

# Tensor Symmetry (43)

$n = 2$	$n = 3$	$n = 4$
$T_{ij} = T_{ji}$	$T_{ijk} = T_{jik} = T_{kji} = T_{ikj}$	$T_{ijkl} = T_{jikl} = T_{kjil} = T_{ljki} = T_{ikjl} = T_{iljk} = T_{ijlk}$
$S_{ij} = -S_{ji}$	$S_{ijk} = -S_{jik} = -S_{kji} = -S_{ikj}$	$S_{ijkl} = -S_{jikl} = -S_{kjil} = -S_{ljki} = -S_{ikjl} = -S_{iljk} = -S_{ijlk}$
$e^{ij}$ $e_{ij}$	$e^{ijk}$ $e_{ijk}$	$e^{ijkl}$ $e_{ijkl}$

Determinants (44)

$n = 2$	$n = 3$	$n = 4$
$Ae^{rs} = e^{ij}a_i^ra_j^s$ $Ae_{rs} = e^{ij}a_{ir}a_{js}$ $Ae^{rs} = e_{ij}a^{ir}a^{js}$	$Ae^{rst} = e^{ijk}a_i^ra_j^sa_k^t$ $Ae_{rst} = e^{ijk}a_{ir}a_{js}a_{kt}$ $Ae^{rst} = e_{ijk}a^{ir}a^{js}a^{kt}$	$Ae^{rstu} = e^{ijkl}a_i^ra_j^sa_k^ta_l^u$ $Ae_{rstu} = e^{ijkl}a_{ir}a_{js}a_{kt}a_{lu}$ $Ae^{rstu} = e_{ijkl}a^{ir}a^{js}a^{kt}a^{lu}$
$A = \frac{1}{2!}e^{ij}e_{rs}a_i^ra_j^s$ $A = \frac{1}{2!}e^{ij}e^{rs}a_{ir}a_{js}$ $A = \frac{1}{2!}e_{ij}e_{rs}a^{ir}a^{js}$	$A = \frac{1}{3!}e^{ijk}e_{rst}a_i^ra_j^sa_k^t$ $A = \frac{1}{3!}e^{ijk}e^{rst}a_{ir}a_{js}a_{kt}$ $A = \frac{1}{3!}e_{ijk}e_{rst}a^{ir}a^{js}a^{kt}$	$A = \frac{1}{4!}e^{ijkl}e_{rstu}a_i^ra_j^sa_k^ta_l^u$ $A = \frac{1}{4!}e^{ijkl}e^{rstu}a_{ir}a_{js}a_{kt}a_{lu}$ $A = \frac{1}{4!}e_{ijkl}e_{rstu}a^{ir}a^{js}a^{kt}a^{lu}$

Delta System (45)

$n = 2$	$n = 3$	$n = 4$
$\delta_{rs}^{ij} = \begin{vmatrix} \delta_r^i & \delta_s^i \\ \delta_r^j & \delta_s^j \end{vmatrix}$	$\delta_{rst}^{ijk} = \begin{vmatrix} \delta_r^i & \delta_s^i & \delta_t^i \\ \delta_r^j & \delta_s^j & \delta_t^j \\ \delta_r^k & \delta_s^k & \delta_t^k \end{vmatrix}$	$\delta_{rstu}^{ijkl} = \begin{vmatrix} \delta_r^i & \delta_s^i & \delta_t^i & \delta_u^i \\ \delta_r^j & \delta_s^j & \delta_t^j & \delta_u^j \\ \delta_r^k & \delta_s^k & \delta_t^k & \delta_u^k \\ \delta_r^l & \delta_s^l & \delta_t^l & \delta_u^l \end{vmatrix}$
$\delta_{rj}^{ij} = \delta_r^i$  $\delta_i^i = 2$	$\delta_{rsk}^{ijk} = \delta_{rs}^{ij}$  $\delta_{rj}^{ij} = 2\delta_r^i$  $\delta_i^i = 3$	$\delta_{rstl}^{ijkl} = \delta_{rst}^{ijk}$  $\delta_{rsk}^{ijk} = 2\delta_{rs}^{ij}$  $\delta_{rj}^{ij} = 3\delta_r^i$  $\delta_i^i = 4$
$\delta_{rs}^{ij} = e^{ij}e_{rs}$	$\delta_{rst}^{ijk} = e^{ijk}e_{rst}$	$\delta_{rstu}^{ijkl} = e^{ijkl}e_{rstu}$
$A = \frac{1}{2!}\delta_{rs}^{ij}a_i^ra_j^s$	$A = \frac{1}{3!}\delta_{rst}^{ijk}a_i^ra_j^sa_k^t$	$A = \frac{1}{4!}\delta_{rstu}^{ijkl}a_i^ra_j^sa_k^ta_l^u$

Relative Tensors (46)

$n = 2$	$n = 3$	$n = 4$
$e^{i'j'} = J e^{ij} J_i^{i'} J_j^{j'}$ $e_{i'j'} = J^{-1} e_{ij} J_i^i J_j^j$	$e^{i'j'k'} = J e^{ijk} J_i^{i'} J_j^{j'} J_k^{k'}$ $e_{i'j'k'} = J^{-1} e_{ijk} J_i^i J_j^j J_k^k$	$e^{i'j'k'l'} = J e^{ijkl} J_i^{i'} J_j^{j'} J_k^{k'} J_l^{l'}$ $e_{i'j'k'l'} = J^{-1} e_{ijkl} J_i^i J_j^j J_k^k J_l^l$
$ a_{\cdot'}'  =  a_{\cdot} $ $ a_{\cdot'}{}_{\cdot'}  = J^2  a_{\cdot\cdot} $ $ a_{\cdot'}{}_{\cdot'}{}_{\cdot'}  = J^{-2}  a_{\cdot\cdot\cdot} $	$ a_{\cdot'}'  =  a_{\cdot} $ $ a_{\cdot'}{}_{\cdot'}  = J^2  a_{\cdot\cdot} $ $ a_{\cdot'}{}_{\cdot'}{}_{\cdot'}  = J^{-2}  a_{\cdot\cdot\cdot} $	$ a_{\cdot'}'  =  a_{\cdot} $ $ a_{\cdot'}{}_{\cdot'}  = J^2  a_{\cdot\cdot} $ $ a_{\cdot'}{}_{\cdot'}{}_{\cdot'}  = J^{-2}  a_{\cdot\cdot\cdot} $

Cofactors (47)

$n = 2$	$n = 3$	$n = 4$
$A_r^i = \frac{\partial A}{\partial a_i^r} = \frac{1}{1!} \delta_{rs}^ij a_j^s$ $A^{ir} = \frac{\partial A}{\partial a_{ri}} = \frac{1}{1!} e^{ij} e^{rs} a_{sj}$ $A_{ir} = \frac{\partial A}{\partial a^{ri}} = \frac{1}{1!} e_{ij} e_{rs} a^{sj}$	$A_r^i = \frac{\partial A}{\partial a_i^r} = \frac{1}{2!} \delta_{rst}^{ijk} a_j^s a_k^t$ $A^{ir} = \frac{\partial A}{\partial a_{ri}} = \frac{1}{2!} e^{ijk} e^{rst} a_{sj} a_{tk}$ $A_{ir} = \frac{\partial A}{\partial a^{ri}} = \frac{1}{2!} e_{ijk} e_{rst} a^{sj} a^{tk}$	$A_r^i = \frac{\partial A}{\partial a_i^r} = \frac{1}{3!} \delta_{rstu}^{ijkl} a_j^s a_k^t a_l^u$ $A^{ir} = \frac{\partial A}{\partial a_{ri}} = \frac{1}{3!} e^{ijkl} e^{rstu} a_{sj} a_{tk} a_{ul}$ $A_{ir} = \frac{\partial A}{\partial a^{ri}} = \frac{1}{3!} e_{ijkl} e_{rstu} a^{sj} a^{tk} a^{ul}$
$A_r^i a_m^r = A \delta_m^i$ $A^{ir} a_{rm} = A \delta_m^i$ $A_{ir} a^{rm} = A \delta_i^m$	$A_r^i a_m^r = A \delta_m^i$ $A^{ir} a_{rm} = A \delta_m^i$ $A_{ir} a^{rm} = A \delta_i^m$	$A_r^i a_m^r = A \delta_m^i$ $A^{ir} a_{rm} = A \delta_m^i$ $A_{ir} a^{rm} = A \delta_i^m$

Volume Element (48)

$n = 2$	$n = 3$	$n = 4$
$\sqrt{Z'} =  J \sqrt{Z}$	$\sqrt{Z'} =  J \sqrt{Z}$	$\sqrt{Z'} =  J \sqrt{Z}$
$\frac{\partial \sqrt{Z}}{\partial Z^i} = \sqrt{Z} \Gamma_{mi}^m$	$\frac{\partial \sqrt{Z}}{\partial Z^i} = \sqrt{Z} \Gamma_{mi}^m$	$\frac{\partial \sqrt{Z}}{\partial Z^i} = \sqrt{Z} \Gamma_{mi}^m$

Voss-Weyl Formula (49)

$n = 2$	$n = 3$	$n = 4$
$\operatorname{div} \vec{T} = \nabla_i T^i = \frac{1}{\sqrt{Z}} \frac{\partial}{\partial z^i} (\sqrt{Z} T^i)$	$\operatorname{div} \vec{T} = \nabla_i T^i = \frac{1}{\sqrt{Z}} \frac{\partial}{\partial z^i} (\sqrt{Z} T^i)$	$\operatorname{div} \vec{T} = \nabla_i T^i = \frac{1}{\sqrt{Z}} \frac{\partial}{\partial z^i} (\sqrt{Z} T^i)$
$\nabla^2 f = \nabla_i \nabla^i f = \frac{1}{\sqrt{Z}} \frac{\partial}{\partial z^i} \left( \sqrt{Z} Z^{ij} \frac{\partial f}{\partial z^j} \right)$	$\nabla^2 f = \nabla_i \nabla^i f = \frac{1}{\sqrt{Z}} \frac{\partial}{\partial z^i} \left( \sqrt{Z} Z^{ij} \frac{\partial f}{\partial z^j} \right)$	$\nabla^2 f = \nabla_i \nabla^i f = \frac{1}{\sqrt{Z}} \frac{\partial}{\partial z^i} \left( \sqrt{Z} Z^{ij} \frac{\partial f}{\partial z^j} \right)$

Levi-Civita Symbols (51)

$n = 2$	$n = 3$	$n = 4$
$\varepsilon^{ij} = \frac{1}{\sqrt{Z}} e^{ij}$ $\varepsilon_{ij} = \sqrt{Z} e_{ij}$	$\varepsilon^{ijk} = \frac{1}{\sqrt{Z}} e^{ijk}$ $\varepsilon_{ijk} = \sqrt{Z} e_{ijk}$	$\varepsilon^{ijkl} = \frac{1}{\sqrt{Z}} e^{ijkl}$ $\varepsilon_{ijkl} = \sqrt{Z} e_{ijkl}$
$\delta_{rs}^{ij} = \varepsilon^{ij} \varepsilon_{rs}$	$\delta_{rst}^{ijk} = \varepsilon^{ijk} \varepsilon_{rst}$	$\delta_{rstu}^{ijkl} = \varepsilon^{ijkl} \varepsilon_{rstu}$
$\varepsilon^{ij} Z_{ir} Z_{js} = \varepsilon_{rs}$ $\varepsilon_{ij} Z^{ir} Z^{js} = \varepsilon^{rs}$	$\varepsilon^{ijk} Z_{ir} Z_{js} Z_{kt} = \varepsilon_{rst}$ $\varepsilon_{ijk} Z^{ir} Z^{js} Z^{kt} = \varepsilon^{rst}$	$\varepsilon^{ijkl} Z_{ir} Z_{js} Z_{kt} Z_{lu} = \varepsilon_{rstu}$ $\varepsilon_{ijkl} Z^{ir} Z^{js} Z^{kt} Z^{lu} = \varepsilon^{rstu}$
$\nabla_m \varepsilon^{ij} = 0$ $\nabla_m \varepsilon_{ij} = 0$	$\nabla_m \varepsilon^{ijk} = 0$ $\nabla_m \varepsilon_{ijk} = 0$	$\nabla_m \varepsilon^{ijkl} = 0$ $\nabla_m \varepsilon_{ijkl} = 0$



Cross Product (52)

$n = 2$	$n = 3$	$n = 4$
$W^i = \varepsilon^{ij} U_j$ $W_i = \varepsilon_{ij} U^j$ $\vec{W} = \varepsilon^{ij} U_j \vec{Z}_i$ $= \varepsilon_{ij} U^j \vec{Z}^i$	$W^i = \varepsilon^{ijk} U_j V_k$ $W_i = \varepsilon_{ijk} U^j V^k$ $\vec{W} = \varepsilon^{ijk} U_j V_k \vec{Z}_i = \varepsilon_{ijk} U^j V^k \vec{Z}^i$	$W^i = \varepsilon^{ijkl} U_j V_k S_l$ $W_i = \varepsilon_{ijkl} U^j V^k S^l$ $\vec{W} = \varepsilon^{ijkl} U_j V_k S_l \vec{Z}_i = \varepsilon_{ijkl} U^j V^k S^l \vec{Z}^i$
	$\varepsilon_{ijk} U^j V^k = -\varepsilon_{ijk} V^j U^k$	$\varepsilon_{ijkl} U^j V^k S^l = -\varepsilon_{ijkl} V^j U^k S^l = -\varepsilon_{ijkl} S^j V^k U^l =$ $-\varepsilon_{ijkl} U^j S^k V^l$
$(\varepsilon_{ij} U^j) U^i = 0$	$(\varepsilon_{ijk} U^j V^k) U^i = (\varepsilon_{ijk} U^j V^k) V^i = 0$	$(\varepsilon_{ijkl} U^j V^k S^l) U^i = (\varepsilon_{ijkl} U^j V^k S^l) V^i = (\varepsilon_{ijkl} U^j V^k S^l) S^i = 0$
$ \vec{W}  =  \vec{U} $	$ \vec{W} ^2 =  \vec{U} ^2  \vec{V} ^2 - (\vec{U} \cdot \vec{V})^2$ $ \vec{W}  =  \vec{U}   \vec{V}  \sin \theta$	$ \vec{W} ^2 =  \vec{U} ^2  \vec{V} ^2  \vec{S} ^2 -  \vec{U} ^2 (\vec{V} \cdot \vec{S})^2 -  \vec{V} ^2 (\vec{S} \cdot \vec{U})^2$ $-  \vec{S} ^2 (\vec{U} \cdot \vec{V})^2 + 2(\vec{U} \cdot \vec{S})(\vec{V} \cdot \vec{U})(\vec{S} \cdot \vec{V})$
$U^i (\varepsilon_{ij} V^j) = V^i (\varepsilon_{ij} U^j)$	$U^i (\varepsilon_{ijk} V^j W^k) = V^i (\varepsilon_{ijk} W^j U^k) = W^i (\varepsilon_{ijk} U^j V^k)$	$U^i (\varepsilon_{ijkl} V^j W^k S^l) = V^i (\varepsilon_{ijkl} W^j S^k U^l) = W^i (\varepsilon_{ijkl} S^j U^k V^l)$ $= S^i (\varepsilon_{ijkl} U^j V^k W^l)$

Curl (53)

$n = 2$	$n = 3$	$n = 4$
	$V^i = \varepsilon^{ijk} \nabla_j U_k$ $V_i = \varepsilon_{ijk} \nabla^j U^k$ $\vec{V} = \varepsilon^{ijk} \nabla_j U_k \vec{Z}_i = \varepsilon_{ijk} \nabla^j U^k \vec{Z}^i$	
	$V^i = \varepsilon^{ijk} \frac{\partial U_k}{\partial Z^j}$	

Integration (54)

$n = 2$	$n = 3$	$n = 4$
$d\Omega = \sqrt{Z} dZ^1 dZ^2$	$d\Omega = \sqrt{Z} dZ^1 dZ^2 dZ^3$	$d\Omega = \sqrt{Z} dZ^1 dZ^2 dZ^3 dZ^4$
$\int_{\Omega} f d\Omega = \iint_{\Omega} f(Z) \sqrt{Z} dZ^1 dZ^2$	$\int_{\Omega} f d\Omega = \iiint_{\Omega} f(Z) \sqrt{Z} dZ^1 dZ^2 dZ^3$	$\int_{\Omega} f d\Omega = \int \iiint_{\Omega} f(Z) \sqrt{Z} dZ^1 dZ^2 dZ^3 dZ^4$