Tecnológico de Costa Rica

Computer Science
Operations Research

Knapsack Problem

Dynamic Programming

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1 Knapsack Problem

The *Knapsack problem* is a classic optimization problem. The goal is to fill a backpack optimally with a set of objects, each with a weight and a profit, in order to maximize the total profit without exceeding the backpack's capacity.

There are a few main types of knapsack problems:

- **0/1 Knapsack:** Each object can be taken or not, only one copy per object.
- **Bounded Knapsack:** Each object has a limited number of copies.
- **Unbounded Knapsack:** Each object can be taken any number of times, as long as the total weight allows.

1.1 Solution

A common way to solve these problems is using dynamic programming. We build a table to keep track of the optimal profit for different capacities and numbers of objects. By filling this table, we can find the maximum profit achievable for the given backpack capacity. The resulting table will show the exact quantity and which objets take in order to archive the optimal weight.

2 Problem

Agua: Amount:30, Profit:1, and Cost:2

Barrita: Amount: ∞, Profit:2, and Cost:13

Linterna: Amount:10, Profit:5, and Cost:5

Encendedor: Amount: ∞, Profit:3, and Cost:3

Kit de Costura: Amount:1, Profit:4, and Cost:7

This translates to:

Maximize $Z = 1X_{\text{Agua}} + 2X_{\text{Barrita}} + 5X_{\text{Linterna}} + 3X_{\text{Encendedor}} + 4X_{\text{Kit de Costura}}$

Subject to:

$20 \geq 2X_{\text{Agua}} + 13X_{\text{Barrita}} + 5X_{\text{Linterna}} + 3X_{\text{Encendedor}} + 7X_{\text{Kit de Costura}}$

$X_{\text{Agua}} \leq 30$

$X_{\text{Barrita}} \leq \infty$

$X_{\text{Linterna}} \leq 10$

$X_{\text{Encendedor}} \leq \infty$

$X_{\text{Kit de Costura}} \leq 1$

3 Costs Table

Capacity	Agua	Barrita	Linterna	Encendedor	Kit de Costura
0	0 x=0	0 x=0	0 x=0	0 x=0	0 x=0
1	0 x=0	0 x=0	0 x=0	0 x=0	0 x=0
2	1 x=1	1 x=0	1 x=0	1 x=0	1 x=0
3	1 x=1	1 x=0	1 x=0	3 x=1	3 x=0
4	2 x=2	2 x=0	2 x=0	3 x=1	3 x=0
5	2 x=2	2 x=0	5 x=1	5 x=0	5 x=0
6	3 x=3	3 x=0	5 x=1	6 x=2	6 x=0
7	3 x=3	3 x=0	6 x=1	6 x=0,2	6 x=0
8	4 x=4	4 x=0	6 x=1	8 x=1	8 x=0
9	4 x=4	4 x=0	7 x=1	9 x=3	9 x=0
10	5 x=5	5 x=0	10 x=2	10 x=0	10 x=0
11	5 x=5	5 x=0	10 x=2	11 x=2	11 x=0
12	6 x=6	6 x=0	11 x=2	12 x=4	12 x=0
13	6 x=6	6 x=0	11 x=2	13 x=1	13 x=0
14	7 x=7	7 x=0	12 x=2	14 x=3	14 x=0
15	7 x=7	7 x=0	15 x=3	15 x=0,5	15 x=0
16	8 x=8	8 x=0	15 x=3	16 x=2	16 x=0
17	8 x=8	8 x=0	16 x=3	17 x=4	17 x=0
18	9 x=9	9 x=0	16 x=3	18 x=1,6	18 x=0
19	9 x=9	9 x=0	17 x=3	19 x=3	19 x=0
20	10 x=10	10 x=0	20 x=4	20 x=0,5	20 x=0

4 Optimal Solutions

$X_{\text{Agua}} = 0\ X_{\text{Barrita}} = 0\ X_{\text{Linterna}} = 4\ X_{\text{Encendedor}} = 0\ X_{\text{Kit de Costura}} = 0$
$X_{\text{Agua}} = 0\ X_{\text{Barrita}} = 0\ X_{\text{Linterna}} = 1\ X_{\text{Encendedor}} = 5\ X_{\text{Kit de Costura}} = 0$