

# Blockchains & Distributed Ledgers

Lecture 06 - Economics and Incentives

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Slide credits: AK, Dionysis Zindros, Christos Nasikas

# Mining parameters

# Mining incentives

A miner is incentivized to mine with 2 ways:

*(1) Fees and (2) rewards*

# Mining fees

*Fees* are the remaining money from the Law of Conservation for confirming transactions:

$$\text{fees} = \sum_{\text{tx} \in \text{block}} \left[ \sum_{i \in \text{in}(\text{tx})} w(i) - \sum_{o \in \text{out}(\text{tx})} w(o) \right]$$

These fees are claimed by the miner who included the respective transaction

$w(.) = \text{value}$

# Mining rewards

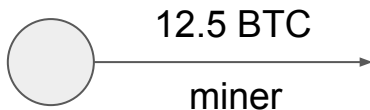
A miner is given a **reward**, today equal to 12.5 BTC

The *reward* and the *fee* are called the **coinbase** and together can be claimed by the miner.

Example: fees = 1 BTC, rewards = 12.5 BTC, total coinbase = 13.5 BTC

# The coinbase transaction

- The **coinbase transaction** is the transaction by which a miner is paid the fees and the rewards
- There can be only **one coinbase transaction** per block
- It is the **first** transaction that appears in the block
- It has *no inputs*
- This is the only way fresh bitcoins are generated



# Coinbase transaction validity

- The **induction basis** for transaction validity  
(we have seen the inductive **step** when we talked about UTXO evolution)
- Has no inputs, so does not conform to the Law of Conservation!
- Never forms part of the mempool -- only included in blocks for the first time
- When a block is confirmed, the coinbase is checked for validity:
  - It is the first in the block
  - There's only one of it
  - **output value  $\leq$  block reward + block fees**
- Hence, a malicious miner cannot generate more money for themselves

# The coinbase transaction

- As it does not have any inputs, scriptSig can be anything for coinbase
- scriptSig is used for certain block metadata:
  - The block height (this is verified for validity)
  - The name of the mining pool that mined the block
  - Extra entropy (nonce) if the block header nonce ("ctr") has been exhausted
  - Signalling for protocol updates  
(whether a miner is in favour of an upgrade or not -- more on this in a later lecture)



# Money supply in bitcoin

- The **money supply** in bitcoin is **algorithmically predetermined**
- Achieved with an **algorithm** known beforehand to all
- Concretely:
  - The coinbase of **genesis** has **reward 50 BTC**
  - Each next block has reward equal to its previous block
  - Every 210,000 blocks (expected every **4 years**) the reward is **halved**
  - The duration during which rewards stay the same is known as an **era**

$$\text{total\_btc\_supply} = \frac{\sum_{i=0}^{32} 210000 \lfloor \frac{50 \cdot 10^8}{2^i} \rfloor}{10^8}$$

number of eras until reward is negligible

era duration in blocks

genesis block reward

total\_btc\_supply = 
$$\frac{\sum_{i=0}^{32} 210000 \left[ \frac{50 \cdot 10^8}{2^i} \right]}{10^8}$$

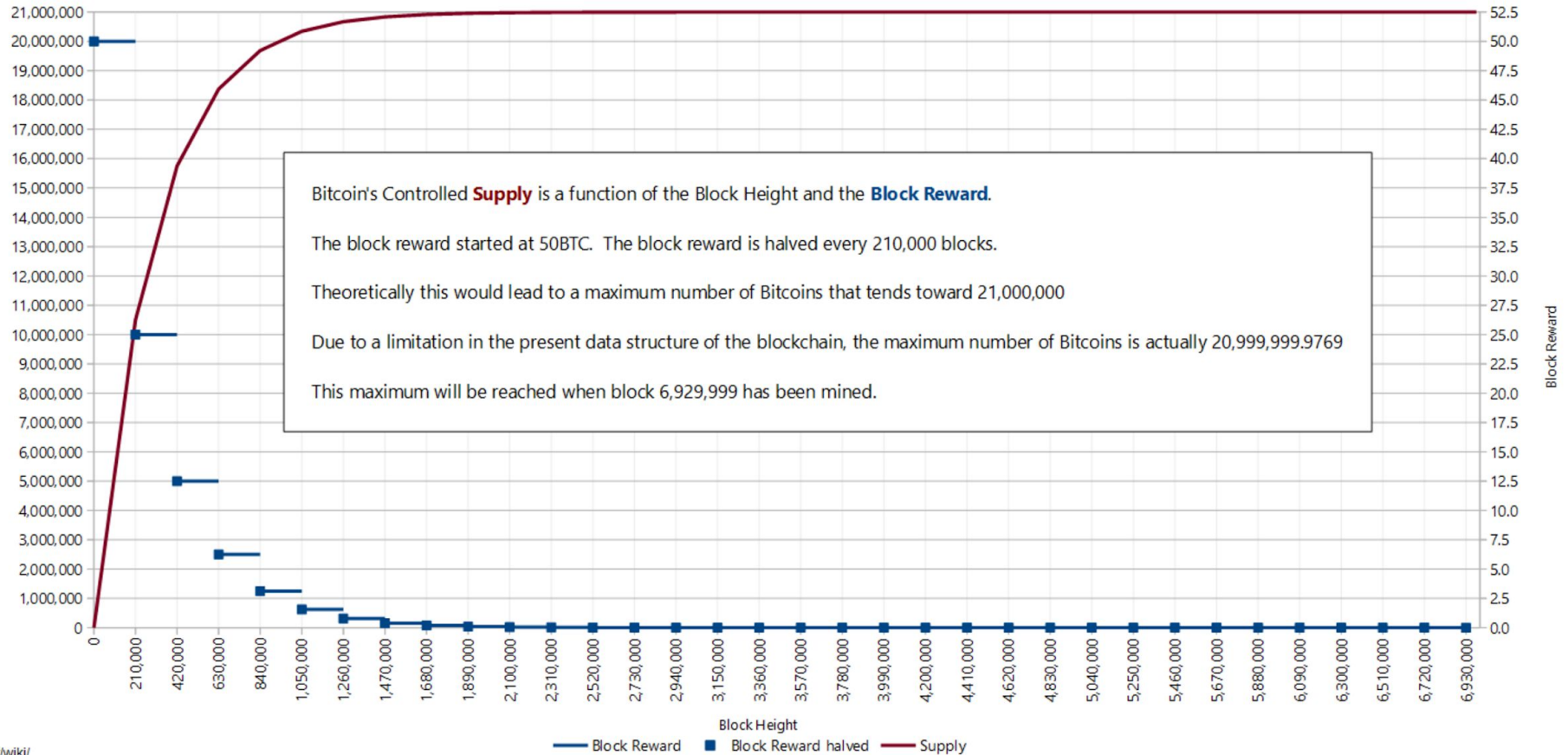
satoshi / bitcoin

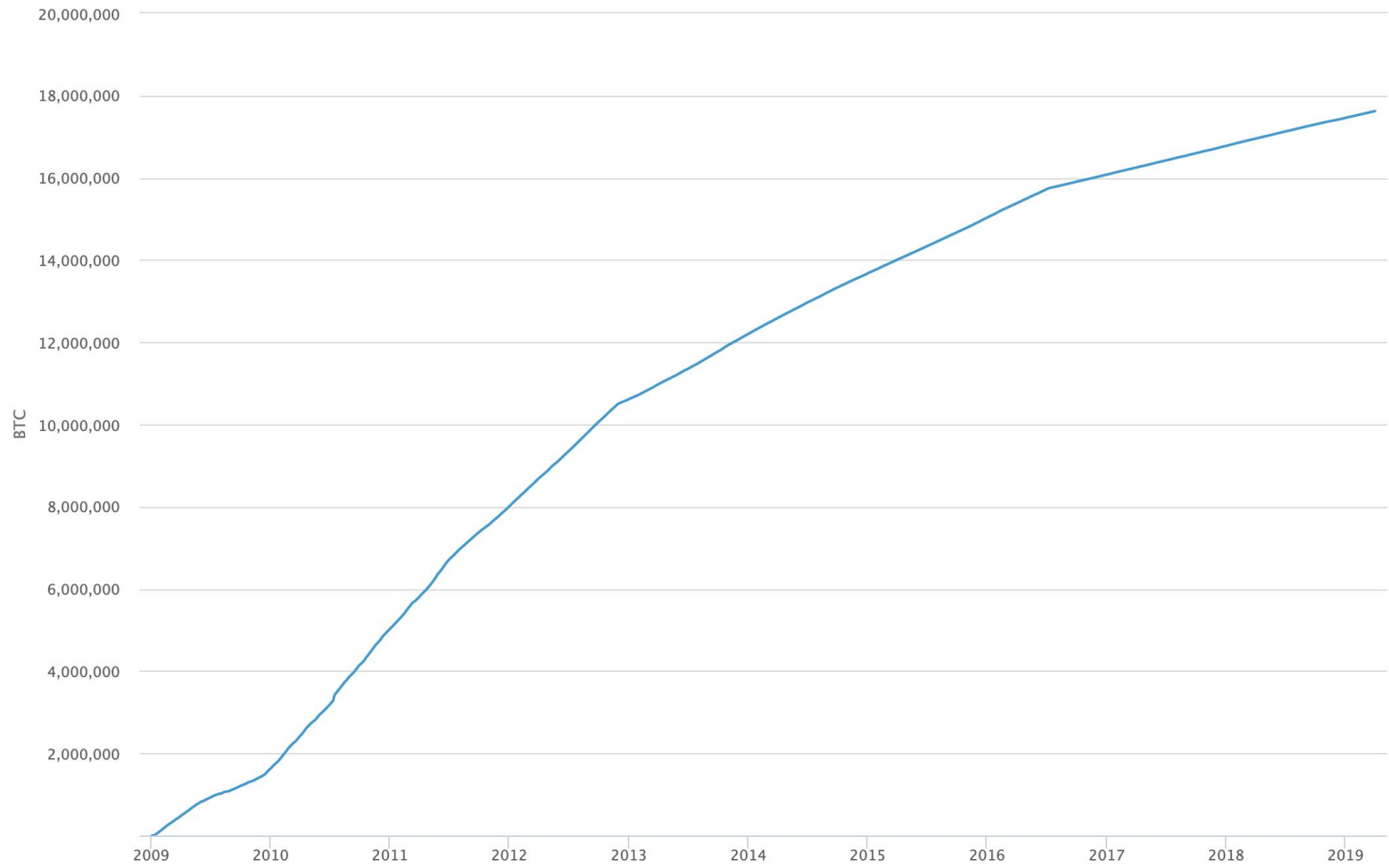
The diagram illustrates the formula for calculating the total Bitcoin supply. The formula is presented as a fraction. The numerator is a summation from i=0 to 32 of the product of 210,000 and the floor of (50 \* 10^8) divided by 2^i. The denominator is 10^8. Annotations with arrows point to specific parts of the formula: an orange arrow points to the upper limit '32' of the summation, labeled 'number of eras until reward is negligible'; a red arrow points to the constant '210000', labeled 'era duration in blocks'; a blue arrow points to the '50' in the numerator's fraction, labeled 'genesis block reward'; and a green arrow points to the '10^8' in the denominator, labeled 'satoshi / bitcoin'.

Date reached	Block	Reward Era	BTC/block	Year (estimate)	Start BTC	BTC Added	End BTC	BTC Increase	End BTC % of Limit
2009-01-03	0	1	50.00	2009	0	2625000	2625000	infinite	12.500%
2010-04-22	52500	1	50.00	2010	2625000	2625000	5250000	100.00%	25.000%
2011-01-28	105000	1	50.00	2011*	5250000	2625000	7875000	50.00%	37.500%
2011-12-14	157500	1	50.00	2012	7875000	2625000	10500000	33.33%	50.000%
<a href="#">2012-11-28</a>	210000	2	25.00	2013	10500000	1312500	11812500	12.50%	56.250%
2013-10-09	262500	2	25.00	2014	11812500	1312500	13125000	11.11%	62.500%
2014-08-11	315000	2	25.00	2015	13125000	1312500	14437500	10.00%	68.750%
2015-07-29	367500	2	25.00	2016	14437500	1312500	15750000	9.09%	75.000%
2016-07-09	420000	3	12.50	2016	15750000	656250	16406250	4.17%	78.125%
2017-06-23	472500	3	12.50	2018	16406250	656250	17062500	4.00%	81.250%
2018-05-29	525000	3	12.50	2019	17062500	656250	17718750	3.85%	84.375%
	577500	3	12.50	2020	17718750	656250	18375000	3.70%	87.500%
	630000	4	6.25	2021	18375000	328125	18703125	1.79%	89.063%
	682500	4	6.25	2022	18703125	328125	19031250	1.75%	90.625%
	735000	4	6.25	2023	19031250	328125	19359375	1.72%	92.188%
	787500	4	6.25	2024	19359375	328125	19687500	1.69%	93.750%

## Bitcoin - Controlled Supply

Number of bitcoins as a function of Block Height





# Bitcoin's price

- Extremely volatile
- April 2019: 1 BTC = 4,429 EUR
- mid 2018: 1 BTC = 5,502 EUR
- late 2017: 1 BTC = 17,000 EUR
- beginning 2015: 1 BTC = 208 EUR
- max 2013: 1 BTC = 900 EUR
- min 2013: 1 BTC = 73 EUR
- 2012: 1 BTC = 4 EUR
- 2010: 1 BTC = 0.06 EUR
- 22 May 2010: First real purchase with bitcoin



22 May 2010: One pizza for 10,000 BTC





# Bitcoin Price (BTC)

\$5,001.55

+\$4,969.94 (37.2K%)

1H

24H

1W

1M

1Y

ALL





# Market capitalization

Total bitcoin supply:

**04 April 2019: 17,626,900 BTC = \$87,655,483,440.00 USD**

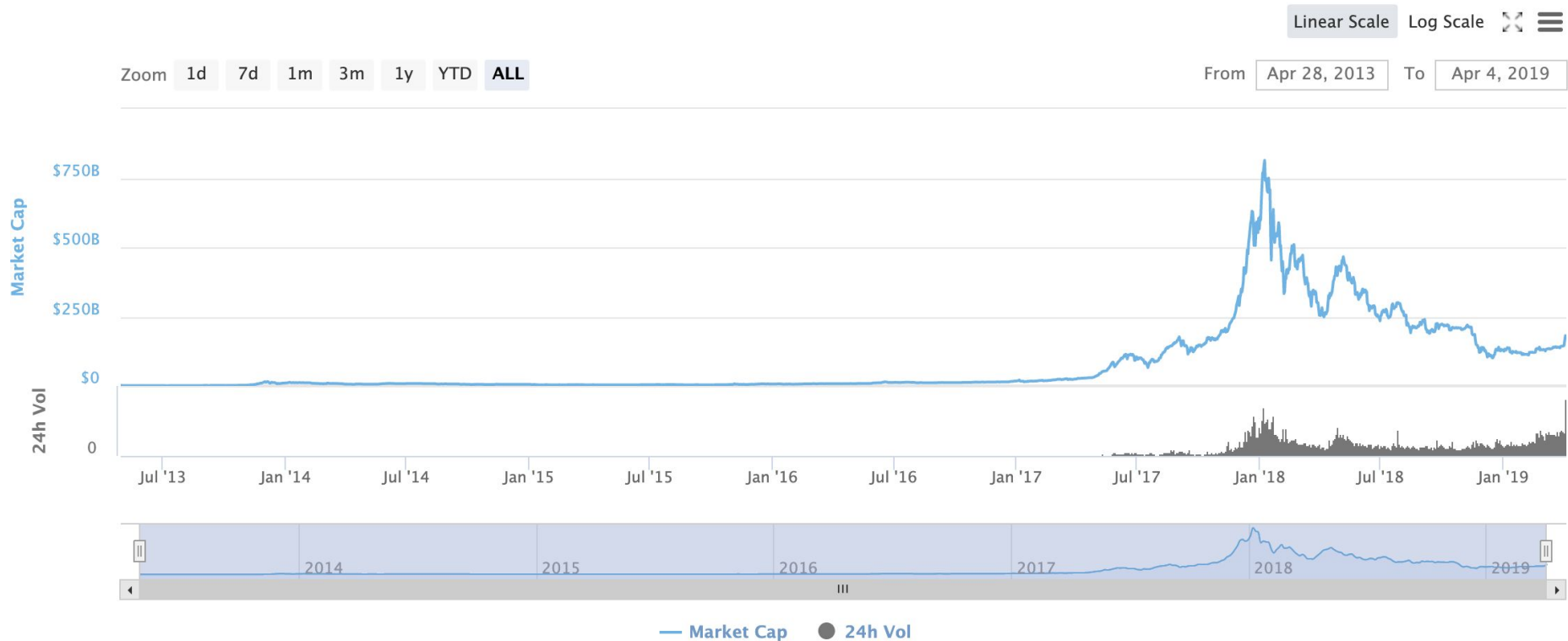
# Bitcoin market capitalization

Zoom 1d 7d 1m 3m 1y YTD **ALL**

From Apr 28, 2013 To Apr 4, 2019



# Total Market Capitalization



# Mining target

Recall the PoW equation:  $H(x \parallel \text{ctr} \parallel \text{previd}) < T$

$\Pr[\text{a query yields a valid block}] = T / 2^\kappa$

- $T$  is **the target**
- In bitcoin,  $\kappa = 256$
- $T$  is the same for all miners
- Block validity **requires proof-of-work validation**
- Adversarial blocks with insufficient proof-of-work are rejected as invalid

# Difficulty adjustment

- $T_0 = 2^{256} - 32$
- The target  $T$  is not constant but **dynamically adjusted**
- **This is the way the network ensures a rate of 1 block per 10 minutes**
- $T$  is adjusted collectively by the network
- Adjustment happens every 2016 blocks (expected 2 weeks)
- The period of 2016 blocks is known as an **epoch**
- Difficulty is adjusted so that if the previous 2016 blocks had been generated with the *new*  $T$ , they would have taken *exactly* 2 weeks

The target adjustment equation

$$T' = T \frac{\text{TS}(\text{block}_n) - \text{TS}(\text{block}_{n-2016})}{2 \text{ weeks}}$$

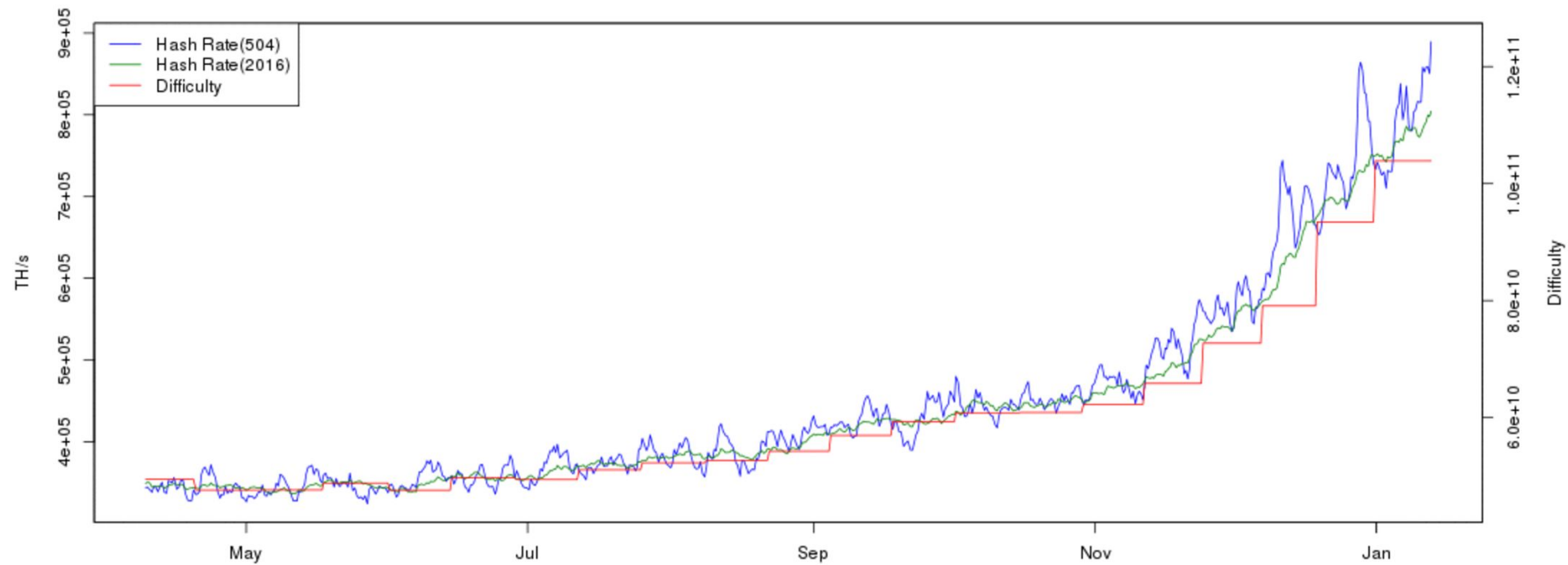
# Difficulty

- The inverse of the target is known as the **difficulty**

$$\text{difficulty} = T_0 / T$$

$$\text{difficulty}_0 = 1$$

Bitcoin Hash Rate vs Difficulty (9 Months)





# Bitcoin denominations

- **1 bitcoin** is divisible up to  **$10^{-8}$**
- $10^{-8}$  bitcoin = 1 **satoshi**
- 1 satoshi = 0.00000001 BTC
- 1 BTC = 100,000,000
- The bitcoin implementation stores **integers** in the output edges, representing the number of satoshis
- Hence, there are no floating-point errors

# Mining pools

- Mining gives a high reward per block (£80,000)
- Has a small probability of success
- The variance is high
  - I prefer to get £1600 per month than £80,000 per 4 years
- Miners typically collaborate in **mining pools**
- If a miner in a pool finds a block, the proceeds are split among the pool members
- Splitting is *pro rata* according to the computational power contributed by each
- Miners that mine outside of pools (very rare) are called **solo**
- Pools are maintained by a trusted, centralized **pool owner**

# Mining inside a pool

- The pool maintains a different **internal** target for proof-of-work  $T_{\text{pool}} > T_{\text{bitcoin}}$
- If a block satisfies  $T_{\text{bitcoin}} < H(B) < T_{\text{pool}}$ , it is called a **share**
- The miners of the pool mine as follows:
  - They include a coinbase tx with output the pool address
  - If  $H(B) < T_{\text{bitcoin}}$ , they **broadcast the block to the bitcoin network**
  - If  $H(B) < T_{\text{pool}}$ , they **broadcast the share inside the pool**

# Pool share verification

Pool owner verifies shares:

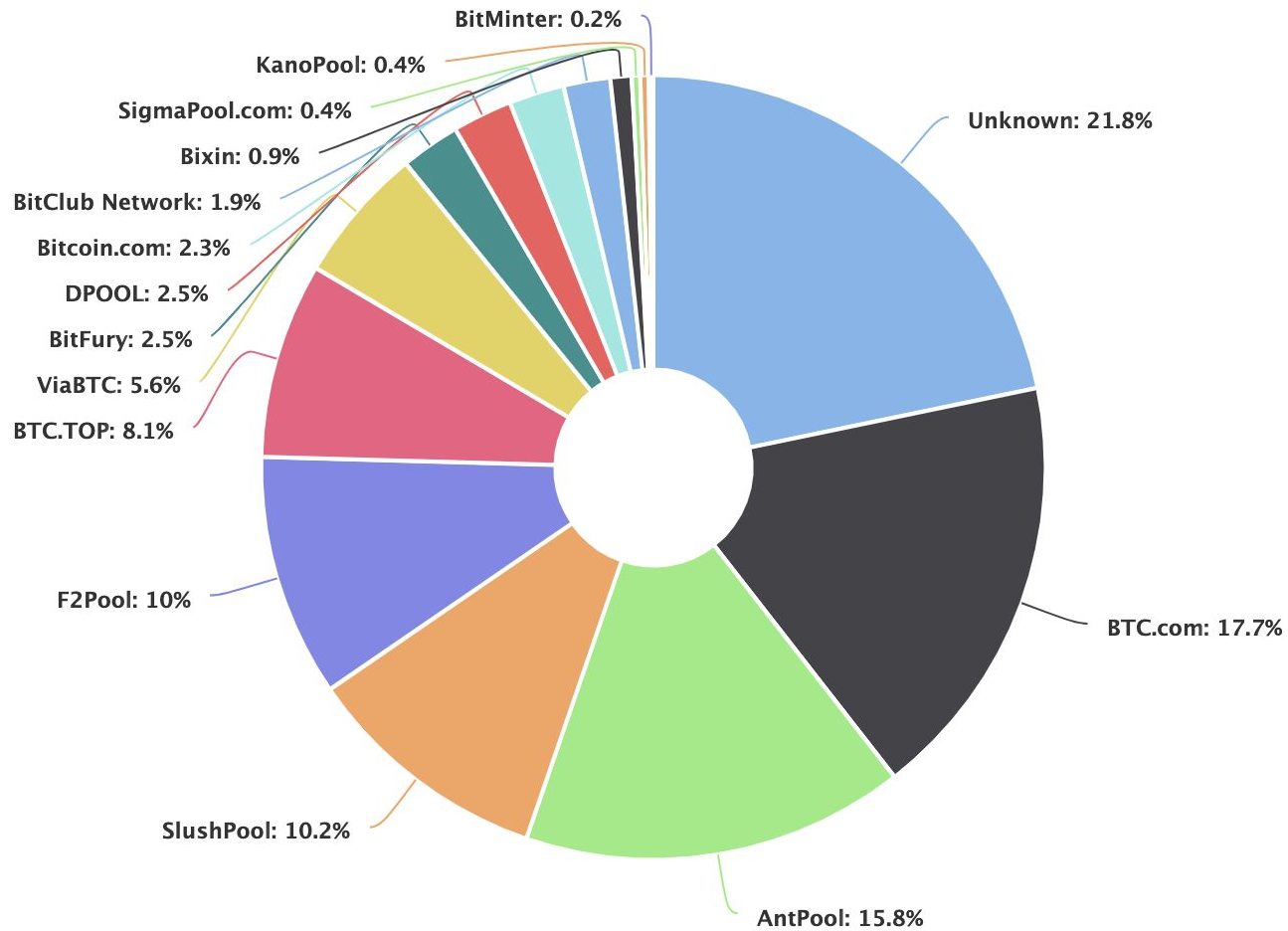
- Check that PoW is achieved with  $T_{\text{pool}}$
- Check that coinbase tx pays to the **pool address** and not some other address

# Pool rewarding

- When a bitcoin block is created, each node in the pool is rewarded *proportionally* to the pool blocks they have recently generated (compared to the total number of pool blocks generated)
- Node participants pay a participation fee to the **owner** of the pool
- Pools are a trusted scheme: Miners trust the pool owner, but the pool owner does not trust the miners. Miners don't trust the other miners in the pool.
- The pool owner can steal money, but they will be detected

- Why can't a pool miner mine shares with the pool address, but blocks with his own address?

- Why can't a pool miner mine shares with the pool address, but blocks with his own address?
- They don't know if it will be a share or a block during mining! After mining is completed, changing the address will invalidate the PoW.





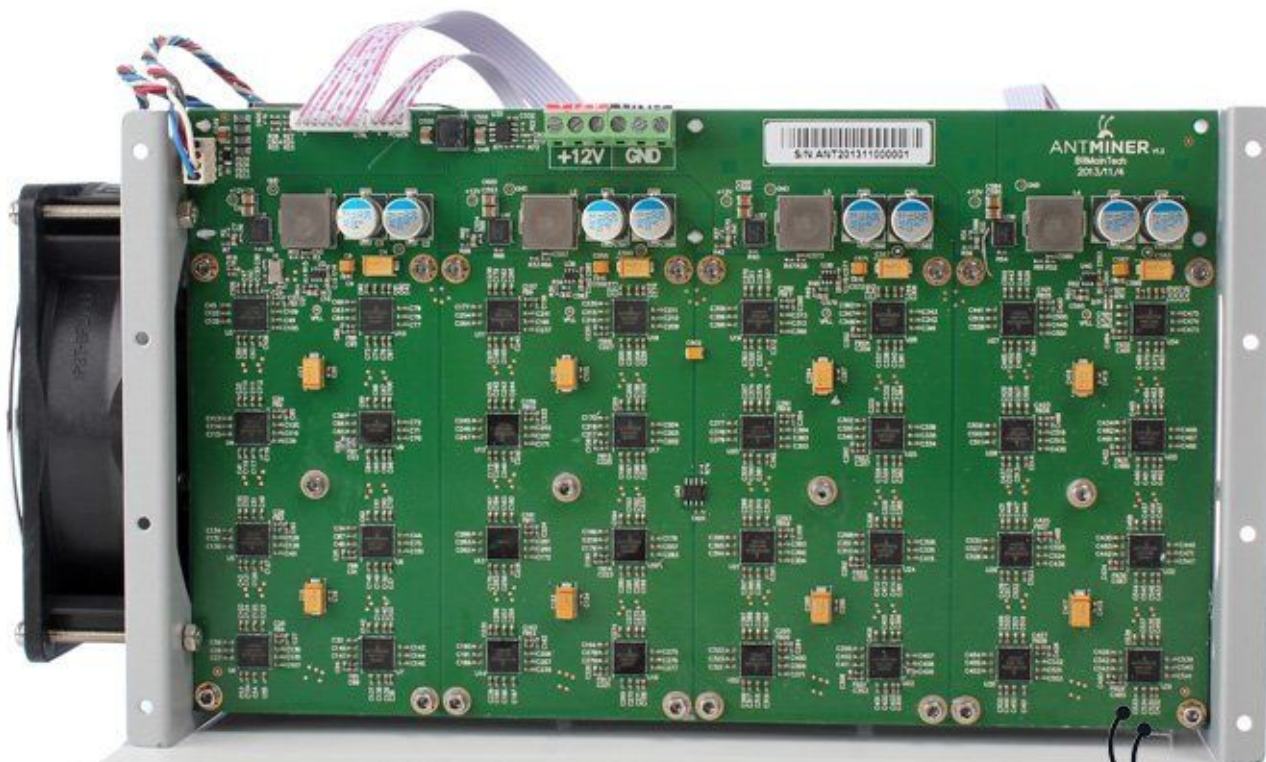
	Pool	Hashrate Share	Hashrate	Blocks Mined	Empty Blocks Count	Empty Blocks Percentage	Avg. Block Size (Bytes)	Avg. Tx Fees Per Block (BTC)	Tx Fees % of Block Reward
0	NETWORK	100.00 %	45.33 EH/s	141	2	1.42 %	1,223,954	1.08225240	8.66 %
1	<a href="#">AntPool</a>	19.15 %	8.68 EH/s	27	0	0.00 %	1,223,365	1.08066490	8.65 %
2	<a href="#">BTC.com</a>	16.31 %	7.39 EH/s	23	1	4.35 %	1,233,210	1.03386865	8.27 %
3	<a href="#">Poolin</a>	10.64 %	4.82 EH/s	15	0	0.00 %	1,225,077	1.11124874	8.89 %
4	<a href="#">SlushPool</a>	9.22 %	4.18 EH/s	13	0	0.00 %	1,248,950	1.13871978	9.11 %
5	<a href="#">F2Pool</a>	8.51 %	3.86 EH/s	12	0	0.00 %	1,191,480	1.10212394	8.82 %
6	<a href="#">unknown</a>	7.80 %	3.54 EH/s	11	0	0.00 %	1,232,072	1.16722453	9.34 %
7	<a href="#">BTC.TOP</a>	7.09 %	3.21 EH/s	10	0	0.00 %	1,189,027	1.05074060	8.41 %
8	<a href="#">ViaBTC</a>	4.96 %	2.25 EH/s	7	0	0.00 %	1,180,631	1.05787656	8.46 %
9	<a href="#">DPOOL</a>	4.26 %	1.93 EH/s	6	0	0.00 %	1,301,065	1.17691249	9.42 %
10	<a href="#">BitClub</a>	3.55 %	1.61 EH/s	5	0	0.00 %	1,254,027	1.15430945	9.23 %

# Ways to mine

**CPU:** Mining takes place in the CPU

**GPU:** Mining takes place in the graphics card (high parallelization)

**ASIC:** Specialized hardware for mining, separate hardware device



  
**ANTMINER**

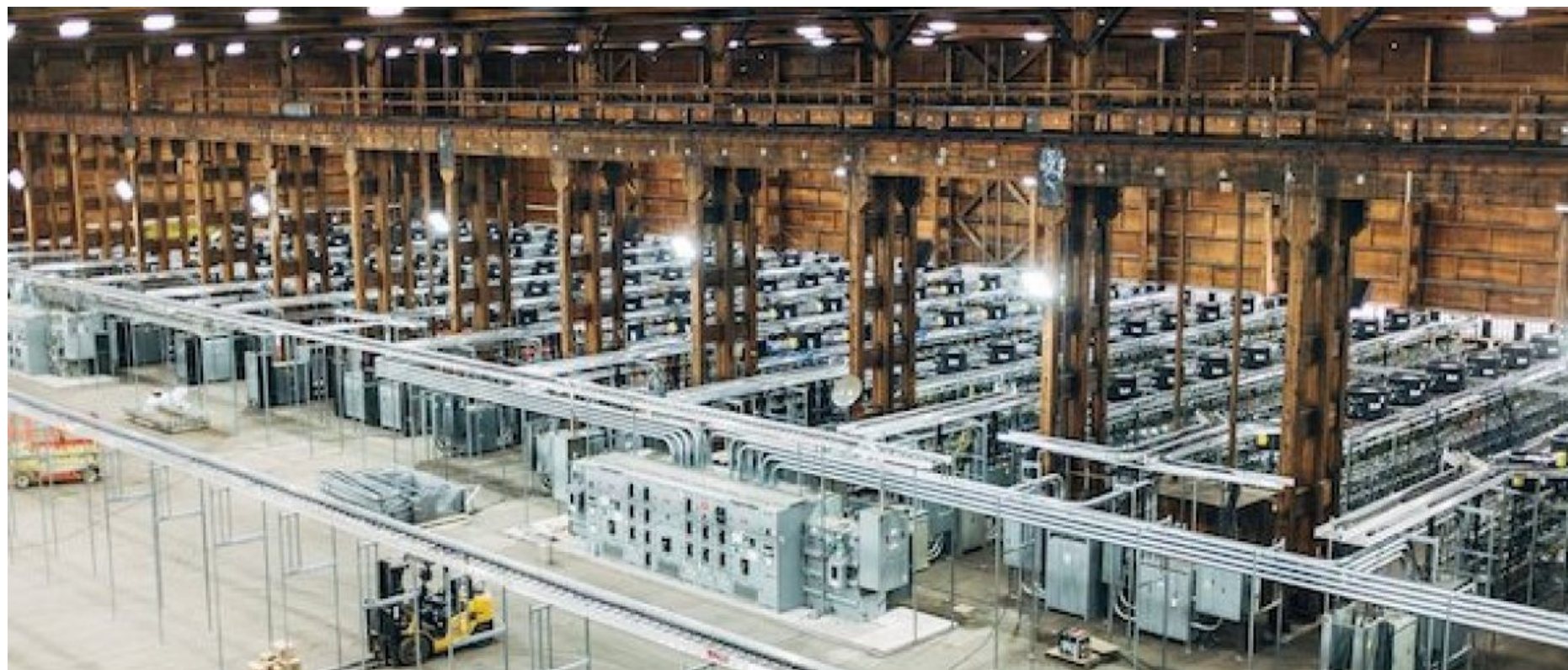
# Is it worth it to mine? Probably not...

- With **CPU** Core i7: income **\$0.04 / year**
- M€ **GPU** NVIDIA GTX590: income **\$0.13 / year**
- M€ **GPU** ATI 6990: **income \$0.58 / year**
- With specialized hardware AntMiner S5+:
  - Income: \$5136 / year
- Electricity cost: \$2409 / year (with \$0.08 / kWh)
- Pool participation cost: \$102.74
- Profit: \$2624 / year
- Initial hardware cost: \$2307
- **Profits the first year: \$317**







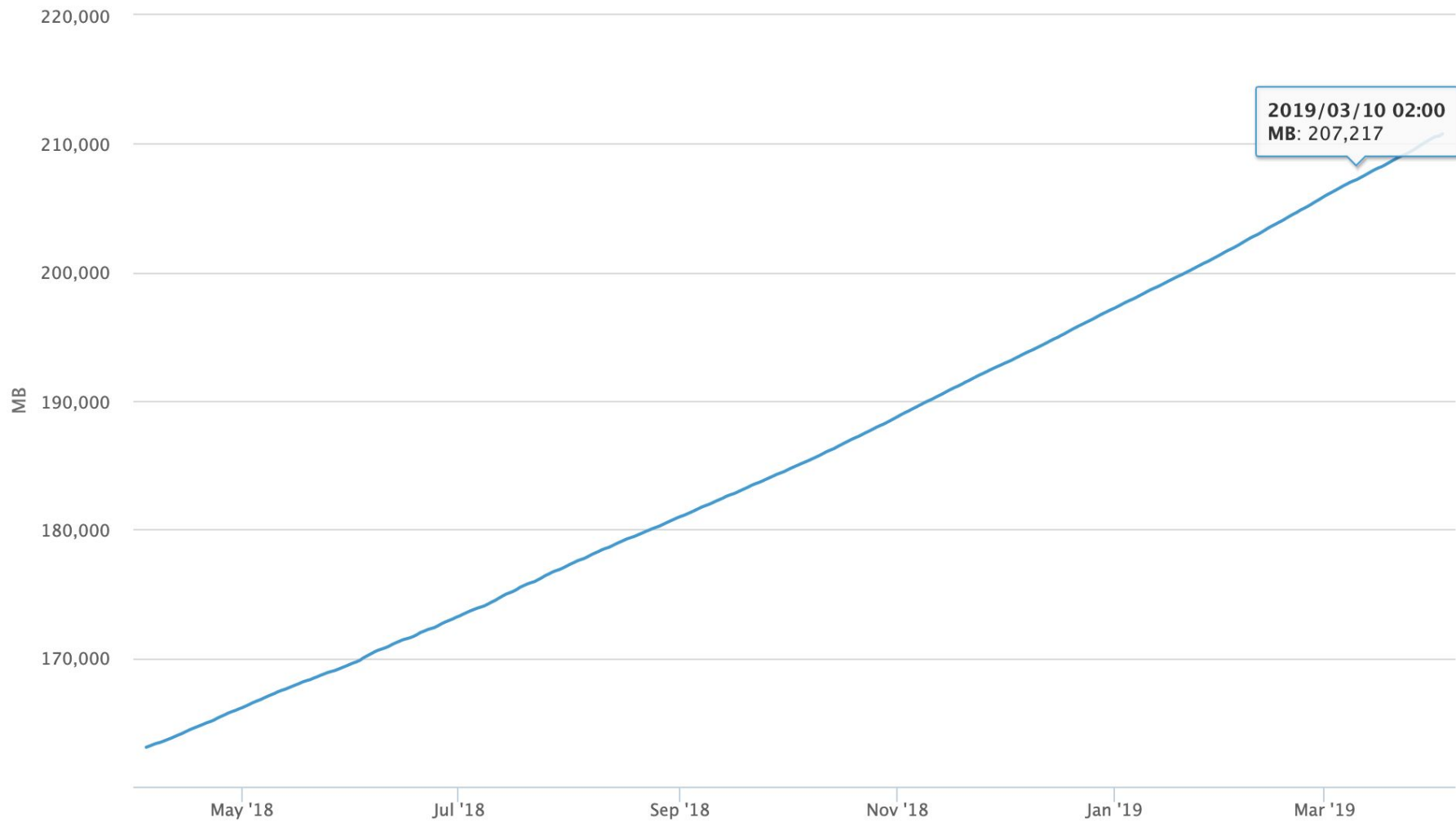


# Wallets

# Full nodes

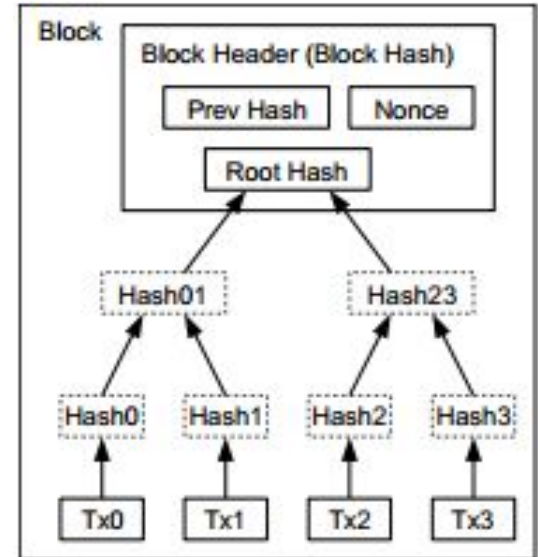
- Some wallets maintain the whole blockchain
- Full nodes:
  - Keep the whole blockchain history (~187 GB)
  - Keep the whole UTXO
  - Verify each tx
  - Verify each block
  - Relay every tx and block

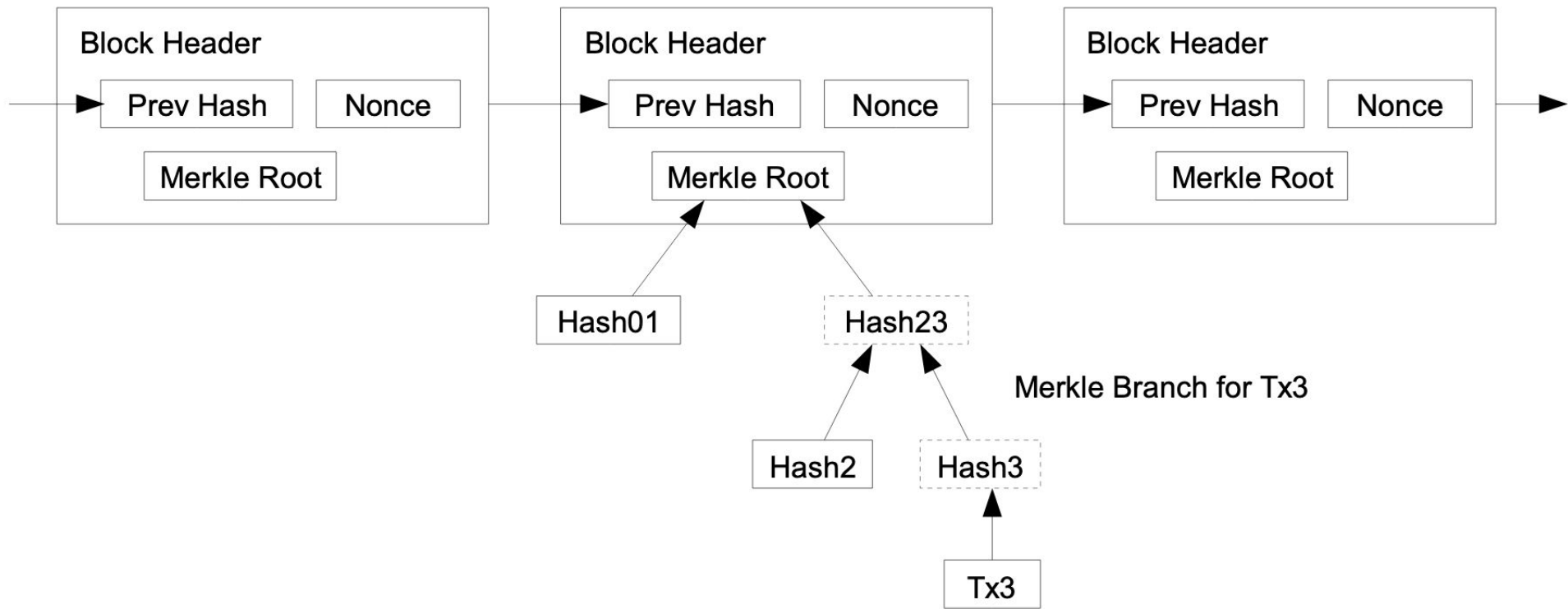




# Merkle trees of transactions

- Transactions not yet confirmed, but received by a full node are collected into a data structure called the **mempool**
- To build a block, the mempool transactions **are collected** into a Merkle Tree in an (arbitrary, but valid) order defined by the miner
- The application data in the **block header** for which the proof-of-work equation is solved only contain **the root** of this Merkle Tree:  
**x is the Merkle Tree Root!**





# Advantages of using a Merkle tree

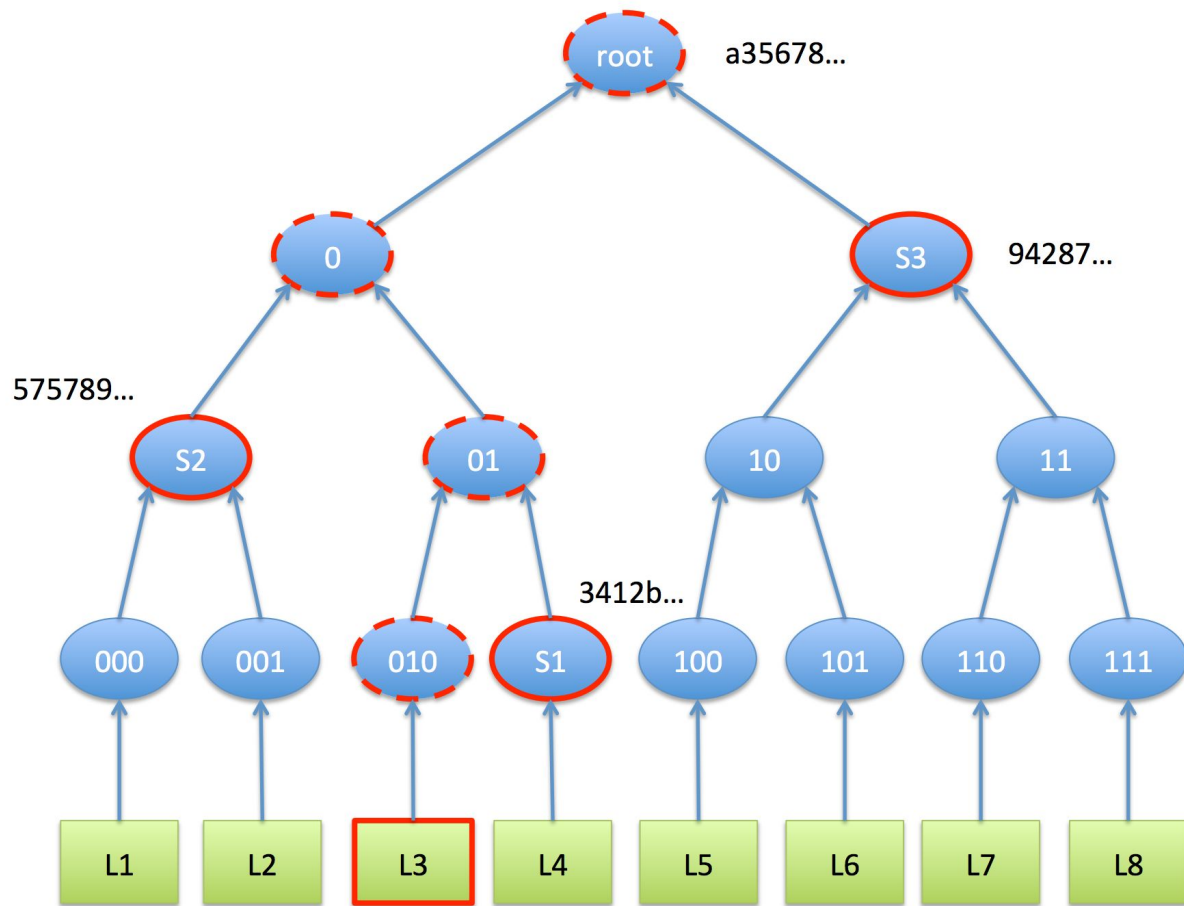
- Proof-of-work difficulty **does not depend** on the number of confirmed transactions
  - Each miner is incentivized to include all transactions they can and have a non-zero fee
- The PoW difficulty **only depends on the target T**
  - This allows better control of the mining rate
- **It enables SPV wallets!**

# SPV

- Simple Payment Verification
- A different type of wallet
- Useful for mobile, laptops etc.
- Doesn't need to download the whole blockchain
  - Does not download all transactions
- Much faster
- Keeps **only the block headers from genesis till today (C)**
- Connects to multiple **untrusted** servers
- Server is a full node which **proves** to the SPV wallet each claim

# SPV

- The SPV wallet sends to the SPV server the bitcoin addresses they have
  - **But not the private keys!**
  - The SPV server knows which transactions to send to the SPV client
  - The bitcoin addresses are shared in the form of a Bloom filter
- The SPV wallet verifies each block's **PoW** and authenticated ancestry
  - Keeps a longest chain as usual -- but without transactions
- The SPV wallet verifies **each tx** it receives
  - Sigs
  - Law of conservation
- The SPV wallet verifies that the tx belongs to the Merkle Tree root of a block



# SPV Security

- SPV wallets
  - **don't keep** a UTXO
  - **don't verify or receive** txs they are not interested in
  - **don't verify** coinbase validity
- Have the **same** level of security as a regular full node  
(under the honest majority assumption)
- What can a malicious SPV server achieve?
  - Temporary fork to invalid block (invalid coinbase, txs, non-existing UTXO, double spending...)



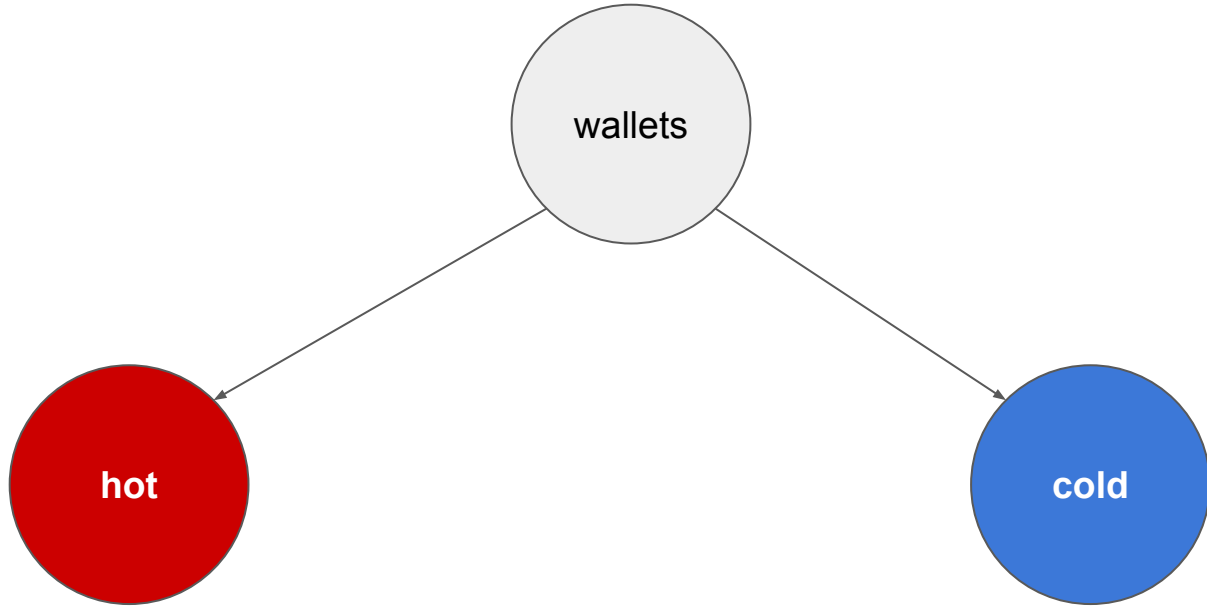
# Wallet seeds and HD wallets

- An infinite sequence of wallet private keys can be generated from a single “master private key” (BIP 0032) -- an **HD wallet**
- A private key can be encoded in the form of a human-readable *seed*
- This seed is sufficient to recover *all* the private keys of a wallet
- Typically backed up on paper and optionally encrypted with password

*Example:*

deal smooth awful edit virtual monitor term sign start home shrimp  
wrestle

# Wallet classification



# Hot and cold wallets

- I can have my keys on an Internet-connected computer
  - “**Hot** wallet”
  - Easy to use
  - I can always spend my money immediately
- I can keep my private keys offline
  - “**Cold** wallet”
  - Kept on a computer not connected to the Internet or a hard drive
  - My keys cannot easily be stolen
  - I can move my keys to a hot wallet when I need to spend it
  - I can see how much money I have using my public keys which can be kept safely online!

# Other ways to store cold wallets

## Paper wallet

- Private key is printed on a piece of paper
- Can be kept in a physical safe or a real bank vault
- Can optionally be encrypted with a secret password (which is remembered)

## Brain wallet

- Private key is literally  $\text{SHA256}(\text{"my dog's name is Barbie"})$  or some other passphrase
- Full private key can be recovered by memory
- Extremely unsafe! (More than \$100,000 stolen due to low entropy passwords)

# Hardware wallets

- Special hardware device used to store private keys
- Cold wallet
- Most popular ones: **Trezor** and **Ledger**
- Connects to a computer via USB
- Keys never leave the device
- Device produces signed transaction, sends transaction to computer
- Addresses you sent to are verified by looking at the screen
- As hardware/software is specialized, much harder to “hack” or have bugs
- Works safely even if host computer is compromised
- Protected by a pin in case of theft
- Can be backed up into paper and/or other hardware wallets



 Confirm sending  
0.0469 BTC  
to  
1MuuZ7S3n7h3ZnCQJ  
CT2HYKTffQjhpXhcw

☐ Cancel

☒ Confirm ✓

## BITCOIN DEPOSIT

Deposit >

EU Bank (SEPA) >

International Bank >

Bitcoin

> Deposit

> HW Wallet



Bitcoins will be sent from your TREZOR HW wallet to your Bitstamp address

3NZ8Gpj9zwbo2Gv6PDAF1ysLfteL3Lw7Gb



Please input the amount of bitcoins you wish to send to your Bitstamp address

Amount (BTC):

DEPOSIT

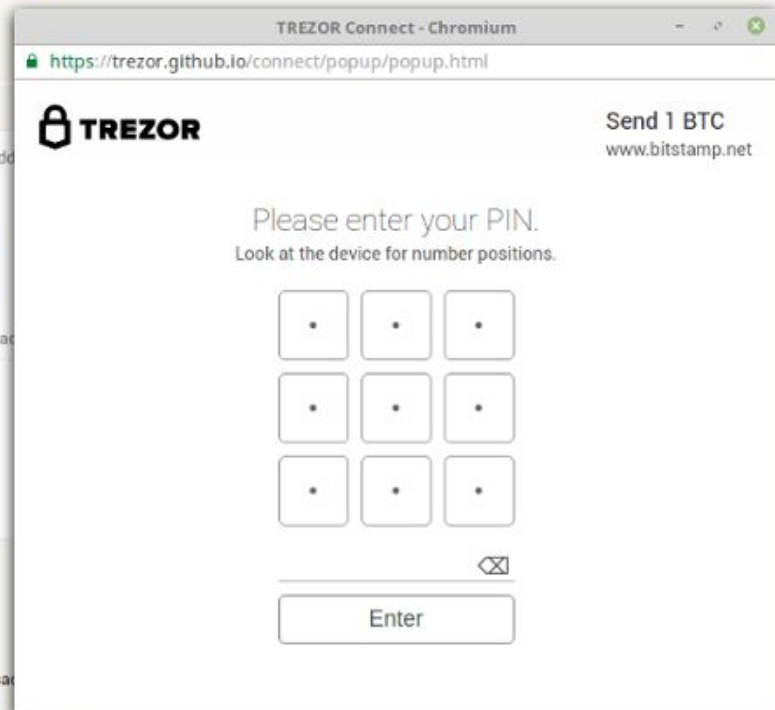
### NOTICE

Please double-check the receiving address before initiating a bitcoin transaction.

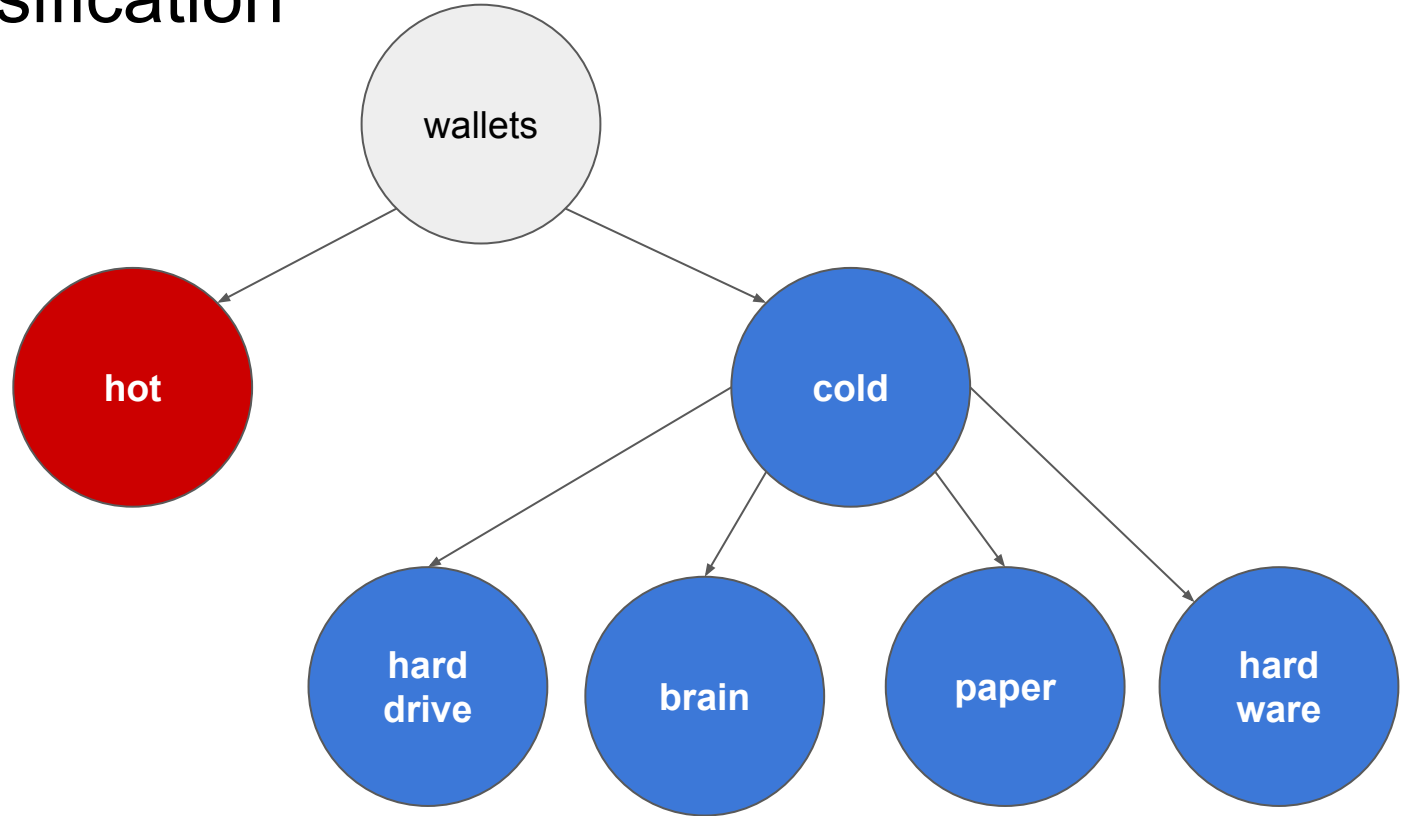
Bitcoin network to confirm the transaction. Please note that 3 network

System's security. Multiple deposits in a single bitcoin transaction always make **ONLY ONE**

transaction



# Wallet classification





# Blockchain Incentives

# Mining Games

- As we have seen: miners are incentivised via rewards to follow the protocol.
- Does this ensure that the protocol will be executed?
  - Is the reward mechanism “incentive compatible”?
  - 
  - Protocol is **dominant strategy**: a party will fare best by following the protocol.
  - Protocol is **Nash equilibrium**: if all parties follow the protocol, you cannot do better by deviating.
  -

# Dominant Strategies: example

Participants want to  
maximise payoff

Split or Steal Game:

	Split	Steal
Split	50/50	0/100
Steal	100/0	1/1

Here stealing is the dominant strategy!

Not necessarily the best outcome...

See prisoner's dilemma  
And the golden balls TV game

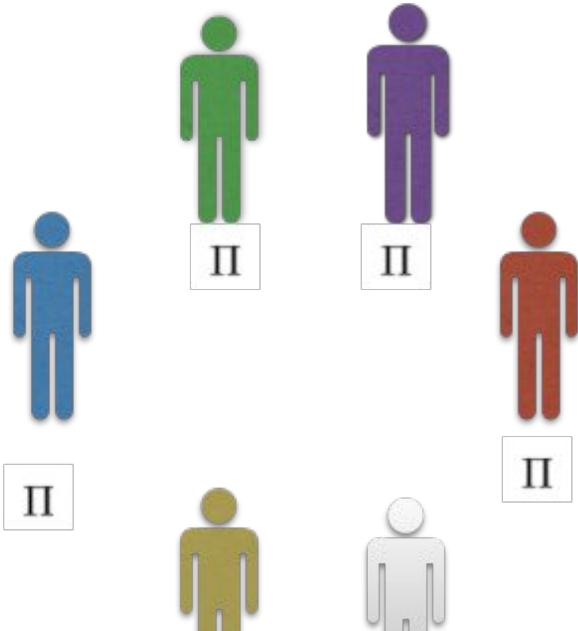
[https://en.wikipedia.org/wiki/Prisoner%27s\\_dilemma](https://en.wikipedia.org/wiki/Prisoner%27s_dilemma)

# Protocol is Nash Equilibrium, I

- All the participants are rational and want to maximize the utility they obtain at the end of the execution.
- Utility of a participant is a function that takes as input the strategies of all the participants and has as output a real number that represents the gains of this participant at the end of the execution.

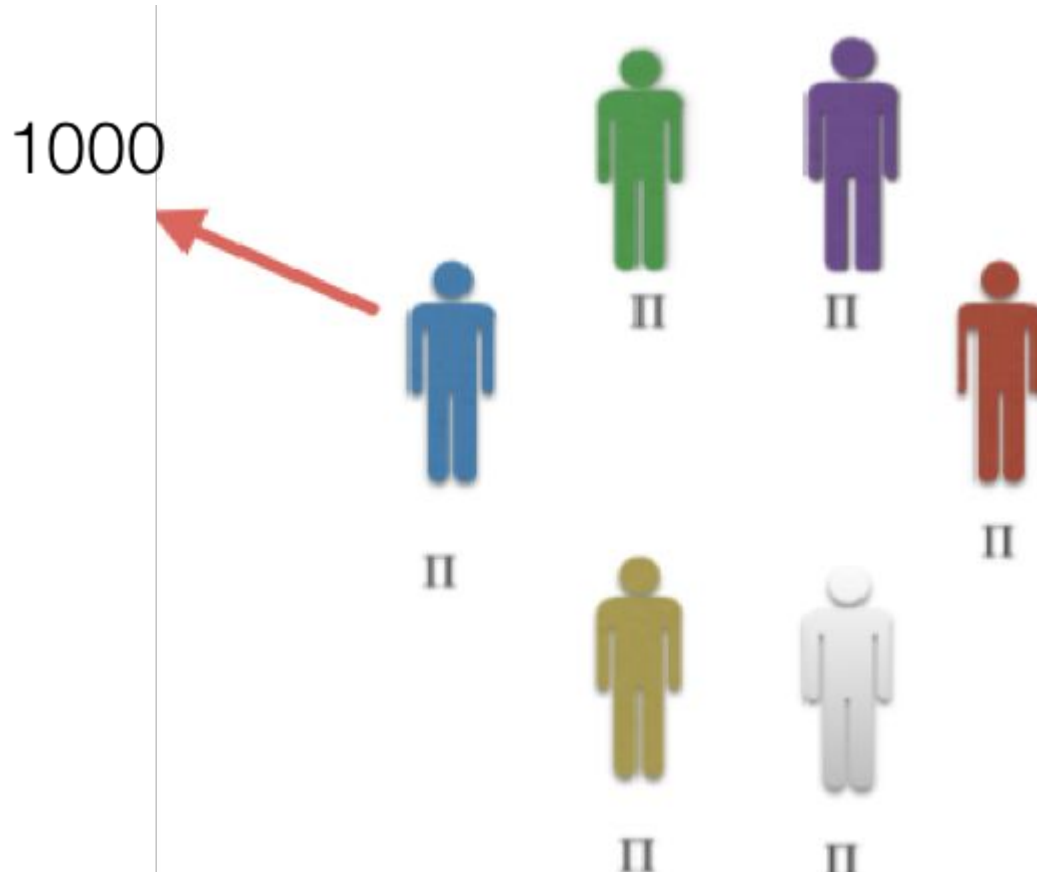
# Protocol is Nash Equilibrium, II

All the participants are rational: they want to maximize their utility.

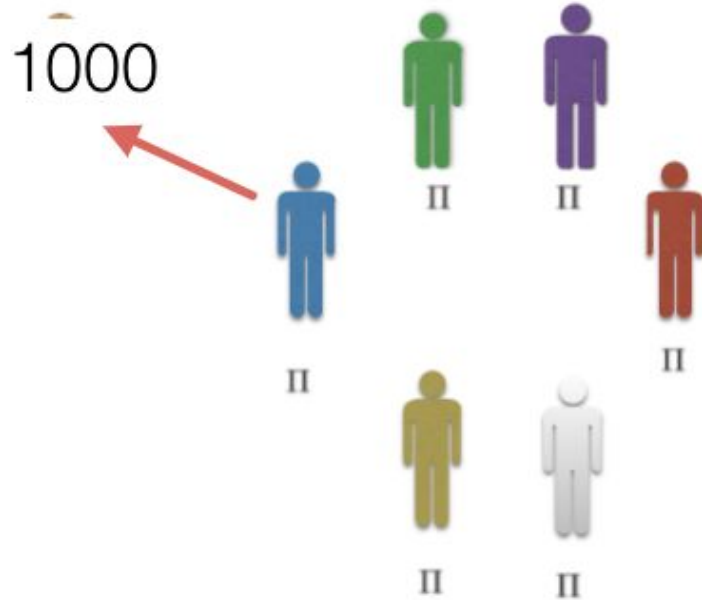


Protocol  $\Pi$  is Nash equilibrium

# Protocol is Nash Equilibrium, III

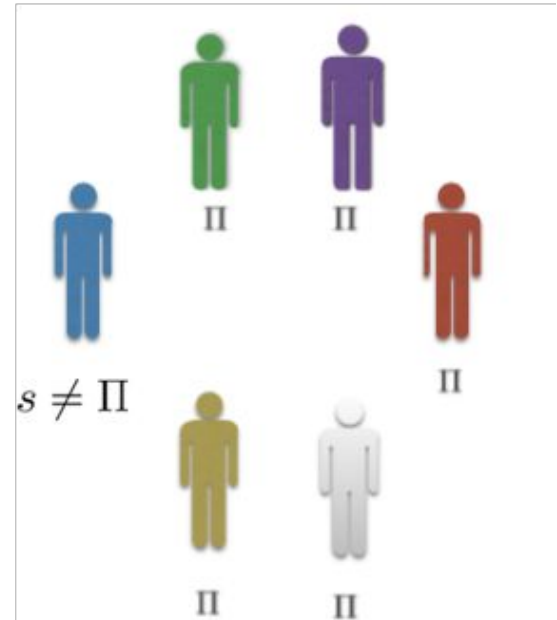


# Protocol is Nash Equilibrium, IV

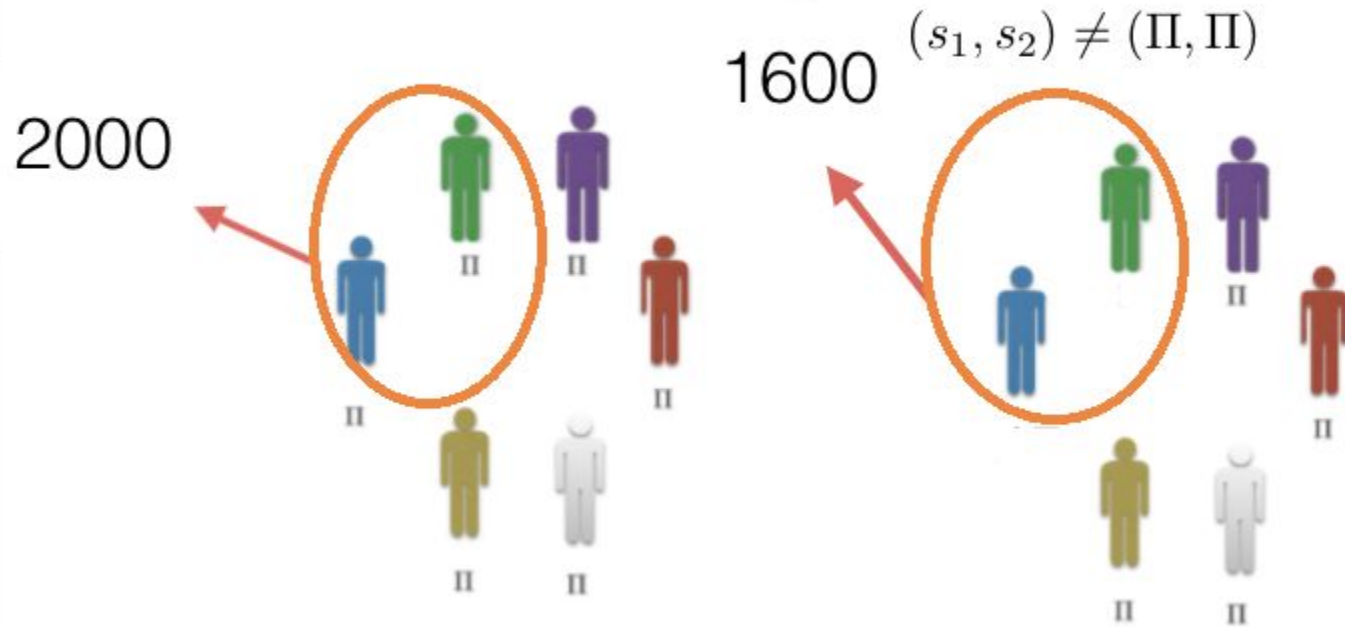


800

A diagram showing six stylized human figures of different colors (blue, green, purple, red, yellow, and grey) arranged in two rows. Each figure has the Greek letter  $\Pi$  below it. A red arrow points from the number 800 to the blue figure in the top row.



# Generalisation to Coalitions



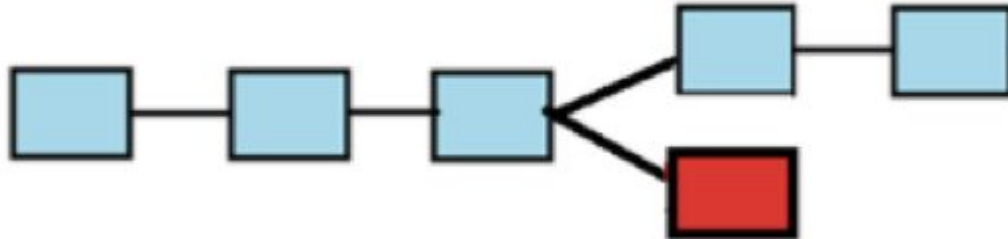


# Is Bitcoin a Nash Equilibrium ?

- What could be the utility in Bitcoin?
- How could utility be defined in a probabilistic protocol?

# Absolute Rewards, I

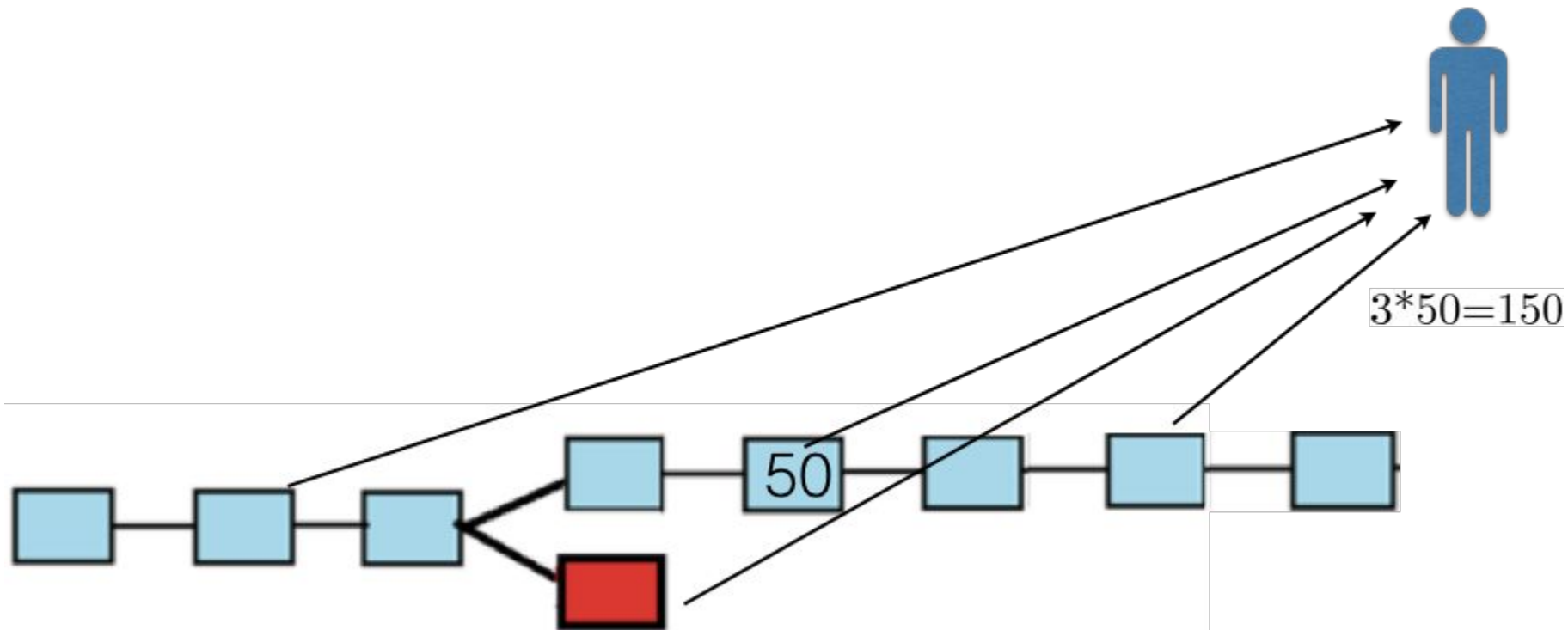
- If we specify the algorithms of all the participants and the outcome of all the randomness used by participants in a finite execution of the Bitcoin protocol then we have a unique outcome.



## Absolute Rewards, II

- Each block of the adopted chain gives a reward to its producer.
- **Absolute rewards utility.** The utility of a coalition is equal to the number of BTC that it has obtained at the end of the execution.

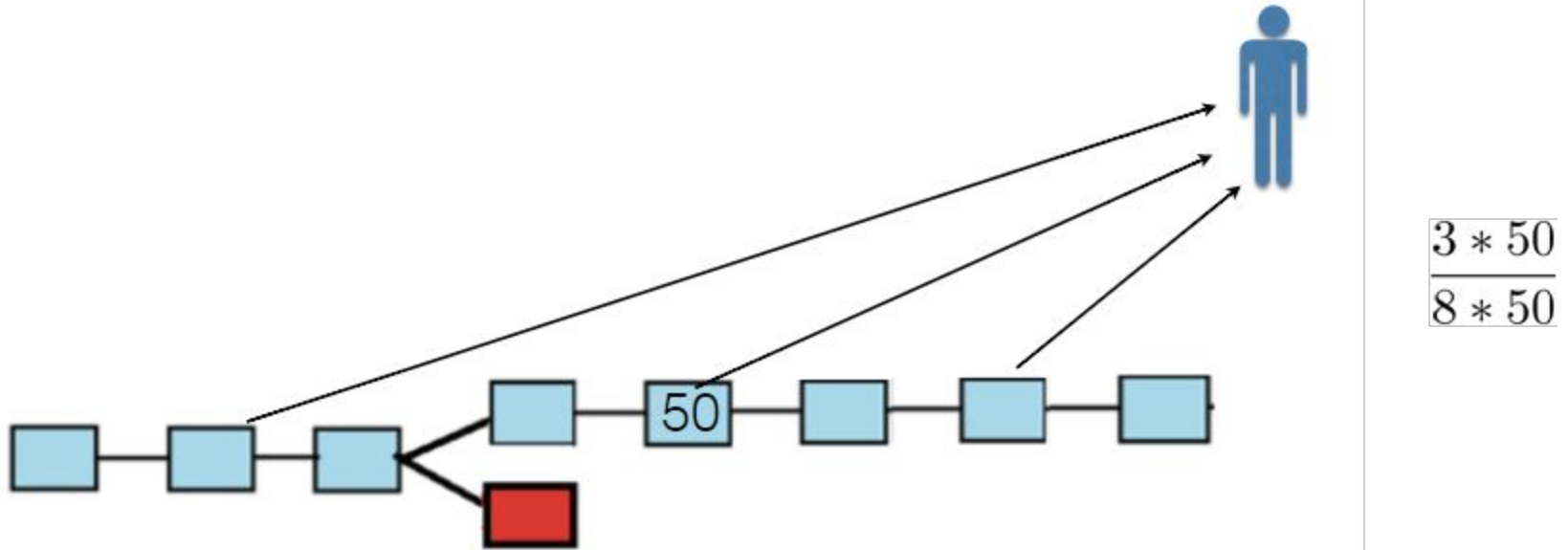
# Absolute Rewards, III



## Relative Rewards, I

- **Relative Rewards.** The utility of a coalition in Bitcoin is equal to relative rewards when it wants to maximize the amount of BTC that it earns divided by the total amount of BTC that all the participants receive at the end of the execution.

## Relative Rewards, II



# Utility in probabilistic protocols

- Given the strategies of all the participants, the outcome of the Bitcoin execution is a random variable. So the utility of a coalition is also a random variable. How to resolve this?
  - Via Expectation: will determine the expected value of utility.
  - Via events that happen with high probability.

# Bitcoin and Equilibria, I

- Kroll et al. in “The economics of Bitcoin mining, or Bitcoin in the presence of adversaries” (2013) show that a certain modeling of the Bitcoin protocol is a Nash equilibrium w.r.t. absolute rewards.

If utility is equal to the expected value of absolute rewards and block difficulty is fixed, then the expected number of blocks is proportional to mining power and this is what is delivered by a Bitcoin execution



## Bitcoin and Equilibria, II

- Eyal and Sirer in “Majority is not enough: Bitcoin mining is vulnerable” (2014) show that Bitcoin is susceptible to a type of attack called selfish mining and the protocol is not a Nash equilibrium w.r.t. relative rewards.

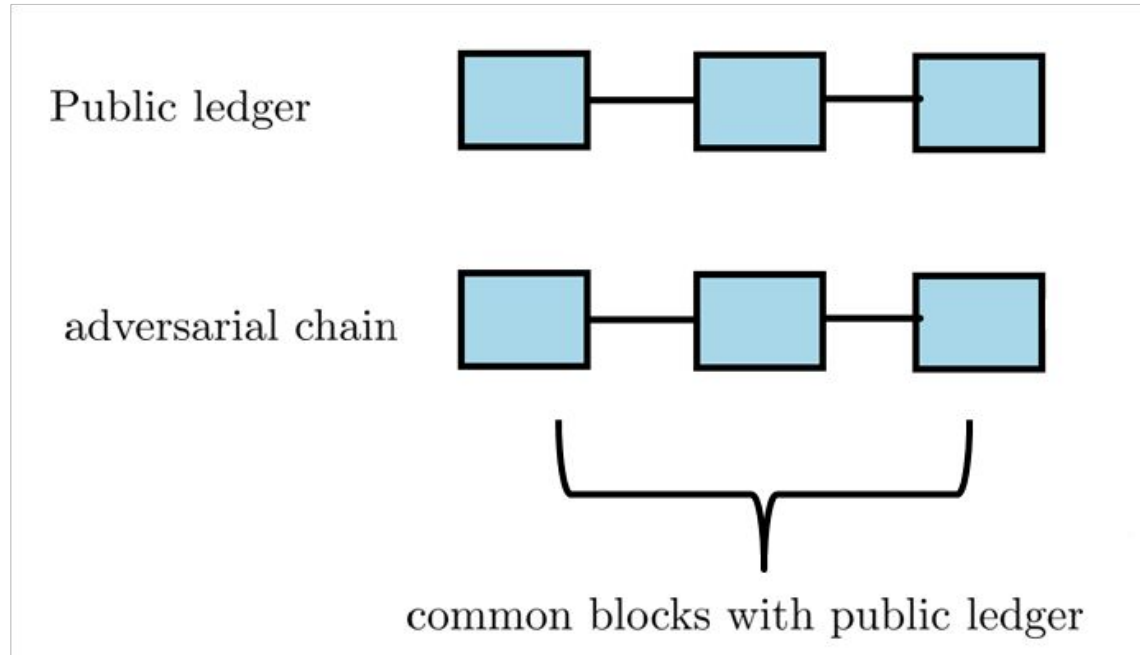
# Selfish Mining, I

- A strategy that enables a coalition to collect more expected relative rewards by deviating from the protocol.
- Attacker maintains a private chain, strategically releasing its blocks to deny honest parties' blocks from being adopted to the “main chain.”

Eyal, Ittay, and Emin Gün Sirer. "Majority is not enough: Bitcoin mining is vulnerable."(2014)

# Selfish Mining, II

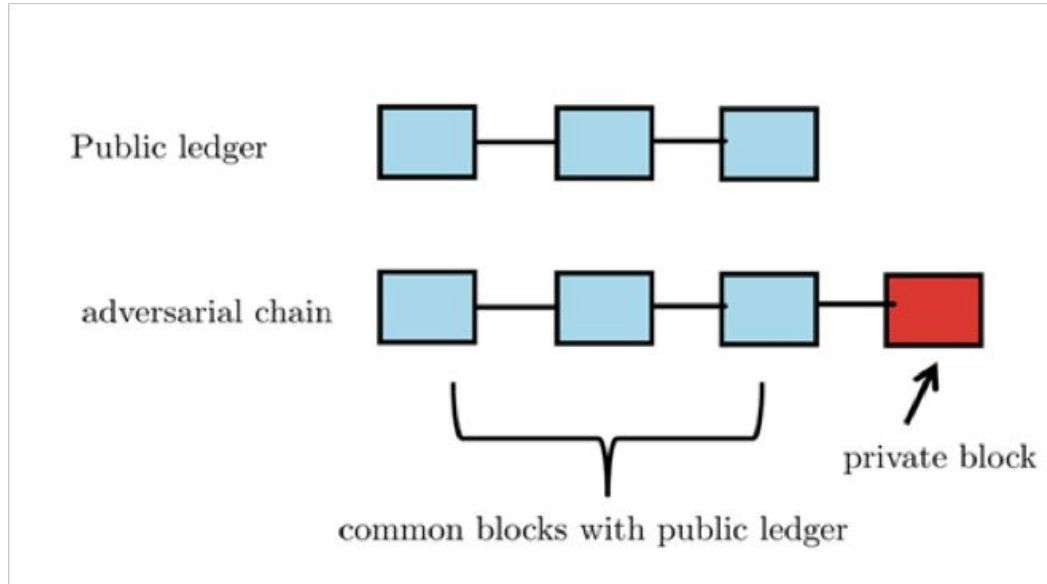
1)The attacker adopts the longest chain and tries to extend it.



# Selfish Mining, III

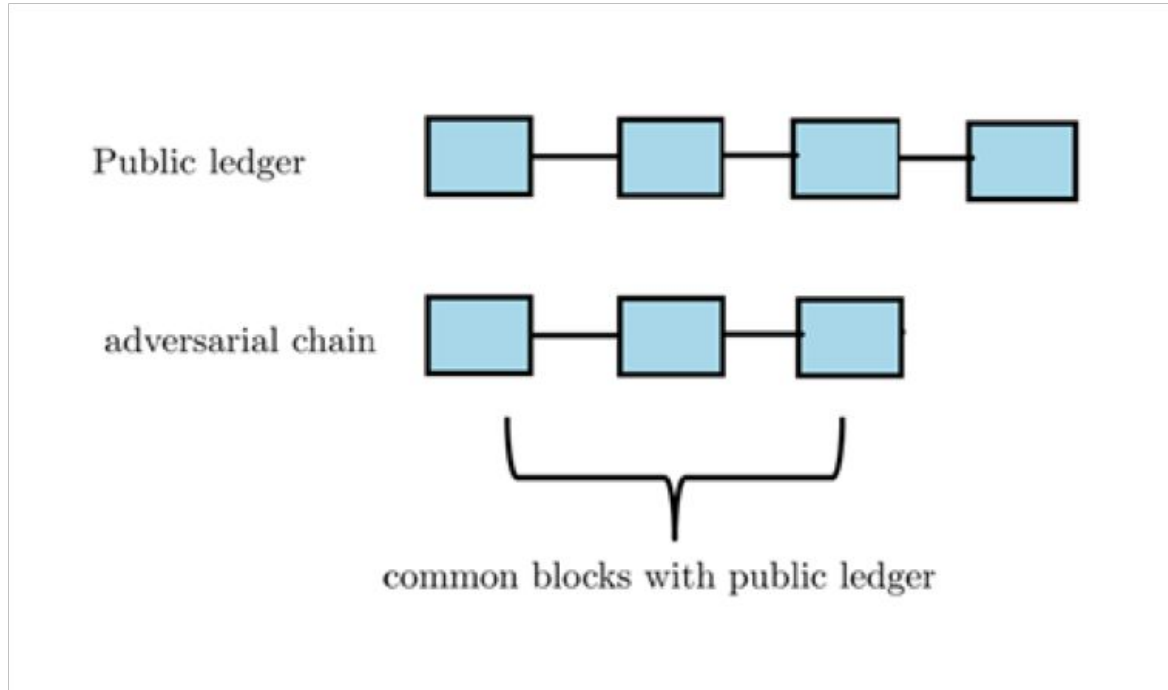
We have the following two cases : 2a or 2b

2a) The attacker is first to produce a block.



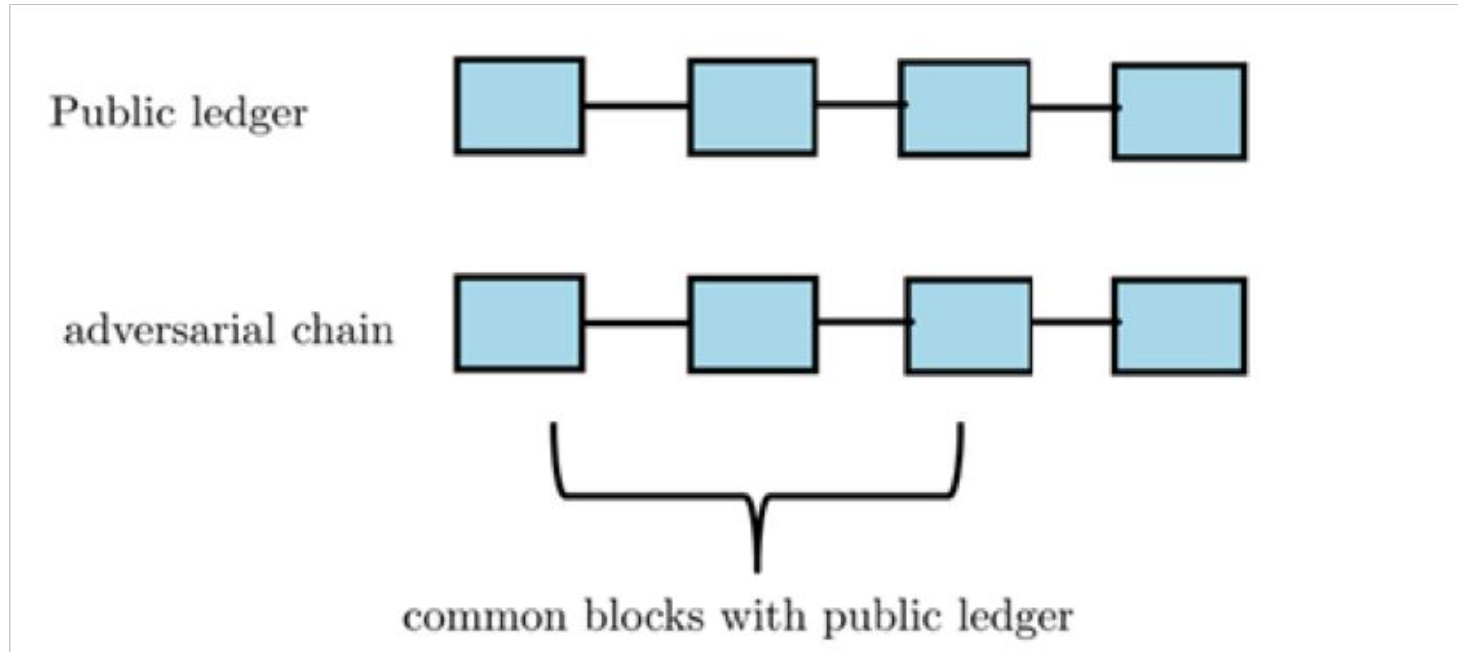
# Selfish Mining, IV

2b) The attacker does not manage to produce first a block.



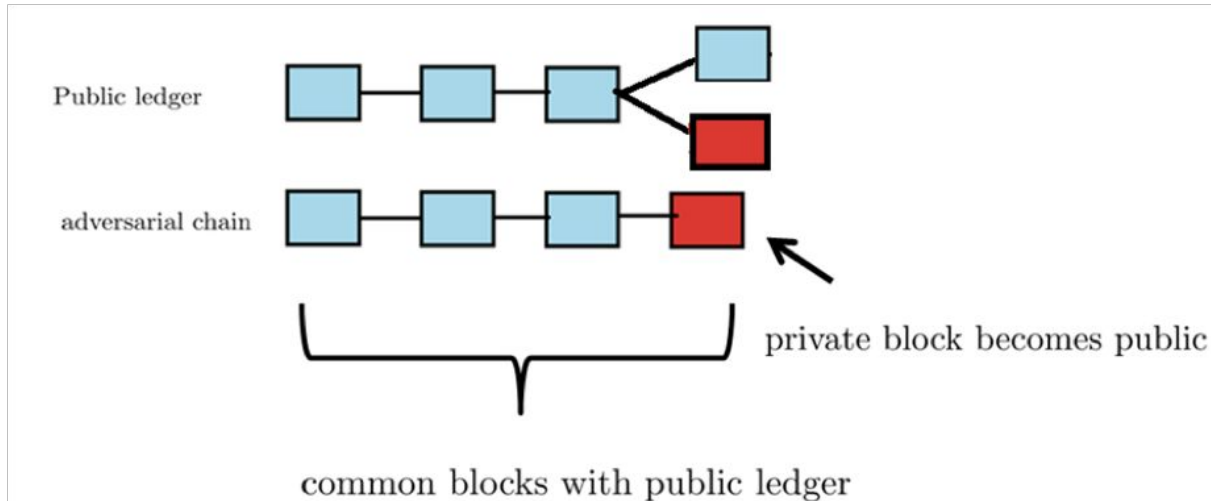
## Selfish Mining - case 2a)

The attacker in this case adopts the public ledger.



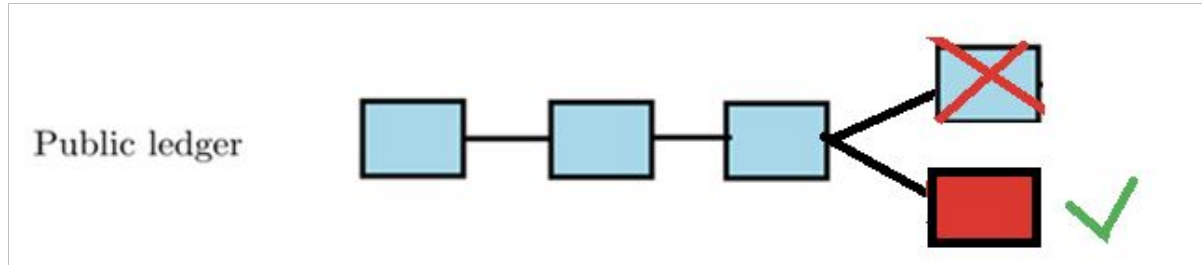
# Selfish Mining - case 2a)

The attacker withholds its block(s) private until the public chain comes to be (i) one block behind the private one, (ii) equal in length to the attackers. (choice can depend on “network dominance” of the adversary)



## Selfish Mining, Case 2a), II

If the other parties choose to extend the adversarial chain then the adversary has managed to censor legitimate blocks from the public ledger.





## Selfish Mining, Case 2a), III

- More generally: the adversary will be capable of censoring blocks, if
  - a. the adversary's chain gets two blocks ahead of the public chain.
  - b. The adversary manages to deliver first its block to the other parties. When an honest party receives two chains of the same length then it chooses the first that it received.

# Selfish Mining, Analysis, I

- The computational power of the attacker contributes only towards censoring blocks and not to extend the public ledger.
- So when it implements the attack, the total number of blocks (expected value) in the public ledger is smaller compared to the total number of blocks in the case where it follows the protocol.
- If the attacker does not manage to deliver its block first, it loses the rewards from this block.

# Selfish Mining, Analysis, II

- Consider a process operating in “block rounds.” Attacker has probability  $\alpha$  to produce the next block. Assume attacker always wins the network race vs. public blocks.
  - Honest play:  $n$  rounds result to  $n$  blocks. Attacker owns  $\alpha n$  blocks in expectation.  
Utility (Relative Rewards) =  $\alpha$ , (Absolute Rewards) =  $\alpha n$ .
  - Selfish play:  $n$  rounds result to  $(1-\alpha)n$  blocks. Attacker owns  $\alpha n$  of those blocks.  
Utility (Relative Rewards) =  $\alpha/(1-\alpha)$ , (Absolute Rewards) =  $\alpha n$ .
- In a static difficulty setting absolute rewards remain the same but relative rewards increase.

# Bitcoin and Equilibria, III

- In the changing difficulty setting.
  - Selfish mining will impact chain growth and thus the difficulty recalculation mechanism will lower the difficulty.
  - This means that block production per actual unit of time will increase.
  - As a result, attacker will also receive higher number of (absolute) rewards compared to honest play.

# Block Reward Zero Attack

- When the block reward becomes zero the following deviation may arise:
  - When a miner receives two blocks of the same height instead of choosing the first one, it has incentives to choose the block that leaves the most transaction fees unclaimed.
  - A selfish miner can take advantage of this behaviour and create a fork with a block including fewer transaction fees compared to the transaction fees in the head of the public ledger.

# Bribery Attack

- The attacker creates a fork and includes in the first block a transaction T0 that gives *bribe* money to miners who will adopt the fork and will extend this block.
- The input of T0 is also transmitted in the public ledger and double spends the bribe money.
- If the chain of the attacker does not manage to become longer than the public ledger then the attacker does not lose the bribe money.
  - a. In this case, miners who adopted this fork will have spent computational power without gaining anything.

# Mining Pool Games

- To create a pool or join an existing one?
- Assuming cost of verification and pool maintenance is non-negligible:
  - Optimal solution is a single dictatorial pool.
    - (reason: offset costs with the player that has the lowest service cost).

