Quartz resonator
SCI-C0200

Joonatan Bergholm 507260
Osama Abuzaid 524832

June 1, 2016

Contents

1 | Introduction

In this assignment we investigated electronic properties of one of the most common electronic components, the quartz tuning fork or quartz resonator, like one in figure 2. It is used in watches and other every day electrical appliances to provide a stable clocking frequency. Typically the frequency is $f_0 = 32\,768\,\mathrm{Hz}$, because is is a round number ($32768_{10} = 2^{15}{}_{10} = 1000000000000000_2$) in base 2, which is commonly used in electrical appliances.

Contrary to conventional tuning fork, one does not need generate mechanical excitation on the quartz tuning fork, because quartz has piezoelectric properties and thus mechanical excitation can be replaced with electronic one.

2 | Tuning fork

Piezoelectricity is a phenomena, where deformation of a material causes electrical current and vice versa. Crystal oscillators are made of piezoelectric materials, and in electric circuits they are used to produce an electrical signal with precise frequency. The Q-value of crystal oscillator is defined to be the oscillators energy per dissipated energy at one oscillation:

$$Q \equiv 2\pi \frac{E}{\Delta E}. \tag{1}$$

In our work we are using quartz tuning forks. The fundamental frequency of a tuning fork is given by equation

$$f_0 \approx 0.16 \frac{T}{L^2} \sqrt{\frac{E}{\rho}}, \tag{2}$$

where $L = 3.98\mathrm{mm}$ is the length of one quartz bar, $T = 0.60\mathrm{mm}$ the width of the bar, $E = 79\mathrm{GPa}$ the Young modulus and $\rho = 2660 \frac{\mathrm{kg}}{\mathrm{m}^3}$ the density of quartz. Also the thickness of fork we are using in our work is $H = 0.30\mathrm{mm}$. Advantage of having two bars instead of one is that the oscillations of two bars at the fulcrum cancel each other, resulting to better Q-value.

We can estimate the Q-value of the tuning fork from figure 3 by using equation[1]

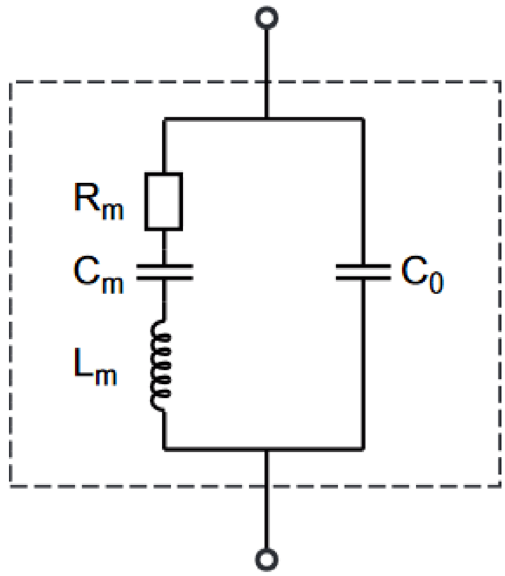

Figure 1: Equivalence circuit of a tuning fork.

$$t = \frac{f_0}{Q}$$
$$\Rightarrow Q = tf_0 \approx 13000$$

where according to figure 3 $t \approx 0.4$s is the time constant, which tells the time it takes for the resonator to stabilize to the equilibrium amplitude.

An equivalence circuit of tuning fork is an electrical circuit which can't be distinguished from tuning fork in a black box. The components of equivalence circuit in figure 1 are obtained from following equations:

$$\begin{cases} L_m = \frac{2\rho L^3}{9e_p^2 HT} \\ C_m = \frac{1}{\omega_0^2 L_m} \\ R_m = \frac{\omega_0 L_m}{Q} = \gamma L_m \\ C_0 = \frac{\epsilon_0 \epsilon_r A}{T} \end{cases}, \tag{3}$$

where $e_p \approx 0.18 \frac{\mathrm{C}}{\mathrm{m}^2}$ is the piezoelectric modulus of tuning fork, ϵ_0 and ϵ_r are the vacuum and relative permittivity, respectively, and $A = TL$ the effective

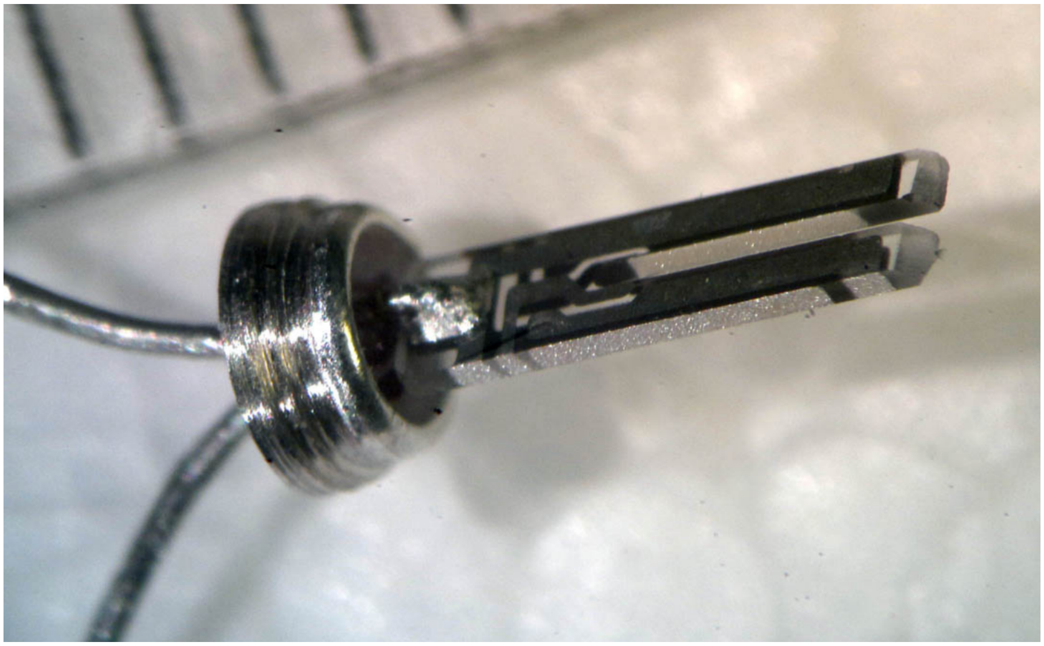

Figure 2: Quartz resonator

area. Equivalence circuit of our tuning fork would have following components:

$$\begin{cases} L_m \approx 3.3 \cdot 10^{-20} \text{ H} \\ C_m \approx 7.1 \cdot 10^{8} \text{ F} \\ R_m \approx 5.3 \cdot 10^{-19} \ \Omega \\ C_0 \approx 3.5 \cdot 10^{-9} \text{ F} \end{cases}$$

References

[1] https://mycourses.aalto.fi/pluginfile.php/211810/mod_resource/content/1/projekti_kertaohje4.pdf, "Kvartsiresonaattorin mittaus", accessed 2016.06.01

A | Source code

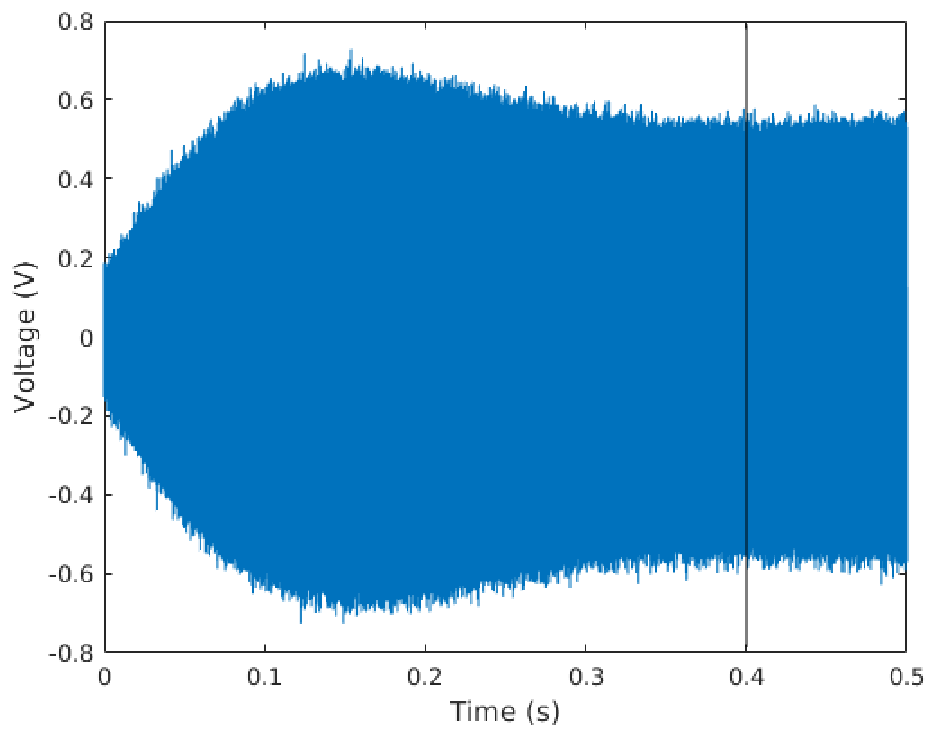

Figure 3: Voltage plotted from connecting the driving voltage to 0.5 s after connecting.

Listing 1: ac.m

```
g_data = [];
ps_data = [];
f_data = linspace(32.74e3, 32.752e3, 100);

for f = f_data
    [g, ps] = DAQreadout(f);
    g_data(end + 1) = g;
    ps_data(end + 1) = ps;
    length(ps_data)
end

figure
```

```
plot(f_data, g_data)
title('Vahvistus')
figure
plot(f_data, ps_data)
title('Vaihe-ero')
```

B | Pretasks

B.1 | Session 1

B.1.1 | Task 1

In this project we are supposed to study electric properties of quartz resonators and build a circuit, which is used to collect data from the resonator. Data is then analyzed with MATLAB. Also we are going to measure some properties of the circuit itself.

B.1.2 | Task 2

V_+ is connected to ground, so $V_+ = 0$.

$$
\begin{aligned}
V_{out} =& A(V_+ - V_-) \\
\Rightarrow V_{out} =& -AV_-
\end{aligned}
$$

Then according to Kirchoff II and Ohm's law,

$$
\begin{aligned}
V_- - V_{out} = R_f I \quad \wedge \quad I = \frac{V_{in} - V_{out}}{R_f + R_{in}} \\
\Rightarrow V_- =& R_f \frac{V_{in} - V_{out}}{R_f + R_{in}} + V_{out} \\
\Rightarrow V_- =& \frac{R_f V_{in} - R_f V_{out}}{R_f + R_{in}} + \frac{R_f V_{out} + R_{in} V_{out}}{R_f + R_{in}} \\
\Rightarrow V_- =& \frac{R_f V_{in} + R_{in} V_{out}}{R_f + R_{in}}
\end{aligned}
$$

Then combining these two equations, we obtain V_{out} as function of V_{in}.

$$
\begin{aligned}
V_{out} &= -A\frac{R_f V_{in} + R_{in} V_{out}}{R_f + R_{in}} \\
\Leftrightarrow V_{out} + A\frac{R_{in} V_{out}}{R_f + R_{in}} &= -\frac{A R_f V_{in}}{R_f + R_{in}} \\
\Leftrightarrow V_{out}\frac{R_f + R_{in} + A R_{in}}{R_f + R_{in}} &= -\frac{A R_f V_{in}}{R_f + R_{in}} \\
\Leftrightarrow V_{out} &= -\frac{A R_f}{R_f + R_{in} + A R_{in}} V_{in}
\end{aligned}
$$

When A is very large, we obtain the following form.

$$V_{out} \approx -\frac{R_f}{R_{in}} V_{in}$$

And thus,

$$G = -\frac{R_f}{R_{in}} \tag{4}$$

B.2 | Session 2

B.2.1 | Task 2

In old western movies the wagon wheel seems to be rotating in the wrong direction because camera's frame rate is so slow that it can't properly get samples of fast rotating wagon wheel, and thus the rods have enough time between frames to move in positions that are closer to the next rod's previous position than rod's original position.

B.2.2 | Task 3

As seen in figure 4 the higher the frequency of the signal is the worse the samples fit to signal. Thus with 10 kHz signal the samples show the shape correctly, but with 250 kHz signal they do not.

B.2.3 | Task 4

Tasks 2 and 3 are actually the same phenomenon, where sampling frequency affects the collected data and thus may make it seem like something else than it is.

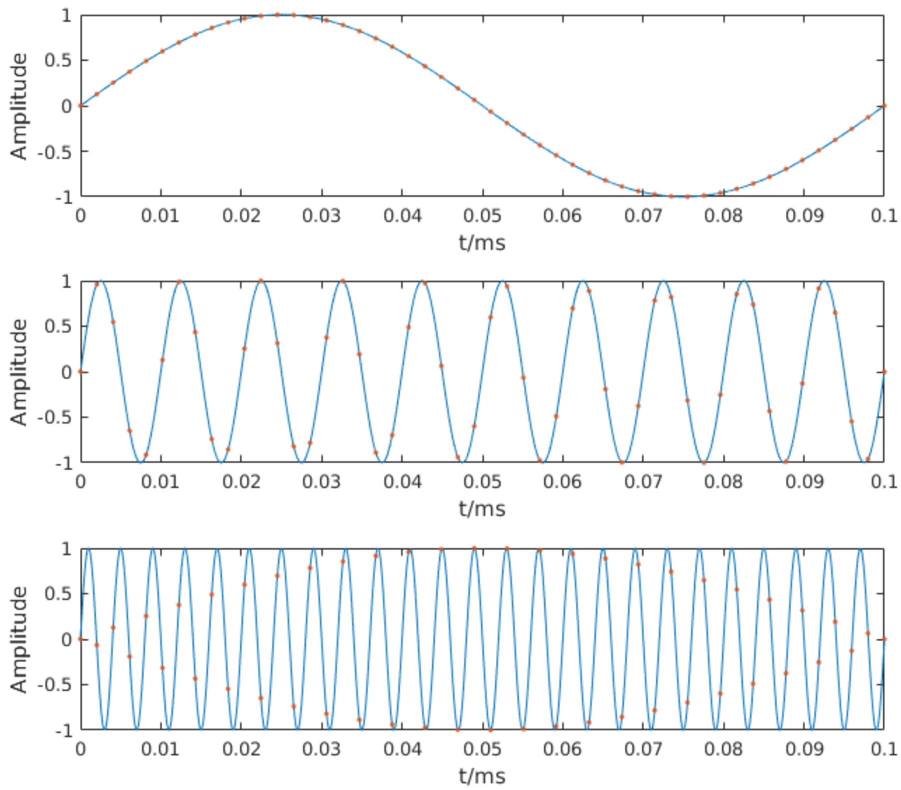

Figure 4: Sine waves (blue) and samples (red) for frequencies 10 kHz, 100 kHz and 250 kHz with sampling frequency of 500 kHz.

B.3 | Session 3

B.3.1 | Task 1

Let signal $s(t) = A\sin(2\pi(nf_s \pm f_1)\cdot t)$, where sampling frequency is f_s and thus after sampling we only take values of t which conform equation $t = k/f_s$, where $k = 0, 1, 2, 3...$ Thus the sampled signal s_s is

$$
\begin{aligned}
s_s(k) &= A\sin(2\pi k \cdot (n \pm \frac{f_1}{f_s})) \\
&= A\sin(2\pi kn \pm 2\pi k\frac{f_1}{f_s}) \\
&= A(\sin(2\pi kn)\cos(\pm 2\pi k\frac{f_1}{f_s}) + \sin(\pm 2\pi k\frac{f_1}{f_s})\cos(2\pi kn)) \\
&= A\sin(\pm 2\pi k\frac{f_1}{f_s}) \\
&= A\sin(\pm 2\pi f_1 t)
\end{aligned}
$$

Thus sampled signal of s looks the same as signal with frequency f_1.

B.3.2 | Task 2

According to Kirchoff's laws and basic operational amplifier properties,

$$
\begin{aligned}
V_- - V_{out} = R_f I_{in} \quad &\wedge \quad V_{out} = -AV_- \\
\Rightarrow -V_{out}(1 + \frac{1}{A}) &= R_f I_{in} \\
\Leftrightarrow V_{out} &= -\frac{A}{A+1} R_f I_{in}
\end{aligned}
$$

And because A is usually large,

$$I_{in} \approx \frac{V_{out}}{R_f}$$

B.3.3 | Task 3

According to previous task,

$$
\begin{aligned}
V_{out} &= -R_f I_{in} \\
\Rightarrow K &= -R_f
\end{aligned}
$$

Thus our circuit works as a current-voltage converter.

B.3.4 | Task 4

A good voltage meter must have a high impedance, because then it doesn't create an alternate route for current and thus affects the circuit as little as possible.

A good current meter on the other hand must have a low impedance, because it must let current flow through itself, and if it has a high impedance it affects the flow of current.

B.3.5 | Task 5

B.4 | Session 4

B.4.1 | Task 1

Let stone be a cube length of a side $d = 0.1\,\mathrm{m}$.

$$C = \epsilon \frac{A}{d} = \epsilon d \quad \wedge \quad Q = CU \quad \wedge \quad \frac{Q}{d^2} = e_p \frac{x}{d}$$
$$\Rightarrow \frac{\epsilon d U}{d} = e_p x$$
$$\Leftrightarrow U = \frac{e_p x}{\epsilon}$$

Also $F = \frac{EAx}{d} = Edx$, thus

$$U = \frac{e_p F}{\epsilon_0 \epsilon_r E d}$$

Assuming some values $E = 79\,\mathrm{GPa}$, $e_p = 0.18\,\mathrm{C\,m^{-2}}$, $F = 500\,\mathrm{N}$, $\epsilon_r = 4.5$ and $\epsilon_0 \approx 9 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ we obtain

$$U \approx 280\,\mathrm{V}$$

B.4.2 | Task 2

To fit to hand the length of tuning fork must be $l \approx 0.1\,\mathrm{m}$ and usually they are made of steel which has density $\rho \approx 7.7 \times 10^3\,\mathrm{kg\,m^{-3}}$ and young's modulus $E = 200\,\mathrm{GPa}$. Also they have frequency $f_0 = 440\,\mathrm{Hz}$. We solve T from equation $f_0 \approx 0.16 \frac{T}{L^2} \sqrt{\frac{E}{\rho}}$ and obtain

$$T = \frac{f_0 L^2}{0.16}\sqrt{\frac{\rho}{E}} \approx 0.5\,\mathrm{cm}$$

The result for thickness T is reasonable, so it is possible to build tuning fork with these characteristics.

B.4.3 | Task 3

We use sampling frequency of 500 kHz, which is over two times as large as signal frequency 33 kHz, so according to 2. session's pretasks it should be easily usable for this measurement.

B.5 | Session 5

B.5.1 | Task 1

B.5.2 | Task 2