Assignment 6: Dr. Guitton

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Note: All figures are generated by running the *MASTER.m* script. An arbitrary RNG seed of 503 was chosen for the assignment for testing and reproducibility

Part 1) assume force F = constant

A) Solve linear differential equation during saccade

$$F = k\theta + r\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} + \frac{k}{r}\theta = \frac{F}{r}, \text{now in form of a first-order linear ODE form}$$

$$(\frac{d\theta}{dt} + \frac{k}{r}\theta)e^{\frac{kt}{r}} = \frac{F}{r}e^{\frac{kt}{r}}, \text{this is from considering the integration factor } \mathrm{e}^{\frac{\mathrm{kt}}{\mathrm{r}}}$$

$$\theta e^{\frac{kt}{r}} = \frac{F}{\mathrm{k}}e^{\frac{kt}{r}} - e^{\frac{kt}{r}}\frac{d\theta}{dt} \Longrightarrow \theta e^{\frac{kt}{r}} = \frac{F}{\mathrm{k}}e^{\frac{kt}{r}} + \mathrm{c}, \text{expand, multiply all by } \frac{\mathrm{r}}{\mathrm{k}}, \text{set} - e^{\frac{kt}{r}}\frac{d\theta}{dt} \text{ as a "constant" c}$$

$$\theta = \frac{F}{\mathrm{k}} + \frac{\mathrm{c}}{\mathrm{e}^{\frac{\mathrm{kt}}{\mathrm{r}}}}, \text{divide all by } \mathrm{e}^{\frac{\mathrm{kt}}{\mathrm{r}}}$$

Now, using the initial conditions given (t=0, theta=0):

$$0 = \frac{F}{\mathrm{k}} + \frac{\mathrm{c}}{1} \Longrightarrow \mathrm{c} = -\frac{F}{\mathrm{k}} \Longrightarrow \boldsymbol{\theta} = \frac{\boldsymbol{F}}{\boldsymbol{k}} - \frac{\frac{\boldsymbol{F}}{\boldsymbol{k}}}{\boldsymbol{e}^{\frac{\boldsymbol{kt}}{\boldsymbol{r}}}}$$

B) Find F for and during

Values of k and r were from Sylvestre & Cullen: k = 4.2g/° and r = 0.42gs/°.
Rearranging the equation from A), the equation for F is:

$$F = \frac{k\theta e^{-\frac{kt}{r}}}{e^{\frac{kt}{r}} - 1} = \frac{k\theta}{1 - e^{\frac{-kt}{r}}} = \frac{4.2\theta}{1 - e^{-10\mathrm{t}}}$$

Hence, for 10°, 45ms

$$F = \frac{4.2 \cdot 10}{1 - e^{-10 \cdot 0.045}} = 115.90g$$

Hence, for 20°, 68ms

$$F = \frac{4.2 \cdot 20}{1 - e^{-10 \cdot 0.068}} = 170.25g$$

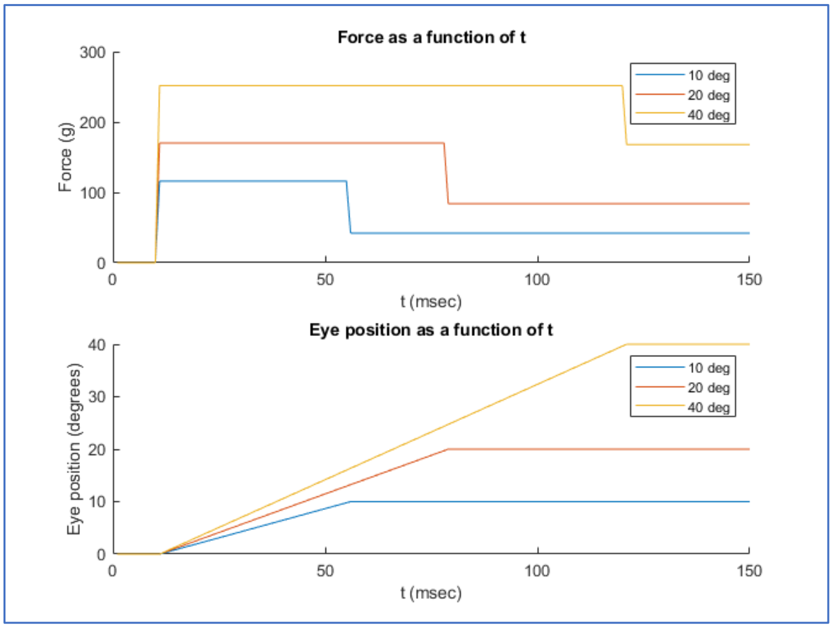
Hence, for 40°, 110ms

$$F = \frac{4.2 \cdot 40}{1 - e^{-10 \cdot 0.110}} = 251.83g$$

C) Solve linear differential equation after saccade

Eye immobile $\Longrightarrow \frac{d\theta}{dt} = 0$. Then the equation $F = k\theta + r\frac{d\theta}{dt} \Longrightarrow F = k\theta \Longrightarrow \boldsymbol{\theta} = \frac{\boldsymbol{F}}{\boldsymbol{k}}$

D) Plot pulse-steps



From the plots, it seems that saccades with increasing amplitudes require pulses (force) that are stronger and last longer to compensate.

Similarly, the plots reveal that saccades with increasing amplitudes require greater magnitude of force after the saccade to immobilize the eye.

Part 2) F ≠ constant, but velocity = constant via fixed BN firing frequency

A) Box2: Constraints A and B relating to k and r

The equation $F = k\theta + r\frac{d\theta}{dt}$ can be rearranged to be: $\theta = \frac{F - r\frac{d\theta}{dt}}{k} = \frac{1}{k}(F - r\frac{d\theta}{dt})$ by factoring out the $\frac{1}{k}$. Similarly, as the Box 2 implements the equation as well, the eye plant can be represented as the following: $\boldsymbol{\theta_{out}} = \boldsymbol{A}(\boldsymbol{F} - \boldsymbol{B}\frac{\boldsymbol{d\theta}}{\boldsymbol{dt}})$. Which "replaces" θ with θ_{out}, $\frac{1}{k}$ with A, and r with B. Therefore, $A = \frac{1}{k}$ and $B = r$ is the relation for the constraints A and B to k and r from equation 2.

B) Equation of Box 1 & what is Box 3 "?"

$$Box\ 1: ff = BG\left(r + \int dx\right)\ and\ F\ = k\theta + r\frac{d\theta}{dt}$$

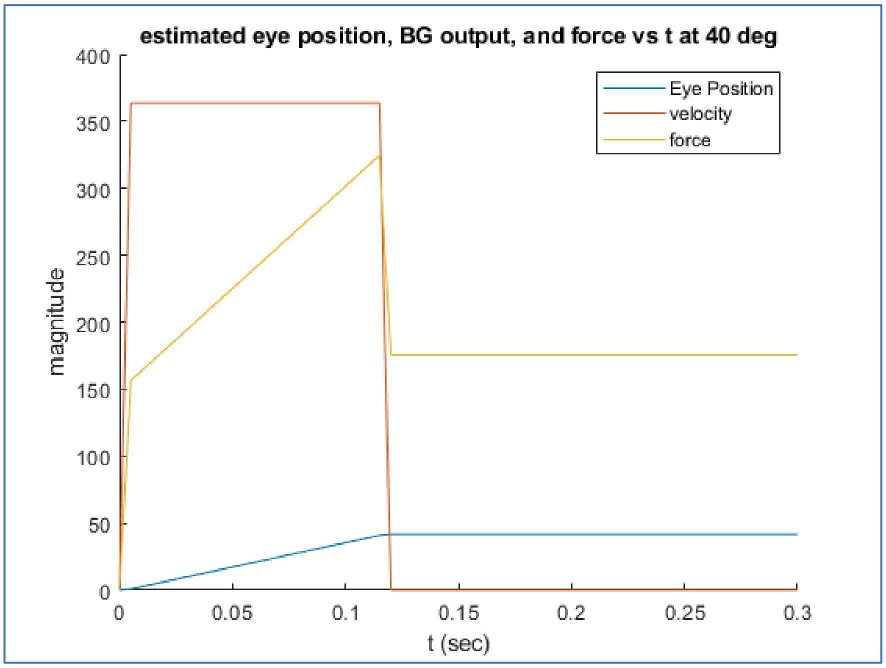
$$Given\ ff = F \Longrightarrow BG\left(r + \int dx\right)\ = k\theta + r\frac{d\theta}{dt}$$

$$\Longrightarrow\ also\ note\ BG \propto \frac{d\theta}{dt}, and\ thus\ after\ simplifying\ \Longrightarrow\ BG\int dt = \theta$$

$$\Longrightarrow Box\ 3\ "?" = \int dt$$

The work above indicates that Box 3 allows for the integration of BG over time to determine the eye position in degrees.

C) Pulse-step for 40 deg saccade



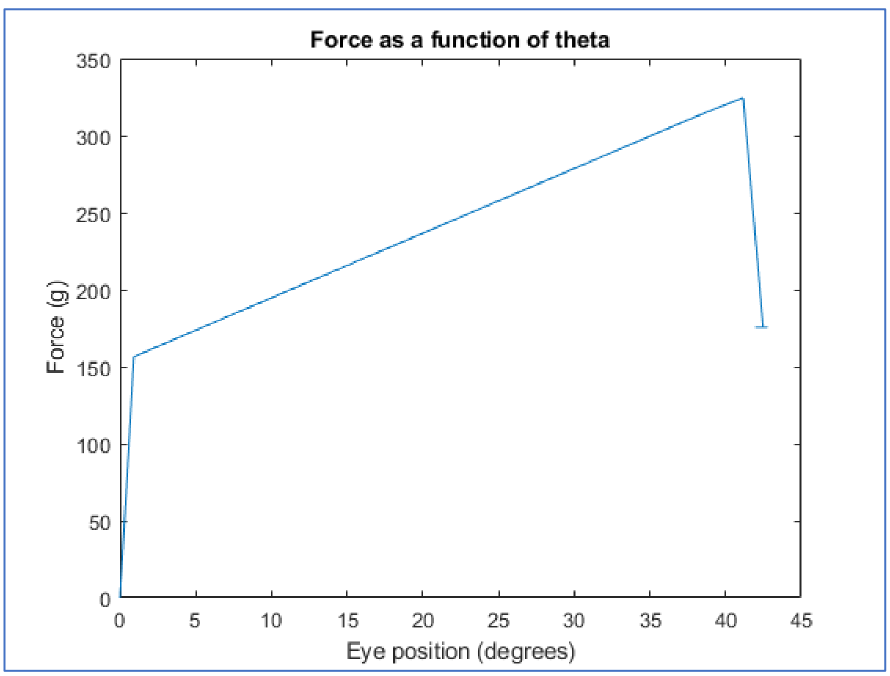
As expected, eye position increases until it reaches 40 degrees.

Velocity is non zero during eye movement, but as soon as the eye position approaches fixed at 40 deg, it decreases greatly to 0 (when eye stops moving)

The force increasing then later maintaining was already discussed in 1D).

D) Solve plant equation 2

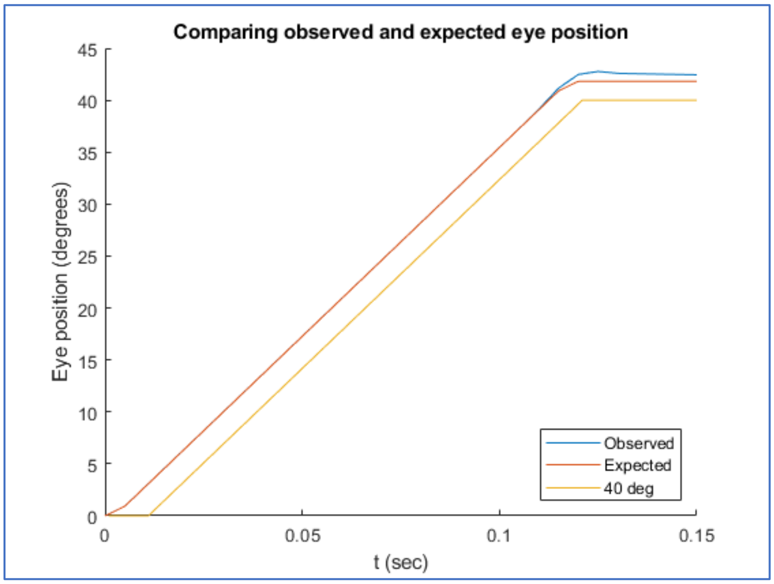
$Given\ that\ \frac{d\theta}{dt} + \frac{k}{r}\theta = \frac{F}{r} \Longrightarrow \frac{d\theta}{dt} = \frac{F - k\theta}{r}.$



The function headers for the derived equation were generated, and F values given t in the timespan range were obtained by interpolation.

The function ode45 was used as requested

E) Plotting Eye position vs time



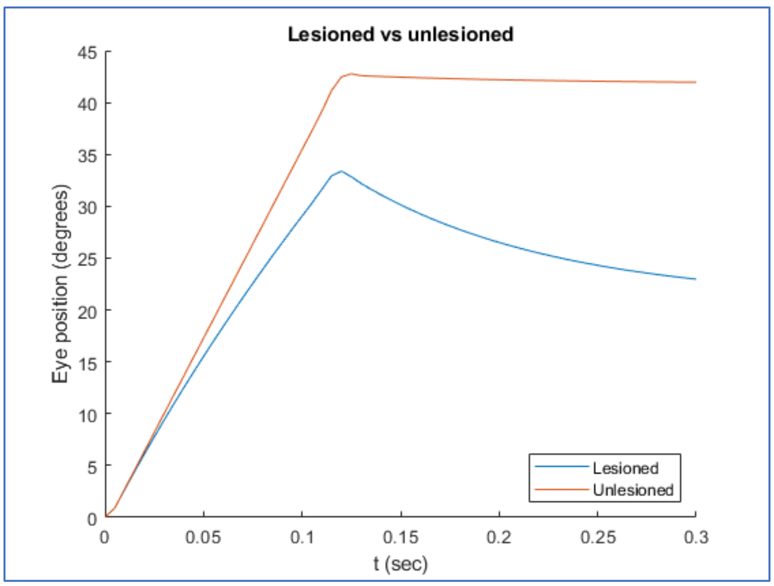
The expected is the output from the BG generator. The observed is the "real" eye position over the timescale.

The similarity of the two observed and expected indicate that Box 1 and 2 are very alike, as it seems to produce almost identical function until ~ 0.1 second mark.

This implies ff from BG ~= force on the eye

Do not much to comment for 40 degree. It is the same from 1d).

F) Resultant eye trajectory from deficient integrator output



Comparing result from lesion and not lesioned.

These are the eye positions for a 40 deg saccade in a normal (not lesioned) and deficient output (lesioned).

The lesioned the failure to reach 40 degrees imply that the integrator fail to consistently produce a firing frequency to reach 40 degrees.