8.012 Problem Set 1
Link

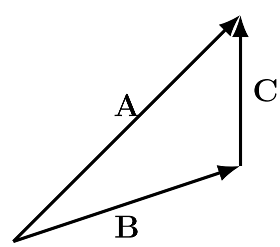
- **1.** *Kleppner & Kolenkow, Problem 1.2*

  Let $\mathbf{A} = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ and $\mathbf{B} = (6\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$.

  - **(a)** $\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A} = 3^2 + (-2)^2 + 5^2 = 9 + 4 + 25 = \mathbf{38}$
  - **(b)** $\mathbf{B}^2 = \mathbf{B} \cdot \mathbf{B} = 6^2 + (-7)^2 + 4^2 = 36 + 49 + 16 = \mathbf{101}$
  - **(c)** $(\mathbf{A} \cdot \mathbf{B})^2 = \big((3*6) + (-2*-7) + (5*4)\big)^2 = (18 + 14 + 20)^2 = 52^2 = \mathbf{2704}$

- **2.** *Kleppner & Kolenkow, Problem 1.7*

  Let **A**, **B**, **C** define a triangle, as below.



We have that

$$\begin{aligned}\mathbf{A} &= \mathbf{B} + \mathbf{C}\\ \mathbf{B} &= \mathbf{A} - \mathbf{C}\\ \mathbf{C} &= \mathbf{A} - \mathbf{B}\end{aligned}$$

then

$$\begin{aligned}|\mathbf{A} \times \mathbf{B}| &= AB \sin\theta_{\mathbf{AB}} = |(\mathbf{B} + \mathbf{C}) \times \mathbf{B}| = |\mathbf{C} \times \mathbf{B}|\\ |\mathbf{C} \times \mathbf{B}| &= CB \sin\theta_{\mathbf{CB}} = |\mathbf{C} \times (\mathbf{A} - \mathbf{C})| = |\mathbf{C} \times \mathbf{A}|\\ |\mathbf{C} \times \mathbf{A}| &= CA \sin\theta_{\mathbf{CA}}\end{aligned}$$

Thus,

$$AB \sin\theta_{\mathbf{AB}} = CB \sin\theta_{\mathbf{CB}} = CA \sin\theta_{\mathbf{CA}}$$

Dividing by ABC, we see:

$$\frac{\sin\theta_{\mathbf{AB}}}{C} = \frac{\sin\theta_{\mathbf{CB}}}{A} = \frac{\sin\theta_{\mathbf{CA}}}{B}$$

as desired.

- **3.** *Kleppner & Kolenkow, Problem 1.12*

  Let $\mathbf{r}_1$ and $\mathbf{r}_2$ be two points separated by distance $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Then $\mathbf{r}_2 - \mathbf{r}_1$ points in the direction from $\mathbf{r}_1$ to $\mathbf{r}_2$. If x is any number, then $(\mathbf{r}_2 - \mathbf{r}_1)x$ points in the same (or opposite) direction, but has magnitude xr (recall that $(\mathbf{r}_2 - \mathbf{r}_1)$ has magnitude r by definition). Then $\mathbf{A} = \mathbf{r}_1 + (\mathbf{r}_2 - \mathbf{r}_1)x$ points from the origin to a point on the line between $\mathbf{r}_1$ and $\mathbf{r}_2$ at a distance x from $\mathbf{r}_1$.

- **4.** *Kleppner & Kolenkow, Problem 1.13*

- **9.**

  - **(a)**
  - **(b)** Observe that the both trains travel at speed v over a distance $d/2$, and meet after a time $d/(2v)$. The bee thus gets crushed at time $d/(2v)$, and travels at speed u until then. It's obvious then that the bee travels a distance of $ud/(2v)$ in total.