AST2000 - Part 3
Preparing for the journey

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We have looked at the habitable zone in our solar system by looking at radiated, and received flux from our star. Our star is a stable black body with a radius of 317552 km and a surface temperature of 3715K. Using Stephan-Boltzmanns law, and the definitions for flux and luminosity we are able to determine the habitable zone in our solarsystem. From this we find that planet 6 is the most interesting planet to visit, since it has a temperature of 350K that can sustain liquid water, and has a high enough density for us to land on it safely. By the same theory we also find that for our landerunit to have enough power to initialize the landing sequence, its solarpanels needs to be atleast 3750 square centimetres.

I. INTRODUCTION

For a planet to be able to harbor life, there is a common understanding that the planet must be "warm" enough to have water in its liquid state. Around a star that radiates energy we can define a "habitable" zone, which will be a belt with a certain radius where planets inside this belt will receive enough energy from the star to have a temperature suitable to sustain liquid water, and thus, might harbor life as we know it.
In this article we will look at our solarsystem to find what planets are in the habitable zone using Stephan-Boltzmanns law to calculate surface temperatures, and choose one of them for our interplanetary mission in search of extraterrestrial life.

After choosing a planet, we will calculate how close we must engage our planet before getting locked inside its gravitational pull, and stay in orbit around the planet, to ready ourselves for an eventual landing.

II. METHOD

To find the habitable zone around our star we will assume that the star is a stable black body, meaning that it radiates thermal energy as a result of the motion on an atomic scale due to its temperature.
Assuming this we can find the total flux

$$F = \frac{\Delta E}{\Delta A \Delta t} \tag{1}$$

(energy per area per time) radiated out from the star as given by Stephan-Boltzmann for a black body

$$F^{out} = \sigma T^4 \tag{2}$$

where T is the temperature of the star, and σ as Stefan Boltzman's constant defined as

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2} = 5.67037 \cdot 10^{-8}$$

where k = Boltzmann-constant, h = Planck-constant and c = speed of light in vacuum. This expression is derived by integrating Planck's law of radiation (see Lecture Notes 1D)

Using the radiated flux from the star we can find the luminosity of our star, which is total energy per time, by

$$L = \frac{\Delta E}{\Delta t} = \text{flux} \cdot \text{area} \tag{3}$$

since flux is energy per time per area.

The luminosity will remain constant when the energy starts spreading out through the solarsystem. By this we mean that if we construct a circular "shell" around our star at a distance r from the stars surface, the energy flowing out from the "shellsurface" will be equal to that of the flow out from the stars surface, and so the luminosity must remain the same.

This means we can find received flux at this "shellsurface" at a distance r using (3), since we know the luminosity of our star and also the radius and geometry (a sphere) of the shell.

$$\text{Flux} = \frac{\text{Luminosity}}{\text{Area}} = \frac{L}{A} = \frac{L}{4\pi r^2}$$

which can be written as a function of r, since the area of a sphere is $4\pi r^2$.

$$F(r) = \frac{L}{4\pi r^2} \tag{4}$$

This is the case when we neglect the fact that other planets may absorb some of the energy. (This will be a very good approximation seeing how small a planets surface will be compared to the area of the shell)

The illustration in figure (1) shows how the initial radiated flux will spread over a larger area as it travels further from the stars surface, as we can see from our formulas.

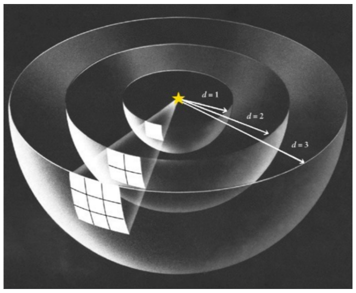

Figure 1. Illustration of flux & luminosity

With this expression for flux received at a distance r, we can find an expression for the total energy received by a planet at the distance r. We will assume that our planets are also black bodies, which means that they will absorb ALL received radiation.

If we now revisit equation (1) we can find how much energy per unit of time a planet of radius R, at a distance r, will absorb since we know the flux it receives (as previously stated). We will assume here that the planets surface is a flat circle, or rather, we will assume that the flux is constant all over the planets surface, meaning that we can find the absorption area by taking the area of a flat circle with radius r, and since astronomical bodies usually are very distant we will also assume that the "energy rays" are parallel as they hit the surface, as illustrated in figure (2).

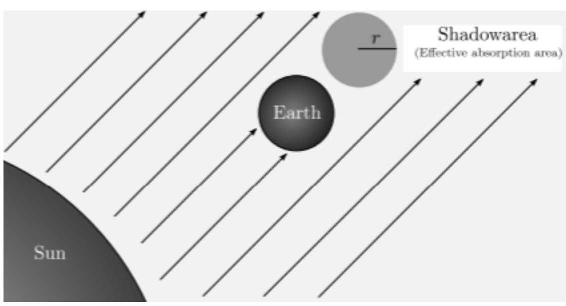

Figure 2. Absorption area (from lecture notes 1D)

This means we can find the total energy per time on the surface using equations 1 and 4 as

$$F(R,r) = \frac{\Delta E}{\Delta A \Delta t} = \frac{L}{4\pi r^2}$$

$$P^{in} = \frac{\Delta E}{\Delta t} = \frac{L^{star} R^2}{4r^2} \tag{5}$$

where we use that the area of a circle $\Delta A = \pi R^2$ and that $L = \frac{\Delta E}{\Delta t}$. We can now find an expression for the surface temperature of a planet.

Since we have assumed that the planet is a black body we know that the planet will emit the same amount of energy that it absorbs, keeping a stable temperature. Thus we have

$$P^{in} = L^{out}$$

although the fluxdensity will be different, since we absorb on a flat circle, but we emit across the whole sphere (again, blackbody radiation), since flux is energy per time per *area* i.e

$$F^{in} \neq F^{out}$$

This means that we can again use Stephan-Boltzmanns law (equation (1)), together with the expression for absorbed energy (5) to find the temperature

$$T = (\frac{F^{out}}{\sigma})^{\frac{1}{4}}$$

which can also be written as a function of r, where the function will give the temperature of a black body planet at a distance r from a star with luminosity L.

$$T(r) = (\frac{F^{out}(R)}{\sigma})^{\frac{1}{4}} = (\frac{L^{out}}{4\pi R^2 \sigma})^{\frac{1}{4}} = (\frac{L^{star}}{16\pi r^2 \sigma})^{\frac{1}{4}} \tag{6}$$

where we have used that absorbed energy equals emitted energy (luminosity of planet) as seen in eq (5) and also the definition of flux = luminosity/area.

For water to be in its liquid state it has to have a temperature within the range of 273K - 373K (0 - 100 °C), assuming the pressure on our planets equals that of earth, i.e 1atm = $101kPa$. This means we now have a criteria for our functions, that we can use to find the habitable zone. Say that our measurements aren't perfect we might extend these boundaries with ±15K, so the upper and lower limit will be 260K - 390K. Meaning that we can use these values to solve for upper and lower limit for the radius in the habitable zone (see result section for table).

We will also estimate the temperature of each of the planets in our solarsystem using the methods and expression we previously found to see which of them can sustain liquid water, and maybe harbor extraterrestrial life!

After we have chosen a planet to explore, we will need to construct our landing unit i.e finding the size of the solarpanel we should put on our lander in order for it to fully function on its way to the planetary surface. We will get our energy from the sun and we have already seen how we can find the energy per time the sun puts out at a given distance r on a surface dA.

With this information, and since we know the distance from our chosen planet to our star, we can find the size using the expressions previously stated in this section by using the same approach as we did to find absorbed energy on a flat, planetary surface with area A to find the absorbed energy on the significantly smaller, square solar panel.

By doing this (See appendix A) we find an expression for the total area of the solar panel

$$\Delta A = \frac{P^{out} 4\pi r^2}{n L^{star}} \tag{7}$$

where P is the wanted energy output from the panels, r the distance from panel to star, n the efficiency factor and L the luminosity of our star.

Next we'll need to generalize the launch algorithm we produced in project part 1, so that we can define a different location to launch and also a different time. We do this by using the program we made in part 2, to find the planetary position of our homeplanet, (x_0, y_0), at our desired time $t = t_0$. With this position, we can then decide at which point on the planetary surface we would like to launch from using an angle θ, and the radius of the planet, r, using polarcoordinates

$$x = x_0 + r\cos\theta$$
$$y = y_0 + r\sin\theta$$

where the angle θ is defined using $\theta = 0$ along the x-axis through the center of the planet, and the coordinates (x_0, y_0) is the center of the planet.

This now gives us a new launch sequence that can take both time, and angle as input to decide when, and where to launch our shuttle.

During our voyage we would like to take some photographs of the planet, to document the voyage, as we are travelling towards it. However, in order for our planet to actually be visible on our pictures we cannot be too far away. It can be shown that we need to be at a distance (See C for derivation, and visualization.)

$$L \lesssim \frac{RP}{F}$$

from the planet in order for it to be resolved in our pictures. We define resolved as the planet being more than 1 pixel in our pixelgrid.
In the equation P is the number of pixels in our PxP grid, F is the anglewidth in which the camera can take picture and R the radius of the planet we're taking picture of.

III. RESULTS

The following table is the collected data about the sun.

Table I. Information about the sun

Info	**Value**
Temperature	3715.4 K
radius	317551.56 km
Surface area	3990470.57 km^2
Flux at the surface	$1.08054 \cdot 10^7 \frac{W}{m^2 s}$
Luminosity	$1.36924 \cdot 10^{25} \frac{W}{s}$

These data have been received from a neighbouring research facility, and from them we can calculate where the habitable zone is in our solarsystem by solving eq (6) for distance r.

$$r_{min} = \left(\frac{L^{star}}{16\pi T_{max}^4 \sigma}\right)^{\frac{1}{2}} = 0.09633 AU$$
$$r_{max} = \left(\frac{L^{star}}{16\pi T_{min}^4 \sigma}\right)^{\frac{1}{2}} = 0.21673 AU$$

since the lower distance, the higher temperature since we are closer to the star.

By using equation (6) in its written form, we can find an estimate of the planetary surface temperatures for all the planets in the system, by knowing the radius of the planets orbits from the ast2000 research center. These values are listed in the table below. The first column in the table provides the index of a given planet, and in the second column we provide the following surface temperature of the planet and in the third column we include the corresponding density of the planet (Check our study of planetary orbits, part 2)

Table II. Received data of surface temperature in a given planet

Planet index (i)	**Surf. Temp.** (K)	**Density** (kg/m^3)
0	478.29	5298.8
1	418.55	3158.0
2	175.98	4783.4
3	305.86	5362.7
4	156.29	772.8
5	229.58	660.9
6	353.79	6154.3

As for the landing unit, our engineers informs us that the lander needs to produce 40 W of electric power, and the solar panels have an efficiency of 12 %. If we implement this, together with the distance from planet

6 to the star and the information from table (I), into the expression for area we found in the method section (ref (7)

$$\Delta A = \frac{P^{out} 4\pi r^2}{n L^{star}}$$

$$\text{Area} = \frac{40[W] \cdot 4 \cdot \pi \cdot (35021396801)^2 [m^2]}{0.12 \cdot 1.36924[W] \cdot 10^{25}} = 0.375[m^2]$$

so we need a solar panel of at least 3750 square centimeters to absorb enough energy from our star to power our landing units instruments.

IV. DISCUSSION

After receiving the temperature data which is given in Table II, we can determine which planet is in the habitable zone around the sun. As explained in method section, the probability of discovering a life on other planet is higher in a planet where water can be liquid. Meaning that the temperature cannot be to high or too low. We assume that the liquid water can exist in the range of temperature between 290K and 390K.

From the data given in table II we can see that there are two planets within the range of the temperature. Planet with index 3 and 6. However, we wish to visit the planet where the surface temperature is closest to the middle value of the temperature range, meaning that the planet with index 3 is too cold for us. This leaves us only with one planet, planet with index 6.

However, we also have to decide wheter the planet is suitable for landing. As for the example it would be quite difficult to land in a gas planet, because of the extreme winds and harsh atmosphere there would be no of not any visibility.

If we want to choose planet with index 6 as the destination, we have to make sure that it is not a gas planet. We therefore find the density of this planet, which is 3158.0 kg/m^3. We then find the relation between the density of the planet and the density of water, which is 997 kg/m^3. If the relation is greater than 4 or 5, then we can conclude that the density of the planet is large enough for it to be solid planet, and not a gas planet. We can see that the relation between the density of planet 6 and density of water is greater than 6, meaning that the planet is dense enough to be defined as a rock planet..

We also found a size for our solar panels that will provide electrical power for our landing unit. This size was $\approx 4000\text{cm}^2$, which may seem small. However, the landing unit only required 40W of power, which is not very much, if we compare it to common household appliances (see table (III)), and the planet we've chosen to visit is very close to it's star.

Appliance	**Watts[W]**
Freezer	60
Fridge	100
Game console	200

Table III. Common household appliances

We found the radius of its orbit to be 0.12 AU which is just 12% of the distance from the sun to our earth. And a typical solar panel on earth produce around 150 W per square meter, so our solar panel at 0.4 square meter producing 40W is a realistic number when we know that we are closer to our star, and our star produce less energy per time than the sun. (About 10 times less, and we are 10 times closer so this will even out somewhat)

Naturally, this number will not be exact seeing as we've made major assumptions about the received flux, and we are also not accounting for the diminishing flux as we travel into the planetary atmosphere, since this will reflect a lot of sunlight.

V. CONCLUSION

We found that planet with index 6 had the properties needed to sustain life as we currently know it, and it was also a suitable candidate for a successful landing. (It was not a gas giant, but had a solid surface). Furthermore it was also near enough to the star that our landing unit could receive enough power to perform its landing sequence with reasonably sized solar panels and this look like a prime candidate for our interplanetary voyage.

When we looked at generalizing the launch sequence we would've rewritten our program to account for launching at different points in time, using a change of coordinatesystem to polarcoordinates so that we can define at which point on the planetary surface we'd like to launch, and we used the array of planetary positions we produced in part 2 of the project to find the planets initial position at a given time according to how we explained it in the method section of this study.

However, since the original program from part 1 was faulty and didn't give us the correct position after launch we are once again reliant on our friends from the ast2000 center when we are performing the actual launch in later studies, and thus we haven't actually written this program - but using the approach we've explained, some brighter programmers should have no trouble in doing so.

ACKNOWLEDGMENTS

Special thanks to the bright minds of ast2000 research center.

REFERENCES

- https://www.uio.no/studier/emner/matnat/astro/AST2000/h20/undervisningsmateriell/lecture_notes/part1d.pdf Lecture notes 1D
- https://www.daftlogic.com/information-appliance-power-consumption.htm Common household appliances - power usage

Appendix A: Derivation of solar panel size for our landing unit

We want to find how large the surface of our solar panels need to be in order to absorb sufficient amounts of energy from the stars radiation for it to be able to power our landing unit.

We will reuse the formulas we derived in the method section, and solve them to find ΔA. As explained in the method section, we know that the fluxdensity from the sun is equal to radiated energy per time per area (ref eq (1)).

$$F = \frac{\Delta E}{\Delta A \Delta t}$$

where we call radiated energy per time for luminosity L. And also, as explained in the method section, we find flux by equation (4) as

$$F = \frac{L^2}{4\pi r^2}$$

where r is the distance from the energy source. Knowing that the fluxdensity F will be constant over the spherical area spanned by $4\pi r^2$, that we can find the same fluxdensity in a smaller "cut" of this surface, which will be our solarpanel.
Thus we can solve these two equations for ΔA

$$\frac{L^{star}}{4\pi r^2} = \frac{\Delta E}{\Delta A \Delta t}$$

$$\Delta A = \frac{\frac{\Delta E}{\Delta t} 4\pi r^2}{L^{star}}$$

Now we have a general expression for the surface area, for a given energy per time, at a distance r with a star with luminosity L. However, we need to take into account that our solar panels can't convert 100% of the absorbed energy to electrical power, it will have some efficiency factor n, meaning output energy = n * input energy which gives

$$\frac{\Delta E^{in}}{\Delta t} = \frac{\frac{\Delta E^{out}}{\Delta t}}{n}$$

If we now define $\frac{\Delta E}{\delta t} = P$, we have that P in our expression for ΔA is the received energy P^{in} and so the final expression for the area of our solar panels will be

$$\Delta A = \frac{P^{out} 4\pi r^2}{n L^{star}} \tag{A1}$$

where again, P is the required output energy, r is distance to star, n the efficiency factor and L the luminosity of our star.

Appendix B: Derivation of the distance between the planet and spacecraft

If we assume that the spacecraft is close to a planet in our solar system, we let the gravitational force from the planet on the spacecraft be k times larger than the gravitational force from the star on to the spacecraft.

The reason is that we want to determine the distance l from the spacecraft to the planet, which would be sufficiently small that the spacecraft could be locked inside the planets gravitational field.

We set up described situation and work ourselves further from that

$$kF_s = F_p$$

F_s is the gravitational forcepull from the sun onto the spacecraft, and F_p is the gravitational forcepull from the planet onto the spacecraft.

$$kG\frac{m_s m_{sc}}{r^2} = G\frac{m_p m_{sc}}{l^2}$$

G is the gravitational constant, m_s is the sun's mass, and m_{sc} is the mass of the spacecraft. r is the distance between the sun and the spacecraft, while l is the distance between the planet and spacecraft.

$$k\frac{m_s}{r^2} = \frac{m_p}{l^2}$$

$$\frac{l^2}{r^2} = \frac{m_p}{km_s}$$

$$\frac{l}{r} = \sqrt{\frac{m_p}{km_s}}$$

We end up with expression for the required length between spacecraft and planet in order to get locked into the planets gravitational field.

$$l = r\sqrt{\frac{m_p}{km_s}}$$

Note that r is the absolute value of the position vector of the spacecraft if we assume that the sun is the origin.

Appendix C: Derivation of minimum distance for visible planet photograph

In order to determine the minimum distance for visible planet photograph, we need to understand the the photograph is set of PxP pixel grid, as figure 3 illustrates.

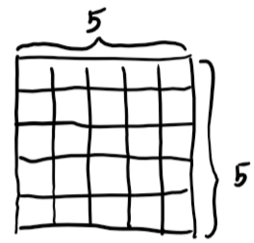

Figure 3. 5x5 pixel grid

Firstly, we define the angular value for the pixel. In order to determine this, we have to know the camera pixel dimension Pxp and field of view FxF measured in degrees. Figure 4 illustrates the meaning of angular value θ for the pixel. If we would draw a line from the top point of an object radial towards us, and draw another line from the most bottom point of the object radial towards us, and find the angle between those two lines, that angle would be angular value of the object.

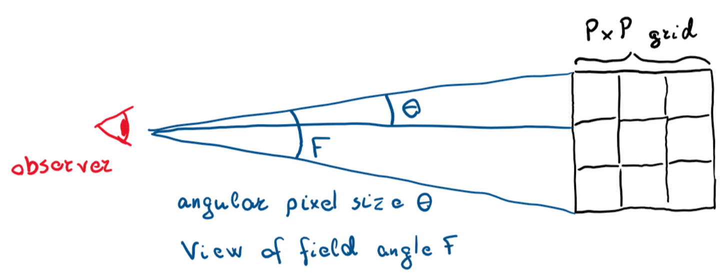

Figure 4. Caption

From the definition of the arc length, we know that

$$s = \omega r$$

where s is the arc lenght, Ω is the angle, and r is the radius of the circle. Figure 5 illustrates the definition of the the arc length

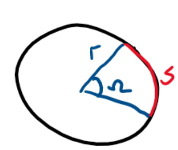

Figure 5. illustration of arc length definition

We can see that we can find the angular pixel value θ from the view angle F and P number of pixels

$$\theta = \frac{F}{P}$$

We can write

$$\tan\theta = \frac{R}{L}$$

where R is the radius of the planet, and L is the distance between us observer and the planet itself.

By assuming that the angular value for the pixel is significally slow, we can assume that

$$\tan\theta = \theta$$

We therefore have now

$$\frac{R}{L} = \frac{F}{P}$$

Meaning that

$$L = \frac{RP}{F}$$

However, this is the expression for the maximum length L between the camera and the planet in order to see planet in the photograph. We can therefore generlise the expression by saying that this must be the maximum length in order to capture the planet.

$$L \lesssim \frac{RP}{F}$$