Computer Science
Operations Research

Knapsack Problem

Dynamic Programming

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1 Knapsack Problem

The *Knapsack problem* is a classic optimization problem. The goal is to fill a backpack optimally with a set of objects, each with a weight and a profit, in order to maximize the total profit without exceeding the backpack's capacity.

There are a few main types of knapsack problems:

- **0/1 Knapsack:** Each object can be taken or not, only one copy per object.
- **Bounded Knapsack:** Each object has a limited number of copies.
- **Unbounded Knapsack:** Each object can be taken any number of times, as long as the total weight allows.

1.1 Solution

A common way to solve these problems is using dynamic programming. We build a table to keep track of the optimal profit for different capacities and numbers of objects. By filling this table, we can find the maximum profit achievable for the given backpack capacity. The resulting table will show the exact quantity and which objets take in order to archive the optimal weight.

2 Problem

A: Amount:1, Profit:7, and Cost:3
B: Amount:1, Profit:9, and Cost:4
C: Amount:1, Profit:5, and Cost:2
D: Amount:1, Profit:12, and Cost:6
E: Amount:1, Profit:14, and Cost:7
F: Amount:1, Profit:6, and Cost:3
G: Amount:1, Profit:12, and Cost:5
This translates to:
Maximize $Z = 7X_{\mathrm{A}} + 9X_{\mathrm{B}} + 5X_{\mathrm{C}} + 12X_{\mathrm{D}} + 14X_{\mathrm{E}} + 6X_{\mathrm{F}} + 12X_{\mathrm{G}}$
Subject to:
$9 \geq 3X_{\mathrm{A}} + 4X_{\mathrm{B}} + 2X_{\mathrm{C}} + 6X_{\mathrm{D}} + 7X_{\mathrm{E}} + 3X_{\mathrm{F}} + 5X_{\mathrm{G}}$
$X_{\mathrm{A}} \leq 1$
$X_{\mathrm{B}} \leq 1$
$X_{\mathrm{C}} \leq 1$
$X_{\mathrm{D}} \leq 1$
$X_{\mathrm{E}} \leq 1$
$X_{\mathrm{F}} \leq 1$
$X_{\mathrm{G}} \leq 1$

3 Costs Table

Capacity	A	B	C	D	E	F	G
0	0 x=0	0 x=0	0 x=0	0 x=0	0 x=0	0 x=0	0 x=0
1	0 x=0	0 x=0	0 x=0	0 x=0	0 x=0	0 x=0	0 x=0
2	0 x=0	0 x=0	5 x=1	5 x=0	5 x=0	5 x=0	5 x=0
3	7 x=1	7 x=0	7 x=0	7 x=0	7 x=0	7 x=0	7 x=0
4	7 x=1	9 x=1	9 x=0	9 x=0	9 x=0	9 x=0	9 x=0
5	7 x=1	9 x=1	12 x=1	12 x=0	12 x=0	12 x=0	12 x=0,1
6	7 x=1	9 x=1	14 x=1	14 x=0	14 x=0	14 x=0	14 x=0
7	7 x=1	16 x=1	16 x=0	16 x=0	16 x=0	16 x=0	17 x=1
8	7 x=1	16 x=1	16 x=0	17 x=1	17 x=0	18 x=1	19 x=1
9	7 x=1	16 x=1	21 x=1	21 x=0	21 x=0	21 x=0	21 x=0,1

4 Optimal Solutions

$X_\mathrm{A} = 1\, X_\mathrm{B} = 1\, X_\mathrm{C} = 1\, X_\mathrm{D} = 0\, X_\mathrm{E} = 0\, X_\mathrm{F} = 0\, X_\mathrm{G} = 0$
$X_\mathrm{A} = 0\, X_\mathrm{B} = 1\, X_\mathrm{C} = 0\, X_\mathrm{D} = 0\, X_\mathrm{E} = 0\, X_\mathrm{F} = 0\, X_\mathrm{G} = 1$