1. LMNOPQRST

(a) $LPR^+ = \{L, P, Q, R, S, T\}$, violation

$LR^+ = \{L, R, S, T\}$, violation

$M^+ = \{L, M, O\}$, violation

$MR^+ = \{M, N, R\}$, violation

(b)

$LPR^+ = \{L, P, Q, R, S, T\}$, violates BNCF, replace with $R_1$ and $R_2$:

$R_1 = \{L, P, Q, R, S, T\}, R_2 = \{L, M, N, O, P, R\}$. Projecting FDs onto R1 and R2:

$R_1 : LR \to ST, LPR \to QST$ and $R2 : M \to LO, MR \to LNO$, where

$LR \to ST, M \to LO$ and $MR \to LNO$ violates BCNF.

Break $R_1$ into $R_{11} = \{L, R, S, T\}$ and $R_{12} = \{L, P, Q, R\}$. Projecting FDs onto $R_{11}$ and $R_{12}$:

$R_{11} : LR \to ST$ and $R_{12} : LPR \to Q$, both satisfying BCNF.

Break R2 into $R_{21} = \{L, M, N, O, R\}$ and $R_{22} = \{M, P, R\}$. Projecting FDs onto $R_{21}$ and $R_{22}$:

$R_{21} : LMR \to NO$ and $R_{22}$ : No FDs, both satisfying BCNF.

Result:

$R_1 = \{L, R, S, T\}, R_2 = \{L, P, Q, R\}, R_3 = \{L, M, N, O, R\}, R_4 = \{M, P, R\}$.

2. ABCDEFGH

(a) Compute a minimal basis for T

Split the RHS of each FD:

$AB \to C, AB \to D, ACDE \to B, ACDE \to F, B \to A, B \to C, B \to D, CD \to A, CD \to F, CDE \to F, CDE \to G, BE \to D$

Remove attribute from LHS:

$B \to C, B \to D, CDE \to B, CDE \to F, B \to A, B \to C, B \to D, CD \to A, CD \to F, CDE \to F, CDE \to G, BE \to D$

Remove excessive FDs:

$CDE \to B, B \to A, B \to C, B \to D, CD \to A, CD \to F, CDE \to G$

(b) Compute all keys for P

Because EH did not appear on RHS, or did not appear at all, they have to be in the key.

Because AFG only appeared on RHS, they can't be part of key.

The keys can be: BEH or CDEH.

(c) 3NF synthesis

For each FD in minimal basis, define a new relation:

$BCDE, AB, BC, BD, ACD, CDF, CDEG$

Because no relation is super-key, add relation BEH whose schema is key:

$BCDE, AB, BC, BD, ACD, CDF, CDEG, BEH$

(d) Does schema allow redundancy?

Yes. It allows FDs with non-superkey on the LHS.