Natural Computing, Assignment 5

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a) A nash equilibrium occurs in the states ¡S,S¿ and ¡H,H¿. If we are in either of those states, no player can gain anything by changing their strategy, and in fact, they both lose 1.

The candidate ESS's are the NE's, so both ¡S,S¿ and ¡H,H¿ are candidates. They both are ESS, since $P(S,S) = 1 > 0 = P(H,S)$ and $P(H,H) = 1 > 0 = P(S,H)$.

b) Here, only the state ¡S,S¿ is a strict Nash equilibrium. The state ¡H,H¿ is a Nash equilibrium, since for both players, the best response to H has 0 payoff, although this payoff is gained by choosing either one of S or H.

The other states are not Nash equilibria, since the response to S should always be S (for both players, because of the symmetry of the game). ¡S,S¿ is also an ESS because $P(S,S) = 1 > 0 = P(H,S)$.

c) In this game the only Nash equilibrium is ¡S,S¿. The player changing to H will lose 1, so this is not the best response. And there is no ESS because $P(S,S) = 0 < 1 = P(H,S)$ and $P(H,H) = -20$ is the lowest value, so none of the condition is correct in order to be an ESS.

d)

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a) The ESSs of this game are ¡A,A¿ and ¡B,B¿ because $P(A,A) = 3 > 0 = P(B,A)$ and $P(B,B) = 1 > 0 = P(A,B)$.

b) A two-player game with strategies A,B and payoffs : $\pi(A,A) = 3$, $\pi(A,B) = 0$, $\pi(B,A) = 0$, $\pi(B,B) = 1$ and x = proportion of individuals using A.
i. The expected payoff are the following :

- $\pi(A,x) = 3x$
- $\pi(A,B) = 1 - x$

ii. $\dot{x} = x(1-x)(\pi(A,x) - \pi(B,x))$ $\dot{x} = x(1-x)(3x - (1-x))$ $\dot{x} = x(1-x)(4x-1)$
iii. Fixed point $x* = 0$, $x* = 1$, $x* = 1/4$
iv. $x* = 1$, $x* = 0$ are the evolutionary end points because everyone uses strategy A or strategy B respectively. For $x* = 1/4$, this is not stable. If there is more A, so the A replicate more fast than the B and in the other way, if there is more B, even if A has a bigger paidoff, the B replicate faster than the A.

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A two-player game with strategies H,D and payoffs : $\pi(H,H) = 1/2(V-C)$, $\pi(H,D) = V$, $\pi(D,H) = 0$, $\pi(D,D) = V/2$

$$\begin{aligned}
\dot{x} &= x(1-x)(1/2(V-C)x + V(1-x) - (V/2(1-x)\\
&= x(1-x)(V(1/2x + 1 - x - 1/2 + 1/2x) + C(-1/2x))\\
&= x(1-x)(1/2V - 1/2Cx)
\end{aligned}$$

If we remplace x with V/C, we obtain $\dot{x} = V/C * (1 - V/C)(1/2 * V - 1/2 * C * V/C) = 0$