1.5 : 갑자포기.

1.6.21 $\lim_{x\to 0} x^4 \cos\frac{2}{x} = 0$

sol) ① $-1 \le \cos\frac{2}{x} \le 1$

② $-x^4 \le x^4 \cos\frac{2}{x} \le x^4$

③ $\lim_{x\to 0} x^4 = 0 \quad \lim_{x\to 0} (-x^4) = 0$

$\therefore \lim_{x\to 0} x^4 \cos\frac{2}{x} = 0$ (by Squeeze Th).

1.6.29 $f(x) = [x] + [-x]$

sol) 1) $x = n \ (n\in\mathbb{Z})$ $\begin{cases} [n] = n \\ [-n] = -n \end{cases}$ $\therefore f(n) = 0$.

2) $n < x < n+1 \ (n\in\mathbb{Z})$ $\begin{cases} [x] = n \\ [-x] = -n-1 \end{cases}$ $\therefore f(x) = -1$.

3) $n-1 < x < n \ (n\in\mathbb{Z})$ $\begin{cases} [x] = n-1 \\ [x] = -n \end{cases}$ $\therefore f(x) = -1$

$\therefore \lim_{x\to 2} f(x) = -1 \ / \ f(2) = 0$

$\therefore \lim_{x\to 2} f(x) \neq f(2)$

1.6.32 $f(x)=\begin{cases} x^2 & (x: \text{유리수}) \\ 0 & (x: \text{무리수}) \end{cases} \Rightarrow \lim_{x\to 0} f(x)=0$

Sol) i) $x$: 유리수. $-1<x<1 \Rightarrow 0\le x^2<1$

$\Rightarrow x\to 0 \Rightarrow x^2\to 0$

$\lim_{x\to 0} f(x)=\lim_{x\to 0} x^2=0$

ii) $x$: 무리수 $\lim_{x\to 0} f(x)=\lim_{x\to 0} 0=0$.

$\therefore \lim_{x\to 0} f(x)=0$

1.6.33 $\nexists \lim_{x\to a} f(x), \nexists \lim_{x\to a} g(x) \Rightarrow \exists \lim_{x\to a} [f(x)g(x)]$

ex)

$f(x)=\begin{cases} 0 & (x<0) \\ 1 & (x\ge 0) \end{cases} \quad g(x)=\begin{cases} 1 & (x<0) \\ 0 & (x\ge 0) \end{cases}$

$\Rightarrow \nexists \lim_{x\to 0} f(x), \nexists \lim_{x\to 0} g(x)$

$\therefore \lim_{x\to 0} [f(x)g(x)]=\lim_{x\to 0} 0=0$.

1.8. 29 ① $f(x)=\cos x - x$ : 연속 on $[0,1]$

② $f(0)=1 \quad f(1)=\cos 1 - 1 \approx -0.46$

$\therefore f(0)f(1)<0$.

$\Rightarrow \exists\, c\in(0,1) \quad f(c)=0$ (by 중간값 Th)

$\therefore \exists\, x\in(0,1) \quad \cos x = x$.

1.8. 30-(a) ① $f(x)=\cos x - x^3$ : 연속 on $[0,1]$

② $f(0)=1$. $f(1)=\cos 1 - 1 \approx -0.46 \Rightarrow f(0)f(1)<0$

$\Rightarrow \exists c\in(0,1) \quad f(c)=0$ (by 중간값 정리)

$\therefore \exists x\in(0,1) \quad \cos x = x^3$