

$$\begin{array}{r|l} \text{אברהם} & \text{נוחמן} \\ \hline \text{אברהם, יוסף, דוד} & \text{דוד, יוסף, אברהם} \\ \hline \text{אברהם} & \text{דוד, יוסף, אברהם} \\ \hline \text{אברהם} & \text{דוד, יוסף, אברהם} \end{array}$$

1, 1, 1

$$S = \{ -1+t^2, -1+2t-t^2, 6+5t \} \quad (1)$$

$$P_2(K) \quad R^3 \quad T = \left\{ \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 5 \\ 0 \end{pmatrix} \right\}$$

$$x \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + y \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} + z \begin{pmatrix} 6 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 & 6 \\ 0 & 2 & 5 \\ 1 & -1 & 0 \end{pmatrix} \xrightarrow{R_1+R_3 \rightarrow R_3} \begin{pmatrix} -1 & -1 & 6 \\ 0 & 2 & 5 \\ 0 & -2 & 6 \end{pmatrix} \xrightarrow{R_2+R_3 \rightarrow R_3} \begin{pmatrix} -1 & -1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 11 \end{pmatrix}$$

$$S'' \text{ ב } T \quad \dim(R^3) = 3$$

$$P_2(K) \quad S \quad R^3$$

$$\begin{pmatrix} -1 & -1 & 6 & | & 1 & 0 & 0 \\ 0 & 2 & 5 & | & 0 & 1 & 0 \\ 1 & -1 & 0 & | & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1+R_3 \rightarrow R_3} \begin{pmatrix} -1 & -1 & 6 & | & 1 & 0 & 0 \\ 0 & 2 & 5 & | & 0 & 1 & 0 \\ 0 & -2 & 6 & | & 1 & 0 & 1 \end{pmatrix} \rightarrow$$

$$\xrightarrow{R_2+R_3 \rightarrow R_3} \begin{pmatrix} -1 & -1 & 6 & | & 1 & 0 & 0 \\ 0 & 2 & 5 & | & 0 & 1 & 0 \\ 0 & 0 & 11 & | & 1 & 1 & 1 \end{pmatrix} \xrightarrow{\begin{matrix} -11R_1+6R_3 \rightarrow R_1 \\ -11R_2+5R_3 \rightarrow R_2 \end{matrix}} \begin{pmatrix} 11 & 11 & 0 & | & -5 & 6 & 6 \\ 0 & -22 & 0 & | & 5 & -6 & 5 \\ 0 & 0 & 11 & | & 1 & 1 & 1 \end{pmatrix} \rightarrow$$

$$\xrightarrow{2R_1+R_2 \rightarrow R_1} \begin{pmatrix} 22 & 0 & 0 & | & -5 & 6 & 17 \\ 0 & -22 & 0 & | & 5 & -6 & 5 \\ 0 & 0 & 11 & | & 1 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & | & -\frac{5}{22} & \frac{6}{22} & \frac{17}{22} \\ 0 & 1 & 0 & | & -\frac{5}{22} & \frac{6}{22} & -\frac{5}{22} \\ 0 & 0 & 1 & | & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 4 & 2 & 2 \\ 1 & 3 & 7 & 3 \\ 3 & 14 & -4 & 4-3k \\ -2 & -8 & -k^2+2k-4 & -4 \end{pmatrix} \begin{array}{l} R_2 - R_1 \rightarrow R_2 \\ \rightarrow \\ -3R_1 + R_3 \rightarrow R_3 \\ 2R_1 + R_4 \rightarrow R_4 \end{array}$$

$$\begin{pmatrix} 1 & 4 & 2 & 2 \\ 0 & -1 & 5 & 1 \\ 0 & 2 & -10 & -2-3k \\ 0 & 0 & -k^2+2k & 0 \end{pmatrix} \xrightarrow{2R_2 + R_3 \rightarrow R_3} \begin{pmatrix} 1 & 4 & 2 & 2 \\ 0 & -1 & 5 & 1 \\ 0 & 0 & 0 & -3k \\ 0 & 0 & -k^2+2k & 0 \end{pmatrix} \rightarrow$$

$$R_3 \leftrightarrow R_4 \rightarrow \begin{pmatrix} 1 & 4 & 2 & 2 \\ 0 & -1 & 5 & 1 \\ 0 & 0 & k(2-k) & 0 \\ 0 & 0 & 0 & -3k \end{pmatrix}$$

$$\text{rank}(A) = 2, \quad k=0 \quad \text{✓/ps}$$

$$\text{rank}(A) = 3, \quad k=2 \quad \text{✓/ps}$$

$$\text{rank}(A) = 4, \quad k \neq 0, 2 \quad \text{✓/ps}$$

$$\dim(\text{Nul}(A)) = 4 - 2 = 2, \quad k=0 \quad \text{✓/ps (2)}$$

$$\dim(\text{Nul}(A)) = 4 - 3 = 1, \quad k=2 \quad \text{✓/ps}$$

$$\dim(\text{Nul}(A)) = 4 - 4 = 0, \quad k \neq 0, 2 \quad \text{✓/ps}$$

$$k=0 \quad \text{✓/ps} \quad \text{5N'0J-N dim(Nul(A)) 12}$$

$$\underline{k=0} \quad \begin{pmatrix} 1 & 4 & 2 & 2 \\ 0 & -1 & 5 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 + 4R_2 \rightarrow R_1} \begin{pmatrix} 1 & 0 & 22 & 6 \\ 0 & -1 & 5 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x = -22z - 6t \\ y = 5z + t \end{cases} \quad \text{508} \quad \begin{pmatrix} -22z - 6t \\ 5z + t \\ z \\ t \end{pmatrix} = z \begin{pmatrix} -22 \\ 5 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -6 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

-4-

$$\begin{pmatrix} -22 \\ 5 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -6 \\ 1 \\ 0 \\ 1 \end{pmatrix} : \text{Null}(A) \text{ } \text{ס'ע } 0'0\text{ן}$$

א/ס'ע/אן א/ס'ע/אן אן - $\dim(\text{Col}(A)) = 2$ (?)

$$\begin{pmatrix} 1 \\ 1 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ 14 \\ -8 \end{pmatrix} : \text{Col}(A) \text{ } \text{ס'ע } 0'0\text{ן}$$

א/ס'ע/אן א/ס'ע/אן אן, $\dim(\text{Null}(A)) = 1$, $\kappa = 2$ א/ס'ע/אן ()
 א/ס'ע/אן א/ס'ע/אן אן

$$|A| = \begin{vmatrix} 2 & 4 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 2 \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} - 4 \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} + 2 \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} =$$

ק'מ'ע'פ'י'ת ה'כ'ח'ל'ק'ו'ן

$$= 2 \cdot 4 - 4 \cdot 5 + 2 \cdot 2 = -8 \neq 0$$

ה'מ'ע'י'כ'ת ה'ח'ל'ק'ו'ן

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 4 & 3 & 5 \\ 2 & 3 & 7 \end{vmatrix} = 2 \begin{vmatrix} 3 & 5 \\ 3 & 7 \end{vmatrix} - \begin{vmatrix} 4 & 5 \\ 2 & 7 \end{vmatrix} + \begin{vmatrix} 4 & 3 \\ 2 & 3 \end{vmatrix} = 2 \cdot 6 - 18 + 6 = 0$$

א'ת ה'מ'ע'י'כ'ת ה'ח'ל'ק'ו'ן א/ס'ע/אן א/ס'ע/אן אן א/ס'ע/אן א/ס'ע/אן אן

$$|A_2| = \begin{vmatrix} 70 & 4 & 2 \\ 6 & 2 & 1 \\ 9 & 2 & 3 \end{vmatrix} = 70 \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} - 4 \begin{vmatrix} 6 & 1 \\ 9 & 3 \end{vmatrix} + 2 \begin{vmatrix} 6 & 2 \\ 9 & 2 \end{vmatrix} =$$

$$= 70 \cdot 4 - 4 \cdot 9 + 2 \cdot (-6) = -8$$

$$y = \frac{|A_2|}{|A|} = \frac{-8}{-8} = 1$$

ה'י'ס $T: M_{2 \times 2}(K) \rightarrow P_2(K)$

(2)

$$T\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right) = 1 + 3t^2, \quad T\left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right) = 3 + t + 7t^2,$$

$$T\left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right) = 4 + 3t + 6t^2, \quad T\left(\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right) = -3 - 2t - 5t^2$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + d \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad | \quad K$$

$$\begin{aligned} T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) &= a T\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right) + b T\left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right) + c T\left(\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right) + d T\left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right) \\ &= a(1 + 3t^2) + b(3 + t + 7t^2) + c(4 + 3t + 6t^2) + d(-3 - 2t - 5t^2) \\ &= (a + 3b + 4c - 3d) + (b + 3c - 2d)t + (3a + 7b + 6c - 5d)t^2 \end{aligned}$$

(2)

$$A = \begin{pmatrix} 1 & 3 & 4 & -3 \\ 0 & 1 & 3 & -2 \\ 3 & 7 & 6 & -5 \end{pmatrix}$$

$$A \cdot \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

(2)

$$\left(\begin{array}{cccc|c} 1 & 3 & 4 & -3 & 1 \\ 0 & 1 & 3 & -2 & 2 \\ 3 & 7 & 6 & -5 & -1 \end{array} \right) \xrightarrow{-3R_1 + R_3 \rightarrow R_3} \left(\begin{array}{cccc|c} 1 & 3 & 4 & -3 & 1 \\ 0 & 1 & 3 & -2 & 2 \\ 0 & -2 & -6 & 4 & -4 \end{array} \right) \rightarrow$$

$$\xrightarrow{+2R_2 + R_3 \rightarrow R_3} \left(\begin{array}{cccc|c} 1 & 3 & 4 & -3 & 1 \\ 0 & 1 & 3 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{R_1 - 3R_2 \rightarrow R_1} \left(\begin{array}{cccc|c} 1 & 0 & -5 & 3 & -5 \\ 0 & 1 & 3 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$a = 5c - 3d - 5, \quad b = -3c + 2d + 2$$

נ"ע א, c

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 5c - 3d - 5 & -3c + 2d + 2 \\ c & d \end{pmatrix}$$

c, d שו

$$v_1 = \begin{pmatrix} 0 \\ 1 \\ -2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} 2 \\ -5 \\ 7 \\ -12 \end{pmatrix}, v_3 = \begin{pmatrix} 2 \\ 0 \\ h \\ -h \end{pmatrix}, v_4 = \begin{pmatrix} -2 \\ k \\ -4 \\ -k \end{pmatrix} \quad \underline{4, \text{ ית}} \quad 1/k$$

$$v_3 = x v_1 + y v_2 \quad \Leftrightarrow v_3 \in \text{Span}(v_1, v_2)$$

$$\left(\begin{array}{cc|c} 0 & 2 & 2 \\ 1 & -5 & 0 \\ -2 & 7 & h \\ 3 & -12 & -h \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c} 1 & -5 & 0 \\ 0 & 2 & 2 \\ -2 & 7 & h \\ 3 & -12 & -h \end{array} \right) \xrightarrow{\begin{array}{l} 2R_1 + R_3 \rightarrow R_3 \\ -3R_1 + R_4 \rightarrow R_4 \end{array}}$$

$$\left(\begin{array}{cc|c} 1 & -5 & 0 \\ 0 & 2 & 2 \\ 0 & -3 & h \\ 0 & 3 & -h \end{array} \right) \xrightarrow{\begin{array}{l} 3R_2 + 2R_3 \rightarrow R_3 \\ -3R_2 + 2R_4 \rightarrow R_4 \end{array}} \left(\begin{array}{cc|c} 1 & -5 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 6+2h \\ 0 & 0 & -6-2h \end{array} \right)$$

$h = -3 \Leftrightarrow 6+2h = 0$ זהו פתרון יחיד
 זהו \mathbb{R}^4 שם 0'ור בלתי תלוי v_1, v_2, v_3, v_4 / י, $\dim(\mathbb{R}^4) = 4$ /
 ש"ת א

$$\left(\begin{array}{cccc} 0 & 2 & 2 & -2 \\ 1 & -5 & 0 & k \\ -2 & 7 & h & 4 \\ 3 & -12 & -h & -k \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cccc} 1 & -5 & 0 & k \\ 0 & 2 & 2 & -2 \\ -2 & 7 & h & 4 \\ 3 & -12 & -h & -k \end{array} \right) \xrightarrow{\begin{array}{l} \frac{1}{2}R_2 \rightarrow R_2 \\ 2R_1 + R_3 \rightarrow R_3 \\ -3R_1 + R_4 \rightarrow R_4 \end{array}}$$

$$\rightarrow \left(\begin{array}{cccc} 1 & -5 & 0 & k \\ 0 & 1 & 1 & -1 \\ 0 & -3 & h & 2k+4 \\ 0 & 3 & -h & -4k \end{array} \right) \xrightarrow{\begin{array}{l} 3R_2 + R_3 \rightarrow R_3 \\ -3R_2 + R_4 \rightarrow R_4 \end{array}} \left(\begin{array}{cccc} 1 & -5 & 0 & k \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 3+h & 2k+1 \\ 0 & 0 & -3-h & 3-4k \end{array} \right) \rightarrow$$

$$\xrightarrow{R_3 + R_4 \rightarrow R_4} \left(\begin{array}{cccc} 1 & -5 & 0 & k \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 3+h & 2k+1 \\ 0 & 0 & 0 & 4-2k \end{array} \right)$$

הקטור v_4 ש"ת א זהו פתרון יחיד
 $k \neq 2$! $h \neq -3$ / י

$$T(f) = f', \quad T: \mathcal{F} \rightarrow \mathcal{F}$$

2

$$T(f_1 + f_2) = (f_1 + f_2)' = f_1' + f_2' = T(f_1) + T(f_2)$$

$$T(k \cdot f) = (k \cdot f)' = k \cdot f' = k \cdot T(f)$$

אם $f, g \in \mathcal{F}$ אז $T(fg) = f'g + fg'$

$$g(x) = x+1, \quad f(x) = x \quad \text{אם } x \in \mathbb{R}$$

$$T(g) = 1, \quad T(f) = 1$$

$$T(fg) = T(x(x+1)) = T(x^2 + x) = 2x + 1$$

$$T(fg) = f'g + fg' = 1 \cdot (x+1) + x \cdot 1 = 2x+1$$

5, 1, 2, 1
/ 7

$$C = \begin{pmatrix} a_1 b_1 & a_1 b_2 & \dots & a_1 b_m \\ a_2 b_1 & a_2 b_2 & \dots & a_2 b_m \\ \vdots & \vdots & \ddots & \vdots \\ a_n b_1 & a_n b_2 & \dots & a_n b_m \end{pmatrix}$$

אם $a_1 = 0$ אז $a_1 \neq 0 \wedge \dots$ אז a_1, \dots, a_n הם וקטור בסיס.
אם $a_1 \neq 0$ אז a_1, \dots, a_n הם וקטור בסיס.

$$\begin{pmatrix} a_1 b_1 & a_1 b_2 & \dots & a_1 b_m \\ a_2 b_1 & a_2 b_2 & \dots & a_2 b_m \\ \vdots & \vdots & \ddots & \vdots \\ a_n b_1 & a_n b_2 & \dots & a_n b_m \end{pmatrix} \xrightarrow{\text{row operations}} \begin{pmatrix} a_1 b_1 & a_1 b_2 & \dots & a_1 b_m \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

אם $a_1 \neq 0$ אז a_1, \dots, a_n הם וקטור בסיס.
rank(C) = 1

-8-

$$x+y+z=1$$

1, 2, 3 (2)

\Downarrow

$$x=4y+4z+1$$

$$y, z \in \mathbb{Z}_5$$

לכנס 25 ע' / > 5

(3)

$$A \cdot \begin{pmatrix} 2 & 1 & 3 \\ 0 & -1 & 5 \\ 0 & 0 & 8 \end{pmatrix} \cdot A = \begin{pmatrix} 5 & 0 & -3 \\ 70 & 0 & 1 \\ 120 & 0 & 2 \end{pmatrix}$$

\Downarrow

$$|A| \cdot \begin{vmatrix} 2 & 1 & 3 \\ 0 & -1 & 5 \\ 0 & 0 & 8 \end{vmatrix} \cdot |A| = \begin{vmatrix} 5 & 0 & -3 \\ 70 & 0 & 1 \\ 120 & 0 & 2 \end{vmatrix}$$

\Downarrow

$$|A|^2 \cdot (-16) = 0 \Rightarrow |A| = 0$$

לכנס 8/10 ע' A > 5 / > 5

$$A^t + 3A^{-1} = 0, |A| = 27, n \times n, 3I = A$$

(4)

$$\Downarrow A \cdot (A^t + 3A^{-1}) = A \cdot A^t + 3I = 0 \Rightarrow A \cdot A^t = -3I$$

$$|A| \cdot |A^t| = (-3)^n$$

\Downarrow

$$27^2 = |A|^2 = (-3)^n$$

\Downarrow

$$n=6$$