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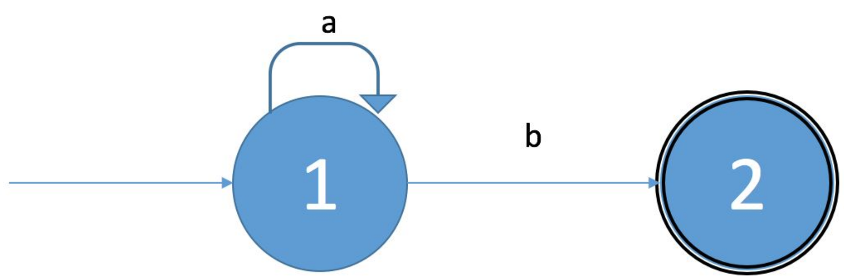
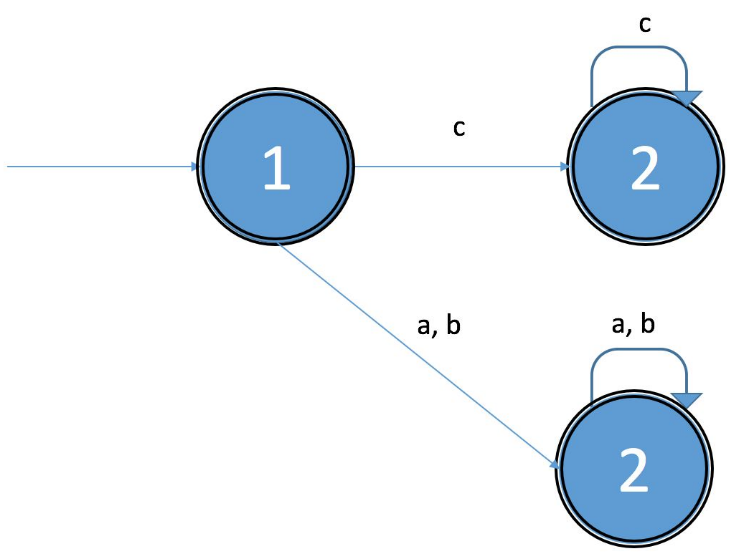
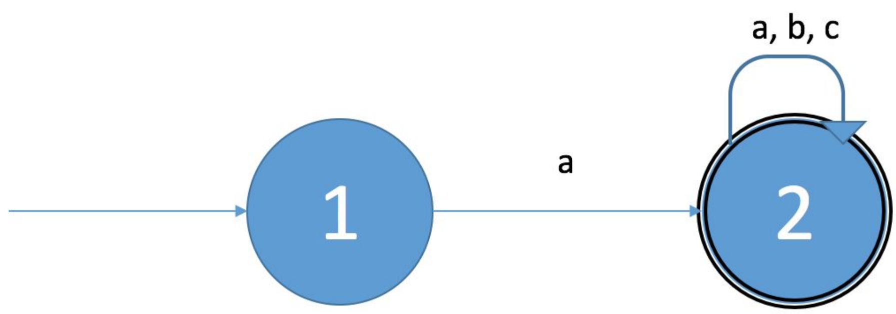
1
a*b



((a|b)* | (cc*))



a(a|(b|c))*



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2.a

If there exist Vertex Cover Set S of size s < k, for the complementary set S' of Vertex Cover Set must satisfy that all edges must have at most one end in S' since at least one end is in S, i.e. S' is an independent set.

S' has size s' = V - s > V - k
for each k, through vertex cover we can determine whether there is size V - k independent set. k ranges from 0 to V, which makes it a strict 1 to 1 mapping in all situations for Vertex Cover to Independent Set.
Q.E.D

2.b

If there exist Independent Set S of size s > k, for the complementary set S' of S must satisfy that all edges must have at least one end in S' since at most one end is in S, i.e. S' is an Vertex Cover Set.

S' has size s' = V - s < V - k
similarly, for each k, through Independent Set we can determine whether there is size V - k Vertex Cover.
Q.E.D

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3

3.1

Given the set of vertices(V), go through all edges to verify whether one end of edge is in the size V/2 set takes O(Elog(V)) time which is polynomial.
=> B \in NP

3.2

A = VERTEX-COVER <G, K>
B <G>

Lemma:
If graph G has vertex cover of size k, the vertex cover of size k + 1 to V all exist for G.
Proof:
Use the vertex set of VC of size k, and add a new vertex to it. All segments now still have at least one end in the new set since the new set is the superset of old set. Q.E.D.

Reduce Problem A to B:
(1)k = |V| / 2

This is a trivial case where problems A and B match. Simply solve problem A with problem B.

(2)k > |V| / 2

Construct new graph G', by adding to original graph G (2k - |V|) node-triplets that are detached from G (a.k.a. (6k - 3|V|) new nodes are appended). Solve this graph with solver of problem B, the result is the same with the original problem.
Proof:

Among each node-triplet, at least one node have to be selected as part of Vertex-Cover.

The size of the new graph G' is 6k-3|V|+|V| = 6k-2|V|. Since at least 2k-|V| nodes in the vertex-cover are in G'-G, and the size of the entire vertex-cover is |G'|/2 = 3k-|V|, at most (3k-|V|) - (2k-|V|) = k nodes are in G.

This means there exists a vertex-cover with size of at most k. According to the lemma proved previously, there exists a vertex-cover with size of k.

(2)k < |V| / 2

Construct new graph G', by adding to original graph G (|V| - 2k) node-quintuples that are detached from G (a.k.a. (5|V| - 10k) new nodes are appended). Solve this graph with solver of problem B, the result is the same with the original problem.
Proof:

Among each node-quintuples, at least three node have to be selected as part of Vertex-Cover.

The size of the new graph G' is 5|V|-10k+|V| = 6|V|-10k. Since at least 3|V|-6k nodes in the vertex-cover are in G'-G, and the size of the entire vertex-cover is |G'|/2 = 3|V|-5k, at most (3|V| - 5k) - (3|V| - 6k) = k nodes are in G.

This means there exists a vertex-cover with size of at most k. According to the lemma proved previously, there exists a vertex-cover with size of k.

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4.
step1 prove that the Longest-simple-cycle problem is NP:
Given a solution to <G, k>(i.e. G contains a simple cycle of k vertices), we just need to check whether it is a simple cycle (i.e. no repeated vertex in the circle) and covers exactly k vertices. The above verification takes polynomial time.
step2 prove that the Ham-Cycle <G, k> can be reduced to Longest-simple-cycle in polynomial time:
Given an instance of Ham-Cycle on a graph G(V, E). We create an instance of Longest-simple-cycle as follows. We use exactly the same graph(i.e. G' = G) and we set k = |V|. Then there exists a simple cycle of length of |V| in G' iff G contains a Hamiltonian path. Since Ham-cycle covers all the vertices, the cycle is the longest one in G'.
It is easy to see that the reduction algorithm takes the input <G, k>, we just use the same graph and length and can easily do in polynomial time. Hence Ham-Cycle ≤p Longest-simple-cycle.
step3 Longest-simple-cycle is NPC because Ham-Cycle is NPC.

Q.E.D