3.2.11 $f(x) = (x-3)^{-2}$ $f'(x) = -2(x-3)^{-3}$

$f(4) - f(1) = f'(c)(4-1)$

$\Rightarrow 1 - \frac{1}{4} = \frac{-2}{(c-3)^3} \cdot 3 \Rightarrow \frac{3}{4} = \frac{-6}{(c-3)^3}$

$\therefore (c-3)^3 = -8$ $c-3 = -2$ $c=1$

$\nexists c \in (1, 4)$

but f: 불연속 on $x=3$ $\therefore$ M.V.T 가정을 만족하지 않으므로.

c가 존재하지 않는다.

3.2.12. $2x + \cos x = 0$ $\exists_1 x$.

1) $f(x) = 2x + \cos x$: 연속 on $(-\infty, \infty)$ / 미분가 on $(-\infty, \infty)$

2) $f'(x) = 2 - \sin x \neq 0$ ($\because 1 \leq 2 - \sin x \leq 3$)

$\therefore$ Rolle's Th 대우 $\Rightarrow$ 한근 or X

3) $f(0) = 1 > 0$ $f(-\frac{\pi}{2}) = -\pi < 0$

$\therefore f(0) f(-\frac{\pi}{2}) < 0$

by 중간값 정리 $\exists c \in (-\frac{\pi}{2}, 0)$ $f(c) = 0$

단 한개의 실근이 존재한다.

3.2.17 $0<x<2\pi$. $\sin x < x$.

M1) Let $f(x)=\sin x - x$

Then ① $f(x)$: 연속 on $[0, 2\pi]$

② $f(x)$: 미·가 on $(0, 2\pi)$

1) $f(0)=0$

2) $f(2\pi)=f(2\pi)-f(0)=f'(c)(2\pi-0)$ $\quad 0<c<2\pi$ (by M.V.T)

but $f'(c)<0$ $\quad \because f'(x)=\cos x-1<0$

$-1\le \cos x<1$

$-2\le \cos x-1<0$ $\quad (0<x<2\pi)$

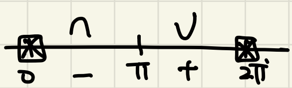
$f(2\pi)<0 \Rightarrow f(x)<0$ $\quad (0<x<2\pi)$

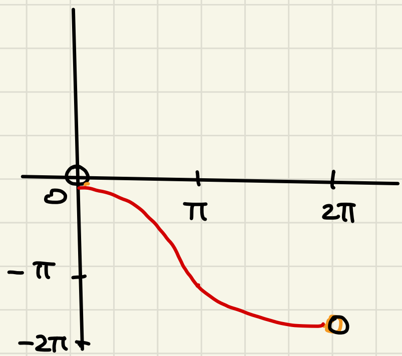
$\therefore \sin x<x$.

M2) $f(x)=\sin x - x$

1) $f'(x)=\cos x-1<0$. 임계수 X ($\because 0<x<2\pi$)

2) $f''(x)=-\sin x=0$ $\quad x=\pi$





$\therefore f(x)<0$

$\Rightarrow \sin x - x<0$

$\sin x<x$.

$(0<x<2\pi)$.

3.2.18. $\forall a, b \in \mathbb{R}$

$|\sin a - \sin b| \le |a-b|$

$f(x) = \sin x$: 연속 on $(-\infty, \infty)$ / 미가 on $(-\infty, \infty)$

1) $a > b$

$f(x) = \sin x$ 연속 on $[b, a]$ / 미가 on (b, a)

by M.V.T $\exists c \in (b, a)$.

$f(a) - f(b) = f'(c)(b-a)$

$\Rightarrow \sin a - \sin b = \cos c\,(b-a)$ $\quad b < c < a$

$\Rightarrow |\sin a - \sin b| = |\cos c|\,|b-a| \le |b-a|$

$\because |\cos c| \le 1$

2) $a < b$ $\quad f(x) = \sin x$ 연속 on $[a, b]$ / 미가 on (a, b)

by M.V.T $\exists c \in (a, b)$ $\quad |\cos c| \le 1$

$|\sin b - \sin a| = |\sin a - \sin b| \le |a-b| = |b-a|$

3) $a = b$ $\quad \underbrace{|\sin a - \sin b|}_{0} = \underbrace{|a-b|}_{0}$.

3.3.7 $f(x)=\dfrac{x^2-24}{x-5}$

1) $D_f=\{x \mid x\neq 5\}$

2) $f'(x)=\dfrac{2x(x-5)-(x^2-24)}{(x-5)^2}=\dfrac{2x^2-10x-x^2+24}{(x-5)^2}$

$=\dfrac{x^2-10x+24}{(x-5)^2}=\dfrac{(x-4)(x-6)}{(x-5)^2}$

임계수: $x=4$ 6.

$+$ 4 $-$ $\boxed{5}$ $-$ 6 $+$

$\nearrow$ $\searrow$ $\searrow$ $\nearrow$

증가구간: $(-\infty\ 4)\cup(6.\ \infty)$

감소구간: $(4\ 5)\cup(5.\ 6)$

극댓값: $f(4)=8$ $f(6)=12$

3.3.9 $f(x)=\sin^2 x-\cos 2x$ $0\le x\le\pi$

$f'(x)=2\sin x\cos x+2\sin 2x$

$=3\sin 2x$

$f''(x)=6\cos 2x$

변곡점의 후보자: $\frac{\pi}{4}$ $\frac{3\pi}{4}$

$\cup$ $\cap$ $\cup$

0 $+$ $\frac{\pi}{4}$ $-$ $\frac{3\pi}{4}$ $+$ π

변곡점 $(\frac{\pi}{4}, \frac{1}{2})$ $(\frac{3\pi}{4}, \frac{1}{2})$

3.3.11 $f(x) = \sin x + \cos x \quad 0 \le x \le 2\pi$

1) $f'(x) = \cos x - \sin x$

임계수 : $\frac{\pi}{4}, \frac{5\pi}{4}$

(a) $0 \; + \; \frac{\pi}{4} \; - \; \frac{5\pi}{4} \; + \; 2\pi$

$\quad \nearrow \quad \searrow \quad \nearrow$

(b) $f(\frac{\pi}{4}) = \sqrt{2} \quad f(\frac{5\pi}{4}) = -\sqrt{2}$

2) $f''(x) = -\sin x - \cos x$

① 변곡점 후보: $\frac{3\pi}{4} \quad \frac{7\pi}{4}$

(c) $\cap \quad \cup \quad \cap$

$0 \; - \; \frac{3\pi}{4} \; + \; \frac{7\pi}{4} \; - \; 2\pi$

(d) $(\frac{3\pi}{4}\ 0)\ (\frac{7\pi}{4}.0)$

3.3.25 $C(x) = x^{\frac{1}{3}}(x+4) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$

(1) $D_f = (-\infty . \infty)$

(2) $C'(x) = \frac{4}{3}x^{\frac{1}{3}} + \frac{4}{3}x^{-\frac{2}{3}} = \frac{4}{3} \cdot \frac{(x+1)}{\sqrt[3]{x^2}}$

ⓘ 임계수: $x=0 \quad -1$

$\lim_{x\to 0+} \frac{4}{3\sqrt[3]{x}} = \infty \;/\; \lim_{x\to 0-} \frac{4}{3\sqrt[3]{x}} = -\infty$ 수직접선.

(a) $- \quad -1 \quad +$

$\searrow \quad \nearrow$

(b) 극솟값 $f(-1) = -3$

극댓값 X

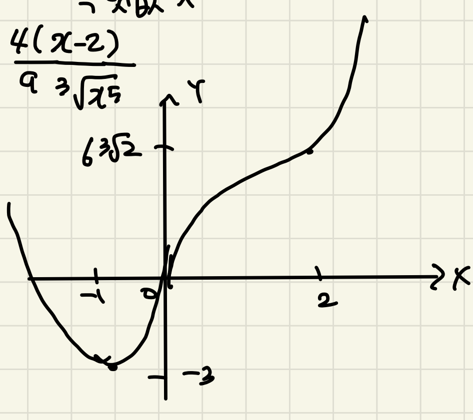
(3) $C''(x) = \frac{4}{9}x^{-\frac{2}{3}} - \frac{8}{9}x^{-\frac{5}{3}} = \frac{4(x-2)}{9\sqrt[3]{x^5}}$

ⓘ 변곡점 후보: $x=0 \quad x=2$

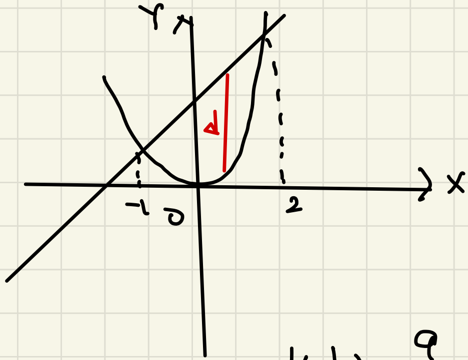
(c) $\cup \quad \cap \quad \cup$

$0 \quad 2.$

변곡점. $(0.0)\ (2.\ 6\sqrt[3]{2})$



3.7.3 $y=x+2 \quad y=x^2 \quad -1 \le x \le 2$



$d(x) = x+2-x^2$

$= -x^2+x+2$

$d'(x) = -2x+1$

$-2x+1 \geq 0$

$-2x > -1$

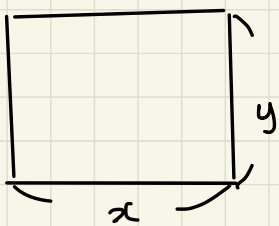
$x \; \frac{1}{2}$

$-1 \quad + \quad \frac{1}{2} \quad - \quad 2$

$\nearrow \qquad \searrow$

$d(\frac{1}{2}) = \frac{9}{4}$

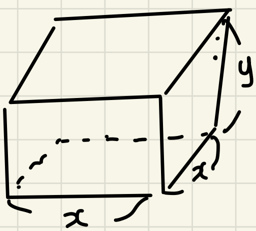
3.7.4



1) $S = xy$

2) $2(x+y) = 100$

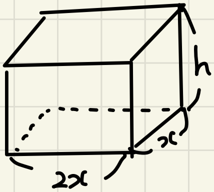
3.7.10



1) $V = x^2y$

2) $x^2+4xy = 1200$

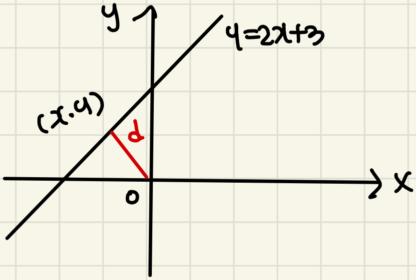
3.7.11



1) $C = 10(2x^2) + 6(6xh)$

2) $2x^2h = 10$

13.



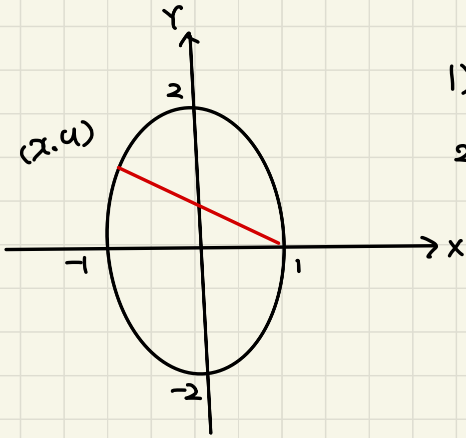
1) $d=\sqrt{x^2+y^2}$

2) $y=2x+3$

$\Rightarrow f(x)=x^2+(2x+3)^2$

$=5x^2+12x+9.$

14.

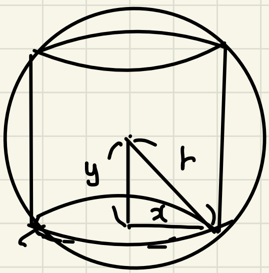


1) $d=\sqrt{(x-1)^2+y^2}$

2) $4x^2+y^2=4$

$\Rightarrow f(x)=(x-1)^2+(4-4x^2).$

20



1) $S=2\pi x^2+4\pi xy$

2) $x^2+y^2=r^2$

$S(x)=2\pi x^2+4\pi x\sqrt{r^2-x^2}$ $0<x<r$

$\Rightarrow$ 뒷장

$$S(x) = 2\pi x^2 + 4\pi x\sqrt{r^2-x^2} \quad (0 \leq x \leq r)$$

$$S'(x) = 4\pi x + 4\pi\left(\sqrt{r^2-x^2} - \frac{x^2}{\sqrt{r^2-x^2}}\right)$$

$$= 4\pi\left(\frac{x\sqrt{r^2-x^2} + r^2-x^2-x^2}{\sqrt{r^2-x^2}}\right)$$

$S'(x)=0$

$$x\sqrt{r^2-x^2} = 2x^2-r^2$$

$$\Rightarrow x^2(r^2-x^2) = (2x^2-r^2)^2$$

$$\Rightarrow 5x^4 - 5x^2r^2 + r^4 = \left(x^2 - \frac{5-\sqrt{5}}{10}r^2\right)\left(x^2 - \frac{5+\sqrt{5}}{10}r^2\right)$$

$$= \cancel{\left(x+\sqrt{\frac{5-\sqrt{5}}{10}}r\right)}\left(x-\sqrt{\frac{5-\sqrt{5}}{10}}r\right)\left(x-\sqrt{\frac{5+\sqrt{5}}{10}}r\right)\cancel{\left(x+\sqrt{\frac{5+\sqrt{5}}{10}}r\right)}$$

$$0 \quad + \quad \sqrt{\frac{5-\sqrt{5}}{10}}r \quad + \quad \sqrt{\frac{5+\sqrt{5}}{10}}r \quad - \quad r$$

$$S\left(\sqrt{\frac{5+\sqrt{5}}{10}}r\right) = 2\pi\cdot\left(\frac{5+\sqrt{5}}{10}r^2\right) + 4\pi\sqrt{\frac{5+\sqrt{5}}{10}}r\cdot\sqrt{\frac{5-\sqrt{5}}{10}}r$$

$$= \pi r^2\left(\frac{5+\sqrt{5}}{5} + \frac{4\sqrt{5}}{5}\right)$$

$$= \pi r^2(1+\sqrt{5})$$

넘 복잡함 ㅠ.ㅠ