AST 2000 - Part 10
Studying your star

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This abstract is abstract.

I. THE ORIGIN AND CURRENT STATE OF YOUR STAR

A. Main sequence Star

1. *Is our star main sequence star?*

We will be answering if our star is in the main sequence. In order to do this we will be determining the Luminocity of our star using its radius and the surface temperature. We know that the HR-diagram is just a plot with x-axis as its surface temperature in Kelvin, and y-axis as the ration between the stars and suns luminocity. The problem we are going to solve is simply to find the Luminocity of our star, and plot it in HR-diagram. By observing the plot, we will then determine if our star is indeed in the main sequence!

By using AST2000-tools we can find our suns surface temperature and radius. We find that the surface temperature and its radius is

$$r = 317551km = 317551000m$$

$$T = 3715, 41K$$

We know that the luminocity is defined as the energy emmited in a given time $L = \frac{dE}{dt}$, and flux is given as that energy emmited per area portion

$$F = \frac{dE}{dtdA}$$

. This means that

$$F = \frac{L}{dA} = \sigma T^4$$

Here we have used the Stefan-Boltzman definition of the flux. By simplifying the expression we end up with

$$L = \sigma T^4 dA = \sigma T^4 4\pi r^2$$

By putting the known values we end up with Luminocity of

$$L = 1.3691e25$$

As for the ration between the stars L and the suns L_s

$$\frac{L}{L_s} = \frac{1.369e25}{3.826e26} = 0.0357$$

By using the documentation and example code from StarPopulation from class of Ast2000tools, we get this plot as the result.

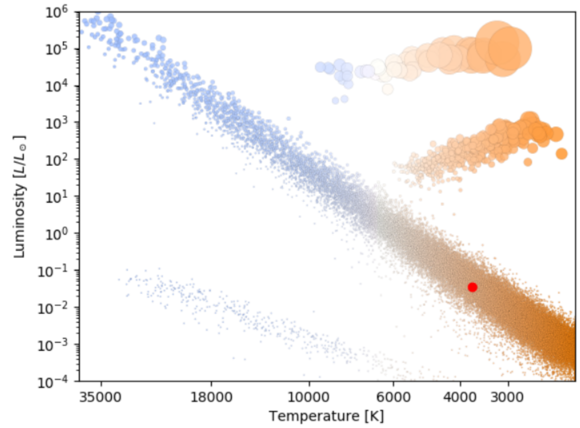

Figure 1. HR-diagram, here we can see our star marked as a red dot in the main sequence

From figure above we can now clearly see that our star is indeed in the main sequence. And therefore we can conclude that our star is a main sequance star.

2. *How long will our star live?*

There is a relationship between the mass of a star and its remaining time on the main sequence. Firstly, the time left for our star to stay in the main sequence, is dependent of the available hydrogen in its core. As long star managed to fuse hydrogen into the helium, and its enough of the fuel (hydrogen), the star has the hydrostatic equilibrium which states the the pressure of the core balances the gravitational pull of the star. Meaning that the pressure and the fusion "fights" back the pull of the gravity. From Lecture notes we can see that we can derive from this hydrastatic equilibrium to the relation of the pressure and the radius, which is proportional to the relation between the mass and the radius of the star. If we assume that there is ideal gas, we can further see that that this pressure is proportional to the temperure times density. Looking at these assumptions and statements, we can get to that this temperature is proportional to the ration between the mass and the radius of the star.

Further, lecture notes states that the luminocity is proportional to RT^4l which means that this energy (emmited energy as the luminocity) is dependent og the temperature, and, and the length that these particles (electrons which travel or emmit its energy towards the surfrace from the core) travel.

By assuming all of this, we end up that this Luminocity is proportional to the mass in power of 3. However, by observation we can tell that if the star is low or medium mass star, then the luminocity must be proportional to the mass in power of 4!

As for the lifetime of the star, we can simply calculate how much energy the star can emmit by its mass, then we find the Luminocity, and simply divide the total energy that the star can emmit, by the luminocity. The expression for this is

$$t = \frac{pMc^2}{L}$$

Here we have p, p is the fraction of the mass of a star that is converted to energy. Here we are going to assume that the value of this p is 10 percent (0.1). We assume this because our star is very similar to the sun.

If we now put all the values to the expression above, we get

$$t = \frac{0.1 \cdot 8.163e29 \cdot (3e8)^2}{3.82e26} = 1.923e19$$

This means that our star will still live for 1.923e10 billion years in the main sequence!

3. *Is our star well-behaved main sequence star?*

By saying well-behaved main sequence star, we mean that all proportionalities works as intended. Previously, we mentioned that the temperature of the surface is proportional to the ratio between the mass and the radius of the sun. Additionaly, we mentioned that the luminocity is proportional to the mass of the star in power of 4 og 3! Because of star is a small star, we therefore use the relation that the luminocity is proportional to the mass in power of 4.
As summary, we have

$$T \propto \frac{M}{R}$$

$$L \propto M^4$$

B. Giant Molecular Cloud

From the theory we know that the starts are born from giant molecular clouds consisting of particles. By the amount of those particles and the density of them, there is possibilities that gravitational force will drag those particles together resulting a star.

Here we will be assuming that our star started out as a spherically symmetric giant molecular cloud. In addition we will also be assuming that our giant molecular cloud began to collapse itself without the help of any other forces than gravitation.

As for more sepcific assumtions, we assume that this cloud has temperature of 10K and consisted of 75 percent of Hydrogen, and 25 percent of Helium atoms.

We are interested in what was the largest possible radius of this cloud.
In order to do this, we will be using Jeans criterion. The Jeans criterion states that in order for the cloud to collapse, this criterion has to be fulfilled

$$2K < U$$

Where K is the kinetic energy and U is the potential energy. This simply states that the potential energy must be double as high than the kinetic energy. If the particles would have high kinetic energy, then these particles would simply scatter away from each other because of their velocity. However, if they have low velocity, then the gravitation will affect those particles from the mass center and make them collapse!

If we want to make this more general, we have to define the kinetic- and potential energy.
From previous parts of lecture notes we know that

$$U = -\frac{3GM^2}{5R}$$

$$K = \frac{3}{2}NkT$$

Where G is the gravitational constat, M and R is the mass and the radius of our star, N is the number of the particles in the gass, T the temperature of the gass and k as the boltzmanns constant.

We can simplify some of those constant such as

$$N = \frac{M}{\mu m_H}$$

We can then rewrite the condition as

$$\frac{3MkT}{\mu m_H} < \frac{3GM^2}{5R}$$

With some algebraic manipulation we can solve this to condition for minimum radius of the cloud. Also called the Jeans lenght

$$R = (\frac{15kT}{4\pi G\mu m_H \rho})$$

Where ρ is density of the particles.

II. NUCLEAR REACTIONS IN YOUR STAR CORE

In this task we are invoking the following assumptions

- Density profile of the star is uniform ρ_0
- The gaspressure inside the star follows ideal gas law
- The star is in hydrostatic equilibrium
- The star consist purely of protons with mass $m_H = 1.673 \cdot 10^{-27}$ kg

A. Core Temperature

1. We can use the assumption of a uniform density to derive an expression for the mass of a spherical part of the star within a radius r, using that $m = \rho V$. In our case, the volume is a function in r
$$V(r) = \frac{4}{3}\pi r^3$$
Paired with the density we find M(r) as
$$M(r) = \rho_0 V(r) = \frac{4}{3}\rho_0 \pi r^3$$

2. Furthermore we can combine the assumptions of hydrostatic equilibrium (2) and pressure (1) (ideal gas law)
$$P = \frac{\rho_0 k T(r)}{\mu m_H} \tag{1}$$
$$\frac{dP}{dr} = -\rho_0 G \frac{M(r)}{r^2} \tag{2}$$
with the expression for mass we derived earlier. Substituting for P and M into equation (2) we get the equation
$$\frac{d}{dr}\left(\frac{\rho_0 k T(r)}{\mu m_H}\right) = -\rho_0 G \frac{\frac{4}{3}\rho_0 \pi r^3}{r^2}$$
$$\frac{d}{dr}T(r) = -\frac{4\pi}{3} G \rho_0 \frac{\mu m_H}{k}$$
where we have used that the derivate is a linear operation, and we have what we wanted to find. ■

3. Next we want to integrate this expression from center of the star to the surface to find an expression for temperature $T(r)$

$$\int_0^R \frac{d}{dr}T(r)dr = \int_0^R -\frac{4\pi}{3} G \rho_0 \frac{\mu m_H}{k} dr = -\frac{2\pi}{3} G \rho_0 \frac{\mu m_H}{k} R^2$$

And from the fundamental theorem of calculus we know that this same integral can be expressed as
$$\int_0^R \frac{d}{dr}T(r) = T(R) - T(0)$$
meaning we can express $T(0) = T_c$, i.e the core temperature of the star as
$$T_c = T(R) + \frac{2\pi}{3} G \rho_0 \frac{\mu m_H}{k} R^2$$
which is what we wanted to find. ■

Solving this using t radius of our star, $R = 317551000$ we find the core temperature to be
$$T_c = 10394543 \,\text{Kelvin}$$
where we used that $\rho_0 = \frac{M_{star}}{V_{star}}$, and that the mean molecular weight $\mu = 1$, since we assumed all particles in the gas are protons of mass m_H, and that $T(R)$ is the surface temperature of the star.

The core temperature of our star is 10 million Kelvin, which corresponds nicely with what we expect from our a star on the main sequence like ours.

B. Energy Production and Luminosity

With the core temperature in hand we can now calculate our stars luminosity, or energyproduction per time, based on the nuclear reactions in the stellar core. We've found our stars core temperature to be $\approx$ 10 million Kelvin, and with this we can make the following assumptions about the stellar core and its nuclear reactions

- Core temperature is constant through the core
- The core radius is 0.2R, where R is the total radius of the star
- Core density is uniform
- Energy production in the core occurs by pp-chain and CNO-cycle
- Core consists of 74.5% Hydrogen, 25.3% Helium and 0.2% Carbon, Nitrogen and Oxygen

Energyproduction in a stellar core is given by the general expression
$$\epsilon_{AB} = \epsilon_{0,AB} X_A X_B \rho^\alpha T^\beta$$
where ϵ_0 is a constant related to the type of reaction, X_A and X_B are the mass fractions of the nuclei in the reaction and T and ρ are temperature and density of the stellar core.

By our assumptions of the stellar core we get the two following equations we need to solve in order to find produced energy in the core from PP-chain and CNO-cycle

$$\epsilon_{PP} = \epsilon_{0,PP} X_H^2 \rho T_6^4 \tag{3}$$
$$\epsilon_{CNO} = \epsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20} \tag{4}$$

The values for ϵ_0 for the two reactions are constants found by experiments, and given to us by our friends at the ast2000 center as

$$\epsilon_{0,pp} = 1.08 \cdot 10^{-12}$$
$$\epsilon_{0,cno} = 8.24 \cdot 10^{-31}$$

and from our assumptions we find the mass fractions $X_H = 0.745$ and $X_{CNO} = 0.002$. Using this with the density, temperature and radius we found in task A we get

$$\epsilon_{pp} = 3.5986 \cdot 10^{-05}\,\text{W/kg}$$
$$\epsilon_{cno} = 7.4205 \cdot 10^{-10}\,\text{W/kg}$$

We see that the pp-chain produces a huge amount of energy compared to the CNO-chain, and it is clearly the dominate energyproduction chain in our stellar core. This follows from what we know about the two nuclear reactions.
The pp-chain is the most dominate with core temperatures $\approx$ 15 million Kelvin, and the CNO-cycle the most dominate with core temperatures $\approx$ 20 million Kelvin. With out stellar core temperature at 10 million Kelvin we expect the PP-chain to be the most dominate, and this corresponds with our calculations.

These values can be used to find the luminosity from the core reactions, knowing that ϵ_{AB} is energy per time per kg, and luminosity is energy per time, with the relation

$$L_{AB} = \epsilon_{AB} M_c$$

where M_c is the mass of the core, found by using that radius of the core is 0.2R and the expression for density.
Thus we find

$$L_{pp} = 2.3496 \cdot 10^{23}\,\text{W}$$
$$L_{cno} = 4.8450 \cdot 10^{18}\,\text{W}$$

and put together we find the total luminosity of our star from the nuclear reactions in the stellar core

$$L = 2.3497 \cdot 10^{23}\,\text{W}$$

and we see that the contribution from the CNO-cycle almost has no effect on the total luminosity from our star. Comparing this result to the one we found in task 1, we see that they are not a match. This result is just $\frac{1}{100}$ of the result in task 1A. This is likely due to the fact that in this task we made major, unrealistic assumptions about the stellar core, like it keeping a constant temperature, or a constant density throughout the entire core. This is just not the case, and will naturally skewer our results away from the correct one.

III. THE DEATH OF YOUR STAR

REFERENCES

- Lecture notes 3C - Nuclear Reactions in stellar cores
- Reference 2

Appendix A: Name of appendix

This will be the body of the appendix.

Appendix B: This is another appendix

Tada.

Note that this document is written in the two-column format. If you want to display a large equation, a large figure, or whatever, in one-column format, you can do this like so:

This text and this equation are both in one-column format. [1]

$$\frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial}{\partial t}\Psi \tag{B1}$$

Note that the equation numbering (this: B1) follows the appendix as this text is technically inside Appendix B. If you want a detailed listing of (almost) every available math command, check: https://en.wikibooks.org/wiki/LaTeX/Mathematics.

And now we're back to two-column format. It's really easy to switch between the two. It's recommended to keep the two-column format, because it is easier to read, it's not very cluttered, etc. Pro Tip: You should also get used to working with REVTeX because it is really helpful in FYS2150.

One last thing, this is a code listing:

```
This will be displayed with a cool programming font!
```

You can add extra arguments using optional parameters:

```
This will be displayed with a cool programming font!
```

You can also list code from a file using `lstinputlisting`. If you're interested, check https://en.wikibooks.org/wiki/LaTeX/Source_Code_Listings.

This is a basic table:

Table I. This is a nice table

Hey	Hey	Hey
Hello	Hello	Hello
Bye	Bye	Bye

You can a detailed description of tables here: https://en.wikibooks.org/wiki/LaTeX/Tables.

This is a more advanced table:

Table II. Tabelleksempel

Partikkelindeks	**Posisjon**	**Hastighet**
(i)	(m)	(m/s)
0	139.22	12.4
1	14.88	18.7
2	233.9	10.10
3	816.12	13.4

I'm not going to delve into Tikz in any level detail, but here's a quick picture:
If you want to know more, check: https://en.wikibooks.org/wiki/LaTeX/PGF/TikZ.

[1] This equation is actually from quantum mechanics. "It's called Schrödinger's Time-Dependent Wave Equation", named after the awesome Austrian physicist Erwin Rudolf Josef Alexander Schrödinger. Yep, the "Schrödinger's cat" guy. Pretty cool dude actually, check his wiki page: https://en.wikipedia.org/wiki/Erwin_Schrodinger. He was physics' no. 1 Ladies' man if there ever was one. Anyway, you will learn more about this equation in FYS2140. You can also find it printed on a glass wall in the UiO Physics Building (it really is that important).

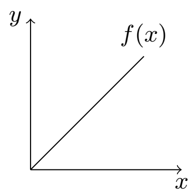

Figure 2. This is great caption