Understanding Neural Network

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1 Understanding the Strategy of a Simple Neural Network

1.1 Abstract

In this Jupyter notebook an attempt is made to uncover the strategy of a simple neural network that is trained for classifying rotating squares and triangles (of the same size). To that end, a symbolic representative of the neural network is being studied.

1.2 About the Study

The neural network that is being studied is from `\models\nn_Linear_1024_2_Rect_Linear_2_2_SoftMax_(batc`
For more details about this particular neural network see the corresponding log file. In `understanding_nn.py` we generated and tested a sympy representative for the neural network that is much easier to study.

There are two possible shapes: squares and triangles. In the following, we assume that the shape color is given by $x \in [0,1]$ and the background is given by color $y \in [0,1]$. So if you would give the shape and the colors x and y we can approximately reproduce the picture except for the correct rotation. In `understanding_nn.py` we averaged over the rotation possibilities to obtain a symbolic representation.

In the following script the symbolic representations are loaded for the squares and triangles, resp., with colors x and y. Afterwards, the representations are printed. Small comment: I did not apply the softmax layer for simplicity. Note that due to the monotonicity of the softmax layer, a relatively large value from the last layer means a relatively large probability (and vice versa).

```
In [109]: import os
          import pickle
          import numpy as np
          from IPython.display import display
          from sympy import init_printing
          init_printing(use_latex=True)

          # load neural network representation for square and triangle, resp.
          root = os.path.dirname(os.getcwd())

          # square (Sq)
          fileNameNNReprSq = root + r'\results\sympyRepresentationOutputLayer3SqNN.pickle'
          with open(fileNameNNReprSq, 'rb') as handle:
```

```
    nnReprSq = pickle.load(handle)

# triangle (Tr)
fileNameNNReprTr = root + r'\results\sympyRepresentationOutputLayer3TrNN.pickle'
with open(fileNameNNReprTr, 'rb') as handle:
    nnReprTr = pickle.load(handle)
```

1.3 Symbolic Representation in case of a Square

```
In [110]: print 'Output for square (positively correlated with probability):'
          display(nnReprSq[0])
          print 'Output for triangle (positively correlated with probability):'
          display(nnReprSq[1])
```

```
Output for square (positively correlated with probability):
```

$$3.0\max\left(0, -9.0x + 9.0y + 0.04\right) + 4.0\max\left(0, 9.0x - 9.0y + 0.04\right) - 1.0$$

```
Output for triangle (positively correlated with probability):
```

$$-3.0\max\left(0, -9.0x + 9.0y + 0.04\right) - 2.0\max\left(0, 9.0x - 9.0y + 0.04\right) + 1.0$$

1.4 Symbolic Representation in case of a Triangle

```
In [111]: print 'Output for square (positively correlated with probability):'
          display(nnReprTr[0])
          print 'Output for triangle (positively correlated with probability):'
          display(nnReprTr[1])
```

```
Output for square (positively correlated with probability):
```

$$3.0\max\left(0, -0.06x - 0.0007y + 0.04\right) + 4.0\max\left(0, -0.04x - 0.3y + 0.04\right) - 1.0$$

```
Output for triangle (positively correlated with probability):
```

$$-3.0\max\left(0, -0.06x - 0.0007y + 0.04\right) - 2.0\max\left(0, -0.04x - 0.3y + 0.04\right) + 1.0$$

1.5 Interpreting the Symbolic Representations

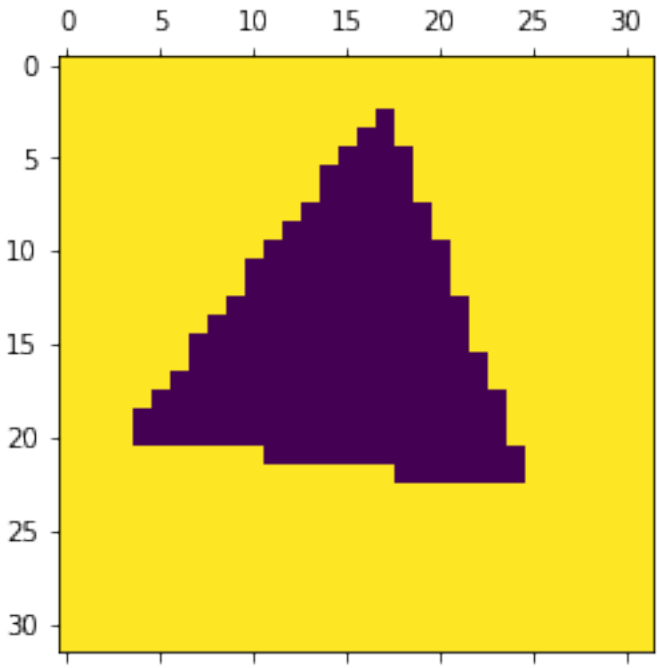
In the following we will demonstrate the symbolic representation for a test image.

1.5.1 Load the data for illustration purposes.

In [112]:
```
# I do not want to load the data every time, therefore the if statement

    # in order to import a package in parent folder
    import os,sys,inspect
    current_dir = os.path.dirname(os.path.abspath(inspect.getfile(inspect.currentframe()))
    parent_dir = os.path.dirname(current_dir)
    sys.path.insert(0, parent_dir)
    import tools, settings
    from tools import data_loader, data_analysis, model_io

    if 'X' not in locals():
        # load data
        X, Y = data_loader.load_data()
```

In [113]:
```
# get a test image (im) with known classification
    imageIdx = 0
    im = X['train'][[imageIdx]]
    classification = Y['train'][[imageIdx]]

    # determine index of a color in shape and background
    idxShapeColor = np.ravel_multi_index([16, 16], (32, 32))
    idxBackgroundColor = 0

    # report on chosen image
    print 'Chosen test image with classification: {}'.format(classification[0])
    print 'Note [1, 0] = square and [0, 1] = triangle.'
    tools.data_analysis.plot_vector_as_image(im)
```

```
Chosen test image with classification: [0. 1.]
Note [1, 0] = square and [0, 1] = triangle.
```



1.5.2 Evaluate image `im` with the neural network

```
In [114]: # load trained neural network (nn)
          nnName = 'nn_Linear_1024_2_Rect_Linear_2_2_SoftMax_(batchsize_10_number_iterations_1
          nn = model_io.read(r'' + parent_dir.replace('\\', '/') + '/' + settings.modelPath + r

          # nn prediction (nnPred)
          nnPred = nn.forward(im)
          layerOutcome = [nn.modules[0].forward(im)]
          for i in range(1, len(nn.modules)):
              layerOutcome.append(nn.modules[i].forward(layerOutcome[-1]))
          print '\n The probabilities of a square and triangle according to neural network:'
          print layerOutcome[-1]
```

```
loading plain text model from C:/Users/berkhout/Desktop/XAI/Software/Python/SimpleLRPExperiment

 The probabilities of a square and triangle according to neural network:
[[0.07858226 0.92141774]]
```

1.5.3 Evaluate image `im` with the sympy representation

```
In [115]: # determine index of a color in shape and background
          idxShapeColor = np.ravel_multi_index([16, 16], (32, 32))
          idxBackgroundColor = 0
```

```
        # determine color of shape and background
        xColor = im[0][idxShapeColor]
        yColor = im[0][idxBackgroundColor]

        # sympy output: the softmax layer is calculated here
        sympyOutputSq = Matrix(1,
                               2,
                               [exp(nnReprSq[0])/(exp(nnReprSq[0]) + exp(nnReprSq[1])),
                                exp(nnReprSq[1])/(exp(nnReprSq[0]) + exp(nnReprSq[1]))])
        sympyOutputTr = Matrix(1,
                               2,
                               [exp(nnReprTr[0])/(exp(nnReprTr[0]) + exp(nnReprTr[1])),
                                exp(nnReprTr[1])/(exp(nnReprTr[0]) + exp(nnReprTr[1]))])

        if classification[0][0] == 1:
            # square
            sympyPred = np.array(sympyOutputSq.subs([('x', xColor), ('y', yColor)])).astype(
        else:
            # triangle
            sympyPred = np.array(sympyOutputTr.subs([('x', xColor), ('y', yColor)])).astype(

        print 'The probabilities according to the symbolic representation are:'
        print sympyPred
```

```
The probabilities according to the symbolic representation are:
[[0.07866254 0.92133746]]
```