Многомерный анализ, интегралы и ряды

Второй семестр
Первое задание

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8.12.2023

C3.3.3

6)

$$f(x,y,z) = \left(\frac{x}{y}\right)^z$$

$$\frac{\partial f}{\partial x} = \frac{z}{x} f \qquad \frac{\partial f}{\partial y} = -\frac{z}{y} f \qquad \frac{\partial f}{\partial z} = f \ln \frac{x}{y}$$

C3.3.44

2)

$$f(x,y) = \operatorname{arctg}\left(\frac{y}{x}\right) \qquad M(1/2; \sqrt{3}/2) \qquad (x-1)^2 + y^2 = 1$$

Так как радиус перпендикулярен касательной, то направлением нормали будет вектор (единичный) $\tau = (-1/2; \sqrt{3}/2)$

$$f_\tau = (\tau, grad(f(M))) = \frac{1}{x^2+y^2}(y/2 + x\sqrt{3}/2)|_M = \frac{\sqrt{3}}{2}$$

C3.3.19

2)

$$f(x,y) = |y| \sin x$$
$$|\Delta f| \le |yx| \le \rho^2$$
$$\Delta f = o(\rho)$$

4)

$$f(x,y) = \operatorname{ch} \sqrt[5]{x^2 y}$$
$$|\Delta f| = |\operatorname{ch} \sqrt[5]{x^2 y} - 1| = |2 \operatorname{sh}^2 \frac{\sqrt[5]{x^2 y}}{2}| \le |x^{\frac{4}{5}} y^{\frac{2}{5}}| \le \rho$$
$$\Delta f = o(\rho)$$

C3.3.20

1)

$$f(x,y) = \sqrt{|xy|}$$
$$y = x \Rightarrow f(x,y) = |x| \ne o(\rho)$$

3) Аналогично

T.1

a)

$$f(x,y) = \operatorname{tg} \sqrt[3]{x^2 y^2} + e^{x+y}$$
$$|\Delta f| = |\operatorname{tg} \sqrt[3]{x^2 y^2} + e^{x+y} - 1| \ge |x^{2/3} y^{2/3} + x + y| \overset{y=x}{\Rightarrow} |x^{4/3} + 2x| \ne o(\rho)$$

b)

$$f(x,y) = \sin(x^\alpha y^{1/3})$$
$$|\Delta f| = |\sin(x^\alpha y^{1/3})| \le |x^\alpha y^{1/3}| \le \rho^{\alpha + \frac{1}{3}} \Rightarrow |\Delta f| = o(\rho) \Leftrightarrow \alpha > \frac{2}{3}$$

C3.4.4

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y} - y = -1$$
$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial x} x = 1$$

C3.4.8

2)

$$f(x, y) = \ln(x + y)$$
$$\frac{\partial^3 f}{\partial y \partial x^2} = \frac{\partial^2 f}{\partial y \partial x} \frac{1}{x + y} = \frac{\partial f}{\partial y} \frac{1}{(x + y)^2} = \frac{1}{(x + y)^3}$$

C3.4.19

$$d^2 f = \frac{\partial^2 f}{\partial x^2} dx^2 + \frac{\partial^2 f}{\partial y^2} dy^2 + \frac{\partial^2 f}{\partial z^2} dz^2 + 2\frac{\partial^2 f}{\partial x \partial y} dydx + 2\frac{\partial^2 f}{\partial z \partial x} dxdz + 2\frac{\partial^2 f}{\partial y \partial z} dzdy$$

1)

$$f(x, y, z) = \frac{z}{x^2 + y^2}$$
$$\frac{2zx^2 + 2zy^2}{(x^2 + y^2)^3} dx^2 + \frac{2zy^2 + 2zx^2}{(x^2 + y^2)^3} dy^2 + 0dz^2 + \frac{8xyz}{(x^2 + y^2)^2} dxdy - \frac{4x}{(x^2 + y^2)^2} dxdz - \frac{4y}{(x^2 + y^2)^2} dydz =$$
$$= \frac{1}{2} dx^2 + \frac{1}{2} dy^2 + 2dxdy - dxdz - dydz$$

2)

$$f(x, y, z) = \left(\frac{x}{y} \right)^{1/z}$$
$$f \frac{z^2 - z}{x^2} dx^2 + f \frac{z^2 + z}{y^2} dy^2 + f \left(\ln^2 z - \frac{1}{z} \right) dz^2 - f \frac{z^2}{zy} dxdy - f \frac{z \ln z}{x} dxdz - f \frac{z \ln z}{y} dydz =$$
$$= 2dy^2 - 2dxdy - 2dxdz + dydz$$

C3.4.25

2)

$$f(x, y, z) = \ln(x + y + z)$$
$$d^n f = \sum C_n^{\alpha,\beta,\gamma} \frac{\partial^n f}{\partial x^\alpha \partial y^\beta \partial z^\gamma} dx^\alpha dy^\beta dx^\gamma = \frac{\partial^n f}{\partial x^\alpha} \sum C_n^{\alpha,\beta,\gamma} dx^\alpha dy^\beta dx^\gamma = \frac{\partial^n f}{\partial x^n} (dx + dy + dz)^n =$$
$$= \frac{(-1)^{n-1}(n - 1)!}{(x + y + z)^{n-1}} (dx + dy + dz)^n$$

C3.4.27

$$\varphi(x,y,z) = f(u)$$
$$d^2\varphi = d^2 f = d(df) = d(f'du) = df'du + f'd^2u = f''du^2 + f'd^2u$$

2)

$$u = \sqrt{x^2+y^2}$$
$$du = \frac{x}{\sqrt{x^2+y^2}}dx + \frac{y}{\sqrt{x^2+y^2}}dy$$
$$d^2u = \frac{x^2}{\sqrt{(x^2+y^2)^3}}dx^2 - 2\frac{xy}{\sqrt{(x^2+y^2)^3}}dxdy + \frac{y^2}{\sqrt{(x^2+y^2)^3}}dy^2$$
$$d^2\varphi = f''\frac{(xdx+ydy)^2}{x^2+y^2} + f'\frac{(xdx-ydy)^2}{\sqrt{(x^2+y^2)^3}}$$

3)

$$u = xyz$$
$$du = yzdx + xzdy + xydz$$
$$d^2u = 2(xdydz + ydxdz + zdxdy)$$
$$d^2\varphi = f''(yzdx + xzdy + xydz)^2 + 2f'(xdydz + ydxdz + zdxdy)$$

C3.4.72

$$f(x,y,z) = \cos x \cos y \cos z - \cos(x+y+z)$$
$$\frac{\partial f(0,0,0)}{\partial x} = \frac{\partial f(0,0,0)}{\partial y} = \frac{\partial f(0,0,0)}{\partial z} = \frac{\partial^2 f(0,0,0)}{\partial x^2} = \frac{\partial^2 f(0,0,0)}{\partial y^2} = \frac{\partial^2 f(0,0,0)}{\partial z^2} = 0$$
$$\frac{\partial^2 f(0,0,0)}{\partial x\partial y} = \frac{\partial^2 f(0,0,0)}{\partial y\partial x} = \frac{\partial^2 f(0,0,0)}{\partial y\partial z} = \frac{\partial^2 f(0,0,0)}{\partial z\partial y} = \frac{\partial^2 f(0,0,0)}{\partial x\partial z} = \frac{\partial^2 f(0,0,0)}{\partial z\partial x} = 1$$
$$f = xy + yz + zx + o(\rho^2)$$

C3.4.73

$$f(x,y,z) = \ln(xy+z^2)$$
$$\frac{\partial f(0,0,1)}{\partial x} = \frac{\partial f(0,0,1)}{\partial y} = \frac{\partial^2 f(0,0,1)}{\partial y^2} = \frac{\partial^2 f(0,0,1)}{\partial x^2} = 0$$
$$\frac{\partial f(0,0,1)}{\partial z} = 2$$
$$\frac{\partial^2 f(0,0,1)}{\partial z^2} = -2$$
$$\frac{\partial^2 f(0,0,0)}{\partial x\partial y} = \frac{\partial^2 f(0,0,0)}{\partial y\partial x} = 1$$
$$\frac{\partial^2 f(0,0,0)}{\partial y\partial z} = \frac{\partial^2 f(0,0,0)}{\partial z\partial y} = \frac{\partial^2 f(0,0,0)}{\partial x\partial z} = \frac{\partial^2 f(0,0,0)}{\partial z\partial x} = 0$$
$$f = 2(z-1) - (z-1)^2 + xy + o(\rho^2)$$

T2

a)$2^{|\mathbb{R}|}$ b) 4 $(\pm|x|, \pm x)$ c) 2 $(|x|, x)$ d) 1 (x)

C3.3.66

$$F(x,y,z,u) = u^3 - 3(x+y)u^2 + z^3 = 0$$
$$f = u(x,y,z)$$
$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial x} = -3u^2$$
$$\frac{\partial F}{\partial z} = 3z^2$$
$$\frac{\partial F}{\partial u} = 3u^2 - 6(x+y)u$$
$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz = -\frac{F'_x}{F'_u}dx - \frac{F'_y}{F'_u}dy - \frac{F'_z}{F'_u}dz = \frac{u^2(x+y) - z^2dz}{u^2 - 2(x+y)u}$$

C3.3.78

$$z = u^3 + v^3 \qquad u + v = x \qquad u^2 + v^2 = y$$
$$z = u^3 + v^3 = (u+v)(u^2 - uv + v^2) = x(x^2 - 3\frac{x^2 - y}{2}) = \frac{3xy - x^3}{2}$$
$$dz = 3\frac{y - x^2}{2}dx + 3\frac{x}{2}dy$$

C3.3.82

2*)

$$\frac{\partial x_i(x^0)}{\partial x_{i+1}} = -\frac{f'_{x_{i+1}}}{f'_{x_i}} \Rightarrow \frac{\partial x_n(x^0)}{\partial x_1}\prod_{i=1}^{n-1}\frac{\partial x_i(x^0)}{\partial x_{i+1}} = (-1)^n$$

C3.3.103

1)

$$u = x(x^2 - 3y^2) \qquad v = y(3x^2 - y^2)$$
$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 3x^2 - 3y^2 & -6xy \\ 6xy & 3x^2 - 3y^2 \end{vmatrix} = 9(x^2 + y^2)^2$$

C3.4.44

2)

$$x + y + u = e^u \Rightarrow u = \ln(x + y + u)$$
$$du = \frac{dx + dy + du}{x + y + u} \Rightarrow du = \frac{dx + dy}{x + y + u - 1}$$
$$d^2u = -\frac{(dx + dy)^2 + du(dx + dy)}{(x + y + u - 1)^2} = \frac{x + y + u}{(1 - x - y - u)^3}(dx + dy)^2$$

3)

$$u = \ln(yu - x)$$

$$du = \frac{udy + ydu - dx}{yu - x} \Rightarrow du = \frac{udy - dx}{yu - y - x}$$

$$d^2u = \frac{dudy(yu - y - x) - (udy + ydu - dy - dx)(udy - dx)}{(yu - y - x)^2}$$

T.3

$$\begin{cases} u(x,y) = e^x \cos y \\ v(x,y) = e^x \sin y \end{cases}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} e^x \cos y & -e^x \sin y \\ e^x \sin x & e^x \cos x \end{vmatrix} = e^{2x} > 0$$

Докажем, что данное отображение не является взаимно однозначным.

$$\begin{cases} u(x_1,y_1) = e^{x_1} \cos y_1 = e^{x_2} \cos y_2 = u(x_2,y_2) \\ v(x,y) = e^{x_1} \sin y_1 = e^{x_2} \sin y_2 = v(x_2,y_2) \end{cases}$$

Данное выражение выполняется, например, когда $\operatorname{tg} y_1 = \operatorname{tg} y_2 \Leftrightarrow y_1 = \pi + y_2$
Также заметим, что $v = u \operatorname{tg} y$, так что $f : \mathbb{R}^2 \to \mathbb{R}^2/\{0\}$

T.4

$$(x_1, x_2) \overset{g}{\mapsto} (u_1, u_2) \overset{h}{\mapsto} (y_1, y_2)$$

Пусть:

$$\frac{\partial(u_1,u_2)}{\partial(x_1,x_2)} = \begin{vmatrix} a & b \\ 0 & 1 \end{vmatrix} \qquad \frac{\partial(y_1,y_2)}{\partial(u_1,u_2)} = \begin{vmatrix} 1 & 0 \\ c & d \end{vmatrix}$$

Тогда:

$$\frac{\partial(y_1,y_2)}{\partial(x_1,x_2)} = \begin{vmatrix} 1 & 2x_2 \\ 2x_1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ c & d \end{vmatrix} \begin{vmatrix} a & b \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} a & b \\ ac & bc + d \end{vmatrix} = \frac{\partial(y_1,y_2)}{\partial(u_1,u_2)} \frac{\partial(u_1,u_2)}{\partial(x_1,x_2)}$$

Решая систему получаем:

$$a = 1 \qquad b = 2x_2 \qquad c = 2x_1 \qquad d = 1 - 4x_1x_2$$

C2.1.2

15)

$$\int \sin^2 \frac{x}{2} dx = \int \frac{1 - \cos x}{2} dx = \frac{x}{2} - \frac{\sin x}{2} + C$$

16)

$$\int \operatorname{ctg}^2 x dx = \int \frac{\cos^2 x}{\sin^2 x} dx = \int \left(\frac{1}{\sin^2 x} - 1 \right) dx = -\operatorname{ctg} x - x + C$$

C2.1.12

2)

$$\int x\sqrt{x+1} dx = \int (t-1)\sqrt{t} dt = \frac{2t^{5/2}}{5} - \frac{2t^{3/2}}{3} + C = \frac{2\sqrt{(x+1)^5}}{5} + \frac{2\sqrt{(x+1)^3}}{3} + C$$

C2.1.17

4)

$$\int x \ln x dx = \frac{1}{2}\int \ln x d(x^2) = \frac{1}{2}x^2 \ln x - \int x dx = \frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) \ln + C$$

C2.1.24

3)

$$\int e^{ax}\sin(bx)dx = \frac{1}{a}e^{ax}\sin(bx) - \frac{b}{a^2}\int e^{ax}\cos(bx)dx = \frac{1}{a}e^{ax}\sin(bx) - \frac{b}{a^2}e^{ax}\cos(bx) - \frac{b^2}{a^2}\int e^{ax}\sin(bx)dx$$

$$\int e^{ax}\sin(bx)dx = e^{ax}\frac{a\sin(bx) - b\cos(bx)}{a^2+b^2} + C$$

C2.2.1

4)

$$\int \frac{dx}{(x+1)(x-2)} = \frac{1}{3}\int \frac{dx}{x-2} - \frac{1}{3}\int \frac{dx}{x+1} = \frac{1}{3}\ln(x-2) - \frac{1}{3}\ln(x+1) + C$$

C2.2.2

1)

$$\int \frac{dx}{(x-1)(x+2)(x+3)} = \frac{1}{12}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{dx}{x+2} + \frac{1}{4}\int \frac{dx}{x+3} =$$

$$\frac{1}{12}\ln(x-1) - \frac{1}{3}\ln(x+2) + \frac{1}{4}\ln(x+3) + C$$

C2.2.3

2)

$$\int \frac{(x^2)dx}{(x-1)(x+1)^2} = \frac{3}{4}\int \frac{dx}{x-1} + \frac{1}{4}\int \frac{dx}{x+1} - \frac{3}{2}\int \frac{dx}{(x+1)^2} =$$

$$= \frac{3}{4}\ln(x-1) + \frac{1}{4}\ln(x+1) + \frac{3}{2(x+1)} + C$$

C2.2.4

2)

$$\int \frac{dx}{x^3+1} = \frac{1}{3}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{x-2}{x^2-x+1}dx =$$

$$\frac{1}{3}\ln(x+1) - \frac{1}{6}\ln(x^2-x+1) + \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}} + C$$

C2.2.5

2)

$$\int \frac{x^4dx}{1-x^4} = -\int dx + \int \frac{dx}{1-x^4} = -x + \frac{1}{2}\left(\int \frac{dx}{x^2+1} + \int \frac{dx}{1-x^2}\right) =$$
$$= -x + \frac{1}{2}\operatorname{arctg} x + \frac{1}{4}\ln\frac{x+1}{x-1} + C$$

C2.3.2

2)

$$\int \frac{dx}{3x+\sqrt[3]{x^2}} = \int \frac{t^2dt}{3t^3+t^2} = \int \frac{3dt}{3t+1} = \ln(3t+1) + C = \ln(3\sqrt[3]{x}+1) + C$$

C2.3.5

1)

$$\int \frac{1-x+x^2}{\sqrt{1+x-x^2}}$$
$$\frac{1-x+x^2}{\sqrt{1+x-x^2}} = A\sqrt{1+x-x^2} + \frac{Ax+C}{2\sqrt{1+x-x^2}} + \frac{\lambda}{\sqrt{1+x-x^2}}$$
$$\int \frac{1-x+x^2}{\sqrt{1+x-x^2}} = -\frac{1}{2}(x+1)\sqrt{1+x-x^2} + \int \frac{7dx}{4\sqrt{1+x-x^2}} =$$
$$= -\frac{1}{4}(x+1)\sqrt{1+x-x^2} + \frac{11}{8}\arcsin\left(\frac{2\sqrt{5}x-\sqrt{5}}{5}\right) + C$$

C2.3.18

3)

$$\int \frac{\sqrt[3]{1+\sqrt[4]{x}}dx}{\sqrt{x}} = \int \frac{\sqrt{1+t}}{t^2}4t^3dt = \int 4t\sqrt{1+t}dt = \int 3s^3(4s^3-4)ds = \frac{12}{7}s^7 - 3s^4 + C =$$
$$= \frac{12}{4}(1+\sqrt[4]{x})^{7/3} - 3(1+\sqrt[4]{x})^{4/3} + C$$

C2.3.19

2)

$$\int \frac{dx}{x^3\sqrt[3]{2-x^3}} = \frac{1}{2}\int \frac{x^4t^2dt}{x^4t} = \frac{1}{4}t^2 + C = \frac{1}{3}\sqrt[3]{\left(\frac{2-x^3}{x^3}\right)^2} + C$$

C2.4.4

1)

$$\int \cos^3 x dx = \int \cos^2 d(\sin x) = \int (1-\sin^2 x)d(\sin x) = \sin x - \frac{\sin^3 x}{3} + C$$

C2.4.15

6)

$$\int \frac{dx}{1-\operatorname{th} x} = \int \frac{dt}{(1-t)^2(1+t)} = \frac{1}{4}\int \frac{dt}{1+t} - \frac{1}{4}\int \frac{dt}{1-t} - \frac{1}{2}\int \frac{dt}{(1-t)^2} =$$

$$= \frac{1}{2}\frac{1}{1-t} + \frac{1}{4}\ln\frac{1+t}{1-t} + C$$

C2.4.16

1)

$$\int \frac{dx}{2\cos^2 x + \sin x\cos x + \sin^2 x} = \int \frac{d(\operatorname{tg} x)}{2+\operatorname{tg} x + \operatorname{tg}^2 x} = \frac{2}{\sqrt{7}}\operatorname{arctg}\frac{2\operatorname{tg} x + 1}{\sqrt{7}} + C$$

C2.4.21

2)

$$\int \frac{dx}{4+\cos x} = \int \frac{dx}{3\sin^2\frac{x}{2} + 5\cos^2\frac{x}{2}} = \int \frac{2dt}{3t^2+5} = \frac{2}{\sqrt{15}}\operatorname{arctg}\left(\sqrt{\frac{3}{5}}\operatorname{tg}\frac{x}{2}\right) + C$$

C2.5.144

$$\int \left(\frac{\sin x}{e^x}\right)^2 dx = \int e^{-2x}\left(\frac{1}{2} - \frac{\cos 2x}{2}\right)dx = \frac{1}{4}e^{-2x}\sin 2x - \frac{1}{2}\int e^{-2x}d(\cos 2x) =$$

$$= \frac{1}{4}e^{-2x}\sin 2x - \frac{1}{4}e^{-2x}\cos 2x + \frac{1}{2}\int e^{-2x}\cos 2x dx$$

$$\int e^{-2x}\frac{\cos 2x}{2}dx = \frac{e^{-2x}}{8}(\sin 2x - \cos 2x) + C$$

$$\int \left(\frac{\sin x}{e^x}\right)^2 dx = \frac{e^{-2x}}{8}(\cos 2x - \sin 2x - 2) + C$$

C2.5.180

$$\int \frac{\arcsin x dx}{(1-x^2)\sqrt{1-x^2}} = \int \frac{\arcsin x d(\arcsin x)}{1-x^2} = \int \frac{tdt}{1-\sin^2 t} = \int td(\operatorname{tg} t) =$$

$$t\operatorname{tg} t - \int \operatorname{tg} tdt = t\operatorname{tg} t + \ln\cos t = \arcsin x \operatorname{tg}\arcsin x + \ln\cos\arcsin x =$$

$$= \frac{x}{\sqrt{1-x^2}}\arcsin x + \frac{1}{2}ln(1-x^2)$$

C2.5.188

$$\int \frac{x^2\arccos(x\sqrt{x})}{(1-x^3)^2}dx =$$

C2.13.2

1)

$$\sum_{k=1}^{n}\frac{1}{(k+2)(k+3)}=\sum_{k=1}^{n}\frac{1}{k+2}-\sum_{k=1}^{n}\frac{1}{k+3}=\frac{1}{3}-\frac{1}{n+3}$$

$$S=\frac{1}{3}$$

C2.13.10

2)

$$\sum_{n=1}^{\infty}a^n\sin n\alpha=\operatorname{Im}\sum_{n=1}^{\infty}a^ne^{in\alpha}=\frac{a\sin\alpha}{1-2a\cos\alpha+a^2}$$

C2.14.25

9)

$$S=\sum_{n=2}^{\infty}\frac{1}{n^\alpha\ln^\beta n}$$

C2.14.2

6)

$$\sum_{n=1}^{\infty}\frac{\ln n+\sin n}{n^2+2\ln n}\le\sum_{n=1}^{\infty}\frac{\sqrt{n}+1}{n^2+0}=\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}+\sum_{n=1}^{\infty}\frac{1}{n^2}$$

Данный ряд сходится.

7)

$$\sum_{n=1}^{\infty}\frac{n+2}{n^2(4+3\sin\left(\frac{\pi n}{3}\right))}\ge\sum_{n=1}^{\infty}\frac{n+2}{7n^2}=\sum_{n=1}^{\infty}\frac{1}{7n}+\sum_{n=1}^{\infty}\frac{2}{7n^2}$$

Данный ряд расходится.

C2.14.9

6) 8)

C2.14.19

8)

$$a_n=\frac{(2n)!!}{n!}\operatorname{arctg}\left(\frac{1}{3^n}\right)$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(2(n+1))!!n!\operatorname{arctg}\frac{1}{3^{n+1}}}{(2n)!!(n+1)!\operatorname{arctg}\frac{1}{3^n}}=\lim_{n\to\infty}2\frac{\operatorname{arctg}\frac{1}{3^{n+1}}}{\operatorname{arctg}\frac{1}{3^n}}=\lim_{n\to\infty}2\frac{3^n}{3^{n+1}}=\frac{2}{3}<1$$

Ряд сходится по признаку Даламбера.

C2.14.20

1)

$$a_n = \frac{(3n)!}{(n!)^3 4^{3n}}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(3(n+1))!(n!)^3 4^{3n}}{(3n)!((n+1)!)^3 4^{3(n+1)}} = \lim_{n\to\infty} \frac{(3n+1)(3n+2)(3n+3)}{64(n+1)^3} =$$

$$= \frac{27}{64} \lim_{n\to\infty} \frac{(n+1/3)(n+2/3)}{(n+1)^2} = \frac{27}{64} < 1$$

Ряд сходится по признаку Даламбера.

C2.14.21

6)

$$a_n = 3^{n+1} \left(\frac{n+2}{n+3} \right)^{n^2}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} 3^{\frac{n+1}{n}} \left(\frac{n+2}{n+3} \right)^{n} = \frac{3}{e} > 1$$

Ряд расходится по признаку Коши. 12)

$$a_n = \left(n \operatorname{sh} \frac{1}{n} \right)^{-n^3}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \left(n \operatorname{sh} \frac{1}{n} \right)^{-n^2} = \frac{3}{e} > 1$$

C2.14.38*

Из монотонности последовательности имеем $S_n \geq n a_n$, откуда:

$$S_{k+n} - S_k \geq (k+n)a_{k+n} - k a_k \geq (k+n)a_{k+n} - k a_{k+n} = n a_{k+n}$$

В предельном переходе:

$$\lim_{k\to\infty} \lim_{n\to\infty} S_{k+n} - S_k = 0 \geq \lim_{k\to\infty} \lim_{n\to\infty} n a_n = \lim_{n\to\infty} n a_n \geq \lim_{k\to\infty} \lim_{n\to\infty} n a_{k+n} \geq 0$$

T.5

C2.15.3

По признаку Лейбница ряды 4) и 5) сходятся.

C2.15.8

3) 4)

C2.15.9

2)

C2.15.15

2*)

C2.16.4

1)Следует из аналогиченой теоремы для последовательностей.
2) Контрпример - $c_n = a_n - b_n = a_n - a_n = 0$.

C2.16.29*

T.6

T.7*