3.2 2022310이성준

11

$f(x) = \frac{1}{(x-3)^2}$

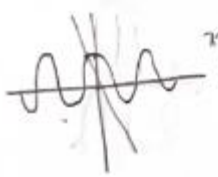
$f(4)=1$ $f(1)=\frac{1}{4}$

$f'(x) = -2(x-3)^{-3}$

$f'(c)$ 에서 $c=3$ 일때 값이 존재하지 않기에 성립하지 않는다.

12.

$\cos x = -2x$ 를 만족하는 x의 값이 실근



제 2사분면 에서 하나의 실근 만이 존재한다

17

에서 $\sin x$ 와 x는 $x=0$ 일때 만난다 0은 구간 포함 x이기에 $\sin x < x$이다.

18.

양변 $|a-b|$ 나누기

$\left|\frac{\sin a - \sin b}{a-b}\right| \le 1$ $\sin x = f(x)$ 라고

$\left|\frac{f(a)-f(b)}{a-b}\right| \le 1$ $|\cos x| \le 1$ 이기때문

$|f'(a)| \le 1$

3.3

3.

(a) 0~1, 3~5 증가

1~3, 5~6 감소

(b)

1, 3, 5

17.

$f'(x) = \frac{2x(x-5) - (x^2-24)}{(x-5)^2}$

$2x^2 - 10x - x^2 + 24$ $x^2 - 10x + 24$

$x=4$, $x=6$

극소: 8 극대: 12

9.

$2\sin\cos + 2\sin 2x$

$2\cos^2 x - 2\sin^2 x + 4\cos 2x = 0$

$\Leftrightarrow$ $6\cos 2x = 0$ $x = \frac{\pi}{4}$ $x = \frac{3}{4}\pi$

11

$\cos x - \sin x$

$(2n\pi - \frac{\pi}{4} \sim 2n\pi + \frac{3\pi}{4})$ 증가 극대: $\sqrt{2}$

$(2n\pi + \frac{3}{4}\pi \sim 2n\pi + \frac{7}{4}\pi)$ 감소 극소: $-\sqrt{2}$

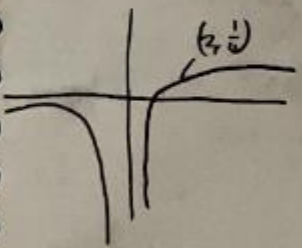
변곡점: $-\frac{\pi}{4} + n\pi$

3.5

9

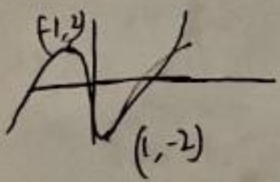
$$\frac{x^2-(x+1)2x}{x^4} = \frac{-x^2+2x}{x^4}$$

x=0, x=2 일때 극대, 극소



15

$1 - x^{-\frac{2}{3}}$ 1, -1에서 극대 극소



25

$y = \frac{x^2+1}{x+1} = x-1+\frac{2}{x+1}$

$y = x-1$

3.7

$x^2y = 100$ $x = y$

$y + \frac{dy}{dx}x = 0$ (10,10)

3.

$2x = 1$ $x = \frac{1}{2}$

$\left(\frac{1}{2}, \frac{5}{2}\right)$ $\left(\frac{1}{2}, \frac{1}{2}\right)$

$\left(\frac{9}{4}\right)^2$ $\frac{81}{16}$

4

$a+b=50$ (25, 25)

ab

10

$1200/5 = 240$

$240\sqrt{240} = 960\sqrt{15}$

11

$2x^2h = 10$ $20x^2 + 3\frac{(2h)}{4}$

$6xh$

$2h = \frac{5}{x}$

$20x \frac{3^{\frac{2}{3}}}{2^{\frac{2}{3}}} + 180 \times \frac{3\sqrt{2}}{3^{\frac{2}{3}}}$

$20x^2 + \frac{180}{x}$

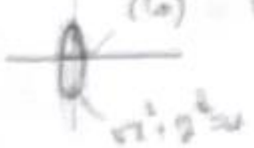
3. 7

13

$\frac{2x+3}{7} = -\frac{1}{2}$ $x = -\frac{6}{5}$

$-\frac{6}{5}, \frac{3}{5}$

14



$(1,0) \Rightarrow (0,4)$

(8)

20

$S = 2\pi x^2 + 2\pi x (2y)$

$= 2\pi x^2 + 2\pi x \times 2y$

$x^2 + y^2 = r^2$

$2\pi(x^2 + x\,y)'$

$2\pi\left(x^2 + x \times \sqrt{r^2 - x^2}\right)$

$2\pi + \sqrt{r^2 - x^2} + x \cdots$

$2\pi\left(\frac{x(2r^2 - 3x^2)}{\sqrt{r^2 - x^2}}\right)$

4. 3

7.

$-\left(\sqrt{1+\sin x} - \sqrt{1+\sin x} \times 0\right)$

$= -\sqrt{1+\sin x}$

9 $F(x-2) - F(1)$

$3 \times \frac{3x+2}{1+(x+2)^2} = \frac{\cdots}{1+(\cdots)^2}$

12

(6)

31

$f(x) = 2x f(x^2) + \frac{1}{2\sqrt{x}} f(\sqrt{x})$

$= f(x) \int \cos t^2 \cdots$ $= 2x \cos x \times 2 \frac{\cos x}{2\sqrt{x}}$

34

$\int_1^4 f'(x)\,dx = f(4) - f(1) = 17$

$f(4) = 29$

(29)

4.4

9.

$\int (2 + \tan^2\theta)\,d\theta = \int (1+\sec^2\theta)\,d\theta$

$= \theta + \tan(\theta) + C$

13

$\left[2t^4 + 6t^{-1}\right]_1^4$

$2\times 256 + \frac{6}{4} - 2 - 6$

$504 + \frac{3}{2}$

17

$4(\tan\theta)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= 4\times\frac{2\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$

23

$\int_0^2 (x^2 - 2x)\,dx - \int_4^0 (x+2x)$

$\left[-\frac{1}{2}x^2\right]_0^2 + \left[\frac{3}{2}x^2\right]_1^0$

$-2 + \frac{3}{2} = -\frac{1}{2}$

$-\frac{1}{2}$

4.5

2.

$\frac{1}{3}du = 3x^2 \frac{1}{3}dx$ $\quad x^2 dx = \frac{1}{3}du$

$\int \frac{1}{3}\sqrt{u}\,du$

$= \frac{1}{3}\times\frac{2}{3}u\sqrt{u} + C$

$= \frac{2}{9}(x^3+1)\sqrt{x^3+1} + C$

4

$du = \frac{1}{2}\times\frac{1}{2}\times t^{-\frac{1}{2}}\,dt$

$2\times t^{\frac{1}{2}}\times du = dt$

$\int \frac{\cos(u)}{\frac{1}{u}}\,du = \sin u$

5.

$-\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$

14

$\int \sqrt{\cot x}\,\csc^2 x\,dx$

$-\csc^2 x\,dx = 2t\,dt$

$= \int t\times(-2t\,dt) = -\frac{2}{3}\cot x\sqrt{\cot x} + C$

15

$\int \sec^3 x \tan x\,dx$ $\quad t = \sec x$

$= \int t^2\,dt$

$\frac{1}{3}\sec^3 x + C$

20

$\int_0^1 \sqrt[3]{1+7x} = \int^1 (1+7x)^{\frac{1}{3}}dx$

$= \frac{3}{4}(1+7x)^{\frac{4}{3}} \times \frac{1}{7} \Big]_0^1$

$= \frac{3}{28}(8^{\frac{4}{3}} - 1)$

$= \frac{3}{28} \times 15 = \frac{45}{28}$

22

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^4 \tan x\,dx) = 0$

$f(x) = x^3 + x^4 \tan x$

$f(-x) = -x^3 - x^4 \tan x$

$f(-x) = -f(x)$ 이므로

$\Delta = 0$

24.

$\left[\cancel{\frac{1}{2}} \times \frac{2}{3} \times (x^2+a^2)^{\frac{3}{2}}\right]_0^a$

$\frac{(2a^2)^{\frac{3}{2}}}{3} - \frac{a^3}{3}$

$\frac{(2\sqrt{2}-1)a^3}{3}$

26

$f(x) = \sin x \quad g(x) = x^{-2}$

$f'(x) = \cos x$

$\cos(x^{-2}) = f'(g(x))$

$x^{-3} \times \cos(x^{-2}) = -\frac{1}{2}g'(x) \times f'(g(x))$

$= -\frac{1}{2}(f(g(x)))'$

$= \left[-\frac{1}{2}\sin(x^{-2})\right]_{\frac{1}{2}}^{1}$

$= \frac{1}{2}(\sin 4 - \sin 1)$

33

$\int_0^{\frac{\pi}{2}} f(\cos x)\,dx = \int_0^{\frac{\pi}{2}} f(\sin x)\,dx$

$\cos(\frac{\pi}{2} - x) = \sin x$ 같은 면적을 가지기에 성립한다

$\int_0^{\frac{\pi}{2}} f(\cos(\frac{\pi}{2} - t)) =$

5.1

11

$y = \pm 2$

$(8, 2)$ ... 2

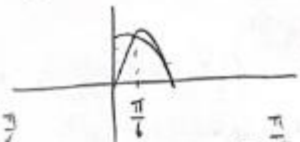
$(8, -2)$

$\left[\frac{2}{3}y^3\right]_0^8 - \left[\frac{2}{3}y^3 + 4y\right]_4^8$

$\frac{2}{3}8^3 - \frac{2}{3}8^3 - 32y + \frac{2}{3}\times 4^3 + 16$

$\frac{128}{3} - 16$ $\quad \frac{80}{3}$

14



$\int_0^{\frac{\pi}{6}} (\cos x - \sin 2x) + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x)\,dx$ $\quad \frac{1}{2}$

$\left[\sin x + \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$

$\frac{1}{2} + \frac{1}{4} - \frac{1}{2} + \left[-\frac{1}{2}\cos 2x - \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$\frac{1}{2} - 1 + \frac{1}{4} + \frac{1}{2}$

15

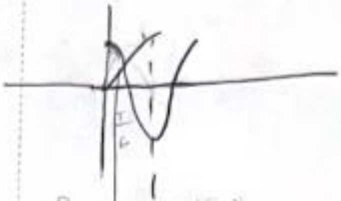
$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} [8\cos x - \sec^2 x]\,dx$

$= [8\sin x + \tan x]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$

$4\sqrt{2} + \sqrt{3} + 4\sqrt{2} + \sqrt{3}$

$8\sqrt{2} + 2\sqrt{3}$

21



$\frac{\pi}{6}$

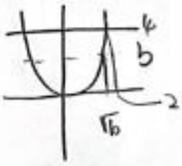
$\int_0^{\frac{\pi}{6}} \cos 2x - \sin x \; dx$

$+ \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x - \cos 2x)\,dx$

$= \left[\frac{1}{2}\sin 2x + \cos x\right]_0^{\frac{\pi}{6}} + \left[-\cos x - \frac{1}{2}\sin 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} - 1 + \left(0 + \frac{\sqrt{3}}{2} + 1\right)$ $\quad \frac{\sqrt{3}}{4}$

32



$\int_0^{\sqrt{b}} x^2\,dx = \int_0^{?}$

$2 \times \frac{1}{3}x^3\Big]_0^{\sqrt{b}}$ $\quad \frac{4}{3}8$ $\quad 4$ $\quad 2^{\frac{4}{3}}$

$b^{\frac{3}{2}} = 2^2$ $\quad b =$

33

576 $\quad 4\times \int_0^c [x^2 - c^2]\,dx$

$= 4\times\left[\frac{1}{3}x^3 - c^2 x\right]_0^c$ $\quad \frac{8}{3}c^3 = 576$

$\frac{1}{3}c^3 - c^3$

$c^3 = 72\times 3 = 3^3 \times 2^3$ $\quad \pm 6$

$= \pm 6$

5.2

1

$$\int_0^3 (x^2+5)^2 \pi \, dx$$

$$\pi \int_0^3 (x^4+10x^2+25)\,dx$$

$$= \pi \left[\frac{1}{5}x^5 + \frac{10}{3}x^3 + 25x\right]_0^3$$

$$= \pi\left(\frac{3^5}{5} + 90 + 75\right)$$

$$= \left(165 + \frac{243}{5}\right)\pi$$

2.

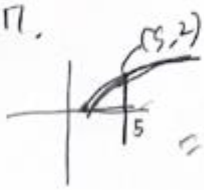
$$\pi \int_0^2 (x^3+1)^2$$

$$= \pi\left[\frac{1}{7}x^7 + \frac{1}{2}x^4 + x\right]_0^2$$

$$\frac{128}{7} + 8 + 2$$

$$= \frac{198}{7}\pi$$

17.



$$\pi \int_1^5 (x-1)\,dx$$

$$= \pi\left[\frac{1}{2}x^2 - x\right]_1^5$$

$$= \pi\left(\frac{25}{2} - 5 - \frac{1}{2} + 1\right) \quad 8\pi$$

11

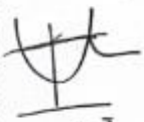
$$2\pi \int_0^1 (1-y)(\sqrt{y}-y^2)\,dy$$

$$= 2\pi\left[\frac{2}{3}y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}} - \frac{y^4}{3+4}\right]_0^1$$

$$= 2\pi \times \frac{11}{60} = \frac{11}{30}\pi$$

$$-\frac{1}{15}$$

12



$$(4-\sec^2 x)$$

$$2\pi \int_0^{\frac{\pi}{3}} (4-\sec^2 x)\,dx$$

$$= 2\pi \times (4x - \tan x)\Big]_0^{\frac{\pi}{3}} = 2\pi\left(\frac{4}{3}\pi - \sqrt{3}\right)$$

14

$$2\pi \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} 1-2y^2 \, dy$$

$$= 2\pi\left[y - \frac{2}{3}y^3\right]_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} = 2\pi\left(\sqrt{2} - \frac{\sqrt{2}}{3}\right)$$

$$= \frac{4\sqrt{2}}{3}\pi$$

31

$$\pi(r^2-y^2)$$

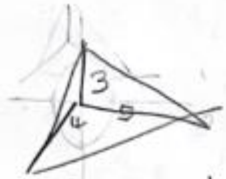
$$\int_0^{r-h} \pi(r^2-y^2)\,dy$$

$$= \pi\left(r^2 y - \frac{1}{3}y^3\right)_0^{r-h}$$

$$= \pi\left(\frac{2}{3}r^3 - rh^2 + \frac{1}{3}h^3\right)$$

$$= \pi\left(rh^2 - \frac{h^3}{3}\right)$$

34



$3 \times 4 \times 5 \times \frac{1}{3} \times \frac{1}{2} = 10$

35

$y = -x + 1$

$\int_0^1 (x-1)^2 dx$

$x^2 - 2x + 1$

$[\frac{1}{3}x^3 - x^2 + x]_0^1$ $\frac{1}{3}$

36

$\frac{1}{2}\int_{-1}^{1} (1-x^2)^2$

$= \int_0^1 (1-x^2)^2 dx$

$= [x^4 - 2x^2 + 1]_0^1$

$= \frac{1}{5}x^5 - \frac{2}{3}x^3 + x$

$= \frac{6}{5} - \frac{2}{3} = \frac{18}{15} - \frac{10}{15}$

$= \frac{8}{15}$

5.3

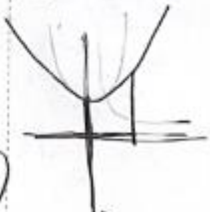
2

$\pi \int_0^{\sqrt{\frac{\pi}{2}}} x \cos x^2 dx$

$[\frac{\sin x^2}{2}]_0^{\sqrt{\frac{\pi}{2}}}$

$\frac{1}{2}\pi$

6



$\int_0^2 (x\sqrt{5+x^2}) dx = [\frac{x}{3} \times \frac{1}{x}(5+x^2)^{\frac{3}{2}}]_0^2$

9π

7.

$\int_1^4 1 \, dx$ 6π

9.

$\pi \int_0^4 x^3 dx$ $\frac{1}{4}x^4$

64π

10

$$\int_1^3 \left[1+(y-2)^2\right]dy$$

y^2-4y+5

$$\left[\frac{1}{3}y^3-2y^2+5y\right]_1^3$$

$= 9-18+15-\frac{1}{3}+2-5$

$= \frac{8}{3}$ $\quad \frac{8\pi}{3}$

12

?....

14

$$\int_1^3 2\pi(x-1)(4x-x^2-3)$$

$= 2\pi\int_1^3 -(x+1)(x-1)(x-3)$

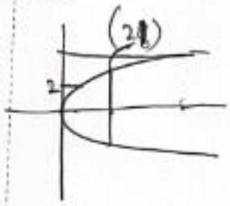
$= -2\pi\left[\frac{x^4}{4}-\frac{5}{3}x^3+\frac{7}{2}x^2-3x\right]_1^3$

$= -2\pi\left[20-51+28+\frac{5}{3}-\frac{1}{4}\right]$

$= +2\pi\left(\frac{4}{3}+\frac{1}{4}\right)$

$= 2\pi\times\frac{19}{12}=\frac{19}{6}\pi$

15



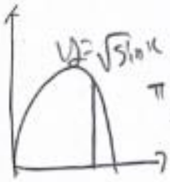
$$\int_0^2 2\pi(3-x)\times(8-x^3)dx$$

$= 2\pi\int_0^2 24-3x^3-8x+x^4\,dx$

$= 2\pi\left[24x-\frac{3}{4}x^4-4x^2+\frac{1}{5}x^5\right]_0^2$

$= \frac{264}{5}\pi$

25



$\pi\int_0^\pi \sin x$

$\pi\left[-\cos x\right]_0^\pi$

2π

32

$2\pi\int_0^b y^2\,dy = \frac{2}{3}b^3\pi$

5.5

2

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3\cos x\,dx$

$= [3\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 6$

$\frac{6}{\pi}$

3.

$\cancel{\frac{1}{3}}\left[\frac{1}{15}(1+t^3)^5\right]_0^2$

$= \frac{9^5}{15}\times\frac{1}{2} = \frac{3\times 9^4}{10}$

4.

$\left[-\frac{1}{5}\cos^5 x\right]_0^\pi$

$\frac{\frac{1}{5}+\frac{1}{5}}{\pi} = \frac{1}{10\pi}$

5.

$\cancel{\frac{1}{2}}\left[-t^{-1}\right]_1^3$

$= \frac{-\frac{1}{3}+1}{2} = \frac{1}{3}$

$\frac{1}{3} = \quad c = \sqrt{3}$