Neural Encoding

Maurice J. Chacron

Depts of physiology and physics,
center for nonlinear dynamics
McGill University

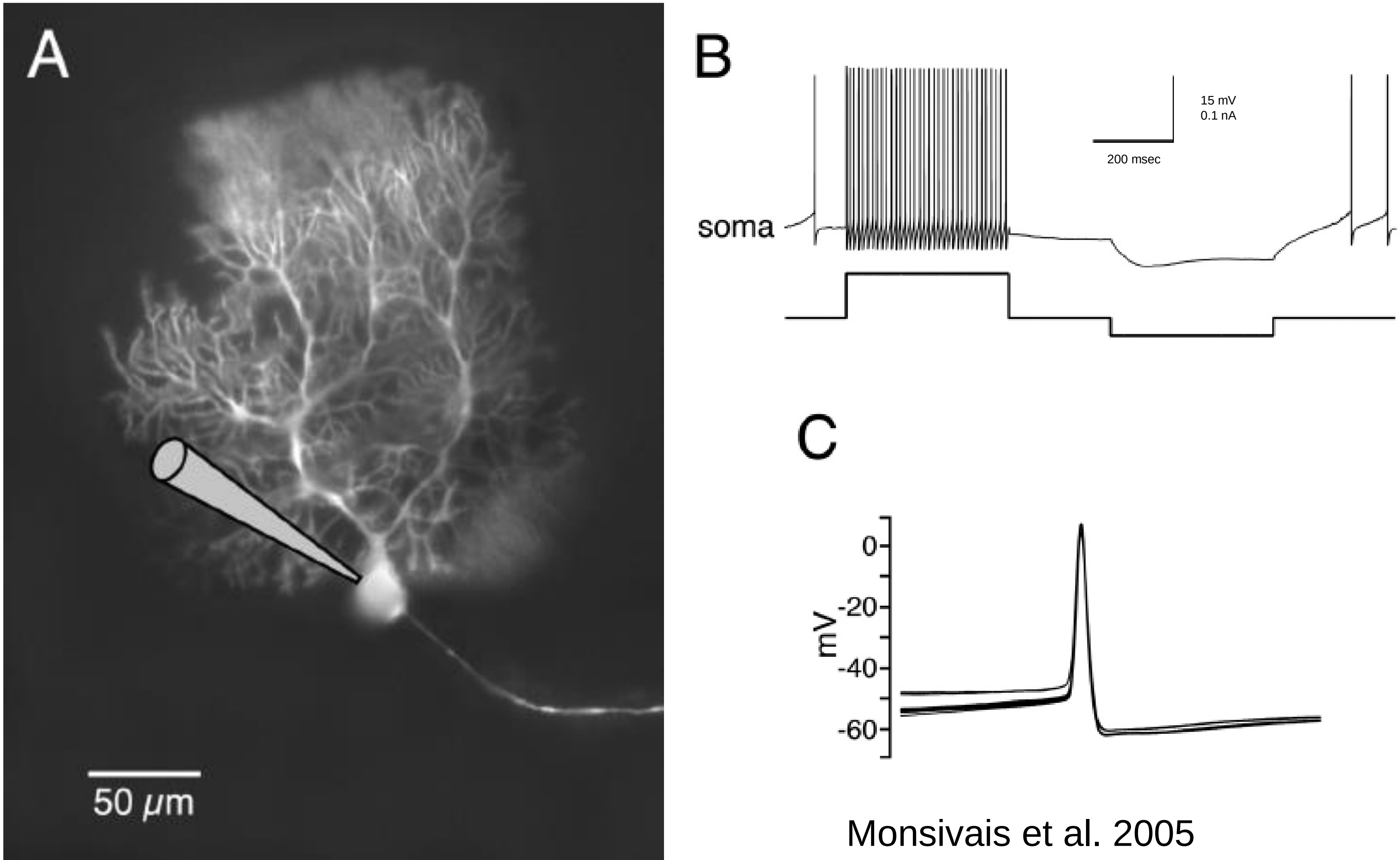
Contact Information:

- Office: 1137 McIntyre, phone: 7493
- E-mail: maurice.chacron@mcgill.ca

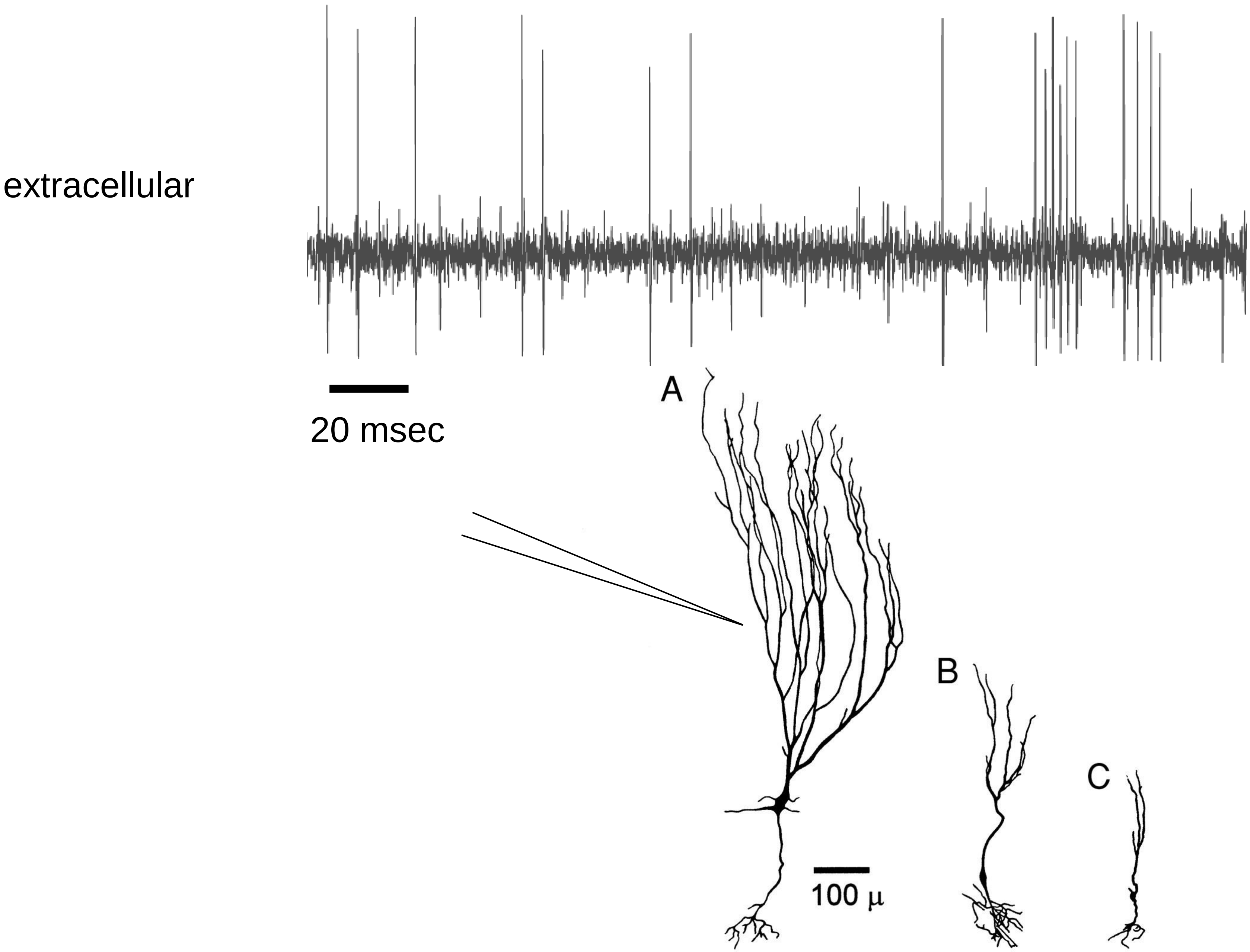
Overview

- Spike Train Statistics
- Measures of Neural Encoding

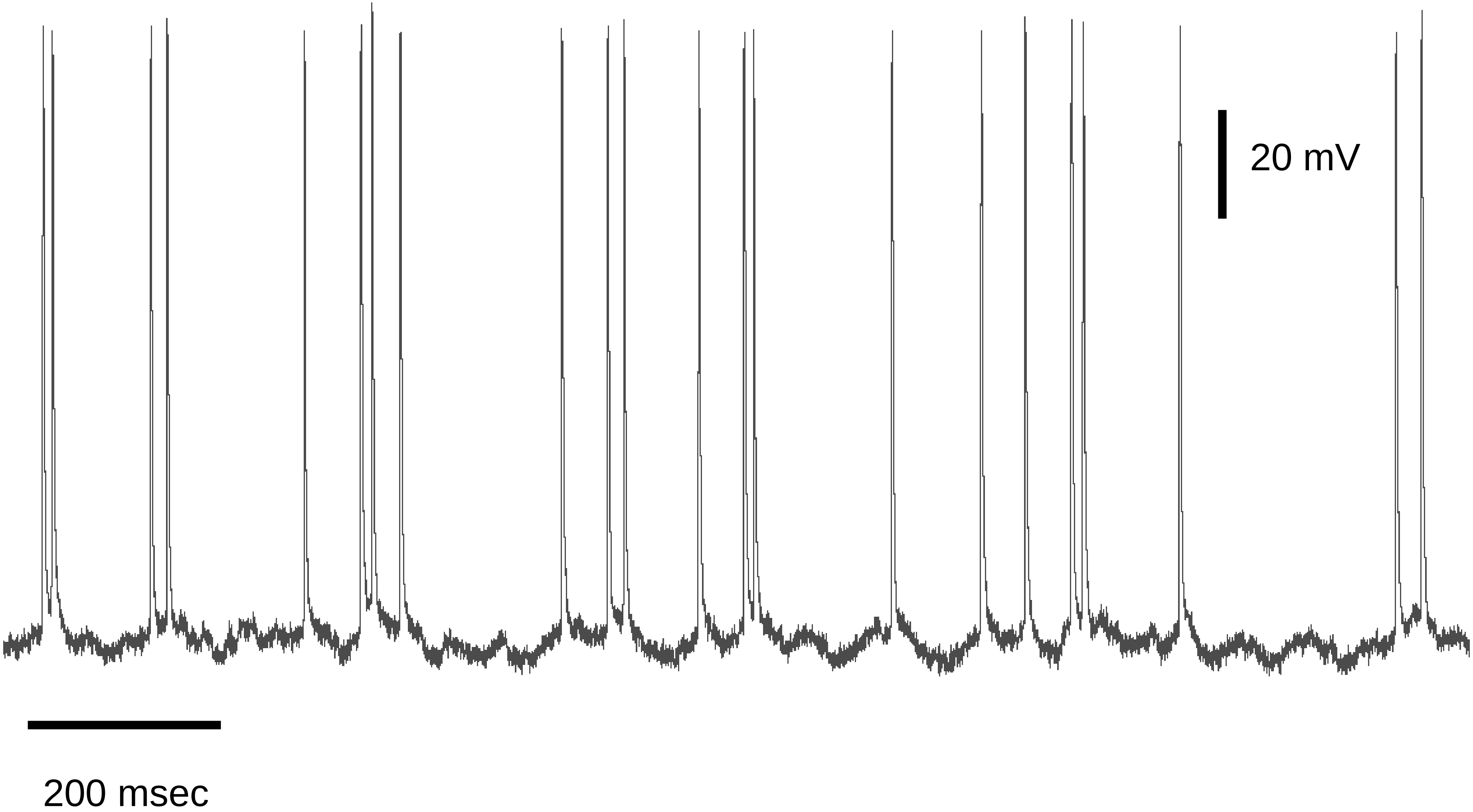
Spike Train Statistics



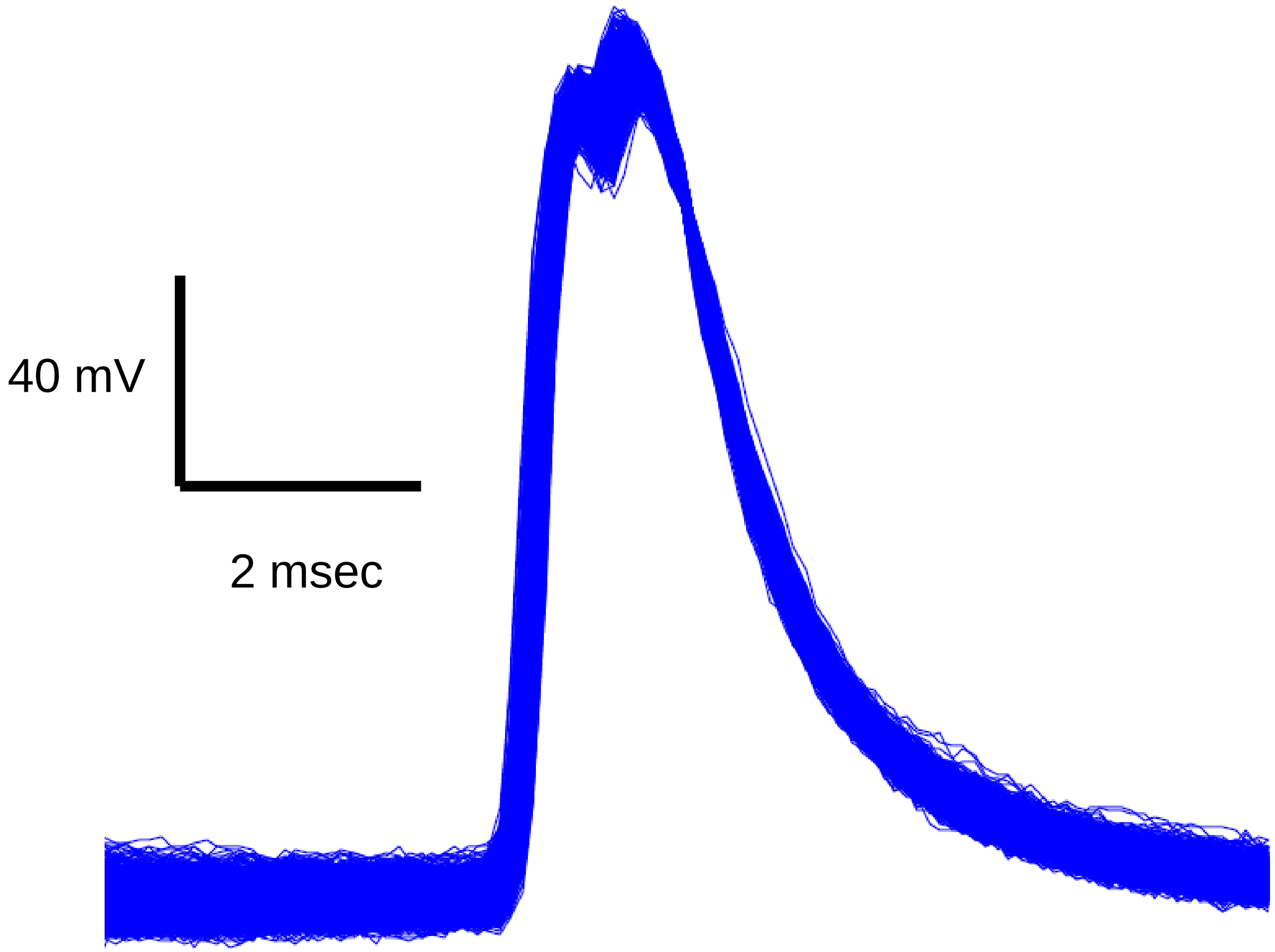
Monsivais et al. 2005



extracellular
20 msec
A
B
C
100 μ



Intracellular recording

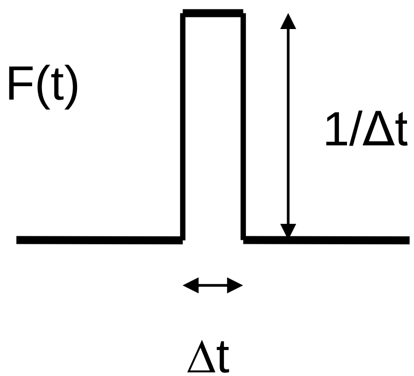


Action potentials are stereotyped

Series of events:

The delta function:

Consider:

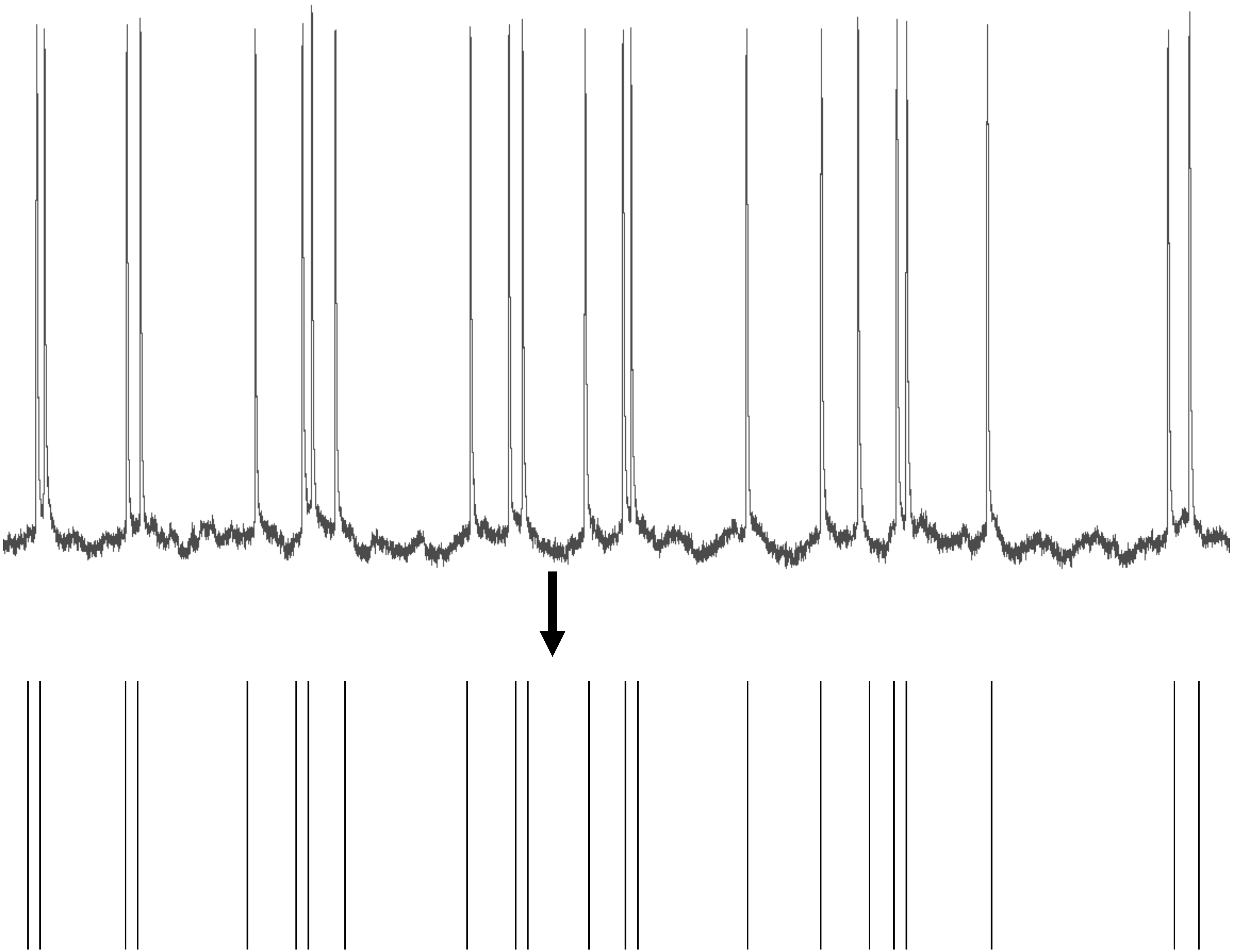


$$\delta(t) = \lim_{\Delta t \to 0} F(t)$$

Properties:

$$\int dt\, G(\tau - t)\delta(t) = G(\tau)$$

If G(t) is “well-behaved”



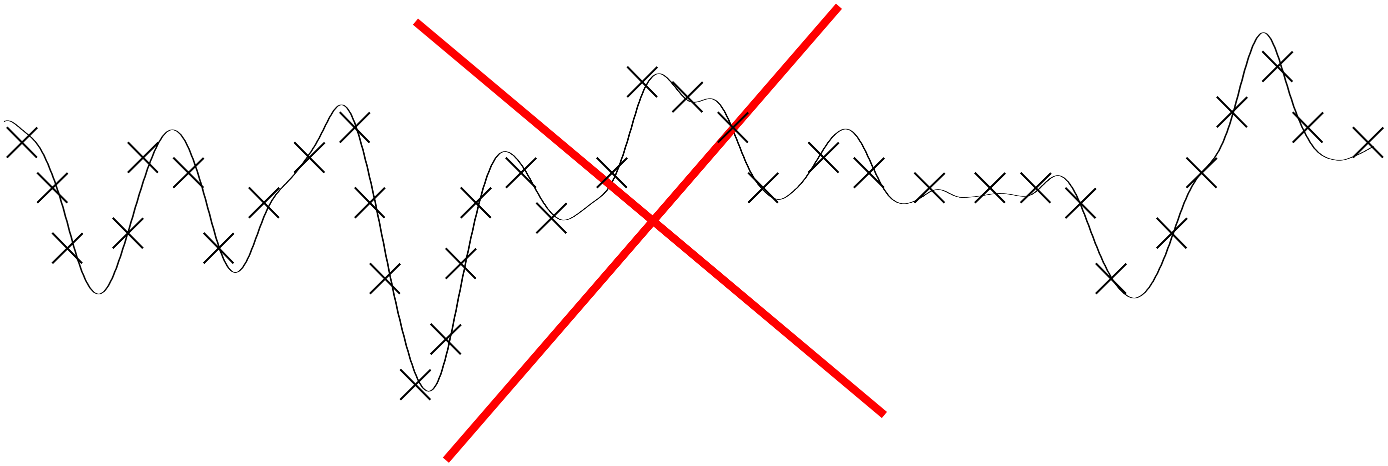
Representing spike trains as series of events

$$X(t) = \sum_i \delta(t - t_i)$$

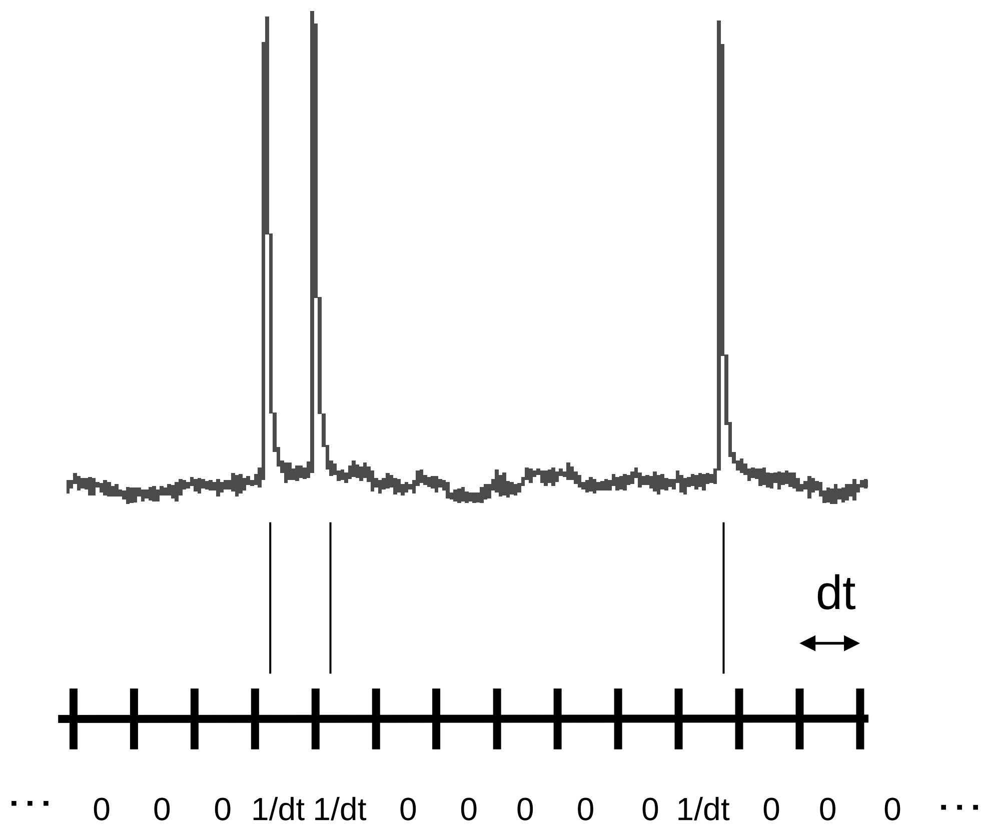
t_i: spike times

In practice

- signals are sampled at a finite rate



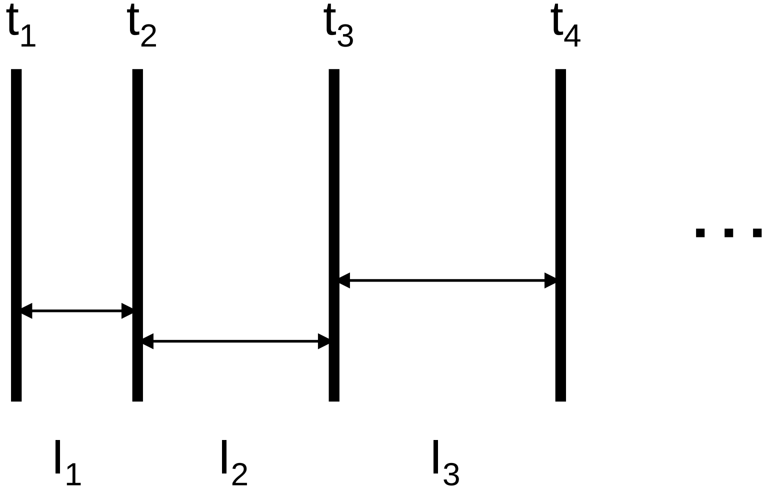
“Binary” Representation of a Spike Train



Interspike Intervals

Interspike interval sequence:

$$I_i = t_{i+1} - t_i$$



Interspike interval probability: P(I)

Probability of observing an ISI with value I

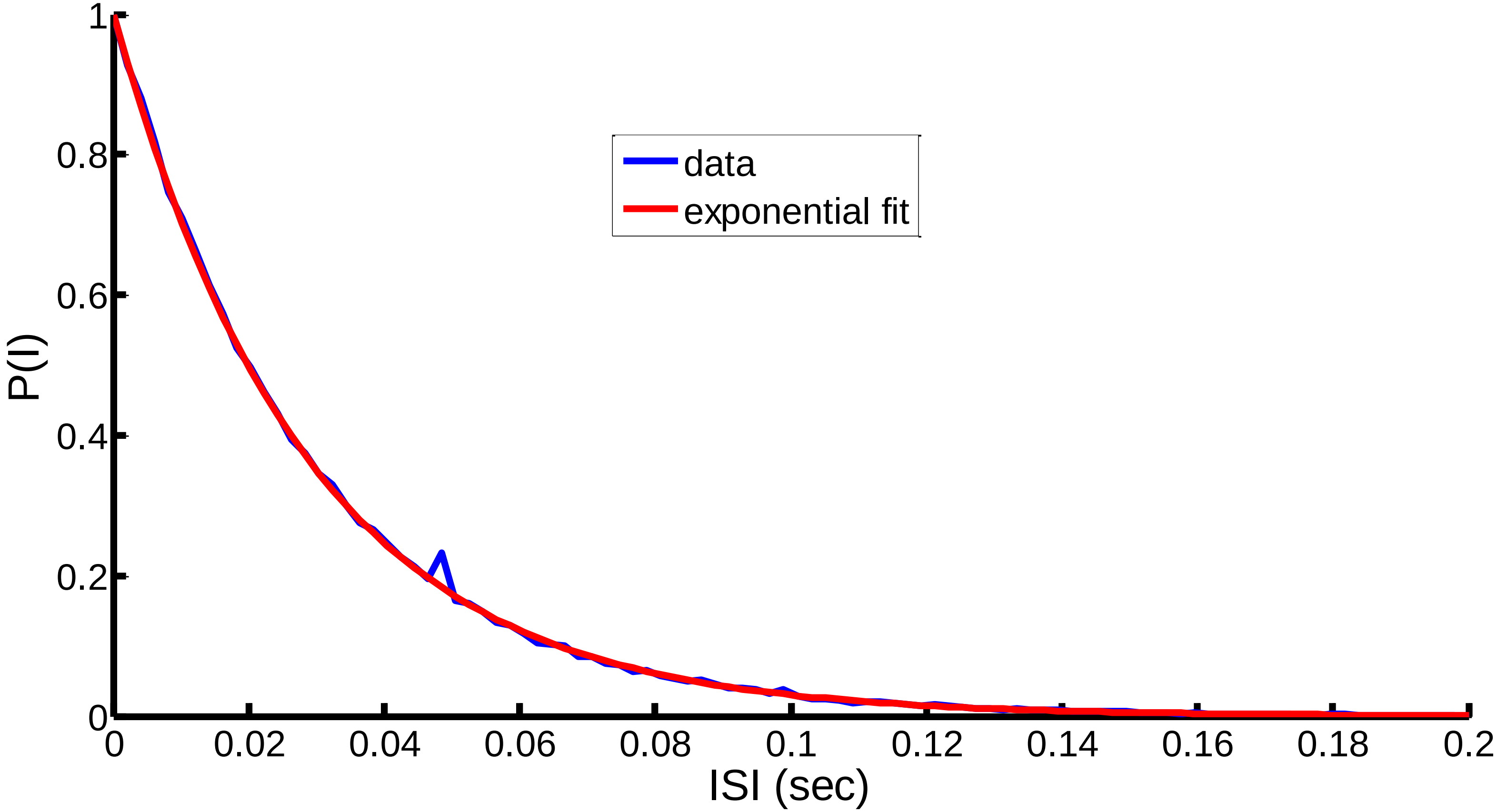
Example: Poisson Process

$$P[(N(t+\tau)-N(t))=k]=\frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}$$

Probability of obtaining k events in an interval of length τ

$$P[(N(\tau)-N(0))=0]=\frac{e^{-\lambda\tau}(\lambda\tau)^0}{0!}=e^{-\lambda\tau}$$

Numerical Simulations:



Coefficient of Variation

- Neurons are stochastic → Variability

$$CV = \frac{STD(I)}{mean(I)}$$

Example: Poisson process

$$CV = \frac{1/\lambda}{1/\lambda} = 1$$

ISI correlations

Normalized autocorrelation function for the ISIs (assume that Var(I)>0)

$$r_j = \frac{< I_i I_{i+j} > - < I_i >^2}{< I_i^2 > - < I_i >^2}$$

$$\rho_0 \equiv 1$$

For a renewal process: $\rho_{i \neq 0} = 0$

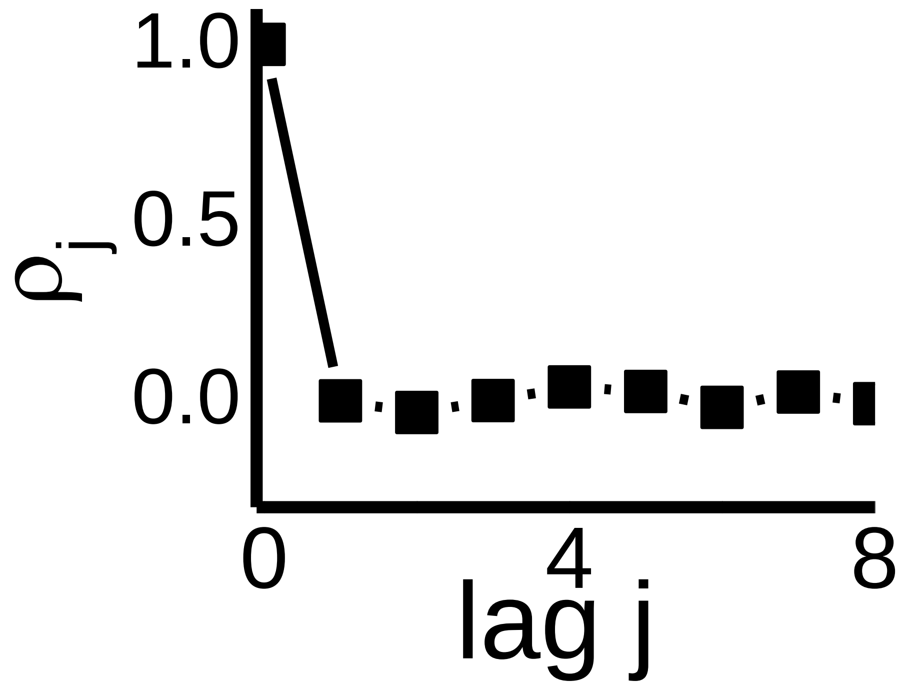
Example: Stochastic IF model driven with white noise

$$\dot{V} = \mu + \xi(t)$$

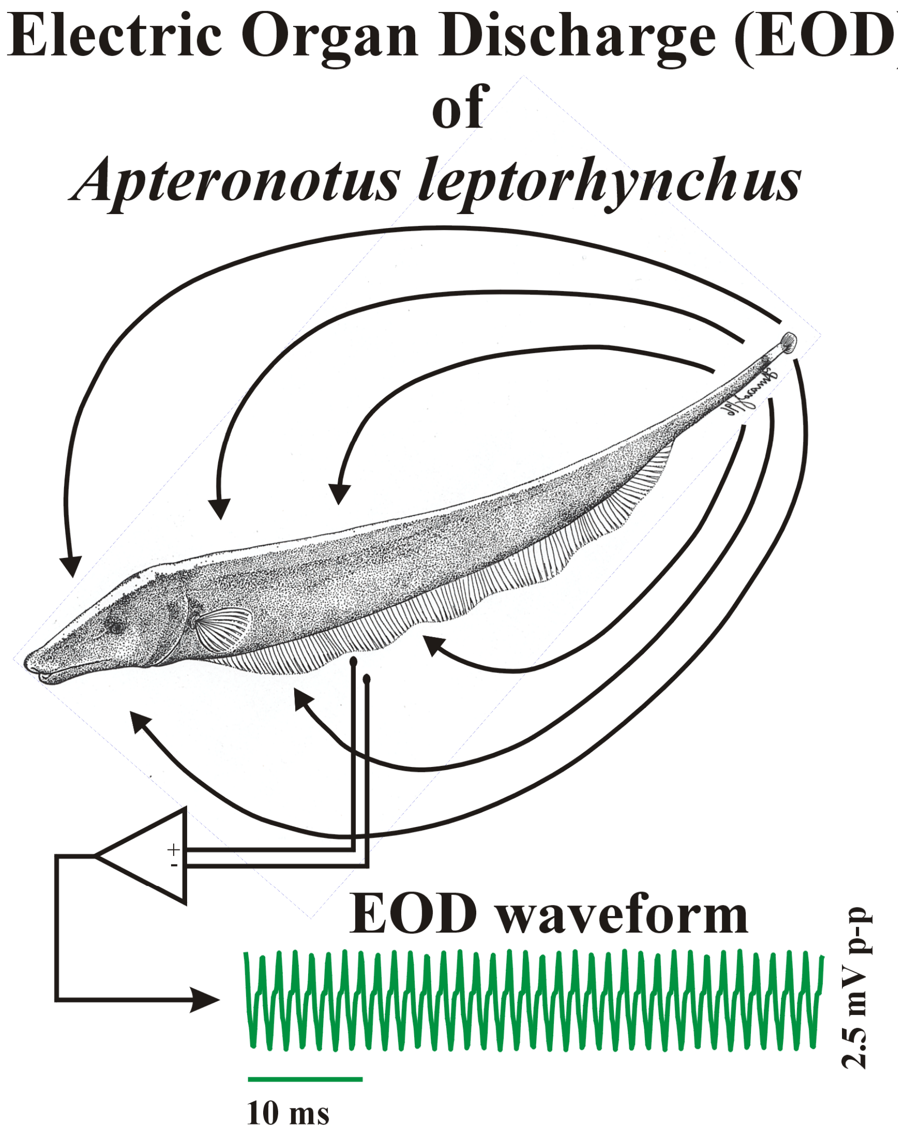
$$V(t_i) = \theta \Rightarrow V(t_i^+) = 0$$

ISIs are independent

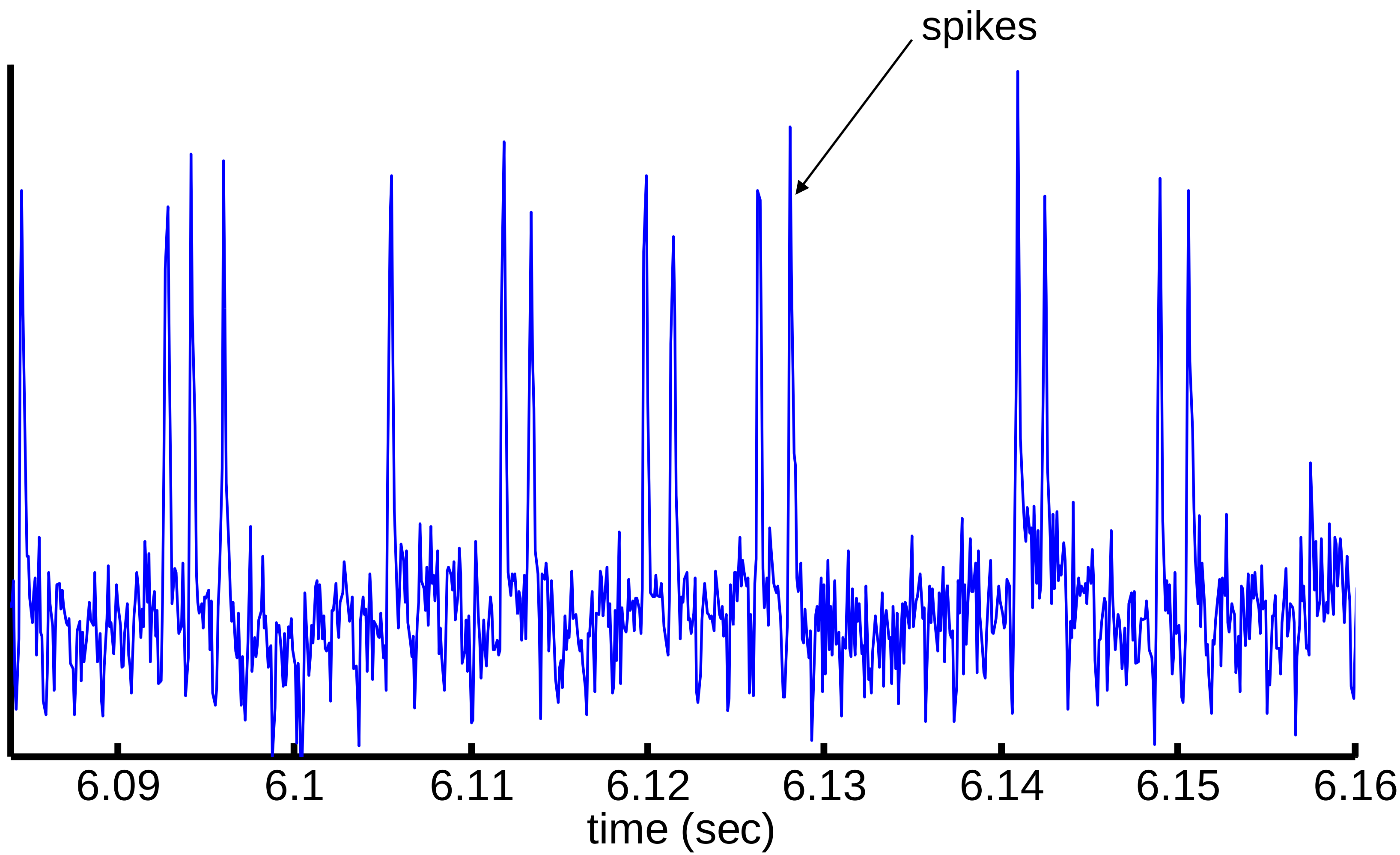
- Reset rule “erases” the history
- White noise is uncorrelated over time



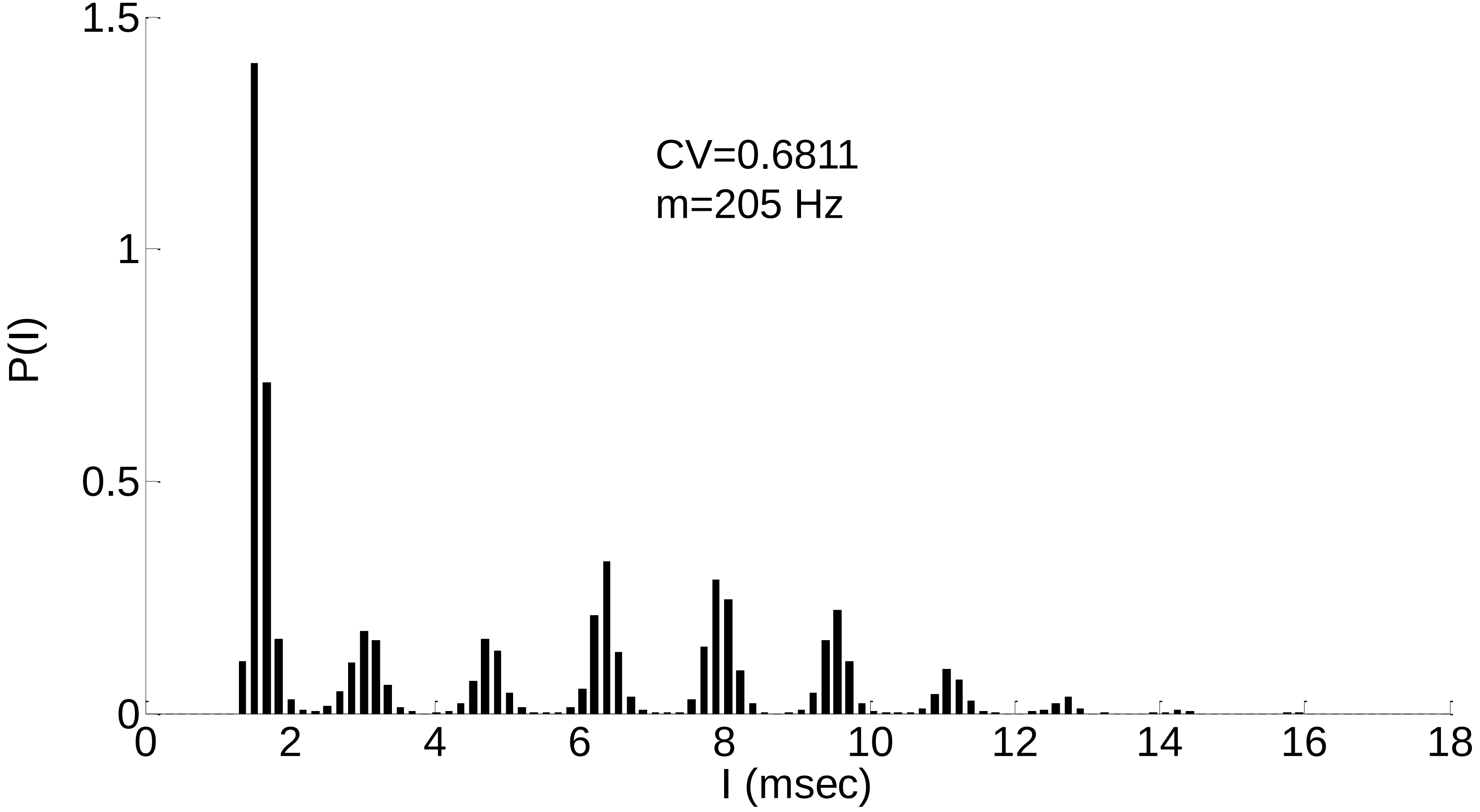
Example: electroreceptor afferent neuron



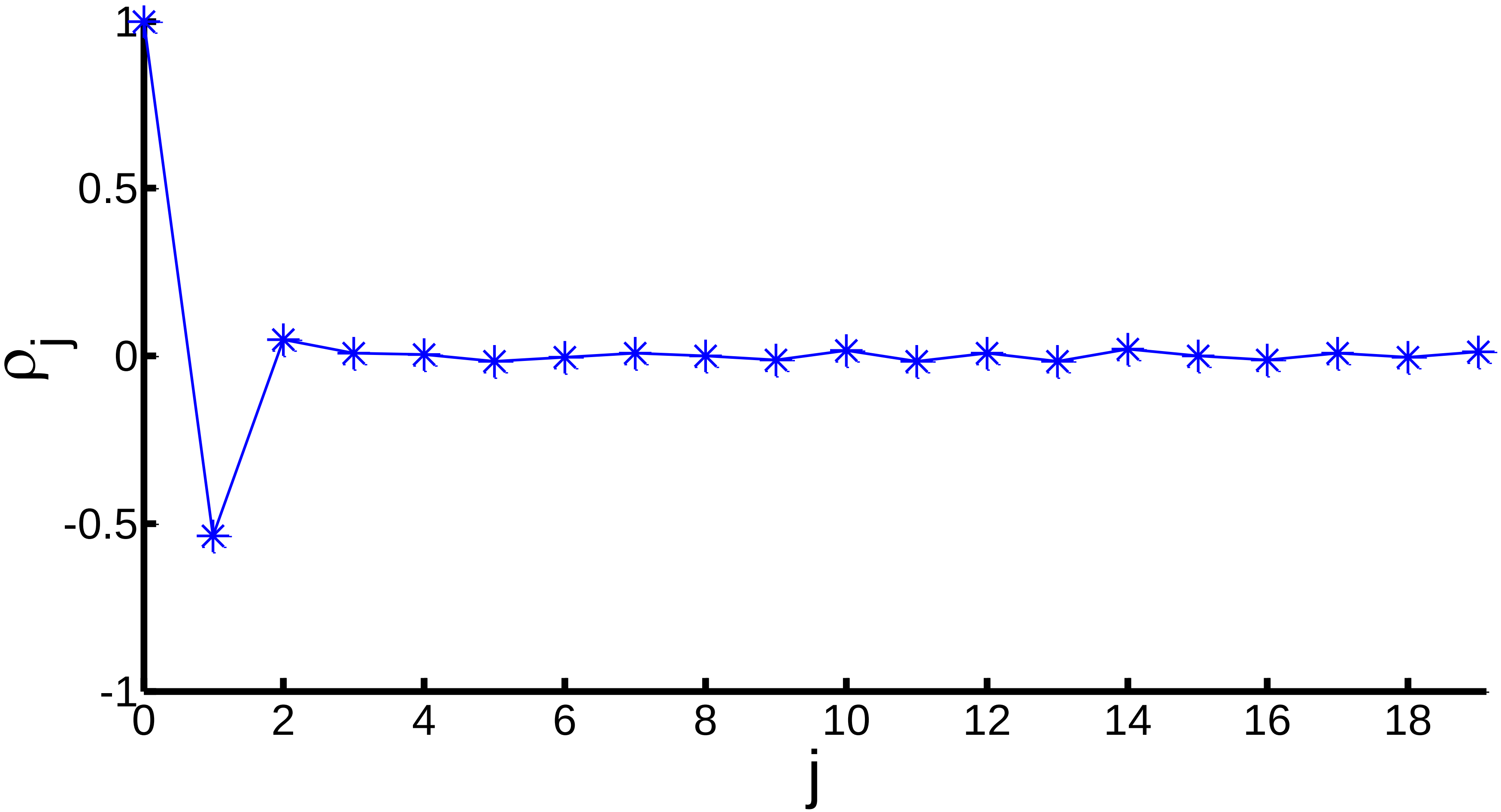
Raw Data



ISI probability density



ISI autocorrelation function



C) Statistics based on event trains

$$X(t) = \sum_i \delta(t - t_i)$$

t_i: spike times

Time varying firing rate:

$$r(t) = < X(t) >$$

For a stationary process: $$r(t) = m$$

Autocorrelation Function

$$A(t,\tau) = < X(t)X(t+\tau) > - < X(t) >< X(t+\tau) >$$

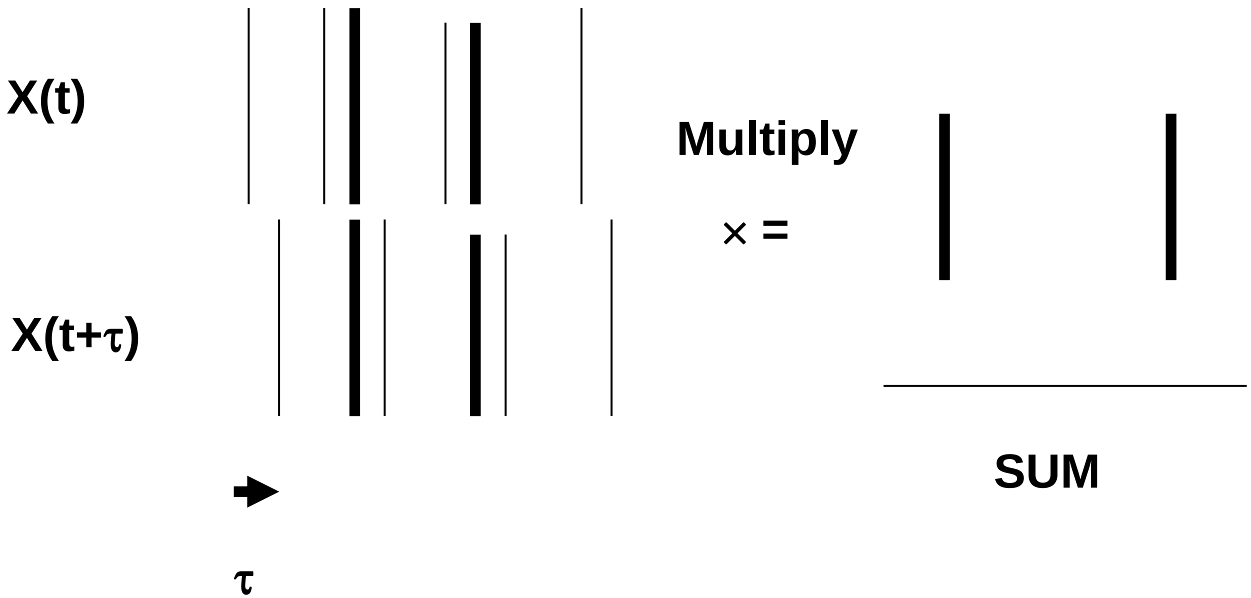
For stationary processes:

$$A(t,\tau) = A(\tau) = < X(t)X(t+\tau) > - m^2$$

$$m = < X(t) >$$

If X(t) and X(t+τ) are uncorrelated, then A(τ)=0

Autocorrelation Function



Example: Poisson Process with firing rate λ

$$A(\tau) = \lambda\delta(\tau)$$

One often removes this delta function:

$$A^{+}(\tau) = A(\tau) - \lambda\delta(\tau)$$

For most processes: $$\underset{\tau \to \pm\infty}{Lim} A(\tau) = 0$$

Power Spectrum

- Fourier Transform of the Autocorrelation function (Wiener-Khintchine Theorem)

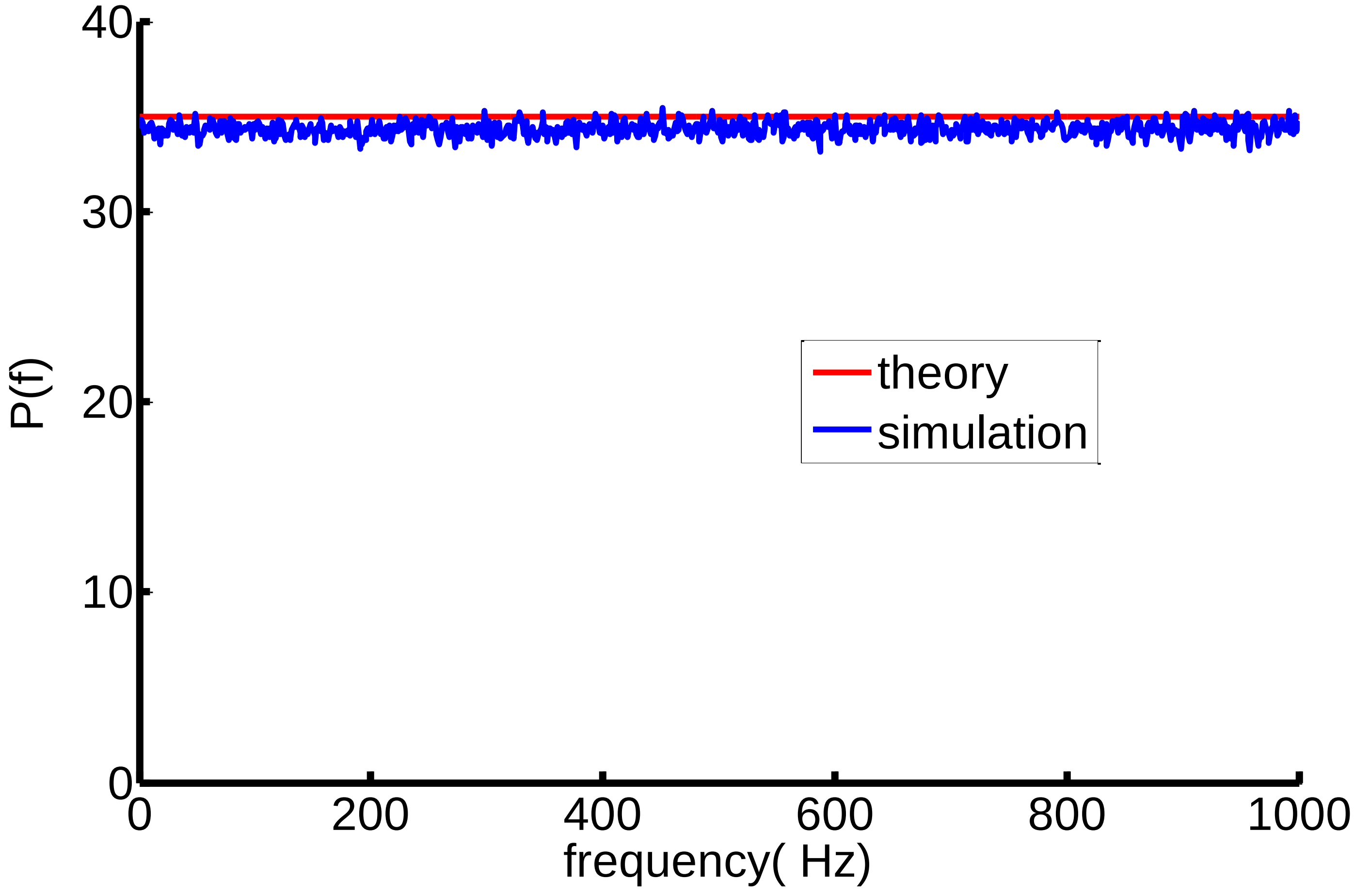
- Same information but represented differently

$$P(f) = <\tilde{X}(f)\, X(f)>$$

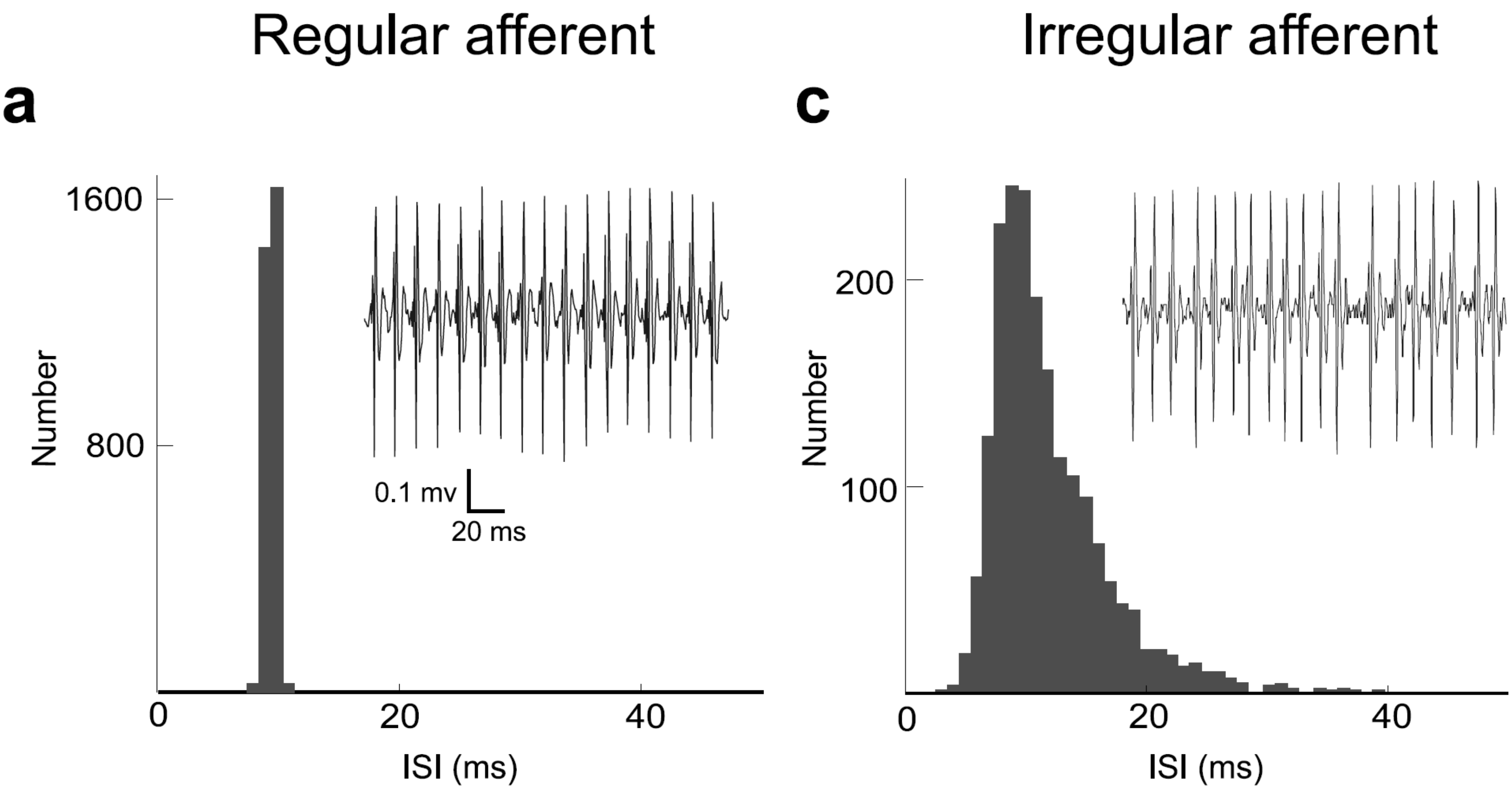
$$\lim_{f \to \infty} P(f) = < X(t) > = m$$ **X(t) stationary**

Example: Poisson Process

$$A(\tau) = \lambda\delta(\tau) \quad \Rightarrow \quad P(f) = \lambda$$



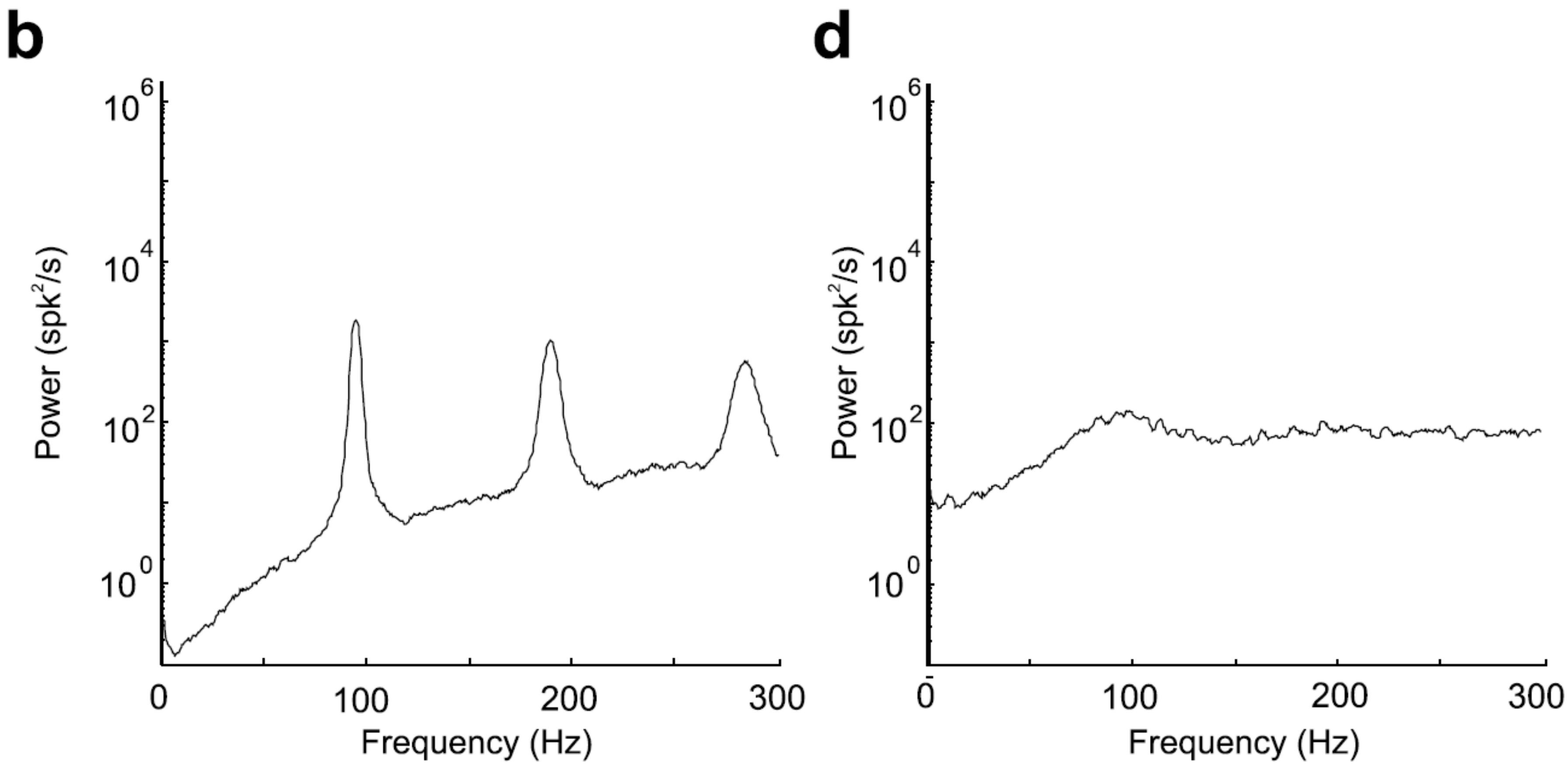
Example: vestibular afferents



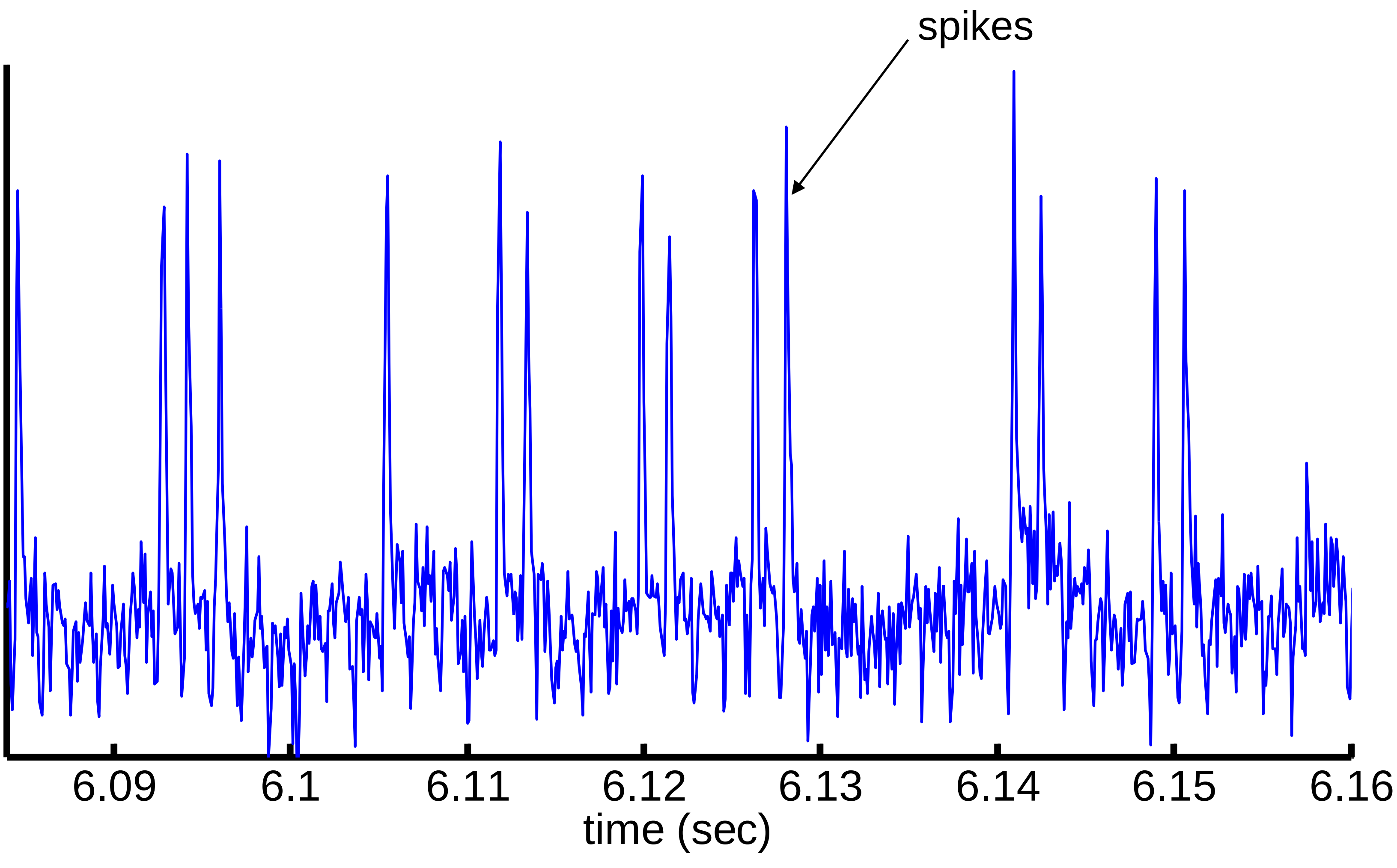
CV=0.044

CV=0.35

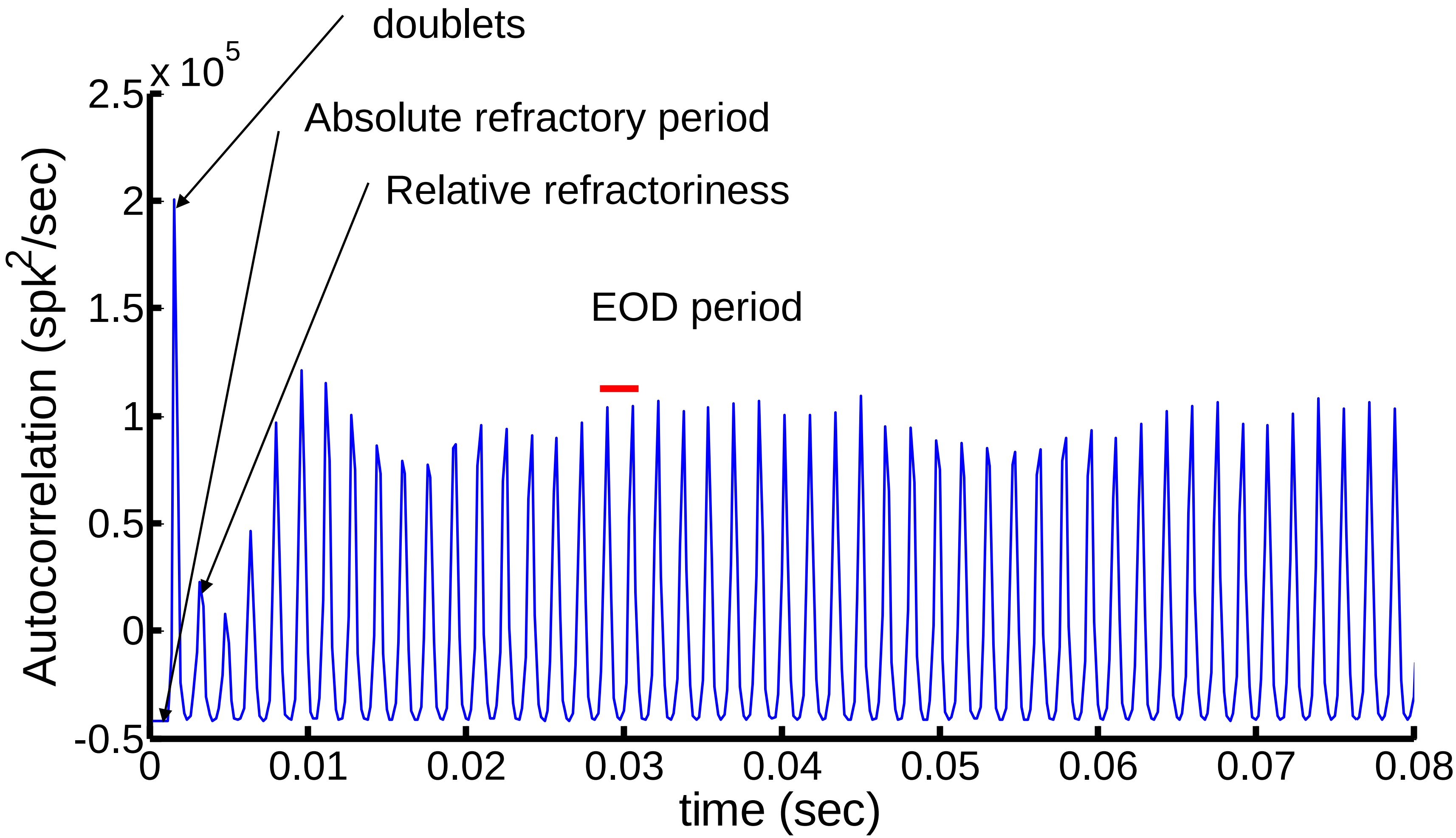
Power Spectra



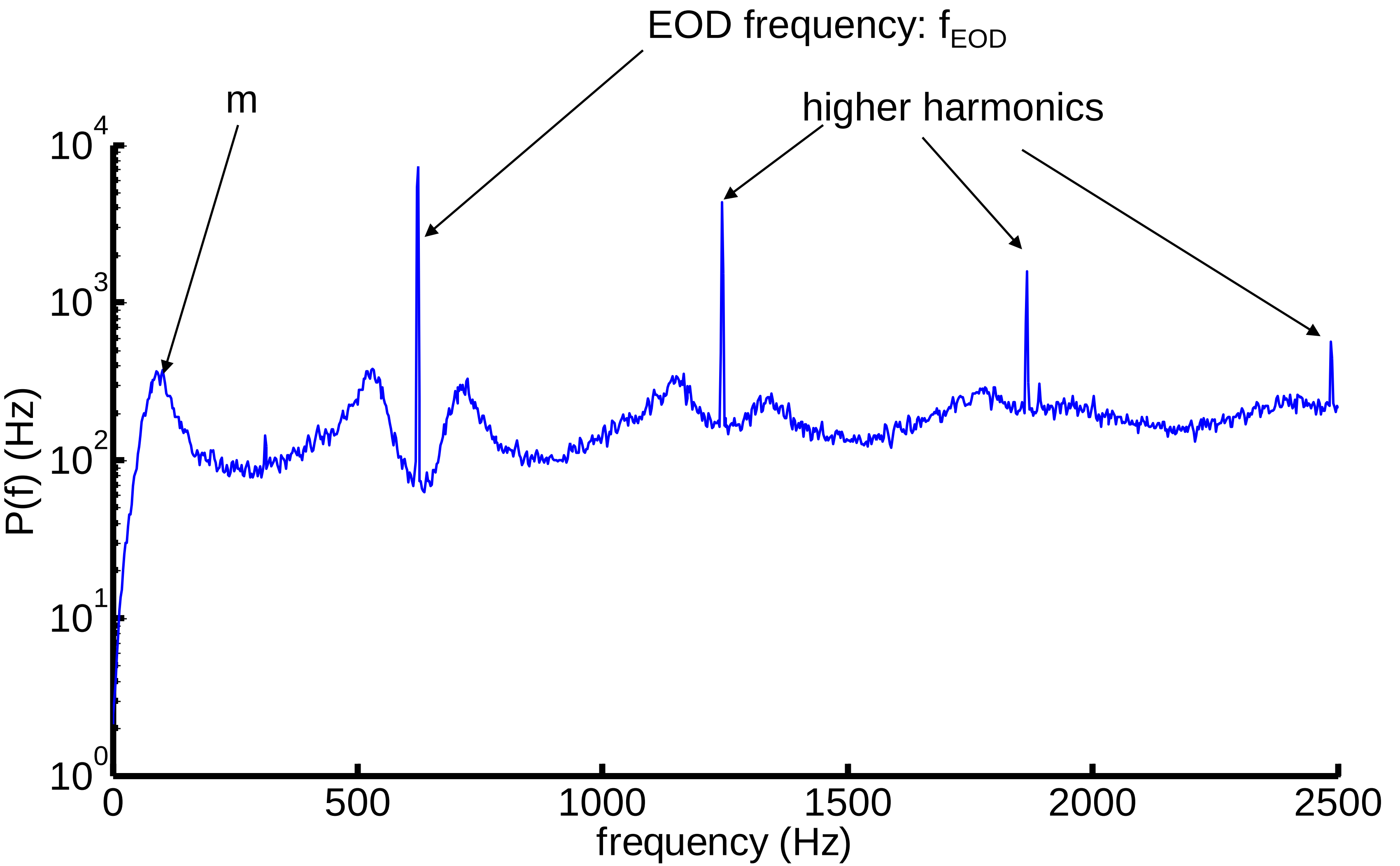
Electroreceptor afferent:



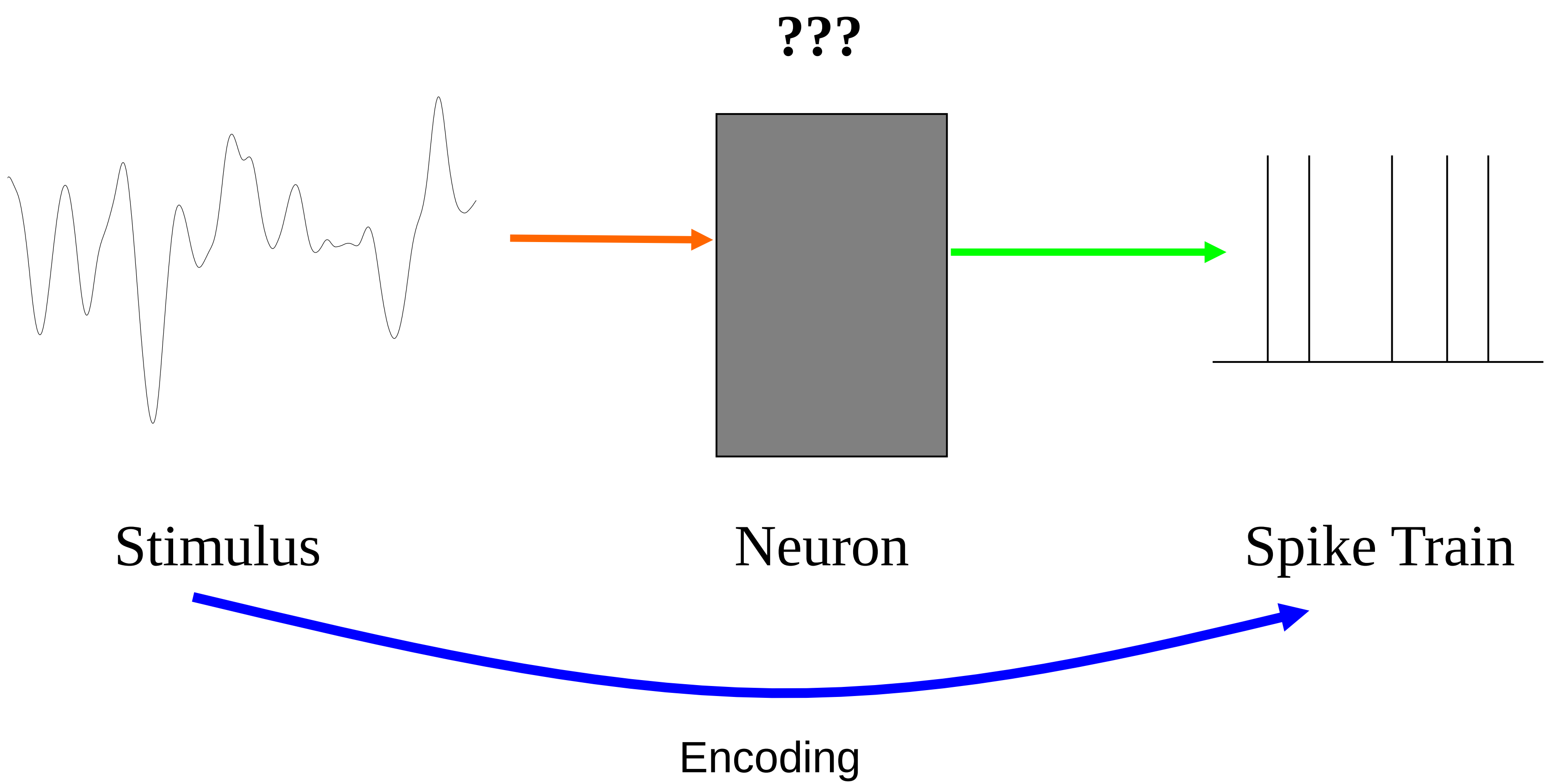
Example: Electroreceptor Afferent



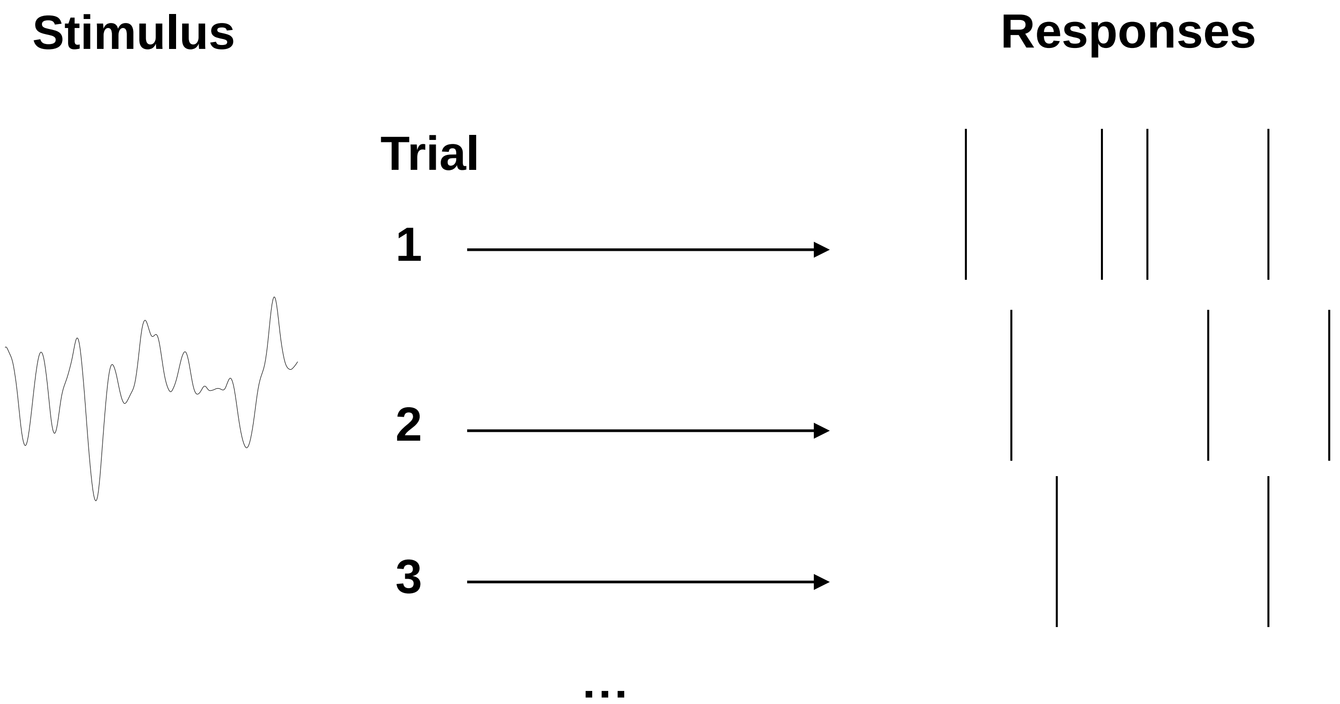
Example: Electroreceptor Afferent



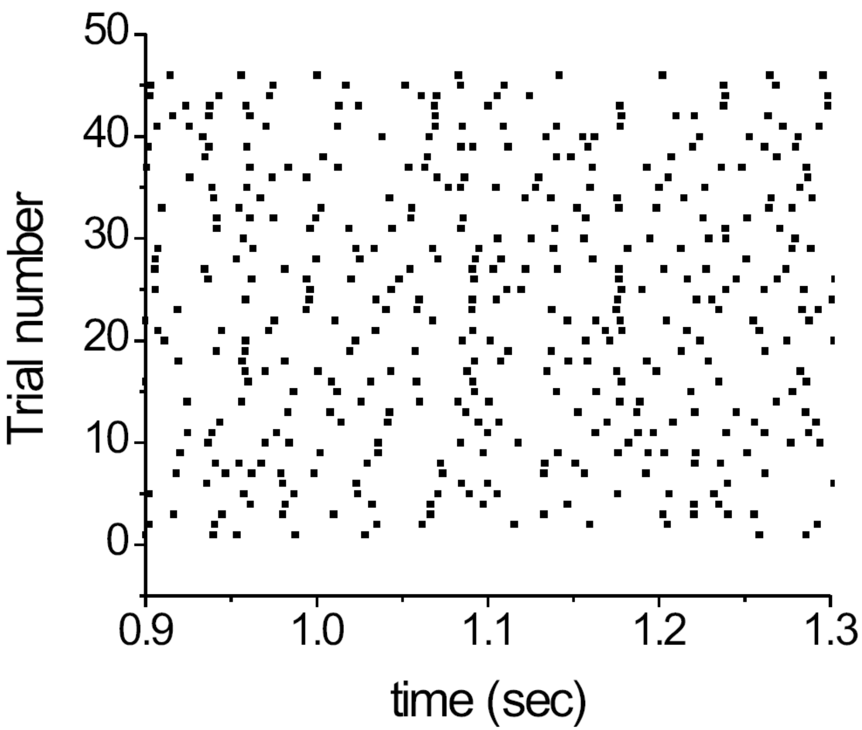
Measures of Neural Encoding



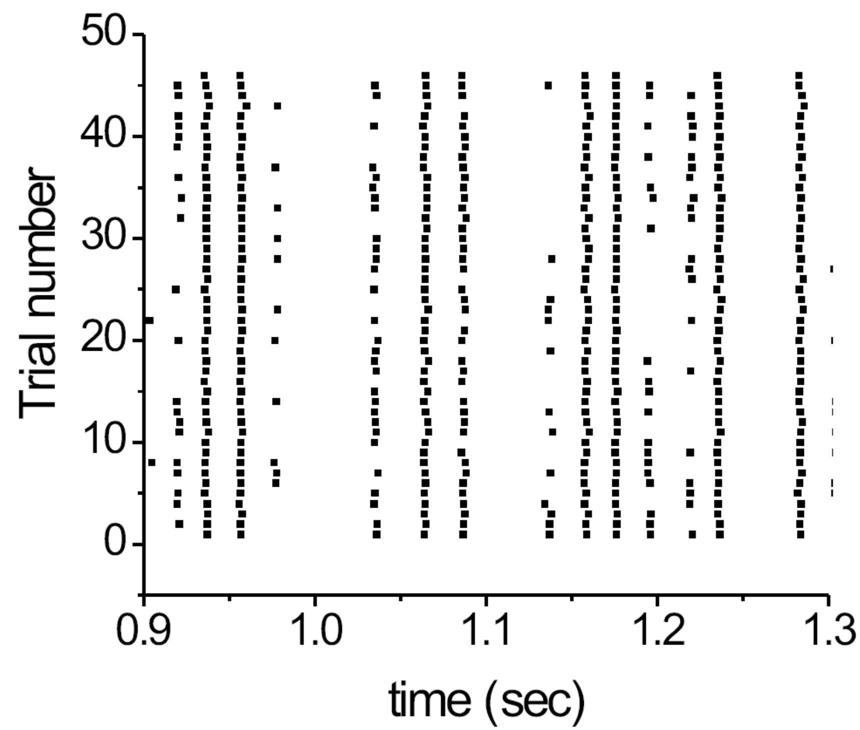
Raster plot



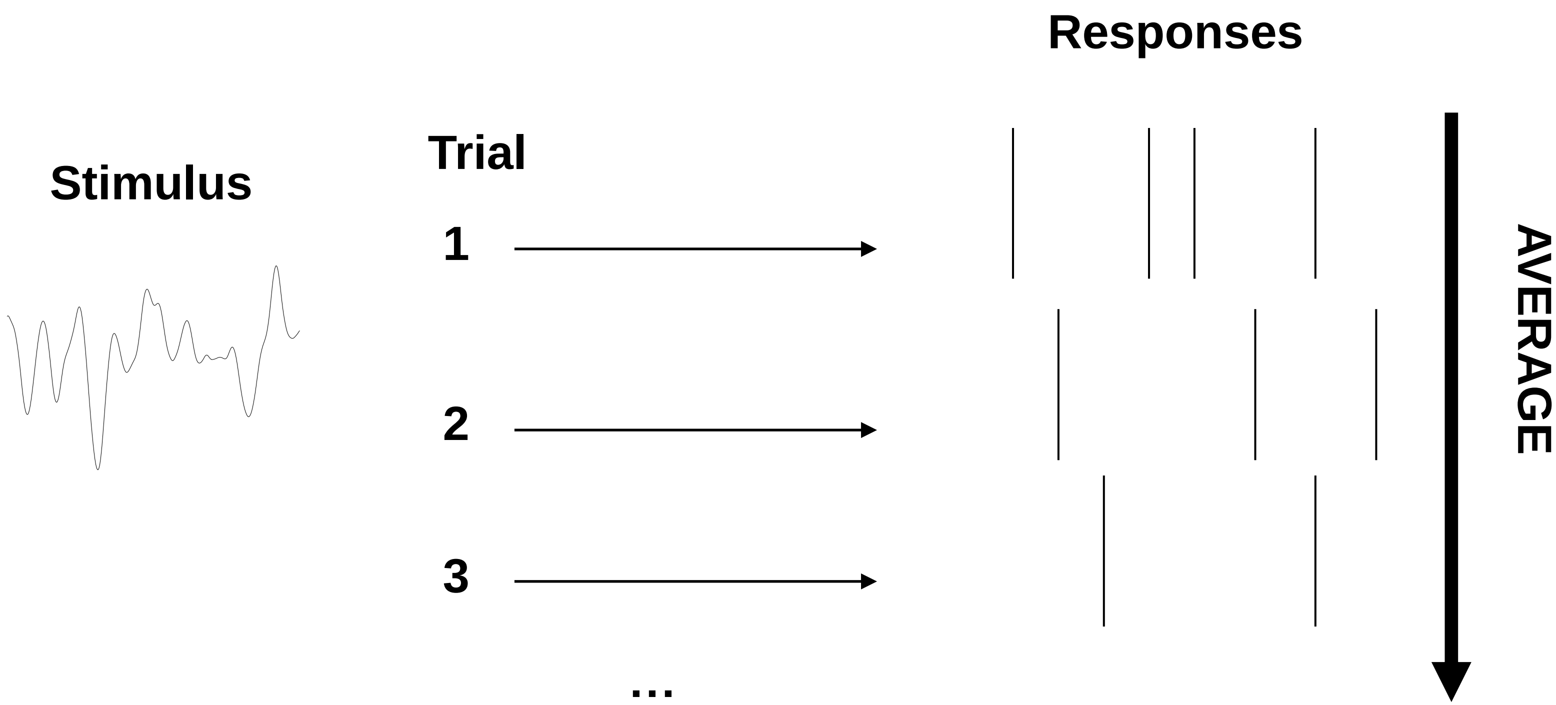
High variability



Low variability

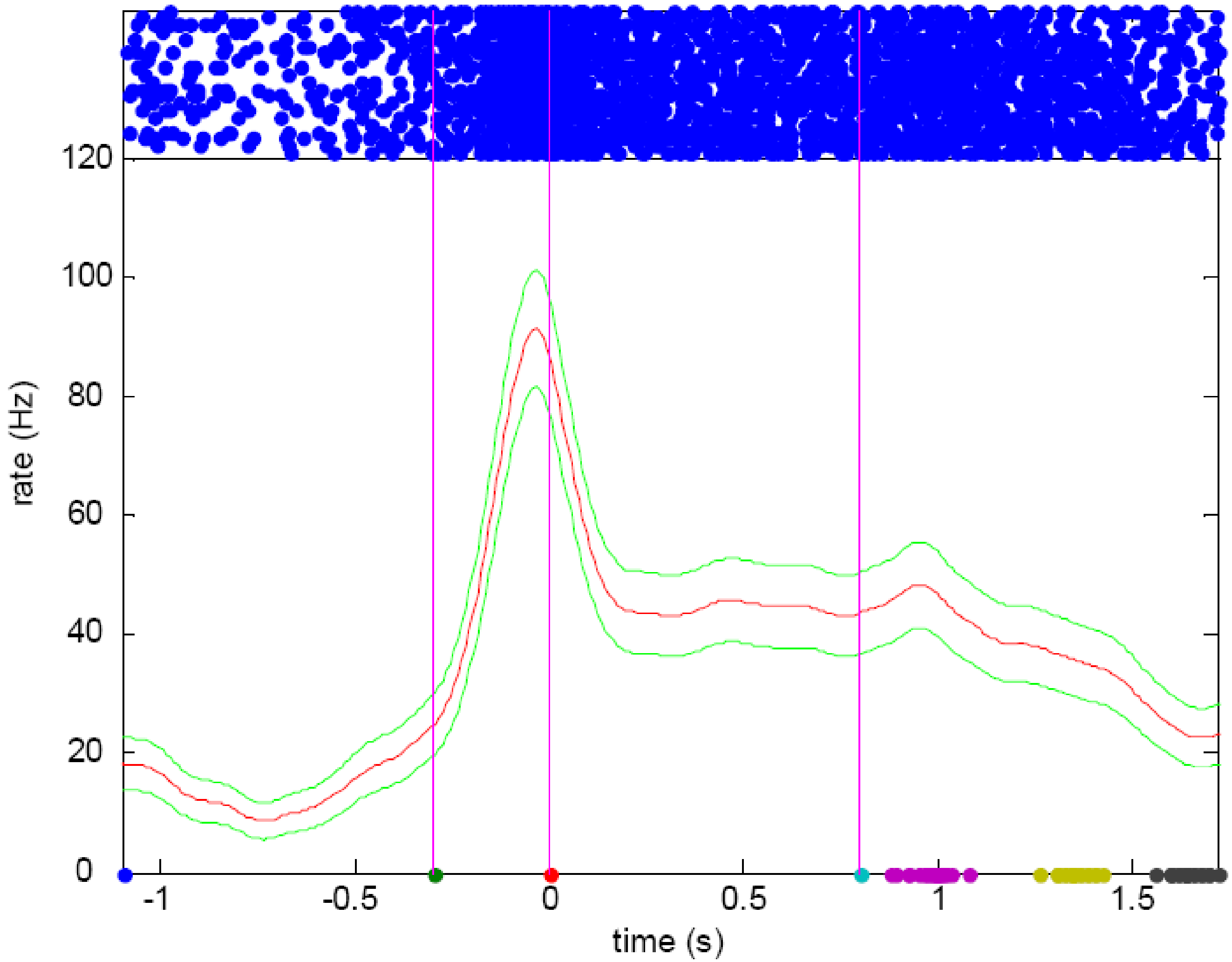


Peri-Stimulus Time Histogram



$$PSTH(t) = \langle X(t) \rangle = \frac{1}{N}\sum_{i=1}^{N} X_i(t)$$

Peri-stimulus time histogram (PSTH)



Cue

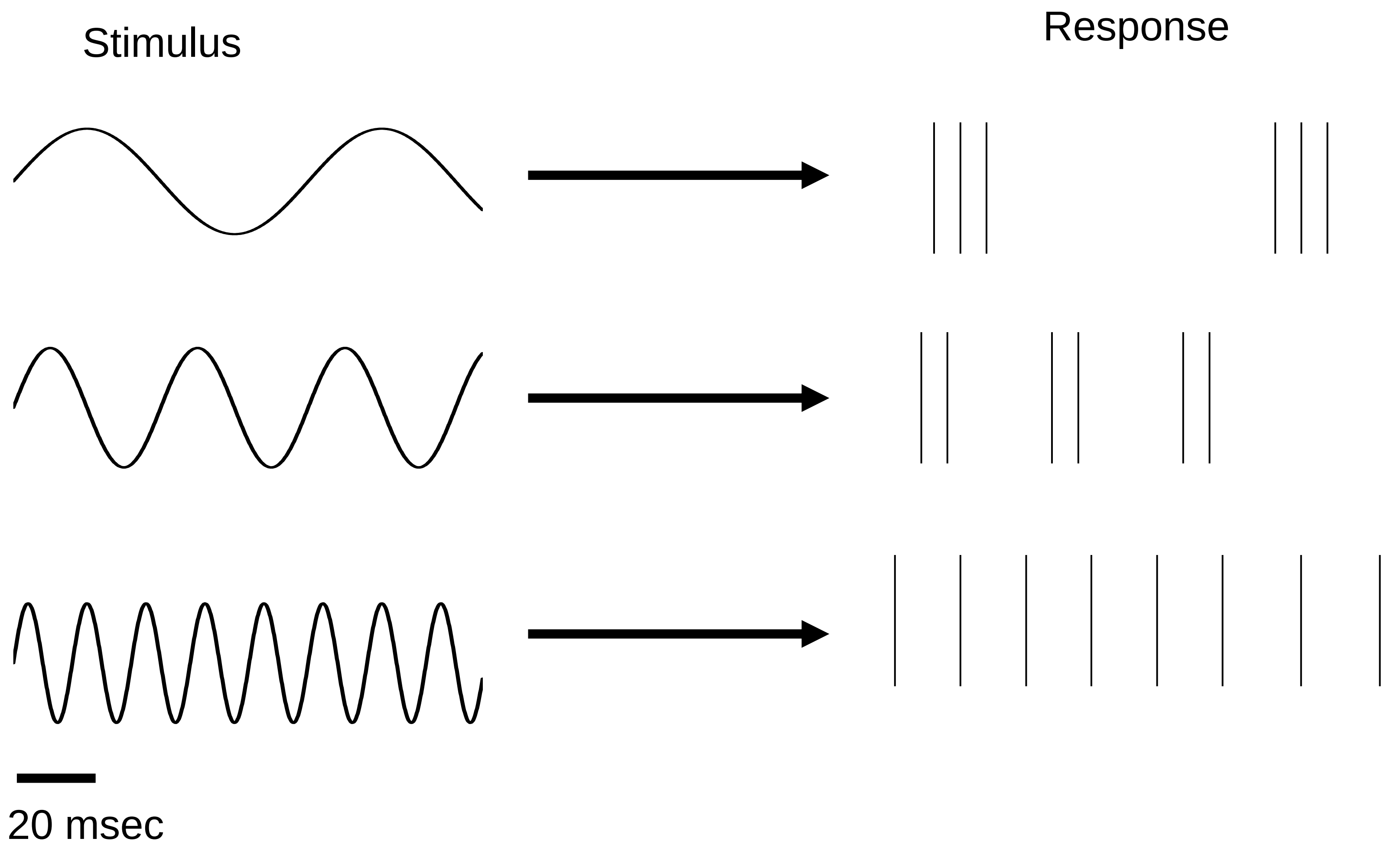
Planning

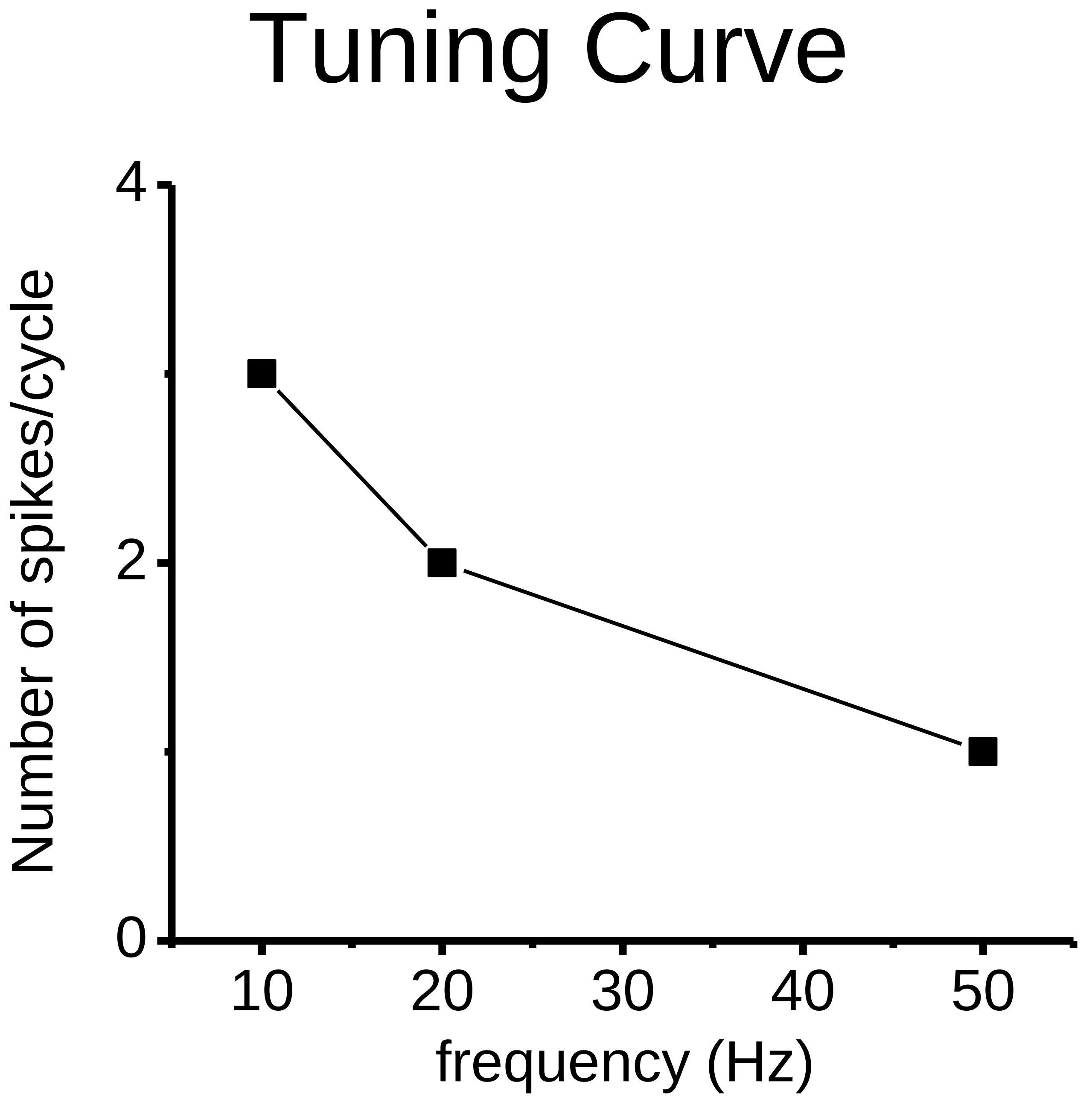
Movement

Tuning Curves

- Plot of response attribute as a function of stimulus attribute

Example: sinusoidal stimulation




Tuning Curve
4
2
0
Number of spikes/cycle
10
20
30
40
50
frequency (Hz)

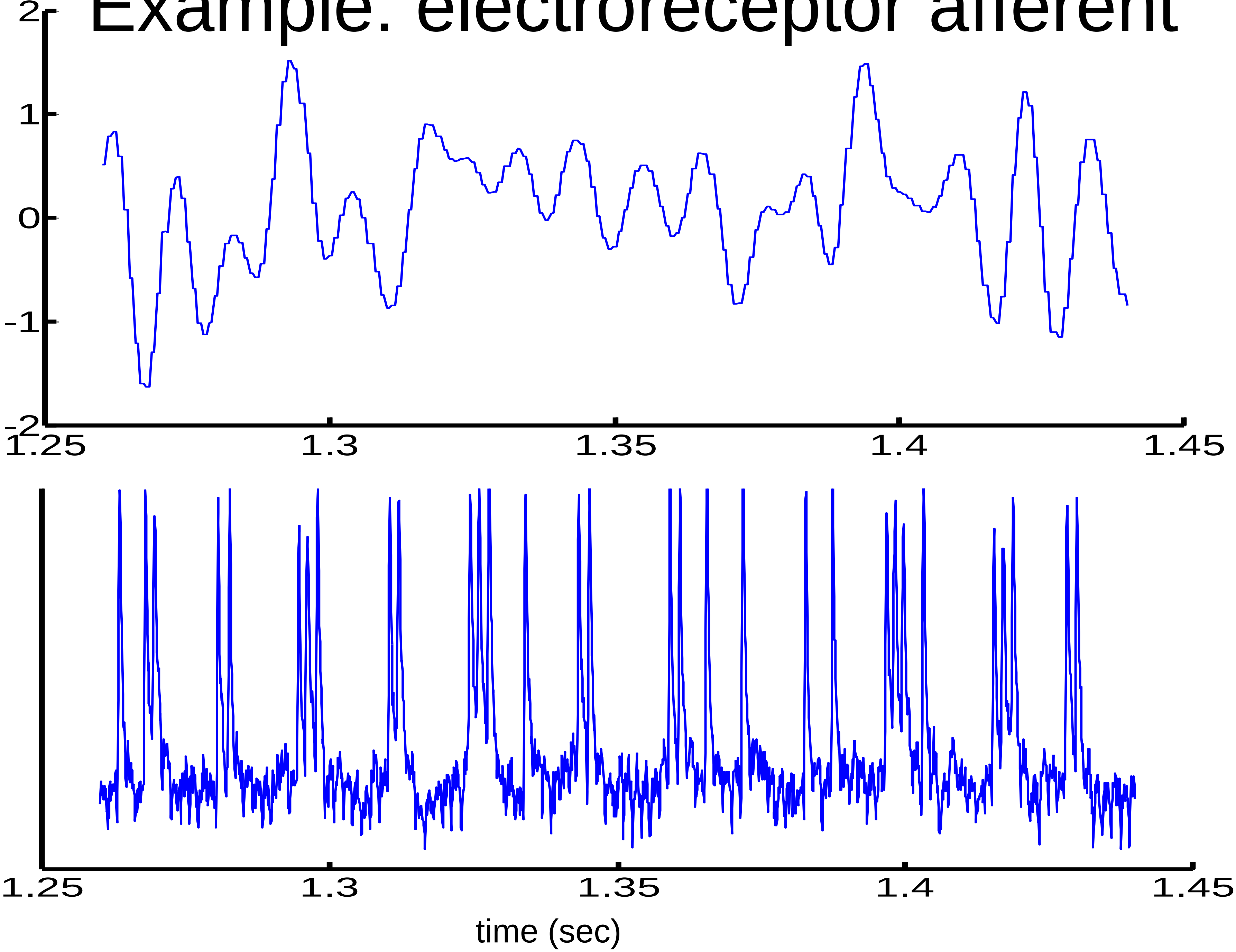
Cross-Correlation Function

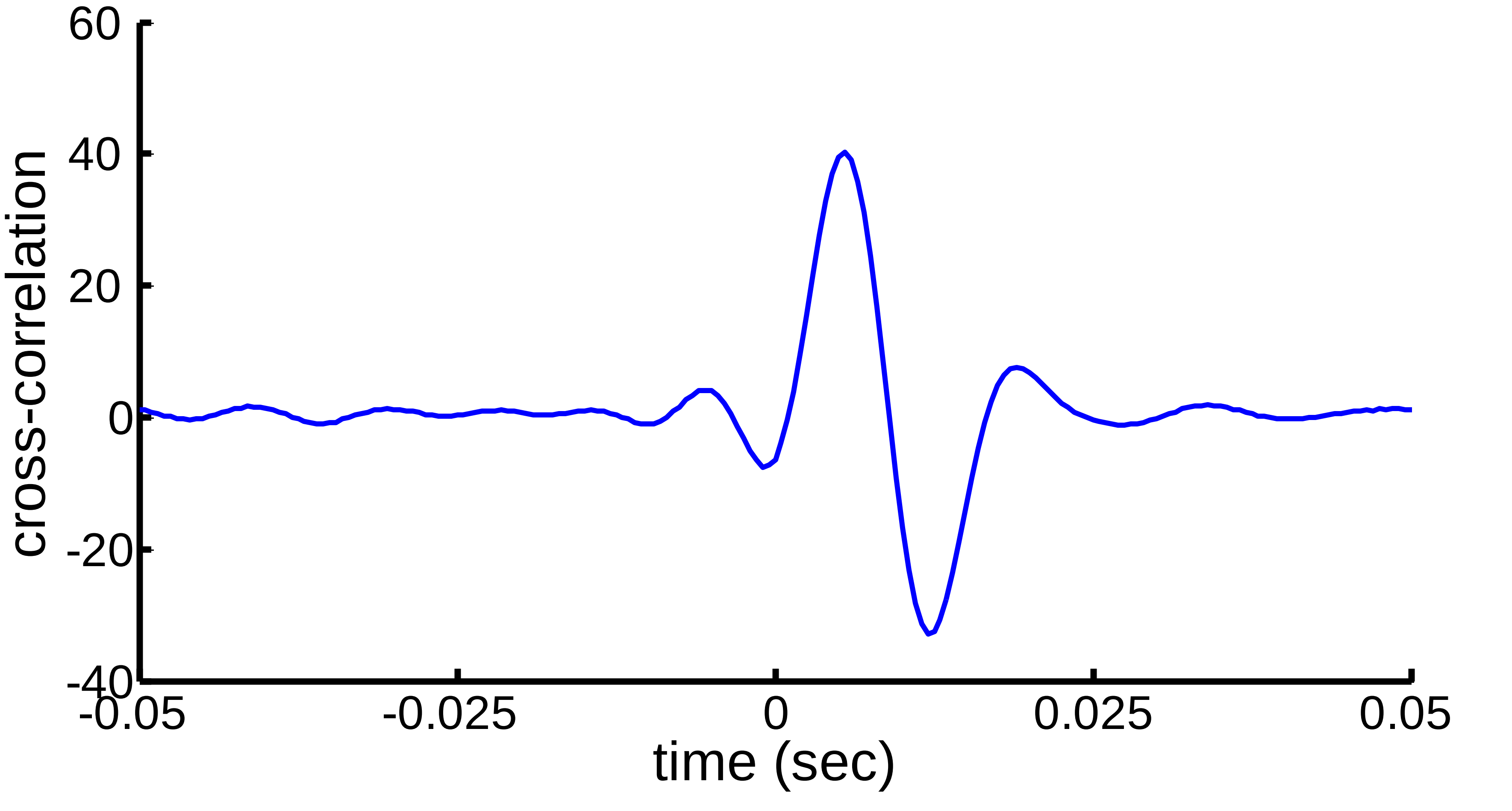
$$C(t,\tau) = \langle X(t)\,S(t+\tau)\rangle - \langle X(t)\rangle\langle S(t+\tau)\rangle$$

For stationary processes: $C(t,\tau) = C(\tau)$

In general, $C(-\tau) \neq C(\tau)$

Example: electroreceptor afferent





60
40
20
0
-20
-40
cross-correlation
-0.05
-0.025
0
0.025
0.05
time (sec)

Cross-Spectrum

- Fourier Transform of the Cross-correlation function
- Complex number in general

$$\tilde{C}(f) = \langle \tilde{X}^*(f)\tilde{S}(f) \rangle - \langle \tilde{X}^*(f) \rangle \langle \tilde{S}(f) \rangle$$

Representing the cross-spectrum:

$$\tilde{C}(f) = |\tilde{C}(f)|\, e^{i\phi}$$

$|\tilde{C}(f)|$: amplitude

$$\phi = \arctan\left(\frac{imag[\tilde{C}(f)]}{real[\tilde{C}(f)]}\right)$$: phase

Transfer functions:

$$OUT(t) = [T * IN](t) + \xi(t)$$

assume output is a convolution of the input with a kernel T(t) with additive noise. We'll also assume that all terms are zero mean.

$$\widetilde{O}UT(f) = \widetilde{T}(f)\,\widetilde{I}N(f) + \widetilde{\xi}(f)$$

Transfer function

Calculating the transfer function

$$\widetilde{IN}^*(f)\widetilde{OUT}(f)=\widetilde{IN}^*(f)\tilde{T}(f)\widetilde{IN}(f)+\widetilde{IN}^*(f)\tilde{\xi}(f)$$

multiply by: $\widetilde{IN}^*(f)$ and average over noise realizations

$$\tilde{C}^*(f)=\tilde{T}(f)\left\langle\widetilde{IN}^*(f)\widetilde{IN}(f)\right\rangle+\underbrace{\cancel{\left\langle\widetilde{IN}^*(f)\tilde{\xi}(f)\right\rangle}}_{=0}$$

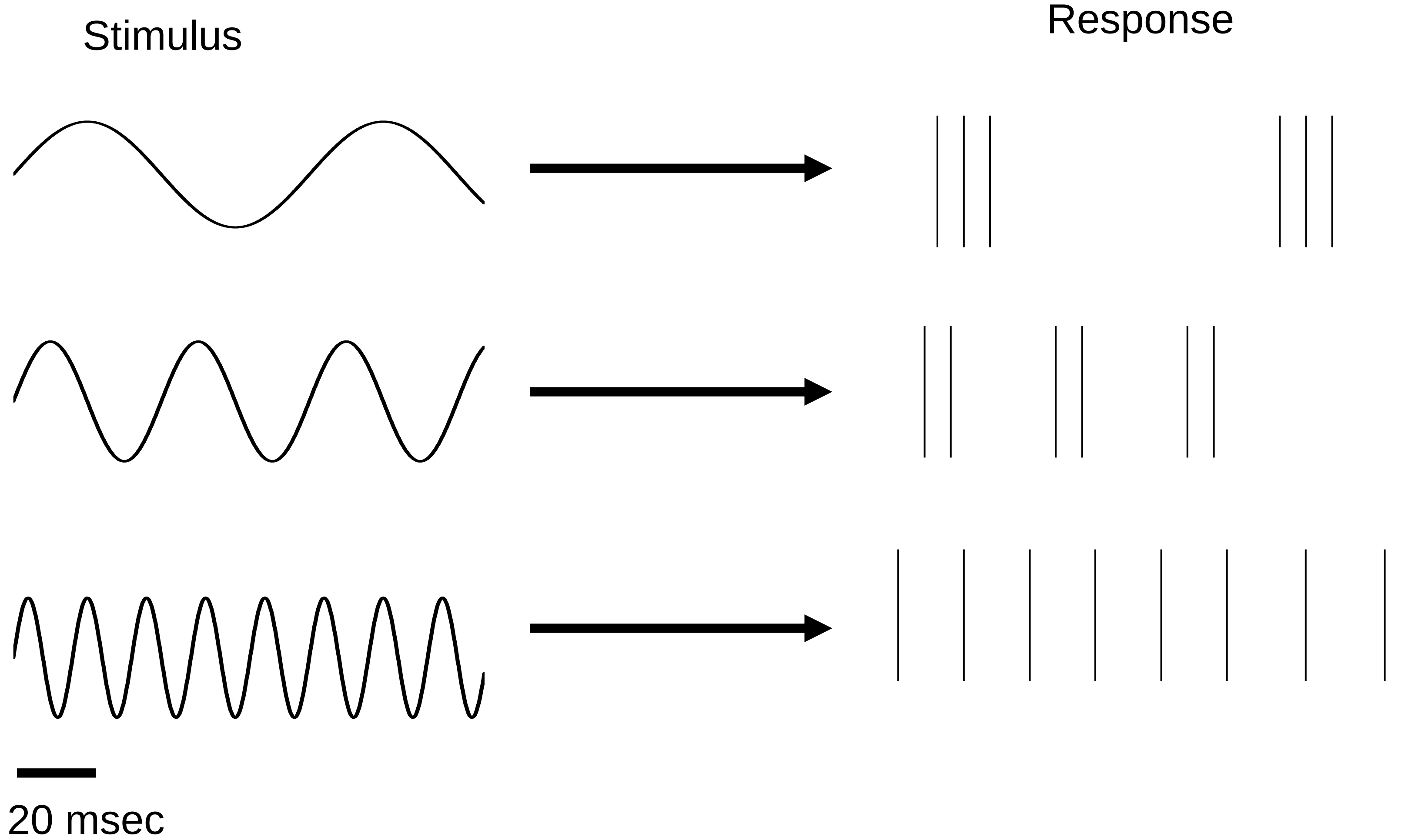
$$\tilde{T}(f)=\frac{\tilde{C}^*(f)}{\tilde{P}_{IN}(f)}$$

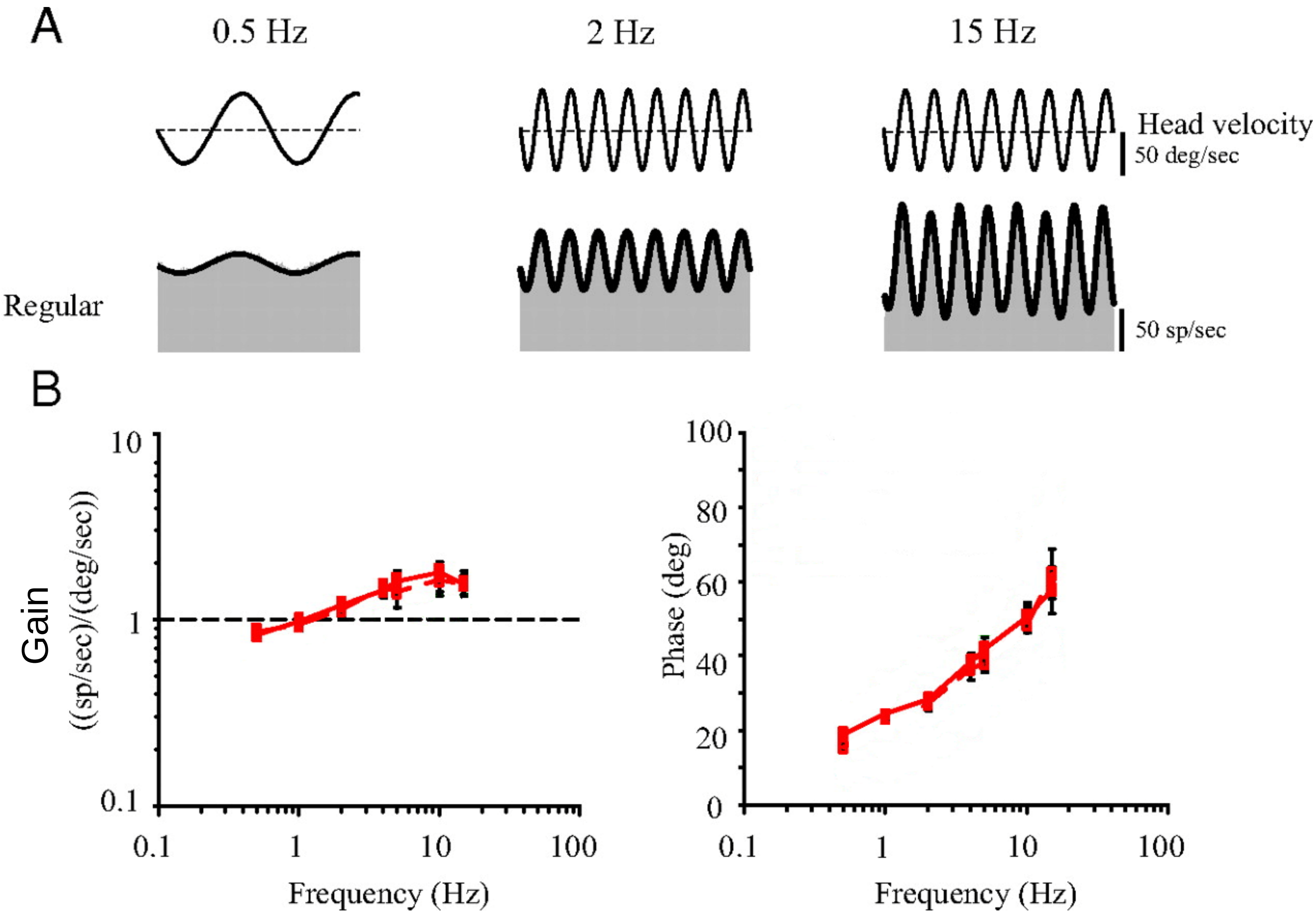
Gain and phase:

$$gain = \left| \tilde{T}(f) \right|$$

$$\phi = \arctan\left(\frac{imag[\tilde{T}(f)]}{real[\tilde{T}(f)]} \right)$$

Sinusoidal stimulation at different frequencies



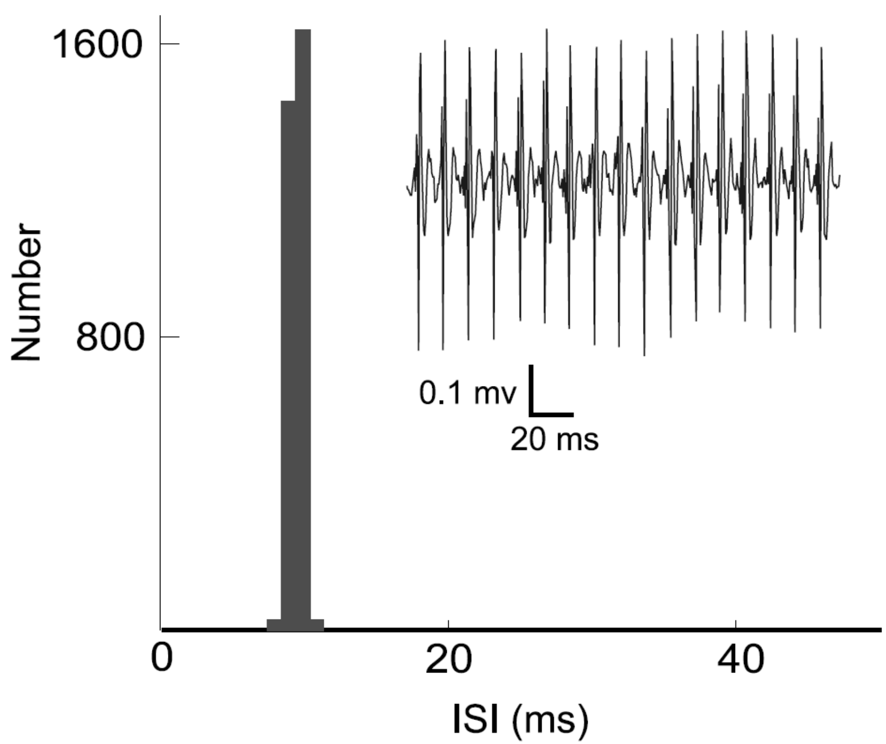


A
0.5 Hz
2 Hz
15 Hz
Head velocity
50 deg/sec
Regular
50 sp/sec
B
Gain
((sp/sec)/(deg/sec))
10
1
0.1
0.1
1
10
100
Frequency (Hz)
Phase (deg)
100
80
60
40
20
0
0.1
1
10
100
Frequency (Hz)

Example: vestibular afferents

Regular afferent

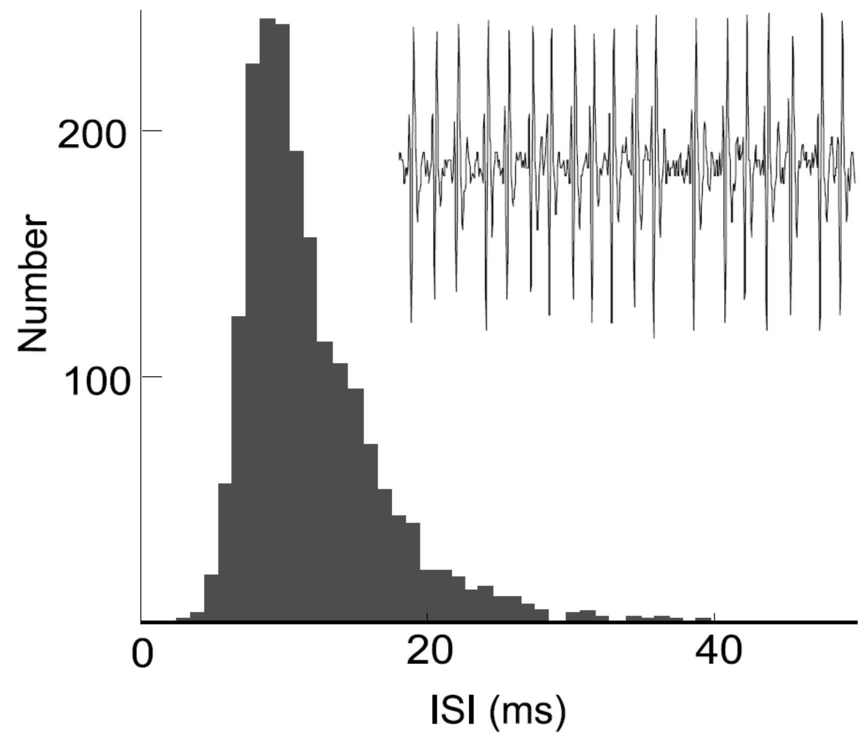
a



CV=0.044

Irregular afferent

c



CV=0.35

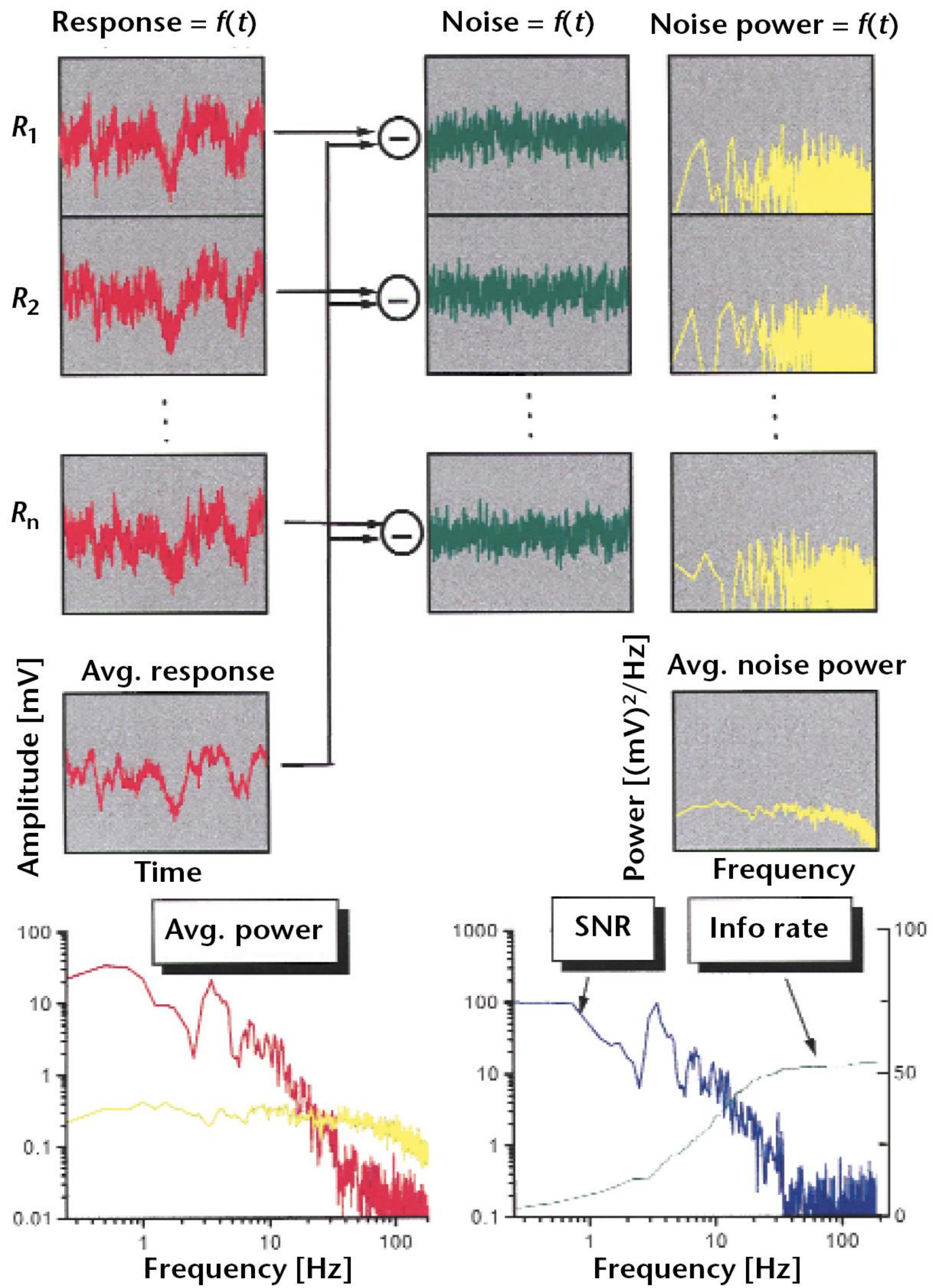
Regular afferent

Irregular afferent

Signal-to-noise Ratio:

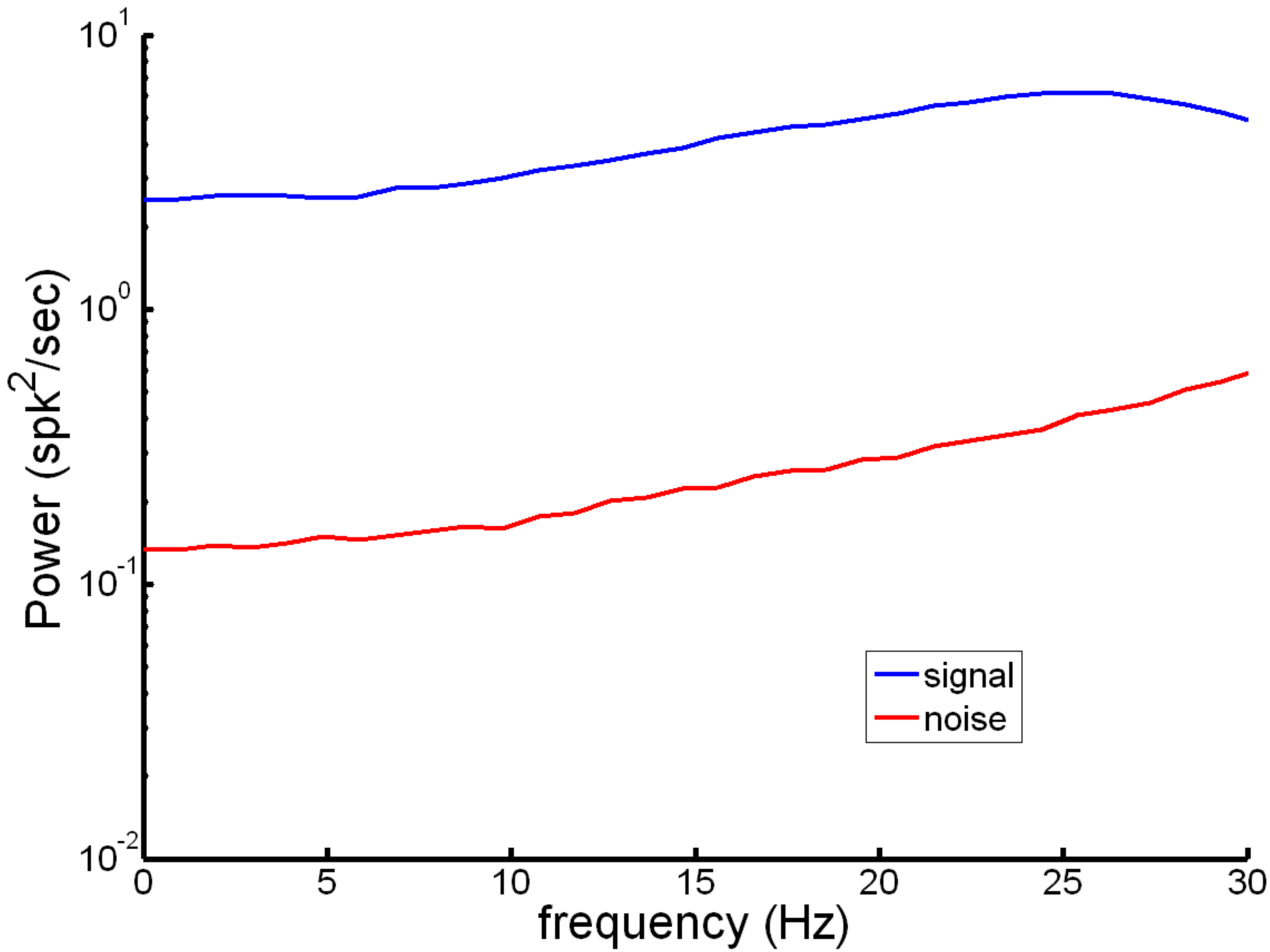
$$SNR(f) = \frac{P_{response}(f)}{P_{noise}(f)}$$

$$noise = response - < response >$$



Borst and Theunissen, 1999

Signal and noise power for the regular afferent model



Signal and noise power for the irregular afferent model