

# FLS 6441 - Methods III: Explanation and Causation

Week 9 - Controlling for Confounding

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April 2020

# Classification of Research Designs

		<b>Independence of Treatment Assignment</b>	<b>Researcher Controls Treatment Assignment?</b>
<b>Controlled Experiments</b>	Field Experiments	✓	✓
	Survey and Lab Experiments	✓	✓
<b>Natural Experiments</b>	Natural Experiments	✓	
	Instrumental Variables	✓	
	Discontinuities	✓	
<b>Observational Studies</b>	Difference-in-Differences		
	Controlling for Confounding		
	Matching		
	Comparative Cases and Process Tracing		

# Section 1

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- ▶ For cross-sectional observational studies, the next-best alternative is...
- ▶ Controls!



## Controlling for Confounding

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  - ▶ **Treatment is *associated* with higher values of the Outcome...for units with the same values of  $X$**
- ▶ **What we don't yet know:** When does controlling allow us to say:
  - ▶ **Treatment *causes* higher values of the Outcome?**

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  - ▶ We have to make an argument and provide supporting evidence

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- Why does controlling for confounders help provide conditional independence?

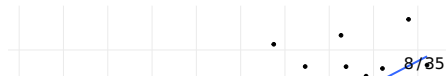
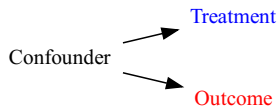
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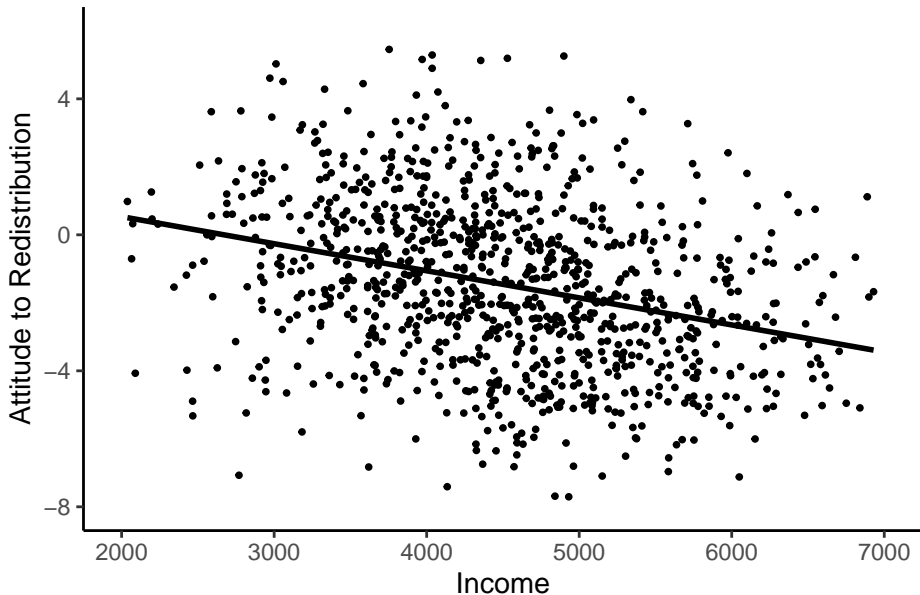
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- ▶ Why does controlling for confounders help provide conditional independence?
- ▶ We need to know what problem - what bias - confounders create:
  - ▶ The problem is of 'fake correlations' -  $D$  and  $Y$  look like they're related, even though treatment does not affect the outcome.
- ▶ Controlling *removes these fake correlations* by only comparing  $D$  and  $Y$  for units with the same value of  $X$

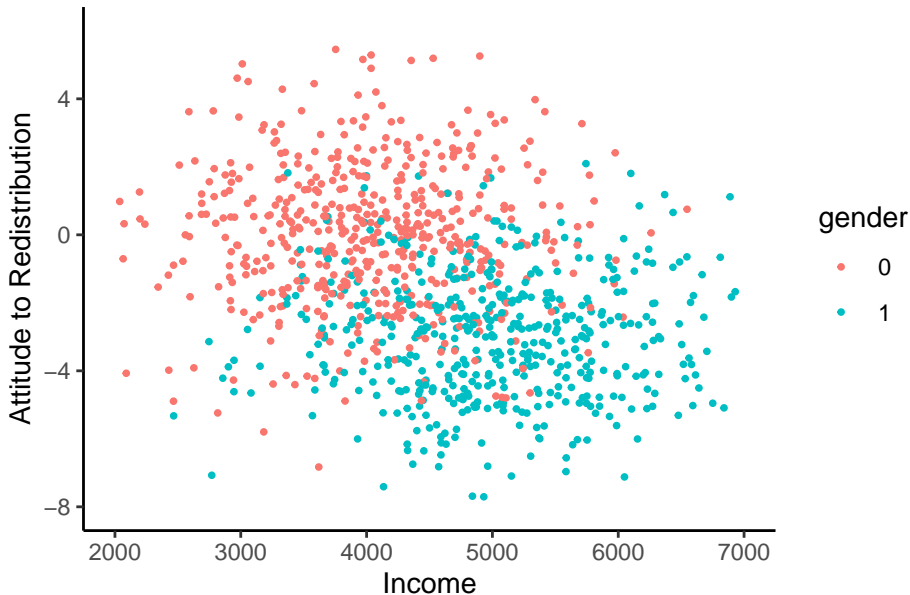
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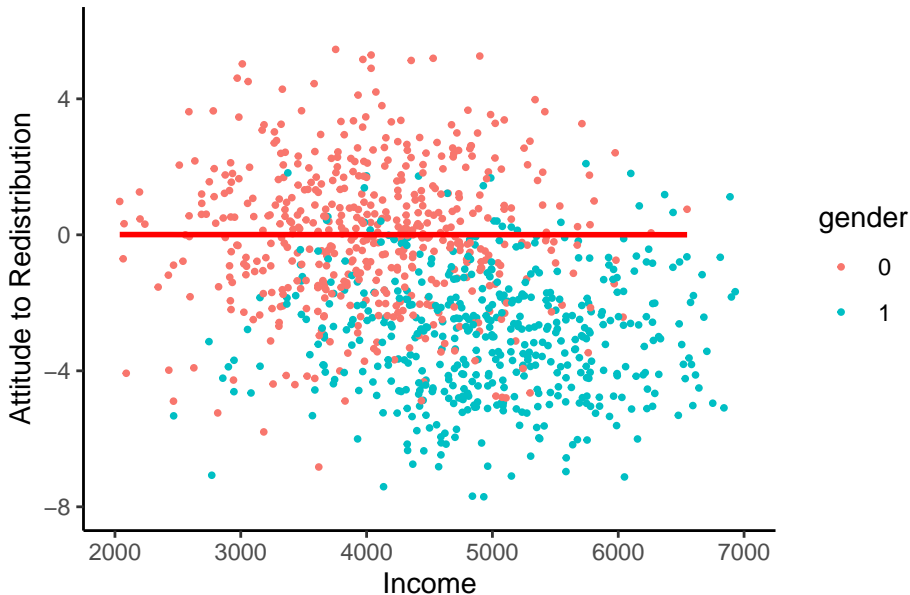


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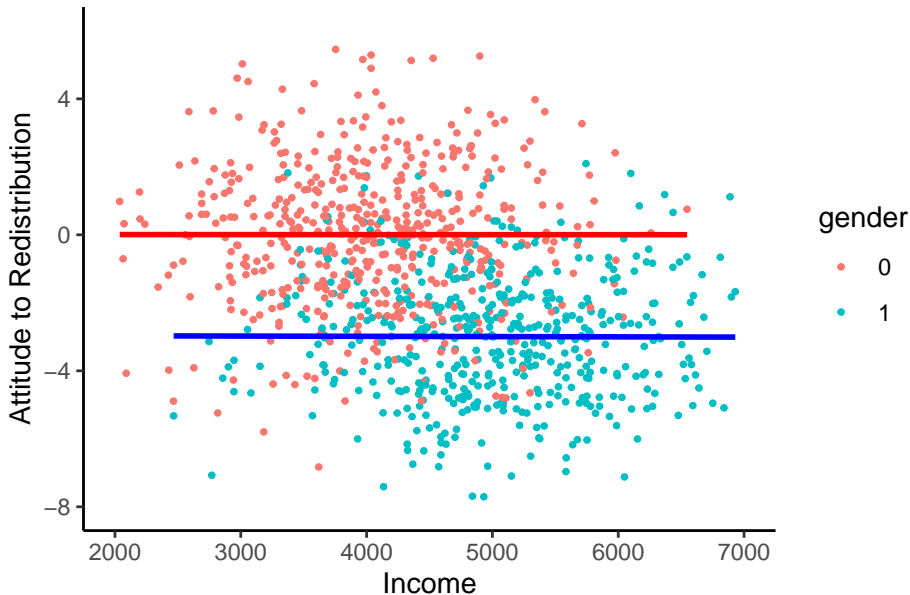




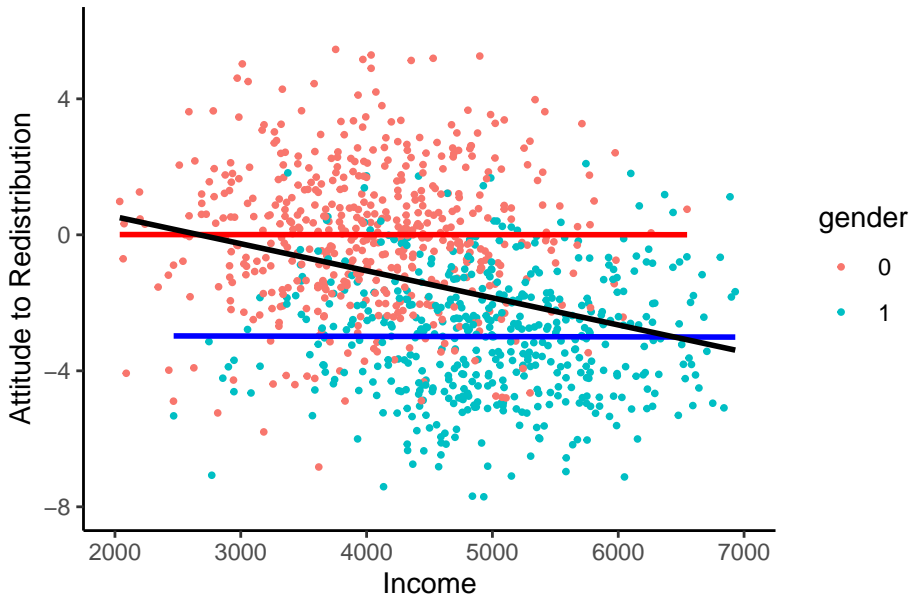
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$$\beta_{wrong} = \beta_{true} + \gamma\delta$$

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  - ▶ Then, within each group, the confounder is **constant** and can't affect the relationship between  $D$  and  $Y$ .
  - ▶ We have **created balance** between the treated and control groups on the confounder

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## Section 2

# Which Variables to Control For

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  - ▶ No circular loops!

## Causal Diagrams (DAGs)

Treatment → Outcome

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- ▶ Causation is like **Water**, flowing along the graph
  - ▶ We want to focus on one 'flow' of causation from treatment to outcomes
  - ▶ Avoiding mixing with the other flows of causation in the network

# Causal Diagrams (DAGs)



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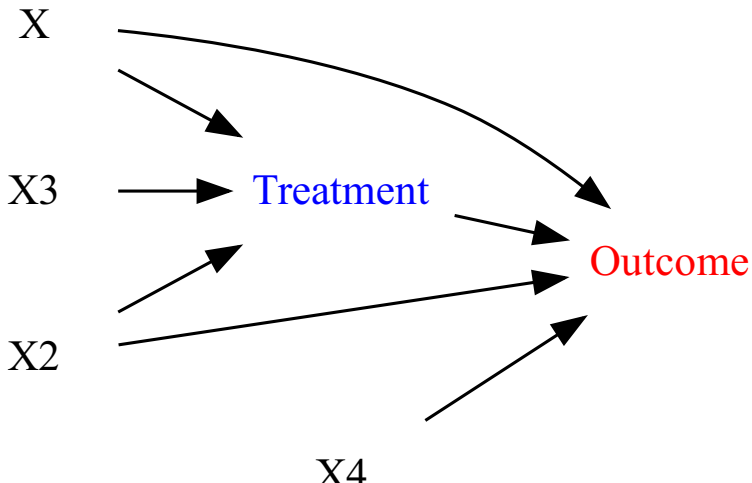
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  - ▶ Include these as control variables in our regression

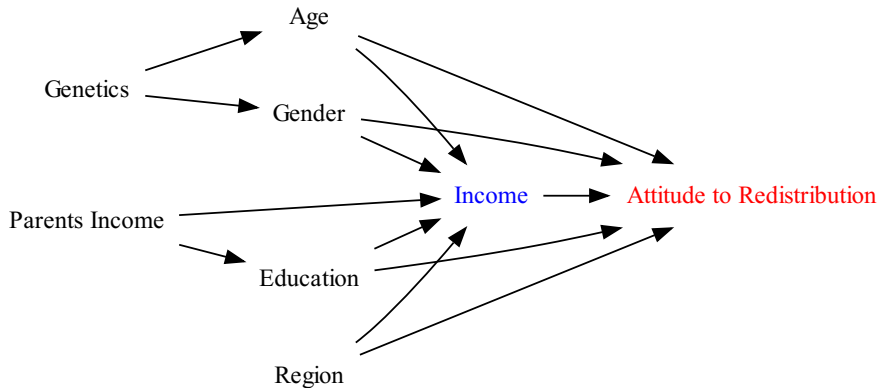
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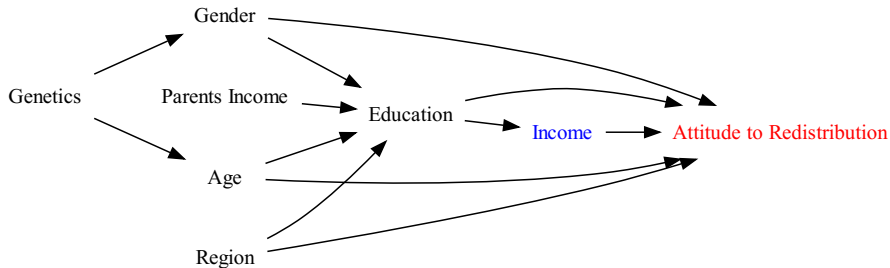


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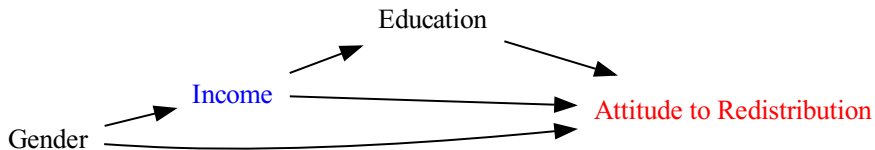
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  - ▶ Controlling for them changes the definition of the causal effect we are estimating

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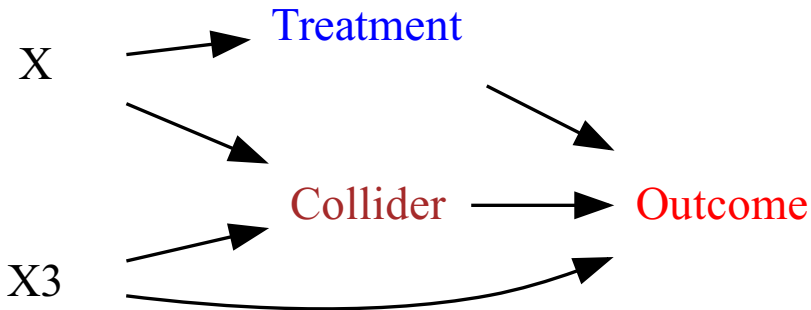
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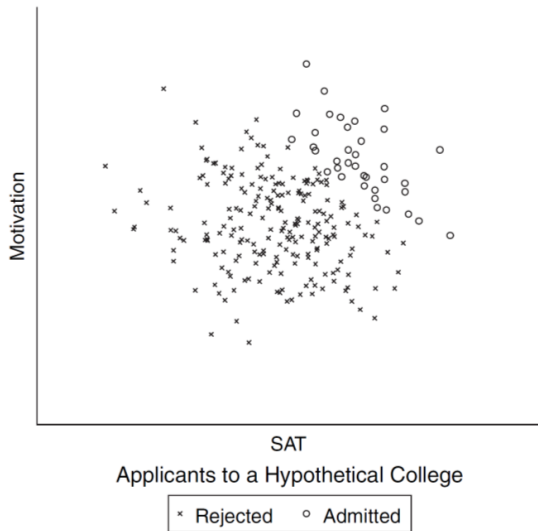
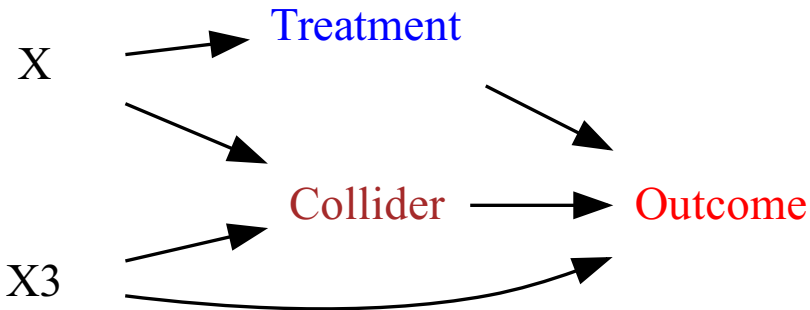


Figure 3.4: Simulation of conditional dependence within values of a collider variable.

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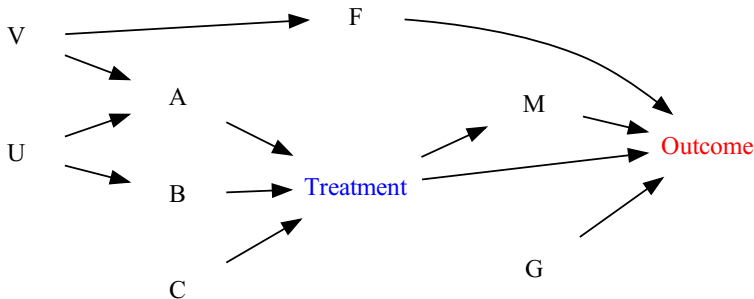
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4. Find a minimum set of variables that blocks all remaining back-door paths
5. Double-check your minimum set of control variables does not contain any post-treatment or collider variables



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    - ▶ In practice, know when your variables were measured

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    - ▶ In practice, variables which theory and past studies identify as potential confounders
  2. Exclude any variables that are **post-treatment**
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## Causal Diagrams (DAGs)

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  3. Exclude any variables that are **colliders**
    - ▶ In practice, don't include unnecessary controls