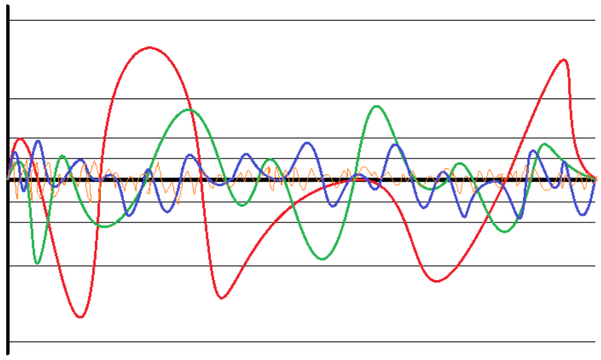
Lazy hypertexture :

Say we have a procedural shape such as a sphere, and we want to displace the surface of that shape using a fractal additive perlin noise.

The displacement D we want to add to the point X(x,y,z) is :

$$D(X)=\sum_{n=0}^{\infty}\frac{1}{2^n}.noise(2^n X)$$



we know that -1 <= noise (X) <= 1 for all X

So we notice that

$$|D(X)|\leq\sum_{n=0}^{\infty}\frac{1}{2^n}$$

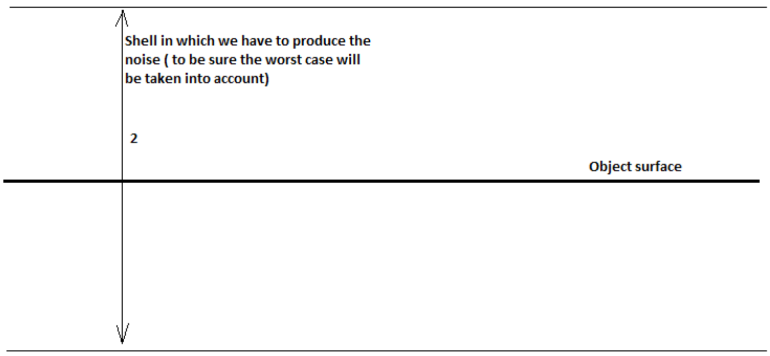
which is the sum of the terms of a geometric suite of first term 1 and raison 1/2.

and the sum S of the N first terms of a geometric suite of first term P and raison Q is :

$$S=P*\frac{Q^N-1}{Q-1}$$

So in our case :

$$S=\lim_{n\to\infty}\left(1*\frac{\frac{1}{2^n}-1}{\frac{1}{2}-1}\right)=2$$



So we have :

$$|D(X)|\leq 2$$

We now can use that fact to say that our producer will never have to write data outside a shell of [+2 ; -2] around the surface of our shape. So we can flag all the nodes ouside that shell as « not conataining any data » never... no way... impossible... maths said so...

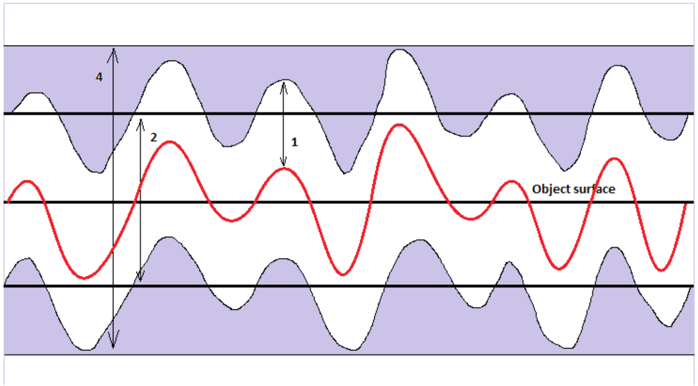
Say now we have produced the first level of LOD and inside, the first octave of noise.
We can use that knowledge to refine the shell for the incoming productions ! ! ! (and that is cool)

let's consider our old point X...
We still want to displace it by :

$$D(X)=\sum_{n=0}^{\infty}\frac{1}{2^n}.noise(2^n X)$$

But we now have more information about D(X)

because we have already computed :



$$N1 = \frac{1}{2^0}.noise(2^0 X)$$

so …

$$D(X) = N1 + \sum_{n=1}^{\infty} \frac{1}{2^n}.noise(2^n X)$$

so...

$$|D(x) - N1| \leq \sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} * \frac{\frac{1}{2^{\infty}} - 1}{\frac{1}{2} - 1} = 1 = 2 - 1$$

Easy to interpret :
Lets assume we are in the case where all the octaves are positives :
it means that :

D(X) < 2 – 1 + N1

We take the former value of the shell (2) we remove the max amplitude of the ocatve number 0 (1) and replace it by the actual value of the amplitude of the octave 0.

Because we don't need to assume that octave 0 will take it's maximum value , we have the information ! ! !

So for the next production, the producer will be able to produce data anly in a thicker shell, of thickness (2-1+N1)

Same when we have already M ocatves already computed (from 0 to M-1) and want to produce the octave number M:

$$D(x) = \sum_{n=0}^{M-1} \frac{1}{2^n}.noise(2^n X) + \sum_{n=M}^{\infty} \frac{1}{2^n}.noise(2^n X)$$

the first sum being numerically known...

leads to :

$$\left| D(x) - \sum_{n=0}^{M-1} \frac{1}{2^n}.noise(2^n X) \right| \leq \sum_{n=M}^{\infty} \frac{1}{2^n} = \frac{1}{2^M} * \frac{\frac{1}{2^{\infty}} - 1}{\frac{1}{2} - 1} = \frac{1}{2^{M-1}}$$

We have again a thicker shell ! ! !