APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) **Why do governments issue bonds and not simply print more money?**

   Governments issue bonds to finance their spending and to raise capital for debt obligations. This doesn't directly impact the supply of money, and thus does not necessarily have an impact on inflation. Finally, bonds allow governments to tap into a large pool of available capital.

   (b) **Give a hypothetical example of why the long-term part of a yield curve might flatten.**

   The long-term portion of a yield curve may flatten for many different reasons, however the most obvious and simple would be lower demand for longer term bonds. As the yield curve relates yields to maturity periods, the relative steepness between short term and long term bond yields will widen as demand for longer terms bonds lowers (without changing demand/yield for shorter term bonds).

   (c) **Explain what quantitative easing is and how the (US) Fed has employed this since the beginning of the COVID-19 pandemic.**

   Quantitative easing is a monetary policy used by central banks to increase money supply whilst decreasing interest rates to stimulate the economy during downturns. The Federal Reserve Bank is one such central bank (in the US) which enacted such a policy during the COVID-19 pandemic. The Federal Reserve aggressively bought treasury bonds and mortgage-backed securities, which in turn improved liquidity and lowered interest rates.

2. **We asked you to pull data for all bonds, but if you'd like to construct a yield a "0-5 year" yield spot curves, as the government of Canada issues all of its bonds with a semi-annual coupon, when bootstrapping you'll only need 10 or 11 bonds to perform this task. Ideally, the bonds in any yield curve should be consistent in some way with one another so that yields are easier to compare. Select (list) 10 bonds that you will use to construct the aforementioned curves with an explanation of why you selected those 10 bonds based on the characteristics we asked you to collect for each bond (coupon, issue date, maturity date, etc.).**

   Selected bonds: [**CAN 0.5 Nov 23, CAN 0.75 Feb 24, CAN 2.5 Jun 24, CAN 3.75 Feb 25, CAN 9 Jun 25, CAN 3 Oct 25, CAN 0.25 Mar 26, CAN 1 Sep 26, CAN 1.25 Mar 27, CAN 2.75 Sep 27**].

   These bonds were selected based on their rating AA+, and their respective differences in maturity time (which were roughly the same in terms of years).

3. **In a few plain English sentences, in general, if we have several stochastic processes for which each process represents a unique point along a stochastic curve (assume points/processes are evenly distributed along the curve), what do the eigenvalues and eigenvectors associated with the covariance matrix of those stochastic processes tell us?**

   PCA is a method for understanding the distribution of high-dimensional data. It uses a covariance matrix to represent the direction and spread of the data. The covariance shows the spread along the eigenvectors and variance shows the spread along the x- and y-axes. The eigenvectors indicate the main lines of force and the axes of greatest variance and covariance highlight where the data is most likely to change. The linear transformation performed by the covariance matrix traces the lines of force and reveals the direction of the principal component. The eigenvalues represent the magnitude of variance corresponding to the eigenvectors and measure the data's covariance. By ranking the eigenvectors based on their eigenvalues, the most significant principal components are obtained, with the eigenvector having the largest eigenvalue determining the first component, which is the direction of the largest variance.

Empirical Questions - 75 points

4.

   (a) **First, calculate each of your 10 selected bonds' yield (ytm). Then provide a well-labeled plot with a 5-year yield curve (ytm curve) corresponding to each day of data superim- posed on-top of each other. You may use any interpolation technique you deem appropriate provided you include a reasonable explanation for the technique used.**

   Please see the GitHub link in the references section for YTM curve plot. The Newton-Raphson method was used for its efficiency in finding a suitable root, but also because it does not require a guess as to the YTM before execution.

   (b) **Write a pseudo-code (explanation of an algorithm) for how you would derive the spot curve with terms ranging from 1-5 years from your chosen bonds in part 2. (Please recall the day convention simplifications provided in part 2 as well.) Then provide a well-labeled plot with a 5-year spot curve corresponding to each day of data superimposed on-top of each other.**

   Please see GitHub link in references section for plots!

   i. Calculate the bond price for each bond in the list by using the bond pricing formula. The coupon rate, notional amount, and the term to maturity of each bond must be inputs to the bond pricing formula.
   ii. Arrange the bonds based on their term to maturity, from shortest to longest.
   iii. For each year from 1 to 5, find the bonds with terms matching the current year.
   iv. Using the bond prices from step 2, and the corresponding yields, calculate the spot rate for the current year by taking the average of the yields.
   v. Repeat steps 4-5 for each year from 1 to 5, until the spot rate for each year has been determined.
   vi. Plot the spot rates against their corresponding terms to obtain the spot curve.

   (c) **Write a pseudo-code for how you would derive the 1-year forward curve with terms ranging from 2-5 years from your chosen bonds in part 2 (I.e., a curve with the first point being the 1yr-1yr forward rate and the last point being the 1yr-4yr rate).**

Then provide a well-labeled plot with a forward curve corresponding to each day of data superimposed on-top of each other.

Please see GitHub link in references section for plots!

i. Calculate the yield of each bond by dividing the coupon payment by the bond price.
ii. Plot the yield of each bond versus its maturity.
iii. Calculate the 1-year forward rate for each bond using the following formula: Forward rate = (Bond yield with (n+1) years maturity) / (Bond yield with n years maturity).
iv. Plot the 1-year forward rate against its maturity. This will give you the 1-year forward curve.
v. Repeat the process for 2-year, 3-year, and 4-year forward rates.
vi. Connect the dots for each forward rate to get the continuous 1-year forward curve.
vii. Extrapolate the curve to estimate the 1-year forward rate for terms ranging from 2 to 5 years.

5. **Calculate two covariance matrices for the time series of daily log-returns of yield, and forward rates (no spot rates). In other words, first calculate the covariance matrix of the random variables χ_i, for i = 1, . . . , 5, where each random variable χ_i has a time series $\chi_{i,j}$ given by: $\chi_{i,j} = log(r_{i,j+1}/r_{i,j})$, j = 1,...,9 then do the same for the following forward rates - the 1yr-1yr, 1yr-2yr, 1yr-3yr, 1yr-4yr.**

   Please see GitHub link in references section for matrix calculations!

6. **Calculate the eigenvalues and eigenvectors of both covariance matrices, and in one sentence, explain what the first (in terms of size) eigenvalue and its associated eigenvector imply.**

   Please see GitHub link in references section for eigen-value calculations!

References and GitHub Link to Code

Code: https://github.com/JonnyI2000/APM466