Homework 3

2. (a) Equation one is first order, equation two is second order, equations three and four are first order.

3. (a) x(0) = 2 μm^3*5/μm^3 = 10 copy numbers
   y(0) = 2 copy numbers

   (b) $c_1 = k_1 = 1s^{-1}$
   $c_3 = k_3/V^6 = 1/64s^{-1}$
   $c_f = k_f/V^3 = 1/8s^{-1}$
   $c_d = k_d = 1s^{-1}$

   (c) There are $M = 4$ reaction channels.

   (d) Two species, x, y

   (e) For each reaction r, $h_1 = 1$ (there are zero reactant molecules). $h_2 = XY$, $h_3 = h_4 = X$. Where the variables denote the copy numbers. Then at the initial copy number we have that
   $a_1 = 1c_1 = 1$
   $a_2 = XYc_3 = 20/64$
   $a_3 = Xc_f = 10/8$
   $a_4 = Xc_d = 10$

   (f) Reaction two is $2x + y \to 3x$. In the initial state we have $R_2([10, 2]) = [11, 1]$. If the decay process $x \to E$ occurs we instead get $[9, 2]$

4. (a)

$$\begin{aligned}
\frac{dc_A}{dt} &= -k_+c_A(t) + k_-c_B(t) \\
&= -k_+c_A(t) + k_-(c_0 - c_A(t)) \\
\implies t &= \int \frac{dc_A}{-k_+c_A(t) + k_-(c_0 - c_A(t))}
\end{aligned}$$

$$\begin{aligned}
t &= \frac{\ln\left(|k_- \cdot (x - c_0) + k_+x|\right)}{-k_+ - k_-} \\
\implies c_A(t) &= \frac{e^{-t(k_+ + k_-)} + k_-c_0}{k_+ + k_-}
\end{aligned}$$

   (b) We assume that the ion channel is a two state system in equilibrium between open and closed. From the figure we can estimate that it takes approx. 10 ms. for the channel to switch from open to closed and 5 ms. to go from closed to open. So if A is defined to be open then we have that $k_+ = 1/10$ms and $k_- = 1/5$ms