Infinite LCL:s

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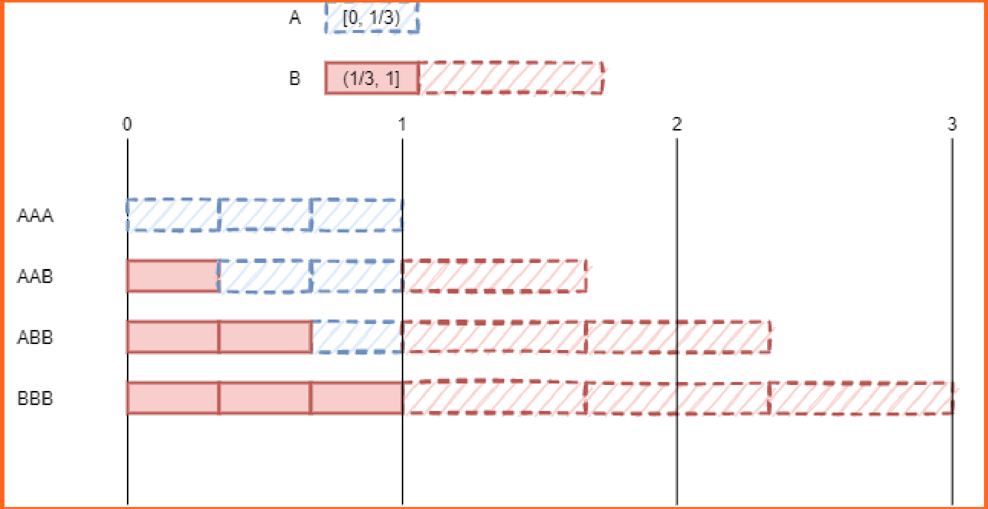
27.8.2021



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In what way infinite?

Locally Checkable Labeling

(Naor, Stockmeyer 1995)

Input label set (finite)

Output label set (finite)

Locally consistent labelings

(finite in radius, finite in count)

In what way infinite?

Locally Checkable Labeling

(Naor, Stockmeyer 1995)

Input label set (infinite)

Output label set (infinite)

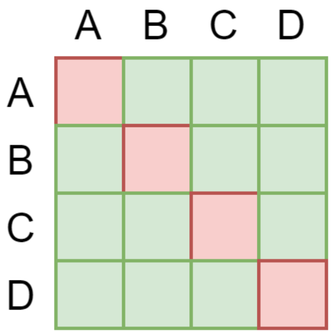
Locally consistent labelings

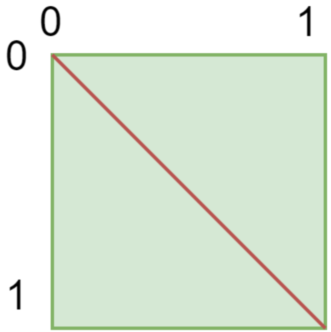
(finite in radius, infinite in count)

Where to start?

First consider paths and cycles

- The set of locally consistent labelings can be interpreted as a binary relation
- Binary relations can be viewed as a 2-dimensional table





Comparison between cyclepath 4-coloring and infinite coloring with colors from $[0, 1]$.

Lack of problems

Coming up with different problems that use infinite labels seems to be very hard.

All constant time cyclepath problems without input are either finite, or ”output radius t-neighborhood” AND produce an infinite coloring.

Packing-Covering problems

One interesting set of problems was packing-covering biregular trees with continuous real valued label sets.

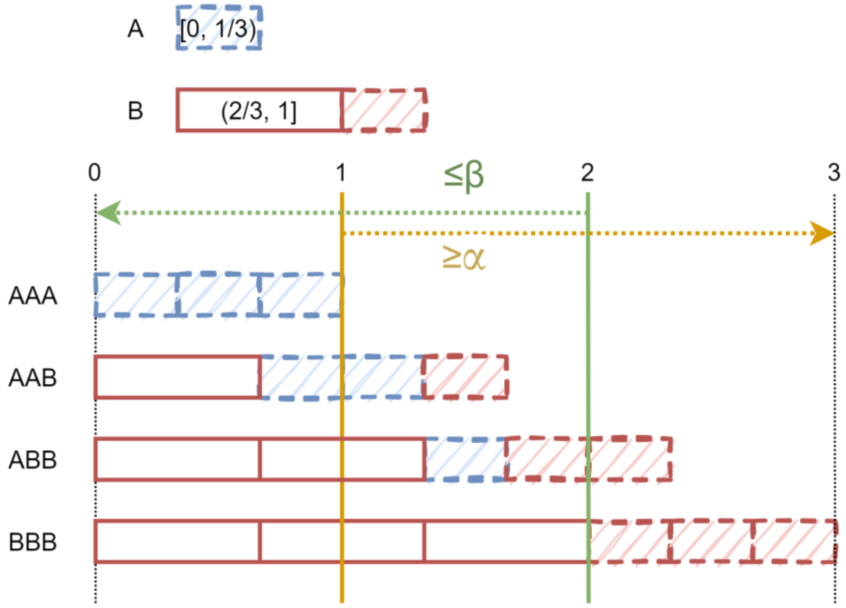
Edge-labeling problem Π:

- $\Sigma \subseteq [0,1]$
- $A = \left\{x \in \Sigma^d \middle| x_1 + x_2 + \cdots + x_d \leq \beta\right\}$
- $P = \left\{x \in \Sigma^\delta \middle| x_1 + x_2 + \cdots + x_\delta \geq \alpha\right\}$

Solving problems

First begin by partitioning Σ. Maybe start with maximal connected subsets.

Construct all possible neighborhoods and calculate which of them are possible for active or passive nodes.



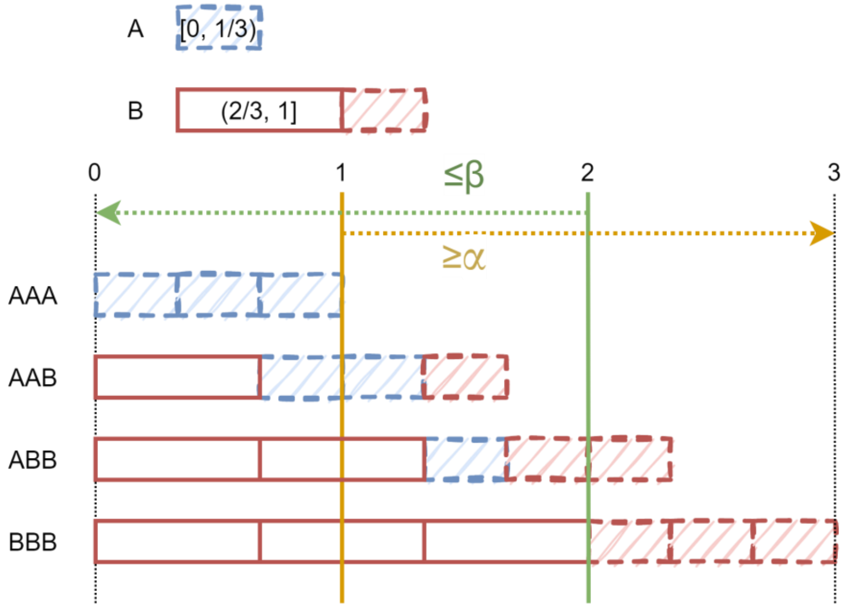
Problem Π

$$\Sigma = \left[0, \frac{1}{3}\right), \left(\frac{2}{3}, 1\right]$$
$$d = \delta = 3$$
$$\alpha = 1, \beta = 2$$

Solving problems

This example problem Π creates the following table:

Neighborhood	Active	Passive
AAA		
AAB		
ABB		
BBB		



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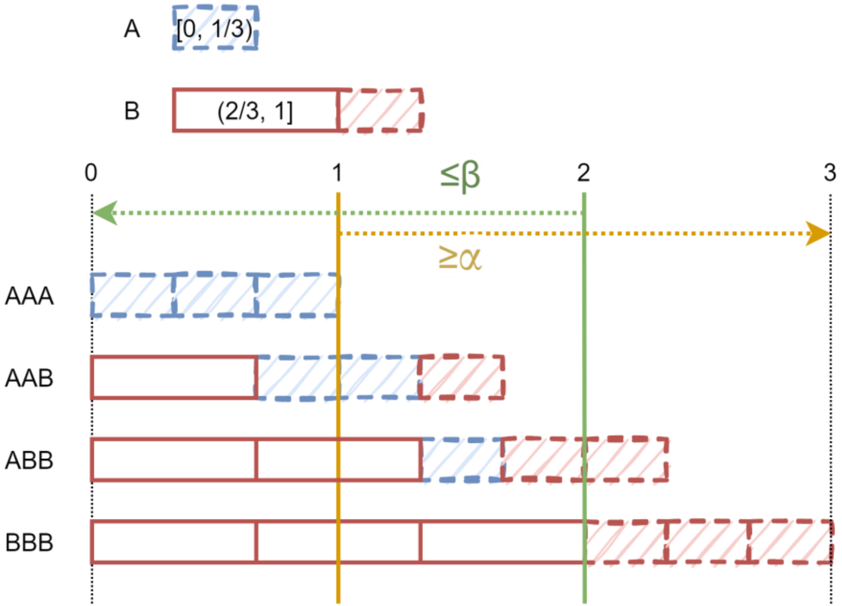

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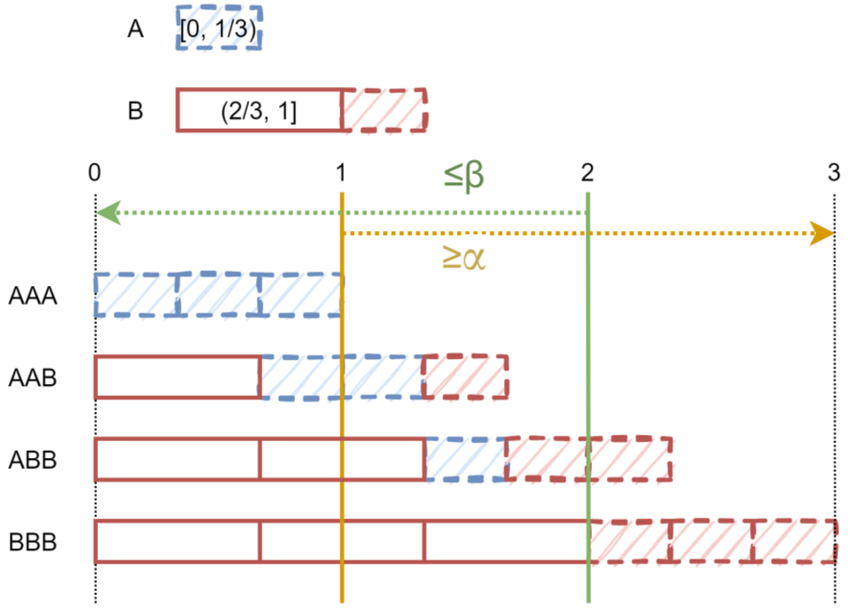

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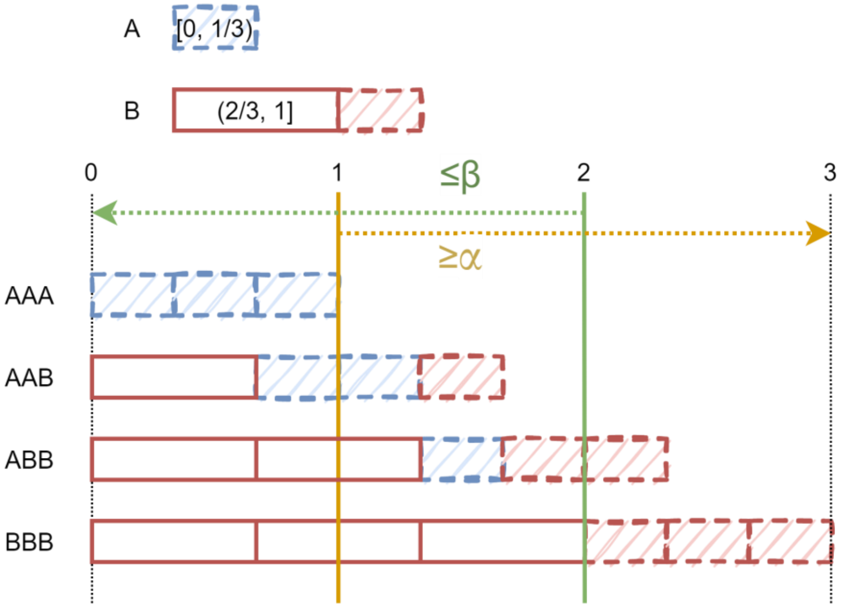

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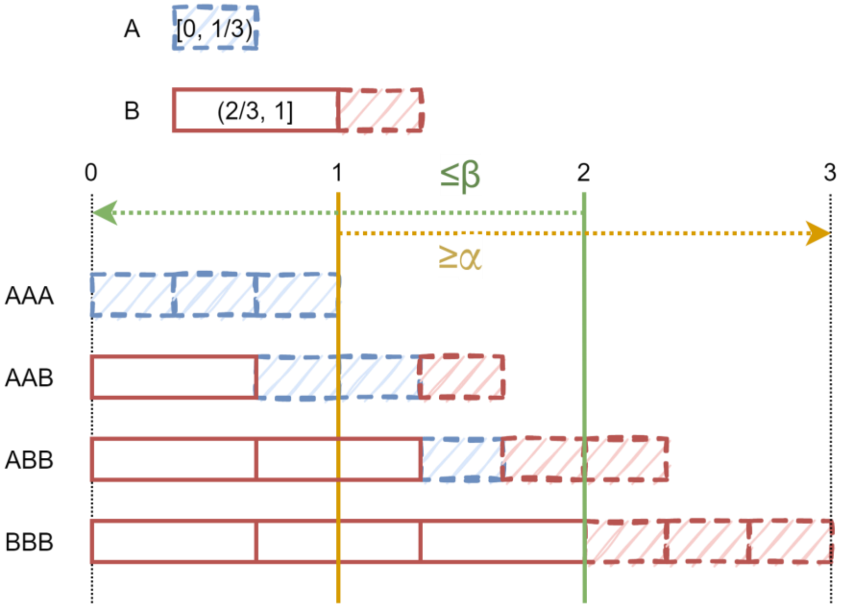

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Active:
A AB AB

Passive:
B AB AB

Sinkless orientation!

Problem Π

$$\Sigma = \left[0, \frac{1}{3}\right), \left(\frac{2}{3}, 1\right]$$
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Solving problems

A solution to the original problem can be mapped in 0 rounds to a solution to the LCL.

The inverse does not hold universally!

Problem Π

$$\Sigma = \left[0, \frac{1}{3}\right), \left(\frac{2}{3}, 1\right]$$

$$d = \delta = 3$$

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Neighborhood table

?

Active:
A AB AB

Passive:
B AB AB

Sinkless orientation!

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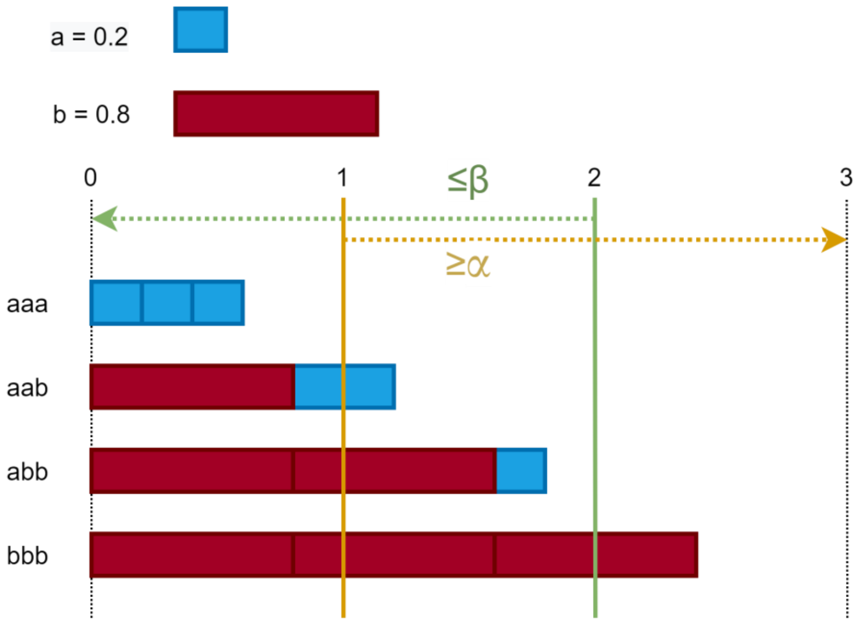
Discretization



Contract each interval into a discrete value so that the neighborhood table is still satisfied.

If found, it will be the 0-round mapping from the LCL to the original problem, which in turn makes them equivalent.

Problem Π

$$\Sigma = \left[0, \frac{1}{3}\right), \left(\frac{2}{3}, 1\right]$$
$$d = \delta = 3$$
$$\alpha = 1, \beta = 2$$

Problem Π'

$$\Sigma' = \{0.2, 0.8\}$$
$$d = \delta = 3$$
$$\alpha = 1, \beta = 2$$

Discretization

How to find it?

Create a set of linear inequalities by using the neighborhood table and interval boundaries.

Neighborhood	Active	Passive
AAA	True	False
AAB	True	True
ABB	True	True
BBB	False	True

Let a, b be the discretization values for intervals A, B.

$$0 \leq a < \frac{1}{3}$$
$$\frac{2}{3} < b \leq 1$$
$$a + a + a \leq \beta$$
$$\alpha \leq a + a + b \leq \beta$$
$$\alpha \leq a + b + b \leq \beta$$
$$\alpha \leq b + b + b$$

Problem Π

$$\Sigma = \left[0, \frac{1}{3}\right), \left(\frac{2}{3}, 1\right]$$
$$d = \delta = 3$$
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$\Rightarrow$

Problem Π'

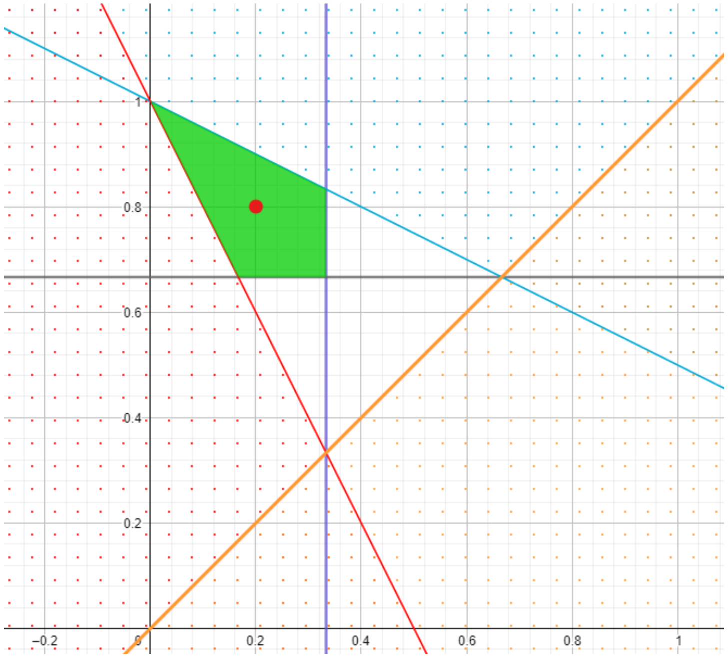
$$\Sigma' = \{0.2, 0.8\}$$
$$d = \delta = 3$$
$$\alpha = 1, \beta = 2$$

Discretization

How to find it?

This set represents a convex polytope in n-dimensional space, where n is the number of intervals.

If it is empty, there are no discretizations for those intervals.



Problem Π

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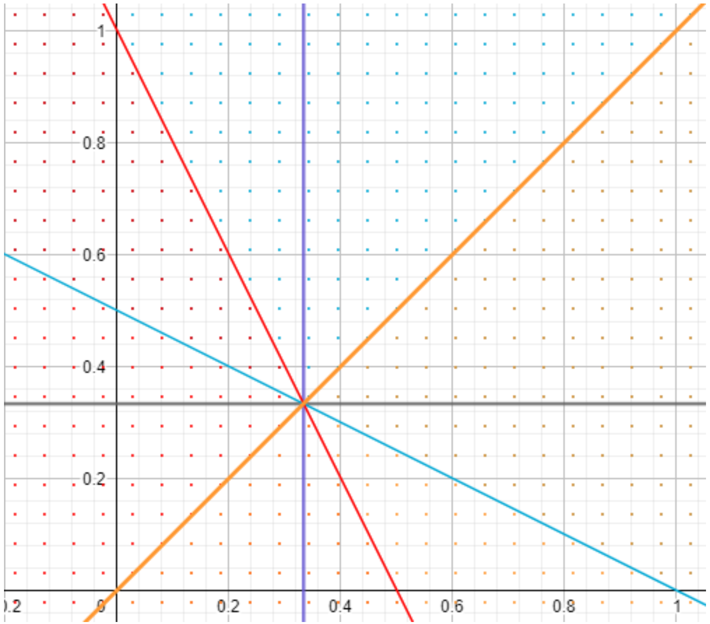


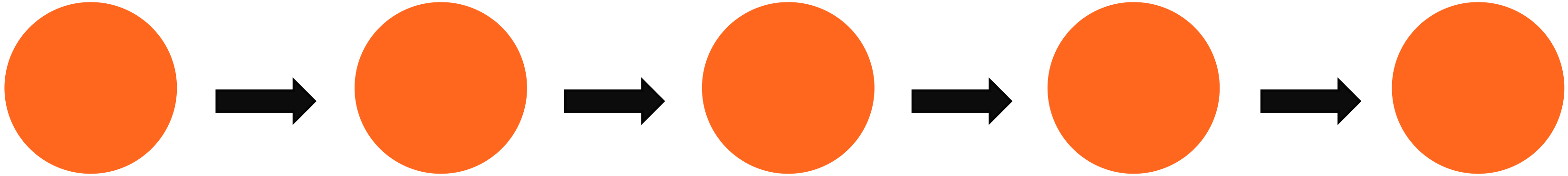
Problem X

$$\Sigma = \left[0, \frac{1}{3}\right), \left(\frac{1}{3}, 1\right]$$
$$d = \delta = 3$$
$$\alpha = 1, \beta = 1$$

Problem X'

$$\Sigma' = ????$$
$$d = \delta = 3$$
$$\alpha = 1, \beta = 1$$

Workflow for solving packing-covering problems



Partition the label set into intervals that are going to be discretized.

Create the neighborhood table for those intervals.

Create the set of inequalities from the neighborhood table.

Solve this set using linear programming.

If the solution set is not empty, choose any one point as the discretization.

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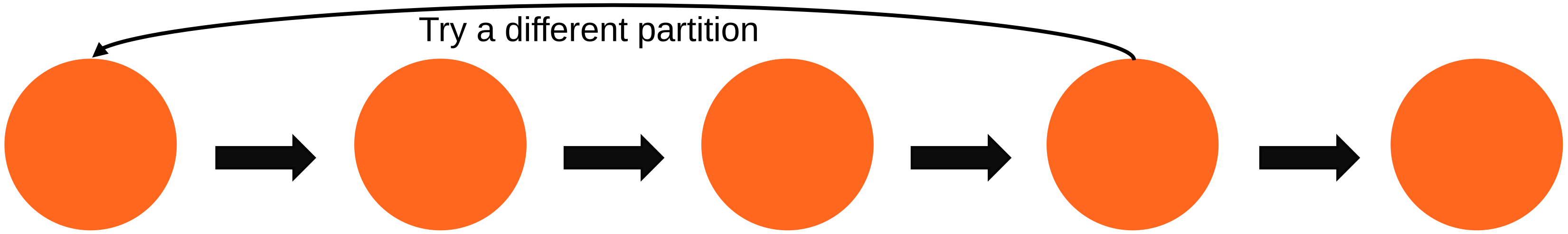
Workflow for solving packing-covering problems



Partition the label set into intervals that are going to be discretized.

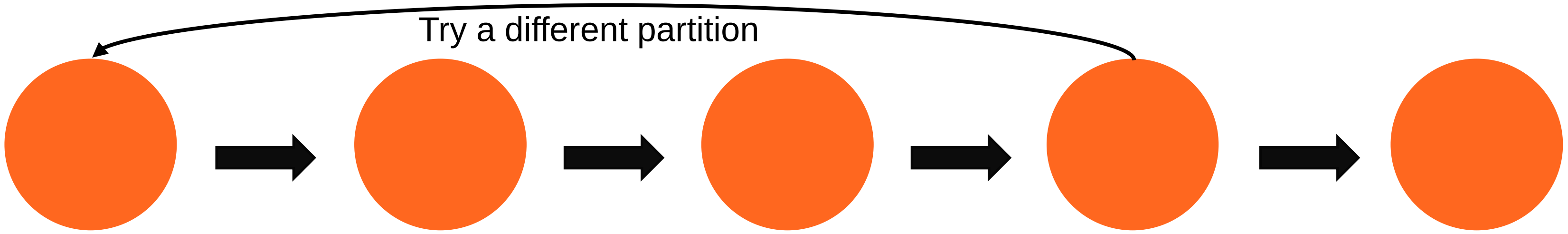
Create the neighborhood table for those intervals.

Create the set of inequalities from the neighborhood table.

Solve this set using linear programming.

If the solution set is not empty, choose any one point as the discretization.

Workflow for solving continuous local problems



Partition the label set into intervals that are going to be discretized.

Create the neighborhood table for those intervals.

Create the set of conditions from the neighborhood table.

Solve this set of conditions.

If the solution set is not empty, choose any one point as the discretization.

Thank you for listening!

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