FLS 6441 - Methods III: Explanation and Causation

Week 1 - Review

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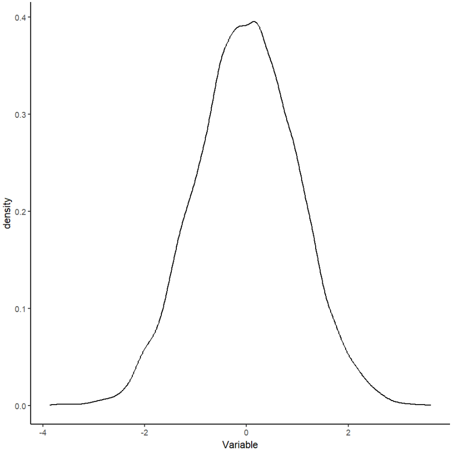
Course Objectives

1. temp

Data

1. We work with variables, which VARY!

	Variable
1	0.39
2	1.69
3	-1.05
4	-1.38
5	0.81
6	2.01
7	0.06
8	0.98
9	-0.98
10	-0.39



Regression

- Regression identifies the line through the data that minimizes the sum of squared vertical distances

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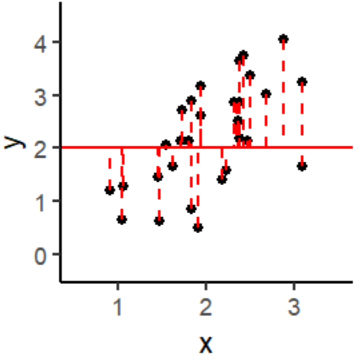
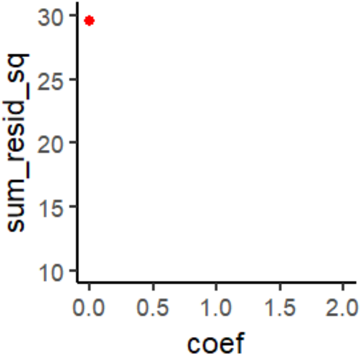
Slope = 0



Sum of Squared Residuals = 29.6



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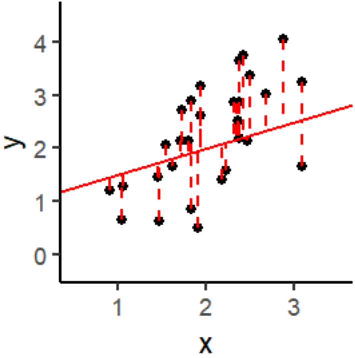
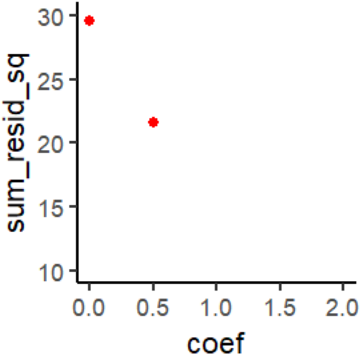
Slope = 0.5



Sum of Squared Residuals = 21.6



Regression

- Regression identifies the line through the data that minimizes the sum of squared vertical distances
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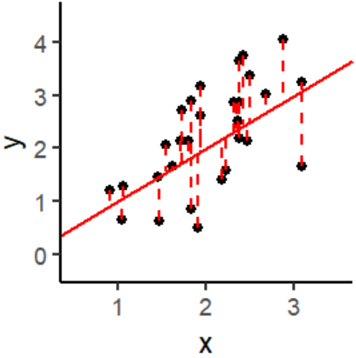
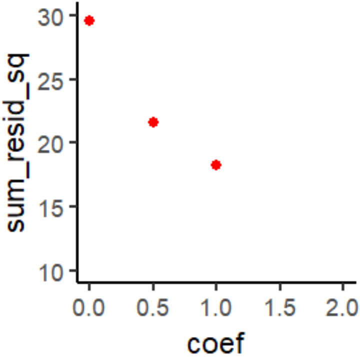
Slope = 1



Sum of Squared Residuals = 18.3



Regression

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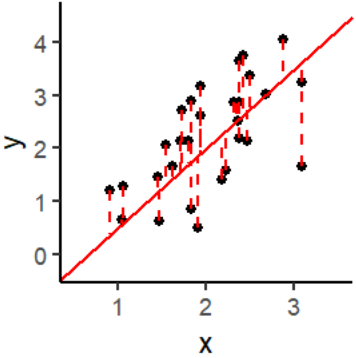
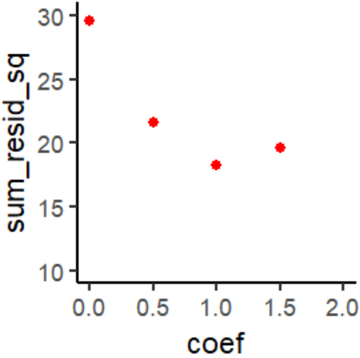
Slope = 1.5



Sum of Squared Residuals = 19.6



Regression

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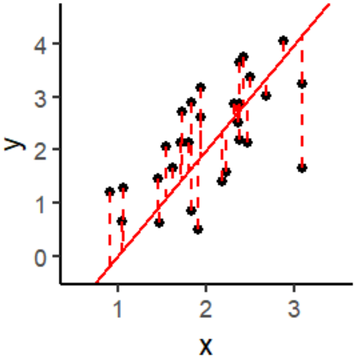
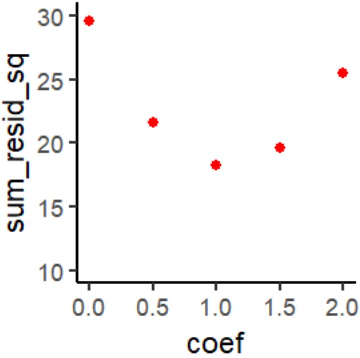
Slope = 2



Sum of Squared Residuals = 25.5



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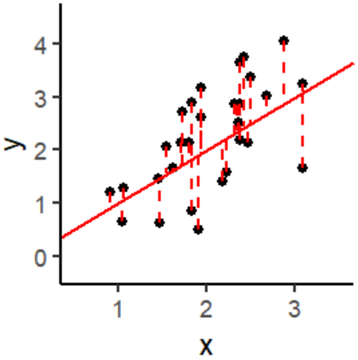
Slope = 1



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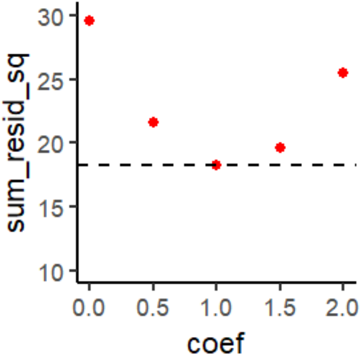
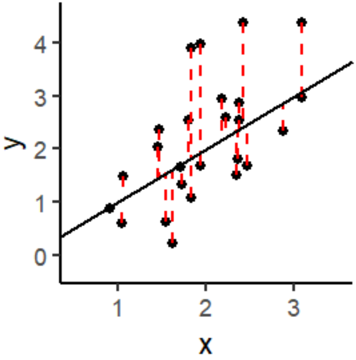


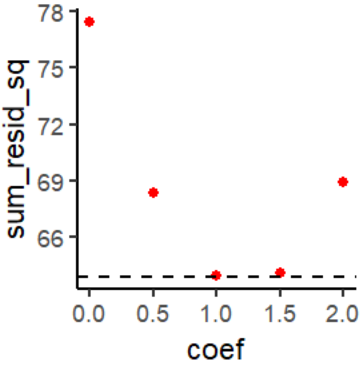
Regression

- If we add pure *noise* to y, our estimate of β is unchanged
  - The residual error increases
- $y_i = \alpha + \beta D_i + \epsilon_i$

Slope = 1

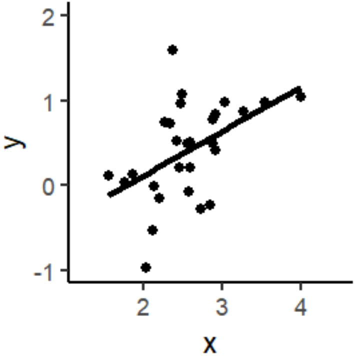


Sum of Squared Residuals = 63.9



Regression

- Dummy control variables *remove variation* associated with specific levels or categories
  - The same for fixed effects
- $y_{ij} = \alpha + \beta D_{ij} + \tau_j + \epsilon_i$



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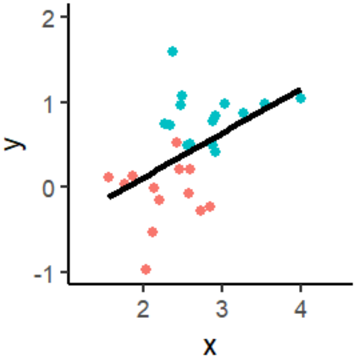

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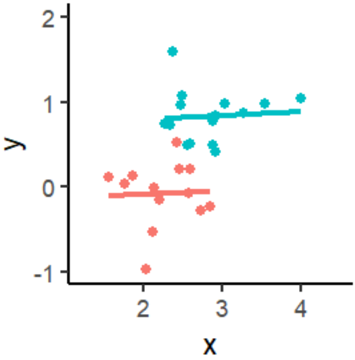

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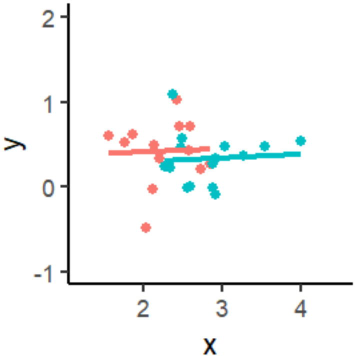

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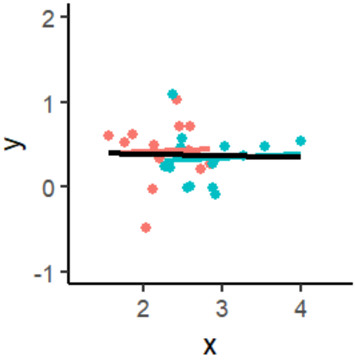
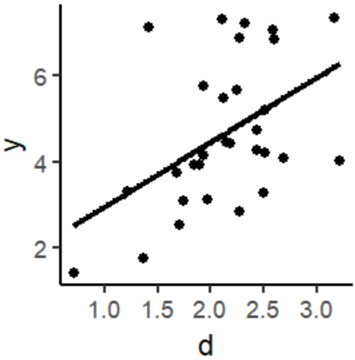


Regression

- Continuous control variables *remove variation* based on how much the control explains y
- $y_i = \alpha + \beta_1 D_i + \beta_2 X_i + \epsilon_i$



The coefficient on D is 1.503
Real coefficient = 1

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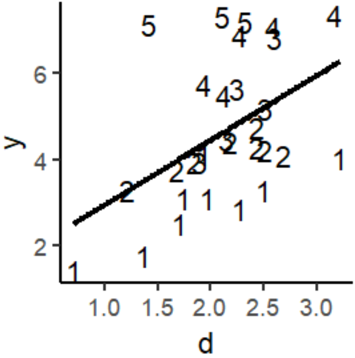

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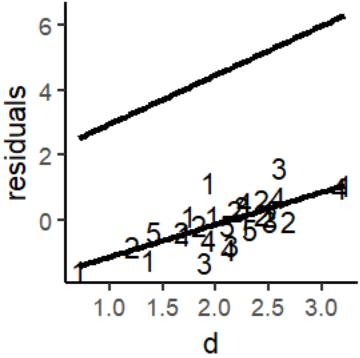

The coefficient on D is 0.991
Real coefficient = 1

Regression

- Regression is a **Conditional Expectation Function**

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- Conditional on x, what is our expectation (mean value) of y?

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- $E(y|x)$

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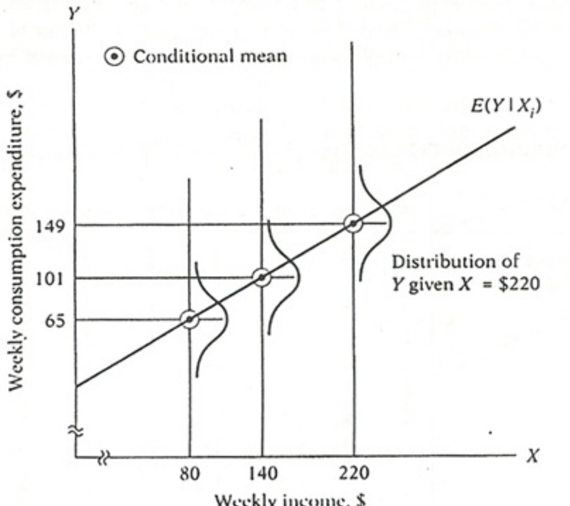
- Conditional on x, what is our expectation (mean value) of y?
- $E(y|x)$
- When age is 20 ($x = 40$), the average salary is R1.000 ($y = 1.000$)
- When age is 40 ($x = 40$), the average salary is R2.000 ($y = 2.000$)

Regression

- Regression is a **Conditional Expectation Function**: $E(y|x)$

Regression

- Regression is a **Conditional Expectation Function**: $E(y|x)$
- It predicts the **mean**, not the median, not the minimum, not the maximum



Regression

$$\hat{\beta_1} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

$$\hat{\beta_0} = \bar{y} - \hat{\beta_1}\bar{x}$$

Regression

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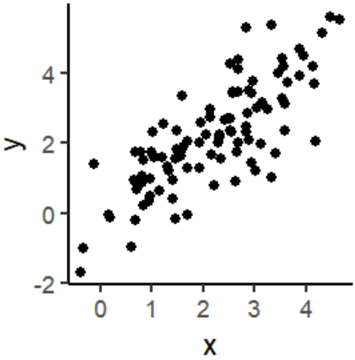

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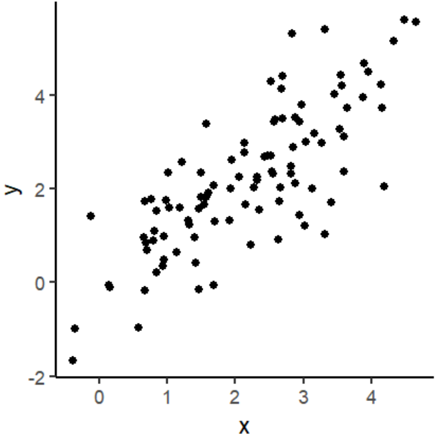

- Correlation is 0.781

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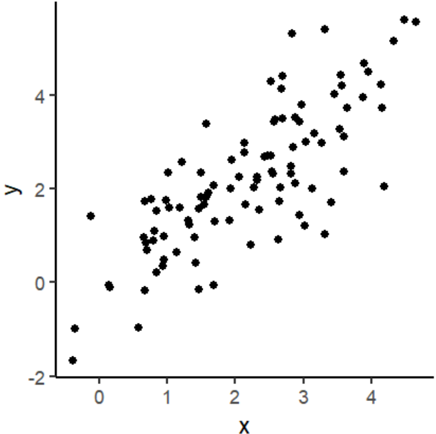

- Correlation is 0.781
- Regression Results:

	term	estimate
1	(Intercept)	0.006
2	x	1.008

Regression

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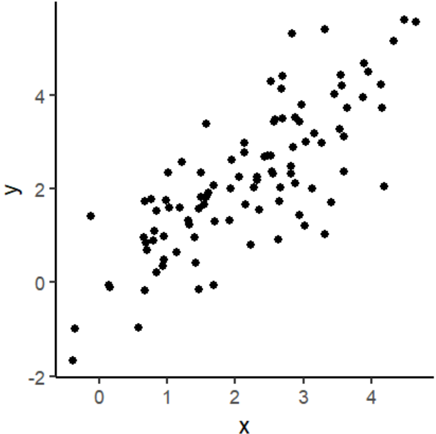

- Correlation is 0.781
- Standardized Regression Results:

	term	estimate
1	(Intercept)	0.000
2	x	0.781

Regression

- Regression with **multiple** variables is very similar to calculating **partial** correlation:

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- Just a small difference in the denominator (how we standardize the measure)

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$$\beta_{x_1} = \frac{r_{yx_1} - r_{yx_2} r_{x_1x_2}}{1 - r^2_{x_1x_2}}$$

$$r_{yx_1|x_2} = \frac{r_{yx_1} - r_{yx_2} r_{x_1x_2}}{\sqrt{(1 - r^2_{yx_2})(1 - r^2_{x_1x_2})}}$$

- **There is no magic in regression, it's just correlation 'extra'**

Regression

- We **NEVER** know the true value of β

Regression

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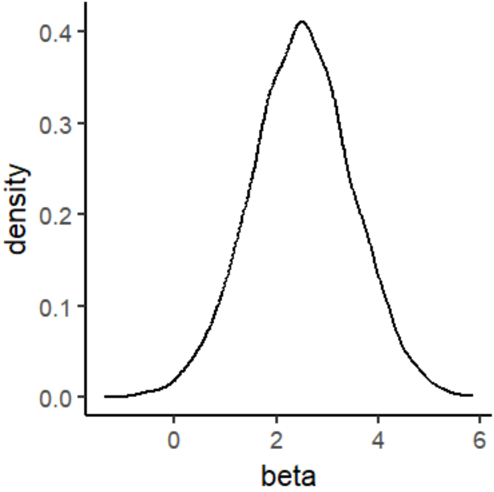
- We **estimate a distribution** for β



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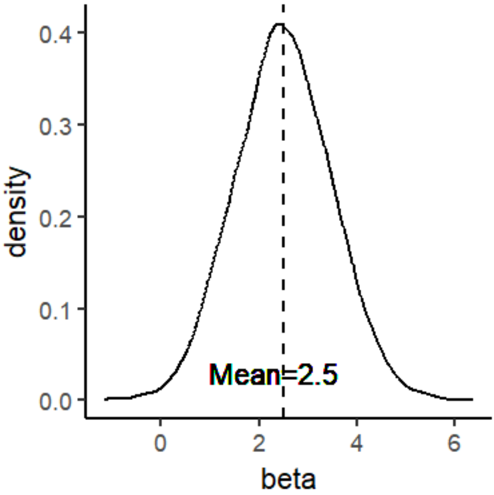

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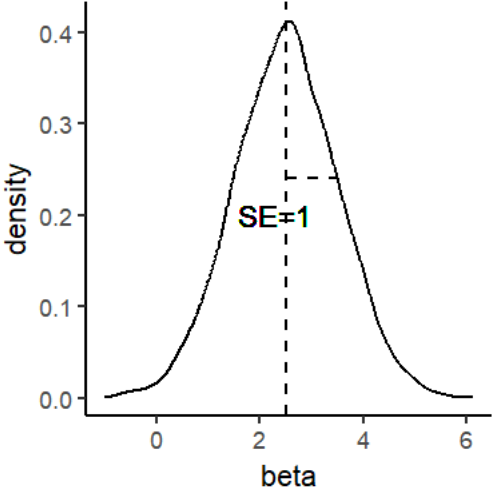

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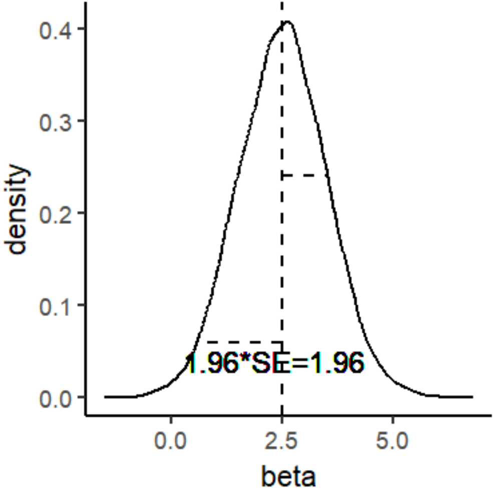

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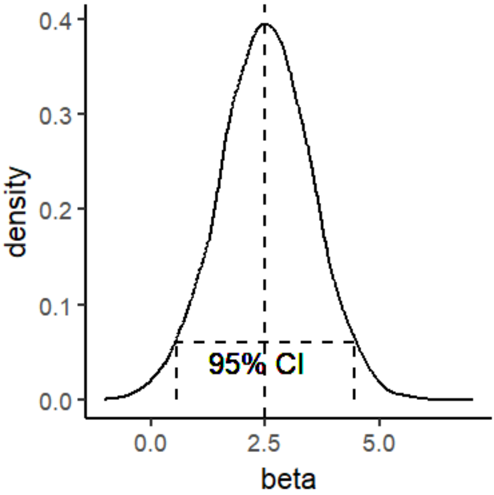

Regression Guide

1. **Choose variables and measures:** To test a specific hypothesis
2. **Choose a Model/Link Function:** Should match the data type of your outcome variable
3. **Choose Covariates:** To match your strategy of inference
4. **Choose Fixed Effects:** To focus on a specific level of variation
5. **Choose Error Structure:** To match known dependencies/clustering in the data
6. **Interpret the coefficients:** Depending on the type/scale of the explanatory variable

2. Regression Models

The Regression Model reflects the data type of the outcome variable:

- Continuous -> Ordinary Least Squares

```
zelig(Y   X,data=d,model="ls")
```

- Binary -> Logit

```
zelig(Y   X,data=d,model="logit")
```

- Unordered categories -> Multinomial logit

```
zelig(Y   X,data=d,model="mlogit")
```

- Ordered categories -> Ordered logit

```
zelig(Y   X,data=d,model="ologit")
```

- Count -> Poisson

```
zelig(Y   X,data=d,model="poisson")
```

6. Interpreting Regression Results

- Difficult! It depends on the scale of the explanatory variable, scale of the outcome, the regression model we used, and the presence of any interaction
- Basic OLS:
  - 1 [unit of explanatory variable] change in the explanatory variable is associated with a β [unit of outcome variable] change in the outcome

Predictions from Regressions

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Predictions from Regressions

- The coefficient on the regression of income on attitude to redistribution is -0.000818
  - So??? What do we learn from this?
  - Coefficients are hard to interpret, and depend on how we measure each variable
  - And p-values are arbitrary
- Better to make specific *predictions* of how changes in X produce changes in Y

Predictions from Regressions

- $$Attitude_i = \alpha + \beta_1 Income_i + \epsilon_i$$

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- $$Attitude_i = \alpha + \beta_1 Income_i + \epsilon_i$$

- $$Attitude_i = 2.235 + -0.000818 Income_i + N(0, 2.378)$$

Predictions from Regressions

- $$Attitude_i = \alpha + \beta_1 Income_i + \epsilon_i$$
- $$Attitude_i = 2.235 + -0.000818 Income_i + N(0, 2.378)$$
- $Income_i = 3000$ -> $Attitude_i = -0.219 + N(0, 2.378)$

Predictions from Regressions

- $$Attitude_i = \alpha + \beta_1 Income_i + \epsilon_i$$
- $$Attitude_i = 2.235 + -0.000818 Income_i + N(0, 2.378)$$
- $Income_i = 3000$ -> $Attitude_i = -0.219 + N(0, 2.378)$
- $Income_i = 6000$ -> $Attitude_i = -2.673 + N(0, 2.378)$

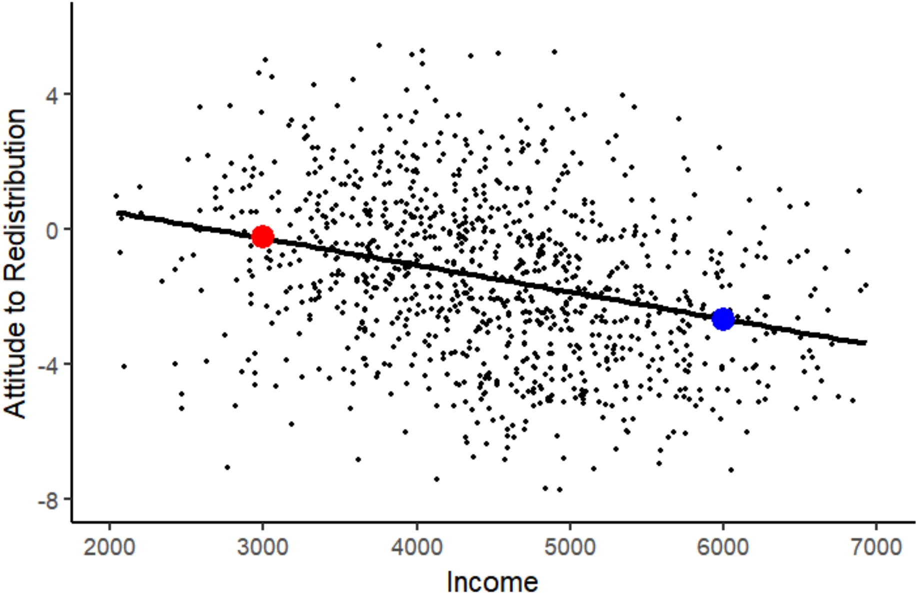
Predictions from Regressions

- $$Attitude_i = \alpha + \beta_1 Income_i + \epsilon_i$$

- $$Attitude_i = 2.235 + -0.000818 Income_i + N(0, 2.378)$$

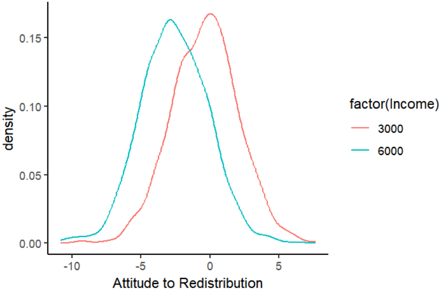
- $Income_i = 3000$ -> $Attitude_i = -0.219 + N(0, 2.378)$
- $Income_i = 6000$ -> $Attitude_i = -2.673 + N(0, 2.378)$
- Increasing Income from 3000 to 6000 -> $\Delta Attitude_i = -0.219 - -2.673$

Predictions from Regressions

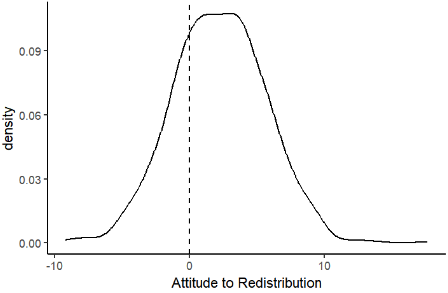


Predictions from Regressions

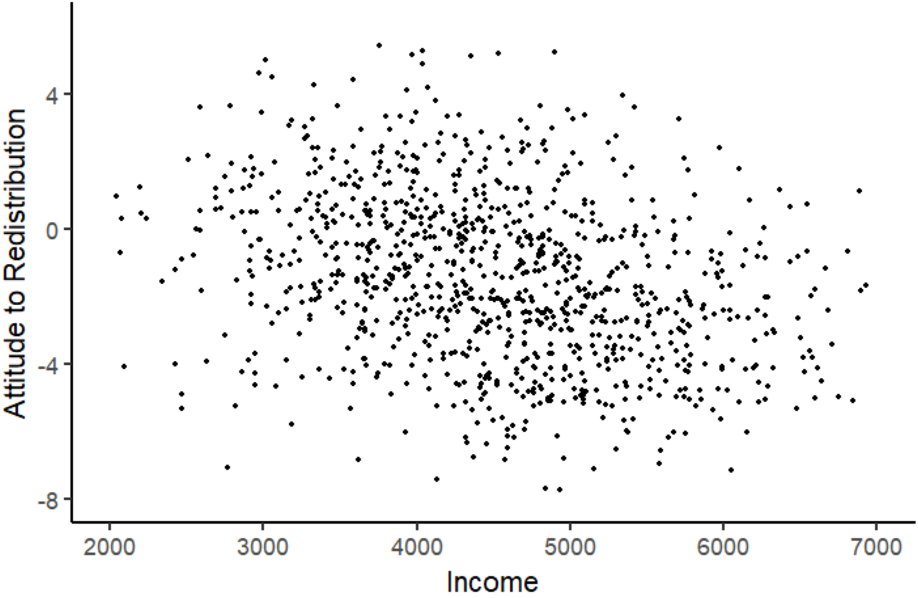
Predicted Values:



First Differences:



Omitted Variable Bias



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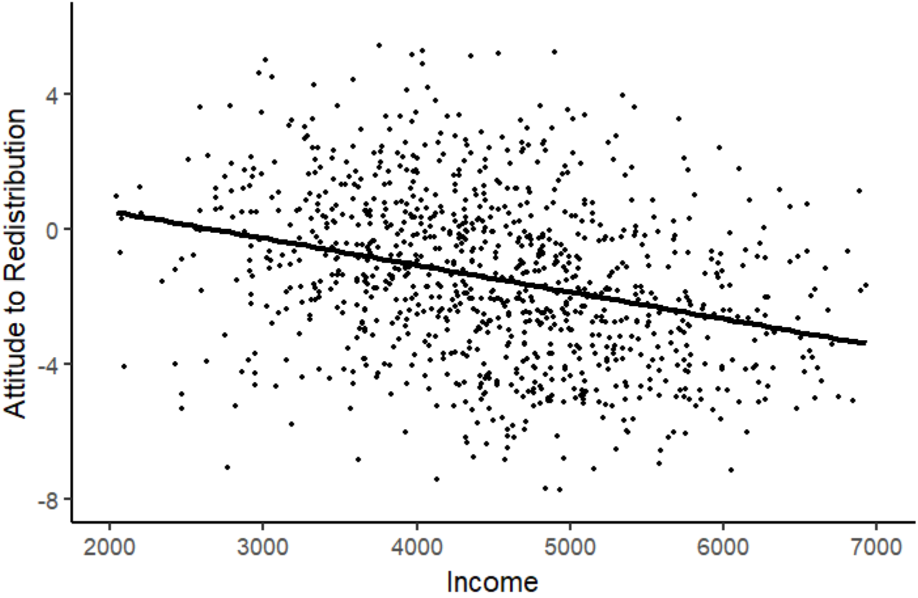

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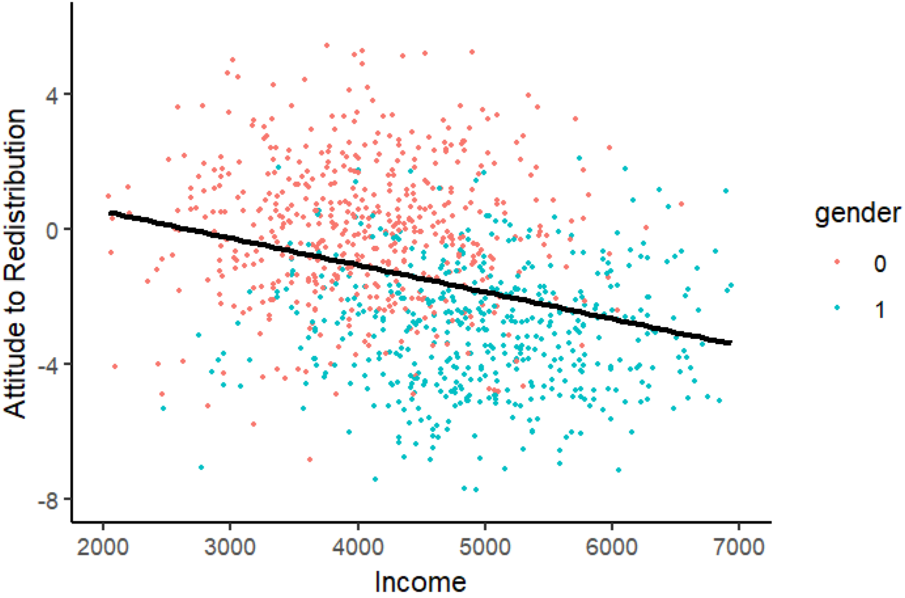

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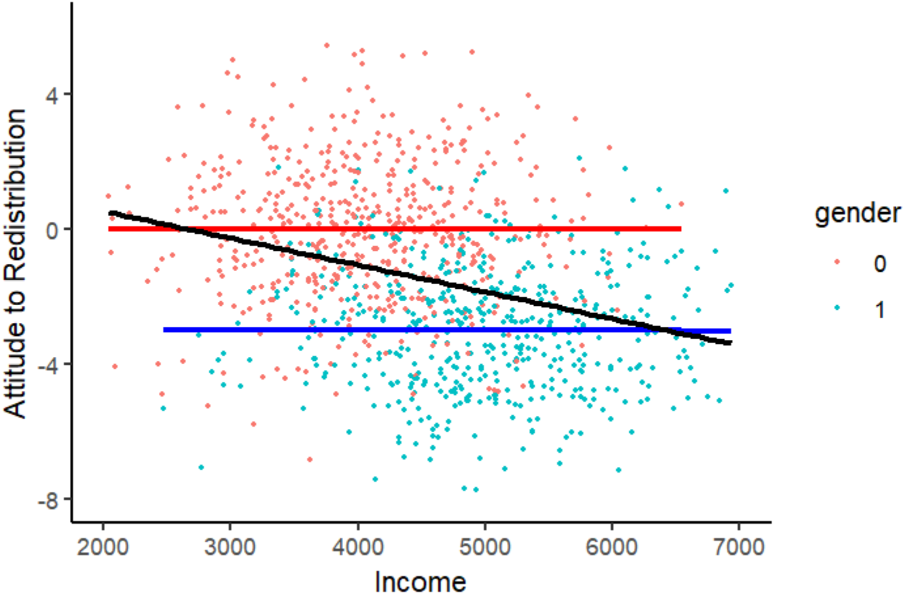

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- Does the ’direction’ of regression matter? I.e. Does regression treat X and Y differently?

Reverse Causation

- Significant regression coefficients just reflect the values in our dataset moving together
- Does the 'direction' of regression matter? I.e. Does regression treat X and Y differently?
- Yes!

	Dependent variable:
	redist
income	−0.011 (0.029)
gender1	−1.201*** (0.058)
Constant	0.589*** (0.038)
Observations	1,000
Note:	*p<0.1; **p<0.05; ***p<0.01

	Dependent variable:
	income
redist	−0.013 (0.034)
gender1	0.993*** (0.069)
Constant	−0.487*** (0.043)
Observations	1,000
Note:	*p<0.1; **p<0.05; ***p<0.01

- Remember, regression measures the *vertical* (not diagonal) distances to the regression line
  - It minimizes the prediction errors for Y
- But that doesn't mean it identifies the direction of causation!

Selection Bias

- There are four selection risks:
  1. **Selection into existence**
  2. **Selection into survival**
  3. **Selection into the dataset**
  4. **Selection into treatment**
- In each case, we don't see the *full* relationship between X and Y
- So our regression estimates are biased

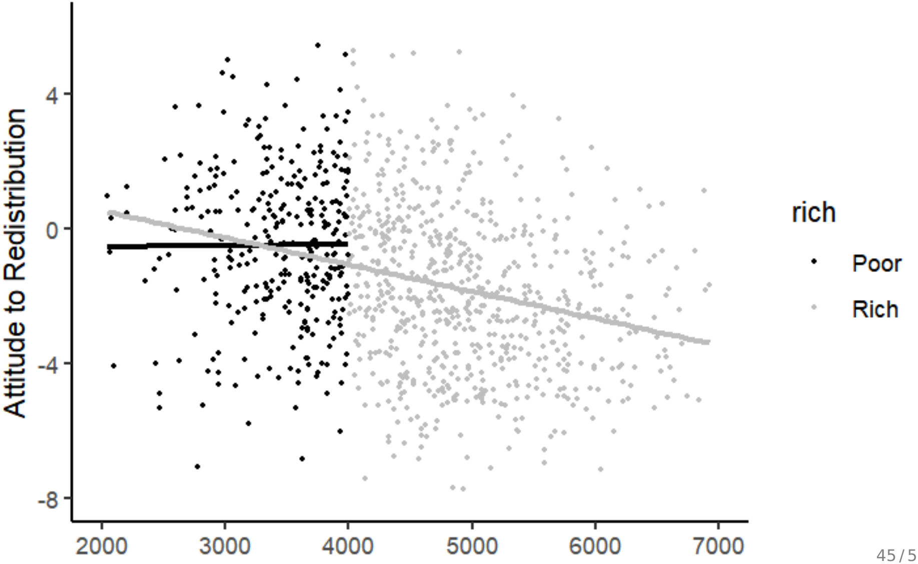
Selection Bias

- Imagine we do not see 'rich' units with high income

```
## Error in FUN(X[[i]], ...):  object 'rich' not
found
```

Selection Bias

- Imagine we do not see 'rich' units with high income

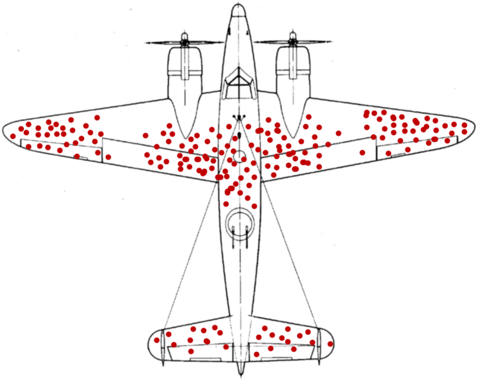


Selection Bias

- There are four selection risks:
  1. **Selection into existence:**
     - Where do units (eg. political parties) come from?
     - Probably only parties that have a chance of success are formed
     - Does forming a party cause electoral success? Not for most people!

Selection Bias

- There are four selection risks:
  2. **Selection into survival:**
     - Certain types of units disappear, so the units we see don't tell the full story



- Where would additional armour protect bombers?
- Returned bombers got hit
- But we do not know where *bombers that did not return* got hit

Selection Bias

- There are four selection risks:
  3. **Selection into the dataset:**
     - Our dataset may not be representative
     - Only units with particular values of X and Y enter the dataset
     - Eg. If survey respondents who refuse are different from those who respond

Selection Bias

- There are four selection risks:
  4. **Selection into treatment:**
     - All units are in our dataset, but they *choose* their treatment value
     - Who chooses treatment? Those with the most to benefit, i.e. depending on Y!
     - Applying treatment to the others would probably have a very different effect

Measurement Bias

- What happens if we measure our variables wrongly?

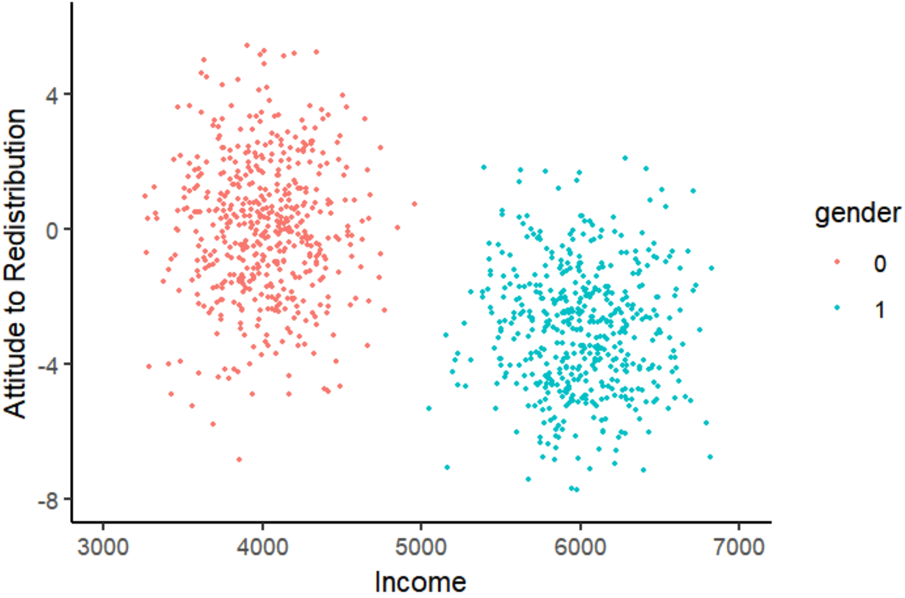
Effects of Measurement Error

	Measured with Bias	Measured with Random Noise
Outcome Variable	Effect biased	No bias but wider standard errors
Treatment Variable	Effect biased	Effect biased to zero

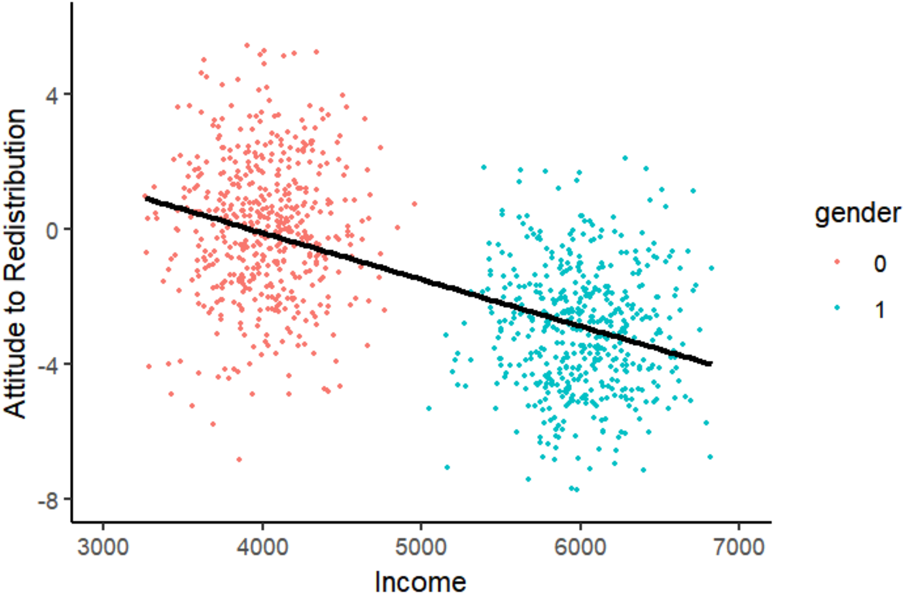
Lack of Overlap

- Regression normally helps us pick appropriate comparisons
  - Eg. Comparing only among men, what is the effect of income on attitudes to redistribution?
- But what if there are no women with high income?
- Regression *creates* comparisons for us
  - How? That's where the functional for of the regression comes in
  - A linear regression interpolates/extrapolates *linearly* to 'create' comparison cases
- Lack of overlap probably means we *cannot* explain outcomes with this data

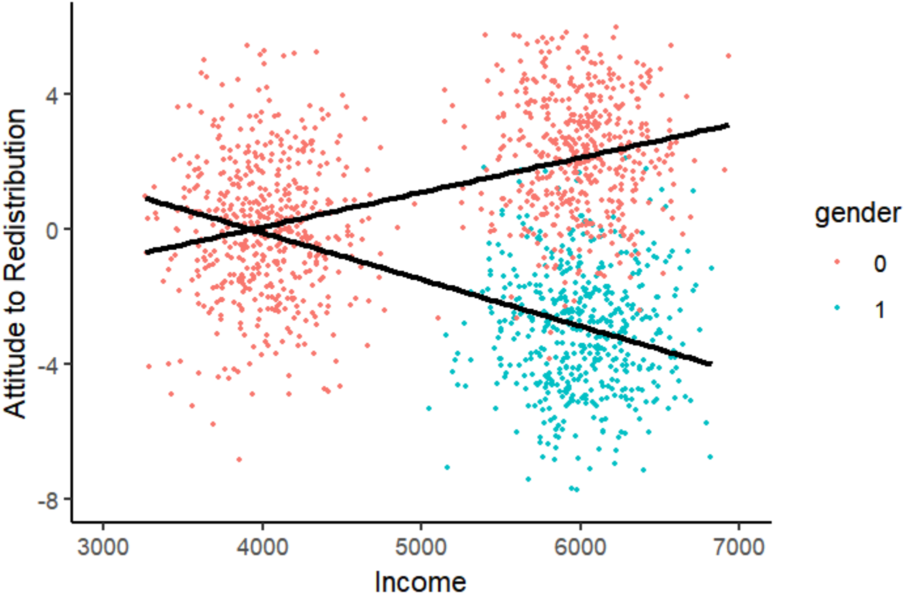
Lack of Overlap



Lack of Overlap



Lack of Overlap



Lack of Overlap

- With more than a few variables, lack of overlap is *guaranteed*
- 6 variables with 10 categories each = $10^6 = 1,000,000$ possibilities, and a sample of maybe 5,000?
- Common datasets have 0% counterfactuals present in the data (King 2006)
    - How many 45 year-old female accountants with a PhD and a cat who live in Centro are there?
    - And we need some that are low-income and some that are high-income
- A problem of **multi-dimensionality**
- And of **model dependence** - our results depend on the functional form in our regression model