Computer Science
Operations Research

Replacement Problem

Dynamic Programming

Group 40
Professor: Francisco Torres Rojas

Carmen Hidalgo Paz
Id: 2020030538

Melissa Carvajal Charpentier
Id: 2022197088

Josué Soto González
Id: 2023207915

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1 Replacement Problem

The *Replacement Problem* involves determining the optimal time to replace a tool or piece of equipment that deteriorates with use. As equipment ages, its efficiency decreases and operating and maintenance costs rise, while replacing it incurs an immediate acquisition cost. The objective is to minimize the total accumulated cost over a given planning horizon by balancing these two factors.

Several variations of the problem may include additional real-world considerations, such as:

- Expected annual profits
- Inflation rate
- Emergence of more modern or efficient equipment

1.1 Solution Approach

To solve this problem, a dynamic programming approach based on Bellman's equation is used. The Bellman equation is defined as:

$$G(t) = \min(C(t, x) + G(x))$$

Where:

- $G(t)$ is the minimum total cost from year t to the end of the time horizon.
- $C(t, x)$ represents the cost of operating the equipment from year t to x, considering maintenance, replacement, and inflation.
- x is the next possible year for replacement.

The algorithm works backwards from the final year, evaluating two options at each step: keeping the current equipment or replacing it. The option with the lower cost is selected to build the optimal policy.

1.2 Data Structures

Three main tables are used in the implementation:

- **C:** Stores individual operating costs between any two years, accounting for inflation and other variables.
- **G:** Contains the minimum accumulated cost from each year onward, computed using dynamic programming.
- **GPOS:** Stores the optimal replacement policy, indicating in which years replacements should occur.

2 Problem

Time of the duration of the project: 5 Lifespan of the equipment: 3
Price of new equipment: 100.00 Earnings: 0.00 Inflation rate: 0.00

Time passed	Maintenance	Maintenance (accumulative)	Selling price	Additional cost for inflation
1	10.00	10.00	50.00	0.00
2	20.00	30.00	30.00	0.00
3	40.00	70.00	10.00	0.00

2.1 Table of Costs $C_{\mathbf{ij}}$

The table represents with a number the cost from buying a new bicicle on the year i and selling it on the year j where the maintenance costs are already included and the ÿear 0m̈arks the start of the project. The table has - where a value is invalid either due to the lifespan of the equipment or the dration of the project.

C	j=0	j=1	j=2	j=3	j=4	j=5
i=0	–	60.00 $	100.00 $	160.00 $	–	–
i=1	–	–	60.00 $	100.00 $	160.00 $	–
i=2	–	–	–	60.00 $	100.00 $	160.00 $
i=3	–	–	–	–	60.00 $	100.00 $
i=4	–	–	–	–	–	60.00 $
i=5	–	–	–	–	–	–

3 Solution

In the following text, you will find the needed operations to determinate the optimal solution for each year.

G(4) = min {
C[4][5] + G[5] = 60.00 + 0.00 = 60.00
}
G(3) = min {
C[3][4] + G[4] = 60.00 + 60.00 = 120.00
C[3][5] + G[5] = 100.00 + 0.00 = 100.00
}
G(2) = min {
C[2][3] + G[3] = 60.00 + 100.00 = 160.00
C[2][4] + G[4] = 100.00 + 60.00 = 160.00
C[2][5] + G[5] = 160.00 + 0.00 = 160.00
}
G(1) = min {
C[1][2] + G[2] = 60.00 + 160.00 = 220.00
C[1][3] + G[3] = 100.00 + 100.00 = 200.00
C[1][4] + G[4] = 160.00 + 60.00 = 220.00

}
G(0) = min {
C[0][1] + G[1] = 60.00 + 200.00 = 260.00
C[0][2] + G[2] = 100.00 + 160.00 = 260.00
C[0][3] + G[3] = 160.00 + 100.00 = 260.00
}

3.1 Results

G(0) = 260.000000
Winners: 1 2 3
G(1) = 200.000000
Winners: 3
G(2) = 160.000000
Winners: 3 4 5
G(3) = 100.000000
Winners: 5
G(4) = 60.000000
Winners: 5
G(5) = 0.000000
Nothing more to be done

4 Graph