2022 3100 성준

1.

$2(x^4+2x^2y^2+y^4)$

$=25(x^4-2x^2y^2+y^4)$

$23x^4-6x^2y^2+23y^4=0$

$(23\times4)x^3-12xy^2-12x^2y\frac{dy}{dx}+(23\times4)y^3\frac{dy}{dx}$

$=0$

$\frac{dy}{dx}(\cancel{92y^3-12x^2y})=\frac{12xy^2-92x^3}{92y^3-12x^2y}$

$\frac{12-92\times3^3}{92-108}=\frac{-2472}{-16}$

$=\frac{309}{2}$

$92 \times 27 = 644 + 184 = 2484$

927

$y=\frac{309}{2}x-\frac{925}{2}$

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2.

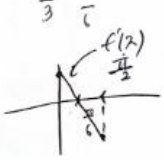
$f'(x)=-2\sin x+2\cos 2x$

$f'(x)=0$ 이 되는 x! $\frac{\pi}{6}$

$\sin x=\cos 2x$

$\frac{\pi}{3}$ $\frac{\sqrt{3}}{2}$ $\frac{\pi}{6}$

$f'(0)=2$



$[0,\frac{\pi}{2}]$ 최대는 $x=\frac{\pi}{6}$

최소는 $x=\frac{\pi}{2}$

최대값: $\frac{3\sqrt{3}}{2}$

최소: 0

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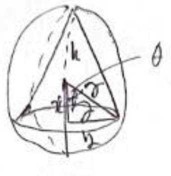
3,



$x^2+y^2=r^2 \qquad x=r\cos\theta$
$\qquad\qquad\qquad\quad y=r\sin\theta$

$h=x+r$

$V=\frac{1}{3}\times\pi\times(r\sin\theta)^2\times$
$\times(r+r\cos\theta)$

$=\frac{1}{3}r^3\underbrace{\sin^2\theta\,(1+\cos\theta)}\,\pi$

$\to'\;2\sin\cos(1+\cos)-\sin^3\theta$

$=\sin\theta(2\cos^2\theta+2\cos\theta-\sin^2\theta)$

$\quad 3 \qquad -1$

$\quad 1 \qquad 1$

$\cos\theta=\frac{1}{3}$
$\sin\theta=\frac{2\sqrt{2}}{3}$

$\frac{1}{3}r^3\times\frac{8}{9}\times\frac{4}{3}=\frac{32\pi r^3}{81}$

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4,

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3,

$x^2+y^2=r^2$ $\quad x=r\cos\theta$

$y=r\sin\theta$

$h=x+r$

$V=\frac{1}{3}\times\pi\times(r\sin\theta)^2\times$

$\times(r+r\cos\theta)$

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$2\sin\cos(1+\cos)-\sin^3\theta$

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5.

$\cos x=t$

$-\sin x=\frac{dt}{dx}$

$\sin 8\theta=2\sin4\theta\cos4\theta$

$\sin4\theta=2\sin2\theta\cos2\theta$

$\sin2\theta=2\cos\theta\sin\theta$

$\sin8\theta=8\times$

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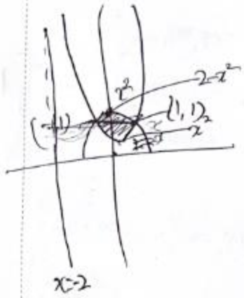
6.



회전축

$\int_{-1}^{1}2\pi(x+1)(2-2x^2)\,dx$

$2\pi\int_{-1}^{1}(2x-2x^3)\,dx$

$=2\pi\left[x^2-\frac{1}{2}x^4\right]_{-1}^{1}$

$1-\frac{1}{2}$

$8-\frac{8}{3}$

$2x+4-2x^3-4x^2$

$\left[4x-\frac{4}{3}x^3\right]_{-1}^{1}$

$4-\frac{4}{3}+4-\frac{4}{3}$

$\frac{32}{3}\pi$

2022760

7.

$avg(x)$

$[a,b]$에서 평균

$$avg(x) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\int_0^{\frac{\pi}{6}} \sec^3 x\,dx$$

$$= \int_0^{\frac{\pi}{6}} \sec x \times \sec^2 x\,dx \text{ (부분적분)}$$

$$= \sec x \times \tan x - \int \sec \tan^2 x\,dx$$

$$= \text{''} - \int \sec x(\sec^2 x - 1)\,dx$$

$$= \text{''} - \int \sec^3 dx + \int \sec\,dx$$

$$\left[\frac{1}{2}\Big(\sec x \tan x + \underbrace{\int \sec x\,dx}_{\ln|\sec x + \tan x|}\Big)\right]_0^{\frac{\pi}{6}}$$

$$\frac{\dfrac{\sqrt{2}\times 1 + \ln(1+\sqrt{2})}{2}}{\frac{\pi}{6} - 0}$$

$$\frac{2\sqrt{2} + 2\ln(1+\sqrt{2})}{2\pi}$$

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8.

$\arctan x$

$=$

$$\int x \arctan \tan^{-1}(x)\,dx$$

$$\tan^{-1}(x) = t$$

$$\frac{1}{1+x^2} = \frac{dt}{dx}$$

$$dx = dt(1+x^2)$$

$$\int (x + x^3)t\,dt$$

$$= \frac{1}{2}($$

부분적분이용

$$\int x \tan^{-1}(x)\,dx = \frac{x^2}{2}\tan^{-1}(x) - \frac{1}{2}\int \frac{x^2}{x^2+1}\,dx + C$$

$$= \frac{1}{2}\left(x^2\tan^{-1}(x) - \int\left(1 - \frac{1}{x^2+1}\right)dx\right) + C$$

$$= \frac{1}{2}\left(x^2\tan^{-1}(x) - x + \tan^{-1}x\right) + C$$

$$= \frac{1}{2}\left((x^2+1)\tan^{-1}(x) - x\right) + C$$