Mathematics Mock Interview

1. Evaluate i^{n^2+n}, where n takes positive integer values.

2. i) Sketch $y = x^3 - x$
ii) Sketch $y = x^5 - x$
iii) Consider the graph of an nth degree polynomial. How many turning points could it have?

3. Consider $\frac{1}{q}$ where q is a positive integer.
i) Under what conditions will the decimal expansion of $\frac{1}{q}$ terminate?
ii) If the decimal expansion of $\frac{1}{q}$ is periodic, what is the maximum length of the repeating block?

4. i) Sketch $y = \frac{\cos(x^2)}{x^2}$
ii) Sketch $y = \frac{1-\cos(x)}{1+\cos(x)}$

5. Let $f(x)$ be a non-constant function satisfying the functional equation $f(x+y) = f(x)f(y)$.
i) Show that $f(n) = a^n$, where $a = f(1)$, for all integers n.
ii) Show also that the same holds for all rational numbers and that $a > 0$.

Notes

Q1. Factorising the quadratic and considering the remainder upon division by 4 will help justify the pattern seen.

Q2. Factorising fully allows the sketches to be deduced in a straightforward manner. The sketches should provide food for thought for the last part. Considering the degree of the derivative and the sign of the polynomial at the extremes of the x-axis should lead to the correct result.

Q3. Considering small values of q may help but ultimately the student must decide what properties q must have if the decimal is to terminate. Working backwards from a terminating decimal to a fraction would be useful. For the second part considering writing $\frac{1}{7}$ as a decimal may help. Looking closely at the remainders each time a division is performed within the long division calculation is vital.

Q4. The first function is even and tends to zero as x tends to infinity in both the positive and negative directions. As x tends to zero (from either side of the origin), the function tends to infinity. The function will oscillate but the oscillations will become smaller and smaller and the distance between turning points will decrease due to the non-linear argument of the cosine function. The second function has many discontinuities; each time cosine takes the value -1 the function will be undefined. The function is periodic and even.

Q5. This is the most challenging and will take time, only start if a student finds other questions straightforward.
Writing n as $(n-1)+1$ and substituting in to the functional equation will lead to the first result upon repetition.
Writing 1 as $(1-\frac{1}{q})+\frac{1}{q}$ substituting in to the functional equation will eventually lead to $f\left(\frac{1}{q}\right) = f(1)^{\frac{1}{q}}$.
Writing $\frac{p}{q}$ as $\frac{p-1}{q}+\frac{1}{q}$ and substituting in to the functional equation will eventually lead to $f\left(\frac{p}{q}\right) = f(1)^{\frac{p}{q}}$.
Writing x as $\frac{x}{2}+\frac{x}{2}$ and substituting in to the functional equation gives $f(x) = [f(\frac{x}{2})]^2$ which leads to the final result.