Circle Geometry Lesson

SLSS Math Club 2019

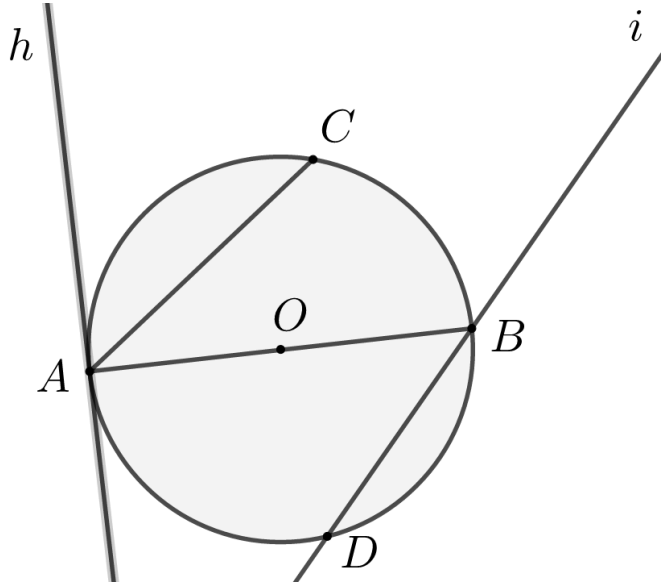
April 17, 2019

Introduction

- The geometry of the circle represents an area of mathematics from which relatively difficult and interesting problems for the Euclid contest are frequently taken.
- You are expected to know some knowledge of the elementary properties of triangles, congruence and similarity, and properties of quadrilaterals.

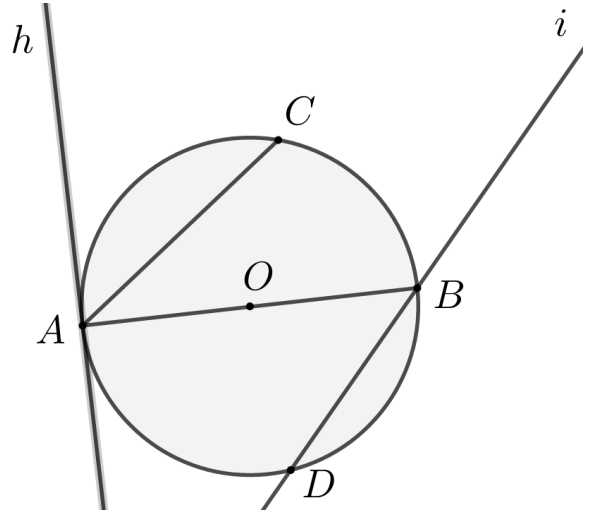
Terminology

- **Radius(OA)** - the segment between **Center(point O)** to each point on the circle.
- **Chord(AC)** - a segment whose endpoints are both on the circle.
- **Diameter(AB)** - a chord which passes through the center of a circle. It is also the longest chord in a circle.



Terminology

- **Tangent(Line h)** - a line which touches the circle in only one place.
- **Secant(Line i)** - a line which passes through the circle, intersecting it in two places.
- **Arc($\overset{\frown}{BD}$)** - the part of the curve of a circle. Since $\overset{\frown}{BD}$ can refer to two different arcs, the long and the short way around the circle, often three points are used to designate the arc, as in $\overset{\frown}{BAD}$. If only two points are used to designate an arc, it is assumed to mean the shorter one.



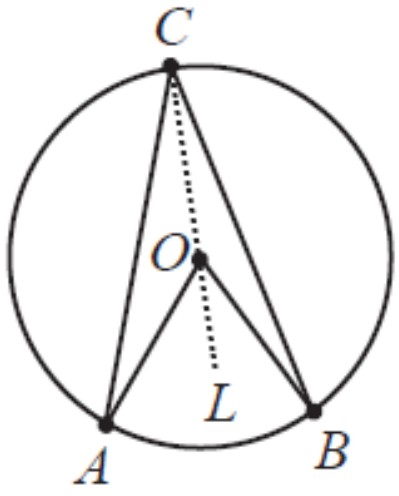
Basic Formulas and Rules

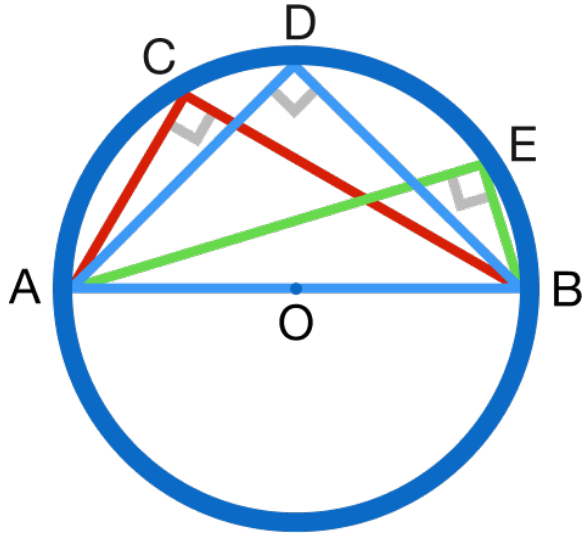
- If d is the diameter of the circle, and r is the radius of the circle, $d = 2r$.
- The area of a circle, denoted by A, is $A = \pi r^2$.
- The formula for the circumference of a circle is $\pi d = 2\pi r$.
- The formula for the arc length of a sector, denoted by "S", is calculated by $S = r\theta$, where θ is the angle between the sides of the arc, in radians. Meaning the arc length of a circle is $(\pi \cdot r)$ which it is a circle's circumference.
- The formula to convert between degrees to radian angles is denoted by $\theta^\circ \cdot \frac{\pi}{180^\circ} = \theta\, rad$.

The "Star Trek" Theorem

The central angle subtended by any arc is twice any of the inscribed angles on that arc. ($\angle AOB = 2\angle ACB$)

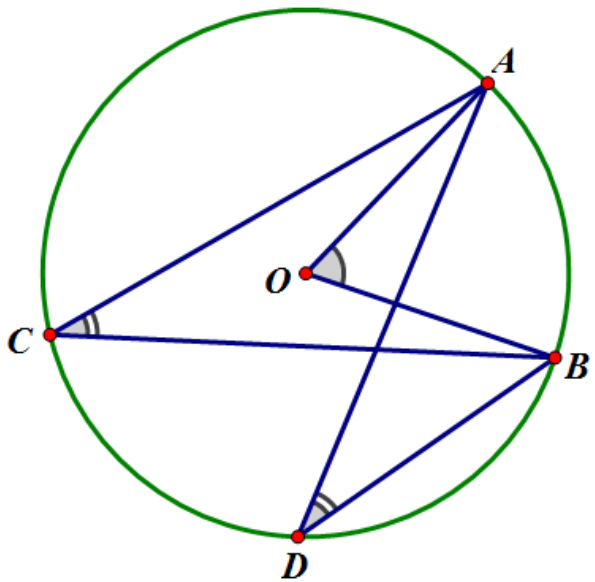
Thales' Theorem: If a triangle is inscribed inside a circle, where one side of the triangle is the diameter of the circle, then the angle opposite to that side is a right angle.





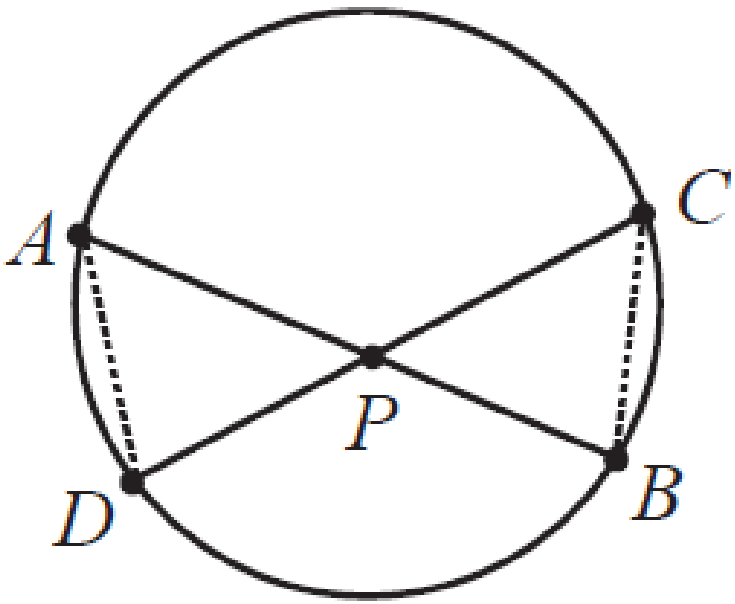
Inscribed Angles

The angles in the same segment from a common chord are equal.
$\angle ACB = \angle ADB = \frac{1}{2}\angle AOB$



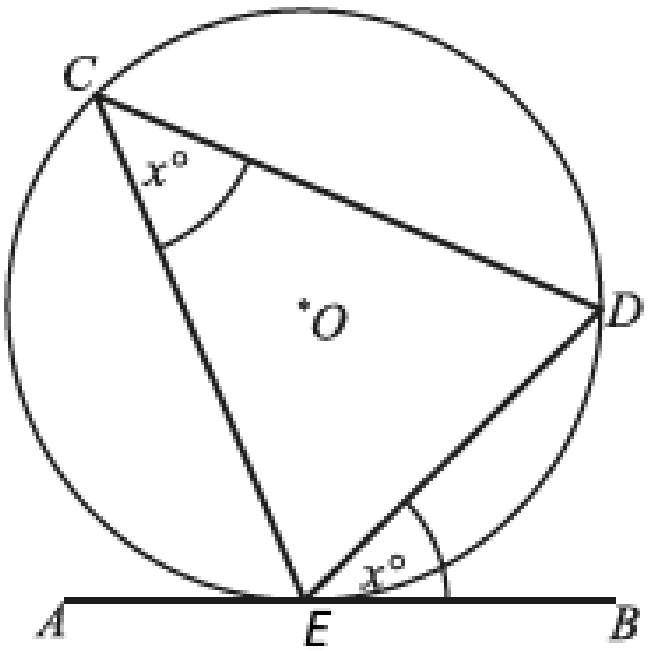
Intersecting Chords

If two chords inside a circle intersect, the products of the intercepts of two intersecting chords are equal.
As shown in the figure below, $(PA)(PB) = (PC)(PD)$



Alternate Segment Theorem

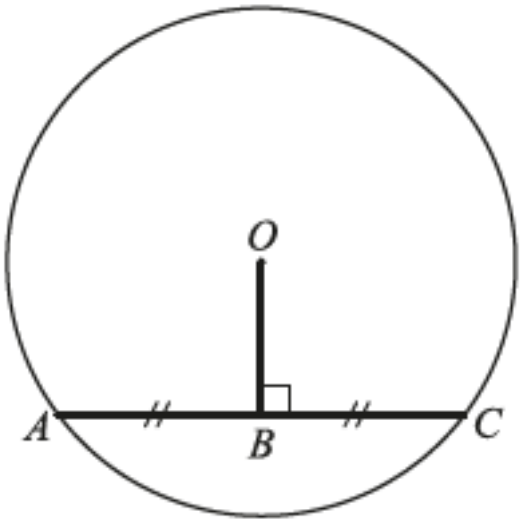
The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.
$\angle DCE = \angle DEB$



NoName Theorem (Perpendicular, Midpoint and Bisect Chords)

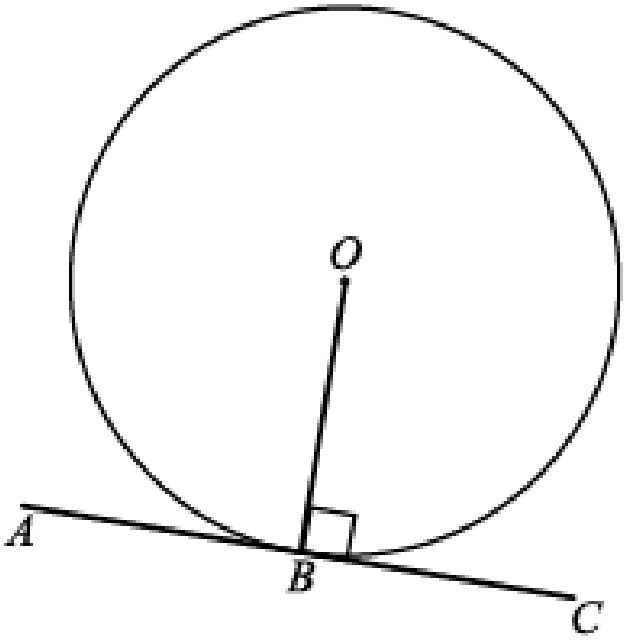
If one of the following statements is true, the other two statements are true as well:

1. The perpendicular from the center of a circle to a chord bisects the chord.
2. The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.
3. The perpendicular bisector of a chord passes through the center of the circle.



Right Angle Tangent

The tangent to a circle is perpendicular to the radius drawn to the point of contact. $\angle OBC = 90^\circ$



Equal Tangents

Tangents to a circle from an external point are equal.
$AC = AB$