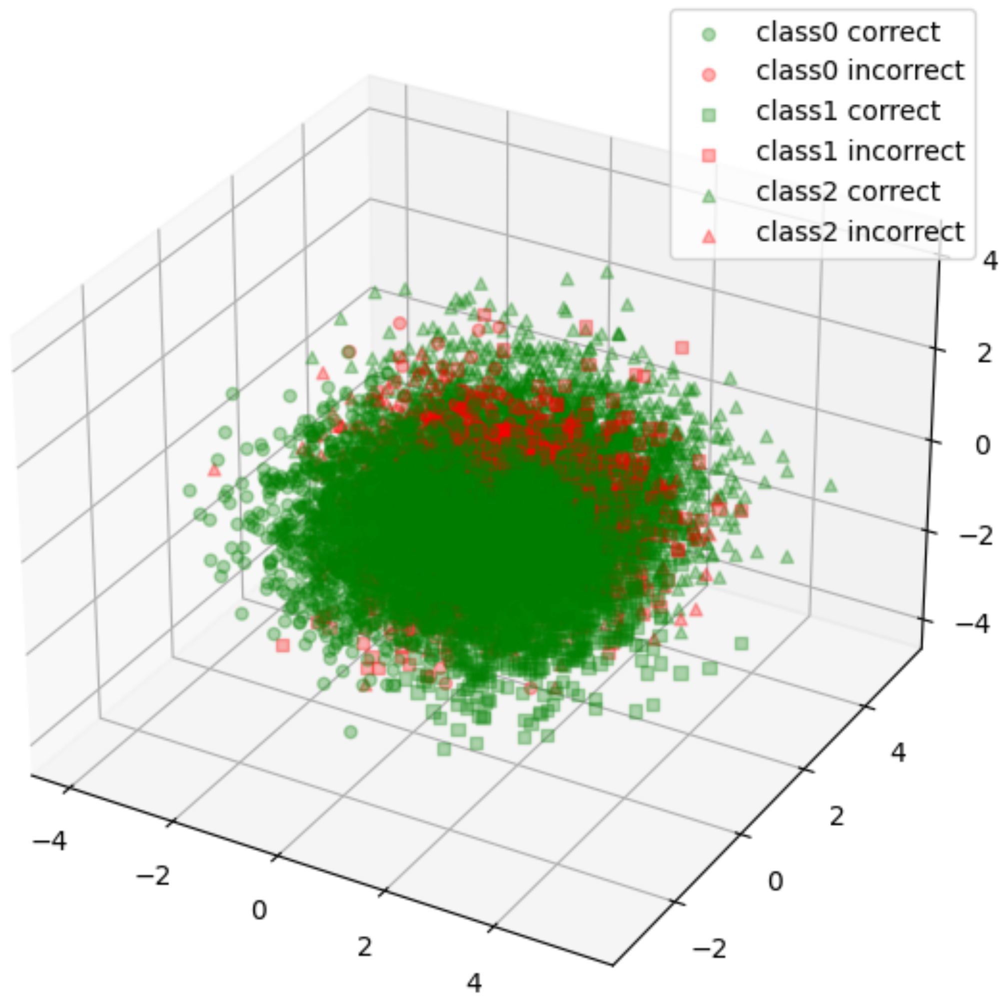
```
(4, 10000) (4, 4)
The empirical estimated minimum probability error is 0.23809999999999998
```



[72]:
```
plt.figure(figsize=(10,7))
ax = plt.axes(projection='3d')
ax.scatter3D(x[y==0,0],x[y==0,1],x[y==0,2],c='r',marker='o',label='class0')
ax.scatter3D(x[y==1,0],x[y==1,1],x[y==1,2],c='g',marker='s',label='class1')
ax.scatter3D(x[y==2,0],x[y==2,1],x[y==2,2],c='b',marker='^',label='class2')
plt.legend()
plt.show()
```




```
[82]: plt.figure(figsize=(10,7))
ax = plt.axes(projection='3d')

marker = ['o','s','^']

for i in range(3):
    idx = np.logical_and(y==i,pred==i)
    idx1 = np.logical_and(y==i,pred!=i)
    ax.scatter3D(x[idx,0],x[idx,1],x[idx,2],c='g',marker=marker[i],alpha=0.
 ↪3,label=f'class{i} correct')
    ax.scatter3D(x[idx1,0],x[idx1,1],x[idx1,2],c='r',marker=marker[i],alpha=0.
 ↪3,label=f'class{i} incorrect')
plt.legend()
plt.show()
```

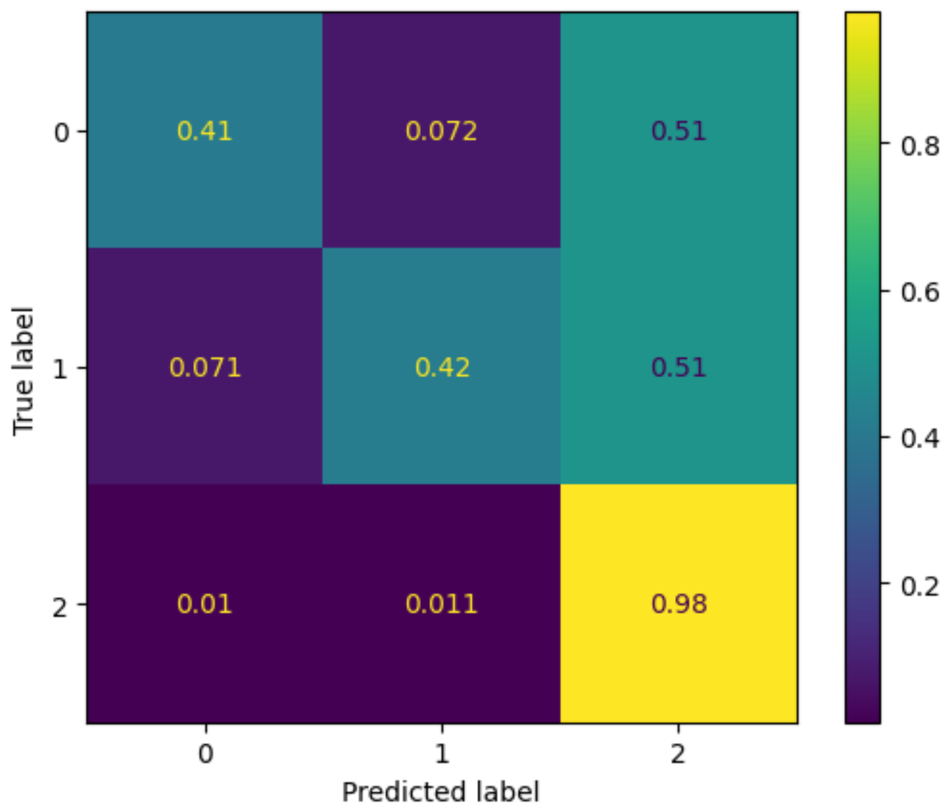


0.2.2 Part B

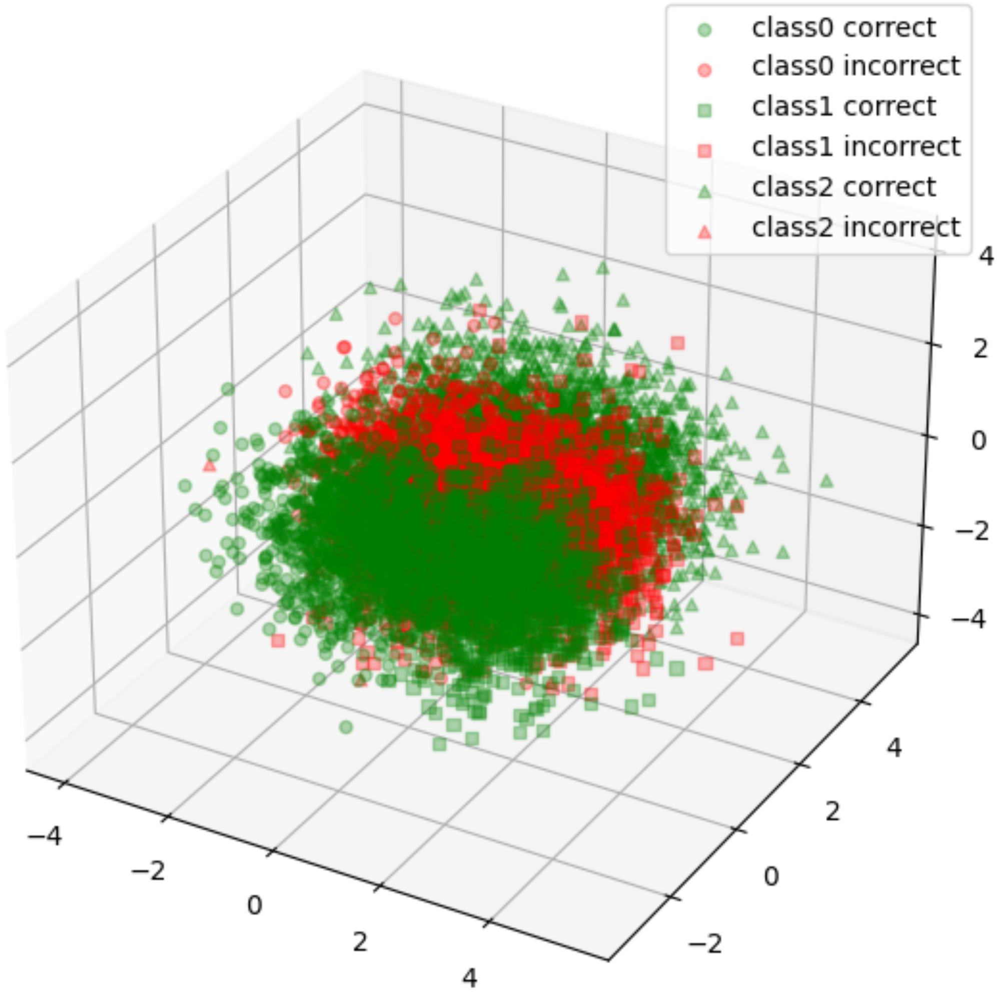
[83]:
```
Lambda10 = np.array([[0,1,10],[1,0,10],[1,1,0]])
Lambda100 = np.array([[0,1,100],[1,0,100],[1,1,0]])
pred_10 = ERM(x,Lambda=Lambda10,gmm_params=gmm_params)
cm = confusion_matrix(y,pred_10,labels=[0,1,2])
prob_err = 1 - np.diag(cm).sum()/cm.sum()
print(f'The empirical estimated minimum probability error is {prob_err}')
cm_prob = cm/cm.sum(axis=1,keepdims=True)
disp = ConfusionMatrixDisplay(confusion_matrix=cm_prob,display_labels=[0,1,2])
disp.plot()
plt.show()
```

```
(4, 10000) (4, 4)
```

```
The empirical estimated minimum probability error is 0.3587
```

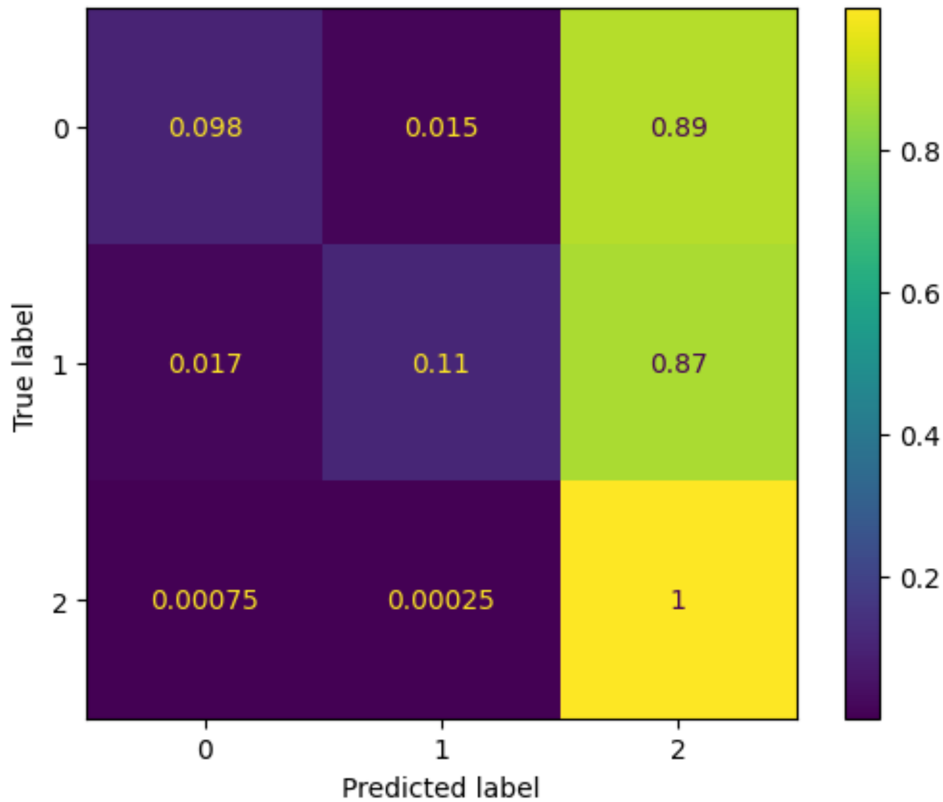


[84]:
```
plt.figure(figsize=(10,7))
ax = plt.axes(projection='3d')

marker = ['o','s','^']

for i in range(3):
    idx = np.logical_and(y==i,pred_10==i)
    idx1 = np.logical_and(y==i,pred_10!=i)
    ax.scatter3D(x[idx,0],x[idx,1],x[idx,2],c='g',marker=marker[i],alpha=0.
 ↪3,label=f'class{i} correct')
    ax.scatter3D(x[idx1,0],x[idx1,1],x[idx1,2],c='r',marker=marker[i],alpha=0.
 ↪3,label=f'class{i} incorrect')
plt.legend()
plt.show()
```

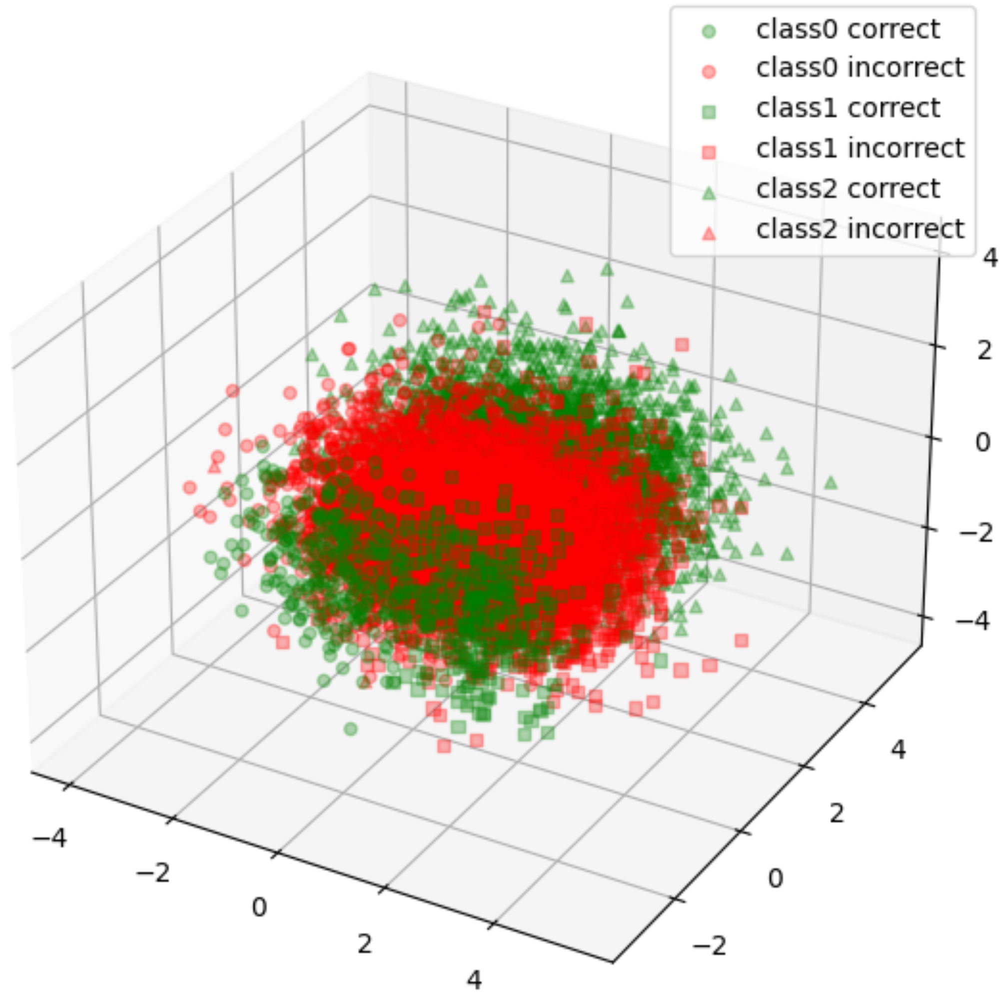


[87]:
```
pred_100 = ERM(x,Lambda=Lambda100,gmm_params=gmm_params)
cm = confusion_matrix(y,pred_100,labels=[0,1,2])
prob_err = 1 - np.diag(cm).sum()/cm.sum()
print(f'The empirical estimated minimum probability error is {prob_err}')
cm_prob = cm/cm.sum(axis=1,keepdims=True)
disp = ConfusionMatrixDisplay(confusion_matrix=cm_prob,display_labels=[0,1,2])
disp.plot()
plt.show()
```

```
(4, 10000) (4, 4)
The empirical estimated minimum probability error is 0.5382
```



[88]:
```
plt.figure(figsize=(10,7))
ax = plt.axes(projection='3d')

marker = ['o','s','^']

for i in range(3):
    idx = np.logical_and(y==i,pred_100==i)
    idx1 = np.logical_and(y==i,pred_100!=i)
    ax.scatter3D(x[idx,0],x[idx,1],x[idx,2],c='g',marker=marker[i],alpha=0.
↪3,label=f'class{i} correct')
    ax.scatter3D(x[idx1,0],x[idx1,1],x[idx1,2],c='r',marker=marker[i],alpha=0.
↪3,label=f'class{i} incorrect')
plt.legend()
plt.show()
```



[ ]:

0.3 Question 3

[102]:
```
def erm_classification(X, Lambda, gmm_params, C):
    class_cond_likelihoods = np.array([multivariate_normal.pdf(X,␣
 ↪gmm_params['mu'][i], gmm_params['sigma'][i]) for i in range(C)])
    class_priors = np.diag(gmm_params['prior'])
    class_posteriors = class_priors.dot(class_cond_likelihoods)
    risk_mat = Lambda@class_posteriors
    return np.argmin(risk_mat, axis=0)

def regCov(X,lam):
    cov = np.cov(X)
```

```
    return cov + lam*np.eye(len(X))
```

[103]:
```
wine = pd.read_csv('winequality-white.csv',delimiter=';')
wine.head()
```

[103]:
```
   fixed acidity  volatile acidity  citric acid  residual sugar  chlorides  \
0            7.0              0.27         0.36            20.7      0.045
1            6.3              0.30         0.34             1.6      0.049
2            8.1              0.28         0.40             6.9      0.050
3            7.2              0.23         0.32             8.5      0.058
4            7.2              0.23         0.32             8.5      0.058

   free sulfur dioxide  total sulfur dioxide  density    pH  sulphates  \
0                 45.0                 170.0   1.0010  3.00       0.45
1                 14.0                 132.0   0.9940  3.30       0.49
2                 30.0                  97.0   0.9951  3.26       0.44
3                 47.0                 186.0   0.9956  3.19       0.40
4                 47.0                 186.0   0.9956  3.19       0.40

   alcohol  quality
0      8.8        6
1      9.5        6
2     10.1        6
3      9.9        6
4      9.9        6
```

[104]:
```
x = wine.drop(columns='quality',axis=1)
y_original = wine['quality']
y_original.value_counts()
```

[104]:
```
6    2198
5    1457
7     880
8     175
4     163
3      20
9       5
Name: quality, dtype: int64
```

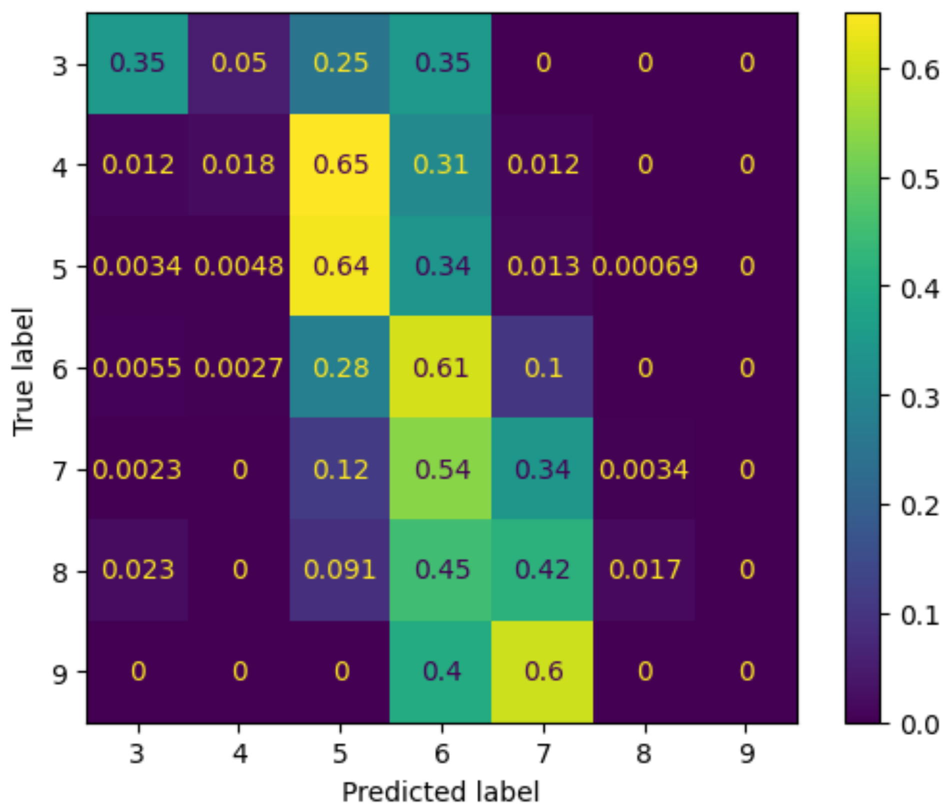
[105]:
```
le = LabelEncoder()
le.fit(y_original)
y = le.transform(y_original)
```

[106]:
```
C = len(np.unique(y))
gmm_params = {}
gmm_params['prior'] = np.array([(y==i).sum() for i in np.unique(y)])/len(y)
gmm_params['mu'] = np.array([x[y==i].mean(axis=0) for i in np.unique(y)])
```

```python
gmm_params['sigma'] = np.array([regCov(x[y==i].T,lam=0.1) for i in np.unique(y)])
Lambda = np.ones((C,C)) - np.eye(C)
pred = erm_classification(X=x,Lambda=Lambda,gmm_params=gmm_params,C=C)
```

[123]:
```python
cm = confusion_matrix(y,pred)
cm_prob = cm/cm.sum(axis=1,keepdims=True)
plt.figure(figsize=(12,12))
disp = ConfusionMatrixDisplay(cm_prob,display_labels=np.unique(y))
disp.plot()
plt.xticks(np.unique(y),le.inverse_transform(np.unique(y)))
plt.yticks(np.unique(y),le.inverse_transform(np.unique(y)))

plt.show()
```

```
<Figure size 1200x1200 with 0 Axes>
```



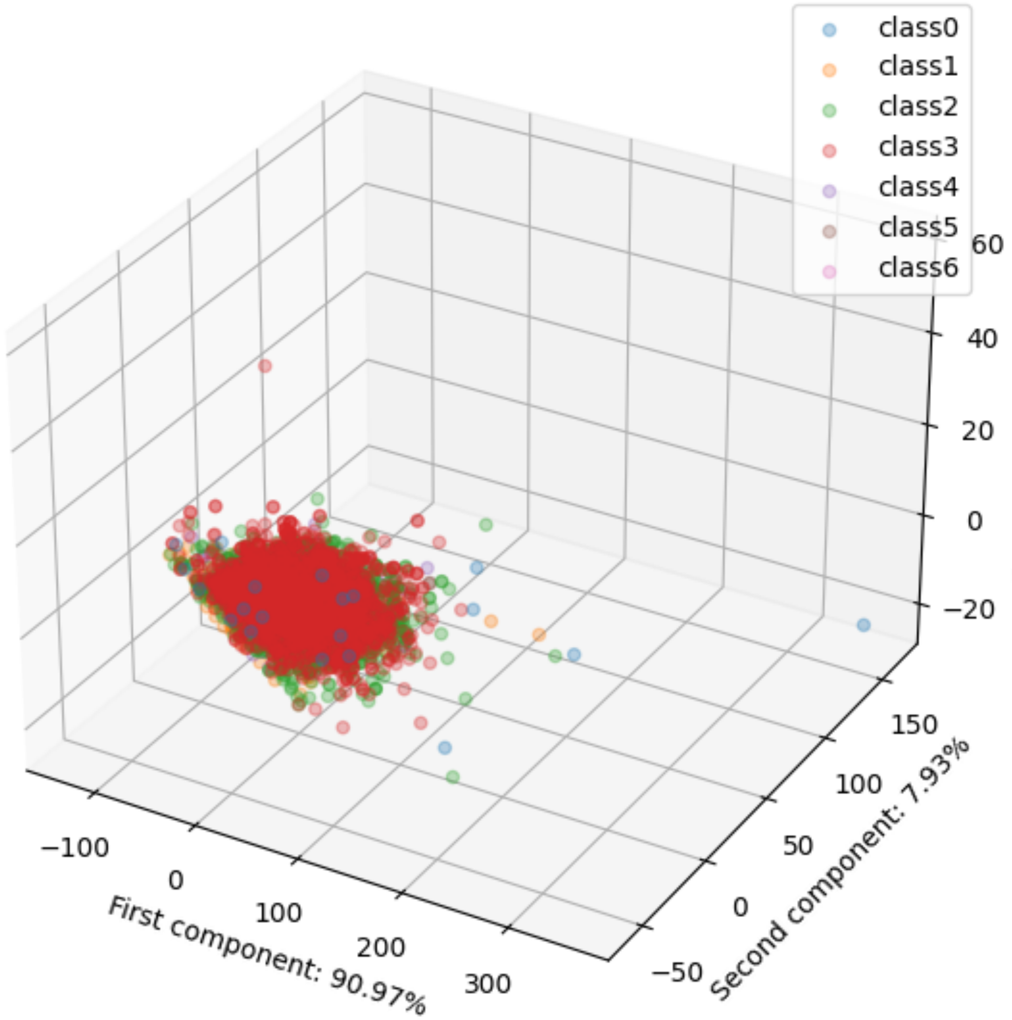
[124]:
```python
prob_err = 1 - np.diag(cm).sum()/len(x)
print(f'The empirical estimated minimum probability error is {prob_err}')
```

```
The empirical estimated minimum probability error is 0.4726418946508779
```

[125]:
```
pca = PCA(n_components=3)
x_trans = pca.fit_transform(x)
var_ratio = pca.explained_variance_ratio_
var_ratio
```

[125]:
```
array([0.90965734, 0.07933386, 0.01015427])
```

[126]:
```
plt.figure(figsize=(10,7))
ax = plt.axes(projection='3d')
for i in np.unique(y):
    ax.scatter3D(x_trans[y==i,0],x_trans[y==i,1],x_trans[y==i,2],alpha=0.
 ↪3,label=f'class{i}')
ax.set_xlabel(f'First component: {np.round(100*var_ratio[0],2)}%')
ax.set_ylabel(f'Second component: {np.round(100*var_ratio[1],2)}%')
ax.set_zlabel(f'Third component: {np.round(100*var_ratio[2],2)}%')

plt.legend()
plt.show()
```



[ ]:

```
```

[127]:

```
x_har_train = pd.read_csv('UCI HAR Dataset/train/X_train.
  ↪txt',delim_whitespace=True,header=None)
x_har_test = pd.read_csv('UCI HAR Dataset/test/X_test.
  ↪txt',delim_whitespace=True,header=None)
y_har_train = pd.read_csv('UCI HAR Dataset/train/y_train.
  ↪txt',delim_whitespace=True,header=None)
y_har_test = pd.read_csv('UCI HAR Dataset/test/y_test.
  ↪txt',delim_whitespace=True,header=None)
x = pd.concat([x_har_train,x_har_test])
y_original = pd.concat([y_har_train,y_har_test])
print(x.shape)
print(y.shape)
```

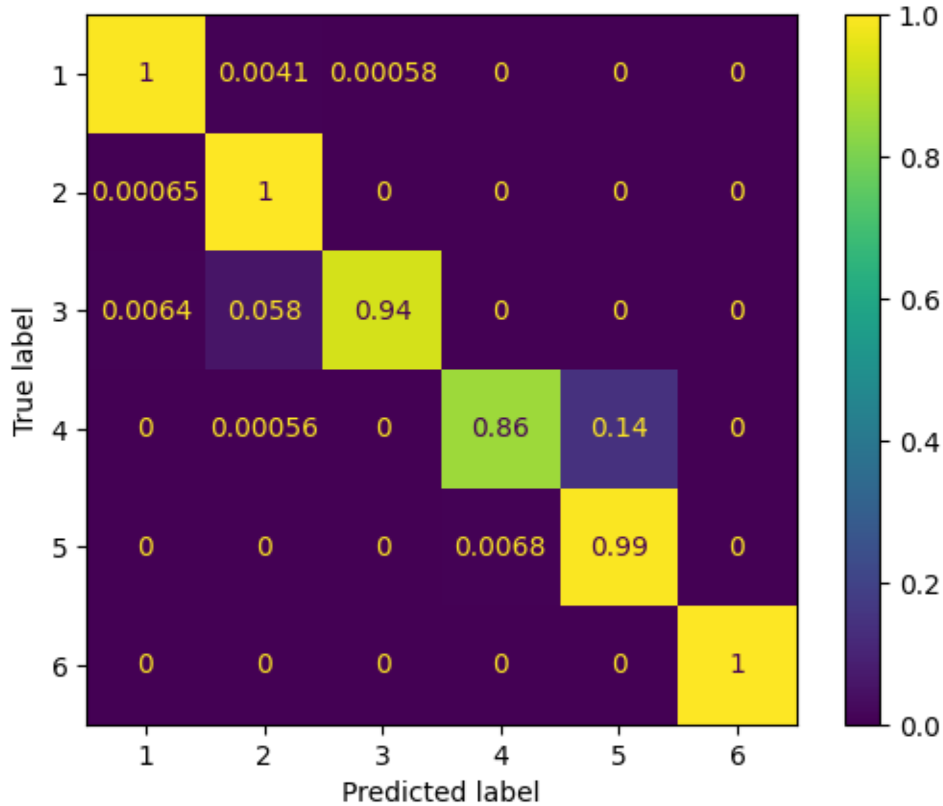
```
(10299, 561)
(4898,)
```

[129]:
```
le = LabelEncoder()
le.fit(y_original.values.flatten())
y = le.transform(y_original.values.flatten())
```

[131]:
```
C = len(np.unique(y))
gmm_params = {}
gmm_params['prior'] = np.array([(y==i).sum() for i in np.unique(y)])/len(y)
gmm_params['mu'] = np.array([x[y==i].mean(axis=0) for i in np.unique(y)])
gmm_params['sigma'] = np.array([regCov(x[y==i].T,lam=0.1) for i in np.
    ↪unique(y)])
Lambda = np.ones((C,C)) - np.eye(C)
pred = erm_classification(X=x,Lambda=Lambda,gmm_params=gmm_params,C=C)
```

[132]:
```
cm = confusion_matrix(y,pred)
cm_prob = cm/cm.sum(axis=1,keepdims=True)
plt.figure(figsize=(12,12))
disp = ConfusionMatrixDisplay(cm_prob,display_labels=np.unique(y))
disp.plot()
plt.xticks(np.unique(y),le.inverse_transform(np.unique(y)))
plt.yticks(np.unique(y),le.inverse_transform(np.unique(y)))

plt.show()
```

```
<Figure size 1200x1200 with 0 Axes>
```



[137]:

```
prob_err = 1 - np.diag(cm).sum()/len(x)
print(f'The empirical estimated minimum probability error is {prob_err}')
```

```
The empirical estimated minimum probability error is 0.03592581804058648
```

[134]:

```
pca = PCA(n_components=3)
x_trans = pca.fit_transform(x)
var_ratio = pca.explained_variance_ratio_
var_ratio
```

[134]:

```
array([0.62227069, 0.04772595, 0.04018191])
```

[135]:

```
plt.figure(figsize=(10,7))
ax = plt.axes(projection='3d')
for i in np.unique(y):
    ax.scatter3D(x_trans[y==i,0],x_trans[y==i,1],x_trans[y==i,2],alpha=0.
 ↪3,label=f'class{i}')
ax.set_xlabel(f'First component: {np.round(100*var_ratio[0],2)}%')
ax.set_ylabel(f'Second component: {np.round(100*var_ratio[1],2)}%')
ax.set_zlabel(f'Third component: {np.round(100*var_ratio[2],2)}%')
```

```
plt.legend()
plt.show()
```