```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Ways to Build New Types

How to build bigger types

- Already know:
  - Have various *base types* like `int bool unit char`
  - Ways to build (nested) *compound types*: tuples, lists, options

- Coming soon: more ways to build compound types

- First: 3 most important type building-blocks in *any* language
  - "Each of": A `t` value contains *values of each of* `t1 t2 … tn`
  - "One of": A `t` value contains *values of one of* `t1 t2 … tn`
  - "Self reference": A `t` value can refer to other `t` values

  Remarkable: A lot of data can be described with just these building blocks

  Note: These are not the common names for these concepts

Examples

- Tuples build each-of types
  - **`int * bool`** contains an **`int`** *and* a **`bool`**

- Options build one-of types
  - **`int option`** contains an **`int`** *or* it contains no data

- Lists use all three building blocks
  - **`int list`** contains an **`int`** *and* another **`int list`** *or* it contains no data

- And of course we can nest compound types
  - **`((int * int) option * (int list list)) option`**

Coming soon

- Another way to build each-of types in ML
  - *Records*: have named *fields*
  - Connection to tuples and idea of *syntactic sugar*

- A way to build and use our own one-of types in ML
  - For example, a type that contains an **`int`** or a **`string`**
  - Will lead to *pattern-matching*, one of ML's coolest and strangest-to-Java-programmers features

- Later in course: How OOP does one-of types
  - Key contrast with procedural and functional programming

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Records

Records

Record values have fields (any name) holding values

```
{f1 = v1, …, fn = vn}
```

Record types have fields (and name) holding types

```
{f1 : t1, …, fn : tn}
```

The order of fields in a record value or type never matters

- REPL alphabetizes fields just for consistency

Building records:

```
{f1 = e1, …, fn = en}
```

Accessing pieces:

```
#myfieldname e
```

(Evaluation rules and type-checking as expected)

Example

```
{name = “Amelia”, id = 41123 - 12}
```

Evaluates to

```
{id = 41111, name = “Amelia”}
```

And has type

```
{id : int, name : string}
```

If some expression such as a variable **`x`** has this type, then get fields with:

```
#id x      #name x
```

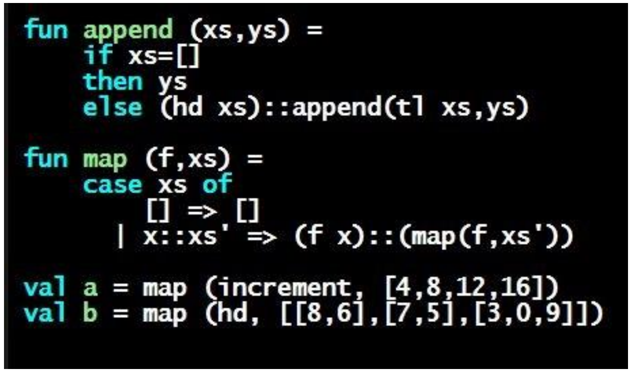
Note we did not have to declare any record types

– The same program could also make a **`{id=true,ego=false}`** of type **`{id:bool,ego:bool}`**

By name vs. by position

- Little difference between **`(4,7,9)`** and **`{f=4,g=7,h=9}`**
  - Tuples a little shorter
  - Records a little easier to remember "what is where"
  - Generally a matter of taste, but for many (6? 8? 12?) fields, a record is usually a better choice

- A common decision for a construct's syntax is whether to refer to things *by position* (as in tuples) or *by some (field) name* (as with records)
  - A common hybrid is like with Java method arguments (and ML functions as used so far):
    - Caller uses *position*
    - Callee uses *variables*
    - Could do it differently; some languages have



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Tuples as Syntactic Sugar

The truth about tuples

Previously, we gave tuples syntax, type-checking rules, and evaluation rules

But we could have done this instead:

- Tuple syntax is just a different way to write certain records
- **`(e1,…,en)`** is another way of writing **`{1=e1,…,n=en}`**
- **`t1*…*tn`** is another way of writing **`{1:t1,…,n:tn}`**
- In other words, records with field names 1, 2, …

In fact, this is how ML actually defines tuples

- Other than special syntax in programs and printing, they don't exist
- You really can write **`{1=4,2=7,3=9}`**, but it's bad style

Syntactic sugar

"Tuples are just syntactic sugar for
records with fields named 1, 2, … n"

- *Syntactic*: Can describe the semantics entirely by the corresponding record syntax
- *Sugar*: They make the language sweeter ☺

Will see many more examples of syntactic sugar

- They simplify *understanding* the language
- They simplify *implementing* the language

Why? Because there are fewer semantics to worry about even though we have the syntactic convenience of tuples

Another example we saw: **`andalso`** and **`orelse`** vs. **`if then else`**

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Case Expressions

Case

ML combines the two aspects of accessing a one-of value with a *case expression* and *pattern-matching*

- Pattern-matching much more general/powerful (soon!)

Example:

```
fun f x = (* f has type mytype -> int *)
    case x of
        Pizza => 3
      | TwoInts(i1,i2) => i1+i2
      | Str s => String.size s
```

- A multi-branch conditional to pick branch based on variant
- Extracts data and binds to variables local to that branch
- Type-checking: all branches must have same type
- Evaluation: evaluate between case … of and the right branch

Patterns

In general the syntax is:

```
case e0 of
      p1 => e1
    | p2 => e2
      …
    | pn => en
```

For today, each *pattern* is a constructor name followed by the right number of variables (i.e., `C` or `C x` or `C(x,y)` or …)

- Syntactically most patterns (all today) look like expressions
- But patterns are not expressions
  - We do not evaluate them
  - We see if the result of `e0` *matches* them

Why this way is better

0. You can use pattern-matching to write your own testing and data-extractions functions if you must
   - But do not do that on your homework

1. You cannot forget a case (inexhaustive pattern-match warning)
2. You cannot duplicate a case (a type-checking error)
3. You will not forget to test the variant correctly and get an exception (like **`hd []`**)
4. Pattern-matching can be generalized and made more powerful, leading to elegant and concise code

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Useful Datatypes

Useful examples

Let's fix the fact that our only example datatype so far was silly…

- Enumerations, including carrying other data

```
datatype suit = Club | Diamond | Heart | Spade
datatype rank = Jack | Queen | King
              | Ace | Num of int
```

- Alternate ways of identifying real-world things/people

```
datatype id = StudentNum of int
            | Name of string
                     * (string option)
                     * string
```

Don't do this

Unfortunately, bad training and languages that make one-of types inconvenient lead to common *bad style* where each-of types are used where one-of types are the right tool

```
(* use the student_num and ignore other
   fields unless the student_num is ~1 *)
{ student_num : int,
  first       : string,
  middle      : string option,
  last        : string }
```

- Approach gives up all the benefits of the language enforcing every value is one variant, you don't forget branches, etc.
- And it makes it less clear what you are doing

That said…

But if instead, the point is that every "person" in your program has a name and maybe a student number, then each-of is the way to go:

```
{ student_num : int option,
  first       : string,
  middle      : string option,
  last        : string }
```

Expression Trees

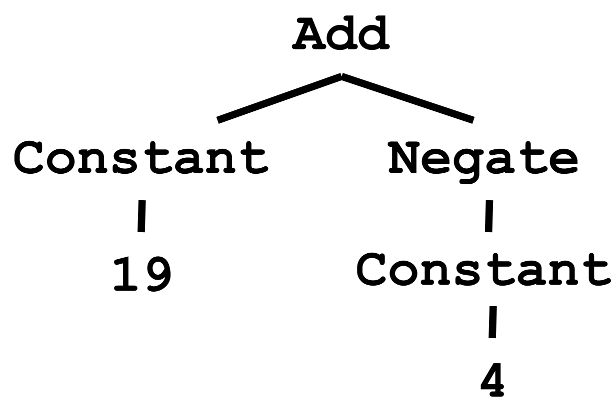
A more exciting (?) example of a datatype, using self-reference

```
datatype exp = Constant of int
             | Negate   of exp
             | Add      of exp * exp
             | Multiply of exp * exp
```

An expression in ML of type **exp**:

```
Add (Constant (10+9), Negate (Constant 4))
```

How to picture the resulting value in your head:



Recursion

Not surprising:

Functions over recursive datatypes are usually recursive

```
fun eval e =
   case e of
        Constant i       => i
      | Negate e2        => ~ (eval e2)
      | Add(e1,e2)       => (eval e1) + (eval e2)
      | Multiply(e1,e2) => (eval e1) * (eval e2)
```

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Pattern-Matching So Far: Precisely

Careful definitions

When a language construct is "new and strange," there is *more* reason to define the evaluation rules precisely…

… so let's review datatype bindings and case expressions "so far"

– *Extensions* to come but won't invalidate the "so far"

Datatype bindings

```
datatype t = C1 of t1 | C2 of t2 | … | Cn of tn
```

Adds type **`t`** and constructors **`Ci`** of type **`ti->t`**

- **`Ci v`** is a value, i.e., the result "includes the tag"

Omit "**`of t`**" for constructors that are just tags, no underlying data

- Such a **`Ci`** is a value of type **`t`**

Given an expression of type **`t`**, use *case expressions* to:

- See which variant (tag) it has
- Extract underlying data once you know which variant

Datatype bindings

```
case e of p1 => e1 | p2 => e2 | … | pn => en
```

- As usual, can use a case expressions anywhere an expression goes
  - Does not need to be whole function body, but often is

- Evaluate **`e`** to a value, call it **`v`**

- If **`pi`** is the first *pattern* to *match* **`v`**, then result is evaluation of **`ei`** in environment "extended by the match"

- Pattern **`Ci(x1,…,xn)`** matches value **`Ci(v1,…,vn)`** and extends the environment with **`x1`** to **`v1`** … **`xn`** to **`vn`**
  - For "no data" constructors, pattern **`Ci`** matches value **`Ci`**

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Another Expression Example

Putting it together

```
datatype exp = Constant of int
             | Negate   of exp
             | Add      of exp * exp
             | Multiply of exp * exp
```

Let's define **`max_constant : exp -> int`**

Good example of combining several topics as we program:

- Case expressions
- Local helper functions
- Avoiding repeated recursion
- Simpler solution by using library functions

See the **`.sml`** file…

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Type Synonyms

Creating new types

- A *datatype binding* introduces a new type name
  - Distinct from all existing types
  - Only way to create values of the new type is the constructors

- A *type synonym* is a new kind of binding

```
type aname = t
```

  - Just creates another name for a type
  - The type and the name are *interchangeable in every way*
  - Do not worry about what REPL prints: picks what it wants just like it picks the order of record field names

Why have this?

For now, type synonyms just a convenience for talking about types

- Example (where **`suit`** and **`rank`** already defined):

```
type card = suit * rank
```

- Write a function of type

```
card -> bool
```

- Okay if REPL says your function has type

```
suit * rank -> bool
```

Convenient, but does not let us “do” anything new

Later in course will see another use related to modularity

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Lists and Options are Datatypes

Recursive datatypes

Datatype bindings can describe recursive structures

- Have seen arithmetic expressions
- Now, linked lists:

```
datatype my_int_list = Empty
                     | Cons of int * my_int_list

val x = Cons(4,Cons(23,Cons(2008,Empty)))

fun append_my_list (xs,ys) =
   case xs of
       Empty => ys
     | Cons(x,xs') => Cons(x, append_my_list(xs',ys))
```

Options are datatypes

Options are just a predefined datatype binding

- **`NONE`** and **`SOME`** are *constructors*, not just functions
- So use pattern-matching not **`isSome`** and **`valOf`**

```
fun inc_or_zero intoption =
    case intoption of
          NONE => 0
        | SOME i => i+1
```

Lists are datatypes

Do not use **`hd`**, **`tl`**, or **`null`** either

- **`[]`** and **`::`** are constructors too
- (strange syntax, particularly *infix*)

```
fun sum_list xs =
   case xs of
        [] => 0
      | x::xs' => x + sum_list xs'

fun append (xs,ys) =
   case xs of
       [] => ys
     | x::xs' => x :: append(xs',ys)
```

Why pattern-matching

- Pattern-matching is better for options and lists for the same reasons as for all datatypes
  - No missing cases, no exceptions for wrong variant, etc.

- We just learned the other way first for pedagogy
  - Do not use **`isSome`**, **`valOf`**, **`null`**, **`hd`**, **`tl`** on Homework 2

- So why are **`null`**, **`tl`**, etc. predefined?
  - For passing as arguments to other functions (next week)
  - Because sometimes they are convenient
  - But not a big deal: could define them yourself

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Polymorphic Datatypes

Finish the story

- Claimed built-in options and lists are not needed/special
  - Other than special syntax for list constructors

- But these datatype bindings are polymorphic type constructors
  - **`int list`** and **`string list`** and **`int list list`** are all types, not **`list`**
  - Functions might or might not be polymorphic
    - **`val sum_list : int list -> int`**
    - **`val append : 'a list * 'a list -> 'a list`**

- Good language design: Can define new polymorphic datatypes

- Semi-optional: Do *not* need to understand this for homework 2

Defining polymorphic datatypes

- Syntax: put one or more type variables before datatype name

```
datatype 'a option = NONE | SOME of 'a

datatype 'a mylist = Empty | Cons of 'a * 'a mylist

datatype ('a,'b) tree =
        Node of 'a * ('a,'b) tree * ('a,'b) tree
      | Leaf of 'b
```

- Can use these type variables in constructor definitions

- Binding then introduces a type constructor, not a type
  - Must say **`int mylist`** or **`string mylist`** or **`'a mylist`**
  - Not "plain" **`mylist`**

Nothing else changes

Use constructors and case expressions as usual

- No change to evaluation rules

- Type-checking will make sure types are used consistently
  - Example: cannot mix element types of list

- Functions will be polymorphic or not based on how data is used

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Pattern-Matching for Each-Of Types: The Truth About Function Arguments

An exciting segment

Learn some deep truths about "what is really going on"

- Using much more syntactic sugar than we realized

- Every val-binding and function-binding uses pattern-matching

- Every function in ML takes exactly one argument

First need to extend our definition of pattern-matching…

Each-of types

So far have used pattern-matching for one of types because we *needed* a way to access the values

Pattern matching also works for records and tuples:

- The pattern **`(x1,…,xn)`**
  matches the tuple value **`(v1,…,vn)`**
- The pattern **`{f1=x1, …, fn=xn}`**
  matches the record value **`{f1=v1, …, fn=vn}`**
  (and fields can be reordered)

Example

This is poor style, but based on what I told you so far, the only way to use patterns

- Works but poor style to have one-branch cases

```
fun sum_triple triple =
   case triple of
      (x, y, z) => x + y + z

fun full_name r =
   case r of
      {first=x, middle=y, last=z} =>
          x ^ " " ^ y ^ " " ^ z
```

Val-binding patterns

- New feature: A val-binding can use a pattern, not just a variable
  – (Turns out variables are just one kind of pattern, so we just told you a half-truth in lecture 1)

```
val p = e
```

- Great for getting (all) pieces out of an each-of type
  – Can also get only parts out (not shown here)

- Usually poor style to put a constructor pattern in a val-binding
  – Tests for the one variant and raises an exception if a different one is there (like `hd`, `tl`, and `valOf`)

Better example

This is okay style

- Though we will improve it again next
- Semantically identical to one-branch case expressions

```
fun sum_triple triple =
   let val (x, y, z) = triple
   in
       x + y + z
   end

fun full_name r =
   let val {first=x, middle=y, last=z} = r
   in
       x ^ " " ^ y ^ " " ^ z
   end
```

Function-argument patterns

A function argument can also be a pattern

- Match against the argument in a function call

```
fun f p = e
```

Examples (great style!):

```
fun sum_triple (x, y, z) =
    x + y + z

fun full_name {first=x, middle=y, last=z} =
    x ^ " " ^ y ^ " " ^ z
```

A new way to go

- For Homework 2:
  - Do not use the `#` character
  - Do not need to write down any explicit types

Hmm

A function that takes one triple of type **`int*int*int`** and returns an **`int`** that is their sum:

```
fun sum_triple (x, y, z) =
    x + y + z
```

A function that takes three **`int`** arguments and returns an **`int`** that is their sum

```
fun sum_triple (x, y, z) =
    x + y + z
```

See the difference? (Me neither.) ☺

The truth about functions

- In ML, every function takes exactly one argument (*)
- What we call multi-argument functions are just functions taking one tuple argument, implemented with a tuple pattern in the function binding
  - Elegant and flexible language design
- Enables cute and useful things you cannot do in Java, e.g.,

```
fun rotate_left (x, y, z) = (y, z, x)
fun rotate_right t = rotate_left(rotate_left t)
```

* “Zero arguments” is the unit pattern `()` matching the unit value `()`

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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A Little Type Inference

A new way to go

- For homework 2:
  - Do not use the **`#`** character
  - Do not need to write down any explicit types

- These are related
  - Type-checker can use patterns to figure out the types
  - With just **`#foo`** or **`#1`** it cannot determine “what other fields”

Why no problem

Easy for type-checker to determine function types:

```
fun sum_triple (x, y, z) =
    x + y + z

fun full_name {first=x, middle=y, last=z} =
    x ^ " " ^ y ^ " " ^ z
```

Get error message without explicit type annotation:

```
fun sum_triple (triple : int*int*int) =
     #1 triple + #2 triple + #3 triple

fun full_name (r : {first:string, middle:string,
                    last:string}) =
      #first r ^ " " ^ #middle r ^ " " ^ #last r
```

Unexpected polymorphism

- Sometimes type-checker is "smarter than you expect"
  - Types of some parts might be less constrained than you think
  - Example: If you do not use something it can have any type

```
(* int * 'a * int -> int *)
fun partial_sum (x, y, z) =
    x + z

(*{first:string, last:string, middle:'a} -> string*)
fun partial_name {first=x, middle=y, last=z} =
    x ^ " " ^ z
```

- This is okay!
  - A more general type than you need is always acceptable
  - Assuming your function is correct, of course
  - More precise definition of "more general type" next segment

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Polymorphic Types and Equality Types

An example

- "Write a function that appends two string lists"

```
fun append (xs,ys) =
    case xs of
        [] => ys
       | x::xs' => x :: append(xs',ys)
```

- You expect `string list * string list -> string list`
- Implementation says `'a list * 'a list -> 'a list`
- This is okay [such as on your homework]: why?

More general

The type

```
'a list * 'a list -> 'a list
```

is more general than the type

```
string list * string list -> string list
```

- It "can be used" as any less general type, such as
  ```
  int list * int list -> int list
  ```

- But it is not more general than the type
  ```
  int list * string list -> int list
  ```

The "more general" rule

Easy rule you (and the type-checker) can apply without thinking:

A type *t1* is more general than the type *t2* if you can take *t1*, replace its type variables consistently, and get *t2*

- Example: Replace each **`'a`** with **`int * int`**
- Example: Replace each **`'a`** with **`bool`** and each **`'b`** with **`bool`**
- Example: Replace each **`'a`** with **`bool`** and each **`'b`** with **`int`**
- Example: Replace each **`'b`** with **`'a`** and each **`'a`** with **`'a`**

Other rules

- Can combine the "more general" rule with rules for equivalence
  - Use of type synonyms does not matter
  - Order of field names does not matter

Example, given

```
type foo = int * int
```

the type

```
{quux : 'b, bar : int * 'a, baz : 'b}
```

is more general than

```
{quux : string, bar : foo, baz : string}
```

which is equivalent to

```
{bar : int*int, baz : string, quux : string}
```

Equality types

- You might also see type variables with a second “quote”
  - Example: `''a list * ''a -> bool`
- These are “equality types” that arise from using the **=** operator
  - The **=** operator works on lots of types: `int`, `string`, tuples containing all equality types, …
  - But not all types: function types, `real`, …
- The rules for more general are exactly the same except you have to replace an equality-type variable with a type that can be used with **=**
  - A “strange” feature of ML because **=** is special

Example

```
(* ''a * ''a -> string *)
fun same_thing(x, y) =
    if x=y then "yes" else "no"

(* int -> string *)
fun is_three x =
    if x=3 then "yes" else "no"
```

(You can ignore the warning about “calling polyEqual”)

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Nested Patterns

Nested patterns

- We can nest patterns as deep as we want
  - Just like we can nest expressions as deep as we want
  - Often avoids hard-to-read, wordy nested case expressions

- So the full meaning of pattern-matching is to compare a pattern against a value for the "same shape" and bind variables to the "right parts"
  - More precise recursive definition coming after examples

Useful example: zip/unzip 3 lists

```
fun zip3 lists =
   case lists of
        ([],[],[]) => []
      | (hd1::tl1,hd2::tl2,hd3::tl3) =>
             (hd1,hd2,hd3)::zip3(tl1,tl2,tl3)
      | _ => raise ListLengthMismatch

fun unzip3 triples =
   case triples of
        [] => ([],[],[])
      | (a,b,c)::tl =>
          let val (l1, l2, l3) = unzip3 tl
          in
              (a::l1,b::l2,c::l3)
          end
```

More examples to come (see code files)

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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More Nested Patterns

Style

- Nested patterns can lead to very elegant, concise code
  - Avoid nested case expressions if nested patterns are simpler and avoid unnecessary branches or let-expressions
    - Example: **`unzip3`** and **`nondecreasing`**
  - A common idiom is matching against a tuple of datatypes to compare them
    - Examples: **`zip3`** and **`multsign`**

- Wildcards are good style: use them instead of variables when you do not need the data
  - Examples: **`len`** and **`multsign`**

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Nested Patterns Precisely

(Most of) the full definition

The semantics for pattern-matching takes a pattern *p* and a value *v* and decides (1) does it match and (2) if so, what variable bindings are introduced.

Since patterns can nest, the definition is elegantly recursive, with a separate rule for each kind of pattern. Some of the rules:

- If *p* is a variable *x*, the match succeeds and *x* is bound to *v*
- If *p* is _, the match succeeds and no bindings are introduced
- If *p* is *(p1,…,pn)* and *v* is *(v1,…,vn)*, the match succeeds if and only if *p1* matches *v1*, …, *pn* matches *vn*. The bindings are the union of all bindings from the submatches
- If *p* is *C p1*, the match succeeds if *v* is *C v1* (i.e., the same constructor) and *p1* matches *v1*. The bindings are the bindings from the submatch.
- … (there are several other similar forms of patterns)

Examples

– Pattern `a::b::c::d` matches all lists with >= 3 elements

– Pattern `a::b::c::[]` matches all lists with 3 elements

– Pattern `((a,b),(c,d))::e` matches all non-empty lists of pairs of pairs

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Optional: Function Patterns

Yet more pattern-matching

[Your instructor has never preferred this style, but others like it and you are welcome to use it]

```
datatype exp = Constant of int
             | Negate   of exp
             | Add      of exp * exp
             | Multiply of exp * exp

fun eval (Constant i) = i
  | eval (Add(e1,e2)) = (eval e1) + (eval e2)
  | eval (Negate e1)  = ~ (eval e1)
  | eval (Multiply(e1,e2)) = (eval e1) + (eval e2)
```

Nothing more powerful

In general

```
fun f x =
  case x of
    p1 => e1
  | p2 => e2
   …
```

Can be written as

```
fun f p1 = e1
  | f p2 = e2
  …
  | f pn = en
```

If you prefer (assuming **x** is not used in any branch)

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Exceptions

Exceptions

An exception binding introduces a new kind of exception

```
exception MyFirstException
exception MySecondException of int * int
```

The **`raise`** primitive raises (a.k.a. throws) an exception

```
raise MyFirstException
raise (MySecondException(7,9))
```

A handle expression can handle (a.k.a. catch) an exception

– If doesn't match, exception continues to propagate

```
e1 handle MyFirstException => e2
e1 handle MySecondException(x,y) => e2
```

Actually…

Exceptions are a lot like datatype constructors…

- Declaring an exception adds a constructor for type **`exn`**
- Can pass values of **`exn`** anywhere (e.g., function arguments)
  - Not too common to do this but can be useful
- Handle can have multiple branches with patterns for type **`exn`**

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Tail Recursion

Recursion

Should now be comfortable with recursion:

- No harder than using a loop (whatever that is ☺)
- Often much easier than a loop
  - When processing a tree (e.g., evaluate an arithmetic expression)
  - Examples like appending lists
  - Avoids mutation even for local variables
- Now:
  - How to reason about *efficiency* of recursion
  - The importance of *tail recursion*
  - Using an *accumulator* to achieve tail recursion
  - [No new language features here]

Call-stacks

While a program runs, there is a *call stack* of function calls that have started but not yet returned

- Calling a function **`f`** pushes an instance of **`f`** on the stack
- When a call to **`f`** finishes, it is popped from the stack

These stack-frames store information like the value of local variables and "what is left to do" in the function

Due to recursion, multiple stack-frames may be calls to the same function

Example

```
fun fact n = if n=0 then 1 else n*fact(n-1)

val x = fact 3
```

fact 3	fact 3: 3*_	fact 3: 3*_	fact 3: 3*_
	fact 2	fact 2: 2*_	fact 2: 2*_
		fact 1	fact 1: 1*_
			fact 0

fact 3: 3*_	fact 3: 3*_	fact 3: 3*_	fact 3: 3*2
fact 2: 2*_	fact 2: 2*_	fact 2: 2*1	
fact 1: 1*_	fact 1: 1*1		
fact 0: 1			

Example Revised

```
fun fact n =
    let fun aux(n,acc) =
            if n=0
            then acc
            else aux(n-1,acc*n)
    in
       aux(n,1)
    end

val x = fact 3
```

Still recursive, more complicated, but the result of recursive calls *is* the result for the caller (no remaining multiplication)

The call-stacks

`fact 3`	`fact 3: _`	`fact 3: _`	`fact 3: _`
	`aux(3,1)`	`aux(3,1):_`	`aux(3,1):_`
		`aux(2,3)`	`aux(2,3):_`
			`aux(1,6)`

`fact 3: _`	`fact 3: _`	`fact 3: _`	`fact 3: _`
`aux(3,1):_`	`aux(3,1):_`	`aux(3,1):_`	`aux(3,1):_`
`aux(2,3):_`	`aux(2,3):_`	`aux(2,3):_`	`aux(2,3):6`
`aux(1,6):_`	`aux(1,6):_`	`aux(1,6):6`	
`aux(0,6)`	`aux(0,6):6`		

Etc…

An optimization

It is unnecessary to keep around a stack-frame just so it can get a callee's result and return it without any further evaluation

ML recognizes these *tail calls* in the compiler and treats them differently:

- Pop the caller *before* the call, allowing callee to *reuse* the same stack space
- (Along with other optimizations,) as efficient as a loop

Reasonable to assume all functional-language implementations do tail-call optimization

What really happens

```
fun fact n =
    let fun aux(n,acc) =
            if n=0
            then acc
            else aux(n-1,acc*n)
    in
       aux(n,1)
    end

val x = fact 3
```

`fact 3`	`aux(3,1)`	`aux(2,3)`	`aux(1,6)`	`aux(0,6)`

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Accumulators

Moral of tail recursion

- Where reasonably elegant, feasible, and important, rewriting functions to be *tail-recursive* can be much more efficient
  - Tail-recursive: recursive calls are tail-calls

- There is a methodology that can often guide this transformation:
  - Create a helper function that takes an *accumulator*
  - Old base case becomes initial accumulator
  - New base case becomes final accumulator

Methodology already seen

```
fun fact n =
    let fun aux(n,acc) =
               if n=0
               then acc
               else aux(n-1,acc*n)
    in
       aux(n,1)
    end

val x = fact 3
```

`fact 3`	`aux(3,1)`	`aux(2,3)`	`aux(1,6)`	`aux(0,6)`

Another example

```
fun sum xs =
   case xs of
       [] => 0
     | x::xs' => x + sum xs'
```

```
fun sum xs =
    let fun aux(xs,acc) =
           case xs of
               [] => acc
             | x::xs' => aux(xs',x+acc)
    in
       aux(xs,0)
    end
```

And another

```
fun rev xs =
   case xs of
       [] => []
     | x::xs' => (rev xs') @ [x]
```

```
fun rev xs =
    let fun aux(xs,acc) =
           case xs of
               [] => acc
             | x::xs' => aux(xs',x::acc)
    in
       aux(xs,[])
    end
```

Actually much better

```
fun rev xs =
   case xs of
        [] => []
      | x::xs' => (rev xs') @ [x]
```

- For **`fact`** and **`sum`**, tail-recursion is faster but both ways linear time
- Non-tail recursive **`rev`** is quadratic because each recursive call uses append, which must traverse the first list
  - And 1+2+…+(length-1) is almost length*length/2
  - Moral: beware list-append, especially within outer recursion
- Cons constant-time (and fast), so accumulator version much better

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
        [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Tail Recursion: Perspective and Definition

Always tail-recursive?

There are certainly cases where recursive functions cannot be evaluated in a constant amount of space

Most obvious examples are functions that process trees

In these cases, the natural recursive approach is the way to go

- You could get one recursive call to be a tail call, but rarely worth the complication

Also beware the wrath of premature optimization

- Favor clear, concise code
- But do use less space if inputs may be large

What is a tail-call?

The "nothing left for caller to do" intuition usually suffices

– If the result of `f x` is the "immediate result" for the enclosing function body, then `f x` is a tail call

But we can define "tail position" recursively

– Then a "tail call" is a function call in "tail position"

…

Precise definition

A *tail call* is a function call in *tail position*

- If an expression is not in tail position, then no subexpressions are

- In **`fun f p = e`**, the body **`e`** is in tail position
- If **`if e1 then e2 else e3`** is in tail position, then **`e2`** and **`e3`** are in tail position (but **`e1`** is not). (Similar for case-expressions)
- If **`let b1 … bn in e end`** is in tail position, then **`e`** is in tail position (but no binding expressions are)
- Function-call *arguments* **`e1 e2`** are not in tail position
- …