1. LMNOPQRST

(a) $LPR^+ = \{L, P, Q, R, S, T\}$, violation

$LR^+ = \{L, R, S, T\}$, violation

$M^+ = \{L, M, O\}$, violation

$MR^+ = \{M, N, R\}$, violation

(b)

$LPR^+ = \{L, P, Q, R, S, T\}$, violates BNCF, replace with R_1 and R_2:

$R_1 = \{L, P, Q, R, S, T\}, R_2 = \{L, M, N, O, P, R\}$. Projecting FDs onto R1 and R2:

$R_1 : LR \rightarrow ST, LPR \rightarrow QST$ and $R2 : M \rightarrow LO, MR \rightarrow LNO$, where

$LR \rightarrow ST, M \rightarrow LO$ and $MR \rightarrow LNO$ violates BCNF.

Break R_1 into $R_{11} = \{L, R, S, T\}$ and $R_{12} = \{L, P, Q, R\}$. Projecting FDs onto R_{11} and R_{12}:

$R_{11} : LR \rightarrow ST$ and $R_{12} : LPR \rightarrow Q$, both satisfying BCNF.

Break R2 into $R_{21} = \{L, M, N, O, R\}$ and $R_{22} = \{M, P, R\}$. Projecting FDs onto R_{21} and R_{22}:

$R_{21} : MR \rightarrow LNO$ and R_{22} : No FDs, both satisfying BCNF.

Result:

$R_1 = \{L, R, S, T\}, R_2 = \{L, P, Q, R\}, R_3 = \{L, M, N, O, R\}, R_4 = \{M, P, R\}$.

2. ABCDEFGH

(a) Compute a minimal basis for T

Split the RHS of each FD:

$AB \rightarrow C, AB \rightarrow D, ACDE \rightarrow B, ACDE \rightarrow F, B \rightarrow A, B \rightarrow C, B \rightarrow D, CD \rightarrow A, CD \rightarrow F, CDE \rightarrow F, CDE \rightarrow G, BE \rightarrow D$

Remove attribute from LHS:

$B \rightarrow C, B \rightarrow D, CDE \rightarrow B, CDE \rightarrow F, B \rightarrow A, B \rightarrow C, B \rightarrow D, CD \rightarrow A, CD \rightarrow F, CDE \rightarrow F, CDE \rightarrow G, BE \rightarrow D$

Remove excessive FDs:

$CDE \rightarrow B, B \rightarrow C, B \rightarrow D, CD \rightarrow A, CD \rightarrow F, CDE \rightarrow G$

(b) Compute all keys for P

Because EH did not appear on RHS, or did not appear at all, they have to be in the key.

Because AFG only appeared on RHS, they can't be part of key.

The keys can be: BEH or CDEH.

(c) 3NF synthesis

Combine FDs with same LHS:

$CDE \rightarrow BG, B \rightarrow CD, CD \rightarrow AF$

For each FD in minimal basis, define a new relation:

$BCDEG, ACDF$

Because no relation is super-key (no H), add relation whose schema is key:

$BCDEG, ACDF, BEH$

(d) Does schema allow redundancy?

Yes. Relation BCDEG satisfies 3NF, but doesn't satisfy BCNF: Functional dependency $B \rightarrow CD$ isn't a super-key. For example, the following table have redundancy:

C1	D1	E1	B1	G1
C1	D1	E2	B1	G2

C1 and D1 are redundant.