

The Book of Statistical Proofs

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This table of contents is necessarily incomplete and only serves as an illustration of the TBSP.

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