

# Blockchains and Distributed Ledgers Lecture 01

Aggelos Kiayias



THE UNIVERSITY  
*of* EDINBURGH

# Lecture 01

- Introduction to blockchain.
- What is a distributed ledger.
- Hash functions and digital signatures.
- Transactions in Bitcoin.

# Understanding DL's

- Distributed ledgers use blockchain protocols as one main means of implementation.
- The blockchain is a distributed database that satisfies a unique set of safety and liveness properties.
- To understand it, we can focus to its first (and so far most successful) application.

# Case study : Money

- What is money?

# Money is useful

(1874) A man offering chicken for a yearly newspaper subscription



# Properties of Money

can be used as medium  
for the exchange  
of goods - no bartering

- a medium of exchange
- a unit of account
- a store of value

can be used for  
pricing of all goods  
and services, for  
accounting purposes  
and debt recording.

storing and retrieving it at a  
point in the future maintains  
its value.

# Creating Money

**Money 1.0** : using a trusted object  
(commodity money)



# Analysis of Money 1.0

**mediocre**

[ok for face to face transactions]

- a medium of exchange
- a unit of account
- a store of value

**mediocre** fungible,  
but not divisible well.  
typically forgeable.

**bad.** some objects may  
deteriorate, others may have  
unknown hidden quantities.



# Creating Money

**Money 2.0** : using a trusted entity  
(fiat money)



Trusted entity issues “IOU”s

# Analysis of Money 2.0

**good**

[for transactions  
within the domain of  
the trusted entity]

- a medium of exchange
- a unit of account
- a store of value

**great!**

fungible & divisible.

**mediocre**

[tied to the availability & reputation  
of the issuing entity]

# Creating Money

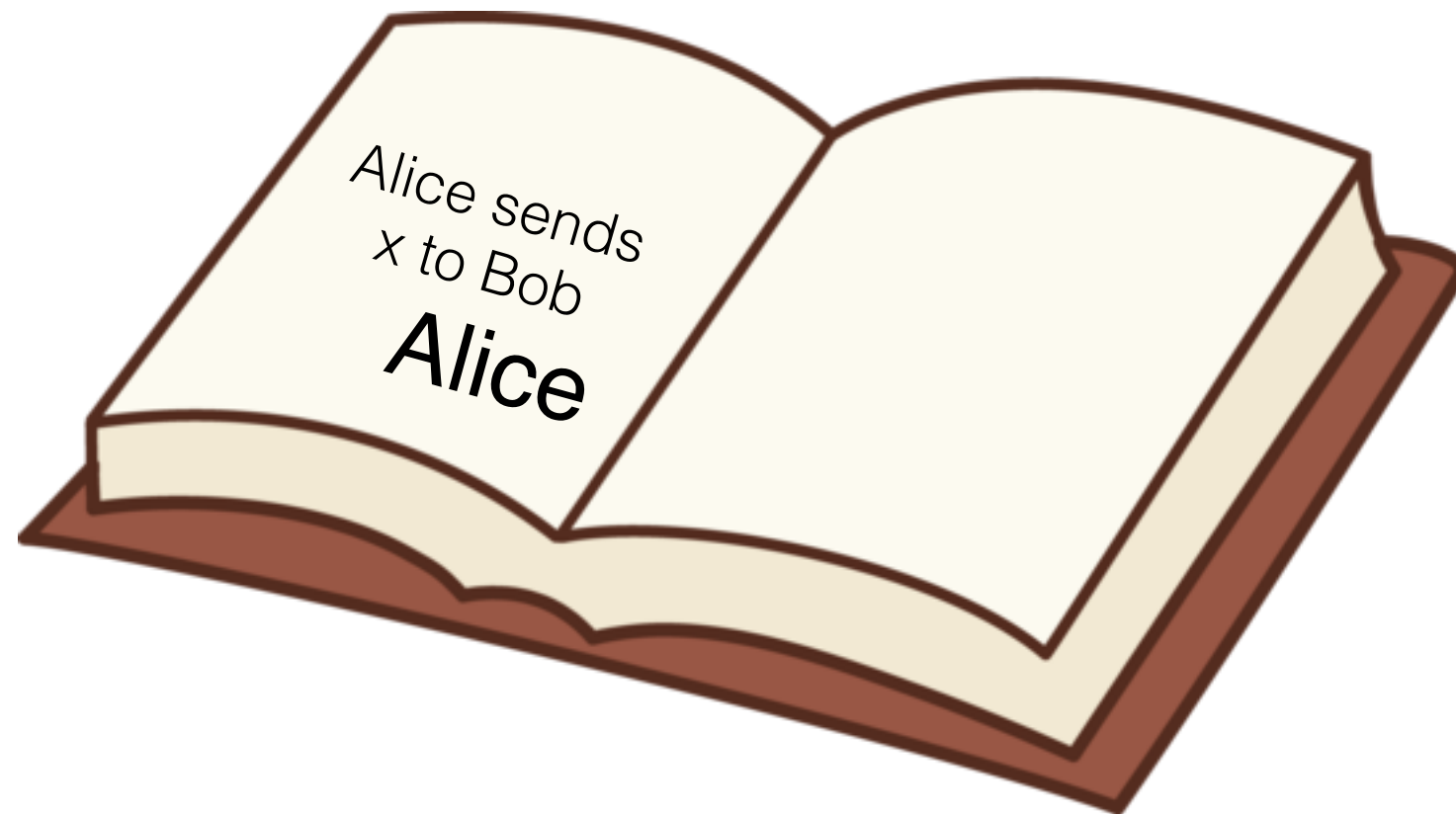
**Money 3.0** : Bitcoin

Enter Blockchains & Distributed Ledgers

# The never-ending book parable



# A “book” of transactions

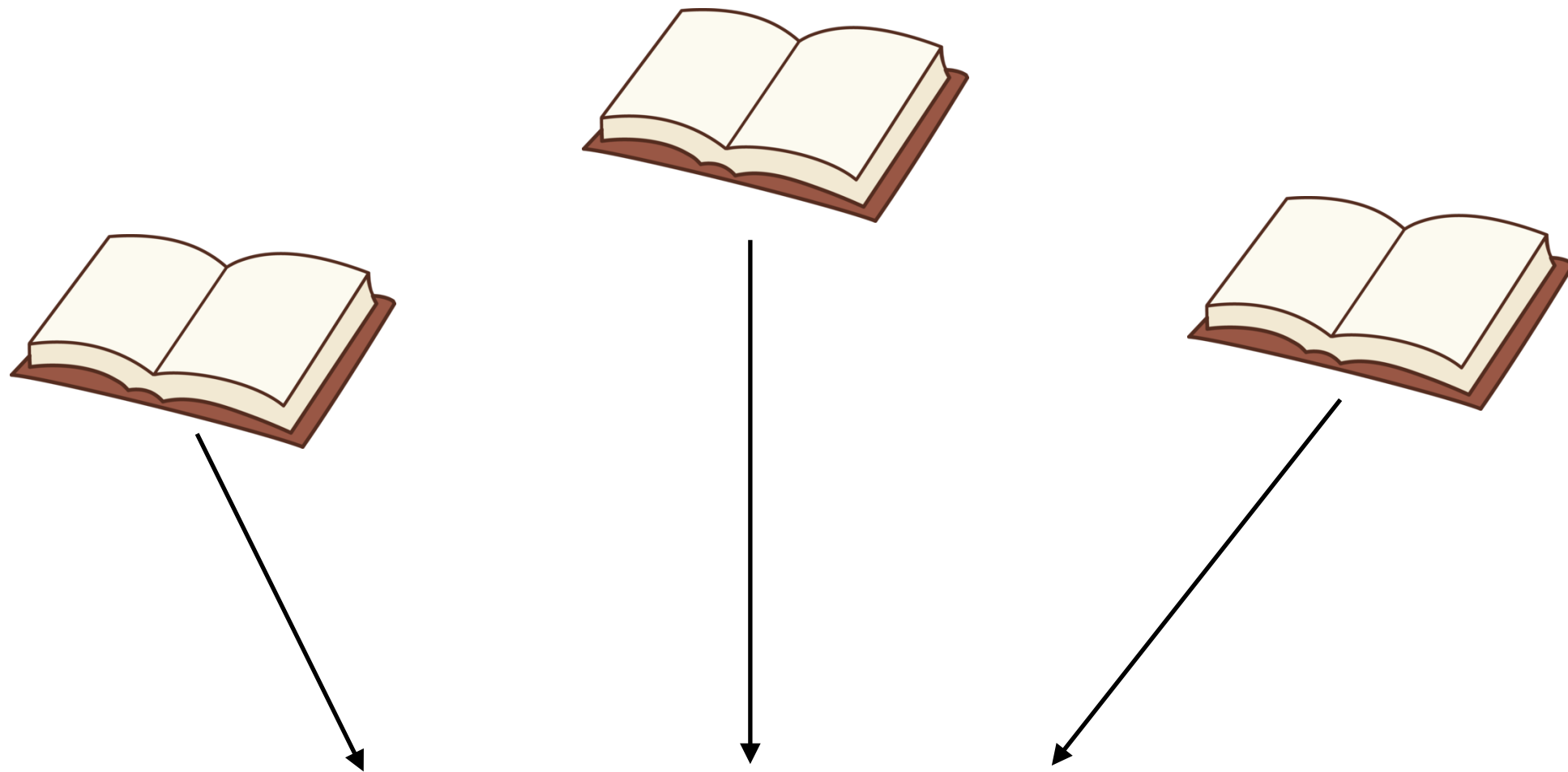


- Each new page requires some effort to produce.
- Anyone can be a scribe and produce a page.
- New pages are produced indefinitely as long as scribes are interested in doing so.

# Importance of Consensus

- If multiple conflicting books exist, which is the “right one”?

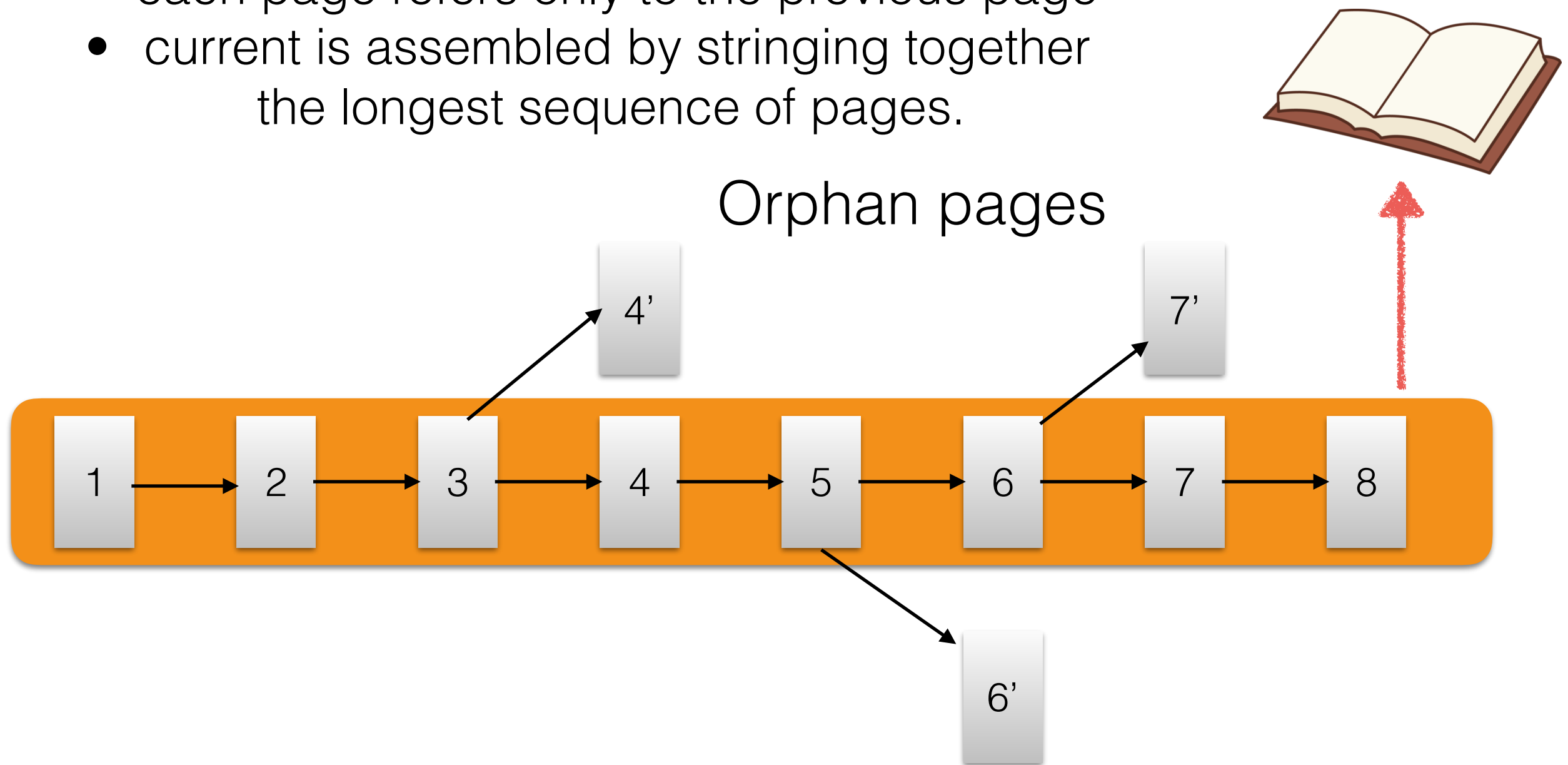
# Choosing the correct book



The **current book** to work on & refer to is the book with the most pages. if multiple exist, just pick one at random.

# Assembling the current book

- each page refers only to the previous page
- current is assembled by stringing together the longest sequence of pages.





# Rules of extending the book

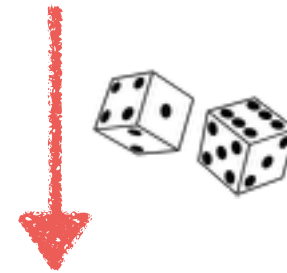
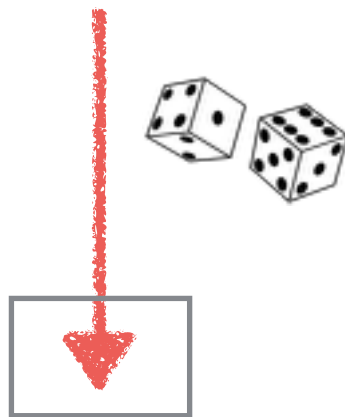
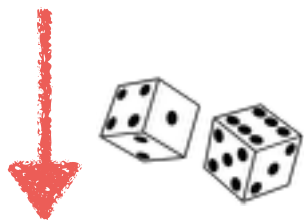


The first scribe that discovers  
a page announces it to everyone else



# Effort is needed to produce a page

equivalent to : each page needs a special combination from a set of dice to be rolled.



The probabilistic nature of the process is paramount to its security

# The benefits of randomness

Imagine two scribes

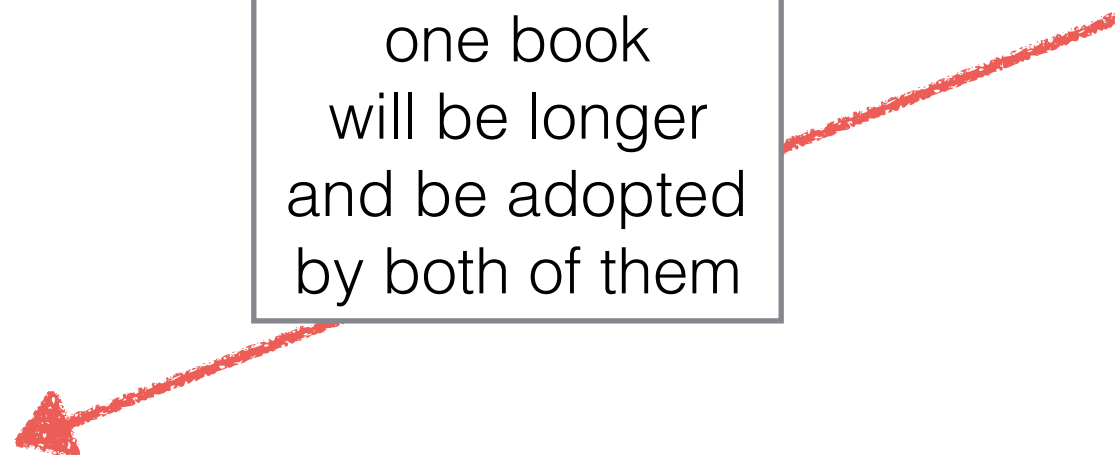
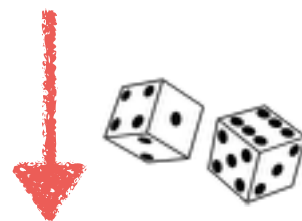
working together



Unlikely  
to continuously  
be lucky  
together



eventually  
one book  
will be longer  
and be adopted  
by both of them

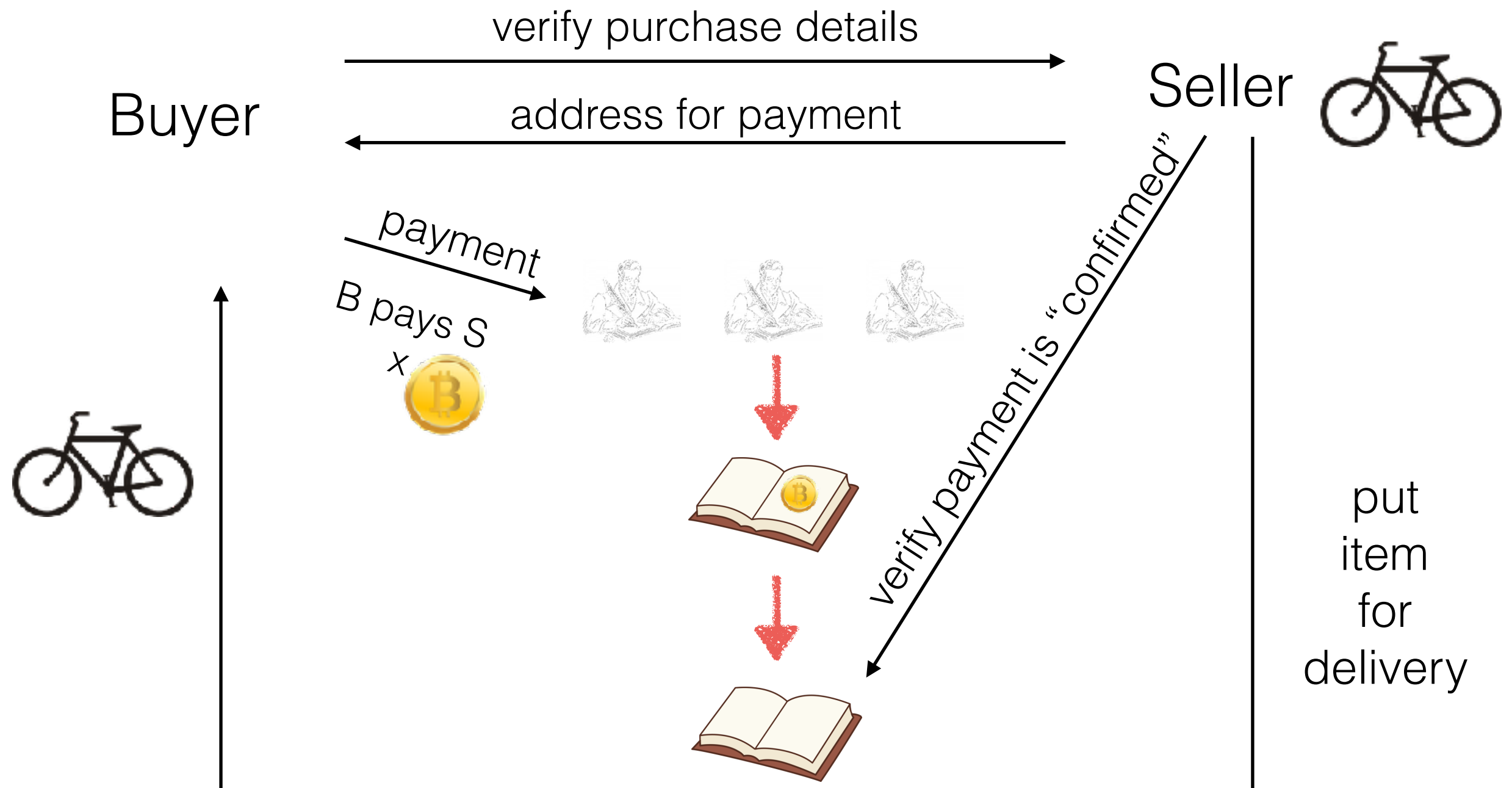


Symmetry Breaking

# Being a scribe

- Anyone can be a scribe for the book.
- As long as one has a set of dice.
- The more dice one has, the higher the likelihood to produce the winning combination to make a page.

# Using the book - Money 3.0



# Parable & Reality

book



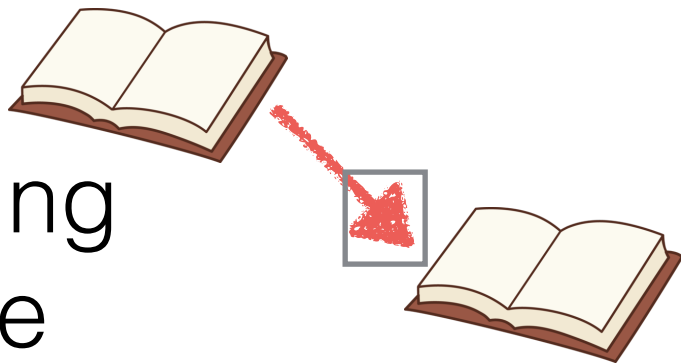
the “blockchain”

scribes



“Miners” / Computer systems  
that organize  
transactions in blocks

producing  
a page



Solving a **cryptographic  
puzzle** that is **moderately  
hard** to solve

rolling  
a set of  
dice



Using a computer to test for a solution  
from a large space of candidate  
solutions

# Analysis of Money 3.0

**improving**

[assuming internet  
connectivity / adoption]

- a medium of exchange
- a unit of account
- a store of value

**great!**

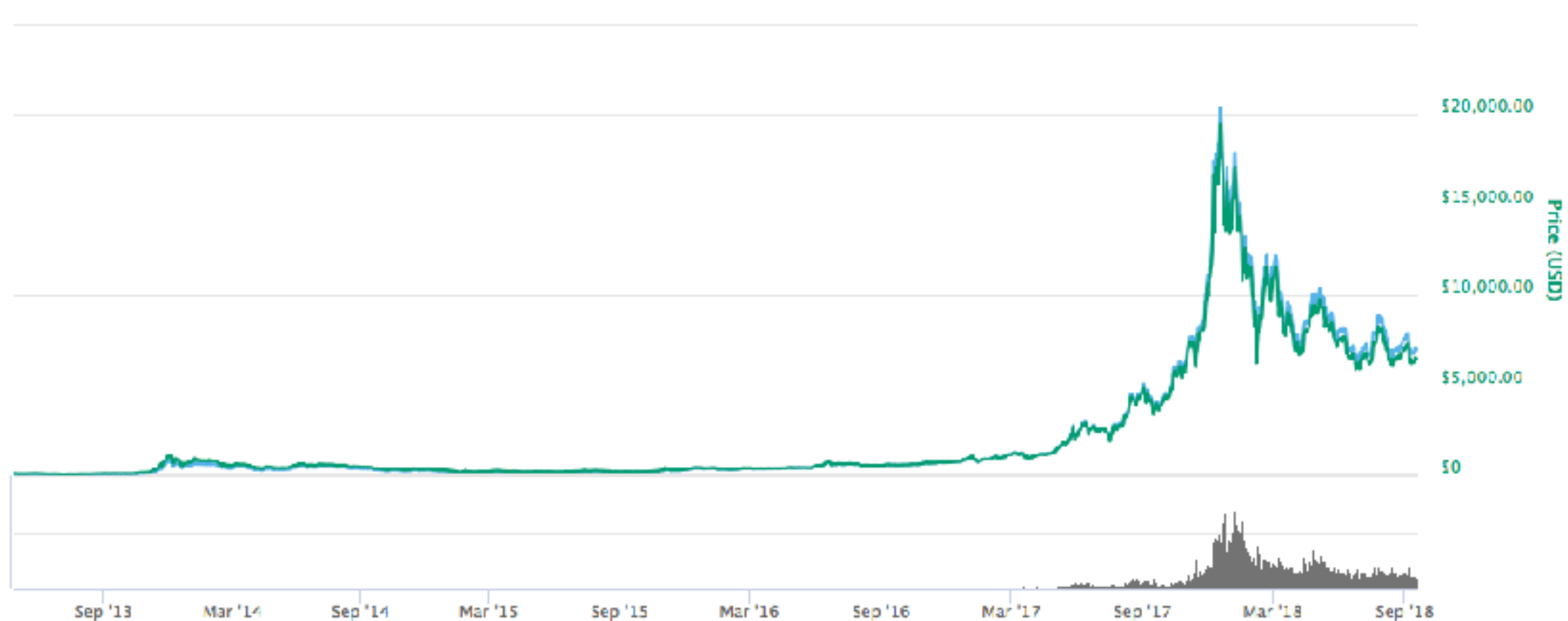
fungible & divisible.

**good**

[no trusted parties -  
no natural deterioration]

# Word of Caution

- Just because something **can** be good as a store of value, it does **not** mean that it **will be** a good store of value in a real world deployment.





# From Money to Smart Contracts

- Since we have created **the book**, why stop at recording monetary transactions?
- We can encode in the book's pages **arbitrary relations** between persons.
- Furthermore, scribes, can perform tasks such as verifying that stakeholders **comply** to contractual obligations ... and **take action** if they do not.

# Smart Contract



# Smart Contract Operation

- A smart contract is a **piece of code** written in a formal language that records all terms for a certain engagement between a set of persons, “stakeholders.”
- Stakeholders are identified by their **accounts**.
- The smart contract has a **public state**.
- The smart contract **self executes** each time a certain trigger condition is fulfilled.

# Questions to Consider

- How are pages created? since the book is empty at the beginning, where do the money come from?

**proofs of work**

- How is it possible to sign something digitally?

**digital signatures**

- How does a page properly refer to the previous page?

**hash functions**

# Hash Functions

# Hash Functions

- An algorithm that produces a fingerprint of a file.

$$\mathcal{H} : \{0, 1\}^* \rightarrow \{0, 1\}^\lambda$$

- what are the required properties (traditionally):  
Efficiency.  
A good spread for various input distributions.
- What are Security/Cryptographic considerations?

# Collisions

- Collision Attack: Find  $x, y : \mathcal{H}(x) = \mathcal{H}(y)$
- Second pre-image Attack: Find  $y : \mathcal{H}(x) = \mathcal{H}(y)$   
For given  $x$

# Birthday Paradox

- How many people should be in a room so that the probability that two of them share a birthday becomes larger than 50% ?



# Paradox Explained

$n$  possible dates

$k$  people

$$\Pr[\neg Col] =$$

$$\frac{n}{n} \frac{n-1}{n} \frac{n-2}{n} \dots \frac{n-k+1}{n} = \prod_{\ell=1}^k \left(1 - \frac{\ell}{n}\right)$$
$$\leq \exp\left(-\frac{1}{n} \sum_{\ell=1}^k \ell\right) = \exp(-k(k+1)/2n)$$

$$\Pr[Col] = \frac{1}{2} \Rightarrow k \approx 1.177\sqrt{n}$$

# Finding Collisions via the BP

- Randomly sample pairs of the form  $\langle x, \mathcal{H}(x) \rangle$
- Store in table and sort according to 2nd coordinate.
- Perform linear pass to see whether there are elements with an equal 2nd coordinate

# Analysis

- For  $k$  elements.
- Running time is  $O(k \log k)$
- Choose  $k$  as  $1.177\sqrt{2^\lambda}$

# Pre-Image Attack

- Given  $\mathcal{H}(m)$   $m \in \{0, 1\}^t$

Find an element of  $\mathcal{H}^{-1}(\mathcal{H}(m))$

- Generic algorithms tries all possible values. How many?  
 $O(2^t)$

# Constructing Hash Functions

- Relates to the notion of one-way functions.

$$f : X \rightarrow Y$$

**easy** :    given  $x$  find  $f(x)$

**hard** :    given  $f(x)$  sample  $f^{-1}(f(x))$

# Word of Caution

Existence of one-way functions  
implies that :

$$P \neq NP$$

The single most important open question  
in Computer Science today

# Hash Function Implementations

- **Retired.** MD5, SHA1.
- **Current.** SHA2, SHA3, available for 224,256,384,512 bits fingerprints.
- **Bitcoin.** Uses SHA2 with 256 bits output, SHA-256.

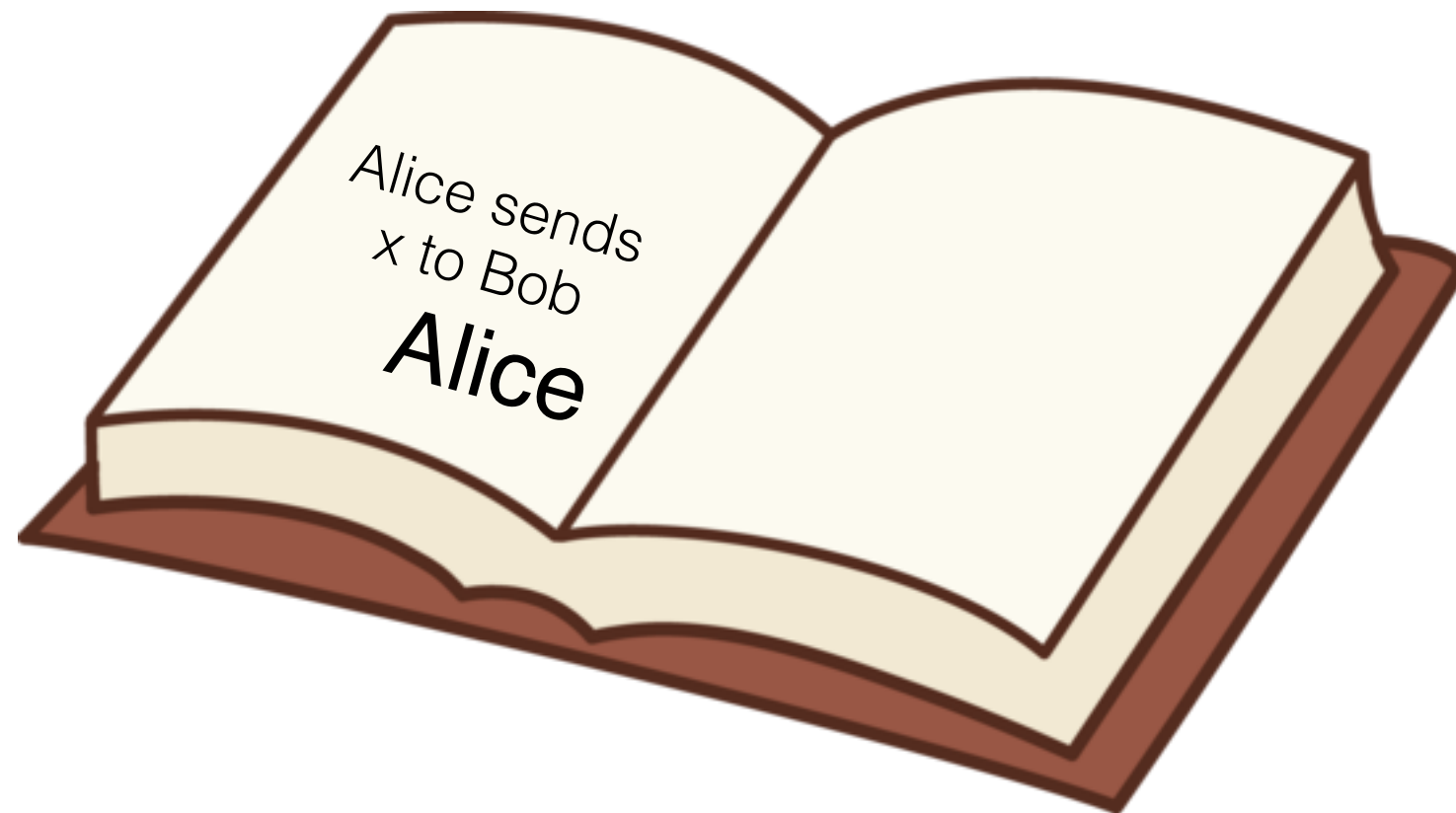
# Digital Signatures



# What is a Signature?

- Can be produced by one specified entity.
- Can be verified by anyone (that is suitably “equipped” and “initialised”).
- Cannot be forged on a new message even if multiple signatures have been transmitted.

# Pen and Paper Signatures



What are the assumptions when you verify them?  
When you produce them ?

# Digital Signatures

Three algorithms (**KeyGen**, **Sign**, **Verify**)

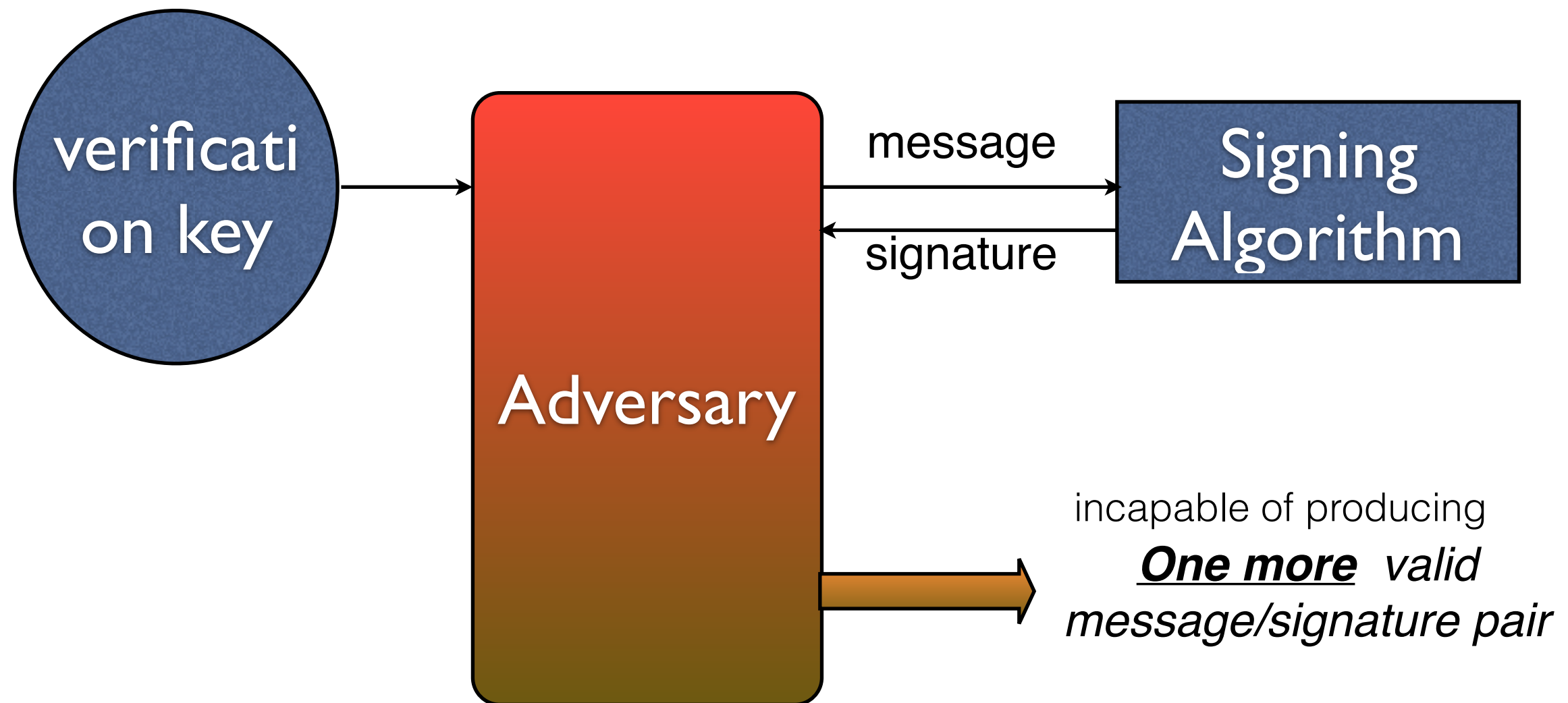
**KeyGen** : takes as input the *security parameter*.  
returns the signing-key and verification-key.

**Sign** : takes as input the *signing-key* and  
the *message* to be signed and  
returns a signature.

**Verify** : takes as input the *verification-key*,  
a *message* and a *signature* on the message and  
returns either True or False.

# Digital Signature Security

Existential Unforgeability under a Chosen Message Attack (EU-CMA)



# Constructing Digital Signatures

- Major challenge:
  - what prevents the adversary from learning how to *sign* messages by analyzing the *verification-key*?

# Example Construction : using hash functions

## One-Time Signatures

$$\mathcal{H} : \{0, 1\}^* \rightarrow \{0, 1\}^\lambda$$

### KeyGen

$$sk = \langle s_{i,b} \rangle_{i \in \{1, \dots, \lambda\}, b \in \{0, 1\}} \quad vk = \langle \mathcal{H}(s_{i,b}) \rangle_{i \in \{1, \dots, \lambda\}, b \in \{0, 1\}}$$

### Sign

$$\mathcal{H}(m) = w = w_1 \dots w_\lambda$$

$$\sigma = \langle s_{1,w_1}, \dots, s_{\lambda,w_\lambda} \rangle$$

**Verify** Hash strings from  $\sigma$  and compare to the hashes stored in the  $vk$  at the positions dictated by  $\mathcal{H}(m)$

# Digital Signature Constructions

- Based on the RSA (Rivest Shamir Adleman), one way trapdoor function (with hardness that relates to the factoring problem).
  - The RSA algorithm
- Based on the discrete-logarithm problem.
  - the DSA algorithm
- **Bitcoin.** Uses ECDSA, a DSA variant over elliptic curve groups.

# bitcoin transactions

- Typical transaction
  - input: contains a signature and public-key
  - output: contains a verification procedure



# General Format

**general format of a Bitcoin transaction (inside a block)**

Field	Description	Size
Version no	currently 1	4 bytes
In-counter	positive integer $VI = \text{VarInt}$	1 - 9 bytes
list of inputs	the first input of the first transaction is also called "coinbase" (its content was ignored in earlier versions)	<in-counter>-many inputs
Out-counter	positive integer $VI = \text{VarInt}$	1 - 9 bytes
list of outputs	the outputs of the first transaction spend the mined bitcoins for the block	<out-counter>-many outputs
lock_time	if non-zero and sequence numbers are < 0xFFFFFFFF: block height or timestamp when transaction is final	4 bytes

# Input and Output Scripts

Input:

Previous tx: f5d8ee39a430901c91a5917b9f2dc19d6d1a0e9cea205b009ca73dd04470b9a6

Index: 0

scriptSig:

304502206e21798a42fae0e854281abd38bacd1aeed3ee3738d9e1446618c4571d10

90db022100e2ac980643b0b82c0e88ffdfec6b64e3e6ba35e7ba5fdd7d5d6cc8d25c6b241501

Output:

Value: 5000000000

scriptPubKey: OP\_DUP OP\_HASH160 404371705fa9bd789a2fcd52d2c580b65d35549d

OP\_EQUALVERIFY OP\_CHECKSIG

The input imports 50 BTC from output #0 of tx f5d..  
and sends them to a Bitcoin address 404...

# Verifying a transaction

```
scriptPubKey: OP_DUP OP_HASH160 <pubKeyHash>  
OP_EQUALVERIFY OP_CHECKSIG
```

```
scriptSig: <sig> <pubKey>
```

# Transaction Processing

- Uses a **stack** data structure:
  - items can be pushed to the stack and popped from the stack.
  - following LIFO (“last in first out”) order.

# Transaction processing

Stack	Script	Description
Empty.	<sig> <pubKey> OP_DUP OP_HASH160 <pubKeyHash> OP_EQUALVERIFY OP_CHECKSIG	scriptSig and scriptPubKey are combined.
<sig> <pubKey>	OP_DUP OP_HASH160 <pubKeyHash> OP_EQUALVERIFY OP_CHECKSIG	Constants are added to the stack.
<sig> <pubKey> <pubKey>	OP_HASH160 <pubKeyHash> OP_EQUALVERIFY OP_CHECKSIG	Top stack item is duplicated.
<sig> <pubKey> <pubHashA>	<pubKeyHash> OP_EQUALVERIFY OP_CHECKSIG	Top stack item is hashed.
<sig> <pubKey> <pubHashA> <pubKeyHash>	OP_EQUALVERIFY OP_CHECKSIG	Constant added.
<sig> <pubKey>	OP_CHECKSIG	Equality is checked between the top two stack items.
true	Empty.	Signature is checked for top two stack items.

# More general transactions

- Transactions can include arbitrary script operations.

e.g., pay to script hash

```
scriptPubKey: OP_HASH160 <scriptHash> OP_EQUAL  
scriptSig: ..signatures... <serialized script>
```

(instead of):

```
scriptPubKey: OP_DUP OP_HASH160 <pubKeyHash> OP_EQUALVERIFY OP_CHECKSIG  
scriptSig: <sig> <pubKey>
```

# script operations

- Some operations that are allowed:
  - basic 32-bit arithmetic (addition subtraction)
  - computation of SHA256[.]
  - verify equality
  - calculate length of string
  - Verify signature (ECDSA)

# End of Lecture 01

- Next time :
  - The consensus layer.
  - Basic Properties.
  - Proof of work.