APS502H1 Report

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Part1

(a) Formulate a portfolio that maximizes expected profit:

Suppose we buy (or sell for option):

x shares of stock ABC of \$20 each

y units of options of \$1000 each

z units bonds of \$90 each

The profit under the three circumstances are:

$$P_1 = 20x + (20 - 15) \times 100y + 100z - 20000$$

$$P_2 = 40x + (40 - 15) \times 100y + 100z - 20000$$

$$P_3 = 12x + 0y + 100z - 20000$$

The total profit is the optimization target:

$$P = \frac{1}{3}(P_1 + P_2 + P_3)$$

The equality constraints is bounded by the total capital:

$$20x + 1000y + 90z = 20000$$

We can buy/sell at most 50 options. The variable bounds are:

$$0 \le x \le \infty$$

$$-50 \le y \le 50$$

$$0 \le z \le \infty$$

Solving optimization problem as above with MATLAB, the maximum profit is attained when we buy:

3500 shares of stock ABC

-50 units of option

0 units of bonds

which gives maximum profit of \$14000.

(b) Does the portfolio result in profit under all circumstances?

Profit in case 1: \$25000.00

Profit in case 2: \$-5000.00

Profit in case 3: \$22000.00

The portfolio doesn't result in profit in all circumstances. A deficit of \$5000 is attained when the stock price remains unchanged.

(c) Maximize expected return subject to at least \$2000

Enforce the following constrains on the optimization problem:

$$P_1 = 20x + (20 - 15) \times 100y + 100z - 20000 \geq 2000$$

$$P_2 = 40x + (40 - 15) \times 100y + 100z - 20000 \geq 2000$$

$$P_3 = 12x + 0y + 100z - 20000 \geq 2000$$

Solving optimization problem as above with MATLAB, the maximum profit is attained when we buy:

2800 shares of stock ABC

-36 units of option

0 units of bonds

Profit in respective cases:

Profit in case 1: \$18000.00

Profit in case 2: \$2000.00

Profit in case 3: \$13600.00

(d) Find portfolio that maximizes risk-less profit:

Introduce new risk-less profit variable p, which is also the optimization target. Then solve the 4-variable optimization problem of x, y, z, p:

The equality constraints is bounded by the total capital:

$$20x + 1000y + 90z + 0p = 20000$$

We can buy/sell at most 50 options. The inequality constrains are:

$$20x + (20 - 15) \times 100y + 100z - p \geq 0$$

$$40x + (40 - 15) \times 100y + 100z - p \geq 0$$

$$12x + 0y + 100z - p \geq 0$$

The variable bounds are:

$$0 \leq x \leq \infty$$

$$-50 \leq y \leq 50$$

$$0 \leq z \leq \infty$$

$$-\infty \leq p \leq \infty$$

Solving optimization problem as above with MATLAB, the maximum profit is attained when we buy:

2272.73 shares of stock ABC

-25.45 units of option

0 units of bonds

which gives maximum profit of $7272.73.

Part2

(a) Calculate expected returns, standard deviations and co-variance of three assets:

According to Yahoo Finance and MATLAB, the expected return of the three assets are:

$\mu_1 = 0.161576319453774$

$\mu_2 = -0.016142707038591$

$\mu_3 = -0.083850395954210$

The standard deviation of the three assets are (sample deviation):

$\sigma_1 = 9.436465492538190$

$\sigma_2 = 0.470579874738465$

$\sigma_3 = 3.563572312667640$

The covariance of the three assets are (sample covariance):

$\sigma_{12} = 1.354063990681560$

$\sigma_{13} = 6.147032932019340$

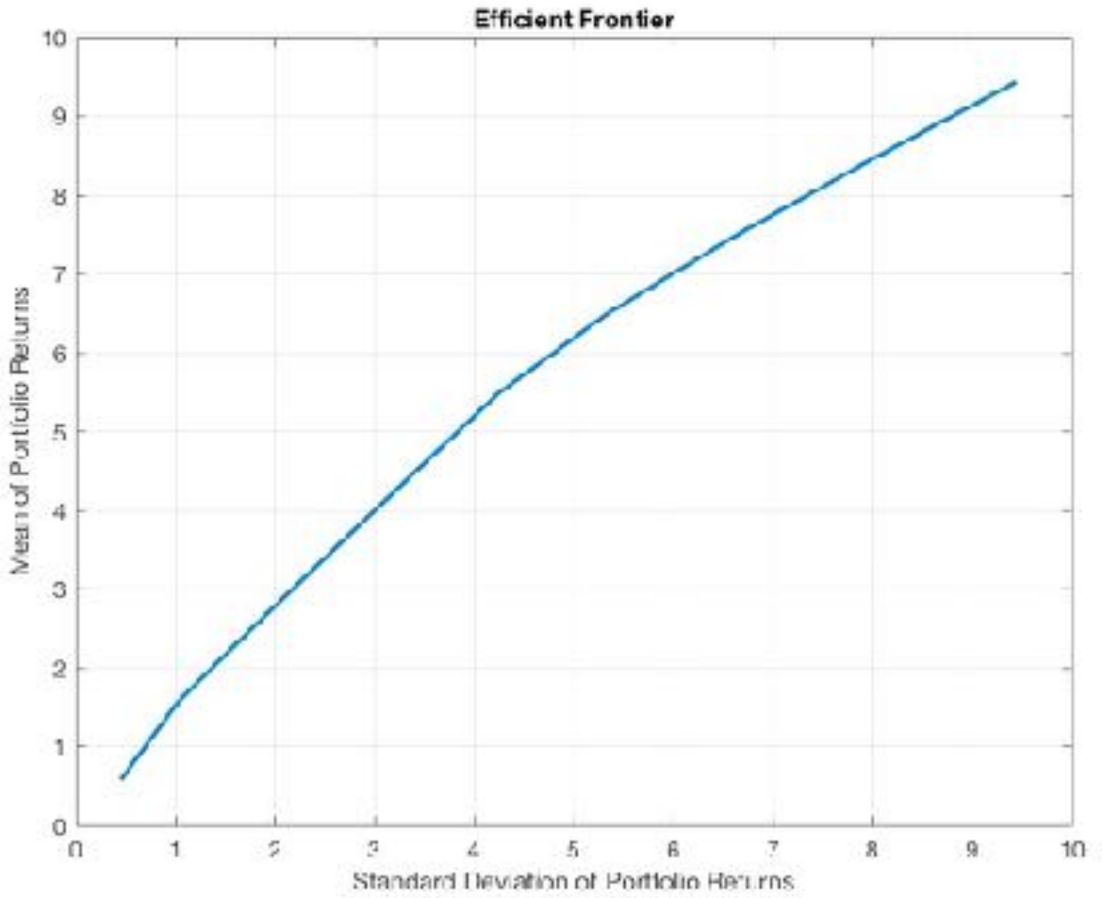
$\sigma_{23} = -0.262925558879088$

(b) Use MVO to generate an efficient frontier of the three assets.

Create portfolio using MATLAB Financial Toolbox, which gives the the efficient frontier. For each expected return, the optimal weights of the assets as well as the portfolio variance value is:

Expected Return	x	y	z	Risk
1	0.0191	0.8652	0.1157	0.6227
2	0.0836	0.6643	0.2521	1.3597
3	0.1481	0.4634	0.3885	2.1716
4	0.2126	0.2626	0.5248	2.9981
5	0.2771	0.0617	0.6612	3.8297
6	0.4149	0	0.5851	4.7601
7	0.5851	0	0.4149	5.9714
8	0.7554	0	0.2446	7.3379
9	0.9257	0	0.0743	8.7874

Plot the efficient frontier with MATLAB Financial Toolbox:



(c) Pick a portfolio from the efficient frontier, and compute the realized returns for the month of January 2017.

According to Yahoo Finance and MATLAB, the realized return of the three assets on January 2017 are:

$$\mu_1 = 1.039291504403617$$
$$\mu_2 = 1.002798835772569$$
$$\mu_3 = 1.019317943695971$$

Choose the 5th portfolio from the table above of $x = 0.271,\ y = 0.0617,\ z = 0.6612$, and a risk of $\sigma = 3.8297$. The realized return of the portfolio is:

$$P = x \times \mu_1 + y \times \mu_2 + z \times \mu_3 = 1.023833$$

which is above the realized return of GOVT and EEMV, but below the realized return of SPY.

Explanation: Among the three assets chosen, SPY is the one with higher volatility, but also higher expected return. GOVT and EEMV provides a lower expected return, but are more stable. Forming a portfolio between the three assets attains less in expense of lower expected returns.

Appendix 1: Code for part 1

```
%% part a: Formulate a linear program

% Objective coefficient (Take inverse to get maximum)
c1 = [20, (20 - 15) * 100, 100]';
c2 = [40, (40 - 15) * 100, 100]';
c3 = [12, 0, 100]';
c = -1 * (c1 + c2 + c3) / 3;

% Inequality constraints
A = [];
b = [];

% Equality constraints
Aeq = [20, 1000, 90];
beq = [20000];

% Variable bounds
ub = [inf, 50, inf]';
lb = [0, -50, 0]';

% Call linprog from matlab
fprintf('Part A\n')
[x, fval] = linprog(c, A, b, Aeq, beq, lb, ub);
fprintf('Investing in:\n\t%.2f shares of stocks\n\t%.2f options\n\t%.2f bonds\n', x(1), x(2), x(3))
fprintf('Total estimated profit:\t$%d\n\n', round(-1 * fval - 20000))

%% part b: Does the portfolio make profit in evert case?

% Calculate the profit in each case
profit_1 = c1' * x - 20000;
profit_2 = c2' * x - 20000;
profit_3 = c3' * x - 20000;
fprintf('Part B\n')
fprintf('Profit in case 1: %.2f\n', profit_1);
fprintf('Profit in case 2: %.2f\n', profit_2);
fprintf('Profit in case 3: %.2f\n\n', profit_3);

%% part c: Have at least 2000 profit in all scenarios
% Inequility constraints (Take inverse to get "greater than")
A = -1 * [c1'; c2'; c3'];
b = -1 * 22000 * ones(3, 1);

% Call linprog from matlab
fprintf('Part C\n')
[x, fval] = linprog(c, A, b, Aeq, beq, lb, ub);
fprintf('Investing in:\n\t%.2f shares of stocks\n\t%.2f options\n\t%.2f bonds\n', x(1), x(2), x(3))
profit_1 = c1' * x - 20000;
profit_2 = c2' * x - 20000;
profit_3 = c3' * x - 20000;
fprintf('Profit in case 1: %.2f\n', profit_1);
fprintf('Profit in case 2: %.2f\n', profit_2);
fprintf('Profit in case 3: %.2f\n\n', profit_3);

%% part d: Max riskless profit under all three scenarios

% Introduce new riskless profit p, which is the objective function. Take
% inverse to maximize
c = -1 * [0, 0, 0, 1]';

% Inequality constraints. Take inverse to get lower limit
A = -1 * [c1', -1; c2', -1; c3', -1];
b = [0, 0, 0]';

% Equality constraints
Aeq = [Aeq, 0];
beq = [20000];
```

```
% Variable bounds
ub = [ub; inf];
lb = [lb; -inf];

fprintf('Part D\n')
[x, fval] = linprog(c, A, b, Aeq, beq, lb, ub);
fprintf('Investing in:\n\t%.2f shares of stocks\n\t%.2f options\n\t%.2f bonds\n', x(1), x(2), x(3))
fprintf('Maximum riskless profit:\t%.2f\n', round(x(4),2) - 20000)
```

Appendix 2: Output for part 1

```
Part A
Optimization terminated.
Investing in:
        3500.00 shares of stocks
        -50.00 options
        0.00 bonds
Total estimated profit: $14000

Part B
Profit in case 1: 25000.00
Profit in case 2: -5000.00
Profit in case 3: 22000.00

Part C
Optimization terminated.
Investing in:
        2800.00 shares of stocks
        -36.00 options
        0.00 bonds
Profit in case 1: 18000.00
Profit in case 2: 2000.00
Profit in case 3: 13600.00

Part D
Optimization terminated.
Investing in:
        2272.73 shares of stocks
        -25.45 options
        0.00 bonds
Maximum riskless profit:        7272.73
```

Appendix 3: Code for part 2

```
%% Part a:
% Calculate expected returns of the portfolio
r_1 = 0.161576319453774;
r_2 = -0.016142707038591;
r_3 = -0.083850395954210;

% Calculate stddev and cov of the portfolio
std_1 = 9.436465492538190;
std_2 = 0.470579874738465;
std_3 = 3.563572312667640;
cov_12 = 1.354063990681560;
cov_13 = 6.147032932019340;
cov_23 = -0.262925558879088;

%% Part b: Generate an efficient frontier between three assets
% Create portfolio
m = [std_1, std_2, std_3]';
C = [std_1^2, cov_12, cov_13;
    cov_12, std_2^2, cov_23;
    cov_13, cov_23, std_3^2];
p = Portfolio('assetmean', m, 'assetcovar', C, 'budget', 1, 'lb', 0);
plotFrontier(p);
```

```
% Estimate portfolio under specific profits
pwgt = estimateFrontierByReturn(p, [1 : 9]);
prsk = estimatePortRisk(p, pwgt);
% Produce table
fprintf('Portfolio and risk under different return goal:\n');
tbl = [pwgt', prsk]


%% Part c: Pick a portfolio and try it on January 2017
% Calculate realized return of each portfolio
r_realized_1 = 1.039291504403617;
r_realized_2 = 1.002798835772569;
r_realized_3 = 1.019317943695971;
r_realized = [r_realized_1, r_realized_2, r_realized_3];
fprintf('The realized individual return of three assets:\n');
fprintf('\tSPT:\t%f\n', r_realized(1));
fprintf('\tGOVT:\t%f\n', r_realized(2));
fprintf('\tEEMV:\t%f\n', r_realized(3));

% Pick a portfolio (#5)
port = pwgt(:, 5);
profit = r_realized * port;
fprintf('Choose the following Portfolio:\n');
fprintf('\tSPT:\t%f\n', port(1));
fprintf('\tGOVT:\t%f\n', port(2));
fprintf('\tEEMV:\t%f\n', port(3));
fprintf('The portfolio have profit %f\n', profit);
```

Appendix 4: Output for part 2

```
Portfolio and risk under different return goal:
tbl =
    0.0191    0.8652    0.1157    0.6227
    0.0836    0.6643    0.2521    1.3597
    0.1481    0.4634    0.3885    2.1716
    0.2126    0.2626    0.5248    2.9981
    0.2771    0.0617    0.6612    3.8297
    0.4149         0    0.5851    4.7601
    0.5851         0    0.4149    5.9714
    0.7554         0    0.2446    7.3379
    0.9257         0    0.0743    8.7874
The realized individual return of three assets:
        SPT:    1.039292
        GOVT:   1.002799
        EEMV:   1.019318
Choose the following Portfolio:
        SPT:    0.277089
        GOVT:   0.061715
        EEMV:   0.661196
The portfolio have profit 1.023833
```