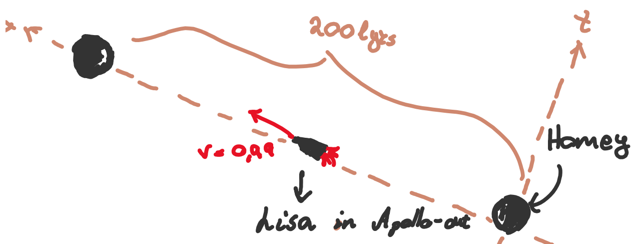
Exercise 3

Wednesday, November 18, 2020 10:15 AM

* Study of twin paradox.
* Use given .xml files, not make uesK.
  ↓
  3 .xml's
* Need 3 people!



Note! Planets are on the same reference system!

2 frames of ref.:

* planets: (x, t), Homey at $x=0=t$
* Apollo-out: (x', t'), at $x'=t'=0$

Events:

A: $x'=x=t'=t=0$ (departure)

B: Apollo-out arrival at it's desting

Part 1:

1. Fra Homey's ref. sys. $t = \frac{\Delta x}{v} = \frac{200\,ly}{0{,}99} = 202{,}02\ ly$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-v^2}} \Rightarrow \Delta t' = \Delta t\sqrt{1-v^2} = 202{,}02\sqrt{1-0{,}99^2} = 202{,}02 \cdot 0{,}14107 = 28{,}498\ ly$$

2. We use the symmetry argument

$\Delta t'_{Back} = 28.498$ ly

$\Delta t_{Back} = 202.02$ ly

Full trave for (x,t) : 204,04 ly
$\downarrow$
$(x',t') = 56,997$ ly

Part 2

We will switch frames now

* (x,t) at Apollo-out
* (x',t') at Homey

1. Apollo-out is at rest now while Homey is departing with $v = 0,99$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-v^2}}$$

$\Delta t = 28,498$

$\Delta t' = \Delta t \sqrt{1-v^2} = 4,0201$ ly

2. Using symmetry:

$4.0201 \cdot 2 = 8,0403$ ly

what?

3.

Firstly, we have defined referense frames for these two observers. We define labaratory frame for the planet at first, and the travelers frame for Lisa. Scientist from the Homey have observed that the distance to the destination planet is 200 light years, by traveling at near lightspeed v = 0.99 traveler would reach the planet in 202 years.

However, by using time dilation equation, we found that the time for the traveler would be 28 years, not 202 years! This means that Lisa's whristwatch tics much slower than the clocks at Homey for Homey's reference.

If Lisa choose to travel back to Homey, we can use symmetry to calculate how much Lisa has aged throughout the travel and compare how much her friends at Homey have aged.

$202 + 202 = 404$
$28 + 28 = 56$

Lisa would have aged by 56 years, while all of her friends would probably be dead :/ (probably not, future AST2000 students would probably have created a mashine that would restore all human memory to computers and this way people would live for eternity!)

However, if we would choose to mix up the reference frames, fishy things would begin to happen.

By mixing, we mean making Lisa's frame as the labaratory fram, and Homey as the traveler. Meaning that Lisa would stay at one place, and the planets would travel at $v = 0.99$ away from her and towards her.

We found out that the time it takes for Lisa to travel to destination planet is 28 years. This means that for Lisa, the destination planet comes towards her and that takes 28 years. However, if we use time dilation we find that for the planet it would now take 4 years! What?! This is where the paradox kicks in

> The planets observe Lisa's travel in two different times, 204 years and 8 years! Which one is the real time for Lisa's travel???

4. Are the two roles really identical?

No, this is because of the Lisa's acceleration. Lisa's has to change direction (and it therefore accelerates) towards Homey. Time dilations expression was accounted for constant velocity. This is why we get these weird results, the error was simply the wrong usage of the time dilation expression.

[Complete Solution To The Twins Paradox]


Complete Solution To The Twins Paradox

Part 3

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* Introducing one more planet and astronaut.
* Third planet (Beyond) is 400 ly from Homey (positive x-axis)
* Second spaceship (Apollo-in) travelling from Beyond with $v=-0.99c$ with Peter

* Three reference frames

  1. Planets (x,t)

     Homey at $x=0$

  2. Apollo-out (travelling from Homey to Destiny)

     (x',t')

     Lisa at $x'=0$

  3. Apollo-in

     Travelling from Beyond to Homey.

     (x'',t'')

     Peter at $x''=0$

Outgoing elevator and returning elevator

(x',t') (x'',t'')

Events:

A: $x_A = x'_A = 0$ $t_A = t'_A$

Lisa jumping on outgoing elevator at Homey.

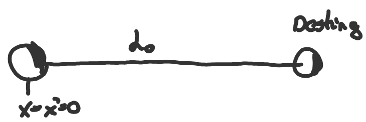
B: Lisa arrives at Destiny and jumps at returning elevator from outgoing elevator.

B': Takes place at

the position of Homey at the same time as Lisa arrives at Destiny in hers frame

* Use Lorentz transformations
* Distance between Homey and Destiny in planet frame is d_0

1. $t_B = \frac{d_0}{v}$



2. Lorentz transformation:

$$t'_B = -v\gamma x_B + \gamma t_B$$

$$t'_B = -v\gamma d_0 + \gamma \frac{d_0}{v}$$

$$= \frac{-vd_0}{\sqrt{1-v^2}} + \frac{d_0}{v\sqrt{1-v^2}} =$$

$$\frac{-0{,}99 \cdot 200}{\sqrt{1-0{,}99^2}} + \frac{200}{0{,}99\sqrt{1-0{,}99^2}}$$

$$= -1403{,}6 + 1432{,}1 = \underline{28{,}5}$$

3. $t_{B'} = \frac{d_0}{v} - vd_0$

* at t'_B , $x_{B'} = 0$
* $t'_{B'} = t'_B$

$$x'_{B'} = \gamma x_{B'} - v\gamma t_{B'}$$

$$x'_{B'} = 0 - v\gamma t_{B'}$$

$$x'_{B'} = -v\gamma t_{B'}$$

$$t_{B'} = -v\gamma x'_{B'} + \gamma t'_{B'}$$

$$= v\gamma(-v\gamma t_{B'}) + \gamma\left(-v\gamma d_0 + \frac{\gamma d_0}{v}\right)$$

$$t_{B'} = -v^2\gamma^2 t_{B'} - v\gamma^2 d_0 + \frac{\gamma^2 d_0}{v}$$

$$t_{B'} = \gamma^2\left(-v^2 t_{B'} - d_0 v + \frac{d_0}{v}\right)$$

$$\frac{t_{B'}}{\gamma^2} = \frac{d_0}{v} - v^2 t_{B'} - d_0 v$$

$$\frac{t_{B'}}{\gamma^2} + v^2 t_{B'} = \frac{d_0}{v} - d_0 v$$

$$t_{B'}\left(\frac{1}{\gamma^2} + v^2\right) = \frac{d_0}{v} - d_0 v$$

$$\frac{1}{\left(\frac{1}{\sqrt{1-v^2}}\right)^2} = \frac{1}{\frac{1}{(1-v^2)}} = (1-v^2)$$

$$t_B\left((1-v^2)+v^2\right) = \frac{d_0}{v} - d_0 v$$

$$t_B(1-v^2+v^2) = \frac{d_0}{v} - d_0 v$$

$$t_B = \frac{d_0}{v} - d_0 v$$

QED

$$t_B = \frac{200}{0{,}99} - 200\cdot 0{,}99 = 202 - 198 = \underline{4}$$

4.

We got introduced to a new planet and a new astronout. The third planet (Beyond) is 400 ly away from Homey in positic x direction.

The second spaceship (Apollo-In) is travelling from the Beyond to Homey witht the same velocity as Apollo-out, but in opposite direction.

As for the events, we have event A, which is when Lisa suddently departures from Homey on her way to Destiny.

Event B, is when Lisa finally arrives at Destiny, and suddently changes direction and departure back to Homey with same velocity.

We are also introduces to a new event, event B'. Event B' takes place at the position of Homey, at the same time as Lisa arrives at Destiny in her frame. This is when our new astronout, named Peter, passes Homey at position x_{B'} = 0. The event itself, is that Peter looks at the clocks on Homey as he passes by. Peter also sends a light signal from his spaceship which is observed at Homey.

In short, Peter traveling in positive x direction with the same velocity as Lisa, at the same position as Homey, observes clocks at Homey, and sends light signal. All this happening at the same time as Lisa arrives at Destiny in their reference frame (spaceship reference).

The very first thing we have done, is that we found out the expression for the time it took for lisa to arrive to destiny in planet ref. frame. Which is 202 years. However, by using Lorentz transformation method, we found out that in Lisa's ref. system, it took only 28.5 years (just as we found out in previous exercise).

However, if we would look at what Peter saw, he saw that clocks at Homey showed that it passed only 4 years since Lisa left Homey in their reference system. By knowing that simulteus events are relativ in all reference system, we know that because Lisa just arrived at Destiny in Peter's ref. system (spaceship ref.sys.), it doesn't mean that it just happened in Homey aswell, it would mean that at Homey, when Peter observed, Lisa haven't traveled a fraction of her journey yet!

If we now look at the animation, we can look at when the light was received at planets ref. system. We receive a blue light at year 4. At year 400, and we can see Lisa coming back at year 404.
As for the second frame, we can follow Lisa on her way toward destiny. We also can notice how lenght of the planets are contracted!

If we look at fram 3, we can follow the spaceship that Lisa jumped back on her way back to Homey, and the time it shows is 57.

> We learned in the previous questions that even if Homey clocks were observed at the same moment as the spaceship/elevator arrived at Destiny (in the outgoing frame), these two events (the observation of Homey clocks and the arrival at Destiny) were *not* simultaneous in the planet frame. For Lisa, only 4 years have passed on Homey when she arrives at Destiny. For observers on Homey on the other hand, Lisa arrived at Destiny when 202 years had passed.

Part 4

Wednesday, November 18, 2020 12:25 PM

Introducing new events

Event D: Peter jumping aboard the returning elevator from beyond.

$t_D = 0$ $x_D = 2d_0$

$x''_D = 0$ $t''_D = 0$

* In the returning elevator from peter is always at origin.

* At planets frame: $t_A = t_D$

Means that Lisa and Peter starts their journey simultaneusly.

Event B'': * Similar to event B'

* $t''_{B''} = t''_B$

* Person in returning elevator at $x'' =$ Homey looking at the clocks at Homey and sending blue light

Cannot use Lorentz transformation

Use space time interval

1: Want to write down Δs of

Δx_{BD} , Δt_{BD}

and

$\Delta x''_{BD}$, $\Delta t''_{BD}$

$$\Delta x_{BD} = |x_B - x_D| = |d_0 - 2d_0|$$

$$\Delta t_{BD} = \left|\frac{d_0}{v} - 0\right|$$

$$\Delta x''_{BD} = |x''_B - x''_D| = |0 - 0|$$

Peter is always at origin!

$$\Delta t''_{BD} = |t''_B - t''_D| = t''_B$$

$$\Delta s^2_{BD} = \left[\frac{d_0}{v}\right]^2 - (d_0 - 2d_0)^2$$

$$= \frac{d_0^2}{v^2} - d_0^2 = (\Delta s''_{BD})^2$$

$$= t''^2_B - 0^2$$

$$\Downarrow$$

$$\frac{d_0^2}{v^2} - d_0^2 = t''^2_B$$

$$\Downarrow$$

$$(t''_B)^2 = d_0^2\left(\frac{1}{v^2} - 1\right) \Rightarrow$$

$$(t''_B) = d_0\sqrt{\frac{1}{v^2} - 1} = \frac{d_0\sqrt{1-v^2}}{v}$$

Found previously that

$$t'_B = -v\gamma d_0 + \frac{\gamma d_0}{v}$$

Which gives us = 28,5

$$t''_B = \frac{d_0}{v\gamma} = \frac{200 \cdot \sqrt{1-0{,}99^2}}{0{,}99} = \frac{28{,}213}{0{,}99} = \underline{28{,}5}$$

Means that $t'_B = t''_B$

Event B happens at the same time both for Peter and Lisa.

2.

The yellow spaceships comes at year 28 to destiny, for yellow frame. As for the red frame, Peter leaves beyond and arrives to destiny after 28 years, same time as for the yello.

As mentioned in ref(1), t'_B = t''_B

Will try to find the time on Homey at the moment when Peter is reaching Destiny in the returning elevator frame.

3.

$$\Delta x_{DB''} = (x_D - x_{B''}) = (2d_0 - 0) = 2d_0$$

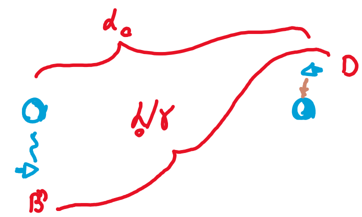
$$\Delta t_{DB''} = (t_D - t_{B''}) = |0 - t_{B''}| = t_{B''}$$

$$\Delta x''_{DB''} = (x''_D - x''_{B''}) = d_0/\gamma$$

0

at Homey's position

$\frac{d_0}{\gamma}$ → But how??



$$\Delta t''_{DB''} = (t''_D - t''_{B''}) = d_0/v\gamma$$

0

$t''_{B''} = t''_B = \frac{d_0}{\gamma v}$

5.

$$\Delta s^2_{DB''} = \Delta t^2_{DB} - \Delta x^2_{DB''} = t^2_{B''} - 4d_0^2$$

$$(\Delta s''_{DB''})^2 = (\Delta t''_{DB})^2 - (\Delta x''_{DB})^2 = \left(\frac{d_0}{\gamma v}\right)^2 - \left(\frac{d_0}{\gamma}\right)^2$$

$$t^2_{B''} - 4d_0^2 = \frac{d_0^2}{\gamma^2 v^2} - \frac{d_0^2}{\gamma^2}$$

$$\gamma^2 = \left(\frac{1}{\sqrt{1-v^2}}\right)^2 = \frac{1^2}{1-v^2} = \frac{1}{1-v^2}$$

$$t^2_{B''} = 4d_0^2 + \frac{d_0^2}{\gamma^2 v^2} - \frac{d_0^2}{\gamma^2}$$

$$t^2_{B''} = 4d_0^2 + \frac{d_0^2(1-v^2)}{v^2} - d_0^2(1-v^2)$$

$$= 4d_0^2 + \frac{d_0^2}{v^2} - d_0^2 - d_0^2 + d_0^2 v^2$$

$$= 4d_0^2 + \frac{d_0^2}{v^2} - 2d_0^2 + d_0^2 v^2$$

$$= d_0^2\left(4 + \frac{1}{v^2} - 2 + v^2\right)$$

$$t_{B''}^2 = d_0^2\left(\frac{1}{v^2} + v^2 + 2\right)$$

$$t_{B''} = d_0\sqrt{\frac{1}{v^2} + v^2 + 2} = \frac{d_0}{v} + d_0 v$$

$$= 200\sqrt{\frac{1}{0{,}99^2} + 0{,}99^2 + 2} = 400$$

The incomming spaceship comes just before Lisa and Peter comes back, it reads clocks as 400 years.