2.4 31. $\dfrac{d^{99}}{dx^{99}}(\sin x)$

sol) $\dfrac{d}{dx}(\sin x) = \cos x$

$\dfrac{d^2}{dx^2}(\sin x) = -\sin x$

$\dfrac{d^3}{dx^3}(\sin x) = -\cos x$

$\dfrac{d^4}{dx^4}(\sin x) = \sin x$

$\therefore\ 99 = 4 \cdot 24 + 3$

$\therefore\ \dfrac{d^{99}}{dx^{99}}(\sin x) = \dfrac{d^3}{dx^3}(\sin x) = -\cos x.$

2.5. 38. $D^{103}\cos 2x = (\cos 2x)^{(103)}$

Let $f(x) = \cos x \Rightarrow y = f(2x) = \cos 2x$

then $y' = 2f'(2x)$

$y'' = 4f''(2x)$

$y^{(3)} = 8f^{(3)}(2x)$

$\vdots$

$y^{(n)} = 2^n f^{(n)}(2x)$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f^{(3)}(x) = \sin x$

$103 = 4 \cdot 25 + 3$

$\therefore\ f^{(103)}(x) = f^{(3)}(x) = \sin x \Rightarrow f^{(103)}(2x) = \sin 2x$

$\therefore\ (\cos 2x)^{(103)} = y^{(103)} = 2^{103} f^{(103)}(2x)$

$= 2^{103}\sin 2x$

2.5.42(a) 우함수의 도함수는 기함수이다.

M1) $f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h}$

$= \lim_{h \to 0} \frac{f(-(x-h)) - f(-x)}{h}$ by 가정 $f(-x) = f(x)$

$= \lim_{h \to 0} \frac{f(x-h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{f(x-h) - f(x)}{-h} (-1)$

$= -f'(x)$

M2) Let $f(-x) = f(x)$

$f'(x) = f'(-x)(-1) = -f'(-x)$ (by chain rule)

$f'(-x) = -f'(x)$.

$\therefore f'$: 기함수.

2.6.3. $\sqrt{x}+\sqrt{y}=1$

(a) $\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}y'=0$. $\therefore y'=-\frac{\sqrt{y}}{\sqrt{x}}=-\frac{1-\sqrt{x}}{\sqrt{x}}=1-\frac{1}{\sqrt{x}}$

(b) $\sqrt{y}=1-\sqrt{x}$ $\quad y=(1-\sqrt{x})^2$

$\Rightarrow y'=2(1-\sqrt{x})(-1)\cdot\frac{1}{2\sqrt{x}}$ $\Rightarrow y'=1-\frac{1}{\sqrt{x}}$

2.6.8. $\tan\left(\frac{x}{y}\right)=x+y$

$\Rightarrow \sec^2\left(\frac{x}{y}\right)\cdot\frac{y-xy'}{y^2}=1+y'$

$\Rightarrow y\sec^2\left(\frac{x}{y}\right)-x\sec^2\left(\frac{x}{y}\right)y'=y^2+y^2y'$

$\therefore y'=\dfrac{y\sec^2\left(\frac{x}{y}\right)-y^2}{y^2+x\sec^2\left(\frac{x}{y}\right)}$

2.6.12 $x^4y^2-x^3y+2xy^3=0$

($x=f(y)$로 생각하고 y에 대해 미분)

$\Rightarrow 4x^3\cdot\frac{dx}{dy}y^2+2x^4y-\left(3x^2\cdot\frac{dx}{dy}y+x^3\right)+2\left(\frac{dx}{dy}y^3+3xy^2\right)=0$

$\Rightarrow (4x^3y^2-3x^2y+2y^3)\frac{dx}{dy}=-2x^4y+x^3-6xy^2$

$\therefore \dfrac{dx}{dy}=\dfrac{-2x^4y+x^3-6xy^2}{4x^3y^2-3x^2y+2y^3}$

2.6.14. $2(x^2+y^2)^2 = 25(x^2-y^2)$, $(3,1)$

(1) $(3,1)$

(2) $4(x^2+y^2)(2x+2yy') = 25(2x-2yy') \Big|_{(3,1)}$

$\Rightarrow 40(6+2y') = 25(6-2y')$

$\therefore y' = -\frac{9}{13}$

$\therefore y-1 = -\frac{9}{13}(x-3)$ or $y = -\frac{9}{13}x + \frac{40}{13}$

2.6.16 $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ $(-3\sqrt{3}, 1)$

(1) $(-3\sqrt{3}, 1)$

(2) $\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0 \Big|_{(-3\sqrt{3}, 1)}$

$\Rightarrow y' = \frac{1}{\sqrt{3}}$

$\therefore y-1 = \frac{1}{\sqrt{3}}(x+3\sqrt{3})$ or $y = \frac{1}{\sqrt{3}}x + 4$.

2.6.21. $xy + y^3 = 1$, $x=0$ / $y'' = ?$

1) $x=0$ $y^3=1 \Rightarrow y=1$.

2) $y + xy' + 3y^2y' = 0 \Big|_{(0,1)} \Rightarrow y' = -\frac{1}{3}$

3) $y' + y' + xy'' + 6yy'y' + 3y^2y'' = 0$.

$-\frac{2}{3} + \frac{2}{3} + 3y'' = 0$ $y'' = 0$.

2.6.23 $2(x^2+y^2)^2 = 25(x^2-y^2)$ — ①

$$y' = \frac{25x - 4x(x^2+y^2)}{25y + 4y(x^2+y^2)}$$

(수평접선 $y'=0$)

$\Rightarrow$ $25x - 4x(x^2+y^2) = 0$

$x(25 - 4(x^2+y^2)) = 0$.

i) $x=0$ $y=0$ ($\neq$ 수평접선)

ii) $x^2+y^2 = \frac{25}{4}$ $\Rightarrow$ ① 대입 $\Rightarrow$ $x^2-y^2 = \frac{25}{8}$.

$\therefore$ $x = \pm\frac{5\sqrt{3}}{4}$. $y = \pm\frac{5}{4}$

$\therefore$ $\left(\frac{5\sqrt{3}}{4}, \frac{5}{4}\right)$ $\left(-\frac{5\sqrt{3}}{4}, -\frac{5}{4}\right)$.

2.6.31 $x^2y^2 + xy = 2$ — ① $y' = -1$

1) $2xy^2 + 2x^2yy' + y + xy' = 0$

$$\Rightarrow y' = \frac{-(2xy^2+y)}{2x^2y+x}$$

2) $$\frac{-(2xy^2+y)}{2x^2y+x} = -1$$

$\Rightarrow$ $2xy^2+y = 2x^2y + x$

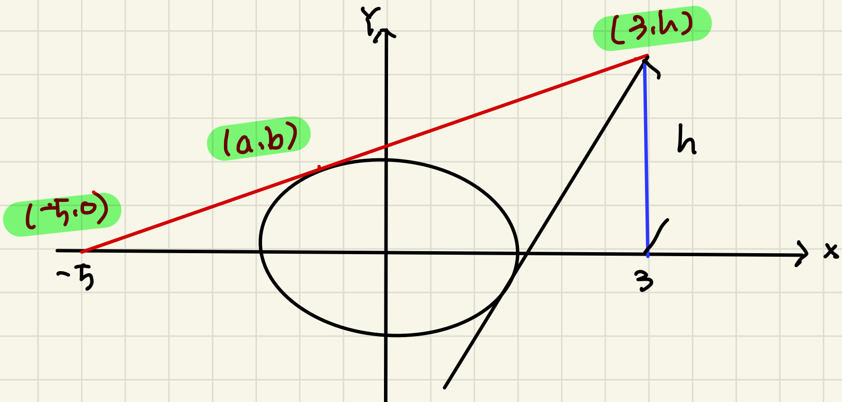
$\Rightarrow$ $(2xy+1)y - (2xy+1)x = 0$

$\Rightarrow$ $(2xy+1)(y-x) = 0$

i) $xy = -\frac{1}{2}$ — ①에 대입 $\frac{1}{4} - \frac{1}{2} \neq 2$ (X)

ii) $y = x$ — ① 에 대입 $x = \pm 1$ $y = \pm 1$ $\therefore$ $(1,1)$ $(-1,-1)$

2.6.33 $\quad x^2+4y^2=5 \cdots ①$



세점

(x_1, y_1) (x_2, y_2) (x_3, y_3)이 한 직선 위에 있는 경우.

$$\frac{y_2-y_1}{x_2-x_1} = \frac{y_3-y_1}{x_3-x_1}$$

sol) 1) $2x+8yy'=0$ $\quad y'=-\frac{x}{4y}\Big|_{(a,b)} = -\frac{a}{4b}$

2) $(-5,0),(a,b) \Rightarrow \frac{b}{a+5}$

3) $(-5,0),(3,h) \Rightarrow \frac{h}{8}$

$\therefore$ 1) = 2) = 3) (∵ 한 직선 위에 있음)

1)=2) $\Rightarrow -\frac{a}{4b} = \frac{b}{a+5} \Rightarrow a^2+4b^2+5a=0$

$\hookrightarrow a^2+4b^2=5$ (by ①)

$\therefore a=-1 \quad b=1$

2)=3) $\Rightarrow \frac{1}{4} = \frac{h}{8}$ $\quad \therefore h=2$