Part 7
The landing

Candidates: 76408 & 83886
(Dated: December 17, 2020)

We've expanded our previous model of planets 6 atmosphere by modelling the drag force experienced by a falling object in order to calculate the required cross-sectional area of our landing unit to make a successful descent (successful meaning no crashlanding) to the planetary surface. We find that our landing unit, including the parachute, needs a cross-sectional area of at least 112 m^2 to safely take us through the atmosphere. This parachute will give us a terminal velocity of < 3m/s which allows us to not use landing thrusters to land safely.

I. INTRODUCTION

In this study we are going to land our shuttle on our destination planets surface. To do so we are going to expand our model for the atmosphere that we built in study part 6, where we are now going to take into account the motion of the atmosphere.

This is useful for us when we are planning the landing sequence, by allowing us to create an accurate model of the drag force experienced by a body with cross-sectional area A descending through the atmosphere with a velocity v. This will give us a way to simulate the landing by solving differential equations for the motion of the shuttle.

II. METHOD

We begin by modeling the air resistance experienced by our lander using the following assumptions

- The entire atmosphere follows the planets rotational motion i.e angular velocity is constant for all part of the atmosphere
- The drag coefficient will be constant $C_d = 1$

Meaning that we can use the air resistance to slow down our landers descent towards the surface considerably. We will model the force acting onto the spacecraft from air resistance, more commonly known as the drag force. The drag force is defined as

$$F_d = \frac{1}{2}\rho C_d A v_d^2$$

where A is the cross-sectional area of the object experiencing the drag force, v_d is the velocity of the object relative to the fluid exerting the force, ρ the density of this fluid and C_d is a drag coefficient that is determined by factors from both object and fluid.

By using the prior assumption, we can express the atmospheric velocity $\mathbf{w}$ as a function of height, r.

We have the relation between angular velocity and velocity in a rotational motion given as

$$\omega = \frac{v}{r}$$

which means, given we assume a constant angular velocity, that we can find the velocity v as

$$v(r) = \omega_0 r$$

where $r = r_0 + h$ is the sum of height above planet surface and planet radius, and $\omega_0 = \frac{2\pi}{T}$ where T is the rotational period of the planet supplied to us by our friends at the ast2000 center.

Before we can find the drag force, we need to know the drag velocity v_d. This is the landers velocity relative to atmospheric motion.

From experience, we know that it is harder to run when the wind is blowing towards us, and easier to run when the wind blows from behind. The same principle applies in our landing situation as well

Meaning, when the atmospheric velocity, $\mathbf{w}$, has the same direction as our landers velocity, $\mathbf{v}$ relative to the planet, it will have a negative effect on the air drag force. This is the concept of a tailwind. This means we can express our drag velocity as

$$\mathbf{v}_d = \mathbf{v} - \mathbf{w}$$

and we can expand the expression for drag force

$$F_d = \frac{1}{2}\rho C_d A|\mathbf{v} - \mathbf{w}|^2$$

Since this drag force is velocity dependent, and the only other external force acting on a falling object through the atmosphere is gravity, there will come a point where the velocity reaches a maximum with only a radial component, called the terminal velocity, $\mathbf{v} = (v_t, 0)$.

There will only be a radial component since the drag force will continuously decelerate the tangential component of the velocity, with no forces applying an acceleration in this direction resulting in the shuttle following the atmospheres motion with zero relative velocity. In the radial direction however, gravity will always accelerate the shuttle on its downward motion and thus, after falling for long enough, the tangential component will be zero and the radial component will have its maximum at v_t.

At this velocity the drag force working *against* the direction of motion cancels out the gravitational force acting *in* the direction of motion and by Newtons 2nd law, $\sum F = F_D - F_G = 0 = ma$, the acceleration of the shuttle will be zero and thus, the velocity will remain constant at v_t.
Furthermore, we can use Newtons 2nd law to find this velocity

$$\begin{aligned} F_d - F_G &= 0 \\ F_d &= F_G \\ \frac{1}{2}\rho C_d A v_t^2 &= G\frac{mM}{r^2} \\ v_t &= \sqrt{2G\frac{Mm}{C_d \rho A r^2}} \end{aligned}$$

This same equation can be rearranged to find the cross-sectional area, A, needed to make a soft landing

$$A = 2G\frac{mM}{r^2 v_t^2 C_d \rho_0} \tag{1}$$

where ρ_0 is the atmospheric density on the surface of the planet, since we can assume this stay fairly uniform in the lower parts of the atmosphere (this is a good enough assumption for an estimate of the area A).

This is an ideal scenario where the terminal velocity aligns with the safety limit on our landing unit. And we need to take into consideration that the velocity our landing unit will have near the surface may be too big for the landers to handle, and thus we need our landing thruster to provide us with some assistance.

We can again setup Newtons 2nd law to find the force needed to change our velocity from terminal velocity, v_t, to the safety threshold, v_{safe}

$$\begin{aligned} F_d + F_L &= F_G \\ F_L &= F_G - F_d \\ &= G\frac{mM}{r^2} - \frac{1}{2}\rho_0 C_d A v_{safe}^2 \\ F_L &= \frac{1}{2}\rho_0 C_d A (v_t^2 - v_{safe}^2) \end{aligned}$$

where we have used the fact that at terminal velocity, v_t, $F_G = F_d(v_t)$ and we find the force needed to compensate the difference from terminal velocity to our threshold velocity.

Our next step is to simulate our landing, using the profiles we derived in part 6 of our studyseries. Before we are delving into constructing this simulation we are going to make some statements that we are basing our simulation on:

- Our landing unit leaves the shuttle at any given time and velocity of our choosing
- The landing thrusters are initiated at any given height above the surface of our choosing
- The drag pressure on our parachute cannot exceed 10^7 Pa
- We invoke all prior assumptions from part 6

III. RESULTS

Knowing that our lander can only handle a velocity of $v = 3$m/s without getting trashed in the landing, we can find the cross-sectional area needed using equation (1)

$$A = 112\,\mathrm{m}^2$$

where we have used that $m = 90$ kg is the mass of the landing unit, ρ_0 is the density on the surface of the planet and the assumption that $C_d = 1$.

IV. DISCUSSION

The cross-sectional area we found that our parachutes + our landing unit need to slow our shuttle to a safe landing seem reasonable when compared to other parachutes from different space mission (see references) where they are often in the circular with diameteres up 100+ feet.
With a parachute of this size our terminal velocity for the final thousand metres (where we assume a uniform density) will be less than 3m/s and we do not require landing thrusters. However, we will still keep them onboard our lander, in case our calculations have been inaccurate due to unforeseen factors.

When we are going to actually descend our landing unit, we will and send it into the atmosphere in the direction of the planetary rotation. This will minimize the factor $|\mathbf{v} - \mathbf{w}|$ in the expression for force exerted from air drag seen in the method section, and thus minimize the pressure on our parachute, since pressure given as

$$P = \frac{F_d}{A}$$

i.e force per area. If this becomes larger than 10^7 kPa our parachute will break and we will surely send our astronauts to an early grave!

V. CONCLUSION

From a lack of time we were not able to implement our ideas into a numerical program to simulate the landing. Neither were we successful in actually performing the landing sequence, but we are firm believers that our methods and procedures from this study would be successful if we had the time to properly implement and test them!

ACKNOWLEDGMENTS

We would like to extend our gratitude to the AST2000 team for all their help and kind words of encouragement during this project, all the way from part 1 to 7 - we would not be here without you guys.

REFERENCES

- https://www.nasa.gov/sites/default/files/atoms/files/orion_parachutes.pdf