Hypothetical Judgement

To know $\mathcal{J}_2 (\mathcal{J}_1)$ is to know $\mathcal{J}_2$ under the circumstances* that you know $\mathcal{J}_1$.

*The evidence for $\mathcal{J}_2$ is defined by induction (pattern matching) on the evidence for $\mathcal{J}_1$.

General Judgement

To know $\big|_x \, \mathcal{J}_x$ is to know $[\mathcal{M}/x]\,\mathcal{J}$ for any expression $\mathcal{M}$ (of the correct arity).

To know $m \Rightarrow m'$ is to know that m' is the value* of m.

*open-ended

To know $\mathcal{A}$ type is to know that $\mathcal{A} \Rightarrow \mathcal{A}'$ such that you know

1) what counts as a canonical verification* of $\mathcal{A}'$, and

2) when two canonical verifications of $\mathcal{A}'$ are equal.

* open-ended

To know $M \in A$ $\left\{ \begin{array}{l} \text{presupposing} \\ A \text{ type} \end{array} \right.$ $A \to A'$

is to know that $M \Rightarrow M'$ such that you know that M' is a canonical verification of A'.

To know $m = n \in \mathcal{A}$ presupposing

$\mathcal{A}$ type $\quad \mathcal{A} \Rightarrow \mathcal{A}'$

$m \in \mathcal{A} \quad m \Rightarrow m'$

$n \in \mathcal{A} \quad n \Rightarrow n'$

is to know that m' and n' are equal canonical verifications of $\mathcal{A}'$.

To know $A = B$ type

presupposing

A type

B type

is to know

1) $\mid_{m}\ m \in B \ (m \in A)$,

2) $\mid_{m}\ m \in A \ (m \in B)$,

3) $\mid_{m,n}\ m = n \in A \ (m = n \in B)$, and

4) $\mid_{m,n}\ m = n \in B \ (m = n \in A)$

Contexts & Sequents

Γ ctx

$\Gamma \gg A$ type

$\Gamma \gg A = B$ type

$x \# \Gamma$

$\Gamma \gg M \in A$

$\Gamma \gg M = N \in A$

explained by mutual induction

To know Γ ctx is to know that $\Gamma \Rightarrow \cdot$; or that $\Gamma \Rightarrow \Delta, x : A$ such that you know Δ ctx, $x \# \Delta$, and $\Delta \gg A$ type.

To know $x \# \Gamma$ (presupposing Γ ctx $\Big\langle$ $\Gamma \Rightarrow \cdot$ or $\Gamma \Rightarrow \Delta, y{:}A$)

is,

if $\Gamma \Rightarrow \cdot$, immediate

| $\Gamma \Rightarrow \Delta, y{:}A$, to know that $x \neq y$,
and $x \# \Delta$.

To know $\Gamma \gg \mathcal{A}$ type $\left\{\begin{array}{l} \text{presupposing} \\ \Gamma \text{ ctx} \end{array}\right.$ $\Gamma \Rightarrow \cdot$ or $\Gamma \Rightarrow \Delta, x{:}B$

is to know,

if $\Gamma \Rightarrow \cdot$, that $\mathcal{A}$ type

$\mid$ $\Gamma \Rightarrow \Delta, x{:}B$, that $\mid_{M}$ $\Delta \gg [M/x]\mathcal{A}$ type

$(\Delta \gg M \in B)$

and $\mid_{M,N}$ $\Delta \gg [M/x]\mathcal{A} = [N/x]\mathcal{A}$ type

$(\Delta \gg M = N \in B)$

To know $\Gamma \gg A = B$ type $\begin{cases} \text{presupposing} \\ \Gamma \text{ ctx} \\ \Gamma \gg A \text{ type} \\ \Gamma \gg B \text{ type} \end{cases}$

Γ ctx: $\Gamma \Rrightarrow \cdot$ or $\Gamma \Rrightarrow \Delta, x{:}C$

is to know,

if $\Gamma \Rrightarrow \cdot$, that $A = B$ type

| $\Gamma \Rrightarrow \Delta, x{:}C$, that

(1) $\Big|_{M} \Delta \gg [M/x]A = [M/x]B$ type

$(\Delta \gg M \in C)$

(2) $\Big|_{M,N} \Delta \gg [M/x]A = [N/x]B$ type

$(\Delta \gg M = N \in C)$

To know $\Gamma \gg M \in A$ presupposing

Γ ctx — $\Gamma \Rightarrow \cdot$ or $\Gamma \Rightarrow \Delta, x{:}B$

$\Gamma \gg A$ type

is to know,

if $\Gamma \Rightarrow \cdot$, that $M \in A$

| $\Gamma \Rightarrow \Delta, x{:}B$, that

(1) $|_{N}\ \Delta \gg [N/x]M \in [N/x]A \ (\Delta \gg N \in B)$

(2) $|_{N,N'}\ \Delta \gg [N/x]M = [N'/x]M \in [N/x]A$

$(\Delta \gg N = N' \in B)$

To know $\Gamma \gg M = N \in A$ (presupposing $\left\{ \begin{array}{l} \Gamma \ \mathsf{ctx} \\ \Gamma \gg M \in A \\ \Gamma \gg N \in A \end{array} \right.$

$\Gamma\ \mathsf{ctx}$: $\Gamma \Rightarrow \cdot$ or $\Gamma \Rightarrow \Delta, x : B$

is to know,

if $\Gamma \Rightarrow \cdot$, that $M = N \in A$

| $\Gamma \Rightarrow \Delta, x : B$, that

(1) $\mid_{O} \quad \Delta \gg [O/x]M = [O/x]N \in [O/x]A$

$(\Delta \gg O \in B)$

(2) $\mid_{O, O'} \quad \Delta \gg [O/x]M = [O'/x]N \in [O/x]A$

$(\Delta \gg O = O' \in B)$