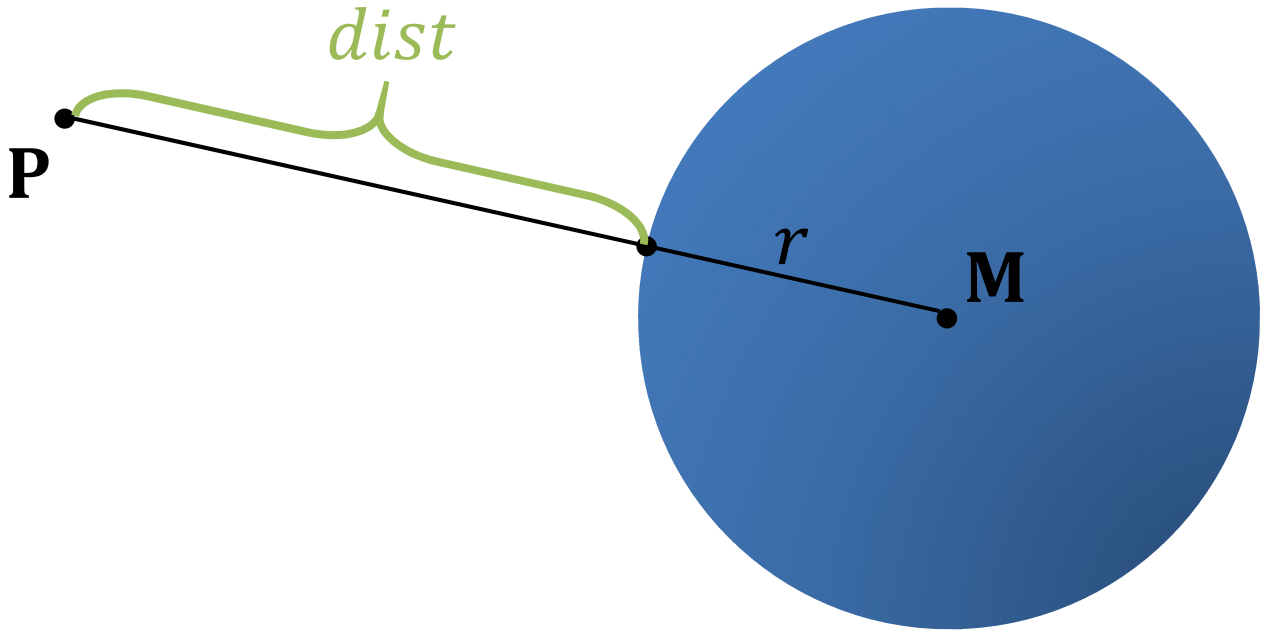
Raymarching Distance Fields

Distance Functions

- Distance function sphere

$$\|\mathbf{P}-\mathbf{M}\|-r = dist_{signed}(\mathbf{P}) = dist_s(\mathbf{P})$$

$$\max(0, \|\mathbf{P}-\mathbf{M}\|-r) = dist_{unsigned}(\mathbf{P}) = dist_u(\mathbf{P})$$

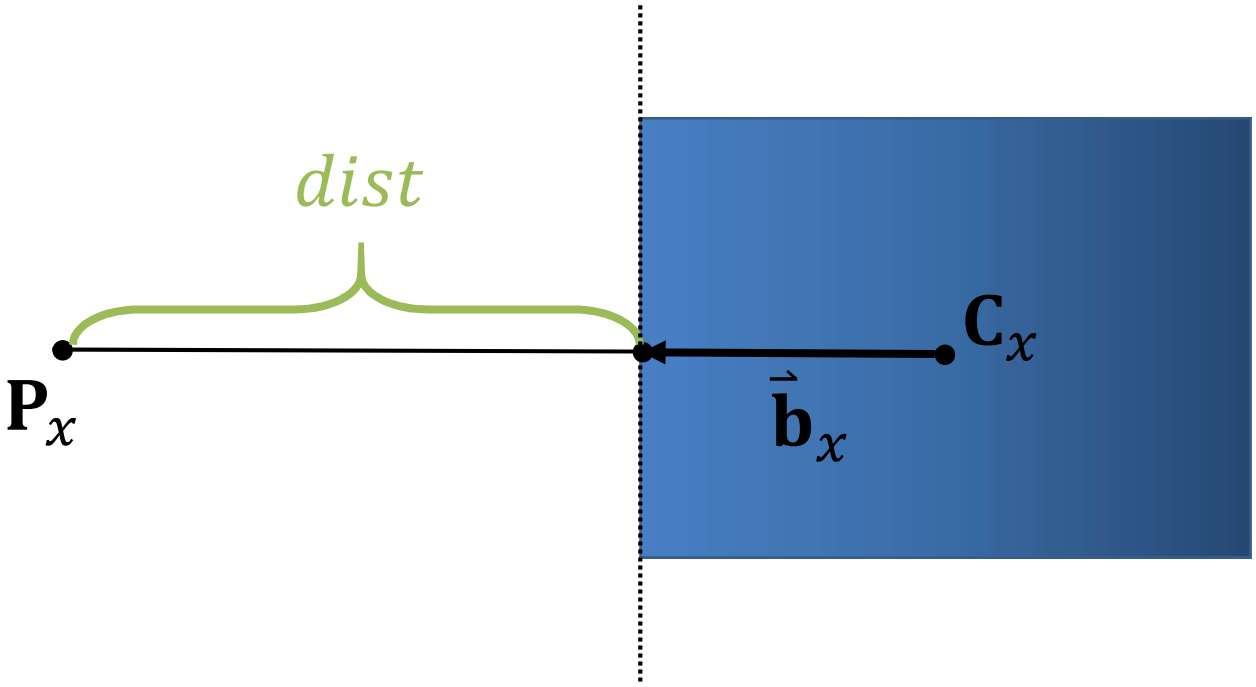


Distance Functions

- Unsigned distance function box – x-direction

  $dist_u(\mathbf{P}_x) = \max\left(abs(\mathbf{P}_x - \mathbf{C}_x) - \vec{\mathbf{b}}_x, 0\right)$

  were $abs(\vec{\mathbf{x}})$ is the component-wise absolute value of $\vec{\mathbf{x}}$

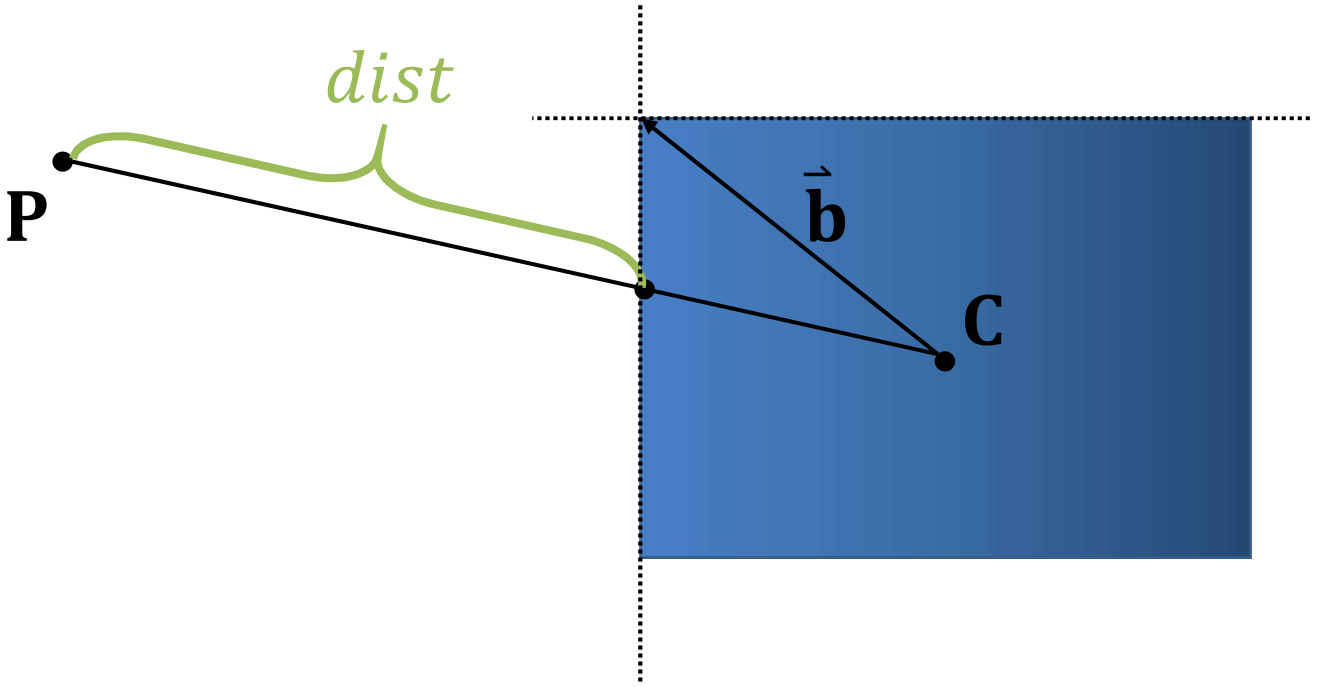


Distance Functions

- Unsigned distance function box

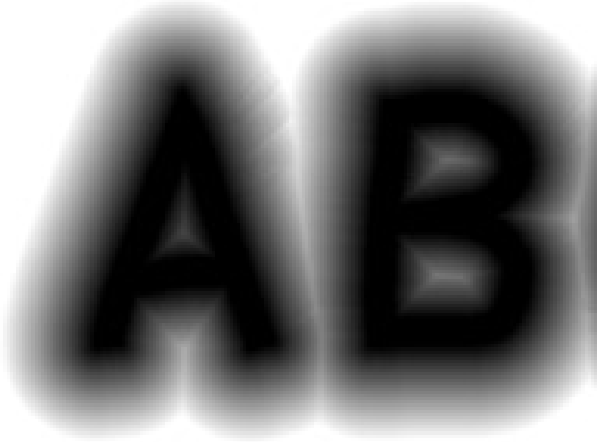
  $dist_u(\mathbf{P}) = \left\|\max\left(abs(\mathbf{P} - \mathbf{C}) - \vec{\mathbf{b}}, \vec{\mathbf{0}}\right)\right\|$

  were $\|\vec{\mathbf{x}}\|$ is the vector absolute value of $\vec{\mathbf{x}}$

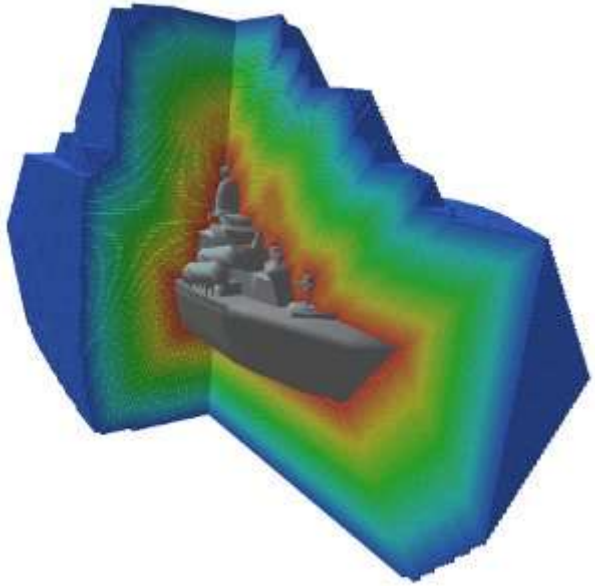


Distance Fields

$\mathbb{R}^2 \rightarrow dist(\mathbb{R}^2)$

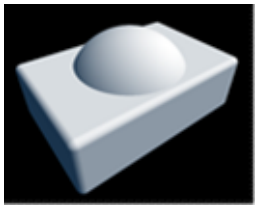
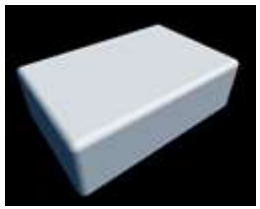


$\mathbb{R}^3 \rightarrow dist(\mathbb{R}^3)$

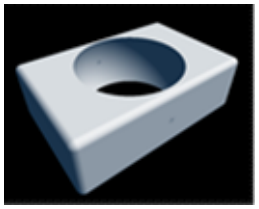
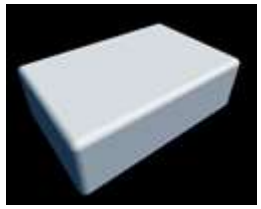


Operations on Distance Fields

- Given $dist_1(\mathbb{R}^3)$ and $dist_2(\mathbb{R}^3)$

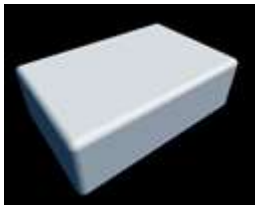
 $= union\left($  ,  $\right)$

- The union is $\min(dist_1(\mathbb{R}^3), dist_2(\mathbb{R}^3))$

 $= substraction\left($  ,  $\right)$

- The substraction is $\max(-dist_1(\mathbb{R}^3), dist_2(\mathbb{R}^3))$

Operations on Distance Fields

- Given $dist(\mathbb{R}^3) =$ 

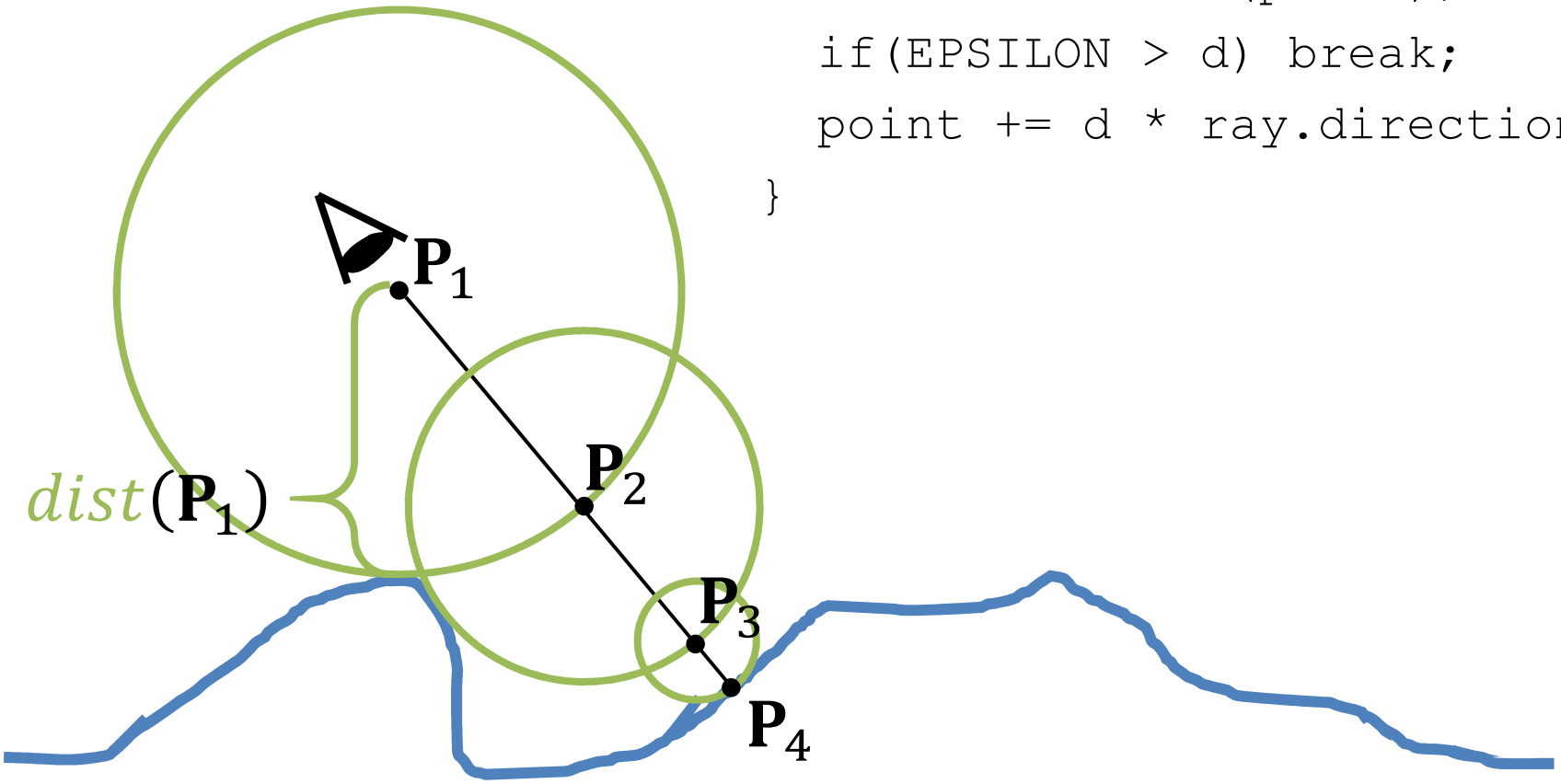
   $= dist(repeat(\mathbb{R}^3))$

- Repeat is $\mathrm{mod}\left(\mathbf{P}, \vec{\mathbf{b}}\right) - \frac{1}{2}\vec{\mathbf{b}}$
  were $\mathrm{mod}(\vec{\mathbf{a}}, \vec{\mathbf{c}})$ is component-wise $\vec{\mathbf{a}}$ modulo $\vec{\mathbf{c}}$

Raymarching Distance Fields

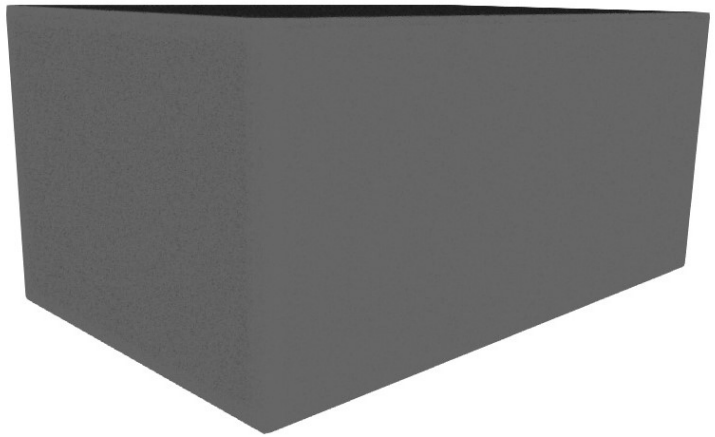
- $dist(\mathbf{P}_i)$

```
vec3 point = ray.origin;
while(--maxSteps) {
  float d = dist(point);
  if(EPSILON > d) break;
  point += d * ray.direction;
}
```



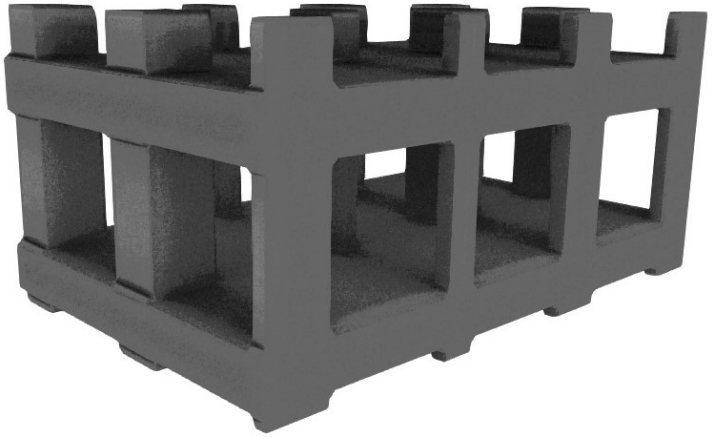
A Box

```
Box(pos, size)
{
  a = abs(pos-size) - size;
  return max(a.x,a.y,a.z);
}
```



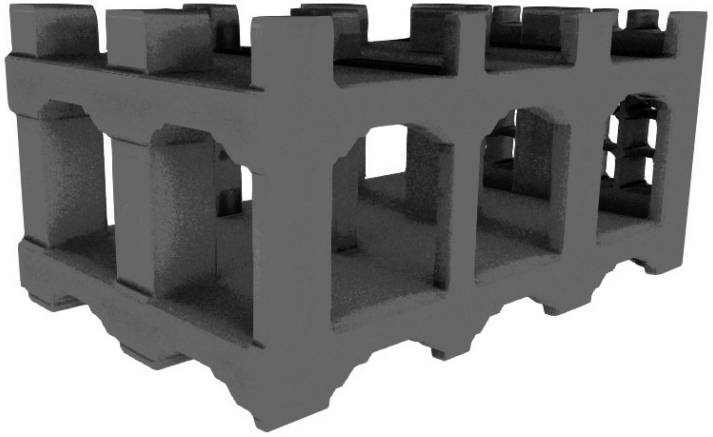
Cutting with Booleans

```
d = Box(pos)
c = fmod(pos * A, B)
subD = max(c.y,min(c.y,c.z))
d = max(d, -subD)
```



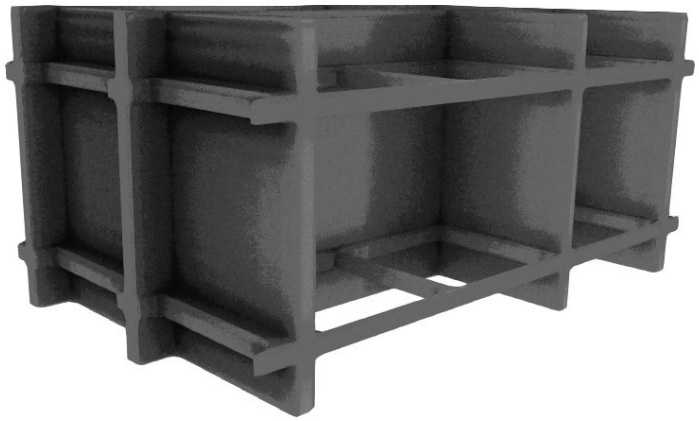
More Booleans

```
d = Box(pos)
c = fmod(pos * A, B)
subD = max(c.y,min(c.y,c.z))
subD = min(subD,cylinder(c))
subD = max(subD, Windows())
d = max(d, -subD)
```



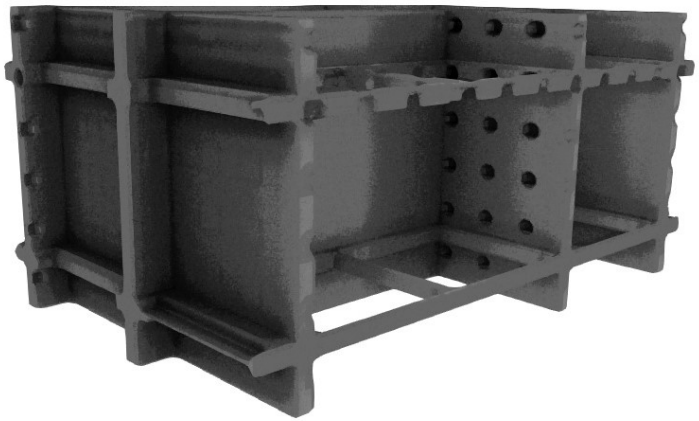
Repeated Booleans

```
d = Box(pos)
e = fmod(pos + N, M)
floorD = Box(e)
d = max(d, -floorD)
```



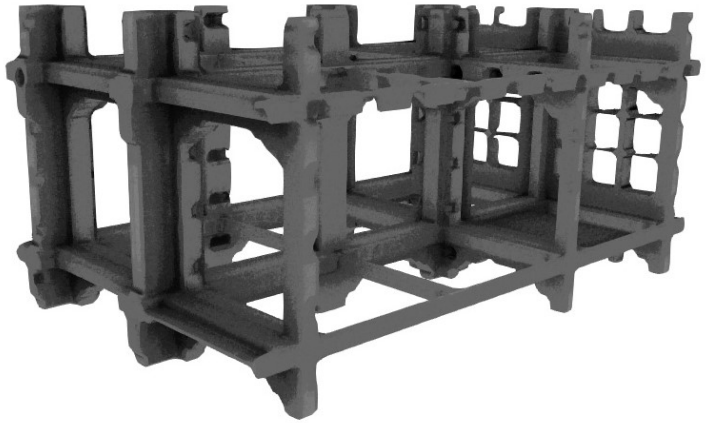
Cutting Holes

```
d = Box(pos)
e = fmod(pos + N, M)
floorD = Box(e)
floorD = min(floorD,holes())
d = max(d, -floorD)
```



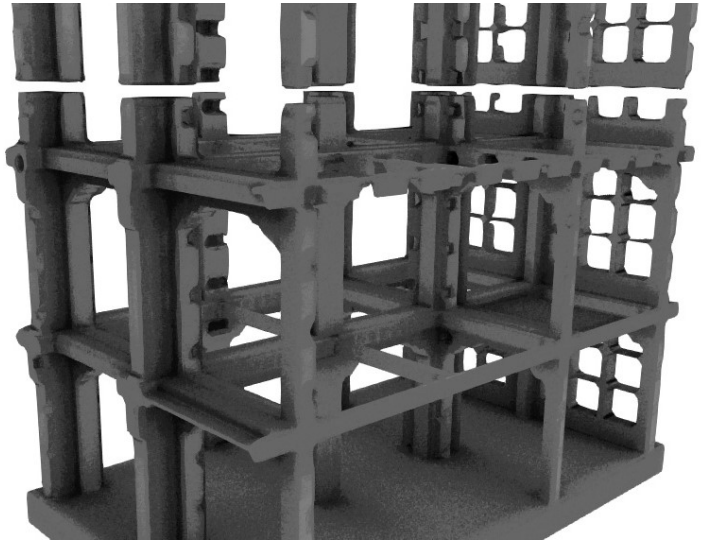
Combined Result

```
d = Box(pos)
c = fmod(pos * A, B)
subD = max(c.y,min(c.y,c.z))
subD = min(subD,cylinder(c))
subD = max(subD, Windows())
e = fmod(pos + N, M)
floorD = Box(e)
floorD = min(floorD,holes())
d = max(d, -subD)
d = max(d, -floorD)
```



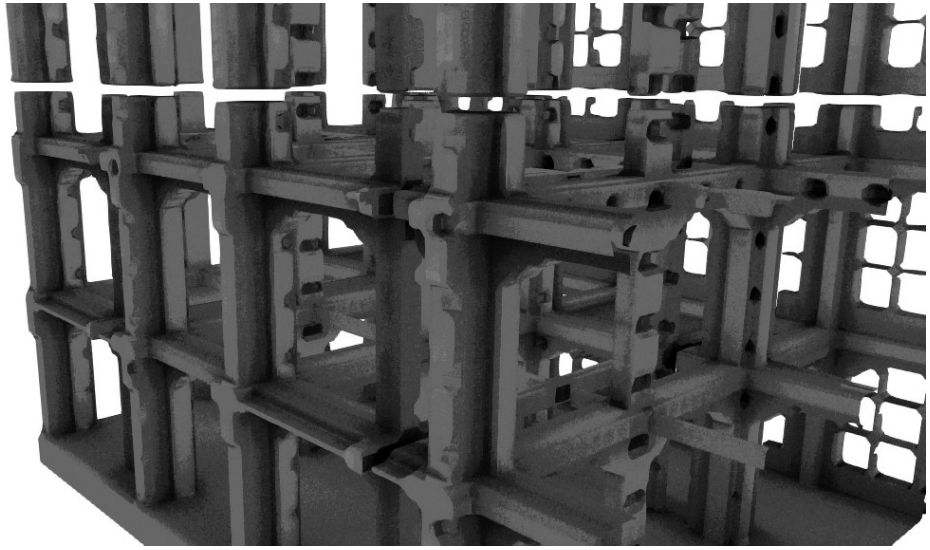
Repeating the Space

```
pos.y = frac(pos.y)
d = Box(pos)
c = fmod(pos * A, B)
subD = max(c.y,min(c.y,c.z))
subD = min(subD,cylinder(c))
subD = max(subD, Windows())
e = fmod(pos + N, M)
floorD = Box(e)
floorD = min(floorD,holes())
d = max(d, -subD)
d = max(d, -floorD)
```



Repeating the Space

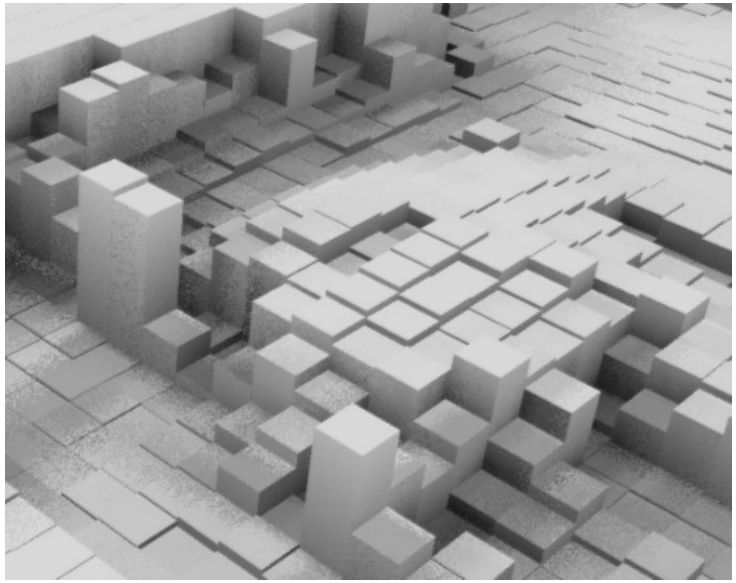
```
pos.xy = frac(pos.xy)
d = Box(pos)
c = fmod(pos * A, B)
subD = max(c.y,min(c.y,c.z))
subD = min(subD,cylinder(c))
subD = max(subD, Windows())
e = fmod(pos + N, M)
floorD = Box(e)
floorD = min(floorD,holes())
d = max(d, -subD)
d = max(d, -floorD)
```



Height Field

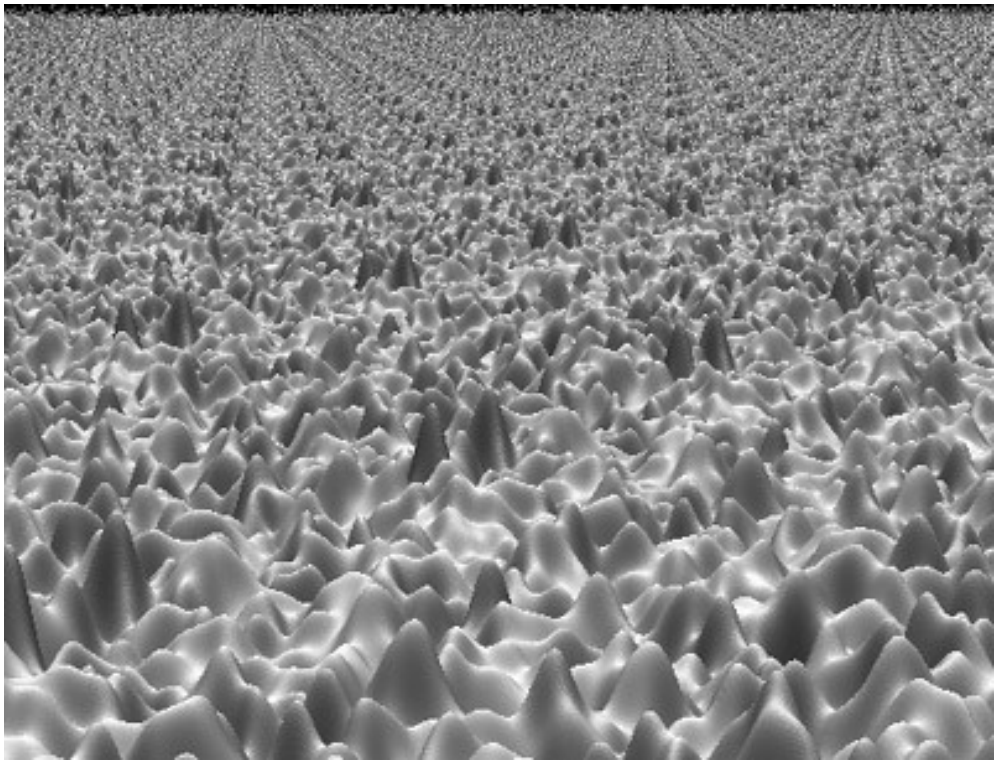
- Grid that stores a height at each position

1	2	1	2	2	1
2	1	2	1	10	1
3	1	2	6	9	0
4	1	2	5	0	0
5	1	2	3	0	0
6	1	2	2	1	0



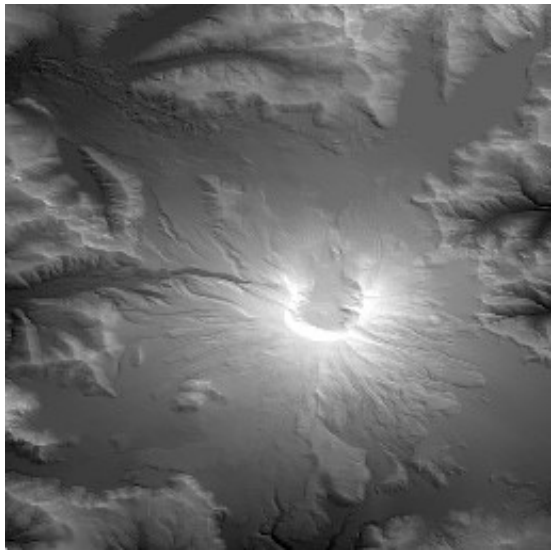
Height Field

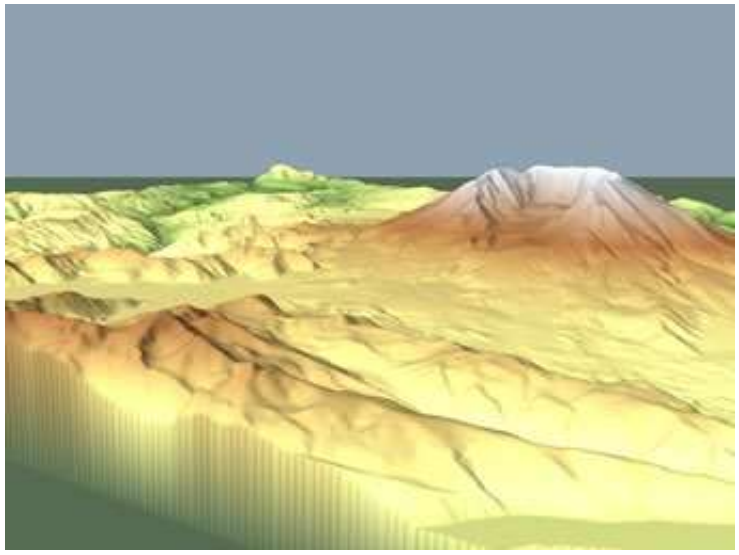
- Can use mathematical function to create grid values



Height Field

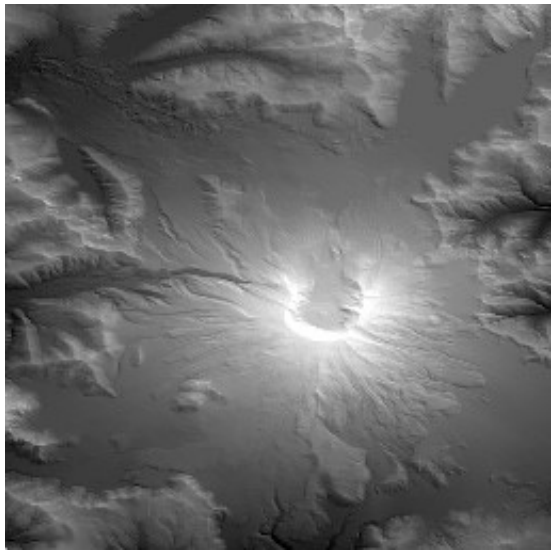
- Can use texture to store grid





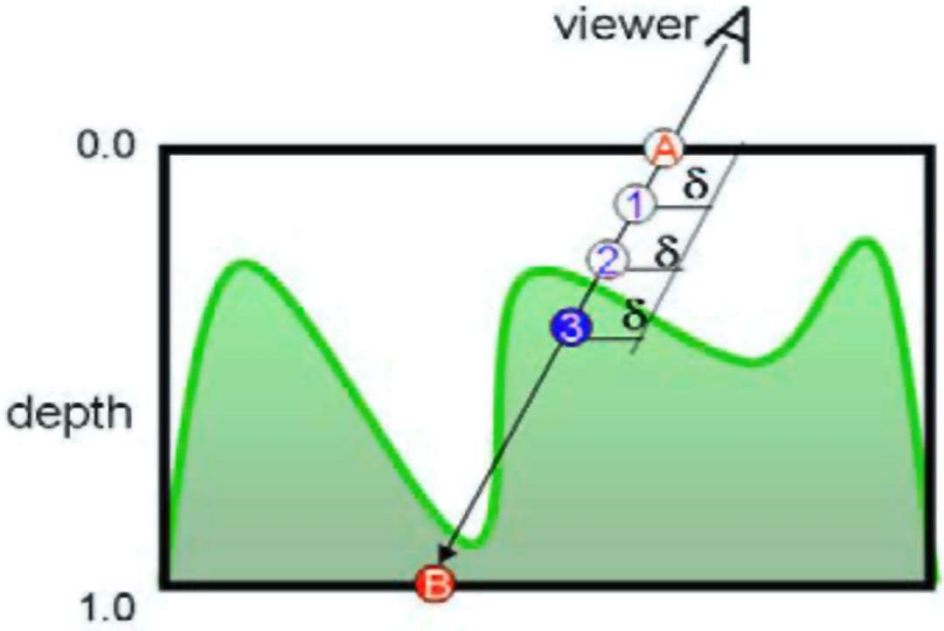
Height Field

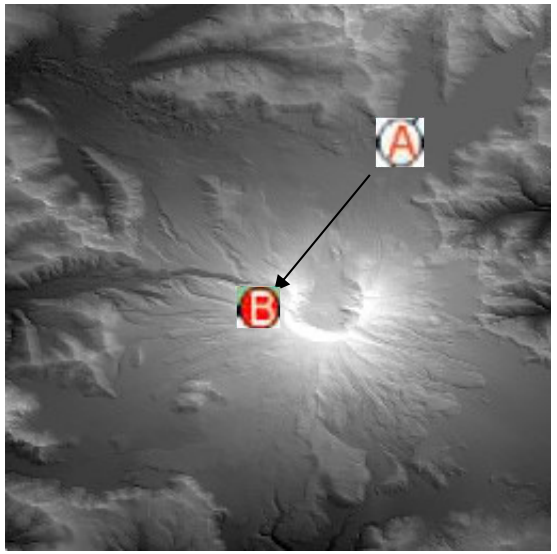
- Can render with different methods



Raymarching Height Field

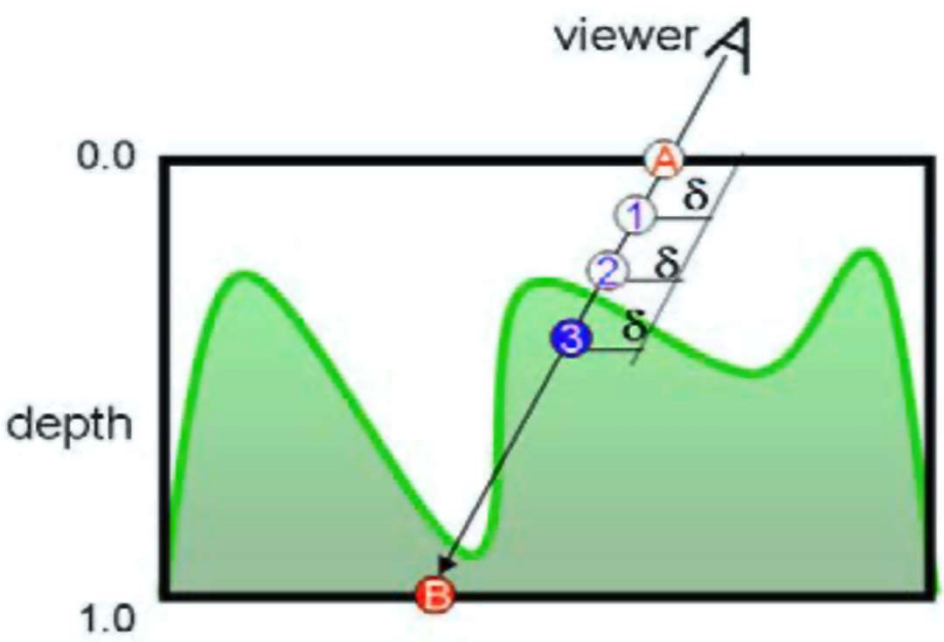
- Step with small increments along ray
- Interval bisection

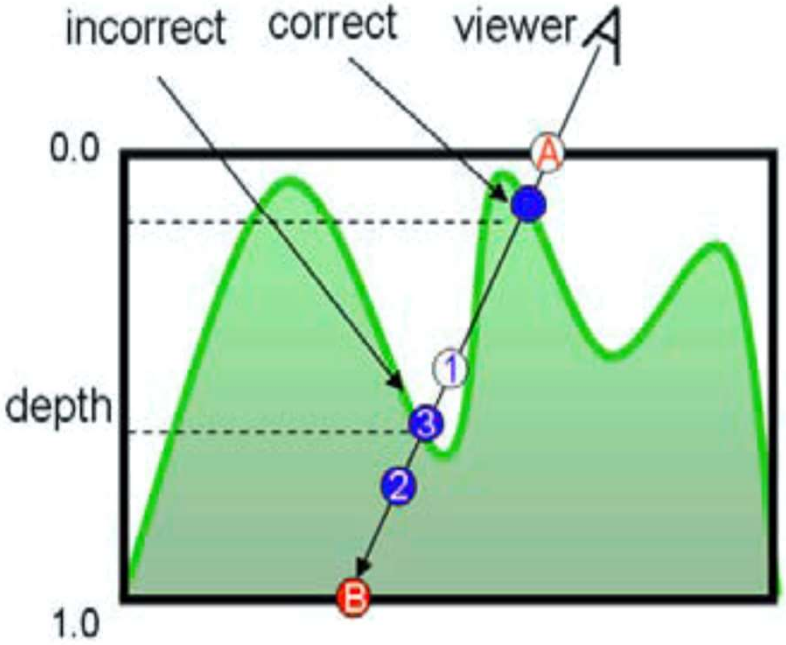




Raymarching Height Field

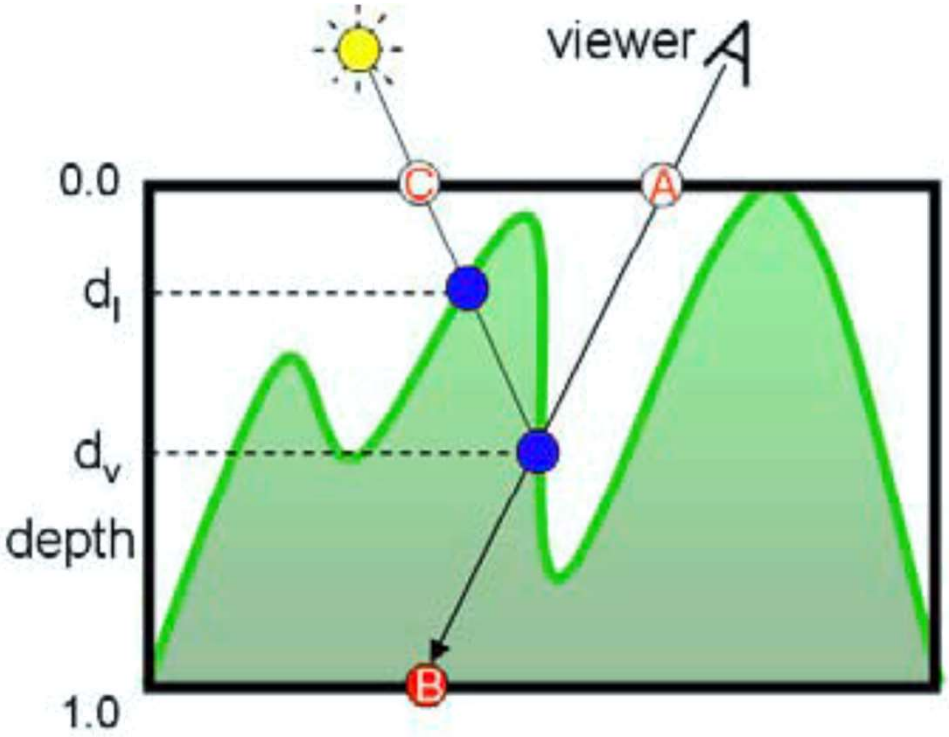
- Step with small increments along ray
- Interval bisection





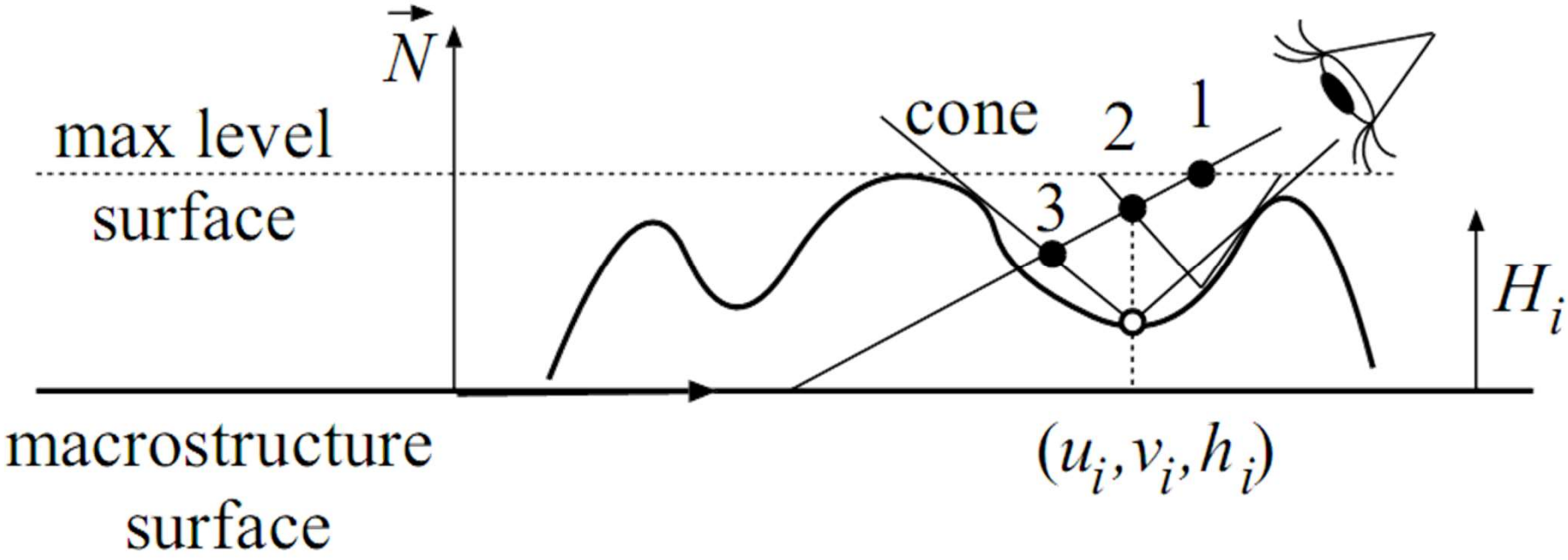
Shadowing

- Send shadow feeler ray



Accelerating Heightfield Rendering

- Help texture
  - Each texel stores cone of empty space above
  - Only store opening angle (2D texture suffices!)



Links

- Overview
  9bitscience.blogspot.de/2013/07/raymarching-distance-fields_14.html
- Distance functions
  www.iquilezles.org/www/articles/distfunctions/distfunctions.htm