AST 2000 - Part 6
Preparing for the landing

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In this study we made a low, stable orbit and made a measurement of the spectrallines radiated from the planets surface. We have analyzed the atmosphere using spectral analysis, and the chimethod to approximate Gaussian curves to fit the measured data. Doing this we've found the concentration in the planet atmosphere to be of 66% CH4, methane gas, and 33% H20, water.

I. INTRODUCTION

Our shuttle is now in an unstable, elliptical orbit around our destination planet. Therefore we need to make adjustments to our orbit and get closer to the planet. We are going to perform the same maneuver to the one we used for the interplanetary trajectory, the Hohmann transfer, in order to reach a more stable, smaller and ideally, circular orbit. We want a circular orbit in order to make accurate measurements of the spectrallines from the radiated flux of the planet.

These spectrallines will be used to deduce the contents of the atmosphere, which we will in turn use to create a model for both temperature, and density of the atmosphere that will allow us to simulate an eventual landing.

To do so however, we will need to study the absorptionlines in this spectrum to deduce what gases are present inside the atmosphere. This method will be thoroughly explained in the method section.

Our final challenge is to find a proper landing site taking into account both our angular velocity and also the planets spin around its center. We would like to land on the sunny side of the planet, and also avoid any big oceans or volcanoes!

II. METHOD

Right now we are orbiting the planet in an elliptical orbit, and as mentioned in the introduction we are going to enter a lower, circular orbit. This means our shuttle needs to have a constant angular velocity around the planet.

Additionally, we are still measuring our velocity in the suns reference frame, and we will have to do a change of coordinate system into the planet-shuttle reference frame. This is done by a standard Galilean transformation where we subtract the planets velocity relative to the old coordinate system, $\mathbf{u}$, as seen here

$$\mathbf{v}_{new} = \mathbf{v}_{old} - \mathbf{u}$$

where the planet will be at the origin of the new coordinate system.

In order to enter a circular orbit, we will be using Hohmann transfer method again. See study 5 in references for more detailed explanation of this transfer method. The main idea is to give a rocket a boost against the velocity direction at a specific time. Illustration in figure 1.

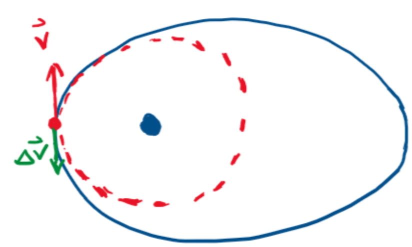


Figure 1: Illustration of an unstable orbit (blue ellipse) around the planet (blue dot) of the rocket (red dot) that boosts in opposite direction of the velocity to gain new circular orbit (red dash line)

The time we choose to give the boost to the rocket is when the rocket is at it's periapsis location to the planet (the point when the rocket is closes to the planet). The boost will decrease the value for semi-major axis a. We define a circular orbit when semi-major axis a and semi-minor axis b are equal. Illustrated in figure 2

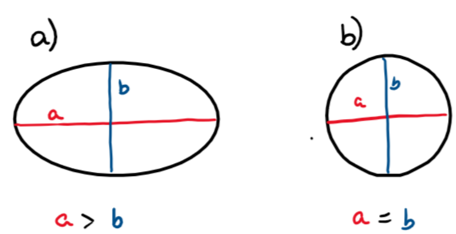


Figure 2: a) shows relation between a and b for an ellipce, while b) shows relation between a and b in a circle

What Hohmann transfer orbit does in this situation, that it simply decreases the length of a til it gets equal to b. We can then conclude that the rocket is in a circular orbit around the planet. We can also choose to test this by looking at the velocity of the rocket. If the orbit is circular, then the absolute value of the velocity must maintain (a near) constant value. When the shuttle is in a circular orbit the potential energy of gravity is also constant, since the distance, r, is the same for all parts of the circular orbit. Due to the conservation of energy in a conservative vectorfield, this means that the kinetic energy must also remain constant, and thus the velocity must be constant as well.

In order to find the boost value, we use the same principles that we derived in part 5 study. When the rocket is at periapsis of its orbit, we boost our engines in the opposite direction of motion by

$$\Delta v = \sqrt{2GM(r_2/r_1(r_1+r_2))} - v_1$$

Where v_1 is the current velocity of the rocket before the boost. M is the mass of the planet. G the gravitational constant. And r_1 and r_2 are the distant relations showed in figure 3

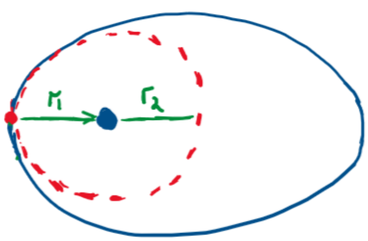


Figure 3: the relation between r_1 and r_2

We can see that for a new circular orbit, r_1 and r_2 must be equal. We therefore can simplify our expression for boost

$$\Delta v = \sqrt{2GM\frac{1}{2r}} - v_1 \tag{1}$$

Where r is the periopsis of the orbit.

However, we need to know when we are at the periopsis. We know that our velocity should be highest at that point (because we have lowest potential energy, and therefore in order to conserve energy, kinetic energy must rise, thus increasing the velocity).

One way we can check when we are at the periopsis, is simply by observing the velocity of the spacecraft. We can write a software that automatically detects when the velocity of the spacecraft is at its peak, and automatically boosts in the opposite direction of the velocity with a boost that we just defined in equation (1).

After boosting, we measure the shuttles velocity in specific points in its orbit, and if there is little to no deviation, i.e near a constant value we can conclude that we have successfully managed a circular orbit.

When we have achieved a circular orbit, we can then use a maneuver illustrated in figure 4.

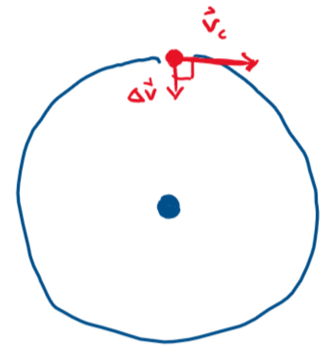


Figure 4: maneuver where we boost our rocket (red dot) towards that direction of the center of the planet (blue dot). vector v_c is a current velocity vector

This maneuver simply boosts our rocket orthogonal of the velocity vector towards the center of the planet.

How much we boost, is simply test and fail situation. We will be making a simulation that tests different boosts and tries to come as close to the surface as possible without falling into the atmosphere.

When our simulation tests a boost value, it then checks if our tangential velocity [ok her var jeg usikker om det heter tangensiell hastighet, sjekk og rett den!]. If our tangensial velocity suddently begins to drop, this means that we have entered the atmosphere and we are being dragged. The goal of the simulation would be to boosts itself as close to the planet without getting into the atmosphere.

After reaching a stable orbit close to our planet we will start the analysis of the planets atmosphere. We do this in order to create a density profile of the atmosphere which will give us an idea of how our landing sequence should happen.
We are going to use spectralanalysis of the spectrum received from the planets radiated flux, which our rocket has already started measuring. However this measurement is really noisy, for many different reason and we cannot expect to find absorptionlines at their natural wavelengths in the spectrum. Some reasons include the atmospheres temperature which gives the molecules thermal movement, also our rockets movement both give rise to a dopplershift by the formula

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

In addition to this, our measuring equipment is not completely trustworthy and the data will be inherently noisy by default.
It will still gives us a good idea of what we can expect the atmosphere.
To be able to minimize the deviations from noise and actual data we will apply the Chimethod, which is a twist on the regular least square method by the formula

$$\chi^2 = \sum_i [\frac{f_i - f(t_i)}{\sigma_i}]^2$$

where the main idea behind the method is to approximate a Gaussian lineprofile to the given data, that minimizes the average deviation from measure data to approximated data.

We are expecting the atmosphere to consist of a combination of the following gases

Gas	Spectral Lines [nm]		
O_2	632	690	760
H_2O	720	820	940
CO_2	1400	1600	—
CH_4	1660	2200	—
CO	2340	—	—
N_2O	2870	—	—

Figure 5: Table

These are the absorption lines we are looking for inside the spectrallines measured in the radiated flux, and these wavelengths will be used as a starting point for the chimethod.
We will now explain the least squared chi method in detail: As mentioned this is a least square method, meaning that we looking to minimize the difference between measured value, and modelled value. In the expression

$$\chi^2 = \sum_i [\frac{f_i - f(t_i)}{\sigma_i}]^2$$

f_i is measured value, σ_i is the noise on wavelength i when we assume the noise is also Gaussian and $f(t_i)$ is the modelled value given by a Gaussian line profile that is given by

$$F = 1 + (F_{min} - 1)e^{-\frac{1}{2}(\frac{\lambda - \lambda_0}{\sigma})^2}$$

where 1 enters the equation since the flux is normalized and $F = 1$ means that there is no flux at this wavelength. When we are trying to approximate this line profile to the measured values we test with different values for the parameters F_{min}, σ and λ_0. We find the intervals by analysing the measured data, and by doing the following assumptions.

For the value of F_min, which is the magnitude of flux on the given wavelength, we expect an actual line to lie in inside the interval

$$F_{min} \in [0.7, 1]$$

For the initial wavelength λ_0 we expect to find this at the same length as the gases, but it will be shifted to a different wavelength by the dopplershift, as a consequence of the shuttles relative velocity to the gas, as well as the thermal velocity of the gas particles. This means we expect λ_0 in the interval

$$\lambda_0 \in [\lambda_0 - \Delta\lambda, \lambda_0 + \Delta\lambda]$$

And lastly we expect the σ, the width of the Gaussian curve, which is given by the formula (see appendix for derivation)

$$\sigma = \frac{2\lambda_0}{c}\sqrt{\frac{2kT}{m}}$$

and we see that this is also related to the temperature T, in which we expect to be in the interval

$$T \in [150K, 450K]$$

With these intervals we can now read our measured values for the flux and noise, and perform the chimethod numerically by iterating over a chosen amount of values within the intervals for the three different parameters to find which of whom gives the best curve fit to the measured data.

With the absorption lines in hand, we can decide which gases the atmosphere consist of and we make the assumption that if we find, e.g two lines of O_2 and one of CO_2 we assume the atmosphere is 66% oxygen and 33% carbondioxide. This gives us a way to calculate the mean molecular weight of the atmosphere, given by the formula

$$\mu = \sum_{i}^{N} f_i \frac{m_i}{m_H}$$

where m_i is the weight of gas i, f_i is the fraction of said gas in the atmosphere and m_H is the mass of a hydrogenatom. This means that the mean molecular weight is given in units of hydrogenmass i.e single proton mass. E.g if we use the same combination as we mentioned previously, μ is computed as follows

$$\mu = \frac{2}{3} \cdot 16 + \frac{1}{3} \cdot (12 + 16) \approx 20$$

Before we begin explaining the method to create the density- and temperatureprofile for the atmosphere we need to make the following assumptions to make this possible

- The atmospheres concentration is uniform
- The atmosphere has a spherical symmetry, i.e $\rho = \rho(r)$
- The atmosphere is in hydrostatic equilibrium
- The atmosphere is an ideal gas
- The mass used in the expression for gravitational pull is constant, meaning that mass of planet >> mass of atmosphere.
- The atmosphere adiabatic with $\gamma = 1.4$ for heights up to $T = T_0/2$ and isothermal otherwise

With these assumptions in mind we have a bunch of tools at our disposal to start creating the profiles we need to model the atmosphere.

Firstly, we start by looking at the formula given to us, by assuming hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho(r)g(r)$$

where $g(r) = \frac{GM}{r^2}$ is the gravitational pull of the planet, P is the atmospheric pressure and $\rho(r)$ is the density we are looking to find.
From the assumption of ideal gas we get the formula

$$P = \frac{\rho(r)kT(r)}{\mu m_H}$$

where k is the Boltzmann constant, μ the mean molecular weight, m_H weight of a hydrogenatom, and from thermodynamics and the assumption of adiabatic and gas we get the formula

$$P^{1-\gamma}T^{-\gamma} = constant$$

and for isothermal gas

$$T = constant$$

We can work these expressions to find a coupled first order differential equation we can solve to find the density and temperature, see the appendix for the full derivation, which gives us the expressions

$$\begin{aligned} \frac{dT(r)}{dr} &= -\frac{(\gamma-1)}{\gamma}\frac{\mu m_H g(r)}{k} \\ \frac{d\rho(r)}{dr} &= -\frac{\rho(r)}{T(r)}\frac{dT(r)}{dr} - \frac{\rho(r)}{T(r)}\frac{g(r)\mu m_H}{k} \end{aligned}$$

We can solve this coupled ODE by normal Euler integration method which we have explained in detail in prior studies, and doing so will give us a set of discrete solutions on an interval from planet surface up to a certain R where we assume the atmosphere "ends" with steplength, dr, of our choosing.

However, we would like a continuous function that describes the system and therefore we are going to interpolate these two set of values to construct polynomials we can evaluate at any given height which we need to model the atmosphere.

To do so we are going to use the SciPy modules interpolation method interp1d (see reference for full documentation).

III. RESULTS

After running our program by iterating over 30 values in the temperature interval, 8 values in the F_{min} interval (that we increased to [0.6,1]) and 40 different values in the sliced arrays of λ_i values (we "slice" out the part that corresponds the the interval we expect to find λ_0 in), we manage to analyze the data given to us from our shuttles onboard equipment and approximate the following absorptionlines using least square chimethod

```
Wavelength |   Fmin   | delta_Lambda | Temperature
[[ 632.        0.7714    0.0004   450.     ]
 [ 690.        0.8857    0.0153   450.     ]
 [ 760.        0.8857    0.0168   150.     ]
 [ 720.        0.9429    0.0122   274.1379]
 [ 820.        0.9429    0.0034   201.7241]
 [ 940.        0.8286    0.0126   450.     ]
 [1400.        0.8286    0.0481   150.     ]
 [1600.        0.9429    0.007    274.1379]
 [1660.        0.9429    0.0313   150.     ]
 [2200.        0.9429    0.01     253.4483]
 [2340.        1.         0.0809   150.     ]
 [2870.        0.9429    0.0024   150.     ]]
```

Figure 6: Table

And we make an educated guess of which of these are actual absorption lines and not just noise. The ones we think to be real are the lines at $\lambda = 720$, $\lambda = 1600$ and $\lambda = 2200$ since they are all in the same range of temperature $\approx 270K$, which seem reasonable knowing from part 3 that our planetary surface should lie around $350K$, and also the dopplershift are all $\Delta\lambda \approx 0.1$. However, the flux from these lines are rather small (< 0.1), but may very well be due to noise at this particular wavelength. This is not an exact science and we are have to resort to making well educated guesses.

This means that the gases we expect to find are the following:

1. $\lambda = 720; H_2O$: This is the chemical composition of water
2. $\lambda = 1600; CH_4$: This is the chemical composition of methane
3. $\lambda = 1600; CH_4$: Another absorption line for methane

and we can see from the plots in figure (7) how the Gaussian line profiles line up with the measured data
This means that we expect our atmosphere to be 33% water and 66% methane and we can thus compute the values for the mean molecular weight by the formula we showed in the method section

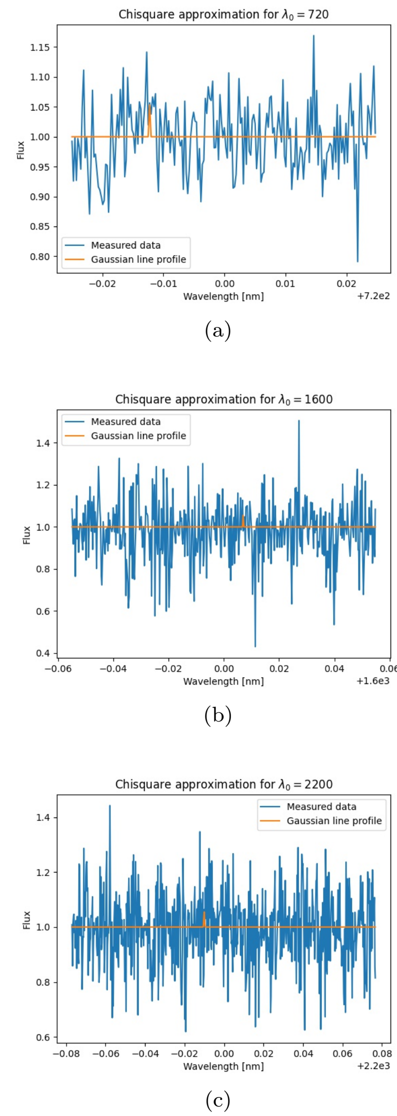
$$\mu = \frac{2}{3}(12 + 4) + \frac{1}{3}(2 + 16) = 16.666... \approx 17$$

We now have everything we need to create the temperature- and densityprofile of the planets atmosphere using the method we explained in the method section.

IV. DISCUSSION

We were unsuccessful in the implementation of the Hohmann transfer into a lower orbit around our destination, but luckily our friends at the ast2000 center helped us using their brand new software. We see from the atmospheric contents that our destination planet could sustain life, as we have found carbon, oxygen and water which are all the building blocks of life on earth - and may very well be the build blocks of life on extraterrestrial planets aswell!

Although we cannot be sure these absorptionlines aren't just noise that by chance fit our approximations well, we still can use this to make good profiles of the atmosphere that will still aid us in making the descent to the planetary surface. Sadly there is not much else we can do to strengthen our results. What we can do however is see that both methane and water consist of hydrogen and oxygen, which are two of the most abundant elements in the universe and we can expect to find traces of this in most planets and stars.



(a) (b) (c)

Figure 7: Absorptionlines

After reaching a low, stable orbit with the help from our friends at the ast2000 center we manage to shoot some great photos of the surface, see appendix (C), and also a nice location for us to land. The location we chose is near the equator, where it is usually warmer, and also it inbetween what looks like lakes, or oceans that might sustain wildlife or vegetation. Whatever it could be, it sure looks interesting.

We found a good spot for landing on the sunny side of the planet, that hopefully isn't a volcano in disguise, but we were not able to construct a model of this site in a coordinatesystem we could use to initiate our landing sequence. Again, we must turn to our friends at the ast2000 center for assistance.

V. CONCLUSION

We were successful in our implementation of the analyzing software, and also in making the profiles for temperature and density our our atmosphere. This means we have what we need to perform the landing sequence. We were not able however, to implement the injection maneuver from unstable to stable orbit, and received assistance from the ast2000 research center.

We did find location on the planetary surface that looked a promising spot for landing, but we did not manage to convert this into a timedependent coordinate system, and so we have no choice but to go in for landing and pray we do not land our dear astronauts in a volcano.

REFERENCES

- https://docs.scipy.org/doc/scipy/reference/generated/scipy.interpolate.interp1d.html
  (SciPy interpolation module, sciPy.interp1d)
- https://www.uio.no/studier/emner/matnat/astro/AST2000/h18/undervisningsmateriell_h2018/forelesningsnotater/part1c.pdf (Lecture notes - Chimethod)

Appendix A: Derivation of change in temperature

We start out with the equation for adiabatic gas

$$P^{1-\gamma}T^{\gamma} = C$$

and we want to take the derivate of this equation by radius r, using the productrule getting

$$(1-\gamma)P^{-\gamma}T^{\gamma}\frac{dP}{dr} + P^{1-\gamma}\gamma T^{\gamma-1}\frac{dT}{dr} = 0$$

doing some algebra, and reorganising the expression gives us the following

$$\frac{dP}{dr} = \frac{P(r)}{T(r)}\frac{\gamma}{\gamma-1}\frac{dT}{dr}$$

If we look at the expression for ideal gas law

$$P(r) = \frac{\rho(r)kT(r)}{\mu m_H}$$
$$\rho(r) = \frac{P(r)\mu m_H}{kT(r)}$$

and if we put this expression for density into equation for hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho(r)g(r)$$
$$\frac{dP}{dr} = -\frac{P\mu m_H g(r)}{kT(r)}$$

Now we have two different equations expressing $\frac{dP}{dr}$, and if we put them together

$$\frac{P(r)}{T(r)}\frac{\gamma}{\gamma-1}\frac{dT}{dr} = -\frac{P(r)\mu m_H g(r)}{kT(r)}$$

and we see that both $P(r)$ and $T(r)$ can be removed from the equation, and reorganising the equation gives us an expression for the change in temperature

$$\frac{dT}{dr} = \frac{1-\gamma}{\gamma}\frac{\mu m_H g(r)}{k}$$

which is what we wanted to find.

Appendix B: Derivation of density diff.eq

When finding the diff.eq for density we use hydrostatic equilibrium and the ideal gas law together. We know that ideal gaslaw gives us the expression

$$P = \frac{\rho(r)kT(r)}{\mu m_H}$$

and put in for P in the equation for hydrostatic equilibrium

$$\frac{d}{dr}\left(\frac{\rho(r)kT(r)}{\mu m_H}\right) = -\rho(r)g(r)$$
$$\frac{d}{dr}(\rho(r)T(r)) = -\frac{\rho(r)g(r)\mu m_H}{k}$$
$$\frac{d\rho(r)}{dr}T(r) + \rho(r)\frac{dT(r)}{dr} = -\frac{\rho(r)g(r)\mu m_H}{k}$$

which, after reorganizing, gives us

$$\frac{d\rho(r)}{dr} = -\frac{\rho(r)}{T(r)}\frac{dT(r)}{dr} - \frac{\rho(r)}{T(r)}\frac{g(r)\mu m_H}{k}$$

and we have what we wanted.

Appendix C: Images of planetary surface

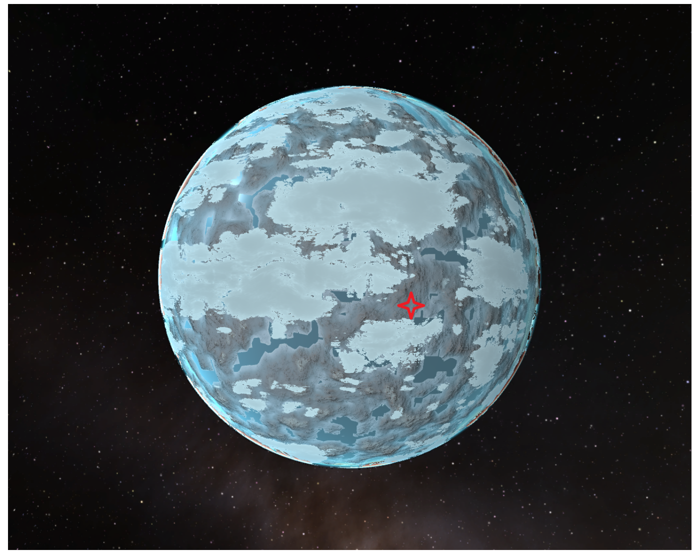


Figure 8: Image with landing site with red cross