1. LMNOPQRST

(a) $LPR^+ = \{L, P, Q, R, S, T\}$, violation

$LR^+ = \{L, R, S, T\}$, violation

$M^+ = \{L, M, O\}$, violation

$MR^+ = \{M, N, R, L, O, S, T\}$, violation

(b)

$LR^+ = \{L, R, S, T\}$, violates BNCF, replace with R_1 and R_2:

$R_1 = \{L, R, S, T\}$, $R_2 = \{L, M, N, O, P, Q, R\}$. Projecting FDs onto R1 and R2:

$R_1: LR \to ST$ which satisfies BCNF and $R2: M \to LO$ which already violates BCNF (as $M^+ = \{L, M, O\}$)

Break R_2 into $R_{21} = \{M, L, O\}$ and $R_{22} = \{M, R, Q, P, N\}$. Projecting FDs onto R_{21} and R_{22}:

$R_{21}: M \to LO$ which satisfies BCNF and $R_{22}: MR \to N$ which already violates BCNF (as $MR^+ = \{M, R, N\}$).

Break R22 into $R_{221} = \{M, N, R\}$ and $R_{222} = \{M, N, P, Q\}$. Projecting FDs onto R_{221} and R_{222}:

$R_{221}: MR \to N$ which satisfies BCNF and $R_{222}: MRP \to Q$ which satisfies BCNF.

Discard relations which violate BCNF, and let $R_{21} = R_2$, $R_{221} = R_3$, $R_{222} = R_4$.

Result:

$R_1 = \{L, R, S, T\}, R_2 = \{M, L, O\}, R_3 = \{M, N, R\}, R_4 = \{M, N, P, Q\}$.

2. ABCDEFGH

(a) Compute a minimal basis for T

Split the RHS of each FD:

$AB \to C, AB \to D, ACDE \to B, ACDE \to F, B \to A, B \to C, B \to D, CD \to A, CD \to F, CDE \to F, CDE \to G, BE \to D$

Remove attribute from LHS:

$B \to C, B \to D, CDE \to B, CDE \to F, B \to A, B \to C, B \to D, CD \to A, CD \to F, CDE \to F, CDE \to G, BE \to D$

Remove excessive FDs:

$CDE \to B, B \to C, B \to D, CD \to A, CD \to F, CDE \to G$

(b) Compute all keys for P

Because EH did not appear on RHS, or did not appear at all, they have to be in the key.

Because AFG only appeared on RHS, they can't be part of key.

The keys are: BEH, CDEH.

(c) 3NF synthesis

Combine FDs with same LHS:

$CDE \rightarrow BG, B \rightarrow CD, CD \rightarrow AF$

For each FD in minimal basis, define a new relation:

$BCDEG, ACDF$

Because no relation is super-key (no H), add relation whose schema is key:

$BCDEG, ACDF, BEH$

(d) Does schema allow redundancy?

Yes. Relation BCDEG satisfies 3NF, but doesn't satisfy BCNF: Functional dependency $B \rightarrow CD$ isn't a super-key. For example, the following table have redundancy:

C1	D1	E1	B1	G1
C1	D1	E2	B1	G2

C1 and D1 are redundant.