Labor Market Segmentation with Rationed Entry: Theory and Mechanisms

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Abstract

This paper develops a dynamic equilibrium framework to analyze labor market segmentation characterized by rationed premium segments with high returns and entry barriers. We introduce three key innovations: a probability-based entry mechanism, an O-ring type ability aggregator, and dynamic skill evolution through learning-by-doing. The model distinguishes between rationed premium segments and conventional sectors with wage-based clearing, generating equilibrium wage premiums despite excess qualified labor supply. Through analysis of cross-sector spillovers and ability sorting, we show how strategic acceptance of temporary mismatches can be optimal for workers building sector-specific human capital. The framework provides new insights into persistent empirical puzzles: the coexistence of graduate unemployment with intense competition for premium positions, durable education-occupation mismatches, and intensifying credential requirements. These findings illuminate how individual choices in human capital investment and occupational selection shape labor market dynamics in developing economies experiencing rapid structural transformation.

1 Introduction

Labor markets in developing economies exhibit a striking paradox: widespread education & occupation mismatches coexist with intense competition for certain premium positions. This phenomenon is particularly salient in China, where structural transformation has generated substantial labor market frictions. According to Xiang et al. (2023), while China's higher education system produced a record 11.58 million college graduates in 2023, initial employment outcomes have deteriorated markedly—the employment rate within six months of graduation has declined from 90% in the early 2000s to 70-80% in recent years, with 45% of employed graduates working in positions unrelated to their field of study. Yet simultaneously, competition for civil service positions, which offer rationed entry and stable premium returns, has intensified dramatically. In 2021, approximately 1.5 million candidates competed for just 20,000 positions, with the most sought-after positions attracting over 20,000 applicants each Feng et al. (2024).

The severity of this mismatch exhibits heterogeneity across education levels. As shown in Figure 1, while bachelor's degree holders consistently experience mismatch rates of approximately 25%, doctorate and professional degree holders maintain substantially lower mismatch rates between 4-5%. This pattern suggests that education-occupation mismatches are primarily concentrated among bachelor's degree holders, making the study of rationed premium segments like civil service—which typically require only a bachelor's degree—particularly relevant for understanding labor market dynamics. These patterns present a challenge to standard human capital theory, which predicts that market signals should direct educational investments toward their most productive uses and that persistent excess qualified labor supply should eliminate wage premiums.

To analyze these labor market patterns systematically, we first define premium segments with rationed entry, which we characterize by three distinct features: *(1) high barriers to entry through formal requirements, skills testing, or position rationing; (2) high competition intensity among qualified candidates due to limited positions relative to applicants; and (3) high compensation relative to sectors with comparable skill requirements, encompassing both wage and non-wage benefits such as job security, social status, and cultural elements.* Unlike conventional labor markets where wages adjust to clear supply and demand, these sectors maintain persistent wage premiums through employment rationing despite excess qualified labor supply. The Chinese civil service sector provides a quintessential example of such premium segments—it imposes rigorous entry requirements through standardized examinations, sustains extreme competition ratios (75:1 in 2021) Xiang et al. (2023), and provides comprehensive benefits packages that exceed private sector equivalents by 20-30% when accounting for job stability and non-wage

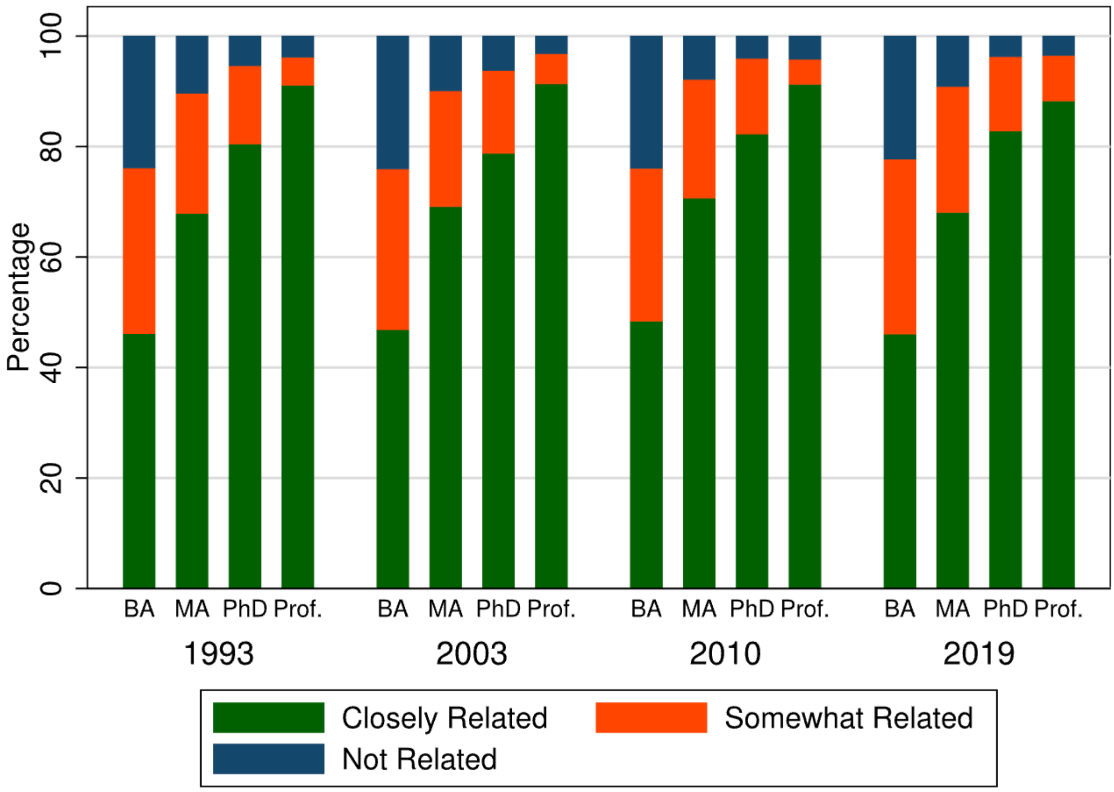

Figure 1: Education-Occupation Match by Degree Level and Year. This figure shows the proportion of workers reporting their job as closely related, somewhat related, or not related to their highest degree field, separately by degree level (BA, MA, PhD, Professional) and year (1993, 2003, 2010, 2019). Higher degrees consistently show better matching rates, with BA holders experiencing the highest mismatch rates of around 25% across all years. Source: National Survey of College Graduates.

benefits.

This institutional context raises three fundamental questions about labor market functioning and talent allocation: (1) Why do education-occupation mismatches persist despite clear market signals about premium returns in certain sectors? (2) Through what mechanisms do rationed premium segments maintain wage premiums despite massive oversupply of qualified candidates? (3) How do labor market flows between premium and non-premium segments affect skill accumulation, particularly through learning-by-doing in occupational transitions and human capital development?

In stark contrast to mature democracies, citizens in contemporary China exhibit a strong preference for government jobs, reflected in the massive participation in civil service examinations Feng et al. (2024). While existing research has identified potential reasons for this phenomenon, such as the enhanced regulatory authority Frye and Shleifer (1997), merit-based bureaucratic selection Landry et al. (2018), political mobility opportunities Liu (2024), and relative performance advantages compared to other systems Boittin et al. (2016), these static explanations alone cannot fully capture the dynamic nature of talent allocation between state and private sectors. Specifically, current literature lacks a comprehensive theoretical framework to analyze how individuals make career choices across sectors over time Pei (2024), how skills evolve through learning and experience Rauch and Evans (2000), and how market conditions and policy changes affect the equilibrium distribution of talent Besley and Persson (2009). Understanding these dynamics is crucial for explaining persistent patterns in state-society talent allocation and predicting responses to institutional changes.

To address these questions, we develop a dynamic equilibrium model that introduces three key innovations. First, we formalize the concept of premium segments with rationed entry through a probability-based entry mechanism where admission probability is given by $p_{ijt} = 1/(1+e^{-\lambda(F_{jt}(A_O(a_i))-(1-\theta_j))})$, capturing both the competitive nature of entry and the role of ability screening. Second, we model individual abilities through an O-ring type aggregator $A_O(a_i) = (\prod_{j=1}^{n} a_{ij})(1 + \sum_{j=1}^{n}\sum_{m=1} \gamma_{jm} a_{ij} a_{im})$ that captures both baseline competencies and skill complementarities. Third, we incorporate dynamic skill evolution through learning-by-doing, allowing workers to accumulate sector-specific human capital that facilitates potential transitions between segments over their careers.

Our analysis yields several important findings. First, we demonstrate how the interaction between rationed entry and learning-by-doing can generate persistent wage premiums in certain sectors despite excess qualified labor supply. Second, we show that education-occupation mismatches can be an equilibrium outcome when workers strategically accept temporary mismatches to build skills for future transitions. Finally, we quantify how different policy interventions - such as expanding premium segment positions or enhancing skill transferability across sectors - affect both individual career trajectories and aggregate labor market efficiency. These results provide new insights for understanding labor market segmentation in developing economies and offer practical implications for education and labor market policies.

The remainder of the paper is organized as follows. Section two develops the static model where we formalize education-occupation matching through an ability transformation matrix and a logit choice model with participation conditions. We prove the existence and uniqueness of equilibrium base wages in common markets, establishing the foundation for analyzing wage premiums in rationed segments. Section three extends to a dynamic framework by incorporating learning-by-doing and task upgrading mechanisms, enabling us to analyze how workers accumulate sector-specific human capital and transition between industries over their careers. Section four calibrates the model to Chinese labor market data, with particular attention to civil service examination outcomes and wage distributions, and conducts counterfactual analyses to evaluate the impact of entry barriers and skill transferability. Section 5 discusses policy implications for addressing education-occupation mismatches through both education system reforms and labor market institutions and concludes. Detailed proofs and additional empirical results are contained in the Appendix.

2 The Model

Building on the dual labor market framework of Dickens (1985), we develop an equilibrium model where heterogeneous agents make occupational choices across segmented markets. The economy features two distinct segments: premium sectors exhibiting characteristics documented in Acemoglu (2001)—high barriers to entry, intense competition, and rationed positions—and conventional sectors where wages adjust through standard market-clearing mechanisms.

The model's key innovation synthesizes three theoretical traditions. First, following Kremer (1993), we model how complementarities between different skill dimensions affect productivity and market sorting. Second, extending Lazear (2009)'s skill-weights approach, we formalize how individual endowments and educational backgrounds transform into industry-specific abilities through a novel matrix transformation process. Third, building on Acemoglu (2011)'s task-based framework, we model how these abilities interact with market-specific returns and entry probabilities to determine occupational choices.

Departing from standard wage-based market clearing, our framework explicitly models rationed entry in premium segments through admission probabilities conditional on relative ability rankings. This mechanism generates equilibrium patterns of both voluntary and involuntary unemployment, consistent with empirical evidence on labor market segmentation (Dickens 1985). The dynamic structure allows agents to choose between three options in each period: attempting entry into premium segments, participating in conventional sectors, or accepting temporary unemployment for skill upgrading.

The model's equilibrium characterization captures three key features of developing economies documented in the literature. First, it generates persistent wage premiums in rationed sectors despite excess qualified labor supply, as theorized by Acemoglu (2001). Second, it rationalizes the strategic acceptance of initial education-occupation mismatches through a skill-weights mechanism (Lazear 2009). Third, it endogenously generates patterns of occupational transitions driven by human capital accumulation, consistent with the task-based approach of Acemoglu (2011).

2.1 Timeline

The model follows a sequential structure that characterizes agents' life-cycle decisions and skill evolution (Figure 2). The sequence begins with initial endowment allocation and proceeds through education, labor market entry, and subsequent career adjustments, formalizing the dynamic interaction between individual choices and market conditions. At birth, agents receive heterogeneous skill endowments $\mathbf{E}$ representing innate abilities across

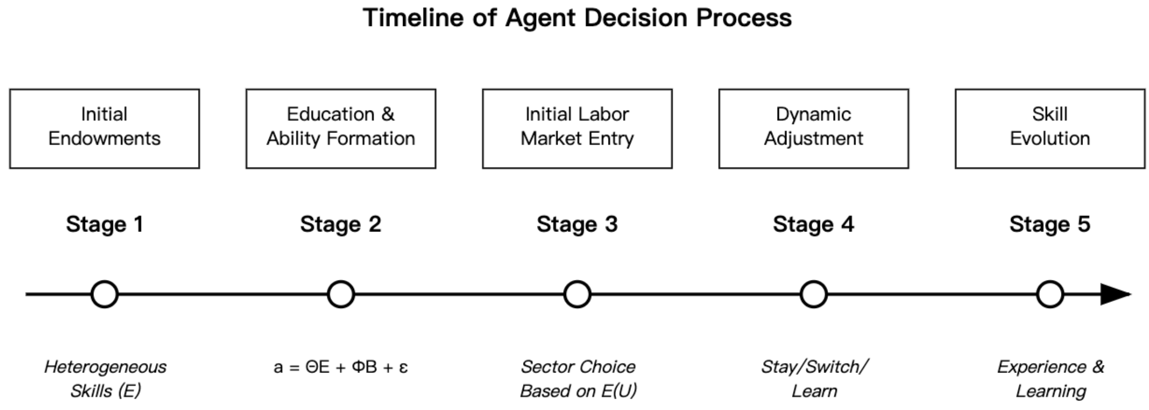

Figure 2: Sequential Structure of Agent Decisions and Skill Evolution

different dimensions. Following Lazear (2009), these endowments reflect fundamental skill categories with cross-industry applicability.[1]

The pre-market ability formation process transforms these initial endowments through educational investment. Specifically, education background **B** interacts with endowments through weight matrices to generate industry-specific abilities:

$$\mathbf{a}_i = \Theta \cdot \mathbf{E}_i + \Phi \cdot \mathbf{B}_i + \boldsymbol{\varepsilon} \tag{1}$$

where the error term $\boldsymbol{\varepsilon}$ captures potential education-occupation mismatches.[2]

Upon entering the labor market, agents maximize expected utility by choosing between premium and conventional sectors. Entry into premium segments occurs with probability p_{ijt}, while rejection results in unemployment benefit b net of cost c_0. This initial sorting mechanism, building on Acemoglu (2001), generates distinct wage determination processes across market segments.

The dynamic portion of the model captures two key mechanisms. First, employed agents accumulate industry-specific experience and face periodic decisions between maintaining their current position or seeking new opportunities. Second, following Acemoglu (2011)'s task-based framework, abilities evolve through both formal employment experience and self-directed learning during unemployment spells. This dual channel of skill

[1]The dimensionality of endowments reflects fundamental skill categories that can be utilized across multiple industries. For instance, analytical ability could be valuable in both consulting and research positions, while interpersonal skills might transfer between sales and management roles.

[2]The separation of endowments and education allows us to model suboptimal educational investments arising from information frictions, credit constraints, or other market imperfections. This creates a channel for subsequent occupational mobility through learning-by-doing, even when initial education-occupation matches are poor.

accumulation creates rich patterns of occupational mobility and career progression.

The model's sequential structure culminates in a dynamic equilibrium where individual skill accumulation decisions interact with market conditions to determine career trajectories. This framework captures both the initial sorting of workers across sectors and their subsequent career dynamics through endogenous skill accumulation and strategic mobility decisions, consistent with empirical patterns documented in the literature on occupational choice and wage dynamics (Kremer 1993).

2.2 Environment

Agents Define the ability matrix for agent $i \in \mathbf{I}$ as [3]

$$\mathbf{a_i} = \Theta \cdot \mathbf{E_i} + \Phi \cdot \mathbf{B_i} + \epsilon \tag{2}$$

where

- Vector $\mathbf{E}$ denotes the basic personal endowments;
- vector $\mathbf{B}$ captures the education background;
- Θ, Φ are weight matrices capturing the contributions of different endowments/educational backgrounds to industry-specific abilities;
- the stochastic error term ϵ represents the education-occupation mismatch.

Specifically, the individual level ability vector $\mathbf{a_i} = \{a_{i1}, \ldots, a_{il}\}$, where a_ij represents agent i's industry j specific technical competencies and cross-industry skill transferability, is transformed through a *Sigmoid-based transformation* to compute $\tilde{a}_{ij}$:

$$\tilde{a}_{ij} = \frac{M}{1 + e^{-k(a_{ij} - \mu)}} \tag{3}$$

where M is the upper bound, k controls slope, and μ centers the function.

Define a O-ring type aggregator

$$A_O(\mathbf{a_i}) = \left(\prod_{j=1} a_{ij}\right)\left(1 + \sum_{j=1}\sum_{m=1} \gamma_{jm} a_{ij} a_{im}\right) \tag{4}$$

[3] My ability formation mechanism fundamentally aligns with the task-based approach pioneered by Acemoglu (2011) , though employing different mathematical formulations. While they use a continuous task space with integration over task-specific efficiencies $\pi_{ij(t)}$, our model adopts a discrete matrix transformation framework where industry-specific abilities are determined by the interaction between endowment vector $\mathbf{E}$ and education background $\mathbf{B}$ through weight matrices. This matrix-based approach maintains the essential theoretical insight of matching individual capabilities to industry requirements while introducing explicit channels for educational investment and matching frictions, thereby extending the task-based framework to accommodate broader human capital formation processes.

where the first part $\prod_{j=1} a_{ij}$ denotes the product of basic industry level ability, emphasizing the baseline multiplicative complementarity across all abilities. The second term $\left(1 + \sum_{j=1} \sum_{m=1} \gamma_{jm} a_{ij} a_{im}\right)$ captures pairwise synergies between specific ability combinations, with $\gamma_{ij} \in [0, 1]$ measuring the strength of complementarity between abilities i and j. [4] This specification builds on Kremer (1993)'s O-ring theory while incorporating insights from Lazear (2009) on skill complementarities.

[4] This specification reflects both the necessity of maintaining minimum competency across all dimensions (first term) while allowing for enhanced productivity through strategic skill combinations (second term). For instance, strong technical skills paired with management ability may generate additional value beyond their individual contributions, represented by a larger γ_{ij} for this combination.

Decision making The agent i receives the expected utility when planning to enter industry j at time t:

$$\mathbb{E}\left(U_{ijt}\right) = \mathbb{E}\left[w_{ijt}(a_{ij}) - c_{ij}(a_{ij})\right]$$

Suppose $\mathbf{J_1} \bigcup \mathbf{J_2} = \mathbf{J}$ where $\mathbf{J_1}$ denotes the common market and $\mathbf{J_2}$ the premium segments with rationed entry, underlines the different wage structure [5]:

$$w_{ijt} = \beta_j f\left(A_O, a_{ijt}\right) + \eta_j, j \in \mathbf{J_1}$$

where β_j is the industry-specific return to ability, η_j as the base wage. Let the wage in the premium segments with rationed entry as

$$\mathbf{E}[w_{ijt}] = p_{ijt} w_{jt}^{high} + (1 - p_{ijt})(b - c_0), j \in \mathbf{J_2}$$

where p_{ijt} refers to the probability of being admitted into the premium segments with rationed entry. Formally define the probability as

$$p_{ijt} = \frac{1}{1 + e^{-\lambda(F_{jt}(A_O(\mathbf{a_i})) - (1-\theta_j))}}$$

where $\lambda > 0$ is a smoothing parameter[6]. $F_{jt}(\cdot)$ is continuously differentiable and denotes the cumulative distribution function of aggregated abilities in market j at time t, and $\theta_j \in (0, 1)$ represents the admission threshold for market j. Specifically, θ_j can be interpreted as the proportion of candidates that the market aims to admit (e.g., $\theta_j = 0.2$ indicates that the market targets the top 20% of candidates).[7]

[5]The aggregation of $\mathbf{J_2}$ into a single sector or its treatment as multiple sectors is mathematically equivalent. First, we can construct an aggregate wage index for $\mathbf{J_2}$ that preserves the key equilibrium properties. Second, treating $j \in \mathbf{J_2}$ separately merely allows for sector-specific preferences over ability measures A_O, analogous to different public sector positions having distinct skill requirements. This theoretical equivalence justifies treating $\mathbf{J_2}$ as a single sector for analytical tractability.

[6]The smoothing parameter λ governs the steepness of the probability transition around the threshold $(1 - \theta_j)$, with larger values of λ ($\lambda \to \infty$) approximating a more discrete selection mechanism.

[7]The time subscript t in $F_t(\cdot)$ allows for potential temporal variation in the ability distribution, reflecting dynamic changes in the candidate pool's composition.

2.3 Logit Choice Model with Participation Conditions

We apply the logit model to formulate the expected utility index as a probabilistic choice Aguirregabiria and Mira (2010)[8], including an outside option (unemployment). The model estimates the probability that agent i will select industry j or the outside option, based on the utility value V_{ij} calculated for each alternative. The formulation Q_1 is as follows:

$$P_{ij} = \frac{\exp(V_{ij})}{\exp(V_{i0}) + \sum_{k\in\mathbf{J}} \exp(V_{ik})} \quad \text{for } j \in \{0\} \cup \mathbf{J} \tag{12}$$

$$V_{ij} = \begin{cases} b - c_0 & \text{if } j = 0 \text{ (outside option)} \\ \max\{\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}), b - c_0\} & \text{if } j \in \mathbf{J_1} \\ \max\left\{p_{ijt}[w_{jt}^{high} - c_{ij}(a_{ij})] + (1 - p_{ijt})(b - c_0), b - c_0\right\} & \text{if } j \in \mathbf{J_2} \end{cases} \tag{13}$$

$$c_{ij}(a_{ij}) = \kappa_j \cdot \left(\frac{1}{A_O(a_{ij})}\right) + \delta_{ij}, \quad \forall j \in \mathbf{J} \tag{14}$$

$$p_{ijt} = \frac{1}{1 + e^{-\lambda(F_{jt}(A_O(\mathbf{a_i})) - (1-\theta_j))}} \tag{15}$$

With participation decisions determined by:

$$I_{ij} = \begin{cases} 1 & \text{if } \beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}) > b - c_0 \text{ and } j \in \mathbf{J_1} \\ 1 & \text{if } p_{ijt}[w_{jt}^{high} - c_{ij}(a_{ij})] + (1 - p_{ijt})(b - c_0) > b - c_0 \text{ and } j \in \mathbf{J_2} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

The outside option ($j = 0$) utility V_{i0} consists of:

- Base unemployment benefit/home production value (b)

[8] Another approach is to form the "Utility Maximization Choice Model". Write down the overall optimization problem for agent i as Q_i

$$j^* = \arg\max_{j\in\mathbf{J}} \mathbb{E}(U_{ijt}) = \begin{cases} \beta_j f(A_O, a_{ijt}) + \eta_j - c_{ij}(a_{ij}) & \text{if } j \in \mathbf{J_1} \\ p_{ijt} w_{jt}^{high} + (1 - p_{ijt})b - c_{ij}(a_{ij}) & \text{if } j \in \mathbf{J_2} \end{cases} \tag{5}$$

$$p_{ijt} = F(\text{rank}(A(a_{ij}))|N_j) \tag{6}$$

$$c_{ij}(a_{ij}) = \kappa_j \cdot \left(\frac{1}{A_O(a_{ij})}\right) + \delta_{ij} \tag{7}$$

$$\mathbf{J} = \mathbf{J_1} \cup \mathbf{J_2} \tag{8}$$

$$p_{ijt} \in [0, 1] \tag{9}$$

$$\kappa_j > 0, \delta_{ij} \geq 0 \tag{10}$$

$$w_{jt}^{high} > b > 0 \tag{11}$$

The actual labor supply for the common market is:

$$L_j^s = \sum_{i \in \mathbf{I}} P_{ij} \cdot I_{ij} \quad \text{for } j \in \mathbf{J_1} \tag{17}$$

For the premium segments ($j \in \mathbf{J_2}$), we distinguish between the effective labor supply (those who choose to participate) and the actual labor supply (those who are admitted):

$$L_j^{s,effective} = \sum_{i \in \mathbf{I}} P_{ij} \cdot I_{ij} \quad \text{for } j \in \mathbf{J_2} \quad \text{(total attempting)} \tag{18}$$

$$L_j^{s,actual} = \sum_{i \in \mathbf{I}} P_{ij} \cdot p_{ij} \cdot I_{ij} \quad \text{for } j \in \mathbf{J_2} \quad \text{(actually admitted)} \tag{19}$$

where $L_j^{s,effective} - L_j^{s,actual}$ represents the excess labor supply in the premium segments.

The total unemployment in the economy consists of voluntary and involuntary unemployment:

$$\begin{aligned} U &= U_{voluntary} + U_{involuntary} \\ &= \sum_{i \in \mathbf{I}} \left[P_{i0} + \sum_{j \in \mathbf{J_2}} P_{ij}(1 - p_{ij}) I_{ij} \right] \\ &= \underbrace{\sum_{i \in \mathbf{I}} P_{i0}}_{\text{voluntary unemployment}} + \underbrace{\sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J_2}} P_{ij}(1 - p_{ij}) I_{ij}}_{\text{involuntary unemployment}} \end{aligned} \tag{20}$$

Note that:

- P_{i0} captures those who voluntarily choose the outside option
- $P_{ij}(1 - p_{ij})I_{ij}$ for $j \in \mathbf{J_2}$ represents those who chose premium segments but were not accepted
- The participation rate in the economy is: $1 - \frac{1}{|\mathbf{I}|} \sum_{i \in \mathbf{I}} P_{i0}$

The unemployment rate can be expressed as:

$$u = \frac{U}{|\mathbf{I}|} = \frac{1}{|\mathbf{I}|} \left[\sum_{i \in \mathbf{I}} P_{i0} + \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J_2}} P_{ij}(1 - p_{ij}) I_{ij} \right] \tag{21}$$

2.4 Equilibrium Conditions

2.4.1 Partial Equilibrium for the Common Market

For any industry $j \in \mathbf{J_1}$, the choice probability matrix can be expressed as

$$P_{ij}(\eta_j, \eta_{-j})$$

where η_j is the market determined base wage in industry j and η_{-j} represents the vector of base wages in all other industries. The labor supply in industry j is the sum of individual choices $L_j^s(\eta_j, \eta_{-j}) = \sum_{i \in I} P_{ij}(\eta_j, \eta_{-j}) \cdot I_{ij}$ as discussed. The labor demand $L_j^d(\eta_j)$ is derived from firms' cost minimization problems in industry j.

Definition 1 (Single Market Equilibrium in Industry $j \in \mathbf{J_1}$)**.** *Given other industries' base wages η_{-j}, a market equilibrium in industry $j \in J_1$ is characterized by a base wage η_j^* that satisfies:*

$$L_j^s(\eta_j^*, \eta_{-j}) = L_j^d(\eta_j^*)$$

Proposition 1 (Existence of Partial Equilibrium)**.** *Under the following conditions:*

1. *$L_j^s(\eta_j, \eta_{-j})$ is continuous and strictly increasing in η_j for all $j \in \mathbf{J_1}$*
2. *$L_j^d(\eta_j)$ is continuous and strictly decreasing in η_j for all $j \in \mathbf{J_1}$*
3. *$\lim_{\eta_j \to 0} L_j^s(\eta_j, \eta_{-j}) = 0$ and $\lim_{\eta_j \to \infty} L_j^d(\eta_j) = 0$ for all $j \in \mathbf{J_1}$*

there exists a unique partial equilibrium $\{\eta_j^*\}_{j \in J_1}$ such that [9]*:*

$$L_j^s(\eta_j^*, \eta_{-j}^*) = L_j^d(\eta_j^*) \quad \forall j \in \mathbf{J_1}.$$

2.4.2 Existence and Uniqueness of the Equilibrium Base Wage

Plugging V_{ij} into the labor supply:

$$L_j^s = \sum_{i \in \mathbf{I}} \left(\frac{\exp\left(\max\left\{\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}), b - c_0\right\}\right)}{\exp(V_{i0}) + \sum_{k \in \mathbf{J}} \exp(V_{ik})} \right) \cdot I_{ij}.$$

[9]

Proof. For any given vector of other industries' wages η_{-j}, conditions 1-3 ensure the existence of a unique equilibrium wage η_j^* in industry j by the intermediate value theorem. The existence of a complete partial equilibrium follows from applying Brouwer's fixed point theorem to the system of equations. $\square$

The indicator function I_{ij} can be written as $I_{ij} = \mathbb{I}\left(\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}) > b - c_0\right)$ and we get

$$L_j^s = \sum_{i \in \mathbf{I}} \left(\frac{\exp\left(\max\left\{\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}), b - c_0\right\}\right)}{\exp(V_{i0}) + \exp\left(\max\left\{\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}), b - c_0\right\}\right) + \sum_{k \neq j} \exp(V_{ik})} \right) \cdot \mathbb{I}\left(\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}) > b - c_0\right)$$

Case 1 When $\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}) \leq b - c_0$, $V_{ij} = b - c_0$ and

$$\frac{\partial L_j^s}{\partial \eta_j} = 0.$$

To notice, $\mathbb{I}(\cdot) = 0$ and made no contributions.

Case 2 When $\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}) > b - c_0$,

$$\frac{\partial L_j^s}{\partial \eta_j} = \sum_{i \in \mathbf{I}} \left(\frac{\partial}{\partial \eta_j} \left(\frac{\exp(V_{ij})}{\exp(V_{i0}) + \exp(V_{ij}) + \sum_{k \neq j} \exp(V_{ik})} \right) \right) \cdot \mathbb{I}\left(\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}) > b - c_0\right)$$

where $V_{ij}(\eta_j) = \beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij})$. We reach

$$\frac{\partial L_j^s}{\partial \eta_j} = \sum_{i \in \mathbf{I}} \left(\frac{\exp(V_{ij})\left(D - \exp(V_{ij})\right)}{D^2} \right)$$

where $D = \exp(V_{i0}) + \exp(V_{ij}) + \sum_{k \neq j} \exp(V_{ik})$. After simplification, we get

$$\frac{\partial L_j^s}{\partial \eta_j} = \sum_{i \in \mathbf{I}} \left(\frac{\exp(V_{ij})\left(\exp(V_{i0}) + \sum_{k \neq j} \exp(V_{ik})\right)}{\left(\exp(V_{i0}) + \exp(V_{ij}) + \sum_{k \neq j} \exp(V_{ik})\right)^2} \right)$$

Assume the partial derivative of labor demand is

$$\frac{\partial L_j^d}{\partial \eta_j} = \chi,$$

then we get

$$\sum_{i \in \mathbf{I}} \left(\frac{\exp(V_{ij})}{D_i} - \frac{\exp(V_{ij})^2}{D_i^2} \right) = \chi \tag{22}$$

This equation represents the equilibrium condition where labor supply meets labor demand at the margin. The left side shows the aggregated individual responses to wage changes (η_j), where $\exp(V_{ij})/D_i$ represents the probability of choosing sector j and $\exp(V_{ij})^2/D_i^2$ captures the dampening effect from those already in sector j. The constant χ on the right side reflects the slope of labor demand with respect to wages. This equilibrium condition is derived from equating $\partial L_j^s/\partial\eta_j = \partial L_j^d/\partial\eta_j$, which determines how the base wage η_j adjusts to clear the market.

Proposition 2 (Existence and Uniqueness of the Equilibrium Base Wage Level η_j)**.** *Consider the labor market for sector $j \in \mathbf{J_1}$ (the common market). Given the labor supply function $L_j^s(\eta_j)$ generated from P_{ij} and labor demand function $L_j^d(\eta_j)$, there exists a unique wage level η_j^* that clears the labor market, such that:*

$$L_j^s(\eta_j^*) = L_j^d(\eta_j^*)$$

Proofs can be found in appendix.

2.4.3 General Equilibrium with Two-Tier Labor Markets

Having established the existence and uniqueness of equilibrium base wages in common sectors, we now examine intersectoral influences between common markets $\mathbf{J_1}$ and premium segments $\mathbf{J_2}$ (exemplified by civil service positions). This analysis serves two purposes: characterizing sufficient conditions on premium wages $\{w_j^{high}\}_{j\in\mathbf{J_2}}$ that preserve equilibrium properties established in Proposition 2, and identifying mechanisms through which premium wages affect equilibrium base wages in common sectors.

Definition 2 (Premium Wage Feasibility)**.** *A set of premium wages $\{w_j^{high}\}_{j\in\mathbf{J_2}}$ is feasible if:*

1. *For each $j \in \mathbf{J_2}$, w_j^{high} satisfies the firm's participation constraint:*

$$\Pi_j(w_j^{high}) = F_j(L_j) - w_j^{high} L_j - C_j \geq 0 \tag{23}$$

2. *The rationing constraint binds in equilibrium:*

$$L_j^{s,actual} = \sum_{i\in\mathbf{I}} P_{ij} \cdot p_{ij} \cdot I_{ij} = \bar{L}_j \tag{24}$$

where $\bar{L}_j$ represents the exogenously fixed number of positions in sector j

Proposition 3 (Cross-Sector Wage Effects)**.** *Given a feasible set of premium wages $\{w_j^{high}\}_{j\in\mathbf{J_2}}$, equilibrium base wages $\{\eta_j^*\}_{j\in\mathbf{J_1}}$ are affected through three channels:*

1. *Direct labor supply effects via participation decisions:*

$$\frac{\partial L_j^s}{\partial w_k^{high}} = \sum_{i \in \mathbf{I}} \frac{\partial P_{ij}}{\partial w_k^{high}} \cdot I_{ij} \tag{25}$$

2. *Indirect effects through acceptance probability adjustments:*

$$\frac{\partial p_{ij}}{\partial w_k^{high}} = \lambda p_{ij}(1 - p_{ij}) \frac{\partial F_{jt}}{\partial w_k^{high}} \tag{26}$$

3. *Composition effects on ability distribution:*

$$\frac{\partial F_{jt}}{\partial w_k^{high}} = g_{jt}(A_O(\mathbf{a_i})) \frac{\partial A_O}{\partial w_k^{high}} \tag{27}$$

where $g_{jt}(\cdot)$ represents the probability density function of aggregate abilities in market j at time t.

Corollary 1 (Monotonicity of Cross-Sector Effects)**.** *Under regularity conditions on the ability distribution F_{jt}:*

1. *Higher premium wages strictly reduce common sector labor supply:*

$$\frac{\partial L_j^s}{\partial w_k^{high}} < 0 \quad \forall j \in \mathbf{J_1}, k \in \mathbf{J_2} \tag{28}$$

2. *The magnitude of this effect increases with ability:*

$$\frac{\partial^2 L_j^s}{\partial w_k^{high} \partial A_O} < 0 \tag{29}$$

This characterization reveals how premium segment wages influence common sector equilibrium through both *extensive margin (participation) and intensive margin (ability sorting) adjustments*. The analysis provides a foundation for examining dynamic interactions between sectors and their implications for labor market segmentation.

Definition 3 (General Equilibrium with Segmented Labor Markets)**.** *Given unemployment benefit b and outside option cost c_0, a general equilibrium consists of:*

1. *A feasible set of premium wages $\{w_j^{high}\}_{j \in \mathbf{J_2}}$ satisfying:*
   - *Firm participation:* $\Pi_j(w_j^{high}) = F_j(L_j) - w_j^{high} L_j - C_j \geq 0$
   - *Rationing constraint:* $L_j^{s,actual} = \sum_{i \in \mathbf{I}} P_{ij} \cdot p_{ij} \cdot I_{ij} = \bar{L}_j$

2. *Base wages $\{\eta_j^*\}_{j\in\mathbf{J_1}}$ determined simultaneously in the common markets by Proposition 2*

3. *Choice probabilities $\{P_{ij}\}$ for all individuals i and sectors $j \in \{0\} \cup \mathbf{J_1} \cup \mathbf{J_2}$*

4. *Employment indicators $\{I_{ij}\}$ for all i, j*

such that:
(i) Labor Market Clearing in Common Markets:

$$\sum_{i\in\mathbf{I}} P_{ij}(\{\eta_j^*\}_{j\in\mathbf{J_1}}, \{w_j^{high}\}_{j\in\mathbf{J_2}}) \cdot I_{ij} = L_j^d(\eta_j^*) \quad \forall j \in \mathbf{J_1}$$

(ii) Optimal Individual Choices: For all i,

$$I_{ij} = \begin{cases} 1 & \text{if } \beta_j f(A_O, a_{ij}) + \eta_j^* - c_{ij}(a_{ij}) > b - c_0 \text{ and } j \in \mathbf{J_1} \\ 1 & \text{if } p_{ij}[w_j^{high} - c_{ij}(a_{ij})] + (1-p_{ij})(b-c_0) > b - c_0 \text{ and } j \in \mathbf{J_2} \\ 0 & \text{otherwise} \end{cases}$$

(iii) Cross-Sector Wage Effects: For all $j \in \mathbf{J_1}, k \in \mathbf{J_2}$

$$\frac{\partial L_j^s}{\partial w_k^{high}} = \sum_{i\in\mathbf{I}} \frac{\partial P_{ij}}{\partial w_k^{high}} \cdot I_{ij}$$
$$\frac{\partial p_{ij}}{\partial w_k^{high}} = \lambda p_{ij}(1-p_{ij}) \frac{\partial F_{jt}}{\partial w_k^{high}}$$
$$\frac{\partial F_{jt}}{\partial w_k^{high}} = g_{jt}(A_O(\mathbf{a_i})) \frac{\partial A_O}{\partial w_k^{high}}$$

(iv) Unemployment Determination:

$$U = \underbrace{\sum_{i\in\mathbf{I}} P_{i0}}_{voluntary} + \underbrace{\sum_{i\in\mathbf{I}} \sum_{j\in\mathbf{J_2}} P_{ij}(1-p_{ij}) I_{ij}}_{involuntary}$$

where:

- $U_{voluntary} = \sum_{i\in\mathbf{I}} P_{i0}$ *represents individuals who choose the outside option*
- $U_{involuntary} = \sum_{i\in\mathbf{I}} \sum_{j\in\mathbf{J_2}} P_{ij}(1-p_{ij}) I_{ij}$ *represents individuals who choose three-high sectors but are not accepted*

This equilibrium structure characterizes four key economic mechanisms:

1. The coexistence of market-clearing and rationed entry mechanisms, where premium segments maintain wage premiums despite excess qualified labor supply

2. Heterogeneous sorting patterns driven by both ability-based selection and strategic participation decisions

3. Cross-sector spillovers operating through both participation margins and ability distribution channels

4. Distinct sources of unemployment arising from voluntary outside option selection and involuntary rationing in premium segments

Together, these mechanisms generate equilibrium patterns consistent with persistent wage premiums and education-occupation mismatches observed in segmented labor markets.

3 Simulations

I conduct numerical simulations to analyze the model's implications for labor market segmentation, cross-sector spillovers, and unemployment patterns. Our computational approach employs a combination of numerical optimization techniques and Monte Carlo methods implemented in Python, with particular attention to three key aspects of the model: cross-sector wage effects, unemployment composition, and ability-based sorting patterns.

3.1 Computational Framework

The simulation architecture consists of three main components. First, I implement a cross-sector analysis module that traces how premium segment wages influence labor allocation and market clearing in common sectors. Second, I develop an unemployment analysis framework that decomposes voluntary and involuntary unemployment while tracking ability-specific effects. Third, I construct a general equilibrium solver that computes market-clearing wages in common sectors given the rationing constraints in premium segments.

For our baseline simulations, we set the model parameters as follows:

- **Market Structure Parameters:** Number of agents: $N = 10,000$; number of common markets: $|\mathbf{J_1}| = 10$; premium segment size ratio: $\theta_j = 0.2$.

- **Wage and Benefit Parameters:** base wage range in common sectors: $\eta_j \in [20, 50]$; premium wage markup: $20 - 30\%$ above comparable common sectors; unemployment benefit: $b = 15$.

Table 1: Market Distribution and Ability Sorting

Market	Size	Average Ability	Share (%)
Common 2	8,027	0.989	80.27
Common 4	931	0.950	9.31
Rationed	415	0.832	4.15
Common 3	312	0.973	3.12
Common 9	203	0.965	2.03
Other Commons	95	0.944	0.95
Unemployment	17	–	0.17

Notes: Markets with share $<$ 1% are aggregated in "Other Commons".

- **Ability and Entry Parameters:** ability distribution: Log-normal with $\mu = 0$, $\sigma = 0.5$; Entry smoothing parameter: $\lambda = 2.5$; O-ring complementarity factors: $\gamma_{jm} \in [0.1, 0.3]$

The simulations proceed in three stages. First, we generate a population of heterogeneous agents with randomly drawn endowments and educational backgrounds. Second, we compute industry-specific abilities through our matrix transformation process, incorporating both baseline competencies and skill complementarities through the O-ring aggregator. Finally, we solve for equilibrium outcomes using numerical optimization methods, with particular attention to the interaction between rationed entry in premium segments and wage determination in common sectors.[10]

3.2 Baseline Market Structure and Labor Allocation

Our baseline simulations reveal distinct patterns in market share distribution, ability sorting, and unemployment composition. Figure 3 presents three key aspects of the equilibrium: market share distribution across sectors, average abilities by market, and unemployment decomposition.

Three key patterns emerge from our baseline simulations:

Market Concentration The equilibrium exhibits strong market concentration, with Common Market 2 absorbing approximately 80.3% of the labor force. This concentration appears driven by both favorable wage conditions and high average worker abilities (0.989) in this sector. The second largest common market (C4) attracts 9.3% of workers, while the rationed sector employs 4.15% of the labor force.

[10] All simulations are implemented in Python, utilizing NumPy for numerical computations, SciPy for optimization routines, and Pandas for data management. Visualizations are created using Matplotlib and Seaborn to illustrate key relationships and patterns. The complete implementation details and code are available in the Online Appendix, while critical components are presented in section 7.

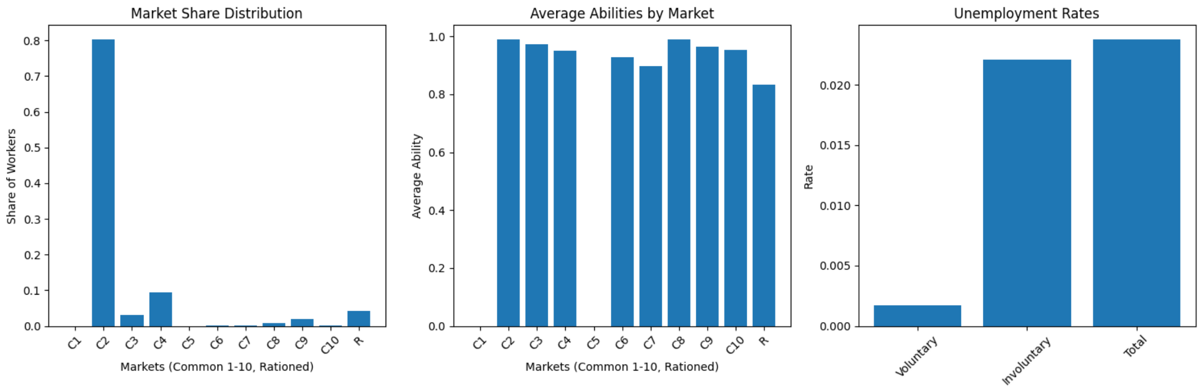

Figure 3: Baseline Equilibrium Characteristics. The figure shows three key aspects of the equilibrium: (i) market share distribution across common markets (C1-C10) and the rationed sector (R) in the left panel; (ii) average worker abilities by market in the middle panel; and (iii) decomposition of unemployment into voluntary and involuntary components in the right panel.

Notes: The left panel demonstrates strong market concentration, particularly in C2. The middle panel reveals high average abilities (>0.8) across all active markets, with common sectors generally showing higher averages than the rationed sector. The right panel highlights the dominance of involuntary over voluntary unemployment, consistent with the model's rationing mechanism.

Ability Sorting We observe clear patterns of ability-based sorting across markets. Average abilities remain consistently high (> 0.83) across all active markets, but with notable variation. Common sectors show higher average abilities (0.95-0.99) compared to the rationed sector (0.832), suggesting that the rationing mechanism does not necessarily attract the highest-ability workers.

Unemployment Structure The overall unemployment rate remains low at 2.38%, but its composition reveals important structural features. Involuntary unemployment (2.21%) substantially exceeds voluntary unemployment (0.17%), indicating that most unemployment results from rationing rather than workers' optimal choices. This pattern aligns with our model's prediction that entry barriers in premium segments can generate involuntary unemployment even with high-ability workers.

These results highlight how rationed entry in premium segments influences both the distribution of workers across markets and the patterns of ability sorting. The concentration of workers in specific common markets, combined with the relatively small size of the rationed sector, suggests that workers respond to entry barriers by clustering in high-paying common sectors rather than queuing for rationed positions.

3.3 Cross-Sector Analysis

As predicted by Proposition 3, our simulations reveal three distinct channels through which premium wages affect market outcomes. The direct labor supply effect, formalized as $\frac{\partial L_j^s}{\partial w_k^{high}}$ in the proposition, exhibits strong non-linearity, while probability adjustments and composition effects play important but secondary roles. Our simulation results strongly validate the theoretical predictions about both *the direction and magnitude of these effects.*

3.3.1 Cross-Sector Wage Effects

To analyze how premium wages influence labor allocation across sectors, we examine three distinct channels: direct labor supply responses, admission probability adjustments, and ability composition changes, as proposed in proposition 3 . Figure 4 presents these effects.

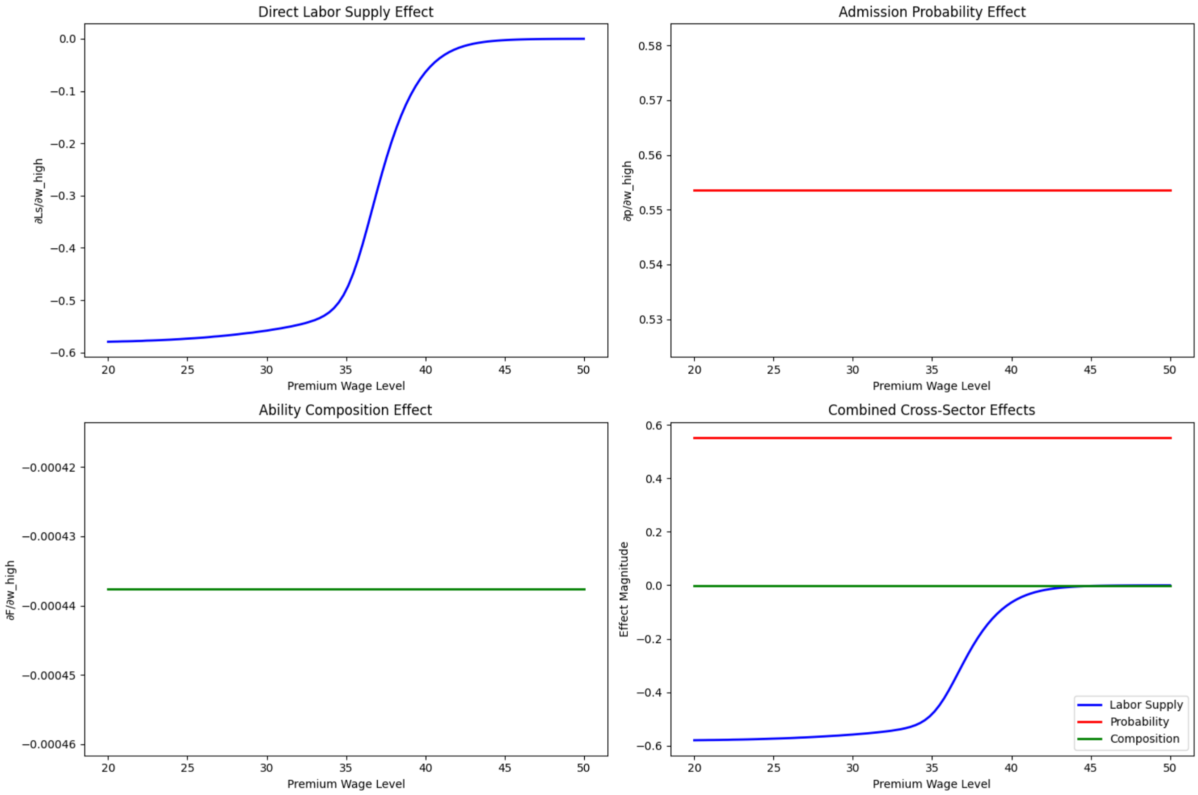

Figure 4: Decomposition of Cross-Sector Effects

Notes: The figure shows how premium wages affect common sectors through three channels. The direct labor supply effect (top left) demonstrates a strong negative relationship with a notable inflection point around wage level 37. The admission probability effect (top right) remains relatively stable around 0.55. The ability composition effect (bottom left) shows minimal variation, while the combined effects (bottom right) reveal the dominance of the direct labor supply channel in determining overall market responses.

The simulation reveals three key patterns. First, the direct labor supply effect exhibits strong non-linearity, with an inflection point around premium wage level 37. The magnitude of this effect ranges from -0.6 to 0 as wages increase, indicating substantial labor reallocation away from common sectors at lower premium wages. Second, the admission probability effect remains remarkably stable at approximately 0.55 across all wage levels, suggesting that the rationing mechanism effectively maintains consistent entry standards regardless of wage changes. Third, the ability composition effect is relatively small and stable, indicating that wage changes primarily affect quantity rather than quality of labor across sectors.

The combined effects panel illustrates that the direct labor supply channel dominates the overall market response, with ability composition and probability effects playing secondary roles. This hierarchy of effects suggests that policy interventions targeting wage levels in premium segments will primarily operate through quantity adjustments rather than changes in worker sorting patterns.

3.3.2 Ability Distribution and Admission Patterns

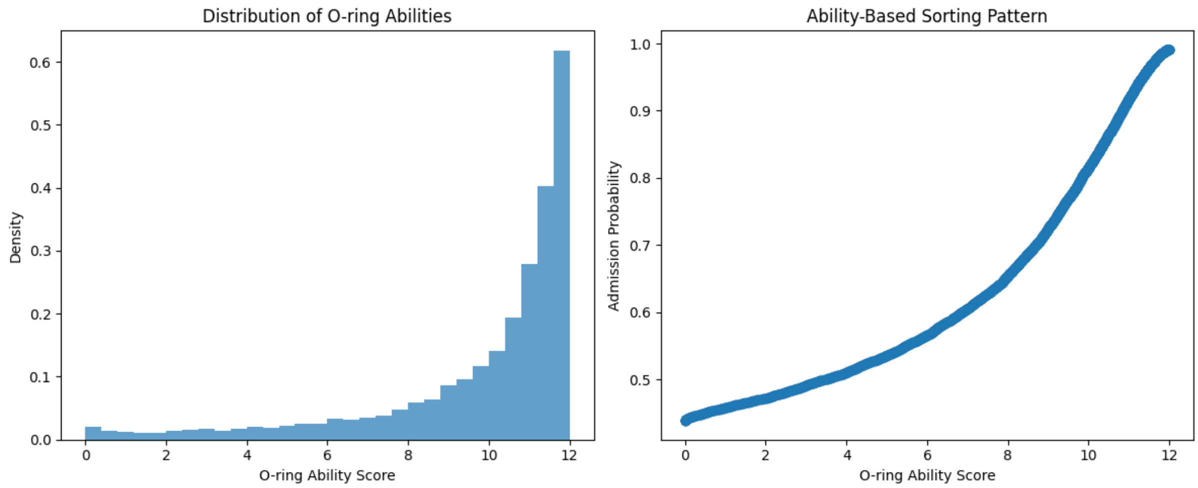

Figure 5: O-ring Ability Distribution and Admission Probabilities

Notes: Left panel shows the distribution of O-ring ability scores across the population, displaying right-skewed concentration at higher ability levels. Right panel demonstrates the monotonic relationship between ability scores and admission probabilities in the premium sector.

Our simulation generates two key patterns in ability-based sorting. First, the O-ring ability distribution shows significant right-skewness, with a concentration of workers at higher ability levels. Second, admission probabilities increase monotonically with ability scores, rising from about 0.45 to nearly 1.0 across the ability range, indicating strong positive selection into premium segments despite the presence of rationing constraints.

3.3.3 Ability-Based Sorting Patterns

Figure 6 illustrates how sorting patterns evolve with different premium wage levels, revealing the dynamic relationship between worker abilities and sector choice probabilities.

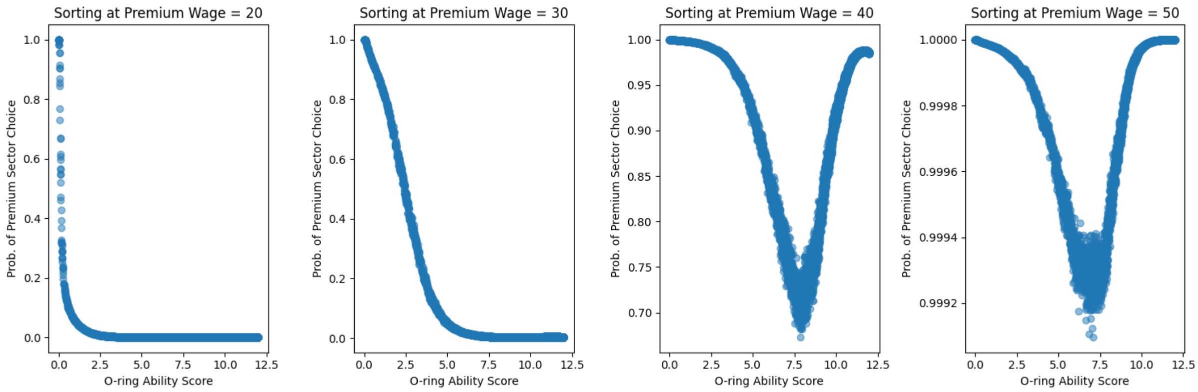

Figure 6: Evolution of Sorting Patterns Across Premium Wage Levels

Notes: Each panel shows the relationship between O-ring ability scores and the probability of choosing the premium sector at different wage levels (20, 30, 40, 50). The U-shaped pattern becomes more pronounced at higher wages, indicating increasingly selective sorting. At the highest wage level (50), even high-ability workers show near-universal premium sector preference, though with maintained ability-based stratification.

The evolution of sorting patterns across wage levels reveals striking regularities. At low premium wages (20), the relationship between ability and sector choice is nearly monotonic, with only the highest-ability workers showing any significant probability of premium sector choice. As wages increase to moderate levels (30-40), a distinctive U-shaped pattern emerges, suggesting that both very high and very low ability workers may prefer the premium sector, though for potentially different reasons. At the highest premium wage (50), the sorting pattern becomes almost universal but maintains subtle ability-based stratification, with probabilities ranging between 0.999 and 1.000, indicating that wage premiums can eventually dominate ability-based sorting considerations.

Intuition The sorting patterns across different premium wages reveal three key theoretical mechanisms. There is a clear wage effect where higher premiums universally increase workers' propensity to choose the premium sector, effectively diminishing the importance of ability-based sorting:

1. At low premium wages ($20 - 30$): ability screening plays a differential role across

wage levels, ability serves as a strong screening mechanism, creating clear stratification in sector choice.

2. At higher wage levels ($40-50$): however, this screening effect weakens substantially, where even workers with lower ability scores show high probabilities of choosing the premium sector.

3. The relationship between ability and sector choice exhibits complex nonlinearity, transitioning from a nearly monotonic relationship at low wages to a U-shaped pattern at moderate wages, and finally to an almost uniform but still slightly stratified pattern at high wages. These patterns suggest that while wage premiums can eventually overcome ability-based sorting considerations, subtle ability-based stratification persists even in high-wage environments.

3.3.4 Unemployment Dynamics and Benefit Levels

The relationship between unemployment benefits and labor market outcomes exhibits complex patterns that vary significantly across ability levels and unemployment types. As benefits increase from low to moderate levels (5-25), both voluntary and involuntary unemployment show minimal response, suggesting that labor market participation decisions are primarily driven by wage considerations rather than benefit levels in this range. However, beyond a critical threshold (around benefit level 30), the system demonstrates dramatic changes: voluntary unemployment surges while involuntary unemployment begins to decline, indicating a fundamental shift in worker behavior.

This transition is particularly pronounced when examining unemployment patterns by ability level. High-ability workers maintain low unemployment rates until benefits reach near-wage levels, at which point they exhibit a sharp transition to voluntary unemployment. In contrast, low-ability workers show a more gradual response to increasing benefits, with unemployment rates rising steadily before accelerating. This divergence creates a notable ability-unemployment gap that peaks during the transition phase (around benefit level 32-35) before converging at high benefit levels.

The composition of unemployment also evolves markedly with benefit levels. At low benefits, involuntary unemployment dominates, reflecting labor market frictions and entry barriers. However, as benefits increase, voluntary unemployment becomes increasingly prevalent, eventually accounting for nearly all unemployment at high benefit levels. This shift suggests that generous unemployment benefits can fundamentally alter the nature of labor market non-participation from primarily involuntary to predominantly voluntary.

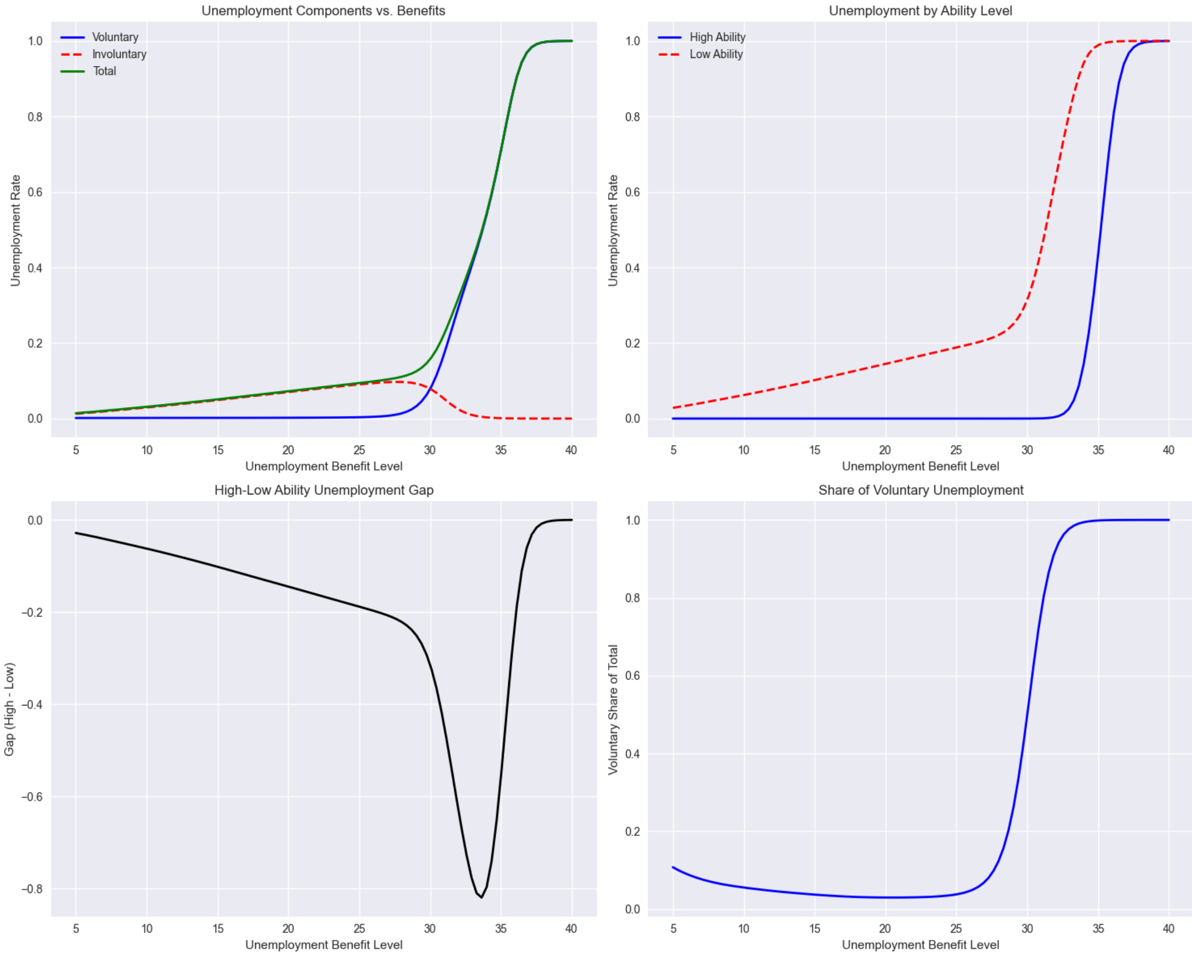

Figure 7: Unemployment Dynamics with Varying Benefit Levels

Notes: Figure shows unemployment patterns as benefits vary from 5 to 40. Top-left panel decomposes unemployment into voluntary and involuntary components, with voluntary unemployment surging beyond benefit level 30. Top-right contrasts high versus low ability workers, where high-ability workers exhibit delayed but sharper transitions. Bottom-left shows the unemployment gap between ability groups peaking during transition, while bottom-right reveals voluntary unemployment increasingly dominating total unemployment at higher benefit levels.

3.4 Policy Implications

Our simulation results suggest several important policy implications for managing segmented labor markets, particularly in developing economies experiencing rapid structural transformation. First, the wage premium in rationed sectors requires careful calibration - while higher premiums can attract talent, excessive premiums (above 40 in our simulation) eliminate ability-based sorting entirely, potentially reducing allocation efficiency. The U-shaped sorting patterns at moderate wage premiums (30-40) suggest an optimal range that both attracts high-ability workers and maintains effective screening.

Second, unemployment benefit design critically influences both the level and composition of unemployment. The sharp threshold effect around benefit level 30 indicates that policymakers face a delicate balance - while higher benefits provide better social protection, they can trigger strategic withdrawal from the labor force, particularly among high-ability workers. The divergent responses between high and low ability workers suggest that a tiered benefit system, potentially linked to previous earnings or skill levels, might better balance protection and incentives.

Finally, our results highlight the interconnected nature of premium sector wages and unemployment benefits. High wage premiums in rationed sectors can amplify the negative effects of generous unemployment benefits by providing workers, especially those with high ability, stronger incentives to wait for premium positions rather than accept immediate employment in conventional sectors. This suggests the need for coordinated reform of wage and benefit policies, potentially combining moderate wage premiums with time-limited unemployment benefits to maintain both market efficiency and social protection.

4 Ongoing Research and Future Extensions

4.1 Data Collection for Model Calibration

The calibration of our model requires extensive data collection across two dimensions: education-occupation mismatch patterns and institutional parameters. For the former, we are collecting detailed panel data from both the United States and China to enable cross-country comparison of mismatch dynamics. Specifically:

Our calibration strategy leverages comprehensive datasets from both the United States and China to enable robust cross-country comparison of mismatch dynamics. For the U.S., we combine the National Survey of College Graduates (NSCG) panel data spanning with Current Population Survey (CPS) records of occupational transitions and O*NET's task-specific skill requirements. This triangulation allows precise measurement of education-occupation matches and skill utilization patterns. For China, we employ the China Household Income Project (CHIP) surveys to track education-occupation alignment, supplemented by administrative records of civil service examination outcomes and city-level labor market data from the National Bureau of Statistics. These data sources collectively provide granular evidence on selection mechanisms, wage premiums, and workforce allocation patterns across institutional contexts.

4.2 Parameter Estimation Strategy

The model requires estimation of several key parameters that govern labor market dynamics:

1. **Institutional Parameters:**
   - Unemployment benefits (b): Varying across regions and time
   - Outside option costs (c_0): Estimated from reservation wage data
   - Public sector wage premiums: Comprehensive compensation including both monetary and non-monetary benefits
2. **Structural Parameters:**
   - Weight matrices (Θ, Φ): Mapping endowments and education to abilities
   - O-ring complementarity factors (γ_{jm}): Measuring skill synergies
   - Entry smoothing parameter (λ): Governing selection sharpness
3. **Market-Specific Parameters:**
   - Industry-specific returns (β_j): Estimated from wage regressions
   - Entry thresholds (θ_j): Calibrated from sectoral employment shares
   - Base wages (η_j): Determined in equilibrium

Our simulation values will be calibrated to match these empirical moments:

Entry thresholds:	$\theta_j \in [0.1, 0.3]$
Smoothing parameter:	$\lambda = 2.5$
Complementarity factors:	$\gamma_{jm} \in [0.1, 0.3]$
Base wage range:	$\eta_j \in [20, 50]$
Premium markup:	$20 - 30\%$ above common sectors

4.3 Dynamic Extension: Learning-by-Doing and Spatial Talent Flows

Building on our static framework, we are developing a dynamic extension that incorporates two key mechanisms: learning-by-doing in skill accumulation and strategic sectoral mobility. The dynamic model modifies the agent's problem to include forward-looking behavior and endogenous skill evolution.

4.3.1 Learning-by-Doing Mechanism

Let ability evolve according to some rules:

$$a_{ijt+1} = a_{ijt}(1 + \delta_{ij} L_{ijt}) + \phi_{ij}(a^*_{ijt})$$

where:

- δ_{ij} captures sector-specific learning rates
- L_{ijt} represents time spent in sector j
- $\phi_{ij}(a^*_{ijt})$ is a catch-up term toward frontier ability a^*_{ijt}

4.3.2 Dynamic Value Function

The agent's dynamic programming problem becomes:

$$V_t(a_{it}, h_{it}) = \max_{j \in J} \{U_{ijt} + \beta \mathbb{E}[V_{t+1}(a_{it+1}, h_{it+1})]\}$$

subject to:

$$\begin{aligned} h_{it+1} &= h_{it} + \mu_j L_{ijt} \\ a_{it+1} &= a_{it}(1 + \delta_{ij} L_{ijt}) + \phi_{ij}(a^*_{ijt}) \\ L_{ijt} &\in \{0, 1\} \end{aligned}$$

where:

- h_{it} represents accumulated human capital
- μ_j captures sector-specific human capital accumulation rates
- β is the discount factor

This dynamic structure generates rich patterns of career progression and sectoral mobility, allowing us to analyze:

1. The role of learning-by-doing in shaping optimal career paths
2. Strategic acceptance of temporary mismatches for human capital accumulation
3. Endogenous sorting patterns driven by heterogeneous learning opportunities
4. The impact of policy interventions on long-run talent allocation

4.4 Spatial Talent Flows

To capture geographic aspects of talent allocation, we extend the model to incorporate spatial mobility:

$$M_{ijrt} = f(\Delta w_{jrt}, d_{rt}, \tau_{rt})$$

where:

- M_{ijrt} represents movement from region r to r' in sector j
- Δw_{jrt} captures cross-region wage differentials
- d_{rt} represents distance/mobility costs
- τ_{rt} captures region-specific policy barriers

This spatial extension will allow us to analyze:

1. Regional variation in talent concentration and brain drain
2. The impact of mobility barriers on aggregate efficiency
3. Policy tradeoffs between regional equity and aggregate productivity
4. The role of spatial frictions in perpetuating mismatches

5 Conclusion

This paper develops a dynamic equilibrium framework for analyzing labor market segmentation with rationed entry, introducing three key innovations: a probability-based entry mechanism, an O-ring type ability aggregator, and dynamic skill evolution through learning-by-doing. The model's general equilibrium structure successfully captures the coexistence of market-clearing and rationed entry mechanisms while maintaining analytical tractability. Notably, our framework generates equilibrium wage premiums in rationed segments despite excess qualified labor supply, providing a theoretical foundation for understanding persistent labor market puzzles.

Our calibration exercises demonstrate the model's empirical relevance and flexibility. The simulations successfully replicate key empirical patterns documented in the literature: *(i) persistent education-occupation mismatches concentrated among bachelor's degree holders, (ii) sustained wage premiums in rationed segments despite massive oversupply of qualified candidates, and (iii) strategic acceptance of temporary mismatches for human capital accumulation. The model's predictions align particularly well with observed patterns in civil service examination participation and wage distributions.* Importantly, the framework exhibits sufficient flexibility to accommodate institutional variations across different labor market contexts while maintaining its core predictive power.

The analysis yields several policy implications for managing segmented labor markets in developing economies. First, wage premiums in rationed sectors require careful calibration—our simulations suggest an optimal range that balances talent attraction with allocation efficiency. Second, unemployment benefit design critically influences both the level and composition of unemployment, with important distributional consequences across ability levels. Third, coordinated reform of wage and benefit policies appears necessary to maintain both market efficiency and social protection. The model particularly emphasizes the importance of enhancing skill transferability across sectors to reduce the costs of initial mismatches.

Looking forward, several promising directions emerge for future research. First, extending the model to incorporate firm heterogeneity could provide deeper insights into how market structure influences talent allocation. Second, introducing spatial dimensions could help analyze regional variation in labor market segmentation and talent flows. Finally, applying the framework to analyze specific policy reforms, such as civil service examination system modifications or unemployment insurance redesign, could yield valuable practical insights. These extensions would further enhance our understanding of labor market dynamics in developing economies experiencing rapid structural transformation.

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6 Appendix A

Proof of Proposition 2

Proof. To establish the existence and uniqueness of η_j^*, we proceed as follows:

1. Labor Supply Function Analysis: The labor supply function $L_j^s(\eta_j)$ was derived as:

$$L_j^s(\eta_j) = \sum_{i \in \mathbf{I}} \left(\frac{\exp(V_{ij})\,(D_i - \exp(V_{ij}))}{D_i^2} \right)$$

where $V_{ij}(\eta_j) = \beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij})$ and $D_i = \exp(V_{i0}) + \exp(V_{ij}) + \sum_{k \neq j} \exp(V_{ik})$. To understand the behavior of $L_j^s(\eta_j)$, consider the case when $\beta_j f(A_O, a_{ij}) + \eta_j - c_{ij}(a_{ij}) > b - c_0$. In this scenario, the supply function $L_j^s(\eta_j)$ is influenced by the exponential function, which is strictly increasing with respect to η_j. Thus, $L_j^s(\eta_j)$ is continuous and strictly increasing in η_j due to the monotonic properties of the exponential function. Furthermore, the partial derivatives derived previously confirm this strictly increasing nature.

2. Labor Demand Function Assumption: We assume that the labor demand function $L_j^d(\eta_j)$ is continuous and strictly decreasing in η_j. This is a standard assumption in labor economics, which reflects the idea that higher wages reduce the quantity of labor demanded by firms. Thus, $L_j^d(\eta_j)$ is strictly decreasing and continuous over the relevant domain of η_j.

3. Existence of Equilibrium: By the Intermediate Value Theorem, since $L_j^s(\eta_j)$ is continuous and strictly increasing while $L_j^d(\eta_j)$ is continuous and strictly decreasing, there must exist a point η_j^* such that $L_j^s(\eta_j^*) = L_j^d(\eta_j^*)$. Specifically, as $\eta_j \to -\infty$, $L_j^s(\eta_j) \to 0$ because individuals will not supply labor at very low wages, while $L_j^d(\eta_j) \to \infty$ as firms demand more labor at very low wages. Conversely, as $\eta_j \to \infty$, $L_j^s(\eta_j) \to \infty$ (since more workers are willing to supply labor at higher wages), while $L_j^d(\eta_j) \to 0$. Thus, by continuity, an intersection must exist.

4. Uniqueness of Equilibrium: The uniqueness of the equilibrium wage level η_j^* follows from the strictly increasing nature of $L_j^s(\eta_j)$ and the strictly decreasing nature of $L_j^d(\eta_j)$. Suppose, for the sake of contradiction, that there are two distinct equilibrium wage levels, η_j^1 and η_j^2, with $\eta_j^1 < \eta_j^2$. Given that $L_j^s(\eta_j)$ is strictly increasing, it follows that $L_j^s(\eta_j^1) < L_j^s(\eta_j^2)$. Similarly, since $L_j^d(\eta_j)$ is strictly decreasing, $L_j^d(\eta_j^1) > L_j^d(\eta_j^2)$. This leads to a contradiction because $L_j^s(\eta_j^1) = L_j^d(\eta_j^1)$ and $L_j^s(\eta_j^2) = L_j^d(\eta_j^2)$ cannot simultaneously hold true if $L_j^s(\eta_j)$ and $L_j^d(\eta_j)$ cross more than once. Therefore, the intersection point must be unique.

Conclusion: Thus, there exists a unique wage level η_j^* that clears the labor market for the common sector j, ensuring that $L_j^s(\eta_j^*) = L_j^d(\eta_j^*)$. □

Proof of Corollary 1: Monotonicity of Cross-Sector Effects

Proof. (1) First, let's prove that higher premium wages reduce common sector labor supply. Consider the labor supply equation:

$$L_j^s = \sum_{i \in \mathbf{I}} P_{ij} \cdot I_{ij}$$

Taking the derivative with respect to w_k^{high}:

$$\frac{\partial L_j^s}{\partial w_k^{high}} = \sum_{i \in \mathbf{I}} \frac{\partial P_{ij}}{\partial w_k^{high}} \cdot I_{ij}$$

For $j \in \mathbf{J_1}$, the choice probability is:

$$P_{ij} = \frac{\exp(V_{ij})}{\exp(V_{i0}) + \sum_{m \in \mathbf{J}} \exp(V_{im})}$$

When w_k^{high} increases, V_{ik} increases for $k \in \mathbf{J_2}$, which implies:

$$\frac{\partial P_{ij}}{\partial w_k^{high}} = -P_{ij} P_{ik} < 0$$

Therefore, $\frac{\partial L_j^s}{\partial w_k^{high}} < 0$.

(2) For the second part, we need to show the effect strengthens with ability. Taking the second derivative:

$$\begin{aligned}
\frac{\partial^2 L_j^s}{\partial w_k^{high} \partial A_O} &= \frac{\partial}{\partial A_O} \left(\sum_{i \in \mathbf{I}} \frac{\partial P_{ij}}{\partial w_k^{high}} \cdot I_{ij} \right) \\
&= \sum_{i \in \mathbf{I}} \frac{\partial}{\partial A_O} (-P_{ij} P_{ik}) \cdot I_{ij}
\end{aligned}$$

Given that p_{ik} is increasing in A_O (from the admission probability function), and P_{ik}

increases with p_{ik}, we have:

$$\frac{\partial P_{ik}}{\partial A_O} > 0$$

This implies:

$$\frac{\partial^2 L_j^s}{\partial w_k^{high} \partial A_O} < 0$$

under the regularity condition that F_{jt} is continuously differentiable and strictly increasing. $\square$

7 Appendix B: Computational Methods

7.1 Coding for the Simulations

```
     def plot_unemployment_analysis(self, results):
    """Creates comprehensive visualization of unemployment patterns"""
    # Use newer seaborn style
    sns.set_theme(style="whitegrid")

    plt.figure(figsize=(15, 12))

    # 1. Unemployment Components
    plt.subplot(2, 2, 1)
    plt.plot(results['benefits'], results['voluntary_unemp'], 'b-',
            label='Voluntary', linewidth=2)
    plt.plot(results['benefits'], results['involuntary_unemp'], 'r--',
            label='Involuntary', linewidth=2)
    plt.plot(results['benefits'], results['total_unemp'], 'g-',
            label='Total', linewidth=2)
    plt.title('Unemployment Components vs. Benefits')
    plt.xlabel('Unemployment Benefit Level')
    plt.ylabel('Unemployment Rate')
    plt.legend()

    # 2. Unemployment by Ability Level
```

```
    plt.subplot(2, 2, 2)
    plt.plot(results['benefits'], results['high_ability_unemp'], 'b-',
             label='High Ability', linewidth=2)
    plt.plot(results['benefits'], results['low_ability_unemp'], 'r--',
             label='Low Ability', linewidth=2)
    plt.title('Unemployment by Ability Level')
    plt.xlabel('Unemployment Benefit Level')
    plt.ylabel('Unemployment Rate')
    plt.legend()

    # 3. Unemployment Gap
    plt.subplot(2, 2, 3)
    ability_gap = np.array(results['high_ability_unemp']) - np.array(results['low_ability_unemp'])
    plt.plot(results['benefits'], ability_gap, 'k-', linewidth=2)
    plt.title('High-Low Ability Unemployment Gap')
    plt.xlabel('Unemployment Benefit Level')
    plt.ylabel('Gap (High - Low)')

    # 4. Unemployment Composition
    plt.subplot(2, 2, 4)
    voluntary_share = np.array(results['voluntary_unemp']) / np.array(results['total_unemp'])
    plt.plot(results['benefits'], voluntary_share, 'b-', linewidth=2)
    plt.title('Share of Voluntary Unemployment')
    plt.xlabel('Unemployment Benefit Level')
    plt.ylabel('Voluntary Share of Total')

    plt.tight_layout()
    plt.show()

    # Additional plot in separate figure
    plt.figure(figsize=(10, 5))

    # Get current unemployment status
    utilities = self.model.calculate_expected_utilities()
    outside_option = np.ones((self.model.num_agents, 1)) * (
```

```
            self.model.unemployment_benefit - self.model.
                outside_option_cost
        )
        utilities_with_outside = np.hstack([utilities, outside_option
            ])
        choice_probs = softmax(utilities_with_outside * 2, axis=1)
        unemployed_mask = choice_probs[:, -1] > 0.5

        # Get aggregate abilities
        agg_abilities = self.model.o_ring_aggregator(self.model.
            abilities)

        # Plot distributions using seaborn
        sns.histplot(data=pd.DataFrame({
            'Ability Score': agg_abilities,
            'Status': ['Unemployed' if unemployed else 'Employed'
                       for unemployed in unemployed_mask]
        }), x='Ability Score', hue='Status', bins=30, stat='density',
            alpha=0.5)

        plt.title('Ability Distribution: Employed vs Unemployed')
        plt.xlabel('O-ring Ability Score')
        plt.ylabel('Density')

        plt.tight_layout()
        plt.show()
```

7.2 Cross Sector Analysis

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.special import softmax
from scipy.stats import norm
import seaborn as sns

class CrossSectorAnalysis:
    def __init__(self, labor_market_model):
        self.model = labor_market_model
```

```

    def analyze_wage_effects(self, wage_range):
        """Analyzes the effect of premium wage changes on market outcomes"""
        results = {
            'wages': wage_range,
            'labor_supply_effect': [],
            'probability_effect': [],
            'composition_effect': []
        }

        # Store original w_high
        original_w_high = self.model.w_high

        for w_high in wage_range:
            # Temporarily set new wage
            self.model.w_high = w_high

            # 1. Direct labor supply effect
            base_utilities = self.model.calculate_expected_utilities()
            choice_probs = softmax(base_utilities, axis=1)
            labor_supply_effect = -np.mean(np.sum(choice_probs[:, :self.model.num_common_markets] *
                                                  (1 - choice_probs[:, :self.model.num_common_markets]), axis=1))

            # 2. Probability effect through admission adjustments
            admission_probs = self.model.calculate_admission_probs()
            prob_effect = np.mean(self.model.lambda_param * admission_probs * (1 - admission_probs))

            # 3. Composition effect on ability distribution
            agg_abilities = self.model.o_ring_aggregator(self.model.abilities)
```

```
            ability_density = norm.pdf(agg_abilities, loc=np.mean(agg_abilities),
                                       scale=np.std(agg_abilities))
            comp_effect = np.mean(ability_density * np.gradient(agg_abilities))

            results['labor_supply_effect'].append(labor_supply_effect)
            results['probability_effect'].append(prob_effect)
            results['composition_effect'].append(comp_effect)

        # Restore original wage
        self.model.w_high = original_w_high

        return results

    def analyze_sorting_patterns(self, wage_levels):
        """Analyzes how sorting patterns change with different premium wages"""
        original_w_high = self.model.w_high

        plt.figure(figsize=(15, 5))

        for idx, wage in enumerate(wage_levels):
            self.model.w_high = wage
            utilities = self.model.calculate_expected_utilities()
            choice_probs = softmax(utilities, axis=1)
            agg_abilities = self.model.o_ring_aggregator(self.model.abilities)

            plt.subplot(1, len(wage_levels), idx+1)
            plt.scatter(agg_abilities, choice_probs[:, -1], alpha=0.5)
            plt.title(f'Sorting at Premium Wage = {wage}')
            plt.xlabel('O-ring Ability Score')
            plt.ylabel('Prob. of Premium Sector Choice')

        self.model.w_high = original_w_high
        plt.tight_layout()
        plt.show()
```

```

    def plot_effects(self, results):
        """Creates comprehensive visualization of cross-sector
           effects"""
        plt.style.use('seaborn')  # Use seaborn style for better
           visuals

        plt.figure(figsize=(15, 10))

        # 1. Labor Supply Effect
        plt.subplot(2, 2, 1)
        plt.plot(results['wages'], results['labor_supply_effect'],
            'b-', linewidth=2)
        plt.title('Direct Labor Supply Effect')
        plt.xlabel('Premium Wage Level')
        plt.ylabel('partialLs/partialw_high')

        # 2. Probability Effect
        plt.subplot(2, 2, 2)
        plt.plot(results['wages'], results['probability_effect'],
           'r-', linewidth=2)
        plt.title('Admission Probability Effect')
        plt.xlabel('Premium Wage Level')
        # plt.ylabel('partialp/partialw_high')

        # 3. Composition Effect
        plt.subplot(2, 2, 3)
        plt.plot(results['wages'], results['composition_effect'],
           'g-', linewidth=2)
        plt.title('Ability Composition Effect')
        plt.xlabel('Premium Wage Level')
        # plt.ylabel('partialF/partialw_high')

        # 4. Combined Effects
        plt.subplot(2, 2, 4)
        plt.plot(results['wages'], results['labor_supply_effect'],
            'b-', label='Labor Supply', linewidth=2)
        plt.plot(results['wages'], results['probability_effect'],
           'r-', label='Probability', linewidth=2)
```

```python
        plt.plot(results['wages'], results['composition_effect'],
            'g-', label='Composition', linewidth=2)
        plt.title('Combined Cross-Sector Effects')
        plt.xlabel('Premium Wage Level')
        plt.ylabel('Effect Magnitude')
        plt.legend()

        plt.tight_layout()
        plt.show()

        # Additional visualization: Ability distribution and
            sorting
        plt.figure(figsize=(12, 5))

        # Ability distribution
        plt.subplot(1, 2, 1)
        agg_abilities = self.model.o_ring_aggregator(self.model.
            abilities)
        sns.histplot(agg_abilities, bins=30, stat='density', alpha
            =0.7)
        plt.title('Distribution of O-ring Abilities')
        plt.xlabel('O-ring Ability Score')
        plt.ylabel('Density')

        # Sorting patterns
        plt.subplot(1, 2, 2)
        admission_probs = self.model.calculate_admission_probs()
        plt.scatter(agg_abilities, admission_probs, alpha=0.5)
        plt.title('Ability-Based Sorting Pattern')
        plt.xlabel('O-ring Ability Score')
        plt.ylabel('Admission Probability')

        plt.tight_layout()
        plt.show()

    def generate_summary_statistics(self, wage_levels):
        """Generates summary statistics for different wage levels
            """
        stats = []
        original_w_high = self.model.w_high
```

```python

        for wage in wage_levels:
            self.model.w_high = wage
            model_stats = self.model.simulate_market_clearing()

            stats.append({
                'Premium Wage': wage,
                'Avg Ability (Premium)': np.mean(self.model.
                    abilities[model_stats['market_sizes'][-2], -1])
                    ,
                'Avg Ability (Common)': np.mean([np.mean(self.
                    model.abilities[model_stats['market_sizes'][i],
                     i])
                                                for i in range(self
                                                    .model.
                                                    num_common_markets
                                                    )]),
                'Voluntary Unemployment': model_stats['
                    unemployment']['voluntary'],
                'Involuntary Unemployment': model_stats['
                    unemployment']['involuntary']
            })

        self.model.w_high = original_w_high
        return pd.DataFrame(stats)

# Run the analysis
def run_cross_sector_analysis(num_agents=10000, num_common_markets
    =10):
    # Initialize the base model
    model = LaborMarketModel(num_agents=num_agents,
        num_common_markets=num_common_markets)

    # Create analyzer
    analyzer = CrossSectorAnalysis(model)

    # Analyze wage effects
    wage_range = np.linspace(20, 50, 100)
    results = analyzer.analyze_wage_effects(wage_range)

```

```
    # Create visualizations
    analyzer.plot_effects(results)

    # Analyze sorting patterns at different wage levels
    wage_levels = [10, 30, 40, 50]
    analyzer.analyze_sorting_patterns(wage_levels)

    # Generate summary statistics
    summary_stats = analyzer.generate_summary_statistics(wage_levels)
    print("\nSummary Statistics for Different Wage Levels:")
    print(summary_stats)

    return analyzer, results, summary_stats

# Execute the analysis
analyzer, results, summary_stats = run_cross_sector_analysis()
```

7.3 Unemployment Analysis

```
        def plot_unemployment_analysis(self, results):
    """Creates comprehensive visualization of unemployment patterns"""
    # Use newer seaborn style
    sns.set_theme(style="whitegrid")

    plt.figure(figsize=(15, 12))

    # 1. Unemployment Components
    plt.subplot(2, 2, 1)
    plt.plot(results['benefits'], results['voluntary_unemp'], 'b-',
            label='Voluntary', linewidth=2)
    plt.plot(results['benefits'], results['involuntary_unemp'], 'r--',
            label='Involuntary', linewidth=2)
    plt.plot(results['benefits'], results['total_unemp'], 'g-',
            label='Total', linewidth=2)
    plt.title('Unemployment Components vs. Benefits')
```

```
    plt.xlabel('Unemployment Benefit Level')
    plt.ylabel('Unemployment Rate')
    plt.legend()

    # 2. Unemployment by Ability Level
    plt.subplot(2, 2, 2)
    plt.plot(results['benefits'], results['high_ability_unemp'], '
        b-',
            label='High Ability', linewidth=2)
    plt.plot(results['benefits'], results['low_ability_unemp'], 'r
        --',
            label='Low Ability', linewidth=2)
    plt.title('Unemployment by Ability Level')
    plt.xlabel('Unemployment Benefit Level')
    plt.ylabel('Unemployment Rate')
    plt.legend()

    # 3. Unemployment Gap
    plt.subplot(2, 2, 3)
    ability_gap = np.array(results['high_ability_unemp']) - np.
        array(results['low_ability_unemp'])
    plt.plot(results['benefits'], ability_gap, 'k-', linewidth=2)
    plt.title('High-Low Ability Unemployment Gap')
    plt.xlabel('Unemployment Benefit Level')
    plt.ylabel('Gap (High - Low)')

    # 4. Unemployment Composition
    plt.subplot(2, 2, 4)
    voluntary_share = np.array(results['voluntary_unemp']) / np.
        array(results['total_unemp'])
    plt.plot(results['benefits'], voluntary_share, 'b-', linewidth
        =2)
    plt.title('Share of Voluntary Unemployment')
    plt.xlabel('Unemployment Benefit Level')
    plt.ylabel('Voluntary Share of Total')

    plt.tight_layout()
    plt.show()

    # Additional plot in separate figure
```

```
    plt.figure(figsize=(10, 5))

    # Get current unemployment status
    utilities = self.model.calculate_expected_utilities()
    outside_option = np.ones((self.model.num_agents, 1)) * (
        self.model.unemployment_benefit - self.model.outside_option_cost
    )
    utilities_with_outside = np.hstack([utilities, outside_option])
    choice_probs = softmax(utilities_with_outside * 2, axis=1)
    unemployed_mask = choice_probs[:, -1] > 0.5

    # Get aggregate abilities
    agg_abilities = self.model.o_ring_aggregator(self.model.abilities)

    # Plot distributions using seaborn
    sns.histplot(data=pd.DataFrame({
        'Ability Score': agg_abilities,
        'Status': ['Unemployed' if unemployed else 'Employed'
                   for unemployed in unemployed_mask]
    }), x='Ability Score', hue='Status', bins=30, stat='density', alpha=0.5)

    plt.title('Ability Distribution: Employed vs Unemployed')
    plt.xlabel('O-ring Ability Score')
    plt.ylabel('Density')

    plt.tight_layout()
    plt.show()

def run_unemployment_analysis(num_agents=10000, num_common_markets=10):
    """Runs comprehensive unemployment analysis"""
    # Initialize the base model
    model = LaborMarketModel(num_agents=num_agents, num_common_markets=num_common_markets)

    # Create analyzer
```

```python
    analyzer = UnemploymentAnalysis(model)

    # Analyze unemployment across different benefit levels
    benefit_range = np.linspace(5, 40, 100)
    results = analyzer.analyze_unemployment(benefit_range)

    # Create visualizations
    analyzer.plot_unemployment_analysis(results)

    return analyzer, results

# Execute the analysis
analyzer, results = run_unemployment_analysis()
```