

# Natural Computing, Assignment 5

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## 1

a) A Nash equilibrium occurs in the states  $(S, S)$  and  $(H, H)$ . If we are in either of those states, no player can gain anything by changing their strategy, and in fact, they both lose 1.

The candidate ESS's are the NE's, so both  $(S, S)$  and  $(H, H)$  are candidates. They both are ESS, since  $P(S, S) = 1 > 0 = P(H, S)$  and  $P(H, H) = 1 > 0 = P(S, H)$ .

b) Here, only the state  $(S, S)$  is a strict Nash equilibrium. The state  $(H, H)$  is a Nash equilibrium, since for both players, the best response to H has 0 payoff, although this payoff is gained by choosing either one of S or H.

The other states are not Nash equilibria, since the response to S should always be S (for both players, because of the symmetry of the game).  $(S, S)$  is also an ESS because  $P(S, S) = 1 > 0 = P(H, S)$ .

c) In this game the only Nash equilibrium is  $(S, S)$ . The player changing to H will lose 1, so this is not the best response. And there is no ESS because  $P(S, S) = 0 < 1 = P(H, S)$  and  $P(H, H) = -20$  is the lowest value, so none of the condition is correct in order to be an ESS.

d)

## 2

a) The ESSs of this game are  $(A, A)$  and  $(B, B)$  because  $P(A, A) = 3 > 0 = P(B, A)$  and  $P(B, B) = 1 > 0 = P(A, B)$ .

b) A two-player game with strategies A,B and payoffs :  $\pi(A, A) = 3$ ,  $\pi(A, B) = 0$ ,  $\pi(B, A) = 0$ ,  $\pi(B, B) = 1$  and  $x$  = proportion of individuals using A.

i. The expected payoff are the following :

- $\pi(A, x) = 3x$
- $\pi(A, B) = 1 - x$

ii.  $\dot{x} = x(1-x)(\pi(A, x) - \pi(B, x))$   $\dot{x} = x(1-x)(3x - (1-x))$   $\dot{x} = x(1-x)(4x - 1)$

iii. Fixed point  $x^* = 0$ ,  $x^* = 1$ ,  $x^* = 1/4$

iv.  $x^* = 1$ ,  $x^* = 0$  are the evolutionary end points because everyone uses strategy A or strategy B respectively. For  $x^* = 1/4$ , this is not stable. If there is more A, so the A replicate more fast than the B and in the other way, if there is more B, even if A has a bigger payoff, the B replicate faster than the A.