

# Surrogate Problem Evaluation and Selection for Optimization with Expensive Function Evaluations

Rodrigo César Pedrosa Silva

Doctor of Philosophy

Department of Electrical and Computer Engineering

McGill University

Montreal, Quebec

February, 2018

A thesis submitted to McGill University in partial fulfillment of the requirements of the  
degree of Doctor of Philosophy

©Rodrigo César Pedrosa Silva, 2018

## **DEDICATION**

This thesis is dedicated to my parents, Marcelo and Maria Lucia.

## **ACKNOWLEDGEMENTS**

First and foremost, I would like to thank Prof. David A. Lowther, for his careful and critical scientific guidance and also for stimulating and giving the time for a personal construction of this work.

My gratitude is also extended to my colleagues in the Computational Electromagnetics Laboratory at McGill for the stimulating discussions and intense knowledge exchange. In particular, I would like to thank Mr. Mohammad Hussain Mohammadi and Dr. Tanvir Rahman for the help with some of the motor designs and simulations, and Dr. Armin Salimi and Mr. Vahid Ghorbanian for providing part of electric motor design data that has greatly enriched this thesis.

Finally, I would like to thank McGill University's staff without whom this work would not have been possible. In particular, I would like to thank Mr. David Robles and Ms. Melanie Gorman for all the patience and help dedicated to my thesis submission and defense.

## ABSTRACT

Despite the advances in simulation, the use of high-fidelity models may limit the application of standard optimization methods due to their high computational cost. To overcome these problems, the literature usually relies on the use of surrogate models or objective reduction. In order to select a surrogate model or a reduced set of objectives, one has to evaluate their suitability as substitutes for the original problem. The number of performance metrics available is vast, but most of them are not generally applicable for both surrogate models and reduced problems. Besides, for most of the metrics, there is no theory that justifies their use in surrogate-based optimization. In this context, this thesis achieves the following: (i) unifies the performance assessment of surrogate models and reduced problems, (ii) investigates different performance metrics and provides formal justification for their use in the scope of optimization; and (iii) proposes a new set of metrics for generalizing the concept of order maintenance for multi-objective problems. The results for a number of well-known benchmark problems show that some of the proposed metrics can identify higher quality surrogate problems in most of the cases and are less sensitive to the sample data when compared to other metrics. Finally, different surrogate problems are evaluated in the context of electric motor design. In this regard, the results show that some of the formulations presented in the literature contain redundant objectives and that low-order polynomials provide high-quality surrogate models for most of the considered objective functions.

## **ABRÉGÉ**

Malgré les progrès en techniques de simulation, l'utilisation des modèles très précis peut être un frein à l'application des méthodes d'optimisation classiques en raison de leur coût élevé en ressources informatiques. Pour résoudre ces problèmes, les solutions proposées dans la littérature sont l'utilisation des modèles de substitution ou une réduction des objectifs. Pour faire un choix de modèle de substitution ou un ensemble d'objectifs réduit, il faut évaluer ses pertinence comme substituts au problème original. Il y a plusieurs paramètres de performance, mais la plupart d'entre eux ne sont généralement pas applicables aux deux solutions proposées dans la littérature. De plus, pour la plupart des paramètres, il n'y a pas de justification théorique d'optimisation basée sur les modèles de substitution. Dans ce cadre, cette thèse vise les points suivants: (i) unifier l'estimation de performance des modèles de substitution et des problèmes réduits, (ii) examiner les différents paramètres de performance et fournir une justification formelle dans le champ l'optimisation et (iii) proposer un nouvel ensemble de paramètres pour généraliser le concept du maintient de l'ordre pour les problèmes multiobjectifs. Les résultats d'un certain nombre de problèmes de référence montrent que certains paramètres proposés peuvent généralement identifier les problèmes de substitution de meilleure qualité dans la plupart des cas et sont moins sensibles aux échantillons donnés en comparaison avec les autres paramètres. Finalement, différents problèmes de substitution sont évalués dans le cadre de la conception de moteurs électriques. À cet égard, les résultats montrent que certaines des formulations présentées dans la littérature contiennent des objectifs superflus et que les polynômes des ordres inférieurs fournissent des modèles de substitution de meilleure qualité pour la plupart des fonctions objectives considérées.

## TABLE OF CONTENTS

DEDICATION . . . . .	ii
ACKNOWLEDGEMENTS . . . . .	iii
ABSTRACT . . . . .	iv
ABRÉGÉ . . . . .	v
LIST OF TABLES . . . . .	ix
LIST OF FIGURES . . . . .	xi
1 Models, Computer-aided Design and Surrogate Optimization Problems . . . . .	2
1.1 Introduction . . . . .	2
1.2 Computer-aided design . . . . .	4
1.2.1 Introducing optimization into the CAD process . . . . .	6
1.3 Surrogate-based optimization . . . . .	7
1.3.1 Surrogate problems . . . . .	9
1.3.2 Surrogate models . . . . .	10
1.4 Quality assessment of surrogate problems: Historical context . . . . .	14
1.4.1 Quality assessment of surrogate models . . . . .	14
1.4.2 Quality assessment of surrogate problems with a reduced number of objectives . . . . .	19
1.4.3 This thesis in the historical context . . . . .	21
1.5 Objective of the thesis . . . . .	21
1.5.1 Specific Objectives . . . . .	21
1.6 Scope of the thesis . . . . .	22
1.7 Contributions . . . . .	23
1.8 Thesis Outline . . . . .	23
2 Background . . . . .	25
2.1 Single objective optimization problems . . . . .	25
2.2 Multi-objective optimization problem . . . . .	26
2.2.1 Comparing non-dominated sets . . . . .	29
2.3 Surrogate optimization problems . . . . .	31
2.4 General structure of Surrogate-based Optimization algorithms . . . . .	32
2.4.1 The sampling module . . . . .	33
2.4.2 The surrogate problem construction module . . . . .	36

3	Surrogate-problem evaluation: theoretical and practical aspects . . . . .	51
3.1	Defining quality for surrogate problems . . . . .	52
3.2	Performance metrics for surrogate models . . . . .	56
3.2.1	Error-based metrics . . . . .	56
3.2.2	Rank-based metrics . . . . .	61
3.3	Performance metrics for reduced problems . . . . .	69
3.3.1	Pareto front preserving metrics . . . . .	70
3.3.2	Correlation-based metrics . . . . .	71
3.4	A summary on the theoretical analysis of performance metrics for surrogate problems . . . . .	75
3.5	A unified approach for the evaluation of surrogate problems . . . . .	76
3.6	Limitations of the Theoretical Analysis . . . . .	80
4	Computational Experiments with Analytical Benchmark Problems . . . . .	82
4.1	Relationship between performance metrics and optimization results . . . . .	82
4.1.1	Objective . . . . .	82
4.1.2	Methodology . . . . .	83
4.1.3	Results and discussion . . . . .	85
4.2	Performance metrics for surrogate model selection . . . . .	89
4.2.1	Objective . . . . .	89
4.2.2	Methodology . . . . .	89
4.2.3	Results and discussion . . . . .	91
4.3	Performance metrics for reduced problem selection . . . . .	95
4.3.1	Objective . . . . .	95
4.3.2	Methodology . . . . .	95
4.3.3	Results and discussion . . . . .	96
5	Computational Experiments with Electrical Motor Design Problems . . . . .	98
5.1	Methodology . . . . .	99
5.2	Interior permanent magnet machine . . . . .	100
5.2.1	Objective reduction . . . . .	101
5.2.2	Surrogate model selection . . . . .	104
5.3	Integrated motor-drive design of an Interior Permanent Magnet Motor . . . . .	106
5.3.1	Objective reduction . . . . .	108
5.3.2	Surrogate model selection . . . . .	110
5.4	Fractional Slot Concentrated Winding Surface Mounted Permanent Magnet Machine (Fractional Slot Concentrated Winding Surface Mounted Permanent Magnet Machine (FSCW-SMPM)) . . . . .	115
5.4.1	Objective reduction . . . . .	117
5.4.2	Surrogate model selection . . . . .	119
5.5	Summary and discussion . . . . .	125

6	Conclusion . . . . .	127
6.1	Extensions . . . . .	128
	Appendix A - Analytical benchmark problems . . . . .	130
	Appendix B - Statistical analysis for the surrogate model selection methods . . . . .	135
	Appendix C - Statistical analysis of surrogate models performances in electric motor design problems . . . . .	149
	References . . . . .	230
	List of Abbreviations . . . . .	243

## LIST OF TABLES

<u>Table</u>		
		<u>page</u>
1–1 Surrogate model performance assessment from previous papers . . . . .	19	
4–1 List of performance metrics . . . . .	84	
4–2 Correlation between performance metrics and IGD using GRNN . . . . .	86	
4–3 Correlation between performance metrics and IGD using RBFNN . . . . .	87	
4–4 Problem set-up . . . . .	90	
4–5 IGD median and interquartile interval (in brackets) obtained by SBO using different performance metrics for model selection . . . . .	92	
4–6 Number of scenarios in which each metric was among the top performing metrics (out of 27 scenarios) . . . . .	93	
4–7 IGD of the best surrogate models combinations selected through NSR . . . .	95	
4–8 IGD of the best surrogate models combinations selected through NKT . . . .	95	
4–9 Median, minimum and maximum cardinality of the reduced problem sets found by each performance metric over 30 independent runs . . . . .	97	
5–1 Selected surrogate models for the machine design problem . . . . .	100	
5–2 IPM motor design objectives . . . . .	101	
5–3 Surrogate model performance for the IPM Average Torque $T_{avg}$ . . . . .	104	
5–4 Surrogate model performance for the IPM Starting Torque $T_{str}$ . . . . .	105	
5–5 Surrogate model performance for the IPM Torque Ripple $T_{rip}$ . . . . .	105	
5–6 Surrogate model performance for the IPM Efficiency $\eta$ . . . . .	105	
5–7 Median and interquartile interval of best surrogate models' performance for the multi-objective IPM motor design problem . . . . .	106	
5–8 Integrated IPM motor-drive design objectives . . . . .	108	
5–9 Surrogate model performance for the IPM motor-drive Inverter Cost ( $Iv_{cost}$ ) .	111	
5–10 Surrogate model performance for the IPM motor-drive Inverter Efficiency ( $\eta_{Iv}$ )	111	

5–11 Surrogate model performance for the IPM motor-drive Start-up Torque ( $T_{str}$ )	111
5–12 Surrogate model performance for the IPM motor-drive Current Density ( $J$ ) . . .	112
5–13 Surrogate model performance for the IPM motor-drive Rising Time ( $rt$ ) . . . .	112
5–14 Surrogate model performance for the IPM motor-drive Power Factor ( $pf$ ) . . .	112
5–15 Surrogate model performance for the IPM motor-drive Speed Overshoot ( $n_o$ )	113
5–16 Surrogate model performance for the IPM motor-drive Motor Efficiency ( $\eta_{motor}$ )	113
5–17 Surrogate model performance for the IPM motor-drive Cogging Torque ( $T_{cog}$ )	113
5–18 Surrogate model performance for the IPM motor-drive Torque Ripple ( $T_{rip}$ ) . .	114
5–19 Median and interquartile interval of best surrogate models' performance for the integrated IPM motor-drive design problem . . . . .	114
5–20 FSCW-SMPM design objectives . . . . .	115
5–21 Surrogate model performance for the FSCW-SMPM motor Torque Ripple ( $T_{rip}$ )	120
5–22 Surrogate model performance for the FSCW-SMPM motor power factor $pf$ . .	121
5–23 Surrogate model performance for the FSCW-SMPM motor Average Torque ( $T_{avg}$ ) . . . . .	122
5–24 Surrogate model performance for the FSCW-SMPM motor Efficiency $\eta$ . . . .	123
5–25 Surrogate model performance for the FSCW-SMPM motor Permanent Magnet Eddy Current Losses $P_{ePM}$ . . . . .	124
5–26 Median and interquartile interval of best surrogate models' performance for the FSCW-SMPM motor multi-objective design problem . . . . .	125

## LIST OF FIGURES

<u>Figure</u>		<u>page</u>
1–1 Modelling according Rosen (1991) . . . . .	3	
1–2 Flowchart description of the Computer-aided Design process . . . . .	5	
1–3 Flowchart description of Surrogate-based optimization methods . . . . .	8	
1–4 A taxonomy for surrogate problems . . . . .	11	
1–5 Meta-model concept . . . . .	12	
2–1 Dominance relation in 2D . . . . .	27	
2–2 Pareto optimal front . . . . .	29	
2–3 The inverse generational distance indicator . . . . .	30	
2–4 The hyper-volume indicator . . . . .	31	
2–5 Surrogate-based optimization . . . . .	34	
2–6 Central composite design . . . . .	35	
2–7 Latin-hypercube designs $n = 8$ samples and $d = 2$ dimensions . . . . .	36	
2–8 Pareto optimal front of a 3 objective problem and its projection onto a two objective space . . . . .	37	
2–9 GRNN block diagram adapted from (Specht, 1991). . . . .	43	
2–10 The median time required to fit different surrogate models . . . . .	45	
2–11 Time required to evaluate 100,000 candidate solutions . . . . .	46	
2–12 Database and surrogate non-dominated sets represented in the design and objective spaces . . . . .	49	
3–1 Traditional approach for quality assessment of surrogate problems . . . . .	53	
3–2 High-fidelity problem, $f(x)$ , and two surrogate problems, $f_{s_1}(x)$ and $f_{s_2}(x)$ . .	53	
3–3 Proposed quality definition for surrogate problems . . . . .	54	
3–4 Perceived surrogate model quality by different metrics. The error function is defined as $\epsilon(x) =  x  + 2$ . . . . .	59	

3–5 Perceived surrogate model performance by $r^2$ . . . . .	60
3–6 Perceived surrogate problem ( $f_s(\cdot)$ ) quality by a rank performance metric (RANK) and the RMSE. The higher the value of RANK the higher the similarity among ranks. The maximum values of RANK is 1 and it happens when $r_s(x) = r_f(x) \forall x$ . . . . .	63
3–7 Parallel coordinates plot for the visualization of ordering preservation . . . . .	67
3–8 Kendall’s $t$ versus Spearman’s $r$ for different distributions of rank dissimilarities. . . . .	68
3–9 Surrogate models with coinciding global optima. . . . .	69
3–10 Projection of $\mathcal{N}^{(\mathcal{F})}$ in a two-objective space with the $\epsilon$ ’s indicated for two dominated solutions. . . . .	71
3–11 Projection of $\mathcal{N}^{(\mathcal{F})}$ in a two-objective space. The set of solutions that becomes dominated $\mathcal{D}^{\{f_a, f_b\}}$ is highlighted . . . . .	72
3–12 Non-dominated sorting . . . . .	78
3–13 Solutions ranks represented in parallel coordinates . . . . .	78
3–14 Normalized Weighted Disagreement (NWD) versus Kendall’s $t$ versus Spearman’s $r$ for different distributions of rank dissimilarities. . . . .	81
4–1 Example of surrogate models generated by GRNNs . . . . .	88
4–2 Example of surrogate models generated by RBFNNs . . . . .	88
4–3 Evolution of rank disruption . . . . .	94
5–1 IPM motor cross-section, design variables and fixed parameters (Adapted from: (Salimi, 2017)) . . . . .	102
5–2 Scatter plots of the Pareto optimal IPM reduced problems and the table representation of the Pareto optimal reduced formulations for NSR and NKT. Highlighted squares indicate the presence and white squares the absence of a given objective function in a given formulation. . . . .	103
5–3 IPM motor cross-section, design variables and fixed parameters (Adapted from: (Ghorbanian et al., 2017)) . . . . .	107
5–4 Scatter plots of the Pareto optimal Integrated IPM motor-drive reduced problems and the table representation of the Pareto optimal reduced formulations for NSR and NKT. Highlighted squares indicate the presence and white squares the absence of a given objective function in a given formulation. A black column indicates that a formulation with that number of objectives is not Pareto optimal. . . . .	109

5–5 FSCW-SMPM motor cross-section, design variables and fixed parameters (Adapted from: (Silva et al., 2017b)) . . . . .	116
5–6 Scatter plots of the Pareto optimal FSCW-SMPM (75 kW) reduced problems under NSR and NKT . . . . .	117
5–7 Table representation of the Pareto optimal reduced formulations for the FSCW-SMPM (75 kW) under NSR and NKT. Highlighted squares indicate the presence and white squares the absence of a given objective function in a given formulation. A black column indicates that a formulation with that number of objectives is not Pareto optimal. . . . .	118

## List of Algorithms

2.1	Infill based on the density in the design space . . . . .	47
2.2	Infill based on the density in the objective space . . . . .	48
2.3	Random infill . . . . .	49
3.1	Clustering algorithm for conflict-based discrepancy measures . . . . .	73
3.2	$\text{conflict}(\mathbf{Y}, \mathcal{K}, f)$ . . . . .	74
3.3	Fast non-dominated sorting . . . . .	77
4.1	Simple Surrogate-based Optimization Method . . . . .	83
4.2	Surrogate-based Optimization Algorithm with Online Surrogate model selection	89

# CHAPTER 1

## Models, Computer-aided Design and Surrogate Optimization Problems

### 1.1 Introduction

Although ubiquitous in science, the term “model” can be applied to very distinct structures in different research fields. Things that are commonly called models can be analogies, such as Bohr’s “solar system” model of the atom, mathematical equations, such as Newton’s law of universal gravitation or even the so called animal models commonly used in biology. In fact, it is difficult to find a broad consensus as to what a model is and their study continues to receive attention in fields such as Science Education and Philosophy of Science (Frigg and Nguyen, 2017).

A view of modelling that is general and fits the meaning that will be used in this thesis is offered by the biologist Rosen (1991) and illustrated by the diagram in Figure 1–1. On the left in Rosen’s diagram we have the system which we want to investigate. We assume that the system is governed by rules which we would like to encode so that they can be used by us in some application. To this end, a set of structures is arranged in a formal system or model. For example, the magnetic field distribution in an electric machine can be translated into a differential equation in a model. While the magnetic field exists in the real device, following the forces of its own systemic causality, our differential equation can be solved using the rules of mathematical calculus.

The intuition brought by Rosen’s diagram is that while the magnetic field and the differential equation in the model both obey rules, whether natural or mathematical, there is no rule dictating how one should map the hypothesized rules that govern the natural system onto the rules in the model. In other words, while the natural system and the model are each internally “entailed” by their own set of rules, nothing entails the natural system with the model. Indeed, practical experience shows that modelers can generate nonequivalent

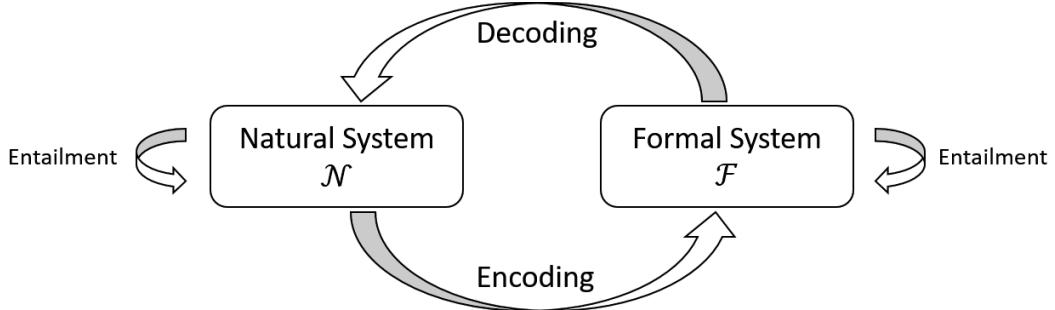


Figure 1–1: Modelling according Rosen (1991)

descriptions of the same system, that is models, whose outputs are compatible with a given set of observations but whose structures are different from one another.

Independently from their structure, models are used in science to represent some aspect of the world in order to make it easier to understand, define, quantify or visualize. But, if different structures can model the same phenomena and there is no rule linking the model and the phenomena, how can we decide which model to use?

A model is evaluated first and foremost by its consistency to the phenomena it is trying to represent. It should be able to explain past observations, predict future observations and, if it is inconsistent with empirical data, it must be modified or rejected. Yet, a fit to empirical data alone is not sufficient to completely characterize a model. Depending on the application, the cost of use can be just as important. A model that has a perfect fit with observed data but takes years to produce predictions may not be useful at all. On the other hand, a model that is not as accurate but can produce estimates very quickly can be extremely helpful. Another criterion that is often taken into account while evaluating a model is its simplicity. In that sense, the Ockham's razor principle is commonly invoked (Gauch, 2003). Loosely speaking, it states that if competing models of a given phenomena are equally acceptable, the simplest model is preferable. The *Ockham's razor* is not an irrefutable principle of logic nor a scientific result. It has, nevertheless, important ties with a model's informativeness and intelligibility which can also be vital in some applications.

Proper model selection has important implications in the Computer-Aided Design (CAD) process. The main tasks of CAD and its connection to scientific models are briefly described in the next section.

## 1.2 Computer-aided design

Some models can explain certain phenomena so elegantly and provide predictions with such a high degree of accuracy that they are raised to the level of theories and laws (Redhead, 1980). Some of these laws are mathematically described as a set of partial differential equations. For instance, in electromagnetics, we have the Maxwell's equations (Maxwell, 1873) that describe how electric and magnetic fields propagate, interact, and how they are influenced by objects; and in fluid mechanics (Landau and Lifshitz, 1987), we have the Navier–Stokes' equations that describe the motion of viscous fluid substances.

The existence of the aforementioned high-fidelity mathematical models, combined with the evolution of digital computers and the development of numerical methods, such as the Finite Element Method (FEM) and the Computational Fluid Dynamics (CFD) method, makes it possible to computationally recreate and simulate devices and processes. Moreover, parameters of the simulated device or process can be tweaked and their effect in the real world can be accurately predicted. Thus, researchers are able to perform virtual experiments which, nowadays, are responsible for significant parts of scientific investigation (Frigg, 2008).

The ability to perform computer experiments leads naturally to the Computer Aided Design (CAD) process which can be defined as “the use of computer systems to aid in the creation, modification, analysis, or optimization of a design” (Sarcar et al., 2008). Figure 1–2 presents a flowchart description of a basic design process in which the fundamental blocks of a CAD system can be identified.

The process starts with the establishment of specifications and requirements which leads to the definition of objectives and constraints for the design process. With the specifications in hand, an initial computational model is designed and the mathematical model is defined. Next, the model is parameterized in order to define the search space within which a final,

possibly improved, design is going to be sought. These initial steps normally require some expert knowledge and usually depend on a human designer. The next step is to specify a (virtual) prototype which is then simulated and evaluated by a numerical analysis package. If the prototype satisfies the specifications and requirements defined by the designer, the design process stops and the designer should proceed to the construction of a physical prototype. Otherwise, if there are resources left, guided by the defined objectives and constraints, a new virtual prototype is defined over the design space and the evaluation process is repeated.

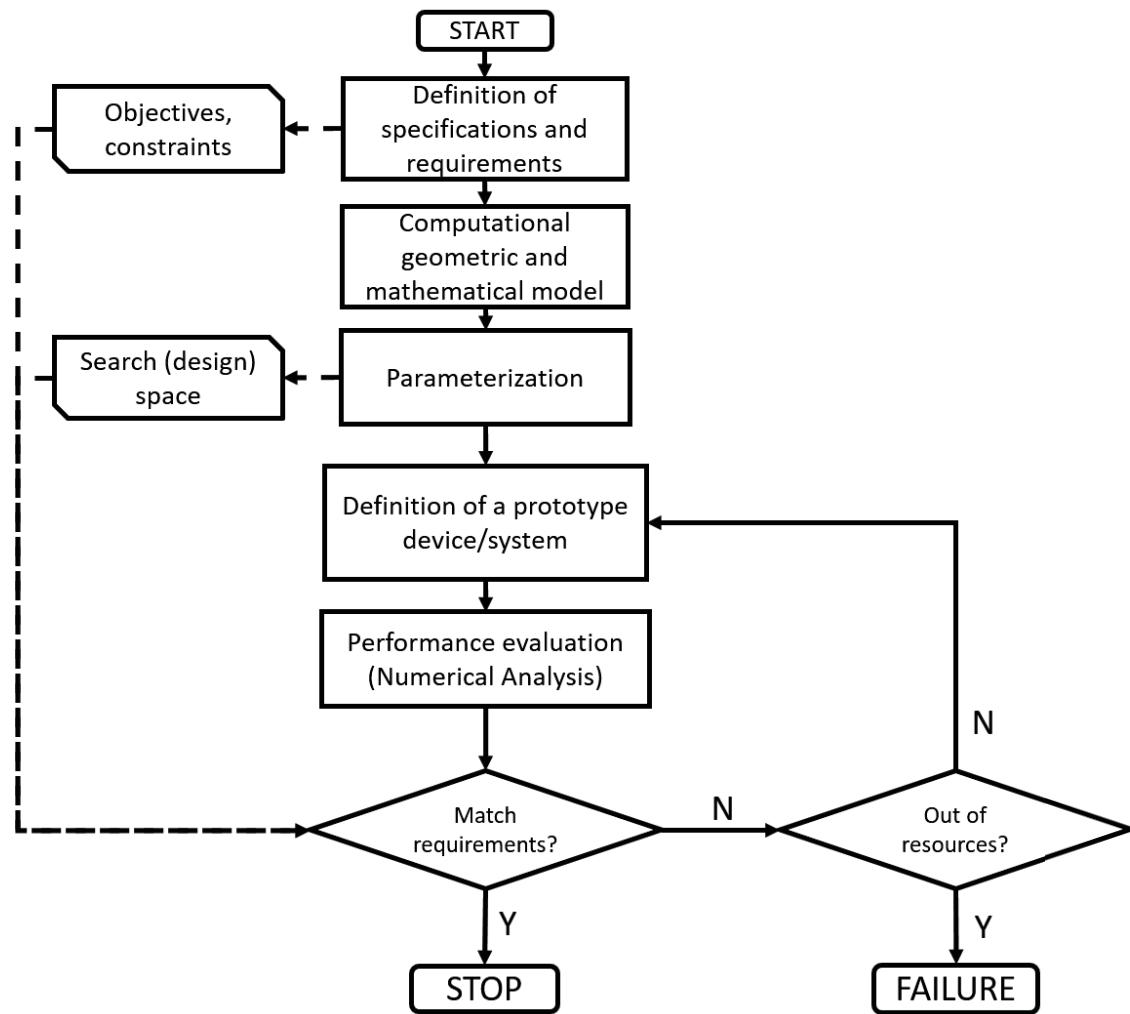


Figure 1–2: Flowchart description of the Computer-aided Design process

### 1.2.1 Introducing optimization into the CAD process

Based on the parameterized prototype and its simulation model, the designer can produce a mathematical definition of the design problem which consists of two basic elements (Guimarães, 2008): (i) the synthesis problem and (ii) the analysis problem.

The synthesis problem consists in mapping the specifications and requirements defined by the designer to objective and constraints functions, and in translating some of the model parameters into design variables in order to define the design space. Alternatively, the synthesis problem can be defined as an optimization problem of the following form:

$$\begin{aligned} \min_{\mathbf{x}} & \mathbf{f}(\mathbf{x}) \\ \text{subject to: } & \mathbf{x} \in \Omega \end{aligned} \tag{1.1}$$

where  $\mathbf{x} = [x_1, x_2, \dots, x_n]$  is the vector of design variables,  $\mathbf{f}(\cdot) = [f_1(\cdot), f_2(\cdot), \dots, f_m(\cdot)]$  is the vector of objective functions and  $\Omega$  the feasible space defined by the constraints.

The analysis problem consists of solving the mathematical equations which describe the laws that govern the behaviour of the device being designed. It aims at simulating a trial design in order to obtain the quantities that will define all or part of the objective and/or constraint functions used in the synthesis problem.

Often, the objective functions can be non-convex and multi-modal. Moreover, the number of objectives to be optimized can be high (Silva et al., 2016b). In these cases, local improvements only are not enough to satisfy the ever increasing performance demands and traditional algorithms such as gradient-based methods and trust-region methods are not satisfactory (Koziel and Leifsson, 2013). Due to their flexibility with respect to the nature of the variables (continuous, discrete or mixed-integer) and the landscape of the objective functions (uni-modal, multi-modal and discontinuous), methods based on evolutionary algorithms and swarm intelligence have become a trend in design optimization (Di Barba, 2010; Dasgupta and Michalewicz, 2013).

These optimization algorithms, however, especially when applied to problems with multiple objectives, frequently require a high number of objective function evaluations (simulations). In fact, it is easy to find in the literature, applications of these methods performing hundreds of thousands of function evaluations (Zhang and Li, 2007). Combined with the fact that the analysis problem often requires sophisticated and computationally expensive models (simulation times may vary from several minutes to several hours or even days (Koziel and Leifsson, 2013)), the use of those algorithms can make the design automation impractical at the present time.

### 1.3 Surrogate-based optimization

In order to make the optimization process possible, something has to be done in order to reduce the associated computational burden.

One of the ways to reduce the optimization time is to work on the improvement of the optimization algorithms themselves such that they converge to the optimal solution in fewer iterations. The issue with this approach is that it is not general. As an implication of the *No Free Lunch Theorems* (Wolpert and Macready, 1997; Corne and Knowles, 2003), as an optimization algorithm becomes more specialized and more efficient in solving a given problem, its performance tends to degrade for other problems. Besides, it is hard to know *a priori* what is the best algorithm for a given problem, and most of the algorithms that do not assume convexity and differentiability will take at least a few hundred function evaluations to produce satisfactory results.

Another alternative, is to change the optimization problem altogether in order to make it computationally cheaper to solve. The aim is, instead of solving the synthesis problem defined in Eq. 1.1, to solve a surrogate problem that is *similar* to the original problem but easier to handle. Surrogate-based Optimization (SBO) is the name given to the family of methods that use surrogate problems to tackle optimization problems with expensive function evaluations.

Figure 1–3 presents a flowchart description of the SBO framework. It starts with the construction of a surrogate optimization problem that has to be computationally cheaper to solve. Next, the surrogate problem is evaluated in order to have its suitability assessed. If the surrogate problem is considered satisfactory it is solved by an optimization algorithm. Next, all or part of the optimized solutions are re-evaluated according to the original problem formulation for validation. If these solutions match the requirements the method stops. Otherwise, the obtained information is used for the construction of an improved surrogate problem and the process is repeated while there are computational and time resources available.

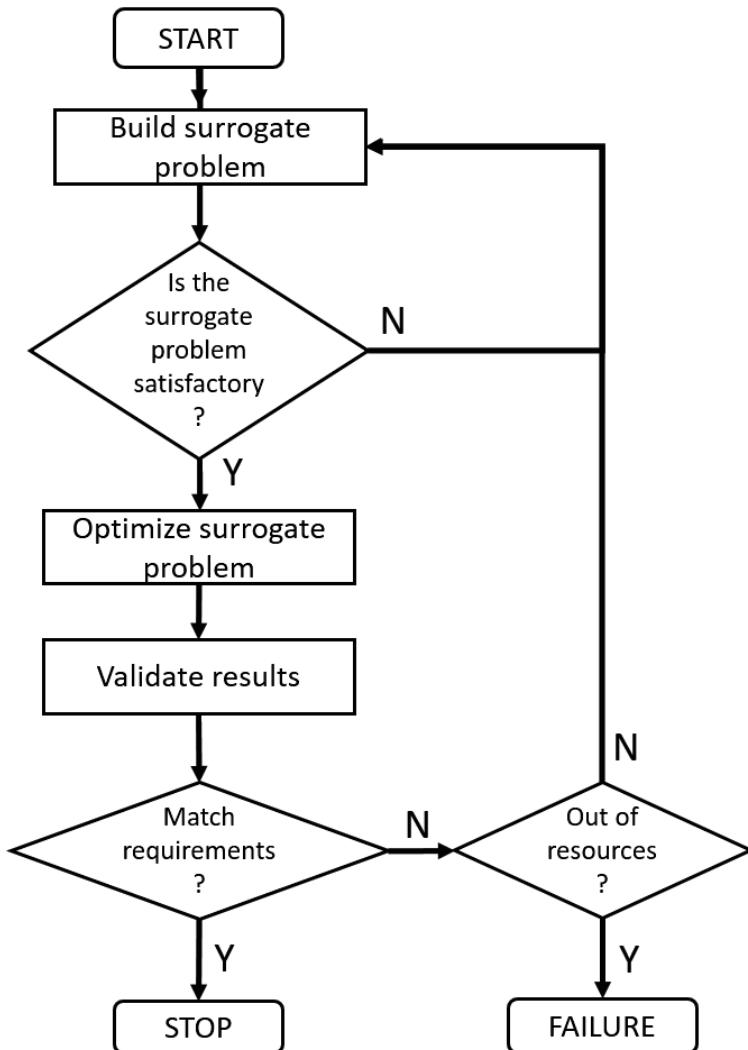


Figure 1–3: Flowchart description of Surrogate-based optimization methods

The literature uses the term SBO to refer to optimization with surrogate models (i.e. models for the analysis problem that are cheaper to compute) only. In this thesis, however, the term is going to be used in a more general way, aggregating other types of surrogate problems under the same name. As we will see, this broader meaning allows for a unified methodology for their evaluation and validation that, apparently, has not been considered before. In the next section, a brief review of methods for the construction of surrogate problems is presented.

### 1.3.1 Surrogate problems

Let's define the surrogate optimization problem as follows:

$$\begin{aligned} & \min_{\mathbf{k}} \mathbf{f}_s(\mathbf{k}) \\ & \text{subject to: } \mathbf{k} \in \Gamma \end{aligned} \tag{1.2}$$

where  $\mathbf{k}$  is the vector of design variables,  $\mathbf{f}_s$  is the vector of surrogate objective functions and  $\Gamma$  the feasible space associated to the surrogate problem.

Surrogate problems can be constructed in several ways. One of them is to reduce the number of design variables in the problem. In this approach, instead of solving the optimization problem for the set of variables  $\mathcal{X} = \{x_1, x_2, \dots, x_n\}$ , the problem is solved for a new set of variables  $\mathcal{K} = \{k_1, k_2, \dots, k_p\}$  which is a subset of  $\mathcal{X}$ . The rationale for selecting a subset of variables is that the original problem contains variables which are either redundant or irrelevant, and can thus be removed without incurring much loss of information. Reducing the number of variables, however, is rarely used as a way to reduce the optimization cost. In fact, at the beginning of any serious design process the designer should guarantee (through, for instance, sensitivity analysis (Saltelli et al., 2008)) that all design variables are relevant for the problem. This approach is presented here for completeness but for the reasons just mentioned it will not be pursued as a way to produce surrogate problems.

A more meaningful approach is to reduce the number of objectives (Brockhoff and Zitzler, 2006; Deb and Saxena, 2006; Brockhoff and Zitzler, 2009a; Saxena et al., 2013;

Freitas et al., 2014). Similar to what happens in variable reduction methods, the aim of objective reduction methods is to eliminate a subset of objectives that is either redundant or irrelevant. Thus, instead of solving the optimization problem for the set of objectives  $\mathcal{F} = \{f_1(\cdot), f_2(\cdot), \dots, f_m(\cdot)\}$ , we solve a new problem with the vector objective function  $\mathbf{s}(\cdot)$  which is composed by the set of objectives  $\mathcal{F}_s = \{f_{s_1}(\cdot), f_{s_2}(\cdot), \dots, f_{s_q}(\cdot)\}$  such that  $\mathcal{S} \subset \mathcal{F}$ . A fundamental difference between the two methodologies is that in variable reduction methods, after a variable is removed, the optimization method loses access to part of the design space. On the other hand, when an objective is removed it does not mean necessarily that it will not be taken into account. Depending on its relationship with other objectives, it may be implicitly accounted for by the optimization method.

A third way to construct a surrogate problem is to use a surrogate model in lieu of the high-fidelity and computationally expensive model over which some of the objectives and constraints are computed. A surrogate model is any construct that is able to produce responses analogous to ones produced by the analysis problem. In general, a surrogate model trades accuracy for computation speed. They are not as accurate as the originally used mathematical model but can be evaluated in a small fraction of the time previously required.

Figure 1–4 presents a taxonomy for surrogate problems. As can be seen, surrogate models can still be subdivided into different categories, each with its own characteristics. In the next section an overview of the main surrogate models used for optimization is presented.

### 1.3.2 Surrogate models

Surrogate models can be divided into two main groups: (i) first principles surrogate models and (ii) meta-models.

A first principles model is built directly from established laws of science without relying on fitted parameters. They are of the same nature and, in general, consist of simplified versions of the original mathematical model. For instance, in the context of low-frequency electromagnetic devices, analytical models are usually obtained through the formal solution

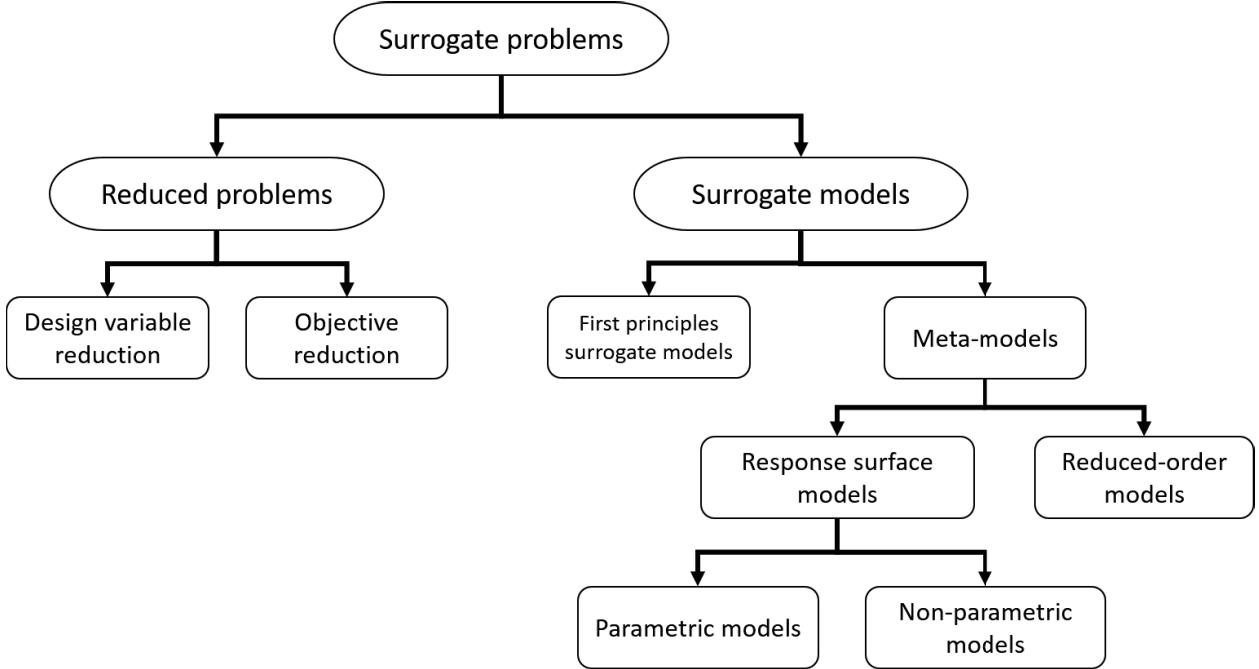


Figure 1–4: A taxonomy for surrogate problems

of Maxwell’s equations in constant permeability regions (Tiegna et al., 2013). Another example, is the Magnetic Equivalent Circuit (MEC) which is based on lumped-parameter magnetic circuit models (reluctance networks). In MECs the field distribution is mapped into a relatively small number of reluctance elements that represent the flux tubes in the geometry of the modeled device (Hammond and Sykulski, 1994; Amrhein and Krein, 2007). Using this concept, the magnetic fields of complex devices can be quickly solved using methods and techniques developed for electric circuits.

Different from the first principles surrogate models, meta-models are not derived directly from the physical principles. As illustrated in Figure 1–5, meta-models are phenomenological models<sup>1</sup> that find the mapping between design variables and model outputs through data

---

<sup>1</sup> “A phenomenological model is a scientific model that describes some phenomena,in a way which is consistent with fundamental theory, but is not directly derived from it. A phenomenological model foregoes any attempt to explain why the variables interact the way they do, and simply attempts to describe the relationship, with the assumption that the relationship extends past the measured values.” - (Hilborn and Mangel, 1997)

obtained from the high-fidelity simulation model. This is what gives them the name meta-model, meaning “model of the model”.

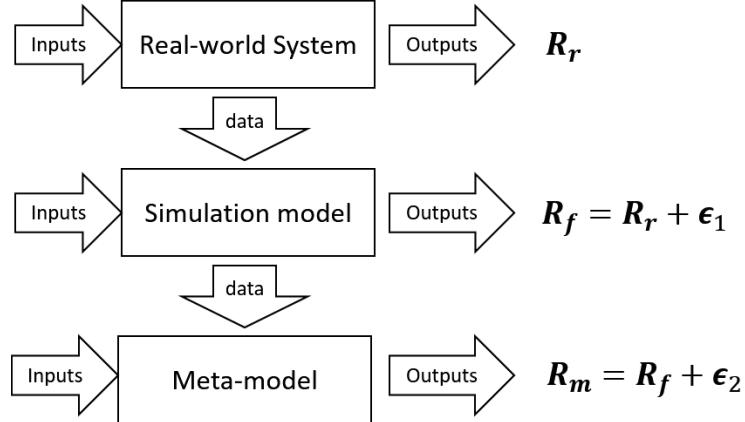


Figure 1–5: Meta-model concept

One way to create meta-models is by generating Response Surfaces which, in turn, can also be divided into two classes: (i) parametric models which assume a priori a specific functional relationship between inputs and outputs; and (ii) non-parametric models that do not assume any particular functional relationship.

Low-order polynomial models fitted to sample data by least squares estimation (Box and Wilson, 1951; Myers and Montgomery, 1995) are probably the simplest and most commonly used type of parametric response surface. Polynomial models were very popular methods for the approximation of expensive computer simulations until the 1990’s (Goldstein and Dushane, 1976; Friedman and Pressman, 1988; Kleijnen, 1993; Yu and Popplewell, 1994). As they became more widely used and better understood, their limitations became more apparent, e.g., the curse of dimensionality (Myers and Montgomery, 1995), the inability to create accurate global approximations in highly nonlinear design spaces (Forrester et al., 2008) and lack of interpolation properties (Lebensztajn, 2004). As a result, by the end of the 1990’s, the interest shifted to more flexible, non-parametric, approximation models, in particular, Radial basis functions (RBF) (Broomhead and Lowe, 1988), and Kriging (Sacks et al., 1989; Lebensztajn, 2004).

RBFs use a weighted sum of simple functions in an attempt to approximate complicated function landscapes, while Kriging uses a global polynomial approximation combined with a localized deviation represented by a Gaussian process. Both models tend to be very flexible with respect to the nature of the landscapes they can emulate (Forrester et al., 2008). Furthermore, confidence intervals for the predictions can be obtained without much additional computational cost (Jin, 2005; Forrester and Keane, 2009). Despite these advantages, Kriging and RBFs have more complex fitting procedures which usually involve the solution of another optimization problem. Besides, Kriging predictions require matrix inversions which, depending on the number of samples, may also be computationally expensive.

In addition to these two, other data-fitting models have also been explored in the literature, e.g, Artificial Neural Networks (Papadrakakis et al., 1998), Genetic Programming (Mendes et al., 2013), Multivariate Adaptive Regression Splines (MARS) (Friedman, 1991), Support Vector Regression (SVR) (Forrester et al., 2008), etc.

Another type of meta-model is the, so called, Reduced Order Model (ROM). ROMs are popular in problems in which the analysis uses FEM. Instead of working on integral values of the field solution (e.g., torque, efficiency, etc.) a ROM tries to reduce the number of nodes in the FEM system. Thus, a ROM mimics the basic structure of the Finite Element Analysis (FEA) and not just a functional relationship between input and output parameters.

Since ROMs operate in the dimensionality of the discretization rather than on the design space, they tend to be less sensitive to the increase in the number of design variables. On the other hand, their application to shape design can be troublesome. In this kind of problem, the physical domain varies according to the design variables. Therefore, the discretized spatial points (mesh) will not be the same for all design candidates (Iuliano and Quagliarella, 2013). Hence, ROMs are rarely used in the context of geometry design.

There is no consensus as to which is the best surrogate modeling technique in the context of CAD. Each family of surrogate models has its own advantages and disadvantages. First principle surrogate models can offer concise descriptions of the device inner physics but are

hard to generalize across different problems. Response surfaces can be easily applied to any kind of problem but offer little insight into the underlying process inner workings. ROMs retain the basic structure of the underlying finite element code but since they need direct access to the mesh structure and field values, they are difficult to couple with proprietary simulation packages.

## 1.4 Quality assessment of surrogate problems: Historical context

As could be seen in the previous sections, the possibilities for the construction of surrogate problems are numerous. A natural question that arises from this is: How do we choose a surrogate problem? This question has multiple answers which, in turn, depend on the quality metric used to evaluate a given surrogate problem. Thus, in this section a historical review on quality metrics for surrogate problems is presented. As mentioned before, until the point at which this document was written, surrogate problems based on surrogate models and objective reduction have been treated as completely different topics. For this reason, in this review, they will also be treated separately.

### 1.4.1 Quality assessment of surrogate models

In the case of surrogate models, all the quality metrics have been imported from the study of models in general. The basic principle is to verify if the responses,  $\mathbf{R}_m$ , produced by a model match the observations obtained from the real system,  $\mathbf{R}_r$ . In practice, however, this simple rationale gets more complex. Some models may produce very accurate predictions for a given set circumstances (inputs) but may perform poorly for another set. In this way, computing the error,  $\epsilon = \mathbf{R}_m - \mathbf{R}_r$ , between prediction/observation pairs does not provide an overall assessment of a model's performance. Model performance evaluation is a problem of longstanding concern. In particular, researchers working on air quality models produced a very thorough discussion on the topic in the late 1970's and early 1980's (Nappo Jr, 1975; Bencala and Seinfeld, 1979; Fox, 1981; Willmott, 1981, 1982).

Bencala and Seinfeld (1979) presented a comprehensive survey on the topic and proposed a package of methods for model performance assessment. Their package included graphical

tools, such as scatter plots and correlograms, the correlation coefficient, and a set of average error (observed - predicted) measures, such as the mean error and the root mean squared error.

Due to their importance in support of laws and regulations, in 1980, the American Meteorological Society (AMS) convened a group of professionals and scientists in air quality models to specifically discuss model performance evaluation metrics and methods. The influential work of Fox (1981) compiles the results of this workshop that, among methods for data collection and visualization, suggests the use of the following metrics for model evaluation:

1. Pearson's product moment correlation ( $r$ )
2. The Root Mean Squared Error (RMSE);
3. The Mean Absolute Error (MAE);
4. Average error; and
5. Error variance.

In a series of papers (Willmott, 1981, 1982, 1984), Willmott adds other metrics, such as the Root Mean Squared Percentual Error (RMSPE), the Mean Absolute Percentual Error (MAPE) and the Maximum Absolute Error (MAX) to Fox's work and generalizes his comments to models other than air quality models.

Willmott agrees with Fox on the use of RMSE and MAE (and their percentual counterparts, the RMSPE and the MAPE) arguing that they are the best overall measures of model performance, as they summarize the mean difference between observations and predictions while keeping the same units. On the other hand, Willmott argues against the error's average and variance. The point made by him is that these two metrics are only useful to characterize the distribution of the error but do not add information about the model's performance. For instance, an average error of 0 could mean a perfect match or could simply mean the model overestimates and underestimates in the same proportion. Another, arguably the strongest, criticism of Willmott with respect to Fox's suggestions is on the use of Pearson's product

correlation  $r$  and the coefficient of determination,  $r^2$ . In (Willmott, 1981) it is shown that these two metrics are completely insensitive to a wide variety of additive and proportional errors. In (Willmott, 1984) this caveat is illustrated with two extreme cases. At one extreme, it is shown that is easy to obtain very high values of  $r$  and  $r^2$  when observations and predictions are very dissimilar. At the other extreme, it is shown that it is also easy to obtain very low values of  $r$  and  $r^2$  when differences between predictions and observation are very small.

In (Friedman and Friedman, 1985), the evaluation of surrogate models is discussed for the first time. The only difference between model and surrogate model validation for them was the frame of reference. Instead of comparing observations and predictions, the comparison is made upon simulation model predictions and surrogate model predictions. In spite of Willmott's discussion, Friedman and Friedman (1985) report the  $r^2$  in addition to the MAE for their linear regression model. By the end of the 1980's to mid 1990's, the discussion on model performance evaluation shifted from metrics to quality distribution estimation. Tichelaar and Ruff (1989) argued that, in order to know how good a model is, one must know how its quality varies as the validation data set changes. In (Willmott et al., 1985; Tichelaar and Ruff, 1989; Cox and Tikvart, 1990; Friedman, 1996) re-sampling methods, such as bootstrapping, jackknife and cross-validation are used in order to estimate quality variance.

In the 1990's, the use of surrogate-models, in particular, response surface models, for optimization was already widespread (Barthelemy and Haftka, 1993; Sobiesczanski-Sobieski and Haftka, 1997; Booker et al., 1999). This popularity led to a large number of papers comparing different surrogate models for various applications. In terms of evaluation methodology, in addition to the aforementioned error metrics, Jin et al. (2001) suggested the following criteria:

- Robustness: indicates to what degree the modelling technique is problem dependent.

- Efficiency: indicates the computation effort required for surrogate model construction and for obtaining predictions.
- Transparency: capability of providing information about the relationships among design variables and objectives.
- Simplicity: ease of implementation.

Except for the *Efficiency*, which is measured in units of time, all the other criteria are qualitative.

In (Runarsson, 2004), instead of using the surrogate model to predict the objective function values (in that case a nearest neighborhood regression model), the author uses it to predict the relative rank of the candidate solutions. Thus, a surrogate model is deemed suitable if the ranking imposed by it on the candidate solutions matches the ranking imposed by the high-fidelity model. Possibly, this was the first time where rank maintenance was explicitly considered to be a desirable characteristic of surrogate models when used for optimization.

In (Runarsson, 2006), the author expands his previous work to other kinds of surrogate models (Polynomials and RBFs) and proposes the use of Kendall's  $t$  for surrogate model performance assessment. While Pearson's  $r$  measures the correlation between the responses produced by the high-fidelity model and the surrogate model, Kendall's  $t$  measures the correlation between the ranks imposed by each of the models. This approach was followed in (Kern et al., 2006), (Loshchilov et al., 2010) and (Díaz-Manríquez et al., 2011) but it was not until 2012 that the rank-based metrics were compared to the classic error-based metrics for the first time in the context of surrogate-based optimization. In Bischl et al. (2012), Kriging, MARS and ANNs were used for the optimization of four single objective optimization problems. Afterwards, the correlation between surrogate-model performance and solution quality was computed. The authors concluded that there was no difference between error-based metrics and rank-based metrics. Both of them presented almost the same correlation with the quality of the optimized solutions.

The use of rank-based performance metrics in SBO was then dormant until 2016 when different researchers revisited the idea. Audet et al. (2016) and Díaz-Manríquez et al. (2016) proposed surrogate model selection methods based on a variation of the Kendall's  $t$  metric, while Cui et al. (2016) used the Spearman's  $r$  rank-correlation metric. Audet et al. (2016) also implemented an error-based version of their framework and demonstrated that the rank-based version produced better solutions for some single objective optimization problems. In parallel, Silva et al. (2016a) applied an SBO method for the design of an Interior Permanent Magnet (IPM) motor and demonstrated that the rank-based performance metric presented higher correlation with the optimization results than the MSE.

Table 1–1, presents an overview of previous papers on the performance assessment of general use surrogate models. It can be seen that Polynomial models, Kriging and RBFs are the preferred choices of surrogate models, and that the RMSE is the preferred choice of performance metric. Interestingly, despite the thorough discussions presented in (Willmott, 1981, 1982, 1984), the  $r^2$  remains popular.

Although all the papers in Table 1–1 refer to optimization as an application for surrogate models, only Gorissen et al. (2009), Li et al. (2010), Bischl et al. (2012), Pilát and Neruda (2013), Díaz-Manríquez et al. (2016), and Audet et al. (2016) report optimization performance. The main conclusions in these papers are as follows: (i) there is no optimal surrogate model for every problem; and (ii) the choice of surrogate model has a significant effect on the optimization performance.

An apparent research gap seems to be whether there is a performance metric that correlates with more strength with the optimization performance. While the results in (Silva et al., 2016a) and Audet et al. (2016) point towards rank-based metrics, in (Bischl et al., 2012) no difference with respect to error-based metrics has been found. Another gap arises from the used test problems. All the presented articles refer to single-objective optimization problems only and as far as this author's knowledge goes, there is no study comparing surrogate models under different performance criteria for multi-objective optimization problems.

Paper	Surrogate Models	Performance metrics
Friedman (1996)	Polynomial	RMSE, $r^2$
Giunta and Watson (1998)	Polynomial, Kriging	RMSE
Varadarajan et al. (2000)	Polynomial, ANN	$r$ , $r^2$
Simpson et al. (2001)	Polynomial, Kriging, RBF, MARS	RMSE, MAX
Jin et al. (2001)	Polynomial, Kriging, RBF	MAPE, Transparency, Simplicity, Efficiency, Robustness
Clarke et al. (2005)	Polynomial, Kriging, SVM	RMSE, MAX, MAE
Queipo et al. (2005)	Polynomial, Kriging, RBF	RMSE
Wang and Lowther (2006)	Polynomial, Kriging, RBF	RMSE, $r^2$
Gano et al. (2006)	Polynomial, Kriging	RMSE, $r^2$ , MAX, Transparency, Simplicity, Efficiency, Robustness
Viana et al. (2009)	Polynomial, Kriging, RBF, SVM	RMSE
Gorissen et al. (2009)	Kriging, RBF, Splines, Rational Functions	MAPE, Convergence rate, Efficiency
Li et al. (2010)	Polynomial, Kriging, RBF, SVM	MSE, MAE, MAX, Transparency, Simplicity, Efficiency, Robustness, Convergence rate
Díaz-Manríquez et al. (2011)	Polynomial, Kriging, RBF, SVM	$r^2$ , Kendall's $t$
Bischl et al. (2012)	Polynomial, Kriging RBF, MARS	RMSE, MAE, Spearman's $r$
Pilát and Neruda (2013)	Polynomial, ANN, SVM	RMSE, Convergence rate
Kleijnen (2015)	Polynomial, Kriging	RMSE, MAX, $r^2$
Díaz-Manríquez et al. (2016)	Polynomial, Kriging, RBF, SVM	$r^2$ , Kendall's $t$
Audet et al. (2016)	Polynomial, Kriging, RBF	RMSE, Kendall's $t$

Table 1–1: Surrogate model performance assessment from previous papers

#### 1.4.2 Quality assessment of surrogate problems with a reduced number of objectives

Even though objective reduction methods can be traced back to the 1970’s (Gal and Leberling, 1977), the topic has only gained widespread attention in the 2000’s. In (Fleming et al., 2005), (Wagner et al., 2007) and (Ishibuchi et al., 2008), among others, it was observed that the state-of-the-art multi-objective optimization algorithms of the time did not scale well with an increasing number of objectives. Although the scalability issue has been reduced in more recent algorithms (Zhang and Li, 2007; Seada and Deb, 2015), the number of objectives

still has a direct relationship with the computational resources consumed by the optimization algorithm (Brockhoff and Zitzler, 2009a).

Gal and Leberling (1977) probably published the first method for objective reduction. However, it was tailored for problems with linear objectives only. More recent approaches, applicable to general black-box multi-objective optimization problems, can be divided into two groups (Yuan et al., 2017): (i) Pareto front preservation methods; and (ii) Correlation-based methods.

In Pareto front preservation methods, first, the optimal solution set of the original problem is estimated. Next, this solution set is projected onto a reduced set of objectives. The reduced problem is then evaluated with respect to how much it “damages” the original Pareto front. In (Singh et al., 2011), (Guo et al., 2016) and (Yuan et al., 2017), the “damage” is defined as the number of solutions that become dominated when a subset of objectives is removed. If no solution becomes dominated, it means that the removed subset of objectives is redundant. In (Brockhoff and Zitzler, 2007) and (Brockhoff and Zitzler, 2009a), the “damage” is defined as the minimum value that has to be subtracted from a solution in the reduced objective space such that all of them remain non-dominated.

Correlation-based methods are built on the idea that if two objectives are not in conflict, in other words, highly correlated, it means that one of them is redundant therefore can be removed. To this end, Saxena et al. (2013) applies the Principal Component Analysis (PCA) to the correlation matrix while López Jaimes et al. (2009), Sinha et al. (2013), Bandyopadhyay and Mukherjee (2015) and Silva et al. (2016b) apply clustering algorithms based on correlation.

The main issue with Pareto front preservation methods is that they require a good approximation of the original Pareto front in order to work properly (Saxena et al., 2013). That defeats the purpose of objective reduction since, in theory, they require the solution of the original problem. Correlation based methods, on the other hand, are more flexible with respect to the quality of the data set. However, these methods fail to recognize redundancy

in very simple cases where one objective is a linear combination of other objectives (Yuan et al., 2017).

### 1.4.3 This thesis in the historical context

At this point, it seems appropriate to locate this thesis in the historical context presented here. As discussed, methodologies for performance assessment of models are well established. However, in the context of SBO, there seems to be no strong theory that allows us to: (i) tie surrogate model and optimization performance; and (ii) explain why some studies show that there is no difference between rank-based and error-based metrics (Bischl et al., 2012) while others show the opposite (Silva et al., 2016b; Audet et al., 2016). Besides, when multi-objective optimization is taken into account, the knowledge gap increases even more, due to the lack of comparative work.

The evaluation of reduced objective problems, on the other hand, in several methods, is directly linked to the optimization results (maintenance of the optimal Pareto front). Thus, if the same methodology applied for reduced problems could be applied to surrogate models, better approaches for the creation of surrogate problems could be achieved. In this context, the aim of this thesis is to establish a link between surrogate models and objective reduction and investigate the performance assessment process for both. Hopefully, this knowledge will lead to the production of better surrogate problems and, consequently, more efficient optimization methods.

## 1.5 Objective of the thesis

The main objective of this thesis is to investigate the question: “what is the best way to assess the performance of a surrogate problem?”

### 1.5.1 Specific Objectives

The specific objectives are:

1. Unify the performance assessment of surrogate models and reduced problems;
2. Investigate the performance assessment of surrogate models for objective functions in the context of optimization;

3. Propose a methodology for the selection of surrogate models for objective functions in multi-objective optimization;
4. Investigate the quality assessment of reduced problems in the context of multi-objective optimization with a high number of objectives;
5. Propose a methodology for the selection of surrogate-problems applied to multi-objective problems;
6. Implementation, test and validation of the proposed methodology in analytical benchmark problems and practical problems comprising the design optimization of electric motors.

## 1.6 Scope of the thesis

The methodology presented in this thesis refers to performance assessment and selection of surrogate problems for surrogate-based optimization. As mentioned before, this family of methods involves: (i) data acquisition, (ii) surrogate problem evaluation, and (iii) surrogate problem construction. Given the additional computational effort in executing these steps, the proposed strategy is only useful in the context of optimization problems with computationally expensive objective and/or constraint functions (such as the ones found in FEA and CFD based optimization engineering problems) in which the additional complexity in the algorithm is justifiable. Hence, it is important to clearly characterize the class of problems that are intended to be treated in this work.

In this thesis we consider:

- Black-box, non-linear single and multi-objective optimization problems with real variables;
- Possibly multi-modal objective function(s);
- Possibly non-linear equality and inequality constraint functions;
- The computation involved in at least one of the objective functions is computationally expensive in such a way that the time required for the evaluation step is dominant in comparison with the time required by the other steps.

## 1.7 Contributions

The contributions of this work are claimed to be:

- The proposition of a general criterion for the evaluation of surrogate-problems that unifies surrogate models and reduced problems.
- A comprehensive evaluation of performance metrics for surrogate model evaluation in the context of optimization.
- A comprehensive evaluation of performance metrics for objective reduction.
- A novel set of metrics for performance assessment of surrogate problems.
- The derivation of theorem results for the suitability of performance metrics for single and multi-objective problems.
- The application of the proposed metrics in the selection of surrogate problems for electric motors.

## 1.8 Thesis Outline

The remainder of this thesis is organized as follows:

Chapter 2 starts with the characterization of single and multi-objective optimization as well as the corresponding surrogate optimization problems. Then, a general framework for surrogate-based optimization is presented with emphasis on the main methods used for each one of its constituent modules.

Chapter 3 is dedicated to the definition of quality for surrogate problems and the analysis of performance metrics used for surrogate problem evaluation. The analysis is done by considering the conditions for which a surrogate problem solution matches the original problem solution, and to what extent a performance metric is able to predict these situations. After assessing the limitations of the current metrics, a unified approach for the evaluation of surrogate models and reduced problems that expands the concept of rank maintenance to multiple objectives is proposed.

Chapter 4 presents an empirical study on the effect of the choice of a particular performance metric in the optimization of well-known benchmark analytical optimization problems.

Chapter 5 presents a thorough evaluation of surrogate problems related to the design optimization of electrical motors.

Finally, Chapter 6 concludes the thesis with an overall discussion and suggestions for future work.

## CHAPTER 2

### Background

In order to design and study Surrogate-based Optimization Methods (SBOM), a series of non-ambiguous definitions are required. These definitions provide a precise set of symbols and formal relationships that permit the formulation and analysis of Single Objective Optimization Problems (SOOP) and Multi-objective Optimization Problems (MOOP), as well as the related surrogate problems.

#### 2.1 Single objective optimization problems

A general single objective optimization problem can be defined as:

$$\begin{aligned} & \text{minimize} && f(\mathbf{x}) \\ & \text{subject to:} && \mathbf{x} \in \Omega \end{aligned} \tag{2.1}$$

Where  $f(\cdot) : \Omega \rightarrow \mathbb{R}$  is the objective function and  $\Omega$  is the feasible set, defined as:

$$\Omega = \{\mathbf{x} \in \mathcal{X} : g_i(\mathbf{x}) \leq 0, i = 1, \dots, p; h_j(\mathbf{x}) = 0, j = 1, \dots, q\} \tag{2.2}$$

The functions  $g_i(\cdot)$  represent the inequality constraints and  $h_j(\cdot)$  the equality constraints which define the sets:

$$\mathcal{G} = \{\mathbf{x} : g_i(\mathbf{x}) \leq 0; i = 1, \dots, p; \} \tag{2.3}$$

$$\mathcal{H} = \{\mathbf{x} : h_j(\mathbf{x}) = 0; j = 1, \dots, q; \} \tag{2.4}$$

The set  $\mathcal{X} \subset \mathbb{R}^d$  represents the, so called, box constraints which define the lower and upper bounds of the design variables,  $\mathbf{x}$ .  $\mathcal{X}$  can be mathematically defined as:

$$\mathcal{X} = \{\mathbf{x} : l_k \leq x_k \leq u_k, k = 1, \dots, d\} \tag{2.5}$$

The optimization process aims at minimizing the objective function within the feasible set  $\Omega$ . Two important concepts in single objective optimization that will be used throughout this thesis are the ones of Local Optimal Solution and Global Optimal Solution.

Locally Optimal Solutions can be formally defined as follows:

**Definition 2.1 (Local optimal solution).** *Given an objective function  $f : \Omega \subseteq \mathbb{R}^d \rightarrow \mathbb{R}$ , a solution  $\mathbf{x}^* \in \Omega$  is said to be a local optimum if there exists an  $\epsilon > 0$  such that for all  $\mathbf{x} \in \Omega$  satisfying  $\|\mathbf{x} - \mathbf{x}^*\| \leq \epsilon$ ,  $f(\mathbf{x}^*) \leq f(\mathbf{x})$*

Definition 2.1 states that it is not possible to find a solution better than  $\mathbf{x}^*$  in the neighborhood defined by  $\epsilon$ . If we choose  $\epsilon$  in a way that the neighborhood of  $\mathbf{x}^*$  encompasses the whole feasible space and  $\mathbf{x}^*$  remains optimal, we say that  $\mathbf{x}^*$  is a global optimal solution. Definition 2.2 formalizes this concept.

**Definition 2.2 (Global optimal solution).** *Given an objective function  $f : \Omega \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$ , for  $\mathbf{x} \in \Omega$ , the value  $f^* \triangleq f(\mathbf{x}^*) > -\infty$  is called a global minimum if and only if:*

$$\forall \mathbf{x} \in \Omega : f(\mathbf{x}^*) \leq f(\mathbf{x}) \quad (2.6)$$

$\mathbf{x}^*$  is by definition a global optimal solution.

## 2.2 Multi-objective optimization problem

A general multi-objective optimization problem is usually defined as:

$$\begin{aligned} & \text{minimize } \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})] \\ & \text{subject to } \mathbf{x} \in \Omega \end{aligned} \quad (2.7)$$

Where  $\mathbf{f}(\cdot)$  is a vector function, composed of  $m$  objectives, that maps the search space  $\Omega \subset \mathbb{R}^d$  into the objective space  $\mathcal{Y} \subset \mathbb{R}^m$ .

Having several objective functions, the problem defined in Eq. 2.7 rarely has a solution. Thus, the goal in multi-objective optimization becomes finding a set of solutions which represents good “trade-offs”. This changes the notion of “optimum” which can be posed in

different ways. The least assuming definition is the one of **Pareto optimality** (Deb, 2001).

Before it can be defined, the concept of Pareto dominance must be introduced.

**Definition 2.3 (Pareto Dominance).** A vector  $\mathbf{u} = [u_1, u_2, \dots, u_m]$  is said to dominate another vector  $\mathbf{v} = [v_1, v_2, \dots, v_m]$ , denoted by  $\mathbf{u} \preceq \mathbf{v}$ , if and only if  $\forall i \in \{1, 2, \dots, m\} : u_i \leq v_i$  and  $\exists j \in \{1, 2, \dots, m\} : u_j < v_j$ .

Figure 2–1 illustrates the concept of dominance. It has two non-dominated solutions with their respective dominated area shaded. Any solution in the shaded area is dominated by the non-dominated solutions depicted in the figure.

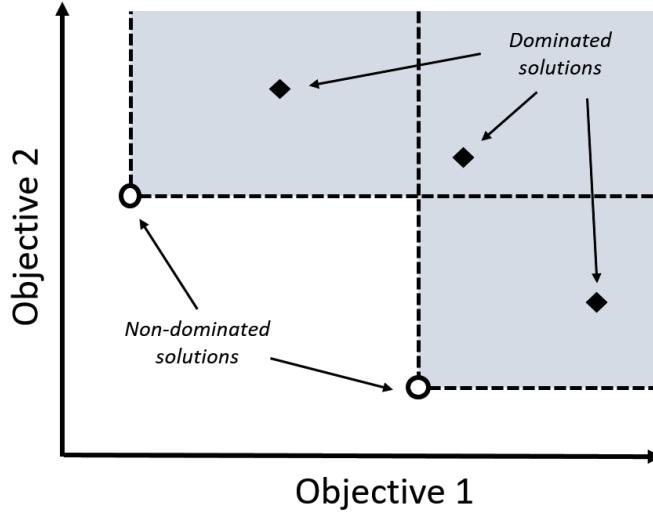


Figure 2–1: Dominance relation in 2D

The Pareto-dominance relationship only establishes a partial order over a given set of candidate solutions. This means that, for certain pairs of elements in the set, one of the elements precedes the other in the ordering but not every pair of elements need be comparable. Thus, there are subsets of solutions that are non-dominated with respect to one another as also indicated indicated in Figure 2–1. Non-dominance can formally defined as follows:

**Definition 2.4 (Non-dominance).** Let  $\mathbf{u}, \mathbf{v} \in \mathbb{R}^m$ . If the relations  $\mathbf{u} \prec \mathbf{v}$  or  $\mathbf{v} \prec \mathbf{u}$  cannot be verified, the vectors  $\mathbf{u}$  and  $\mathbf{v}$  are said to be non-dominated by each other, or incomparable.

In the context of optimization, dominance relations are imposed over the search space according to the following definition:

**Definition 2.5.** Let  $\mathbf{x}_1, \mathbf{x}_2 \in \Omega$ .  $\mathbf{x}_1$  is said to dominate  $\mathbf{x}_2$  if and only if  $\mathbf{f}(\mathbf{x}_1) \prec \mathbf{f}(\mathbf{x}_2)$

Now that the ways of comparing solutions in a multi-dimensional objective space have been described, optimality definitions can finally be presented.

**Definition 2.6 (Pareto Optimal Solution).**  $\mathbf{x}^*$  is a Pareto optimal solution if and only if  $\nexists \mathbf{x} \in \Omega$  such that  $\mathbf{f}(\mathbf{x}) \prec \mathbf{f}(\mathbf{x}^*)$ .

It is possible that many solutions in the feasible search space meet this definition. Thus, as mentioned before, a MOOP has a set of optimal solutions, called Pareto Optimal Set, whose definition is given as follows:

**Definition 2.7 (Pareto Optimal Set).** The Pareto optimal set of a multi-objective optimization problem is defined as:

$$\mathcal{X}^* = \{\mathbf{x} \in \Omega \mid \nexists \mathbf{z} \in \Omega, \mathbf{f}(\mathbf{z}) \prec \mathbf{f}(\mathbf{x})\}. \quad (2.8)$$

This definition means that  $\mathbf{x}$  is a Pareto optimal solution if there exists no other feasible solution,  $\mathbf{z}$ , which would decrease some criterion without causing a simultaneous increase in at least one other criterion (assuming minimization). In other words, the Pareto optimal set is the set of all the non-dominated feasible solutions. When this set is mapped to the objective space, it receives the name of Pareto optimal front,  $\mathcal{Y}^*$  (see Figure 2–2), which can be formally defined as:

**Definition 2.8 (Pareto Optimal Front).** The Pareto optimal front of a multi-objective optimization problem is defined as:

$$\mathcal{Y}^* = \{\mathbf{f}(\mathbf{x}) \in \mathcal{Y} : \mathbf{x} \in \mathcal{X}^*\} \quad (2.9)$$

For the rest of this thesis, the problem defined in Eq. 2.7 will represent in fact the problem of finding the Pareto Optimal Set described by Definition 2.7.

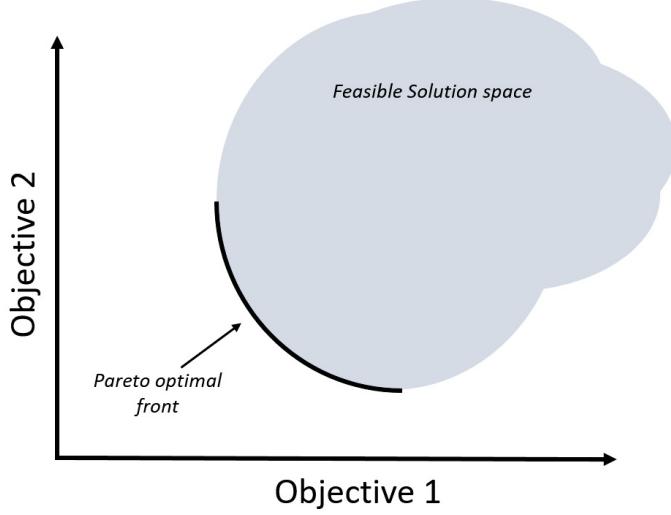


Figure 2–2: Pareto optimal front

### 2.2.1 Comparing non-dominated sets

The result of a multi-objective optimization algorithm is a set of non-dominated solutions. In order to compare algorithms, different indicators can be used to assess the quality of the obtained non-dominated sets. The goal of this section is not to present a comprehensive review on quality indicators but, instead, to present the ones that are most commonly used in the literature. For detailed reviews on the topic see (Coello et al., 2006) and (Ishibuchi et al., 2015).

First we introduce the Inverse Generational Distance (IGD). As illustrated in Figure 2–3, given a reference or target set  $Z$ , the IGD of a set  $A$  is the average distance between each reference point and its nearest solution. The closer to 0 the value of IGD the better is the solution set  $A$ . A formal definition for the IGD is given below.

**Definition 2.9 (Inverse generational distance).** Let  $A = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{|A|}\}$  be a non-dominated solution set obtained by an optimization algorithm and  $Z = \{\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_{|Z|}\}$  a reference set. Then the IGD indicator is computed in the objective space as follows:

$$I_{IGD}(A, Z) = \frac{1}{|Z|} \sum_{i=1}^{|Z|} \left( \min_{\mathbf{a}_j \in A} \|\mathbf{z}_i - \mathbf{a}_j\|_2 \right) \quad (2.10)$$

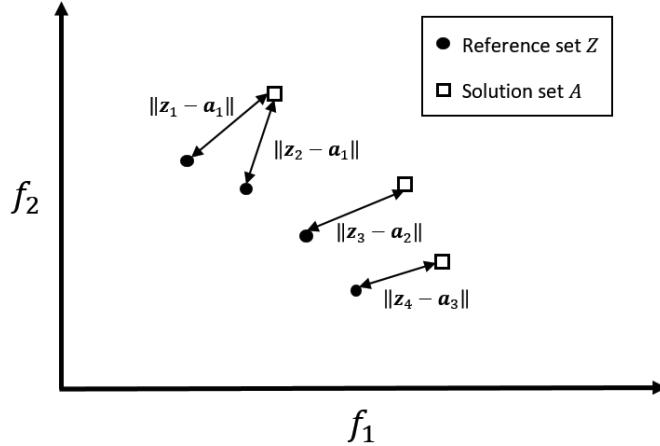


Figure 2–3: The inverse generational distance indicator

The IGD metric requires a reference set that in many cases is hard to obtain. When this set does not exist, the so called hyper-volume indicator is usually preferred. As with the IGD, the hyper-volume indicator (Zitzler and Thiele, 1998) is a measure of the quality of a set  $A = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{|A|}\}$  of non-dominated objective vectors. Assuming minimization, this indicator consists of the measure of the region which is simultaneously dominated by  $A$  and bounded above by a reference point  $\mathbf{z} \in \mathbb{R}^m$  as shown in Figure 2–4. The larger the hyper-volume the better the set  $A$  is.

A formal definition for the hyper-volume is given below.

**Definition 2.10 (Hyper-volume).** Let  $A = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{|A|}\}$  be a non-dominated solution set obtained by an optimization algorithm and  $\mathbf{z} = \{z_1, z_2, \dots, z_m\}$  a reference point. Then the hyper-volume indicator is computed in the objective space as follows:

$$I_{HV}(A, \mathbf{z}) = \sum_{i=1}^{|A|} \int_{\mathbf{a}_i}^{\mathbf{z}} \alpha(\mathbf{y}, \mathbf{a}_i) d\mathbf{y} \quad (2.11)$$

where,

$$\alpha(\mathbf{y}, \mathbf{a}_i) = \begin{cases} 1 & \text{if } \mathbf{a}_i \preceq \mathbf{y} \\ 0 & \text{otherwise} \end{cases} \quad (2.12)$$

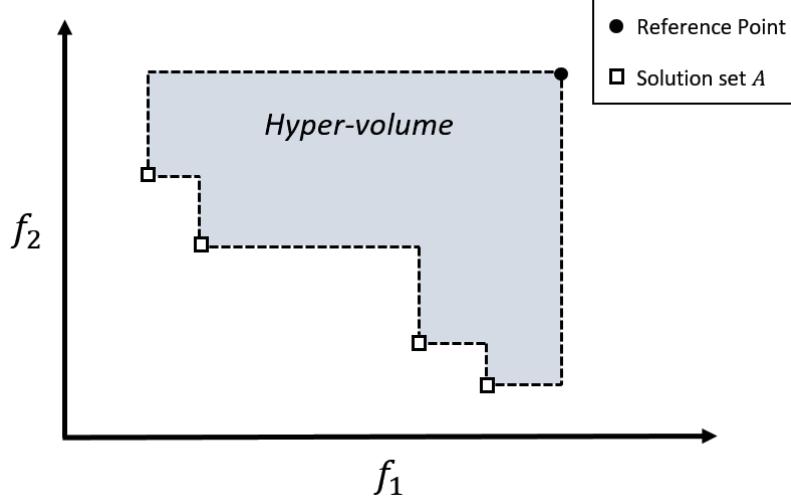


Figure 2–4: The hyper-volume indicator

### 2.3 Surrogate optimization problems

Let  $\mathbf{R}_f : \mathbb{R}^d \rightarrow \mathbb{R}^r$  denote the response vector of the high-fidelity model used to solve the analysis problem. In this work,  $\mathbf{R}_f$  is assumed to be accurate and noise-free. Thus, any prediction computed through  $\mathbf{R}_f$  is considered to be trustworthy.

In this framework, we define the high-fidelity optimization problem as:

$$\begin{aligned} & \text{minimize } \mathbf{f}(\mathbf{x}, \mathbf{R}_f(\mathbf{x})) = [f_1(\mathbf{R}_f(\mathbf{x})), \dots, f_i(\mathbf{R}_f(\mathbf{x})), f_{i+1}(\mathbf{x}), \dots, f_m(\mathbf{x})] \\ & \text{subject to: } \mathbf{x} \in \Omega \end{aligned} \quad (2.13)$$

Where,  $\mathbf{f} : \Omega \rightarrow \mathbb{R}^m$  and  $\Omega \subset \mathbb{R}^n$  defines the feasible set, now defined as:

$$\Omega = \Omega_e \cup \Omega_c \quad (2.14)$$

where

$$\Omega_e = \{\mathbf{x} \in \mathcal{X} : g_i(\mathbf{R}_f(\mathbf{x})) \leq 0, i = 1, \dots, p_e; h_j(\mathbf{R}_f(\mathbf{x})) = 0, j = 1, \dots, q_e\} \quad (2.15)$$

and

$$\Omega_c = \{\mathbf{x} \in \mathcal{X} : g_i(\mathbf{x}) \leq 0, i = (p_e + 1), \dots, (p_c + p_e); h_j(\mathbf{x}) = 0, j = (q_e + 1), \dots, (q_c + q_e)\} \quad (2.16)$$

This formulation means that at least one of the objective functions or at least one of the constraint functions is computed with the high-fidelity model,  $\mathbf{R}_f$ .

Sometimes, the evaluation of  $\mathbf{R}_f$  is computationally expensive which makes the large number of function evaluations required by common optimization methods impractical. In those cases, we wish to use a surrogate problem that is cheaper to solve. A general surrogate optimization problem can be defined as:

$$\begin{aligned} \text{minimize } & \mathbf{f}_s(\mathbf{x}, \mathbf{R}_s(\mathbf{x})) = [f_{s_1}(\mathbf{R}_s(\mathbf{x})), \dots, f_{s_j}(\mathbf{R}_s(\mathbf{x})), f_{s_{j+1}}(\mathbf{x}), \dots, f_{s_{m_s}}(\mathbf{x})] \\ \text{subject to: } & \mathbf{x} \in \Theta \end{aligned} \quad (2.17)$$

Where,  $\mathbf{f}_s : \Theta \rightarrow \mathbb{R}^{m_s}$ ,  $m_s \leq m$ , is the surrogate vector function.  $\mathbf{R}_s : \Theta \rightarrow \mathbb{R}^d$  is a surrogate model that produces responses analogous to the ones produced by  $\mathbf{R}_f$  but is cheap(er) to compute, and  $\Theta \subset \mathbb{R}^d$  is the surrogate feasible set, defined as:

$$\Theta = \Theta_e \cup \Theta_c \quad (2.18)$$

where

$$\Theta_e = \{\mathbf{x} \in \mathcal{X} : g_i(\mathbf{R}_s(\mathbf{x})) \leq 0, i = 1, \dots, p_e; h_j(\mathbf{R}_s(\mathbf{x})) = 0, j = 1, \dots, q_e\} \quad (2.19)$$

and

$$\Theta_c = \{\mathbf{x} \in \mathcal{X} : g_i(\mathbf{x}) \leq 0, i = (p_e + 1), \dots, (p_c + p_e); h_j(\mathbf{x}) = 0, j = (q_e + 1), \dots, (q_c + q_e)\} \quad (2.20)$$

For the sake of conciseness, only the multi-objective formulation of a surrogate problem has been presented. The single objective versions, nevertheless, can be trivially derived by assuming that there is only one objective function.

## 2.4 General structure of Surrogate-based Optimization algorithms

Figure 2–5 shows the general structure of a surrogate-based optimization algorithm. The process starts with the problem specification where the objective functions, constraints,

simulation model and design variables are defined. These specifications constitute the inputs of the sampling module which is responsible for acquiring data about objective functions and constraints. Its output is a database composed of a set of design variables  $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N]^T$  and their respective values of objective functions,  $\mathbf{Y} = [\mathbf{f}(\mathbf{x}_1), \mathbf{f}(\mathbf{x}_2), \dots, \mathbf{f}(\mathbf{x}_N)]^T$ , and constraints,  $\mathbf{G} = [\mathbf{g}(\mathbf{x}_1), \mathbf{g}(\mathbf{x}_2), \dots, \mathbf{g}(\mathbf{x}_N)]^T$  and  $\mathbf{H} = [\mathbf{h}(\mathbf{x}_1), \mathbf{h}(\mathbf{x}_2), \dots, \mathbf{h}(\mathbf{x}_N)]^T$ . The database is used by the surrogate problem selection module for objective reduction, which generates the reduced vector objective function,  $\mathbf{f}_s(\cdot)$ , and surrogate model construction which generates the surrogate model  $\mathbf{R}_s$ . Often, due to the diverse physical nature of the objective and constraint functions, a different surrogate model is constructed for each expensive function, objective or constraint. Thus, objective reduction is usually performed first so the number of surrogate models to be created can be minimized.

Next, the infill point selection module uses the surrogate problem to enrich the database with high-quality points. This module has the ultimate goal of adding to the database the high-fidelity problem optimal solution (or a Pareto optimal set if the problem is multi-objective). Depending on the resource allocation, the enriched database can be used to generate a new surrogate problem and the process is repeated until the algorithm has exhausted the computational budget. Finally, the updated database is returned to the designer who has to pick the solution to be implemented.

Each one of the presented modules is a research field on its own. In the next sections an overview of the main methods used in each module is presented.

#### 2.4.1 The sampling module

In order to construct and validate surrogate problems, data has to be acquired from the high fidelity model through a sampling plan, also called Design of Experiment (DOE). The simplest type of sampling plan is the so called factorial design (Montgomery, 2006) in which the  $d$  design variables are discretized into  $k$  levels and the samples consist of all  $k^d$  possible combinations of all variables at all levels. The Central Composite Design (CCD) is another classic DOE which consists of a  $2^d$  factorial design augmented with axial samples

and a center point. CCDs are used extensively for fitting second order response surfaces. Figure 2–6 shows a CCD that starts from a  $2^2$  factorial design.

It can be seen that the number of samples generated by a factorial design quickly increases with the number of variables and levels. Besides, since a given variable is sampled in the same level more than once, it can be argued that the sampling coverage can be improved if a different strategy is applied. In this context, especially when the underlying function is not expected to behave like a low order polynomial in the region of interest, space filling DOEs tend to be more adequate (Jin et al., 2003; Forrester et al., 2008).

As shown in (Viana et al., 2013), the Latin-hypercube Sampling (LHS) design is the most used space filling DOE in surrogate based optimization. The LHS allows the creation

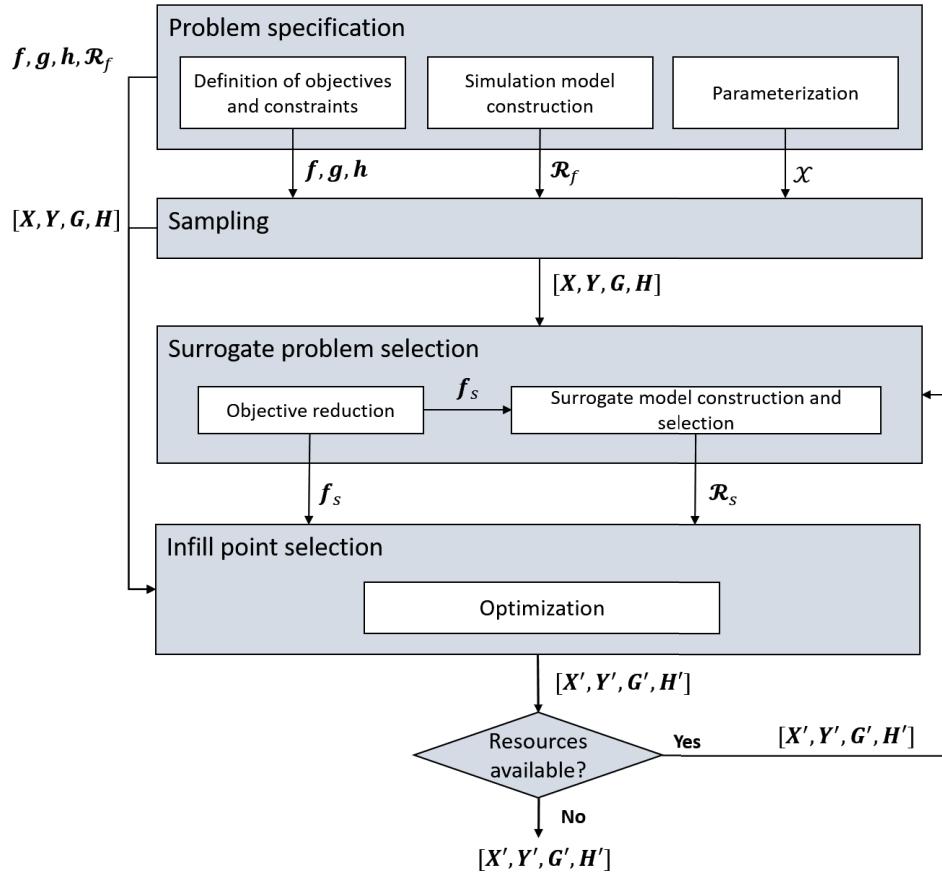


Figure 2–5: Surrogate-based optimization

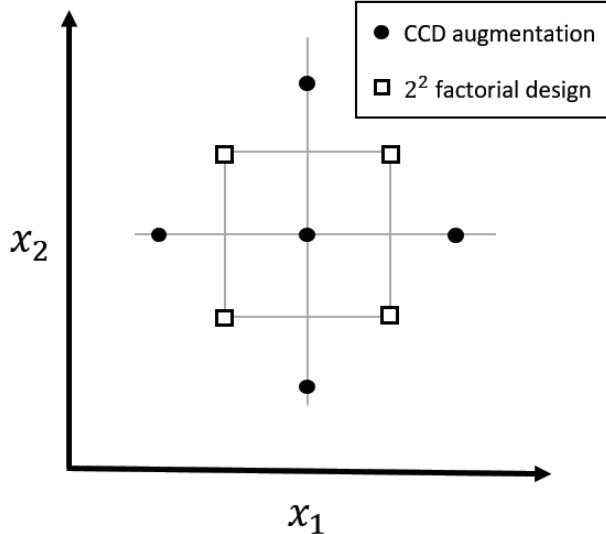
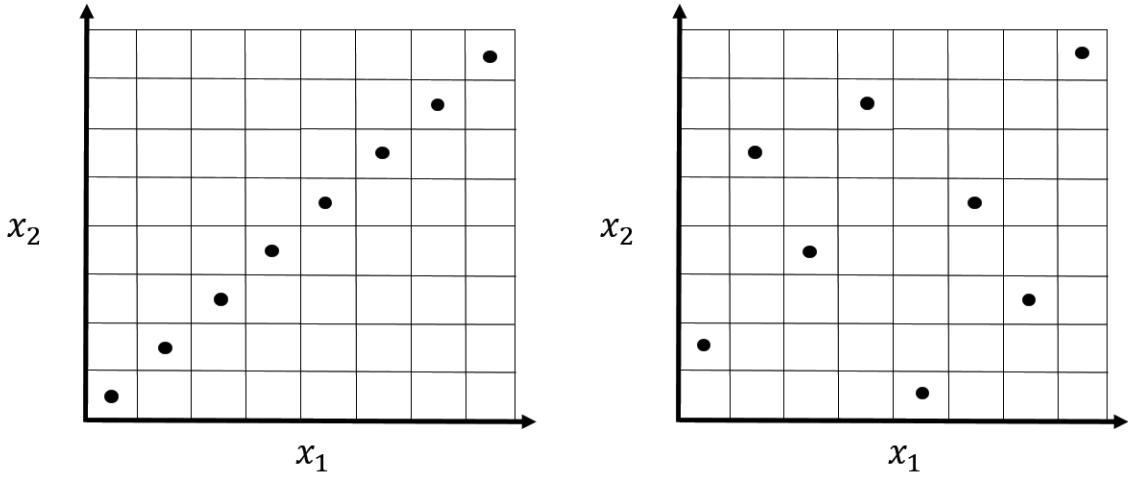


Figure 2–6: Central composite design

of experimental designs with as many points as desired and all the points generated by it are non-collapsing, meaning that a given level is sampled only once per variable.

Let the matrix  $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n]^T$  be an experimental design of  $n$  samples where each column represents a variable and each row  $\mathbf{x}_i = [x_{i,1}, x_{i,2}, \dots, x_{i,d}]$  a sample. In a LHS design each of the  $d$  dimensions is divided into  $n$  levels. Then, the  $n$  samples are randomly distributed such that there is only one sample per level.

In Figure 2–7 it can be seen that the Latin-hypercube design does not necessarily guarantee a good coverage of the design space. This lack of uniformity can be overcome by optimizing the initial plan with respect to some spreading measure of the sample points. Common choices are the maximization of the minimum distance between samples and minimization of the correlation between variables (Queipo et al., 2005; Viana, 2013). Random LHS designs, however, tend not to be as bad as the one shown in Figure 2–7a and, in general, provide an acceptable initial set of samples. Due to the aforementioned characteristics, the LHS design will be used as the main sampling plan in this thesis. For a comprehensive discussion on the topic see (Simpson et al., 2001) and (Montgomery, 2017).



(a) Design with poor space filling properties. (b) Design with good space filling properties.

Figure 2-7: Latin-hypercube designs  $n = 8$  samples and  $d = 2$  dimensions

## 2.4.2 The surrogate problem construction module

### 2.4.2.1 Objective Reduction

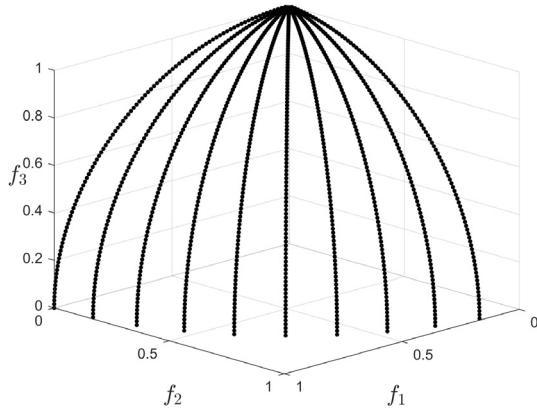
Methods for objective reduction are intrinsically linked with the idea of detecting essential and redundant objectives subsets which will be defined below.

Given a standard MOOP such as the one defined in Eq. 2.7, let  $\mathcal{F} = \{f_1, f_2, \dots, f_m\}$  denote the original objective set and  $\mathcal{Y}^*$  the Pareto optimal front of the original MOOP. Now, let  $\mathcal{F}_s$  be a subset of objectives such that  $\mathcal{F}_s \subset \mathcal{F}$  and  $\mathcal{Y}_s^*$  the Pareto optimal front of the MOOP defined over  $\mathcal{F}_s$ .  $\mathbf{u} \in \mathbb{R}^m$  is a vector in the objective space defined over  $\mathcal{F}$  and  $\mathbf{u}^{(s)} \in \mathbb{R}^k$  its projection onto the objective space defined over  $\mathcal{F}_s$  where  $k = |\mathcal{F}_s|$ .

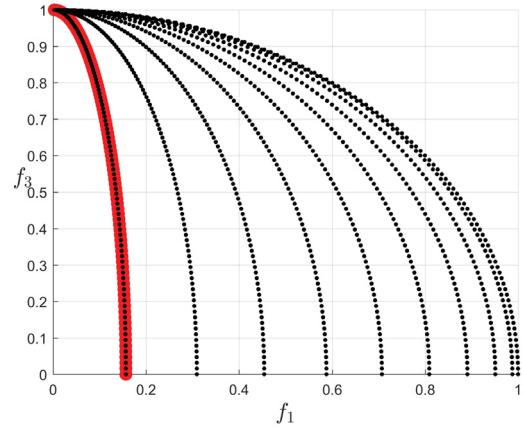
**Definition 2.11 (Redundant Objective Set).** An objective subset  $\mathcal{F}_s \subset \mathcal{F}$  is said to be redundant if and only if the Pareto front of  $\mathcal{F}_r = \mathcal{F} \setminus \mathcal{F}_s$  is  $\mathcal{Y}_r^* = \{\mathbf{u}^{(r)} | \mathbf{u} \in \mathcal{Y}^*\}$ .

In other words, an objective subset  $\mathcal{F}_s$  is redundant if and only if the projection of the Pareto optimal front,  $\mathcal{Y}^*$ , onto the remaining objectives,  $\mathcal{F}_r = \mathcal{F} \setminus \mathcal{F}_s$ , is also Pareto optimal. This means that removing the objectives in  $\mathcal{F}_s$  from the problem does not change the Pareto optimal set.

Figure 2-8a shows a set of Pareto optimal solutions for a hypothetical 3-objective problem defined for  $f_1$ ,  $f_2$  and  $f_3$ . Figure 2-8b shows what happens when  $f_2$  is removed. It can



(a) Pareto optimal front of a 3 objective problem.



(b) Pareto optimal front projected onto objective  $f_1$  and  $f_3$ . The Pareto optimal front of the reduced problem is highlighted in red.

Figure 2–8: Pareto optimal front of a 3 objective problem and its projection onto a two objective space

be seen that, in the reduced formulation, only the points highlighted in red remain Pareto optimal. This “distortion” or, as it is called by Brockhoff and Zitzler (2009a), “error” in the projected front, implies that  $f_2$  is not a redundant objective.

After all the redundant objectives are removed from the objective set we are left with the essential objective set that is formally defined below.

**Definition 2.12 (Essential Objective Set).** *An objective set  $\mathcal{F}_s$  is said to be an essential objective set, if and only if  $\mathcal{F} \setminus \mathcal{F}_s$  is a redundant objective subset with maximum cardinality.*

Alternatively,  $\mathcal{F}_s$  is said to be an essential objective set if it is the subset of lowest possible cardinality such that the remaining objectives,  $\mathcal{F}_r = \mathcal{F} \setminus \mathcal{F}_s$ , are all redundant.

**Definition 2.13 (MOOP Dimensionality).** *The dimensionality of a given MOOP is the cardinality of its essential objective set.*

Given the original set of objectives,  $\mathcal{F} = \{f_1, f_2, \dots, f_m\}$ , a set of samples  $Y \in \mathbb{R}^m$ , and an **error metric  $\epsilon$**  that quantifies the distortion caused by the removal of objectives, the objective reduction problem can be formulated as the following optimization problem:

$$\begin{aligned}
& \text{minimize} && |\mathcal{F}_s| \\
& \text{subject to:} && \mathcal{F}_s \subseteq \mathcal{F} \\
& && \epsilon(\mathcal{F}_s, Y) \leq \tau
\end{aligned} \tag{2.21}$$

In Eq. 2.21 one is looking to minimize the cardinality of the objective set such that the “error” is less than a user-defined threshold,  $\tau$ .

As we will see in Chapter 3 the error function can be formulated in various ways and the definition of the threshold value can be tricky. Besides, if the value of  $\tau$  is too conservative, it may result in overly large objective sets. In order to avoid this problem, Yuan et al. (2017) proposed the following multi-objective formulation for the objective reduction problem:

$$\begin{aligned}
& \text{minimize} && [|\mathcal{F}_s|, \epsilon(\mathcal{F}_s, A)] \\
& \text{subject to:} && \mathcal{F}_s \subseteq \mathcal{F}
\end{aligned} \tag{2.22}$$

It seems clear that there is a compromise between the number objectives in the reduced set and the error. This compromise however is not known a priori and using the multi-objective approach allows the user to visualize it and make an informed decision about the final set of objectives. The only drawback when using this formulation is the need of a decision maker. Hence, human interaction will be needed in the surrogate problem selection module.

The number of possible subsets in a set of size  $m$  is  $2^m$ . Thus, for moderate sizes of objective sets ( $\leq 20$ ), the problems defined by equations 2.21 and 2.22 can be solved exactly. When the number of objectives is high, heuristics, such as Evolutionary Algorithms, are commonly used (Yuan et al., 2017).

#### 2.4.2.2 Surrogate model construction

**Polynomial models.** Arguably, the simplest form of surrogate model encountered in the literature is the Polynomial model. Broadly speaking, it consists of fitting the response data to a low-order polynomial using least squares regression.

Consider the case of the first order polynomial. Given a vector of responses,

$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \quad (2.23)$$

and a matrix of observed variables,

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & \dots & x_{1d} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & \dots & x_{nd} \end{bmatrix} \quad (2.24)$$

where  $n$  is the number of samples, the relationship between  $\mathbf{y}$  and  $\mathbf{X}$  can be described as:

$$\mathbf{y} = \mathbf{X}\beta + \epsilon \quad (2.25)$$

where,  $\beta$  is the vector of regression coefficients, and  $\epsilon$  the error vector.

The model “training” consists of finding the least squares estimators,  $\mathbf{b}$ , that minimize the loss function defined in Eq. 2.26.

$$L = \sum_{i=1}^n \epsilon_i^2 = \epsilon' \epsilon = (\mathbf{y} - \mathbf{X}\mathbf{b})^T(\mathbf{y} - \mathbf{X}\mathbf{b}) \quad (2.26)$$

By taking the derivatives of Eq. 2.26 with respect to the regression coefficients it is possible to derive (see (Myers and Montgomery, 2002) for the detailed derivation) that the least squares estimators of  $\beta$  are given by:

$$\mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} \quad (2.27)$$

Thus, a prediction at an unseen point  $\mathbf{x}$  is:

$$\hat{y}(\mathbf{x}) = \mathbf{x}\mathbf{b} \quad (2.28)$$

In order to fit higher order polynomials, the matrix  $\mathbf{X}$  has to be augmented with higher order terms. Eq. 2.29 defines a generic data matrix for a  $2^{nd}$ -order polynomial of two

variables.

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & x_{11}x_{12} & x_{11}^2 & x_{12}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n_1} & x_{n_2} & x_{n_1}x_{n_2} & x_{n_1}^2 & x_{n_2}^2 \end{bmatrix} \quad (2.29)$$

The minimum number of samples required to fit a  $k^{th}$ -order polynomial of  $d$  variables is:

$$\frac{(k+d)!}{k! d!} \quad (2.30)$$

**Kriging.** Kriging (Krige, 1953; Jones, 2001) is an interpolation method that expresses the sought, unknown, function  $y(\mathbf{x})$  as a combination of a global model  $\beta$  with local deviations  $Z(\mathbf{x})$ , that is:

$$y(\mathbf{x}) = \beta + Z(\mathbf{x}) \quad (2.31)$$

where,  $\beta$  approximates the global trend of the original function while  $Z(\mathbf{x})$  creates local deviations in order to approximate a possible multimodal behavior.

Mathematically,  $Z(\mathbf{x})$  is the realization of a stochastic process with zero mean, variance  $\sigma^2$  and covariance given by:

$$Cov[Z(\mathbf{x}_i), Z(\mathbf{x}_j)] = \sigma^2 \mathbf{R} \quad (2.32)$$

$\mathbf{R}$  is the correlation matrix of all the observed data defined as:

$$\mathbf{R} = \begin{bmatrix} R(\mathbf{x}_1, \mathbf{x}_2) & \cdots & R(\mathbf{x}_1, \mathbf{x}_n) \\ \vdots & \ddots & \vdots \\ R(\mathbf{x}_1, \mathbf{x}_n) & \cdots & R(\mathbf{x}_n, \mathbf{x}_n) \end{bmatrix} \quad (2.33)$$

where,  $R(\mathbf{x}_i, \mathbf{x}_j)$  is the correlation function defined as:

$$R(\mathbf{x}_i, \mathbf{x}_j) = \exp \left( - \sum_{l=1}^k \theta_l |x_{i_l} - x_{j_l}|^2 \right) \quad (2.34)$$

Predicted values at untried points are given by:

$$\hat{y}(\mathbf{x}) = \hat{\beta} + \mathbf{r}(\mathbf{x})^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{1}\hat{\beta}) \quad (2.35)$$

where  $\mathbf{1}$  is the unit vector,  $\hat{\beta}$  is the estimated value of  $\beta$  given by Eq. 2.39,  $\mathbf{y}$  contains the response values of the sample points and  $\mathbf{r}^T$  is the correlation vector between an untried point  $\mathbf{x}$  and the sampled data points  $\mathbf{x}^{(i)}$ ,  $i = 1, \dots, n$ .

$$\mathbf{r}(\mathbf{x})^T = [R(\mathbf{x}, \mathbf{x}_1), R(\mathbf{x}, \mathbf{x}_2), \dots, R(\mathbf{x}, \mathbf{x}_n)] \quad (2.36)$$

Kriging also provides an estimate for the prediction variance given by:

$$s^2(\mathbf{x}) = \hat{\sigma}^2 \left[ 1 - \mathbf{r}(\mathbf{x})^T \mathbf{R}^{-1} \mathbf{r}(\mathbf{x}) + \frac{(1 - \mathbf{r}(\mathbf{x})^T \mathbf{R}^{-1} \mathbf{r}(\mathbf{x}))^2}{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1}} \right] \quad (2.37)$$

Where,  $\hat{\sigma}^2$  is the estimated value of  $\sigma^2$  given by Eq. 2.40.

Training the kriging model consists of maximizing the likelihood function, given by Eq. 2.38, in order to find the unknown parameters  $\theta_l$ .

$$\begin{aligned} \ln(L(\theta)) &= -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln(\sigma^2) - \frac{n}{2} \ln(|\mathbf{R}(\theta)|) \\ &\quad - \frac{(\mathbf{y} - \mathbf{1}\hat{\beta})^T \mathbf{R}(\theta)^{-1} (\mathbf{y} - \mathbf{1}\hat{\beta})}{2\sigma^2} \end{aligned} \quad (2.38)$$

$$\hat{\beta} = (\mathbf{1}^T \mathbf{R}(\theta)^{-1} \mathbf{y}) / (\mathbf{1}^T \mathbf{R}(\theta)^{-1} \mathbf{1}) \quad (2.39)$$

$$\hat{\sigma}^2 = ((\mathbf{y} - \mathbf{1}\hat{\beta})^T \mathbf{R}(\theta)^{-1} (\mathbf{y} - \mathbf{1}\hat{\beta})) / n \quad (2.40)$$

**Radial basis functions neural networks.** Radial basis functions neural networks (RBFNN) can be implemented in a number of different ways. In this thesis, we are going to focus on the classical RBFNN interpolation model with Gaussian basis functions presented in Powell (1988). This neural network has one hidden layer with  $n$  neurons, where  $n$  is also the number of samples, and one output layer. Each neuron in the hidden layer has the following form:

$$\phi_k(\mathbf{x}, \mathbf{c}_k) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{c}_k\|_2^2}{\sigma_k^2}\right) \quad 1 \leq k \leq N \quad (2.41)$$

where  $\mathbf{x}$  is some input vector,  $\mathbf{c}_k$  is the  $k^{th}$  training point which is also the center of the basis function  $\phi_k(\cdot)$ , and  $\sigma_k^2$  controls the basis function width.

The output layer is simply the weighted sum of the outputs of the hidden layer neurons. Given a user provided set of width parameters  $\sigma_k$  and training points  $T$ , composed by input vectors  $\mathbf{c}_i$  and targets  $y_i$ , the RBFNN can be concisely expressed in matrix form as:

$$\Phi \mathbf{w} = \mathbf{Y} = [y_1, y_2, \dots, y_N]^T \quad (2.42)$$

where,

$$\Phi = \begin{bmatrix} \phi(\mathbf{c}_1, \mathbf{c}_2) & \cdots & \phi(\mathbf{c}_1, \mathbf{c}_n) \\ \vdots & \ddots & \vdots \\ \phi(\mathbf{c}_1, \mathbf{c}_n) & \cdots & \phi(\mathbf{c}_n, \mathbf{c}_n) \end{bmatrix} \quad (2.43)$$

If all the centers are pairwise different, the matrix  $\Phi$  is positive-definite and invertible (Yao et al., 2014). Thus, the weight vector  $\mathbf{w}$  can be computed through Eq. 2.42. Once the weights are computed, predictions at untried points  $\mathbf{x}$  are given by:

$$\begin{aligned} \hat{y}(\mathbf{x}) &= \mathbf{r}(\mathbf{x}) \cdot \mathbf{w} \\ \mathbf{r}(\mathbf{x}) &= [\phi(\mathbf{x}, \mathbf{c}_1), \phi(\mathbf{x}, \mathbf{c}_2), \dots, \phi(\mathbf{x}, \mathbf{c}_N)]^T \\ \mathbf{w} &= [w_1, w_2, \dots, w_N]^T \end{aligned} \quad (2.44)$$

**Generalized regression neural network.** The Generalized Regression Neural Network (GRNN) is a simple yet powerful regression model proposed in Specht (1991). Different from other artificial neural networks (ANNs), GRNNs do not use backpropagation for training. Instead, the predictions are directly derived from the sampled data using the following formula:

$$\hat{y}(\mathbf{x}) = \frac{\sum_{i=1}^n y_i \exp\left(-\frac{(\mathbf{x}-\mathbf{x}_i)^T(\mathbf{x}-\mathbf{x}_i)}{2\sigma^2}\right)}{\sum_{i=1}^n \exp\left(-\frac{(\mathbf{x}-\mathbf{x}_i)^T(\mathbf{x}-\mathbf{x}_i)}{2\sigma^2}\right)} \quad (2.45)$$

where,  $\mathbf{x}_i$ s and  $y_i$ s are the sampled input vector and the respective response values.  $\sigma$  is known as the smoothing parameter and controls the width of the Gaussian functions. Notice that the estimate  $\hat{y}(\mathbf{x})$  is simply the weighted average of all the sample observations  $y_i$ , where the weight for each observation is proportional to the exponential of the squared Euclidean distance between sample  $\mathbf{x}$  and  $\mathbf{x}_i$ .

Defining the kernel function as  $k(\mathbf{a}, \mathbf{b}) = \exp\left(-\frac{(\mathbf{a}-\mathbf{b})^T(\mathbf{a}-\mathbf{b})}{2\sigma^2}\right)$ , GRNNs can also be represented using the block diagram shown in Figure 2–9. It can be seen that the GRNNs have one input unit, two summation units, one output unit and  $n$  pattern units, where  $n$  corresponds to the number of samples in the database. The weights for the connections between neurons are either given by the sample data or 1. It is important to highlight that unlike Kriging and the adopted RBFNNs, GRNNs do not necessarily interpolate the sample data. Thus, when used as surrogate models in the context of optimization, GRNNs may smooth out the objective functions, and, in some cases, make the optimization problem easier to solve by removing unimportant local minima.

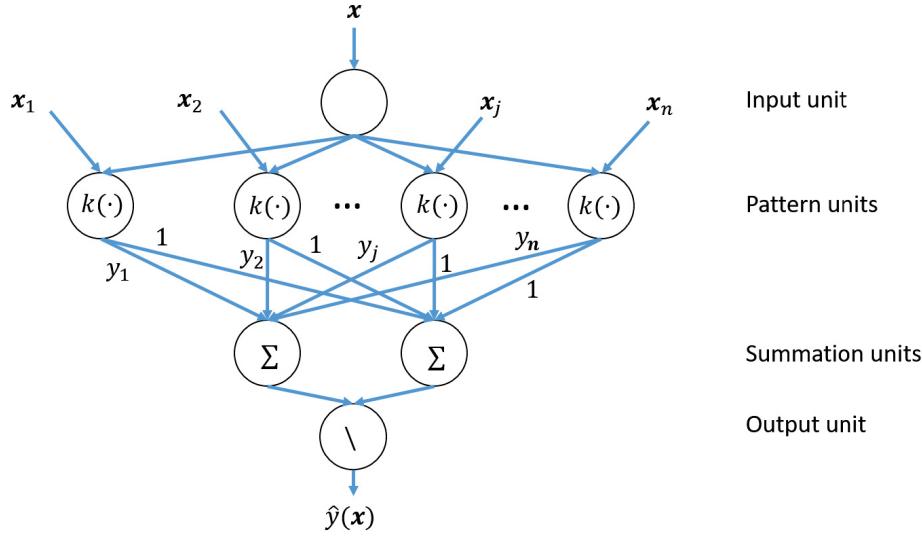


Figure 2–9: GRNN block diagram adapted from (Specht, 1991).

#### 2.4.2.3 Time required for the construction and evaluation of surrogate models

In surrogate-model based optimization it is important to know how much time is used for surrogate construction and evaluation. Thus, in this section, MATLAB implementations of each one of the presented surrogate models are compared in an 8-core Intel Xeon E5 1650 CPU (3.5 GHz) with 32 GB RAM. The goal for each model was to approximate the quadratic function defined below:

$$f(\mathbf{x}) = \sum_{i=1}^d x_i^2 \quad (2.46)$$

Figure 2–10 presents the time required to fit the surrogate models with sample sizes varying from 100 to 1500 and number of variables varying from 2 to 40. The exception is the 3<sup>rd</sup> polynomial which requires more than 1700 samples for functions with more than 20 variables. Figure 2–11 presents the time required for evaluating 100,000 samples.

**It can be seen that** the polynomial models are the fastest for both training and sample evaluation (up to 8 variables). Nevertheless, as the number of variables increases, the number of samples required to fit these models becomes impractical. GRNNs do not require training, meaning that their construction cost is simply the time required to allocate the data-structures that store the samples. RBFNNs have higher training costs, but the sampling evaluation cost of both GRNNs and RBFNNs is of the same order of magnitude. Kriging presented the highest costs for both training and sample evaluation which was orders of magnitude higher than the costs presented by the other meta-models.

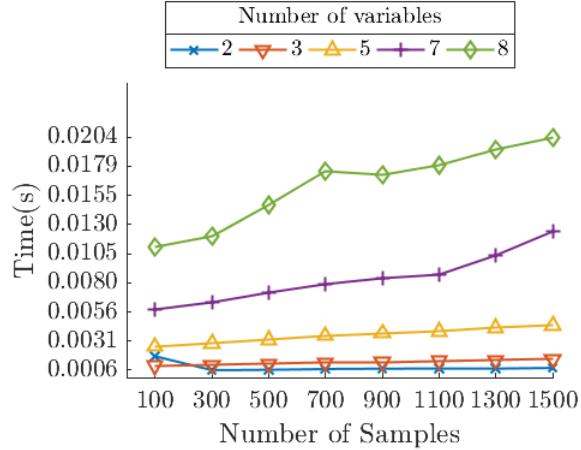
Given the times required to solve the analysis problem, is it important that practitioners keep this information in mind while considering a surrogate-model based approach for a given problem.

#### 2.4.2.4 The infill point selection module

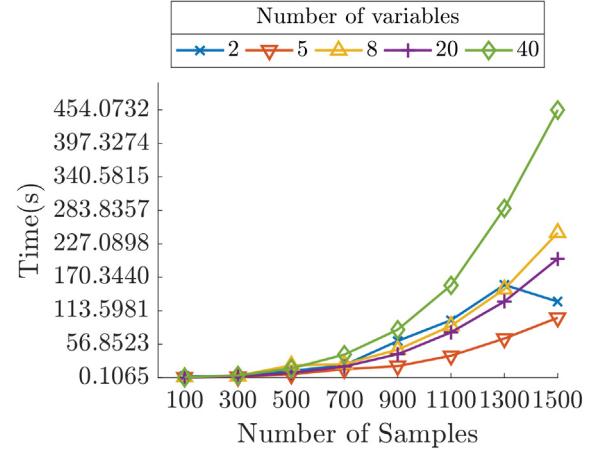
Given a single objective surrogate problem, the infill point selection problem can be defined as finding the solution,  $\mathbf{x}_s^*$ , that optimizes it. That is:

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} \quad f_s(\mathbf{x}) \\ & \text{subject to: } \mathbf{x} \in \Theta \end{aligned} \tag{2.47}$$

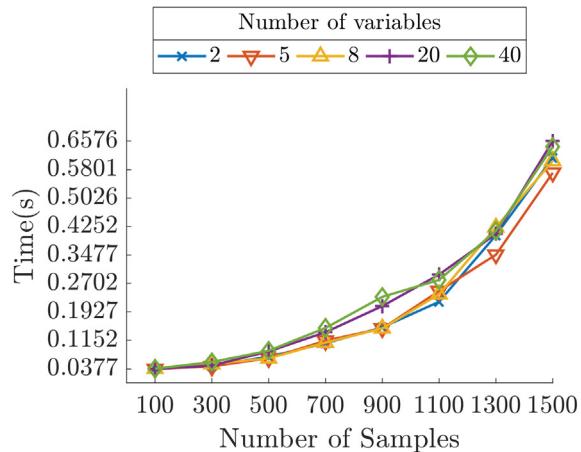
When the problem is multi-objective, a set of non-dominated solutions is obtained instead of a single point. Depending on the set size, **it may not be realistic to evaluate every single solution with the high-fidelity model.** Therefore, auxiliary objectives have to be introduced in order to find, among the set of surrogate problem solutions, the ones that generate the most improvement for the non-dominated set that already exists in the database. Most



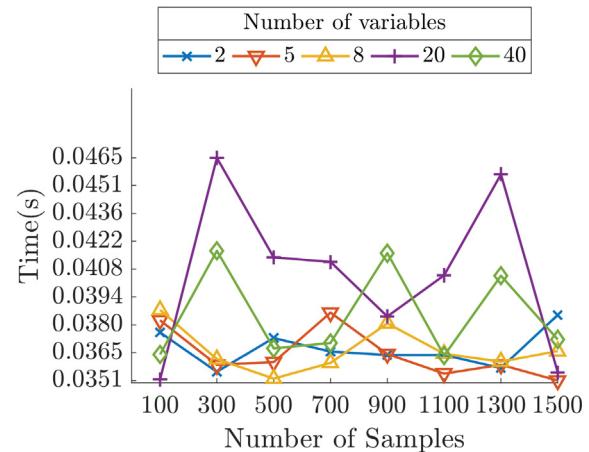
(a) 3<sup>rd</sup> order polynomial model



(b) Kriging



(c) Radial basis functions neural network



(d) Generalized regression neural network

Figure 2–10: The median time required to fit different surrogate models

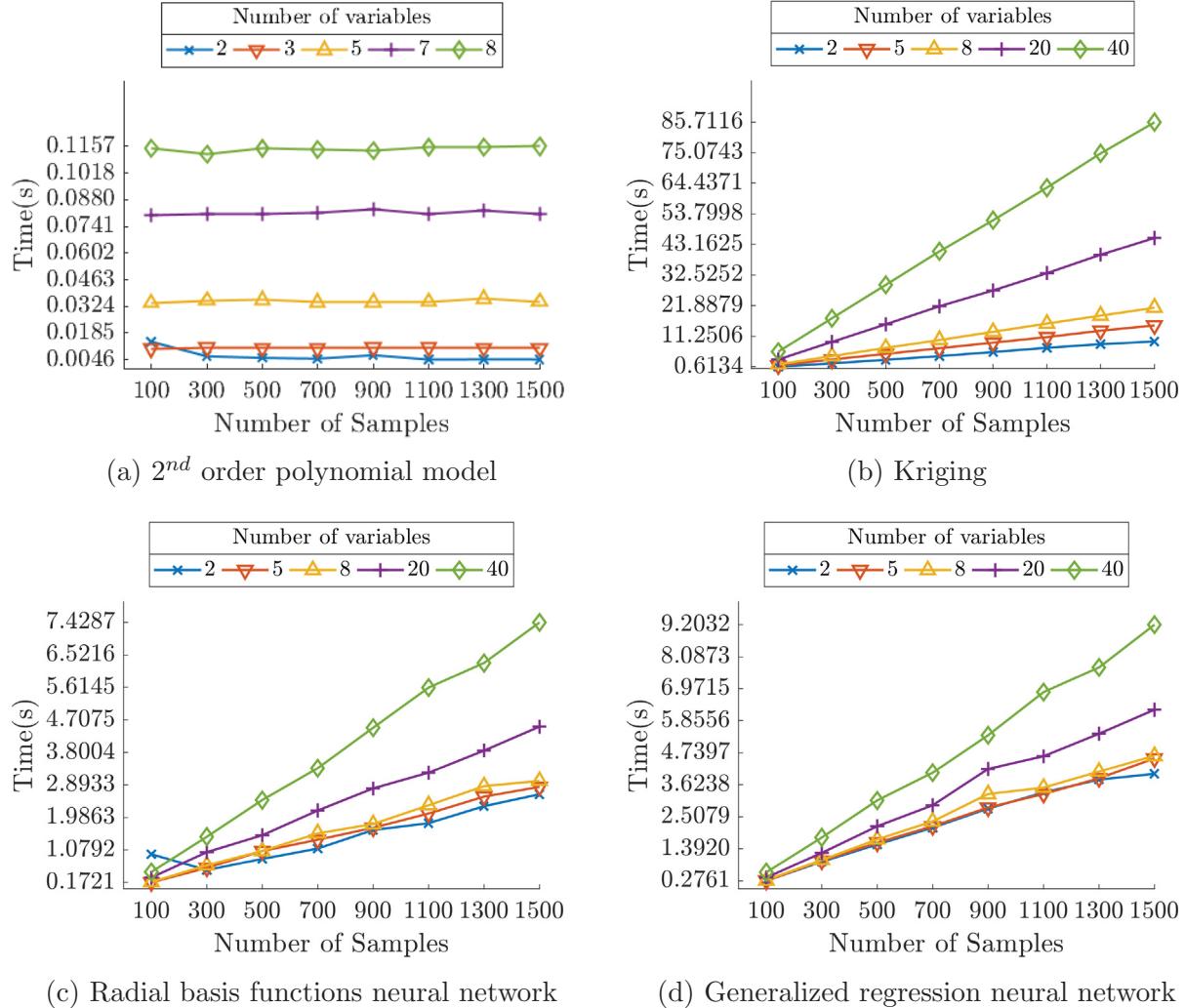


Figure 2–11: Time required to evaluate 100,000 candidate solutions

of the infill point selection strategies in the literature are based on the prediction variance information given by surrogate models like Kriging (Jones, 2001; Forrester et al., 2008; Zhan et al., 2017). For most surrogate models, however, this information is not available. In this context, Silva et al. (2017a) proposed and compared three infill point selection algorithms that only depend on the current database and on the solution obtained through the surrogate problem optimization.

Let  $\mathbf{X}_s^+ = [\mathbf{x}_{s_1}, \dots, \mathbf{x}_{s_k}]^T$  be the set of feasible non-dominated solutions found by solving the problem defined by Eq. 2.17 and  $\mathbf{Y}_s^+ = [\mathbf{f}_s(\mathbf{x}_{s_1}), \dots, \mathbf{f}_s(\mathbf{x}_{s_k})]^T$  the corresponding surrogate objective values.  $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_n]^T$  is the database of candidate solutions and  $\mathbf{Y} = [\mathbf{f}(\mathbf{x}_1), \dots, \mathbf{f}(\mathbf{x}_n)]^T$  the corresponding objective values evaluated with the original formulation.

The first selection method proposed by Silva et al. (2017a) is described in Algorithm 2.1. This algorithm is based on the density in the search space and the rationale is that the original problem is well represented in the regions densely populated by the samples in the database. Therefore, from the points seen as the best ones according to the surrogate problem,  $\mathbf{X}_s^+$ , one should select the ones that are the farthest from other points in the database. By applying this rationale, the method exploits the regions considered promising from the surrogate problem point of view and at the same time explores regions that are under represented by the database.

---

**Algorithm 2.1:** Infill based on the density in the design space

---

**Data:**  $\mathbf{X}, \mathbf{X}_s^+, \mathbf{Y}, p$   
**Result:** Updated database:  $\mathbf{X}, \mathbf{Y}$

```

1 for i = 1 to p do
2   |   x ← arg maxxi ∈ Xs+ minxj ∈ X ||xi - xj||2;
3   |   X ← X ∪ x;
4   |   Y ← Y ∪ f(x);
5   |   Xs+ ← Xs+ \ x;
6 end

```

---

The next selection method proposed by (Silva et al., 2017a) is described in Algorithm 2.2 and is based on the density in the objective space. The rationale is similar to the previous method but instead of computing the distance in the design space, it is now computed in the objective space. Another difference is that this method requires that the infill points are not dominated with respect to any point in the database. If the number of objectives in  $\mathbf{f}(\cdot)$  is less than in  $\mathbf{f}_s(\cdot)$ , the  $\mathbf{f}(\cdot)$  values are projected onto the reduced set.

---

**Algorithm 2.2:** Infill based on the density in the objective space

---

**Data:**  $\mathbf{X}, \mathbf{X}_s^+, \mathbf{Y}, \mathbf{Y}_s^+, p$   
**Result:** Updated database:  $\mathbf{X}, \mathbf{Y}$

```

1  $\mathbf{X}' \leftarrow$  non-dominated solutions in  $\mathbf{Y}^{(s)} \cup \mathbf{Y}_s^+$  ;
2 for  $i = 1$  to  $p$  do
3    $\mathbf{x} \leftarrow \arg \max_{\mathbf{x}_i \in \mathbf{X}' \cap \mathbf{X}_s^+} \min_{\mathbf{x}_j \in \mathbf{X}' \cap \mathbf{X}} \|\mathbf{f}_s(\mathbf{x}_i) - \mathbf{f}(\mathbf{x}_j)\|_2$ ;
4    $\mathbf{X} \leftarrow \mathbf{X} \cup \mathbf{x}$ ;
5    $\mathbf{Y} \leftarrow \mathbf{Y} \cup \mathbf{f}(\mathbf{x})$ ;
6    $\mathbf{X}_s^+ \leftarrow \mathbf{X}_s^+ \setminus \mathbf{x}$ ;
7 end
```

---

Figure 2–12 helps to illustrate the difference between the two methods. If the method has to select 2 infill points, according to Algorithm 2.1, points 3 and 4 would be selected. According to Algorithm 2.2, solution 3 cannot be selected since it is dominated by another solution in the database. In this case, solutions 1 and 4 would have been selected.

Finally, the last method proposed by Silva et al. (2017a) is given by Algorithm 2.3. In this method, the infill points are randomly selected with uniform distribution from the set  $\mathbf{X}_s^+$ . This operation is represented as  $\mathcal{U}_{\mathbf{X}_s^+}$ .

As expected, according to (Silva et al., 2017a), Algorithms 2.1 and 2.2 presented better results (better non-dominated sets for the original problem) when compared to Algorithm 2.3. Since the Algorithm 2.2 is more "aggressive" towards the the non-dominated set, the authors suggest its application if the number of SBOM iterations is small. On the other hand, if the number of iterations is high ( $> 20$ ), Algorithm 2.1 is recommended due to its higher exploratory capabilities.

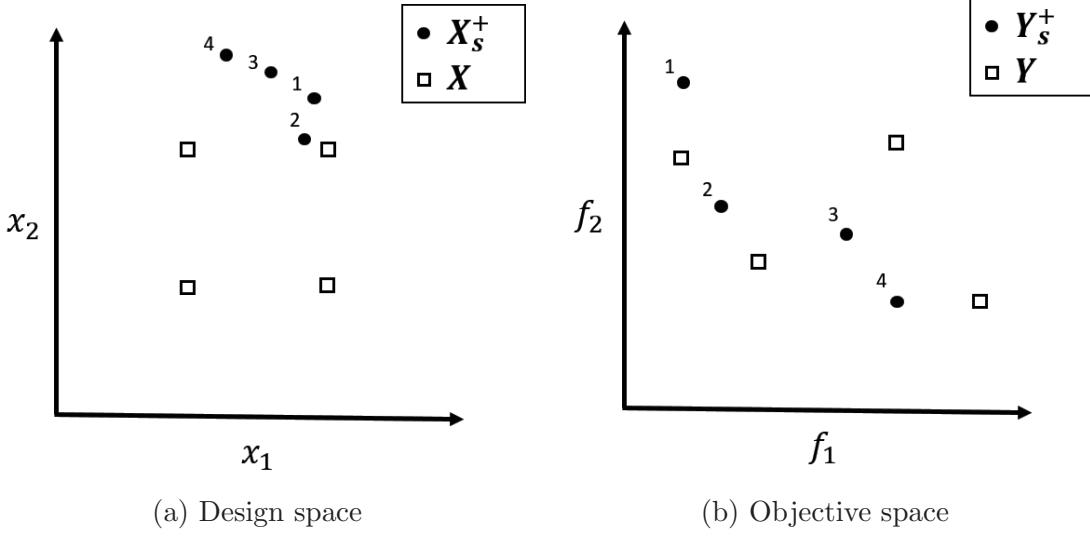


Figure 2–12: Database and surrogate non-dominated sets represented in the design and objective spaces

---

**Algorithm 2.3:** Random infill

---

**Data:**  $\mathbf{X}, \mathbf{X}_s^+, \mathbf{Y}, p$   
**Result:** Updated database:  $\mathbf{X}, \mathbf{Y}$

```

1 for  $i = 1$  to  $p$  do
2    $\mathbf{x} \leftarrow \mathcal{U}_{\mathbf{X}_s^+};$ 
3    $\mathbf{X} \leftarrow \mathbf{X} \cup \mathbf{x};$ 
4    $\mathbf{Y} \leftarrow \mathbf{Y} \cup \mathbf{f}(\mathbf{x});$ 
5    $\mathbf{X}_s^+ \leftarrow \mathbf{X}_s^+ \setminus \mathbf{x};$ 
6 end

```

---

When no feasible solutions are found after optimization Voutchkov and Keane (2010) recommend sampling the solutions with minimum total constraint violation,  $v(\cdot)$ , which is computed as follows:

$$v(\mathbf{x}) = \sum_{i=1}^{p_e} \max(g_i(\mathbf{R}_s(\mathbf{x})), 0) + \sum_{j=1}^{p_c} \max(g_j(\mathbf{x}), 0) + \sum_{i=1}^{q_e} |h_i(\mathbf{R}_s(\mathbf{x}))| + \sum_{j=1}^{q_c} |h_j(\mathbf{x})| \quad (2.48)$$

At this point, all the necessary building blocks for the construction and analysis of surrogate-based optimization methods have been presented. In the next chapter, we tackle the issue of evaluating surrogate problems. Using the definitions provided here, we attempt to establish a definition for surrogate problem quality that is aligned with the goal of attaining the optimal solution(s) of the original problem. Under the proposed definition, a

theoretical analysis of many performance metrics is presented, and new performance metrics are introduced.

## CHAPTER 3

### Surrogate-problem evaluation: theoretical and practical aspects

Independently from the technique used for their construction (surrogate modeling, objective reduction or both), surrogate problems have to be evaluated before they can be used to solve a given optimization task. Their quality directly affects the feasibility, cost, and effectiveness of the optimization.

As could be seen in Chapter 1, the questions, “What is the best surrogate model?” and “What is the best reduced problem?” have been extensively investigated in the literature. A question that seems to be overlooked however is “What does ‘best’ mean?”. The answer to this question is of tantamount importance and not answering it properly can lead to poor decisions regarding the choice of surrogate problems.

Clearly, the definition of “best” depends on the application and this is where confusion arises. For instance, in the field of surrogate-model based optimization, the definition of “best” has been, most of the time, imported from the general theory of model evaluation which focuses on the accuracy of the model predictions. Prediction and optimization, however, are very distinct tasks with very different goals.

One of the main issues when choosing a metric to evaluate surrogate problems is the lack of a rigorous definition of quality that links surrogate problem and optimization results. Without this definition, the selection of a performance metric becomes purely speculative which, as mentioned before, may give rise to poor surrogate problems.

Given this background, Section 3.1 presents a discussion on the meaning of quality for surrogate problems. This discussion results in a set of formal definitions of quality that are applicable for both surrogate models and reduced problems. Under the proposed definitions, in Section 3.2, a thorough theoretical analysis of commonly used performance metrics for surrogate models is developed. In Section 3.3, the same kind of analysis is developed for

performance metrics used for reduced problems. After a summary of the main findings in Section 3.4, finally in Section 3.5, new performance metrics intended to unify the evaluation of surrogate problems and overcome some of the issues encountered in the analyzed metrics are presented.

### 3.1 Defining quality for surrogate problems

In order to define quality for surrogate problems we have to go back to the problem definitions and identify the surrogate-based optimization main goal.

In SBO, one is trying to solve the high-fidelity problem defined in Eq. 1.1 and reproduced below.

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{f}(\mathbf{x}) \\ \text{subject to: } & \mathbf{x} \in \Omega \end{aligned} \tag{1.1}$$

Since the computational cost involved in solving this problem is too high, instead, a surrogate problem (given in Eq. 2.17 and concisely represented by Eq. 3.1) which is computationally cheaper to handle is solved.

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{f}_s(\mathbf{x}) \\ \text{subject to: } & \mathbf{x} \in \Theta \end{aligned} \tag{3.1}$$

The traditional way to look at how good a surrogate problem is with respect to a given high-fidelity problem is to verify how well the former matches the responses of the latter. By this rationale, quality is defined as a summary (e.g. the average) of some norm of the difference between responses. Figure 3–1 illustrates this approach.

This view about quality however seems to be too restrictive. For instance, by definition, it already gives a very low selection chance to surrogate-models that do not interpolate the sample data. Besides, if Figure 3–1 is inspected again, it is possible to see that the minimization problems defined by both functions have the exact same solution. Since, by the SBO algorithm shown in Figure 1–3, all the solutions returned to the user have to be validated against the high-fidelity problem, requiring matching responses seems to be counter productive. In this respect, consider the example shown in Figure 3–2 in which there is a

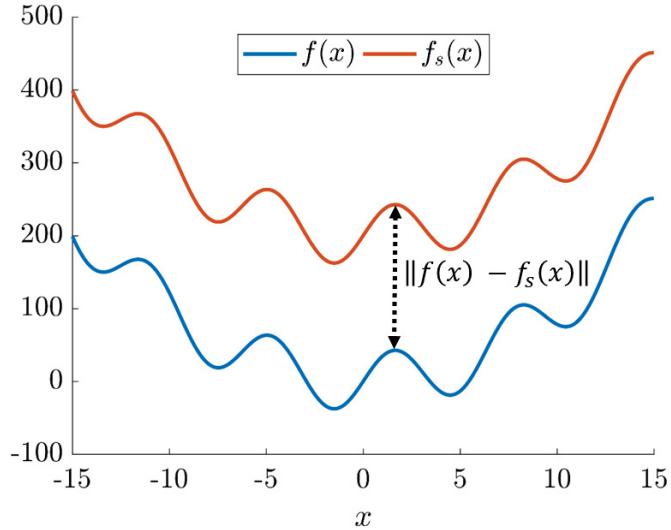


Figure 3–1: Traditional approach for quality assessment of surrogate problems

high-fidelity problem defined over  $f(x)$  and two surrogate problems  $f_{s1}(x)$  and  $f_{s2}(x)$ . Using the traditional definition of quality,  $f_{s1}$  is always preferred over  $f_{s2}$ . This is why, in principle, this definition should not be used in SBO. It can be seen, that  $f_{s2}$  and  $f$  have matching optima. Therefore, optimizing  $f_{s2}$  would result in the high-fidelity problem optimal solution which is the ultimate goal. Motivated by these examples, it is possible to infer that in SBO a definition of quality that gives preference to  $f_{s1}$  over  $f_{s2}$  is not adequate.

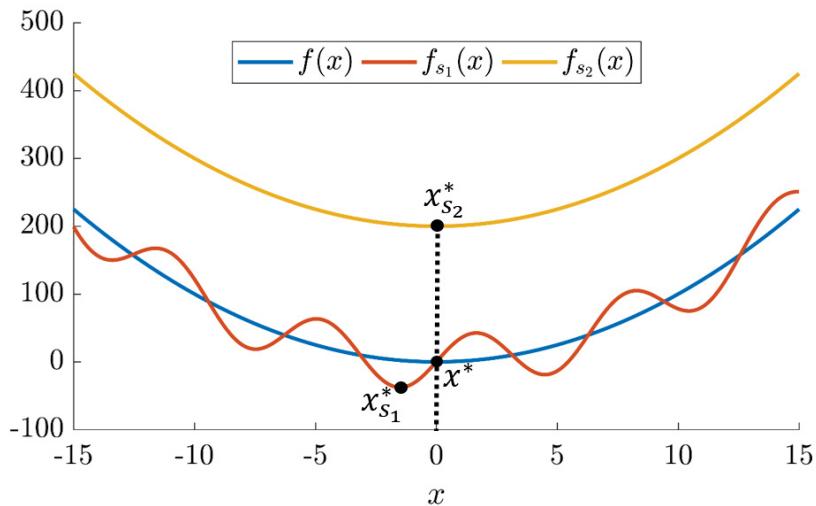


Figure 3–2: High-fidelity problem,  $f(x)$ , and two surrogate problems,  $f_{s1}(x)$  and  $f_{s2}(x)$

In summary, given that in SBO one is trying to find the solution of a high fidelity problem (Eq. 1.1) through the solution of a surrogate problem (Eq. 3.1), on the assumption of *weak locality*<sup>1</sup>, this thesis proposes that an ideal measure of quality should reflect, in fact, the distance between optimal points, i.e.,  $\|x^* - x_s^*\|$ , and not between responses, i.e.,  $\|f(x) - f(x_s)\|$ . Thus, the quality of a surrogate problem in the context of single objective optimization (see Figure 3–3) can be defined as follows:

**Definition 3.1.** Let  $\mathbf{x}^*$  be the optimal solution of a single objective high-fidelity problem and  $\mathbf{x}_s^*$  the solution of the corresponding surrogate problem. The quality of the surrogate problem, for  $p \geq 1$ , is given by:

$$\|\mathbf{x}^* - \mathbf{x}_s^*\|_p \quad (3.2)$$

where,  $p \geq 1$ .

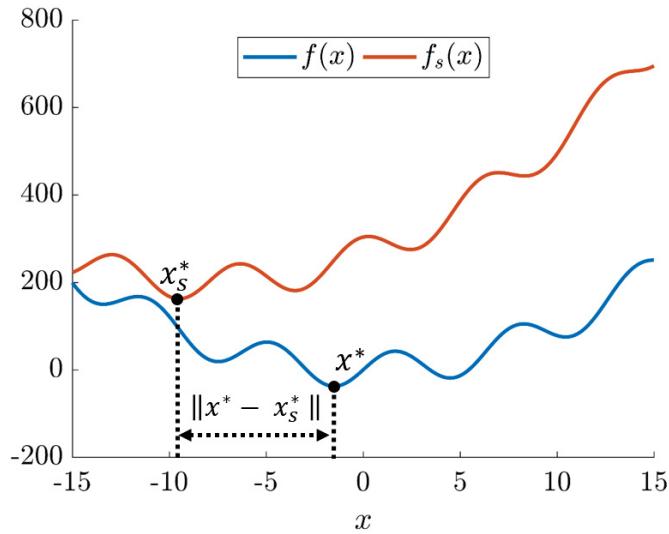


Figure 3–3: Proposed quality definition for surrogate problems

---

<sup>1</sup> “The weak locality concept means that the probability distribution of the objective function value at a given point is conditioned by the values in the neighborhood of that point” - (Guimarães, 2008). In other words, it means that candidate solutions that are close to each other in the search space are also close to each other in the objective space.

In multi-objective problems instead of a single optimal solution, a set of Pareto-optimal solutions is obtained. Thus, a quality metric for surrogate problems in this context can be any metric that is able to estimate the distance between Pareto-sets. One option is the use the following variation of the IGD metric (see Definition 2.9) which is defined in the design space rather than in objective space.

**Definition 3.2.** Let  $\mathcal{X}^*$  be a reference Pareto-optimal set of a multi-objective high-fidelity problem and  $\mathcal{X}_s^*$  the Pareto-optimal set of the corresponding surrogate problem. The quality of the surrogate problem, for  $p \geq 1$ , is given by:

$$\frac{1}{|\mathcal{X}^*|} \sum_{i=1}^{|\mathcal{X}^*|} \left( \min_{\mathbf{x}_j \in \mathcal{X}_s^*} \|\mathbf{x}_i - \mathbf{x}_j\|_p \right), \mathbf{x}_i \in \mathcal{X}^* \quad (3.3)$$

The use of the presented quality measures in practice would involve finding the high-fidelity problem solution (or Pareto-optimal solutions) which defeats the purpose of using surrogate problems. Despite their lack of practical value, these metrics are rigorous quality definitions which are in perfect agreement with the SBO main objective. Thus, due to their theoretical value, they will be used as a base of comparison for other metrics. In other words, any metric proposed for the evaluation of surrogate problems should reflect definitions 3.1 and 3.2 as well as possible.

Given the proposed quality definitions, in the following sections a comprehensive analysis of the performance metrics generally used for surrogate problems is presented. In addition to reflecting definitions 3.1 and 3.2, the following desirable properties will also be considered in the analysis:

- Robustness to the sample data: A performance metric should reflect the performance of a surrogate problem as well as possible, independent of the sample data.
- Easiness of interpretation: A performance metric should be intelligible from a human perspective since a human decision maker may be responsible for the selection a surrogate problem or setting performance thresholds for a machine selector.

## 3.2 Performance metrics for surrogate models

### 3.2.1 Error-based metrics

Traditionally, different measures of model accuracy are used for quality assessment of surrogate models (Wang and Lowther, 2006; Wang and Shan, 2007; Pilát and Neruda, 2013; Díaz-Manríquez et al., 2016; Cai et al., 2017). Let  $\mathbf{y} = [y_1, y_2, \dots, y_n]$  and  $\mathbf{y}_s = [y_{s_1}, y_{s_2}, \dots, y_{s_n}]$  be the arrays of responses computed with a high-fidelity model and its surrogate model, respectively. The most popular measures of error are defined as follows:

- The Mean Squared Error (MSE)

$$\epsilon_{MSE}(\mathbf{y}, \mathbf{y}_s) = \frac{\sum_{i=1}^n (y_i - y_{s_i})^2}{n} \quad (3.4)$$

- The Root Mean Squared Error (RMSE)

$$\epsilon_{RMSE}(\mathbf{y}, \mathbf{y}_s) = \sqrt{\epsilon_{MSE}(\mathbf{y}, \mathbf{y}_s)} = \sqrt{\frac{\sum_{i=1}^n (y_i - y_{s_i})^2}{n}} \quad (3.5)$$

- The Mean Absolute Error (MAE)

$$\epsilon_{MAE}(\mathbf{y}, \mathbf{y}_s) = \frac{\sum_{i=1}^n |y_i - y_{s_i}|}{n} \quad (3.6)$$

- The Mean Absolute Percentage Error (MAPE)

$$\epsilon_{MAPE}(\mathbf{y}, \mathbf{y}_s) = \frac{1}{n} \sum_{i=1}^n \left| \frac{y_i - y_{s_i}}{y_i} \right| \times 100 \quad (3.7)$$

- The Maximum Absolute Error (MAX)

$$\epsilon_{MAX}(\mathbf{y}, \mathbf{y}_s) = \max_{i \in 1 \dots n} |y_i - y_{s_i}| \quad (3.8)$$

These metrics have the following attractive properties (Wang and Bovik, 2009):

1. They are simple and inexpensive to compute;
2. They can be evaluated at each sample, independently;
3. They satisfy the following conditions which allow for direct interpretation of similarity;
  - non-negativity:  $\epsilon(\mathbf{y}, \mathbf{y}_s) \geq 0$ ;

- identity:  $\epsilon(\mathbf{y}, \mathbf{y}_s) = 0$  if and only if  $\mathbf{y} = \mathbf{y}_s$ ; and
  - symmetry:  $\epsilon(\mathbf{y}, \mathbf{y}_s) = \epsilon(\mathbf{y}_s, \mathbf{y})$ .
4. In the context of error minimization, the MSE possesses the very satisfying properties of convexity, symmetry, and differentiability.

When the errors are normally distributed the MSE gives the variance of the error and the RMSE the standard deviation of the error (Chai and Draxler, 2014). The RMSE is generally favoured over the MSE because its values have the units of the response which makes it easier to interpret.

The MAE also returns values in the units of the response. It differs from the RMSE on the type of distribution it tries to describe. While the RMSE is suitable for normal distributions, the MAE is suitable for uniform distributions (Chai and Draxler, 2014). Another difference between these two metrics is that while the MAE gives the same weight to all errors, the RMSE gives more weight to errors with larger absolute values than errors with smaller absolute values (Willmott and Matsuura, 2005).

The MAPE is a scale independent measure that is useful to compare models across different data-sets. When a given surrogate model is used to approximate different physical quantities with large variations in range, the MAPE offers a more intuitive interpretation of accuracy than the aforementioned metrics.

The lower the values obtained by each of these metrics, the more accurate the surrogate model is. While MSE, RMSE, MAE and MAPE are used to gauge the overall accuracy of the model, MAX estimates upper bounds for the prediction errors. It is important to highlight that the MAX is the least robust statistic, i.e., it is maximally sensitive to outliers. Thus, a surrogate model that produces very accurate predictions for the whole data-set except for one sample would still receive a low score.

### 3.2.1.1 Error-based performance metrics in SBO

As could be seen in the previous section, a lot of effort has been previously put into investigating the properties of these metrics. However, they still lack a formal analysis in the context of SBO, and this is what this section intends to provide.

In order to know how good these error-based metrics are in finding high quality surrogate models, it is important to get acquainted with the underlying assumptions that they imply.

Consider that the surrogate model response,  $R_s(\cdot)$ , consists of the fine model response,  $R_f(\cdot)$ , plus an error function,  $\epsilon(\cdot)$ , that is:

$$R_s(\mathbf{x}) = R_f(\mathbf{x}) + \epsilon(\mathbf{x}) \quad (3.9)$$

When the MSE, RMSE or the MAE are used the following assumptions are made:

1. The quality of  $R_s$  is independent of any relationship between  $R_f$  and the error function  $\epsilon$ . In other words, for a given  $\epsilon$ , the quality of  $R_s$  remains unchanged, regardless of which  $R_f$ ,  $\epsilon$  is added to, and;
2. All the samples are equally important in the estimation of the  $R_s$  quality.

Both the above implicit assumptions are very strong, since they impose significant limitations on how the surrogate models are perceived by the model selector, human or machine. The question is: are they useful or damaging in the context of measuring suitability for surrogate-based optimization?

Figure 3–4 shows how these assumptions can distort the perception about the surrogate model quality. In both examples, the error function,  $\epsilon(x)$ , is the same. As expected from Assumption 1, despite the fact that in Figure 3–4a the surrogate model has maximum quality (e.g.  $\|x_f^* - x_s^*\| = 0$ ) and in Figure 3–4b it does not, both receive the same scores according to MSE, RMSE and MAE. Besides, it can be seen that the error observed near  $R_f^*$  in Figure 3–4b is greater than the error observed near  $R_f^*$  in Figure 3–4a. Ideally, for optimization purposes, a  $R_s$  that is more accurate near the optimum region should receive better scores.

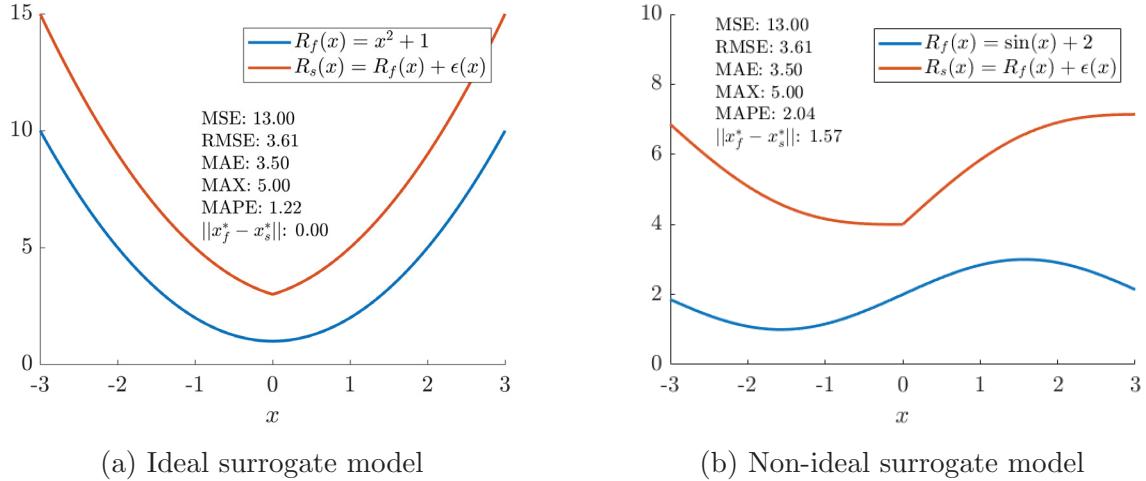


Figure 3–4: Perceived surrogate model quality by different metrics. The error function is defined as  $\epsilon(x) = |x| + 2$

However, as expected from Assumption 2, again, both  $R_s$ 's receive the same score when they should not.

Figure 3–4 also demonstrates an interesting property of MAPE that may be useful in the context of SBO. Differently from the other metrics, **MAPE gives more weight to errors when the values of  $R_f(x)$  are small.** Since  $x_f^*$  happens at the minimum value of  $R_f$ , penalizing more severely the error in regions of small  $R_f$  may be beneficial.

One issue shared by all the error-based metrics is the fact that **they do not provide any information about how well  $R_s$  approximates the  $R_f$  landscape which in turn defines the positions of the optimal solutions.** When using an error-based performance, a surrogate model defined as  $R_s(\mathbf{x}) = R_f(\mathbf{x}) + c$  where  $c$  is a constant, would have its quality exclusively determined by  $c$ . Meanwhile, from the optimization point of view, they are the exact same problem. The optimal points are the same, the gradients (if they exist) are the same and even the hessian matrices (if they exist) are the same.

### 3.2.1.2 The Coefficient of Determination ( $r^2$ )

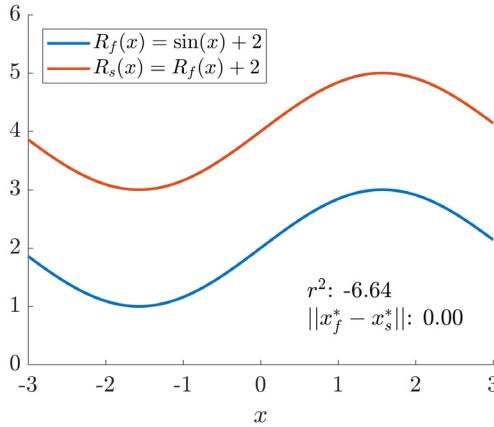
The coefficient of determination,  $r^2$ , is another measure of model accuracy and can be defined as:

$$r^2 = 1 - \frac{\sum_{i=1}^m (y_i - y_{s_i})^2}{\sum_{i=1}^m (y_i - \bar{y})^2} \quad (3.10)$$

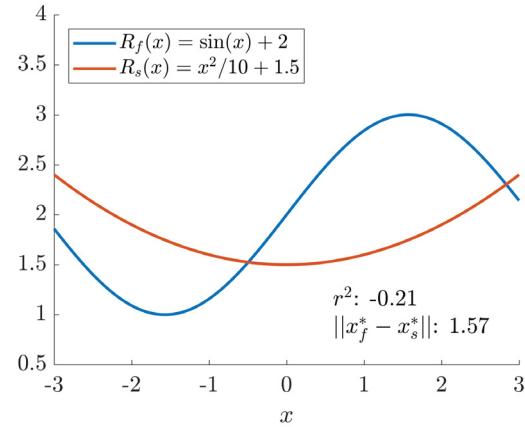
where,  $\bar{y}$  is the mean of the set  $\mathbf{y}$ .

The  $r^2$  is interpreted as “the proportion of variance ‘explained’ by the regression model” (Nagelkerke, 1991). The term  $\sum_{i=1}^m (y_i - y_{s_i})^2$ , as in the MSE, is the error variance. The term  $\sum_{i=1}^m (y_i - \bar{y})^2$  is the sample variance. Thus, a  $r^2$  of 1 means that the sum of the errors is 0 which, in turn, means that the model explains 100% of the variance contained in the data. Different from the other metrics, the higher the values of  $r^2$  the more accurate the surrogate model is.  $r^2$  ranges from  $(-\infty, 1]$  and it is maximum only when the error,  $y_i - y_{s_i}$ , is equal to 0.

The  $r^2$  is insensitive to scaling, i.e., if  $\mathbf{y}$  and  $\mathbf{y}_s$  are scaled by the same factor,  $k \in \mathbb{R}$ , the  $r^2$  value does not change. This property is aligned with definitions 3.1 and 3.2 since scaling does not affect the position of the optima. Other than that, the  $r^2$  seems to have a weak relationship with the presented quality definitions. In fact, is not hard to build an example for which  $r^2$  can be extremely misleading. For instance, in Figure 3–5a,  $R_s$  presents maximum quality nevertheless it is evaluated as worse than the  $R_s$  depicted in Figure 3–5b when it should not be.



(a) Ideal surrogate model



(b) Non-ideal surrogate model

Figure 3–5: Perceived surrogate model performance by  $r^2$

### 3.2.2 Rank-based metrics

Although the idea that order maintenance is an important property is not very new (Runarsson, 2004), only recently have rank-based performance metrics been seriously used for the evaluation of surrogate models (Silva et al., 2016a; Díaz-Manríquez et al., 2016; Audet et al., 2016). These metrics are used to measure the similarity of the orderings of the sample data when ranked by different objective functions. They are based on the hypothesis that by maintaining the order imposed by the objective function defined over the high-fidelity model, a surrogate problem has a better chance of leading to high quality solutions. Though successful in some applications and empirical experiments, their superiority over error-based metrics is unclear. Besides, their use in SBO lacks a formal theoretical justification, in particular, for multi-objective problems. In order to fulfill this gap, Section 3.2.2.1 provides such a theoretical justification by proving that ordering preservation guarantees matching optima in both single and multi-objective surrogate model based optimization. Then, in Section 3.2.2.2, metrics used to measure rank preservation are defined and analyzed.

#### 3.2.2.1 Theoretical analysis

Before deriving the theorems that will be used to justify rank-based metrics, consider the formal definition of ordering preservation given below:

**Definition 3.3 (Ordering preservation).** Let  $f_a : \Omega \rightarrow \mathbb{R}$  and  $f_b : \Omega \rightarrow \mathbb{R}$  be the objective functions defined over models  $a$  and  $b$ , respectively.  $f_b(\cdot)$  is said to preserve the ordering imposed by  $f_a(\cdot)$  if and only if  $\forall \mathbf{u}, \mathbf{v} \in \Omega$  at least one of the following conditions is satisfied:

1. If  $f_a(\mathbf{u}) < f_a(\mathbf{v})$  then  $f_b(\mathbf{u}) < f_b(\mathbf{v})$ ; or
2. If  $f_a(\mathbf{u}) > f_a(\mathbf{v})$  then  $f_b(\mathbf{u}) > f_b(\mathbf{v})$

It can be seen that ordering preservation is insensitive to the difference,  $\|f(\cdot) - f_s(\cdot)\|$ . Moreover, it is possible to prove that if the  $f_s(\cdot)$  preserves the ordering imposed by  $f(\cdot)$ , then  $\mathbf{x}^* = \mathbf{x}_s^*$ . This property is stated in Theorem 3.1.

**Theorem 3.1.** Given an objective function,  $f : \Omega \rightarrow \mathbb{R}$ , and its corresponding surrogate,  $f_s : \Omega \rightarrow \mathbb{R}$ ,  $\mathbf{x}^* = \arg \min_{\mathbf{x} \in \Omega} f(\mathbf{x})$  and  $\mathbf{x}_s^* = \arg \min_{\mathbf{x} \in \Omega} f_s(\mathbf{x})$ , then  $\mathbf{x}^* = \mathbf{x}_s^*$  if  $\forall \mathbf{u}, \mathbf{v} \in \Omega$ ,  $f_s(\cdot)$  preserves the ordering imposed by  $f(\cdot)$ .

*Proof.* Assume that  $\forall \mathbf{u}, \mathbf{v} \in \Omega$ ,  $f_s(\cdot)$  preserves the ordering imposed by  $f(\cdot)$ . Now, suppose that  $\mathbf{x}_f^* \neq \mathbf{x}_s^*$ . That implies  $f(\mathbf{x}_f^*) < f(\mathbf{x}_s^*)$  and  $f_s(\mathbf{x}_f^*) > f_s(\mathbf{x}_s^*)$ . Since, as stated,  $f_s(\cdot)$  preserves the ordering imposed by  $f(\cdot)$ ,  $f(\mathbf{x}_f^*) < f(\mathbf{x}_s^*)$  and  $f_s(\mathbf{x}_f^*) > f_s(\mathbf{x}_s^*)$  is a contradiction, therefore,  $\mathbf{x}_f^* \neq \mathbf{x}_s^*$  must be FALSE.  $\square$

As mentioned before, rank-based performance metrics measure the similarity between the orderings imposed by  $f(\cdot)$  and  $f_s(\cdot)$ . When the performance measured by these metrics is maximized, both original and surrogate problems present matching optima as a consequence of Theorem 3.1. This is in alignment with Definition 3.1 since every time that a rank-based performance metric is maximized,  $\|x^* - x_s^*\| = 0$ . Although this is also true for the error-based metrics (i.e., when the error is 0,  $\|x^* - x_s^*\| = 0$ ), rank-based metrics are much less restrictive once they do not require  $f_s(\cdot)$  to match  $f(\cdot)$ . Besides, as illustrated in Figure 3–6, there is a relationship between the ordering similarity and how the surrogate problem approximates the original problem “landscape”.

The literature has a few studies which empirically compare rank and error-based performance metrics for single-objective SBO (Bischl et al., 2012; Silva et al., 2016a; Audet et al., 2016). The same is not true for multi-objective problems.

From an error point of view, it is obvious that if the surrogate objective functions  $\mathbf{f}_s(\cdot) = [f_{s_1}(\cdot), f_{s_2}(\cdot), \dots, f_{s_m}(\cdot)]$  and high-fidelity objective functions  $\mathbf{f}(\cdot) = [f_1(\cdot), f_2(\cdot), \dots, f_m(\cdot)]$  have matching responses, they will also have matching Pareto-sets. Again, requiring matching responses seems to be too restrictive. Besides, error-based performance metrics do not embody any information about how well the original problem “landscape” is approximated by the surrogate objective functions. In this context, we have to ask if measures of ordering preservation are also meaningful for multi-objective problems. In order to demonstrate this “meaningfulness”, consider the following theorems.

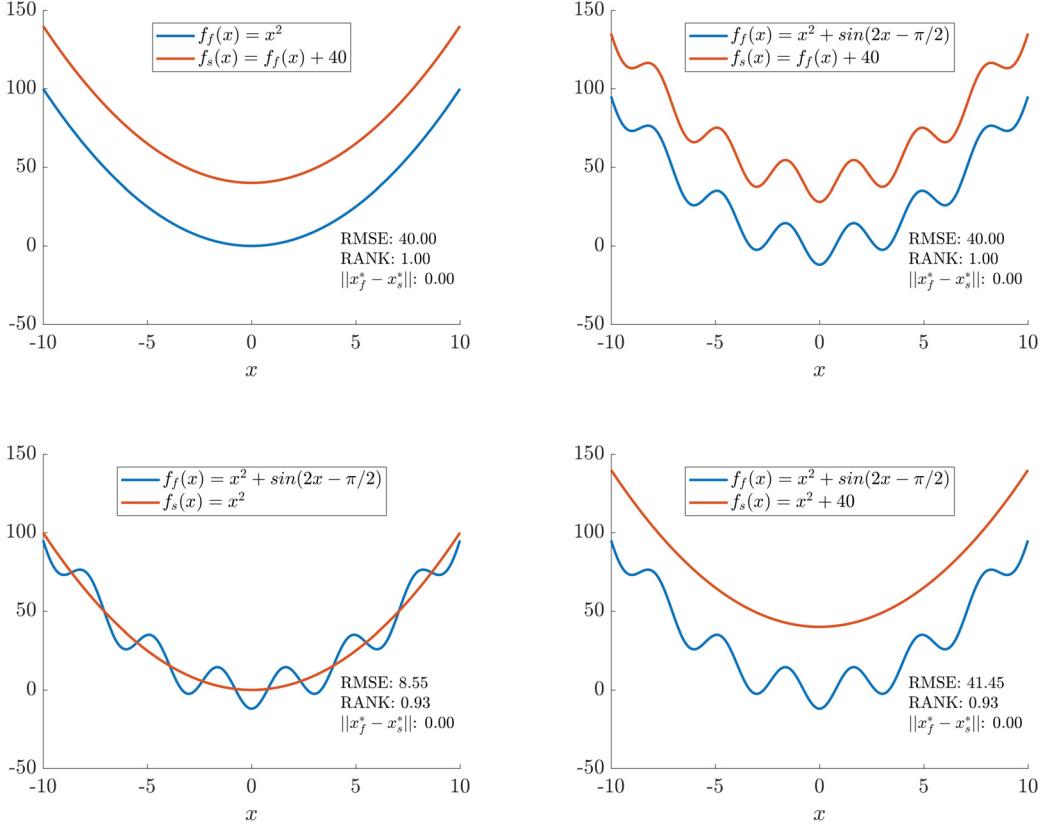


Figure 3-6: Perceived surrogate problem ( $f_s(\cdot)$ ) quality by a rank performance metric (RANK) and the RMSE. The higher the value of RANK the higher the similarity among ranks. The maximum values of RANK is 1 and it happens when  $r_s(x) = r_f(x) \forall x$

**Theorem 3.2.** Given a vector objective function,  $\mathbf{f} : \Omega \rightarrow \mathbb{R}^m$  and its corresponding surrogate  $\mathbf{f}_s : \Omega \rightarrow \mathbb{R}^m$ . If  $\forall \mathbf{u}, \mathbf{v} \in \Omega$   $f_{s_k}(\cdot)$  preserves the ordering imposed by  $f_k(\cdot)$ ,  $k \in \{1, \dots, m\}$  then  $\mathbf{f}_s(\mathbf{x}_i) \prec \mathbf{f}_s(\mathbf{x}_j) \Rightarrow \mathbf{f}(\mathbf{x}_i) \prec \mathbf{f}(\mathbf{x}_j)$ .

*Proof.* Assume that  $\forall \mathbf{x}_a, \mathbf{x}_b \in \Omega$ ,  $f_{s_k}(\cdot)$  preserves the ordering imposed by  $f_k(\cdot)$ ,  $k \in \{1, \dots, m\}$  and  $\mathbf{f}_s(\mathbf{x}_a) \prec \mathbf{f}_s(\mathbf{x}_b)$ . By the Pareto dominance definition (Definition 2.3):

- $\forall k \in \{1, \dots, m\}$ ,  $f_{s_k}(\mathbf{x}_a) \leq f_{s_k}(\mathbf{x}_b)$  and  $\exists k : f_{s_k}(\mathbf{x}_a) < f_{s_k}(\mathbf{x}_b)$

Thus, by the Ordering preservation definition: (Definition 3.4):

- $\forall k \in \{1, \dots, m\}$ ,  $f_k(\mathbf{x}_a) \leq f_k(\mathbf{x}_b)$  and  $\exists k : f_k(\mathbf{x}_a) < f_k(\mathbf{x}_b)$

Therefore,  $\mathbf{f}_s(\mathbf{x}_a) \prec \mathbf{f}_s(\mathbf{x}_b) \Rightarrow \mathbf{f}(\mathbf{x}_a) \prec \mathbf{f}(\mathbf{x}_b)$  if  $\forall \mathbf{u}, \mathbf{v} \in \Omega$   $f_{s_k}(\cdot)$  preserves the ordering imposed by  $f_k(\cdot)$ ,  $k \in \{1, \dots, m\}$ .

□

**Theorem 3.3.** *Given a vector objective function,  $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ , its corresponding surrogate  $\mathbf{f}_s : \mathbb{R}^n \rightarrow \mathbb{R}^m$  and the respective Pareto optimal sets,  $\mathcal{X}^*$  and  $\mathcal{X}_s^*$ . If  $\forall \mathbf{x}_a, \mathbf{x}_b \in \Omega$ ,  $\mathbf{f}_s(\mathbf{x}_a) \prec \mathbf{f}_s(\mathbf{x}_b) \Leftrightarrow \mathbf{f}(\mathbf{x}_a) \prec \mathbf{f}(\mathbf{x}_b)$  then  $\mathcal{X}^* = \mathcal{X}_s^*$ .*

*Proof.* Assume that  $\forall \mathbf{x}_a, \mathbf{x}_b \in \Omega$ ,  $\mathbf{f}(\mathbf{x}_a) \prec \mathbf{f}(\mathbf{x}_b) \Rightarrow \mathbf{f}_s(\mathbf{x}_a) \prec \mathbf{f}_s(\mathbf{x}_b)$ . Now, suppose that  $\mathcal{X}^* \neq \mathcal{X}_s^*$ . That implies that there exists an  $\mathbf{x} \in \mathcal{X}$  such that:

$$\nexists \mathbf{z} \in \Omega \text{ such that } \mathbf{f}(\mathbf{z}) \prec \mathbf{f}(\mathbf{x}), \mathbf{x} \in \mathcal{X}^*. \quad (3.11)$$

$$\exists \mathbf{z} \in \Omega \text{ such that } \mathbf{f}_s(\mathbf{z}) \prec \mathbf{f}_s(\mathbf{x}), \mathbf{x} \in \mathcal{X}^*. \quad (3.12)$$

or there exists an  $\mathbf{x} \in \mathcal{X}_s$  such that

$$\nexists \mathbf{z} \in \Omega \text{ such that } \mathbf{f}_s(\mathbf{z}) \prec \mathbf{f}_s(\mathbf{x}), \mathbf{x} \in \mathcal{X}_s^*. \quad (3.13)$$

$$\exists \mathbf{z} \in \Omega \text{ such that } \mathbf{f}(\mathbf{z}) \prec \mathbf{f}(\mathbf{x}), \mathbf{x} \in \mathcal{X}_s^*. \quad (3.14)$$

Since, as stated,  $\forall \mathbf{x}_a, \mathbf{x}_b \in \Omega$ ,  $\mathbf{f}(\mathbf{x}_a) \prec \mathbf{f}(\mathbf{x}_b) \Rightarrow \mathbf{f}_s(\mathbf{x}_a) \prec \mathbf{f}_s(\mathbf{x}_b)$  either 3.11 and 3.12 are contradictory or 3.13 and 3.14 are contradictory, therefore,  $\mathcal{X}^* \neq \mathcal{X}_s^*$  must be FALSE.

□

**Theorem 3.2 shows** that if the objective-wise ordering imposed by the original objective functions and their corresponding surrogates are the same, the Pareto-dominance relationships observed in the original problem are preserved in the surrogate problem. **Theorem 3.3** implies that if the Pareto-dominance relationships imposed by two problems are the same then the respective Pareto-optimal sets are also the same. This analysis provides theoretical justification for the use of objective-wise rank-based performance metrics in the context of multi-objective problems.

### 3.2.2.2 Measures of rank agreement

In the previous section, the theoretical justification for the use of rank-based performance metrics for surrogate models in single and multi-objective optimization was presented.

The problem now is to measure the similarity of the ordering imposed by original and surrogate objective functions. Fortunately, the field of statistics provides a few candidates, known as a rank correlation statistics, to measure the relationship between different rankings of the same variable. Following previous work (Bischl et al., 2012; Díaz-Manríquez et al., 2011; Díaz-Manríquez et al., 2016; Audet et al., 2016), here *Kendall's t* (Lipps, 1905; Kendall, 1938) and *Spearman's r* (Spearman, 1904) are considered.

The Kendall's  $t$  between two sets defined over a sample of size  $n$  can be computed by:

$$t = P_c - P_d \quad (3.15)$$

where,  $P_c$  is the probability of concordant pairs and  $P_d$  the probability of discordant pairs.

A pair of observations  $(f_a(\mathbf{x}_i), f_b(\mathbf{x}_i))$  and  $(f_a(\mathbf{x}_j), f_b(\mathbf{x}_j))$ ,  $i \neq j$ , is said to be concordant if both,  $f_a(\mathbf{x}_i) < f_a(\mathbf{x}_j)$  and  $f_b(\mathbf{x}_i) < f_b(\mathbf{x}_j)$ ; or if both  $f_a(\mathbf{x}_i) > f_a(\mathbf{x}_j)$  and  $f_b(\mathbf{x}_i) > f_b(\mathbf{x}_j)$ . They are considered to be discordant if  $f_a(\mathbf{x}_i) > f_a(\mathbf{x}_j)$  and  $f_b(\mathbf{x}_i) < f_b(\mathbf{x}_j)$ ; or if  $f_a(\mathbf{x}_i) < f_a(\mathbf{x}_j)$  and  $f_b(\mathbf{x}_i) > f_b(\mathbf{x}_j)$ . Thus, the Kendall's  $t$  rank correlation can be alternatively defined as:

$$t = \frac{n_c - n_d}{\frac{1}{2}n(n-1)} \quad (3.16)$$

where,  $n_c$  is number of concordant pairs,  $n_d$  is number of discordant pairs and  $n$  the number of samples.

One important characteristic of any quality metric is its ease of interpretation. Fortunately,  $t$  has a straightforward interpretation in the context of surrogate-based optimization. It is possible to say that  $t$  is the probability that an improvement in the surrogate problem will translate into an improvement in the original problem ( $P_c$ ) minus the probability that an improvement in the surrogate problem will translate into a deterioration in the original problem ( $P_d$ ). To simplify the interpretation even further, some authors such as Díaz-Manríquez et al. (2016) and Audet et al. (2016) report only  $P_d$ .

The Spearman's  $r$  rank correlation requires the explicit computation of the ranks (ordering) imposed by  $f(\cdot)$  and  $f_s(\cdot)$ . Ranks can be computed through the use of the following definition:

**Definition 3.4 (Rank).** Let  $f_{m,i}(\cdot)$  be the objective function value defined over the model  $m$ , such that:

$$f_{m,1} < f_{m,2} < f_{m,3} < \dots < f_{m,n} \quad (3.17)$$

The rank,  $r_m$ , of a candidate solution  $\mathbf{x}_i$  according to the model  $m$  is given by:

$$r_m(\mathbf{x}_i) = j \text{ such that } f_{m,j} = f_m(\mathbf{x}_i) \quad (3.18)$$

After the ranks are computed, the Spearman's  $r$  can be defined as:

$$r_{r_s, r_f} = \frac{\text{cov}(r_s, r_f)}{\sigma_{r_s} \sigma_{r_f}} = \frac{\sum_{i=1}^n (r_{s,i} - \bar{r}_s)(r_{f,i} - \bar{r}_f)}{\sqrt{\sum_{i=1}^n (r_{s,i} - \bar{r}_s)^2} \sqrt{\sum_{i=1}^n (r_{f,i} - \bar{r}_f)^2}} \quad (3.19)$$

where,  $\text{cov}(r_s, r_f)$  is the covariance of the rank variables.  $\sigma_{r_s}$  and  $\sigma_{r_f}$  are the standard deviations of the rank variables.

While all the presented error-based metrics can be computed in  $O(n)$ , the computation of  $t$  using Knight's algorithm (Knight, 1966) is  $O(n \log n)$  and the computation of  $r$  is  $O(n \log n + n)$ .

As with  $t$ ,  $r$  is defined in the range  $[-1, 1]$  where,  $-1$  means 100% negative association, or perfect inversion and  $+1$  means 100% positive association, or perfect agreement. Unfortunately, the interpretation of  $r$  is not as straightforward as the interpretation of  $t$  and it can only be said that it measures the strength of the monotone association between two variables (Pirie, 2004).

By using rank-correlation measures, we are looking for a  $R_s$  for which, given any two candidate solutions  $\mathbf{x}_i$  and  $\mathbf{x}_j$ , if  $f(\mathbf{x}_i) < f(\mathbf{x}_j) \forall i, j$  then  $f_s(\mathbf{x}_i) < f_s(\mathbf{x}_j) \forall i, j$  and vice-versa.

An insightful way to visualize ordering preservation is to use the so-called parallel coordinates plots as shown in Figure 3–7.

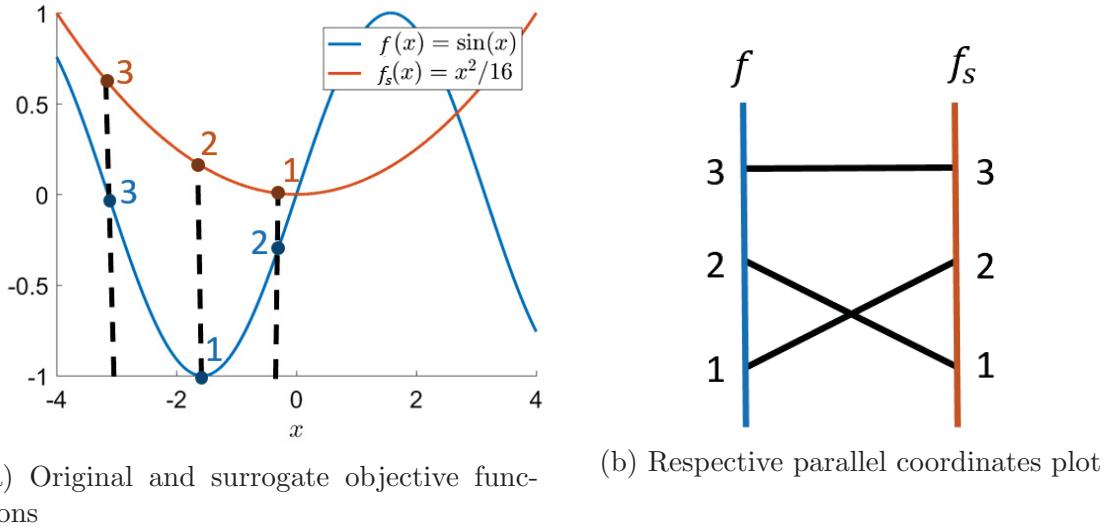


Figure 3–7: Parallel coordinates plot for the visualization of ordering preservation

Figure 3–7a depicts the ranks imposed by original and surrogate problems over three sample points. Figure 3–7b presents each of those samples as a separate line and the ranks imposed by each objective function in a different coordinate. When the ordering is preserved, all the lines are in parallel. When there are dissimilarities, crossing lines are observed.

Through the use of parallel coordinates plots, Figure 3–8 presents a comparison between  $r$  and  $t$  under different scenarios and provides the [following insights on the metrics behavior](#):

1.  $r$  and  $t$  are not equivalent;
2. Both metrics are insensitive to the location of the rank dissimilarities;
3.  $r$  seems to be more sensitive to the “amount” of dissimilarity than  $t$ . This property can be observed in Figure 3–8g where the fact that top solutions of a model became the bottom solutions of the other affected  $r$  more than  $t$ .

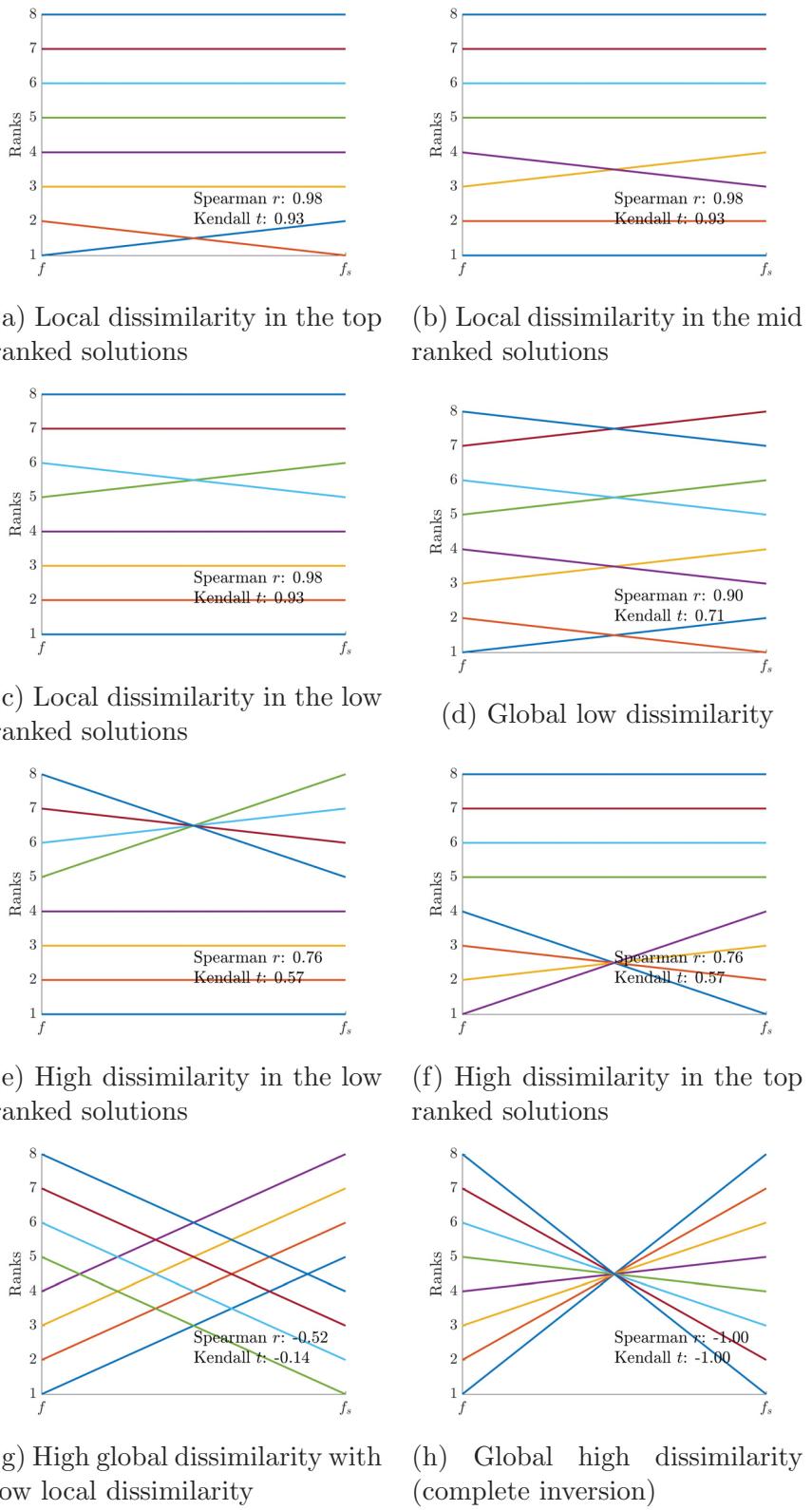


Figure 3-8: Kendall's  $t$  versus Spearman's  $r$  for different distributions of rank dissimilarities.

To conclude this section, it is important to highlight that using rank correlation metrics to evaluate surrogate models does not necessarily translate into improved SBO performance. For instance, in Figure 3–9, any rank correlation between  $f_{s_1}(\cdot)$  and  $f(\cdot)$  would result in

1. Meanwhile, the correlation between  $f_{s_2}(\cdot)$  and  $f(\cdot)$  is  $< 1$ . In this scenario however selecting  $f_{s_2}(\cdot)$  would increase the optimization performance since it is uni-modal and easier to optimize. Nevertheless, since the optimal solution position cannot be predicted without solving both optimization problems, original and surrogate, it is expected that, on average, surrogate models that maintain the candidate solutions ordering will lead to better results.

When MOOPs are considered, disrupting the original ordering (or ranking) may be even more harmful since the Pareto-optimal solutions can be spread across the design space.

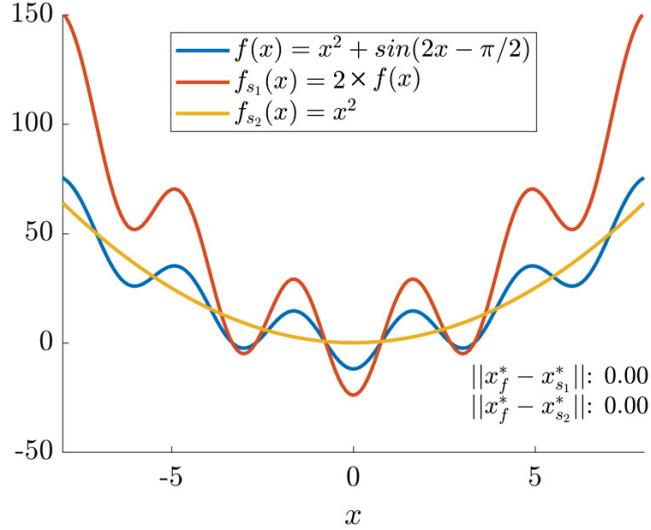


Figure 3–9: Surrogate models with coinciding global optima.

### 3.3 Performance metrics for reduced problems

As discussed in Section 2.4.2.1, performance metrics for reduced problems should be able to estimate the distortion caused by the objective removal on the original Pareto set  $\mathcal{Y}^*$ . These metrics can be divided into two groups: (i) Pareto front preserving metrics and (ii) correlation-based metrics.

### 3.3.1 Pareto front preserving metrics

The Pareto front preserving metrics are based in the following definition (Yuan et al., 2017):

**Definition 3.5.** An objective subset  $\mathcal{F}_r \subset \mathcal{F}$  is redundant if and only if  $\forall \mathbf{u} \in \mathcal{Y}^*$ ,  $\#\mathbf{v} \in \mathcal{Y}$  such that  $\mathbf{v}^{(\mathcal{F}_s)} \prec \mathbf{u}^{(\mathcal{F}_s)}$ , where  $\mathcal{F}_s = \mathcal{F} \setminus \mathcal{F}_r$ .

Definition 3.5 means that if a redundant set of objectives is removed from the MOOP formulation, any Pareto optimal solution in the original problem,  $\mathcal{Y}^*$ , remains Pareto optimal in the reduced problem defined over  $\mathcal{F}_s$ . For this to happen,  $\mathcal{X}_s^*$  has to be equal to  $\mathcal{X}^*$  which aligns perfectly with the proposed definitions of quality given in Section 3.1. Pareto front preserving methods, therefore, try to find a subset of objectives,  $\mathcal{F}_s \subset \mathcal{F}$ , which minimizes the discrepancy between  $\mathcal{Y}^*$  and  $\mathcal{Y}_s^*$ . Brockhoff and Zitzler (2009b) define the discrepancy between original and reduced problems as:

**Definition 3.6 ( $\epsilon$ -discrepancy).** Let  $\mathcal{N}^{(\mathcal{F})} \subset \mathbb{R}^m$  be a non-dominated solution set in the objective space defined by the set of objectives  $\mathcal{F}$  and  $\mathcal{N}^{(\mathcal{F}_s)}$  its projection in the objective space defined by  $\mathcal{F}_s$ . The  $\epsilon$ -discrepancy can be defined as:

$$\begin{aligned} d_\epsilon(\mathcal{N}^{(\mathcal{F})}, \mathcal{N}^{(\mathcal{F}_s)}) &= \max(\epsilon) \\ \text{subject to } \mathbf{v}^{(\mathcal{F}_s)} &\preceq \mathbf{u}^{(\mathcal{F}_s)} \\ \mathbf{v}^{(\mathcal{F}_s)} &= \arg \min_{\mathbf{v}^{(\mathcal{F}_s)}} \|\mathbf{v}^{(\mathcal{F}_s)} - \mathbf{u}^{(\mathcal{F}_s)}\| \\ (\mathbf{u}^{(\mathcal{F}_s)} - \epsilon) &\preceq \mathbf{v}^{(\mathcal{F}_s)} \\ \mathbf{u}, \mathbf{v} &\in \mathcal{N}^{(\mathcal{F})} \end{aligned} \quad (3.20)$$

As illustrated in Figure 3–10, Eq. 3.20 defines discrepancy as the maximum  $\epsilon$  such that all Pareto dominated solutions in  $\mathcal{N}^{(\mathcal{F}_s)}$  become non-dominated.

$d_\epsilon$  is difficult to interpret when the objectives are defined with different units. In order to avoid this issue Yuan et al. (2017) proposed an alternative measure of discrepancy based on Definition 3.5 which is defined as follows:

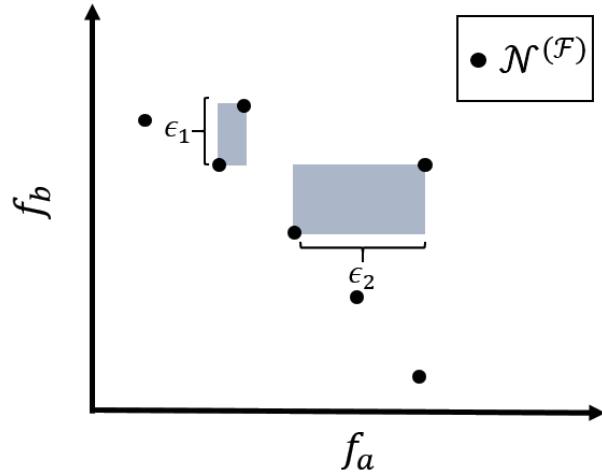


Figure 3–10: Projection of  $\mathcal{N}^{(\mathcal{F})}$  in a two-objective space with the  $\epsilon$ 's indicated for two dominated solutions.

**Definition 3.7 ( $\eta$ -discrepancy).** Let  $\mathcal{N}^{(\mathcal{F})} \subset \mathbb{R}^m$  be a non-dominated solution set in the objective space defined by the set of objectives  $\mathcal{F}$  and  $\mathcal{D}^{(\mathcal{F}_s)}$  the set of solutions that become dominated when  $\mathcal{N}^{(\mathcal{F})}$  is projected onto  $\mathcal{F}_s$ . Thus, the  $\eta$ -discrepancy can be defined as:

$$d_\eta(\mathcal{N}^{(\mathcal{F})}, \mathcal{D}^{(\mathcal{F}_s)}) = \frac{|\mathcal{D}^{(\mathcal{F}_s)}|}{|\mathcal{N}^{(\mathcal{F})}|} \quad (3.21)$$

Broadly speaking, Eq. 3.21 defines discrepancy as the proportion of solution in  $\mathcal{N}^{(\mathcal{F})}$  that become Pareto dominated when the problem is reduced. Figure 3–11 illustrates the formulation.

Both definitions of discrepancy aim at finding a reduced objective set such that  $\mathcal{X}^* = \mathcal{X}_s^*$ . As shown in (Yuan et al., 2017), in order for these metrics of discrepancy to work properly,  $\mathcal{N}^{(\mathcal{F})}$  must be a good representation of  $\mathcal{Y}^*$  which, in turn, can be hard to guarantee.

### 3.3.2 Correlation-based metrics

In the literature, the use of correlation-based metrics for objective reduction is informally justified by the idea that if two objectives are highly correlated they are non-conflicting and therefore one of them can be considered redundant. In order to formalize this idea and to prove its ability to identify redundant objectives, non-conflicting objectives can be defined as follows:

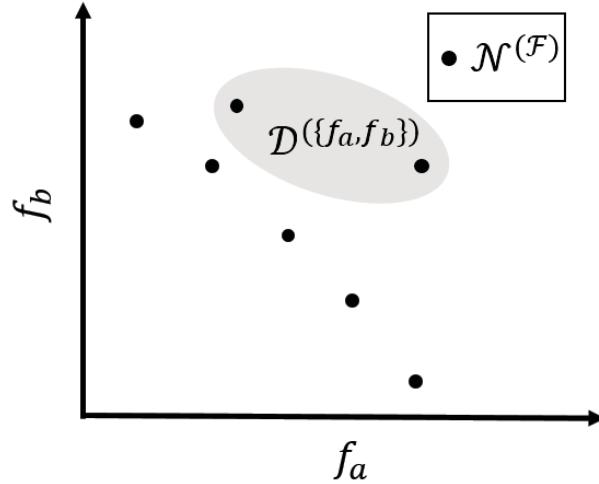


Figure 3–11: Projection of  $\mathcal{N}^{(\mathcal{F})}$  in a two-objective space. The set of solutions that becomes dominated  $\mathcal{D}^{\{f_a, f_b\}}$  is highlighted

**Definition 3.8 (Non-conflicting objectives).** Let  $f_a(\cdot)$  and  $f_b(\cdot)$  be two objective functions in a MOOP.  $f_a$  and  $f_b$  are non-conflicting if and only if  $\forall \mathbf{x}, \mathbf{y} \in \Omega, f_a(\mathbf{x}) > f_a(\mathbf{y}) \Leftrightarrow f_b(\mathbf{x}) > f_b(\mathbf{y}) \vee f_a(\mathbf{x}) < f_a(\mathbf{y}) \Leftrightarrow f_b(\mathbf{x}) < f_b(\mathbf{y})$ .

Definition 3.8 means that a pair of objectives  $f_a$  and  $f_b$  is considered to be non-conflicting if an improvement in  $f_a$  translates into an improvement in  $f_b$  and vice versa, or if a deterioration  $f_a$  translates into a deterioration of  $f_b$  and vice versa. With this definition, we can prove that one of the non-conflicting objectives is redundant as follows:

**Theorem 3.4.** If  $f_a$  and  $f_b$  are non-conflicting objectives, either  $f_a$  or  $f_b$  is redundant.

*Proof.* Given two non-conflicting objectives  $f_a$  and  $f_b$ , consider the reduced objective set  $\mathcal{F}_s = \mathcal{F} \setminus f_b$ . Assume that  $f_b$  is not redundant according to Definition 2.11. This implies that  $\exists \mathbf{u} \in \mathcal{Y}^*$  and  $\exists \mathbf{v} \in \mathcal{Y}$  such that  $\mathbf{v}^{(\mathcal{F}_s)} \prec \mathbf{u}^{(\mathcal{F}_s)}$  meaning that  $\forall i \in \{1, \dots, |\mathcal{F}_s|\}, v_i^{(\mathcal{F}_s)} \leq u_i^{(\mathcal{F}_s)}$  which, in turn, implies that:

1.  $v_a \leq u_a$  since  $f_a \in \mathcal{F}_s$ ; and
2.  $v_b > u_b$  because, in the original formulation,  $\mathbf{u}$  was not dominated by  $\mathbf{v}$ .

By the definition of non-conflicting objectives (Definition 3.8),  $u_a \leq v_a$  and  $u_b > v_a$  cannot be simultaneously true. Therefore the assumption that  $f_b$  is not redundant must be FALSE.  $\square$

In the methods presented in (López Jaimes et al., 2008; López Jaimes et al., 2009; Sinha et al., 2013; Bandyopadhyay and Mukherjee, 2015), the conflict is estimated by some measure of correlation, in general, the Pearson's correlation coefficient (Pearson, 1895). It is important to highlight that the Pearson's correlation coefficient is a measure of linear correlation, therefore, more restrictive than Definition 3.8. Because of that, a pair of objectives that would have been considered non-conflicting by Definition 3.8 will present some level of conflict according to this correlation measure. In this context, rank correlation measures such as the ones presented in Section 3.2.2 are considered to be more adequate measures of conflict (Freitas et al., 2014; Silva et al., 2016b; Kou et al., 2017; Yuan et al., 2017).

A correlation measure,  $r(\mathbf{y}_a, \mathbf{y}_b)$ , can only estimate the conflict between two objectives. In order to use it as a measure of discrepancy between objective function sets, in general, a clustering algorithm based on the level of conflict is applied (Yuan et al., 2017). A generic clustering procedure based on conflict is presented in Algorithm 3.1.

---

**Algorithm 3.1:** Clustering algorithm for conflict-based discrepancy measures

---

```

Data:  $\mathbf{Y}, \mathcal{F}, \mathcal{F}_s$ 
Result:  $\mathcal{K} = \{\mathcal{K}_1, \dots, \mathcal{K}_{|\mathcal{F}_s|}\}$ 
// Initialize clusters
1  $i \leftarrow 1;$ 
2 foreach  $f_s \in \mathcal{F}_s$  do
3    $\mathcal{K}_i \leftarrow f_s;$                                 // clusters of objectives
4    $\mathcal{F} \leftarrow \mathcal{F} \setminus f_s;$ 
5    $i \leftarrow i + 1;$ 
6 end
7 while  $\mathcal{F} \neq \emptyset$  do
8   foreach  $f \in \mathcal{F}$  do
9      $i = \arg \min_i (\text{conflict}(f, \mathcal{K}_i, \mathbf{Y})) ;$ 
10     $\mathcal{K}_i \leftarrow \mathcal{K}_i \cup f;$ 
11     $\mathcal{F} \leftarrow \mathcal{F} \setminus f;$ 
12  end
13 end

```

---

In Algorithm 3.1, each cluster,  $\mathcal{K}_i$ , is initialized with one of the objectives in the reduced set,  $\mathcal{F}_s$ . Then, one by one, each objective in the original objective set,  $\mathcal{F}$ , is added to the cluster with which it has the least amount of conflict.

The conflict between an objective,  $f$ , and a cluster,  $\mathcal{K}_i$ , is defined as the maximum conflict between  $f$  and each of the other objectives in  $\mathcal{K}_i$ . Algorithm 3.2 presents a procedure to compute the conflict among objectives and clusters.

---

**Algorithm 3.2:**  $\text{conflict}(\mathbf{Y}, \mathcal{K}, f)$ 


---

**Data:**  $\mathbf{Y}, \mathcal{K}, f$

**Result:**  $c_{max}$

```

1  $c_{max} \leftarrow 0;$ 
2 foreach  $f_k \in \mathcal{K}$  do
3    $c \leftarrow \text{conflict}(\mathbf{Y}^{(f_k)}, \mathbf{Y}^{(f)})$  ;
4   if  $c > c_{max}$  then
5      $c_{max} \leftarrow c$ ;
6   end
7 end

```

---

Given the set of clusters  $\mathcal{K} = \{\mathcal{K}_1, \dots, \mathcal{K}_{|\mathcal{F}_s|}\}$  obtained with Algorithm 3.1 and a database of samples  $\mathbf{Y}$ , the discrepancy between original and reduced objective sets is defined as the maximum conflict between 2 objectives in the same cluster. This measure of discrepancy is formally defined in Eq. 3.22.

$$d_\kappa(\mathcal{K}, \mathbf{Y}) = \max_{\mathcal{K}_i \in \mathcal{K}, f_j \in \mathcal{K}_i} (\text{conflict}(\mathcal{K}_i, f_j, \mathbf{Y})) \quad (3.22)$$

Measures of correlation are usually defined in the range  $[-1, 1]$ . In order to simplify their interpretation when used as conflict measures, a transformation is applied to them such that they are defined in the range  $[0, 1]$  where, 0 represents no conflict and 1 maximum conflict (total inversion). Thus, given a a correlation measure,  $r(\mathbf{y}_a, \mathbf{y}_b)$ , the conflict,  $c(\mathbf{y}_a, \mathbf{y}_b)$ , based on that measure is given by:

$$c(\mathbf{y}_a, \mathbf{y}_b) = (1 - r(\mathbf{y}_a, \mathbf{y}_b))/2 \quad (3.23)$$

According to (Yuan et al., 2017),  $d_\kappa$  is more robust with respect to the sample set when compared with  $d_\epsilon$  and  $d_\eta$ .

Although Theorem 3.4 justifies the use of conflict-based clustering algorithms, reducing objectives by this kind of analysis is not completely consistent with the original intention of objective reduction. Consider the set  $\mathcal{F} = \{f_1, f_2, f_3\}$  of objectives where,  $f_1$  and  $f_2$  have maximum conflict ( $c(\mathbf{y}_{f_1}, \mathbf{y}_{f_2}) = 1$ ) and  $f_3(\mathbf{x}) = f_1(\mathbf{x}) + f_2(\mathbf{x})$ . Clearly,  $f_3$  is a redundant objective. However, the conflict between  $f_3$  and  $f_1$  is by no means close to 0 because of the  $f_2$  term and the same can be said about the conflict between  $f_3$  and  $f_2$ . Since the correlation, as employed in these methods, can only capture the relationship between pairs of objectives, even for a simple example as the one just presented, it fails to find redundant objective sets when the redundancy arises from the relationship of subsets of objectives.

### 3.4 A summary on the theoretical analysis of performance metrics for surrogate problems

In this chapter, the idea that a surrogate problem should be evaluated with respect to how well it predicts the optimal solution rather than from its response has been introduced. Under this rationale, the theoretical justification for the use of different performance metrics was presented. Through the theoretical analysis, the following conclusions could be derived:

1. For surrogate models:
  - 1.1. Error-based metrics focus on the response differences rather than on the ability of the surrogate model to predict the location of the high quality solutions. For this reason, it is hard to establish a link between error-based metrics and optimization performance.
  - 1.2. Rank maintenance metrics are insensitive to monotone transformations and present links with order maintenance and Pareto-dominance maintenance, which in turn, can be linked back to optimality. On the other hand, rank maintenance metrics are insensitive to the region where the rank disagreement happens. In principle,

disagreements found among high quality solutions should be treated more severely than the disagreements among low quality solutions.

2. For reduced problems:

- 2.1. Metrics for the evaluation of reduced problems focus on the discrepancy between Pareto-fronts which is directly aligned with proposed quality definitions.
- 2.2. Pareto-front preserving metrics, however, are sensitive to the sample set requiring a good representation of the original Pareto optimal front. This demand seems to be counter productive since it requires the solution of the original problem before it can be reduced.
- 2.3. In principle, correlation-based metrics are more robust with respect to the sample set. Nevertheless, they are not able to identify redundancies when they arise from combinations of subsets of objectives.

The theoretical analysis points towards the use of rank correlation for the performance assessment of surrogate models. However, as shown in Section 3.3.2, their use for reduced problems faces important limitations.

With this in view, in the next section, more natural extensions of rank-correlation metrics for reduced problems are proposed. In addition to reduce the drawbacks observed in other performance metrics, they generalize the evaluation of surrogate problems. The methods presented below can be seamlessly used for the evaluation of surrogate problems generated by surrogate models, objective reduction or both.

### 3.5 A unified approach for the evaluation of surrogate problems

In order to generalize the rank-based performance metrics for reduced objective problems, the notion of “ordering” itself has to be generalized for multi-dimensional responses. A significant number of Evolutionary Algorithms for multi-objective optimization are based on the idea of non-dominated sorting (Deb, 2001; Coello et al., 2006) which can rank solutions according to Pareto-dominance.

In a naive implementation of the non-dominated sorting algorithm, given a set of solutions,  $\mathbf{Y} = [\mathbf{y}^{(1)}, \dots, \mathbf{y}^{(n)}]^T$ , all the non-dominated solutions in  $\mathbf{Y}$  are assigned rank 1. The rank 1 solutions are removed from  $\mathbf{Y}$  and the non-dominated solutions of the resulting set are assigned rank 2. The rank 2 solutions are now removed from  $\mathbf{Y}$  and the process is repeated until  $\mathbf{Y}$  is empty. Algorithm 3.3 presents a version of the non-dominated sorting algorithm that avoids the recalculation of Pareto-dominance at each step.

---

**Algorithm 3.3:** Fast non-dominated sorting

---

**Data:**  $\mathbf{Y}$

**Result:** Sample ranks:  $r(\mathbf{y})$

```

1  $s \leftarrow$  number of samples in  $\mathbf{Y}$ 
2 for  $i \in \{1, 2, \dots, s\}$  do
3    $c(i) \leftarrow$  number of samples in  $\mathbf{Y}$  that dominate  $\mathbf{y}^{(i)}$ ;
4    $L(i) \leftarrow$  set of solutions in  $\mathbf{Y}$  that  $\mathbf{y}^{(i)}$  dominates ;
5 end
6  $k \leftarrow 1$ ;
7 while  $\neg$  all individuals are classified do
8    $\mathcal{I} = \{i : c(i) = 0\}$ ;
9   Put non-dominated solutions  $\mathbf{y}^{(\mathcal{I})}$  in front  $\mathbf{Y}_{(k)}$ ;
10   $r(\mathbf{y}^{(\mathcal{I})}) = k$ ;
11   $\mathcal{J} = \{j : \mathbf{y}^{(j)} \in L(\mathcal{I})\}$ ;
12   $c(\mathcal{I}) \leftarrow c(\mathcal{I}) - 1$ ;
13   $k \leftarrow k + 1$ ;
14 end

```

---

Figure 3–12a shows a set of candidate solutions for a hypothetical 3-objective problem and their respective ranks according to Algorithm 3.3. Figure 3–12b shows the same set of solutions projected onto  $f_1$  and  $f_2$ . By observing how each subset of objectives ranks the solutions according to non-dominated sorting, one can easily apply rank correlations measures to assess the performance of a reduced objective set.

In order to make the visualization simpler, in Figure 3–13, the solutions ranks imposed by each subset of objectives is plotted in parallel coordinates. It can be easily seen that the  $f_3$  removal does not affect the ranks implying that it does not affect the dominance relationships. Thus, as a consequence of Theorem 3.3,  $f_3$  is a redundant objective.

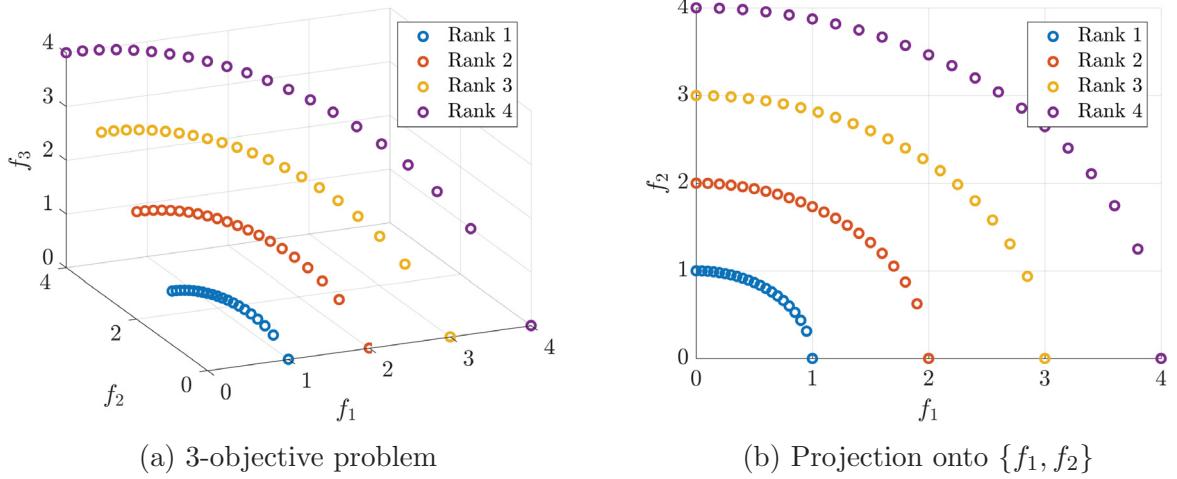


Figure 3–12: Non-dominated sorting

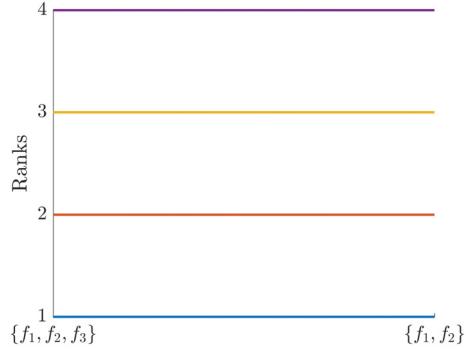


Figure 3–13: Solutions ranks represented in parallel coordinates

As an alternative to the use of non-dominated ranking, the Kendall's  $t$  statistic can be computed with a tweak on the definition of concordant and discordant pairs such that they become Pareto-dominance compliant. More specifically, consider that  $\mathbf{y}^{(i)} = \mathbf{f}(\mathbf{x}_i)$  and  $\mathbf{y}_s^{(i)} = \mathbf{f}_s(\mathbf{x}_i)$  are the vector responses obtained when  $\mathbf{x}_i$  is evaluated by the original and surrogate objective functions, respectively. A pair of observations  $(\mathbf{y}^{(a)}, \mathbf{y}^{(b)})$  and  $(\mathbf{y}_s^{(a)}, \mathbf{y}_s^{(b)})$  are said to be concordant if both,  $\mathbf{y}^{(a)} \prec \mathbf{y}^{(b)}$  and  $\mathbf{y}_s^{(a)} \prec \mathbf{y}_s^{(b)}$ ; or if both  $\mathbf{y}^{(a)} \succ \mathbf{y}^{(b)}$  and  $\mathbf{y}_s^{(a)} \succ \mathbf{y}_s^{(b)}$ . They are considered to be discordant if  $\mathbf{y}^{(a)} \prec \mathbf{y}^{(b)}$  and  $\mathbf{y}_s^{(a)} \succ \mathbf{y}_s^{(b)}$ ; or if  $\mathbf{y}^{(a)} \succ \mathbf{y}^{(b)}$  and  $\mathbf{y}_s^{(a)} \prec \mathbf{y}_s^{(b)}$ . By this definition, a pair of evaluations is considered discordant only if there is an inversion in the dominance relationship. Sometimes, it is possible that a dominated solution becomes indifferent when evaluated through a surrogate problem. It

is not clear, if considering this scenario as a discordance would play a significant role on the perception of the performance of a surrogate problem. In order to investigate this question the Adapted Kendall's Tau (AKT) defined in Eq. 3.24 is also included in the set of performance metrics.

$$AKT = \frac{n_c - n_d}{\text{total number of pairs}} \quad (3.24)$$

where, the set of concordant pairs is defined as  $\mathcal{C} = \{(\mathbf{y}^{(a)}, \mathbf{y}^{(b)}) | \mathbf{y}^{(a)} \prec \mathbf{y}^{(b)} \wedge \mathbf{y}_s^{(a)} \prec \mathbf{y}_s^{(b)} \vee \mathbf{y}^{(a)} \succ \mathbf{y}^{(b)} \wedge \mathbf{y}_s^{(a)} \succ \mathbf{y}_s^{(b)}\}$ , the set of discordant pairs  $\mathcal{D} = \{(\mathbf{y}^{(a)}, \mathbf{y}^{(b)}) | (\mathbf{y}^{(a)}, \mathbf{y}^{(b)}) \notin \mathcal{C}\}$ ,  $n_c = |\mathcal{C}|$  and  $n_d = |\mathcal{D}|$ .

Both methodologies are completely consistent with the principles of objective reduction (see Section 2.4.2.1) and are able to measure the distortions in dominance relationships caused by the removal of objectives. Again, as a consequence of Theorem 3.3, if the dominance relationships are not altered the Pareto-optimal set remains the same.

Differently from the other Correlation-based approaches for objective reduction, the proposed methodologies are able to assess relationships among subsets of objectives and not just pairwise relationships. Moreover, since they do not focus exclusively on the non-dominated solutions, they are expected to be more robust to the sample set. Another advantage is the fact that the dimensionality of the data sets obtained through the original and surrogate problems do not have to be the same which, as mentioned before, generalizes the process of surrogate problems evaluation.

As raised before, a possible weakness of rank-based performance measures is the fact that they are insensitive to the quality of the solutions for which the rank disagreements occur. To remedy this issue the Normalized Weighted Disagreement (NWD) is introduced.

Given two sets of ranked variables  $\mathbf{r}_a = \{r_a^{(1)}, \dots, r_a^{(n)}\}$  and  $\mathbf{r}_b = \{r_b^{(1)}, \dots, r_b^{(n)}\}$ , the NWD is given by:

$$NWD(\mathbf{r}_a, \mathbf{r}_b) = \frac{\sum_{i=1}^{n-1} \sum_{j=i}^n \phi(r_a^{(i)}, r_a^{(j)}, r_b^{(i)}, r_b^{(j)}) \times (n - \min(r_a^{(i)}, r_a^{(j)}, r_b^{(i)}, r_b^{(j)}))}{\sum_{i=0}^{\lfloor \frac{n-1}{2} \rfloor} (4i + 1) \times (\lfloor \frac{n-1}{2} \rfloor + i + 1)} \quad (3.25)$$

where,

$$\phi(r_a^{(i)}, r_a^{(j)}, r_b^{(i)}, r_b^{(j)}) = \begin{cases} 0 & , \text{if } (r_a^{(i)} - r_a^{(j)}) \times (r_b^{(i)} - r_b^{(j)}) > 0 \\ 1 & , \text{if } (r_a^{(i)} - r_a^{(j)}) \times (r_b^{(i)} - r_b^{(j)}) < 0 \end{cases} \quad (3.26)$$

The NWD is a tweak on the probability of disagreement of Kendall's  $t$  in which each disagreement is penalized by the difference between the maximum rank,  $n$ , and the minimum rank in the discordant pair of observations. Thus, disagreements in the region of top rank solutions are perceived as more severe than disagreements in low ranked solutions. The NWD is bounded in  $[0, 1]$  where 0 means no disagreement and 1 total disagreement or complete inversion.

Figure 3–14 presents a comparison among the NWD and the conflict measures based on  $r$  and  $t$  (Eq.3.23) and demonstrates the sensitivity of the NWD with respect to the location of the disagreements.

### 3.6 Limitations of the Theoretical Analysis

Despite providing valuable insight into the suitability of the different performance assessment metrics, the theoretical analysis assumes that the performance is known without uncertainty. In practice, however, the performance is assessed through a limited set of sample points which may introduce error in the evaluation. Thus, the theoretical results presented in this chapter alone are not enough to present a reliable picture on the ability of the presented performance metrics in assessing the performance of surrogate problems within realistic environments. Hence, in order to reduce this gap, in the next chapter the different performance metrics will be used in more realistic scenarios including benchmark analytical problems and electric machine design problems.

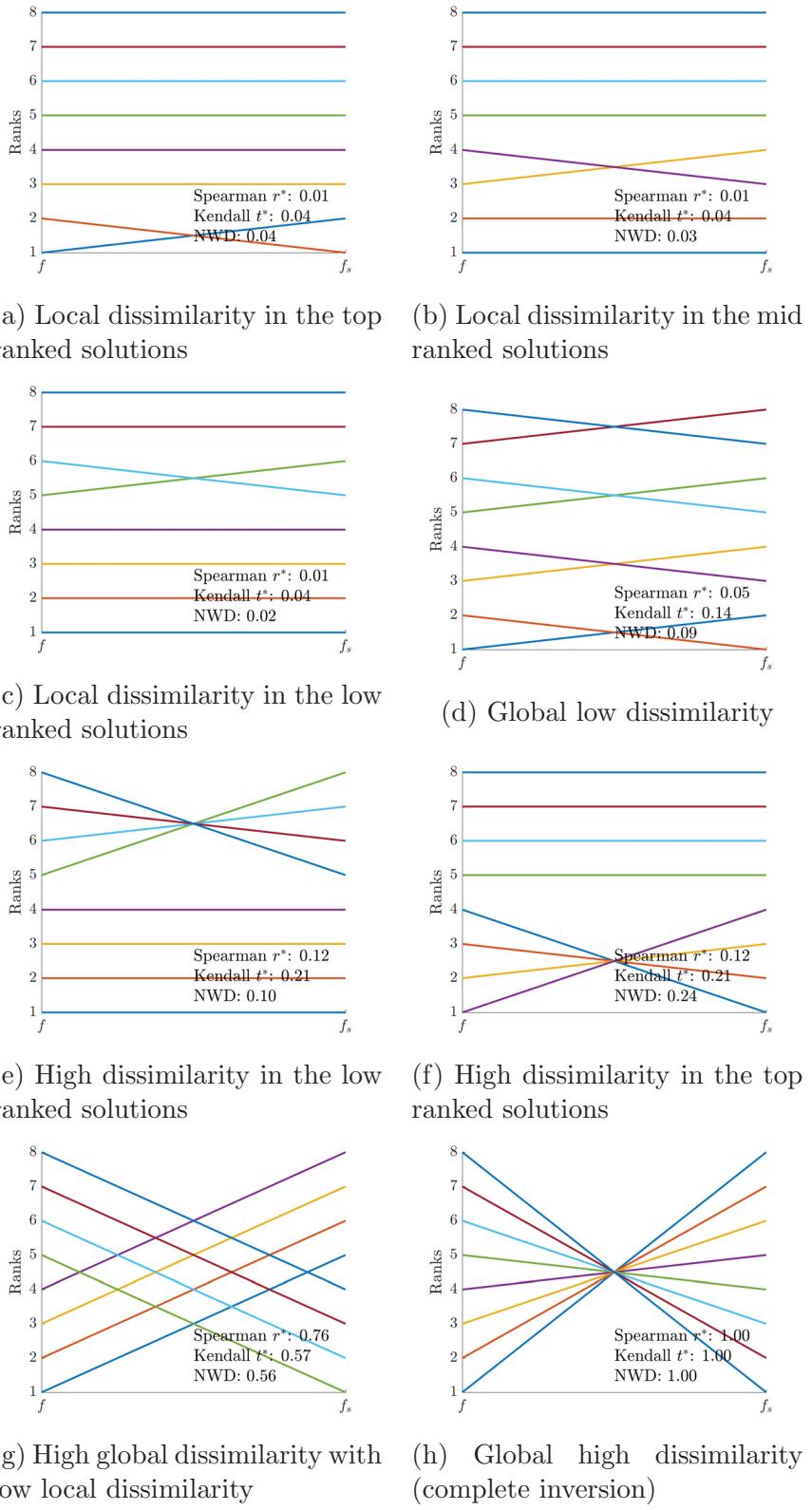


Figure 3-14: Normalized Weighted Disagreement (NWD) versus Kendall's  $t$  versus Spearman's  $r$  for different distributions of rank dissimilarities.

## CHAPTER 4

### Computational Experiments with Analytical Benchmark Problems

In the previous chapter, theoretical and practical aspects of performance metrics were discussed. The presented analysis has allowed us to identify a number of weaknesses and strengths of different performance metrics and to propose a meaningful and general methodology for the evaluation of surrogate problems.

The goal of this chapter is to verify whether the theoretical results hold in practical situations where performance estimates are obtained through sampling and, hence, are uncertain. In order to do this, the suitability of the performance metrics for the evaluations of surrogate problems is tested under different scenarios.

In the first experiment, the relationship between performance metrics and optimization is assessed for surrogate model based optimization. This experiment is similar to the one performed in (Bischl et al., 2012) with the difference that the surrogate models are used for multi-objective problems instead of single objective problems. In the second experiment, the different performance metrics are used for the selection of surrogate models. Finally, the third set of experiments tests the ability of each performance metric to identify essential sets of objectives in the context of objective reduction methods.

#### **4.1 Relationship between performance metrics and optimization results**

##### **4.1.1 Objective**

The main goal of this experiment is to verify how the performance metrics are associated with the optimization results in the context of surrogate model based optimization. In other words, the objective is to assess the correlation between performance metrics and the quality of the obtained non-dominated set.

### 4.1.2 Methodology

Versions with 5, 10, 20 and 30 variables of four bi-objective problems from the well known **ZDT benchmark** (Zitzler et al., 2000a) set were used in this experiment. The simple surrogate based optimization method given by Algorithm 4.1 was applied in order to obtain a non-dominated set of solutions.

---

**Algorithm 4.1:** Simple Surrogate-based Optimization Method

---

```

Data:  $\mathbf{X}, \mathbf{Y}, p_1, \dots, p_k, \mathbf{f}(\cdot)$  /* Database ( $\mathbf{X}, \mathbf{Y}$ ), performance metrics ( $p_i$ ),
       and original objective functions ( $\mathbf{f}(\cdot)$ ) */  

Result:  $\mathcal{Y}^+$  // Set of non-dominated solutions  

1  $\mathbf{f}_s(\cdot) \leftarrow \text{RandomSurrogateModel}(\mathbf{X}, \mathbf{Y})$ ; // Generate a random surrogate model  

2 for  $i = 1 : k$  do  

3    $\pi_i \leftarrow p_i(\mathbf{f}_s(\cdot), \mathbf{X}, \mathbf{Y})$ ; // Evaluate surrogate model  

4 end  

5  $\mathcal{X}^+ \leftarrow \text{optimize}(\mathbf{f}_s(\cdot))$ ; // Optimize surrogate problem  

6  $\mathcal{Y}^+ \leftarrow \mathbf{f}(\mathcal{X}^+)$ ; // Validate optimization results

```

---

The inputs of Algorithm 4.1 are: (i) a set of data samples ( $\mathbf{X}$ ) and the corresponding responses ( $\mathbf{Y}$ ); (ii) a set of performance metrics,  $p_1, \dots, p_k$ ; and (iii) the original objective functions  $\mathbf{f}(\cdot)$ .

The database,  $\mathbf{X}$  and  $\mathbf{Y}$ , was generated with a Latin-hypercube design (See Section 2.4.1) and the number of samples was randomly chosen from 100 to 200 for each execution. Table 4–1 defines the nomenclature which will be used through the rest of the thesis to refer to each performance metric.

Given the inputs, in Algorithm 4.1:Line 1, a surrogate model is generated by randomly setting the width of the radial basis function to a number in the interval  $[0.1, 8]$ . Two types of surrogate models were used, RBFNNs and GRNNs. The neural networks are flexible enough to adapt to the selected benchmark problems, have cheap training and evaluation, and are intrinsically different. While RBFNNs necessarily interpolate the data samples, the GRNNs do not. This fundamental difference may be useful to point out differences among the performance metrics.

Metric	Abbreviation
Mean Squared Error	MSE
Root Mean Squared Error	RMSE
Coefficient of Determination	$r^2$
Maximum absolute error	MAX
Mean Average Percentual Error	MAPE
Mean Absolute Error	MAE
Spearman's $\rho$	SR
Kendall's $\tau$	KT
Adapted Kendall's $\tau$	AKT
Non-dominated ranking with Spearman's $\rho$	NSR
Non-dominated ranking with Kendall's $\tau$	NKT
Normalized Weighted Discrepancy $\tau$	NWD
$\epsilon$ -discrepancy	DEPI
$\eta$ -discrepancy	DETA

Table 4–1: List of performance metrics

In Algorithm 4.1:Lines 2-4, following the work of Bischi et al. (2012), the performance was estimated using 10-fold cross-validation with each one of the performance metrics.

In Algorithm 4.1:Line 5, the Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D) is used to obtain a non-dominated set of solutions. As can be seen in (Zhang and Li, 2007), the  $ZDT$  problems are easily solved by the MOEA/D, thus, any difficulty in obtaining high-quality Pareto sets for these problems comes exclusively from the use of the generated surrogate models. For the same reason, no infill point selection algorithm is used.

Finally, in Algorithm 4.1:Line 6, the complete non-dominated set found during the optimization stage is validated and returned as a possible solution for the problem. For each type of surrogate model (GRNN or RBFNN), Algorithm 4.1 was executed 100 times for each version of each problem.

### 4.1.3 Results and discussion

Tables 4–2 and 4–3 present the correlations between the estimated average performance and the IGD<sup>1</sup> of the obtained non-dominated sets. A statistical correlation test<sup>2</sup> was used to determine, with a confidence level  $\alpha = 95\%$ , if the computed correlation is statistically different from zero. In order to facilitate the results interpretation, from this point on, the conflict transformation given in Eq. 3.23 is applied to all rank correlation metrics. Thus, in the ideal case, the correlation between performance measures and IGD should be positive for all metrics. In Tables 4–2 and 4–3, negative and statistically significant correlations are shown in red, positive and statistically significant correlations are shown in blue, and non-significant correlations are shown in black.

Table 4–2 presents the correlations between IGD and the different performance metrics when GRNNs were used as surrogate models. The results show that the rank-correlation metrics, including the ones based on non-dominated sorting and DETA, are much better predictors of optimization performance (i.e. IGD) than the error-based metrics, including DEPI. Moreover, for a considerable number of scenarios, the correlation between IGD and the error-based metrics was significantly negative.

Table 4–3 presents the correlations between IGD and the different performance metrics when RBFNNs were used as surrogate models. It can be seen that no significant correlation was found for the majority of the cases. This can indicate that independent of the generated surrogate model, the optimization results are equally good or equally bad. When a significant correlation was found, the behaviour observed for the different metrics was similar.

---

<sup>1</sup> Reference sets for the computation of the IGD are available online at <http://jmetal.sourceforge.net/problems.html> (Accessed in August 20, 2017)

<sup>2</sup> The correlation test description can be found online at <https://stat.ethz.ch/R-manual/R-devel/library/stats/html/cor.test.html> (Accessed in August 20, 2017)

GRNN							
Problem	MSE	RMSE	$r^2$	MAX	MAPE	MAE	SR
zdt2 - 5D	-0.127	-0.127	-0.119	-0.211	-0.012	-0.120	0.171
zdt3 - 5D	-0.198	-0.198	-0.204	<b>-0.359</b>	<b>-0.416</b>	-0.199	0.010
zdt4 - 5D	<b>0.604</b>	<b>0.603</b>	<b>0.607</b>	<b>0.504</b>	0.038	<b>0.606</b>	<b>0.553</b>
zdt6 - 5D	-0.261	-0.261	-0.261	-0.246	-0.246	-0.235	0.089
zdt2 - 10D	<b>-0.392</b>	<b>-0.392</b>	<b>-0.394</b>	<b>-0.467</b>	<b>-0.420</b>	<b>-0.397</b>	<b>0.385</b>
zdt3 - 10D	<b>-0.396</b>	<b>-0.396</b>	<b>-0.393</b>	<b>-0.444</b>	-0.167	<b>-0.410</b>	0.266
zdt4 - 10D	-0.160	-0.160	-0.159	-0.106	-0.200	-0.178	-0.134
zdt6 - 10D	-0.207	-0.208	-0.207	-0.273	-0.225	-0.216	0.266
zdt2 - 20D	<b>-0.511</b>	<b>-0.511</b>	<b>-0.509</b>	<b>-0.486</b>	<b>-0.282</b>	<b>-0.499</b>	<b>0.497</b>
zdt3 - 20D	<b>-0.290</b>	<b>-0.290</b>	-0.280	-0.119	<b>-0.392</b>	<b>-0.551</b>	<b>0.532</b>
zdt4 - 20D	-0.061	-0.061	-0.060	-0.047	-0.003	-0.045	-0.098
zdt6 - 20D	-0.206	-0.207	-0.204	-0.280	-0.197	-0.253	0.270
zdt2 - 30D	<b>-0.557</b>	<b>-0.557</b>	<b>-0.558</b>	<b>-0.453</b>	<b>-0.504</b>	<b>-0.679</b>	<b>0.685</b>
zdt3 - 30D	<b>-0.350</b>	<b>-0.350</b>	<b>-0.359</b>	-0.055	-0.147	<b>-0.387</b>	<b>0.482</b>
zdt4 - 30D	-0.013	-0.013	-0.016	-0.054	0.024	-0.001	0.010
zdt6 - 30D	-0.261	-0.261	-0.257	-0.271	-0.267	<b>-0.310</b>	<b>0.318</b>
Problem	KT	AKT	NSR	NKT	NWD	DETA	DEPI
zdt2 - 5D	0.182	0.184	0.193	0.193	0.187	<b>0.631</b>	<b>-0.288</b>
zdt3 - 5D	-0.023	0.060	0.075	0.055	0.067	0.050	-0.255
zdt4 - 5D	<b>0.559</b>	<b>0.538</b>	<b>0.533</b>	<b>0.539</b>	0.195	0.110	-0.071
zdt6 - 5D	0.035	0.176	0.163	0.175	0.162	0.082	-0.209
zdt2 - 10D	<b>0.392</b>	<b>0.367</b>	<b>0.365</b>	<b>0.370</b>	<b>0.368</b>	<b>0.513</b>	-0.131
zdt3 - 10D	0.266	0.246	<b>0.287</b>	0.247	0.268	0.038	<b>-0.472</b>
zdt4 - 10D	-0.136	-0.057	-0.048	-0.055	-0.087	0.079	0.176
zdt6 - 10D	0.221	<b>0.340</b>	<b>0.335</b>	<b>0.337</b>	<b>0.345</b>	0.076	-0.120
zdt2 - 20D	<b>0.497</b>	<b>0.517</b>	0.514	<b>0.521</b>	<b>0.517</b>	<b>0.452</b>	<b>-0.326</b>
zdt3 - 20D	<b>0.565</b>	<b>0.537</b>	<b>0.544</b>	<b>0.538</b>	<b>0.536</b>	<b>0.423</b>	<b>-0.501</b>
zdt4 - 20D	-0.100	0.111	0.113	0.112	0.153	0.171	-0.186
zdt6 - 20D	<b>0.282</b>	0.256	0.280	0.255	0.264	0.073	<b>-0.364</b>
zdt2 - 30D	<b>0.686</b>	<b>0.659</b>	<b>0.661</b>	<b>0.658</b>	<b>0.659</b>	<b>0.451</b>	<b>-0.328</b>
zdt3 - 30D	<b>0.480</b>	<b>0.470</b>	<b>0.445</b>	<b>0.468</b>	<b>0.466</b>	0.245	<b>-0.610</b>
zdt4 - 30D	0.005	-0.175	-0.172	-0.173	-0.066	<b>0.364</b>	-0.223
zdt6 - 30D	<b>0.347</b>	<b>0.399</b>	<b>0.395</b>	<b>0.400</b>	<b>0.398</b>	0.304	<b>-0.364</b>

Table 4–2: Correlation between performance metrics and IGD using GRNN

RBFNN							
Problem	MSE	RMSE	$r^2$	MAX	MAPE	MAE	SR
zdt2 - 5D	-0.048	-0.048	0.115	0.065	-0.110	-0.077	0.124
zdt3 - 5D	<b>-0.302</b>	<b>-0.302</b>	<b>-0.306</b>	<b>-0.332</b>	-0.067	-0.290	<b>-0.374</b>
zdt4 - 5D	-0.078	-0.078	-0.076	-0.168	<b>0.336</b>	-0.047	<b>0.334</b>
zdt6 - 5D	-0.193	-0.193	-0.193	<b>-0.328</b>	-0.283	-0.159	-0.201
zdt2 - 10D	<b>0.301</b>	<b>0.301</b>	<b>0.309</b>	<b>0.299</b>	<b>0.345</b>	<b>0.305</b>	<b>0.306</b>
zdt3 - 10D	-0.223	-0.223	-0.222	-0.246	0.135	-0.187	-0.227
zdt4 - 10D	0.050	0.050	0.049	0.012	<b>0.322</b>	0.043	0.091
zdt6 - 10D	-0.197	-0.197	-0.196	-0.284	<b>-0.457</b>	-0.123	-0.044
zdt2 - 20D	<b>0.648</b>	<b>0.648</b>	<b>0.649</b>	<b>0.642</b>	<b>0.693</b>	<b>0.653</b>	<b>0.645</b>
zdt3 - 20D	-0.167	-0.167	-0.170	-0.197	-0.217	-0.146	-0.202
zdt4 - 20D	-0.203	-0.203	-0.203	-0.146	0.160	-0.201	-0.096
zdt6 - 20D	-0.131	-0.131	-0.134	-0.132	<b>-0.502</b>	0.003	-0.147
zdt2 - 30D	<b>0.572</b>	<b>0.572</b>	<b>0.572</b>	<b>0.581</b>	<b>0.572</b>	<b>0.574</b>	<b>0.566</b>
zdt3 - 30D	-0.065	-0.065	-0.069	-0.023	-0.018	0.012	-0.131
zdt4 - 30D	0.041	0.041	0.041	0.089	0.353	0.032	-0.047
zdt6 - 30D	-0.259	-0.259	-0.261	<b>-0.555</b>	<b>-0.427</b>	-0.112	-0.251
Problem	KT	ATK	NSR	NKT	NWD	DETA	DEPI
zdt2 - 5D	-0.089	-0.067	-0.064	-0.072	-0.067	-0.030	0.012
zdt3 - 5D	<b>-0.327</b>	-0.263	-0.254	-0.264	-0.244	<b>0.390</b>	<b>-0.367</b>
zdt4 - 5D	<b>0.339</b>	0.263	0.248	0.253	0.028	<b>-0.488</b>	0.030
zdt6 - 5D	-0.067	-0.271	-0.286	-0.263	<b>-0.297</b>	<b>0.519</b>	0.118
zdt2 - 10D	<b>0.303</b>	<b>0.340</b>	<b>0.346</b>	<b>0.341</b>	<b>0.316</b>	0.304	0.271
zdt3 - 10D	-0.087	-0.173	-0.204	-0.173	-0.188	0.194	-0.278
zdt4 - 10D	0.080	-0.170	-0.151	-0.158	-0.304	<b>-0.331</b>	<b>0.322</b>
zdt6 - 10D	0.236	-0.019	-0.044	-0.020	-0.027	<b>0.322</b>	-0.047
zdt2 - 20D	<b>0.652</b>	<b>0.673</b>	<b>0.672</b>	<b>0.673</b>	<b>0.671</b>	<b>0.551</b>	0.068
zdt3 - 20D	-0.182	-0.231	-0.240	-0.229	-0.199	0.043	-0.125
zdt4 - 20D	-0.069	-0.230	-0.148	-0.151	-0.051	-0.092	-0.106
zdt6 - 20D	0.145	-0.145	-0.155	-0.144	-0.176	0.144	<b>0.348</b>
zdt2 - 30D	<b>0.578</b>	<b>0.561</b>	<b>0.561</b>	<b>0.560</b>	<b>0.572</b>	<b>0.423</b>	0.095
zdt3 - 30D	0.040	0.056	0.046	0.057	0.030	0.098	0.099
zdt4 - 30D	-0.005	<b>-0.403</b>	-0.313	-0.308	-0.168	-0.090	0.100
zdt6 - 30D	-0.018	-0.237	-0.269	-0.245	-0.244	0.189	0.213

Table 4–3: Correlation between performance metrics and IGD using RBFNN

In order to understand these results it is useful to consider Figure 4–1. Depending on the choice of the hyper-parameters (i.e. the radial basis function width), GRNNs can produce models with very low errors which, due to the generated flat regions, are problematic for optimization. On the other hand, GRNNs can also produce surrogate models, like the one in Figure 4–1a, which present higher error but preserve the solutions' ranks. Thus, the used quality metric may lead to very different choices of surrogate models. The same phenomenon does not happen with RBFNN for which low error is usually linked to high rank-correlation as illustrated in Figure 4–2a and high error is linked with low rank-correlation as in Figure 4–2b.

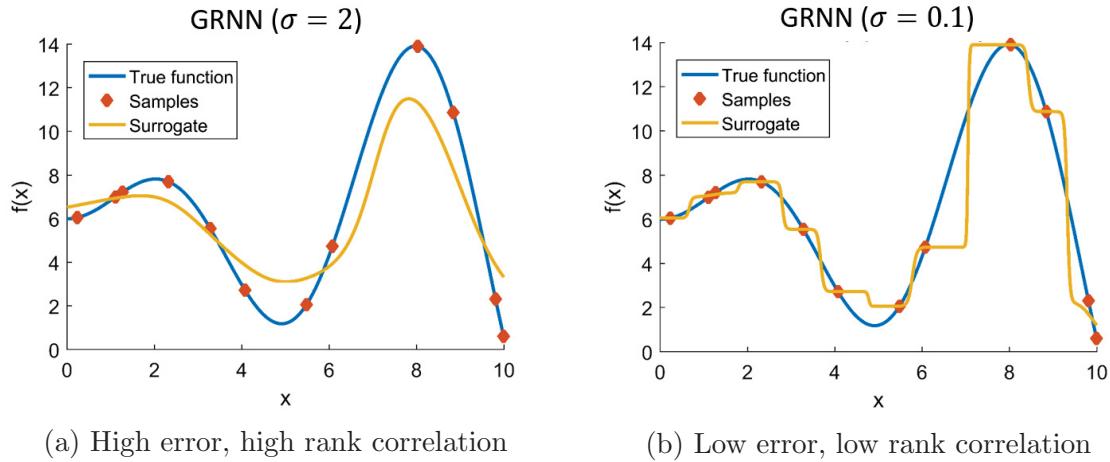


Figure 4–1: Example of surrogate models generated by GRNNs

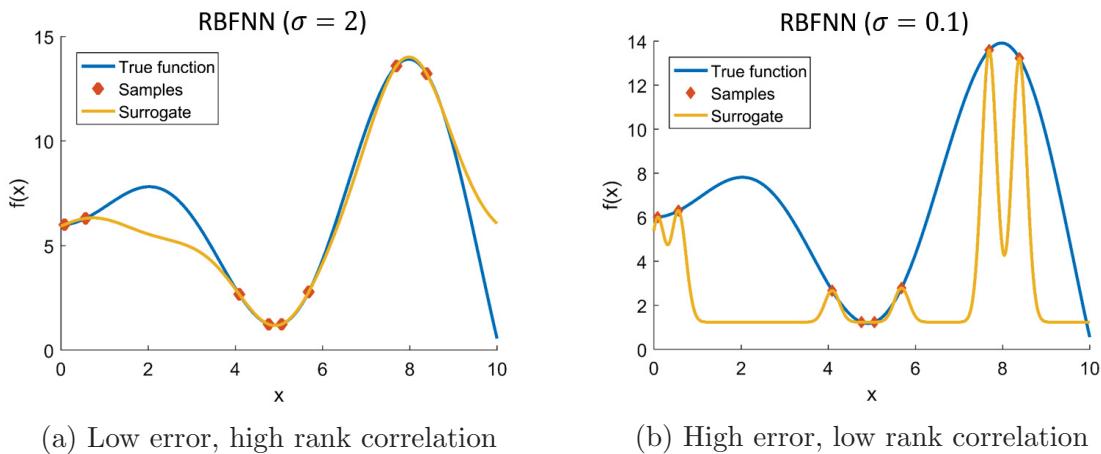


Figure 4–2: Example of surrogate models generated by RBFNNs

These results give support to the theoretical analysis and show that, for the tested scenarios, the performance metrics that are more aligned with the definitions of quality presented in Chapter 3 are better or, at least, not worse predictors of optimization performance when compared to the other metrics.

## 4.2 Performance metrics for surrogate model selection

### 4.2.1 Objective

The objective of this experiment is to verify how the use of the different performance metrics affects the optimization performance when used for surrogate model selection.

### 4.2.2 Methodology

In this experiment the Surrogate model based optimization method with online surrogate selection described in Algorithm 4.2 is used to obtain non-dominated fronts.

---

**Algorithm 4.2:** Surrogate-based Optimization Algorithm with Online Surrogate model selection

---

**Data:**  $\mathbf{X}, \mathbf{Y}, p, \mathbf{f} : \mathcal{X} \rightarrow \mathbb{R}^m, \mathcal{L} = \{l_1(\cdot), \dots, l_m(\cdot)\}$  /\* Database ( $\mathbf{X}, \mathbf{Y}$ ),  
performance metrics ( $p$ ), original objective functions ( $\mathbf{f}(\cdot)$ ) and  
list of surrogate models ( $\mathcal{L}$ ) \*/

**Result:**  $\mathcal{Y}^+$  // Set of non-dominated solutions

```

1 if  $p$  is computed objective-wise then
2   for  $i = 1 : m$  do
3     // For each objective
4      $f_{s_i} \leftarrow \arg \min_{l_j \in \mathcal{L}} p(l_j(\cdot), f_i(\cdot), \mathbf{X}, \mathbf{Y});$            // Evaluate surrogate model
5   end
6 else
7    $\mathbf{f}_s \leftarrow \arg \min_{\mathbf{l} \subseteq \mathcal{L}} p(\mathbf{l}(\cdot), \mathbf{f}(\cdot), \mathbf{X}, \mathbf{Y})$  s.t.  $|\mathbf{l}| = m;$ 
8 end
9  $\mathcal{X}^+ \leftarrow \text{optimize}(\mathbf{f}_s(\cdot));$                                      // Optimize surrogate problem
10  $\mathcal{Y}^+ \leftarrow \mathbf{f}(\mathcal{X}^+);$                                          // Validate optimization results

```

---

Algorithm 4.2 has as inputs a set of samples,  $\mathbf{X}$ , their respective responses,  $\mathbf{Y}$ , the original objective functions,  $\mathbf{f}(\cdot)$ , a performance metric  $p$ , and a list of surrogate models  $\mathcal{L}$ .

Again, the database ( $\mathbf{X}, \mathbf{Y}$ ) was generated with a Latin-hypercube design and the number of samples was randomly chosen from 100 to 200 for each execution. Three sets

of surrogate models were used in this experiment,  $\mathcal{L}_{GRNN}$ ,  $\mathcal{L}_{RBFNN}$  and  $\mathcal{L}_{GRNN+RBFNN}$ .  $\mathcal{L}_{GRNN}$  consists of 4 GRNNs with  $\sigma \in \{0.1, 1, 3, 6\}$  and  $\mathcal{L}_{RBFNN}$  consists of 4 RBFNNs with  $\sigma \in \{0.1, 1, 3, 6\}$ .  $\mathcal{L}_{GRNN+RBFNN}$  consists of the union between  $\mathcal{L}_{GRNN}$  and  $\mathcal{L}_{RBFNN}$ .

The surrogate models which minimize a given performance measure,  $p$ , are selected to compose the surrogate problem. Surrogate models can be selected objective-wise, when the metrics presented in Sections 3.2.1 and 3.2.2 are used, or as a set when the metrics presented in Sections 3.3.1 and 3.5 are used. As in the previous experiment, the surrogate models were evaluated using a 10-fold cross-validation. After the surrogate problem is created, MOEA/D is used for optimization and the validated non-dominated front is returned.

In addition to the *ZDT* problems used in previous section, six problems from the *DTLZ* (Deb et al., 2002) benchmark were include in these experiments. Table 4–4 shows the problem specifications and their definitions can be found in the Appendix A.

Problem	Number of Variables	Number of Objectives
<i>zdt2</i>	20	2
<i>zdt3</i>	20	2
<i>zdt4</i>	8	2
<i>zdt6</i>	8	2
<i>dtlz1</i>	7	3
<i>dtlz2</i>	12	3
<i>dtlz3</i>	12	3
<i>dtlz4</i>	12	3
<i>dtlz5</i>	12	3

Table 4–4: Problem set-up

For each combination of performance metrics, list of surrogate models and test problems, Algorithm 4.2 was executed 30 times. For each surrogate list, the Kruskall-Wallis test<sup>3</sup> was used to verify if there was statistical difference among the results obtained for each problem.

---

<sup>3</sup> The documentation for the R-implementation of the Kruskall-Wallis test can be found online at <https://www.rdocumentation.org/packages/stats/versions/3.4.1/topics/kruskal.test> (Accessed in August 20, 2017)

When the Kruskall-Wallis test detected a difference, Dunn's test<sup>4</sup> was used for the pairwise comparison. The differences among methodologies were considered significant when the P-value was less than 0.05, i.e, when confidence level was 95%.

#### 4.2.3 Results and discussion

Table 4–5 presents the median and the interquartile interval of the IGD obtained by each one of the versions of Algorithm 4.2. The values in boldface show the results obtained by the top-performing methods in each problem. Among the results in boldface, there is no significant difference. The values that are not in boldface are statistically worse than at least one of top-performing methods. The results show that the performance metric used to select a surrogate model does have a significant influence in the optimization. As expected from the previous experiment, the differences between error-based metrics and rank-based metrics (including the ones based on non-dominated ranking) are more accentuated for GRNNs for which rank maintenance and low error are not strongly linked.

Table 4–6 shows the number of times each metric was among the top-performing ones. DETA, which simply computes the proportion of solutions that became dominated presented the worst results closely followed by the metrics based on average or maximum values of error (i.e. RMSE, MAE, MAX and DETA). Metrics which give more weight for the discrepancies among the highest quality solutions (i.e. MAPE, NWD and DETA) presented slightly better results followed by the objective-wise rank correlation metrics (KT and SR). Overall, the rank-correlation metrics computed with non-dominated ranks (AKT, NKT and NSR) have been shown to be the most reliable measure of surrogate model performance. Even when they were not among the top-performing metrics, the obtained IGDs were not too far off the best ones.

---

<sup>4</sup> The documentation for the R-implementation of the Dunn's test can be found online at <https://cran.r-project.org/web/packages/PMCMR/index.html> (Accessed in August 20, 2017)

$\mathcal{L}_{GRNN}$									
Metric	Problems								
	zdt2	zdt3	zdt4	zdt6	dtlz1	dtlz2	dtlz3	dtlz4	dtlz5
RMSE	3.22[1.79,5.39]	16.80[9.66,22.98]	68.01[5.15,130.20]	39.52[28.76,53.27]	<b>80.30</b> [65.05,101.53]	<b>5.11</b> [4.23,7.02]	<b>1.20</b> [1.10,1.43]	<b>6.67</b> [5.39,8.88]	0.25[0.17,0.32]
MAE	4.53[3.02,7.36]	15.22[10.71,23.24]	90.87[20.11,243.86]	39.47[28.50,54.60]	<b>77.84</b> [57.47,107.63]	<b>4.42</b> [3.37,5.31]	<b>1.22</b> [0.94,1.51]	<b>7.25</b> [5.56,8.56]	0.21[0.16,0.25]
MAPE	6.76[4.34,10.92]	21.96[17.37,27.82]	58.86[33.89,125.88]	51.69[39.82,66.20]	<b>88.18</b> [69.82,115.55]	<b>3.98</b> [3.46,4.60]	2.39[1.93,3.33]	8.16[5.54,9.05]	0.17[0.13,0.19]
MAX	4.17[2.33,8.70]	17.50[11.69,21.94]	43.73[14.55,86.04]	38.09[31.24,55.79]	<b>91.66</b> [81.71,106.68]	7.24[6.64,8.31]	<b>1.25</b> [1.12,1.67]	<b>7.04</b> [5.01,7.91]	0.28[0.25,0.33]
KT	<b>0.02</b> [0.01,0.20]	<b>0.91</b> [0.06,10.02]	<b>5.18</b> [3.25,120.57]	<b>23.29</b> [9.90,35.06]	262.82[207.77,320.00]	9.21[7.87,11.12]	<b>1.85</b> [1.52,2.37]	<b>6.50</b> [5.54,10.15]	0.36[0.33,0.40]
SR	<b>0.03</b> [0.02,0.09]	<b>3.96</b> [0.06,10.04]	30.75[3.06,209.25]	<b>22.25</b> [9.72,34.25]	242.33[209.23,269.04]	9.55[7.99,10.57]	<b>1.81</b> [1.54,2.41]	<b>7.35</b> [5.77,9.31]	0.37[0.32,0.41]
NKT	<b>0.05</b> [0.01,0.42]	<b>3.89</b> [0.12,14.99]	<b>6.80</b> [3.00,17.09]	<b>20.23</b> [9.94,33.91]	<b>110.22</b> [84.15,186.00]	5.80[4.54,7.46]	<b>1.23</b> [0.96,1.57]	<b>7.75</b> [4.83,9.12]	0.27[0.19,0.36]
NSR	<b>0.02</b> [0.01,1.13]	<b>1.95</b> [0.05,5.70]	<b>3.81</b> [2.52,6.34]	<b>20.27</b> [11.29,34.30]	<b>111.66</b> [82.00,156.13]	6.01[5.02,7.41]	<b>1.31</b> [0.93,1.91]	<b>7.56</b> [5.56,9.44]	0.27[0.22,0.37]
AKT	<b>0.02</b> [0.01,0.14]	<b>2.82</b> [0.05,12.19]	<b>4.89</b> [3.19,10.19]	<b>27.25</b> [5.89,38.89]	143.59[121.59,169.42]	7.62[5.63,9.32]	2.02[1.23,2.92]	<b>7.73</b> [5.22,9.09]	<b>0.22</b> [0.17,0.27]
NWD	<b>0.04</b> [0.01,1.57]	<b>3.70</b> [0.61,11.45]	<b>10.94</b> [3.44,42.13]	<b>23.00</b> [11.32,34.68]	263.47[221.03,303.71]	9.28[8.52,11.30]	2.01[1.68,2.80]	<b>7.35</b> [6.48,9.19]	0.33[0.31,0.38]
DEPI	6.20[2.65,8.72]	27.43[19.19,35.37]	22.75[9.18,41.49]	45.41[30.68,70.75]	<b>85.17</b> [64.11,119.58]	<b>4.22</b> [3.17,4.83]	<b>1.71</b> [1.38,2.10]	6.94[6.10,7.95]	<b>0.19</b> [0.14,0.22]
DETA	<b>0.02</b> [0.01,0.07]	<b>1.22</b> [0.05,7.10]	106.01[7.96,171.44]	<b>20.30</b> [7.94,38.53]	249.61[194.99,310.92]	10.24[8.70,11.59]	1.91[1.71,2.39]	<b>6.74</b> [5.57,7.70]	0.38[0.34,0.46]
$\mathcal{L}_{RBFNN}$									
Metric	Problems								
	zdt2	zdt3	zdt4	zdt6	dtlz1	dtlz2	dtlz3	dtlz4	dtlz5
RMSE	<b>0.05</b> [0.04,0.07]	<b>99.54</b> [61.07,142.69]	17.96[4.00,325.83]	67.76[42.63,148.44]	327.57[262.97,420.21]	<b>0.31</b> [0.28,0.45]	<b>2.03</b> [1.08,3.47]	<b>3.48</b> [2.45,4.62]	<b>0.01</b> [0.01,0.01]
MAE	<b>0.04</b> [0.03,0.07]	<b>90.07</b> [64.88,132.44]	23.26[4.00,795.94]	75.74[39.63,106.11]	<b>260.84</b> [172.54,367.95]	<b>0.33</b> [0.26,0.42]	<b>1.50</b> [0.86,4.18]	<b>4.00</b> [3.43,4.72]	<b>0.01</b> [0.01,0.01]
MAPE	<b>0.04</b> [0.03,0.09]	<b>72.49</b> [56.65,93.05]	<b>3.00</b> [2.00,5.00]	<b>20.40</b> [8.16,30.42]	<b>228.96</b> [126.80,324.34]	<b>0.34</b> [0.27,0.41]	<b>0.98</b> [0.52,1.57]	<b>4.07</b> [2.85,9.09]	<b>0.01</b> [0.01,0.01]
MAX	<b>0.05</b> [0.03,0.08]	<b>83.68</b> [55.26,142.08]	30.27[4.63,1017.41]	57.89[29.06,115.45]	<b>286.01</b> [171.73,406.09]	<b>0.35</b> [0.28,0.42]	<b>2.16</b> [1.32,3.25]	<b>3.85</b> [2.65,4.84]	<b>0.01</b> [0.01,0.01]
KT	<b>0.05</b> [0.03,0.07]	<b>87.74</b> [61.19,137.86]	12.55[3.00,40.76]	50.32[13.77,115.07]	<b>206.81</b> [135.94,396.18]	<b>0.31</b> [0.27,0.40]	<b>2.05</b> [0.99,3.93]	<b>4.09</b> [3.14,4.73]	<b>0.01</b> [0.01,0.01]
SR	<b>0.05</b> [0.03,0.09]	<b>87.93</b> [58.35,141.17]	<b>4.00</b> [3.00,69.21]	48.55[20.79,94.38]	<b>268.73</b> [192.79,435.32]	<b>0.33</b> [0.27,0.43]	<b>1.65</b> [0.84,2.52]	<b>3.69</b> [3.12,4.64]	<b>0.01</b> [0.01,0.01]
NKT	<b>0.05</b> [0.03,0.08]	<b>80.03</b> [60.43,119.12]	<b>4.15</b> [3.00,18.24]	<b>33.57</b> [7.99,85.28]	<b>213.62</b> [117.62,355.59]	<b>0.37</b> [0.30,0.56]	<b>1.66</b> [0.75,3.37]	<b>3.81</b> [3.18,4.89]	<b>0.01</b> [0.01,0.01]
NSR	<b>0.05</b> [0.03,0.07]	<b>101.82</b> [68.48,125.66]	<b>10.57</b> [2.00,62.91]	71.11[27.53,137.63]	<b>168.34</b> [126.38,262.01]	<b>0.39</b> [0.32,0.49]	<b>1.51</b> [0.85,3.41]	<b>2.86</b> [2.03,4.04]	<b>0.01</b> [0.01,0.01]
AKT	<b>0.05</b> [0.04,0.08]	<b>120.21</b> [71.05,151.95]	<b>3.91</b> [3.00,45.09]	69.09[33.47,165.48]	<b>156.66</b> [94.71,295.71]	<b>0.43</b> [0.34,0.55]	<b>1.79</b> [0.93,3.02]	<b>4.69</b> [3.34,6.23]	<b>0.01</b> [0.01,0.02]
NWD	<b>0.04</b> [0.03,0.07]	<b>96.30</b> [68.05,129.10]	<b>4.00</b> [3.00,46.60]	66.62[23.18,105.17]	<b>221.81</b> [176.83,372.06]	<b>0.35</b> [0.28,0.56]	<b>3.88</b> [1.50,6.21]	<b>3.49</b> [2.31,5.27]	<b>0.01</b> [0.01,0.01]
DEPI	<b>0.05</b> [0.03,0.08]	<b>69.86</b> [52.14,97.78]	<b>6.37</b> [3.25,66.25]	42.89[9.83,66.80]	<b>214.62</b> [143.92,258.99]	<b>0.34</b> [0.30,0.47]	<b>1.36</b> [0.75,2.30]	<b>3.89</b> [2.40,5.40]	<b>0.01</b> [0.01,0.01]
DETA	<b>0.05</b> [0.03,0.14]	<b>91.16</b> [63.52,109.30]	<b>5.87</b> [3.11,57.22]	50.27[25.30,125.04]	<b>252.72</b> [158.16,370.87]	<b>0.47</b> [0.31,1.21]	4.40[3.48,5.92]	<b>4.44</b> [2.72,6.55]	<b>0.01</b> [0.01,0.02]
$\mathcal{L}_{GRNN+RBFNN}$									
Metric	Problems								
	zdt2	zdt3	zdt4	zdt6	dtlz1	dtlz2	dtlz3	dtlz4	dtlz5
RMSE	<b>0.04</b> [0.03,0.06]	115.58[82.17,137.92]	108.59[10.51,358.17]	<b>30.63</b> [15.79,53.60]	<b>108.46</b> [81.09,128.31]	<b>0.28</b> [0.24,0.41]	<b>1.77</b> [1.19,2.70]	<b>3.58</b> [2.51,4.84]	<b>0.01</b> [0.01,0.01]
MAE	<b>0.04</b> [0.03,0.05]	112.03[72.46,125.93]	110.76[4.19,299.70]	49.96[24.69,77.55]	<b>113.60</b> [69.47,138.97]	<b>0.27</b> [0.21,0.41]	<b>1.90</b> [0.90,3.26]	4.24[3.45,6.37]	<b>0.01</b> [0.01,0.01]
MAPE	<b>0.04</b> [0.03,0.05]	66.03[32.40,97.60]	<b>4.00</b> [2.23,24.96]	<b>13.53</b> [7.66,27.54]	182.98[100.44,299.32]	<b>0.32</b> [0.24,0.45]	<b>1.20</b> [0.61,1.71]	4.42[3.48,5.48]	<b>0.01</b> [0.01,0.01]
MAX	<b>0.04</b> [0.03,0.06]	113.99[77.48,138.66]	110.65[15.47,274.63]	<b>30.11</b> [23.70,55.27]	<b>109.01</b> [82.22,134.70]	<b>0.28</b> [0.24,0.48]	<b>1.81</b> [1.11,2.43]	<b>3.47</b> [1.69,5.24]	<b>0.01</b> [0.01,0.01]
KT	<b>0.04</b> [0.03,0.05]	<b>8.76</b> [2.18,59.11]	<b>5.25</b> [3.00,100.57]	<b>28.78</b> [15.59,49.90]	234.46[183.30,307.03]	<b>0.35</b> [0.24,0.51]	<b>2.04</b> [1.54,2.54]	<b>2.76</b> [1.67,3.78]	<b>0.01</b> [0.01,0.01]
SR	<b>0.04</b> [0.03,0.06]	<b>13.26</b> [3.32,36.50]	<b>3.21</b> [2.00,133.67]	<b>30.40</b> [18.07,48.96]	277.54[200.13,311.54]	<b>0.36</b> [0.23,0.49]	<b>1.87</b> [1.68,2.45]	<b>2.81</b> [1.27,4.32]	<b>0.01</b> [0.01,0.01]
NKT	<b>0.04</b> [0.03,0.06]	<b>22.97</b> [6.53,40.77]	4.42[3.00,42.24]	44.69[28.42,55.82]	<b>155.05</b> [99.52,224.27]	<b>0.35</b> [0.26,0.62]	<b>1.65</b> [0.89,2.77]	<b>3.03</b> [1.41,3.91]	<b>0.01</b> [0.01,0.01]
NSR	<b>0.04</b> [0.03,0.07]	<b>29.12</b> [5.65,78.65]	4.00[2.10,38.68]	<b>32.28</b> [16.06,51.11]	<b>155.83</b> [112.29,249.43]	<b>0.45</b> [0.26,0.77]	<b>1.51</b> [0.85,2.63]	<b>2.90</b> [1.65,4.86]	<b>0.01</b> [0.01,0.01]
AKT	<b>0.03</b> [0.03,0.05]	<b>27.62</b> [4.97,67.36]	<b>5.63</b> [3.00,47.15]	<b>28.33</b> [16.09,51.96]	201.01[154.77,229.18]	<b>0.41</b> [0.38,0.51]	<b>1.86</b> [0.92,3.27]	<b>2.91</b> [1.85,4.56]	<b>0.01</b> [0.01,0.02]
NWD	<b>0.04</b> [0.03,0.06]	<b>14.34</b> [3.86,86.19]	<b>4.50</b> [2.00,42.19]	<b>28.85</b> [14.30,48.34]	218.12[186.10,287.13]	9.18[8.44,10.42]	<b>2.11</b> [1.79,2.32]	<b>3.51</b> [2.10,4.68]	0.29[0.05,0.40]
DEPI	<b>0.03</b> [0.03,0.05]	66.89[33.23,109.47]	<b>7.43</b> [3.50,33.68]	49.02[24.44,97.38]	<b>169.13</b> [98.24,249.92]	<b>0.30</b> [0.26,0.45]	<b>1.00</b> [0.56,3.07]	<b>4.07</b> [2.83,5.18]	<b>0.01</b> [0.01,0.01]
DETA	<b>0.04</b> [0.03,0.07]	<b>27.22</b> [5.48,86.73]	<b>4.50</b> [2.00,124.09]	48.89[24.61,72.21]	237.45[188.32,264.33]	9.51[8.38,10.79]	<b>2.05</b> [1.72,2.43]	<b>3.60</b> [2.86,5.24]	0.36[0.27,0.44]

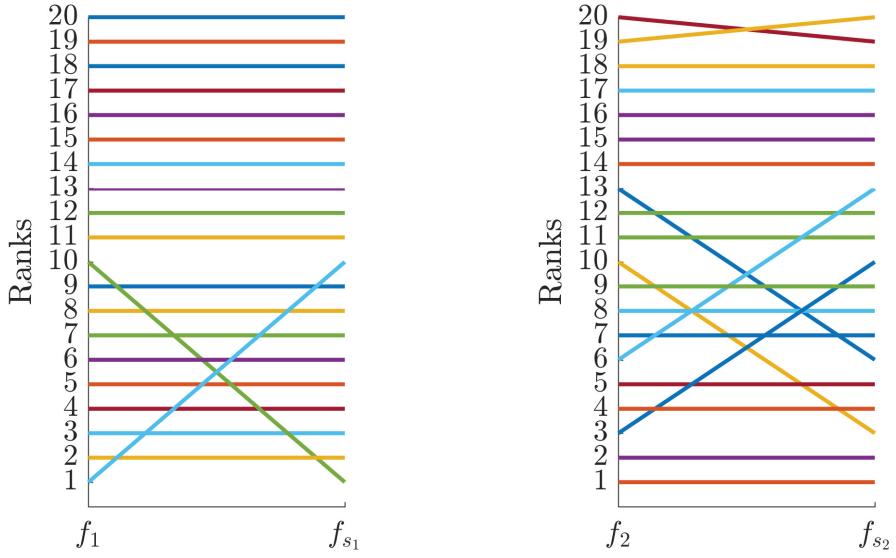
Table 4–5: IGD median and interquartile interval (in brackets) obtained by SBO using different performance metrics for model selection

Metric	Number of times among the top performing metrics
RMSE	17
MAE	17
MAPE	19
MAX	17
SR	21
KT	21
NSR	24
NKT	24
AKT	22
NWD	19
DEPI	20
DETA	16

Table 4–6: Number of scenarios in which each metric was among the top performing metrics (out of 27 scenarios)

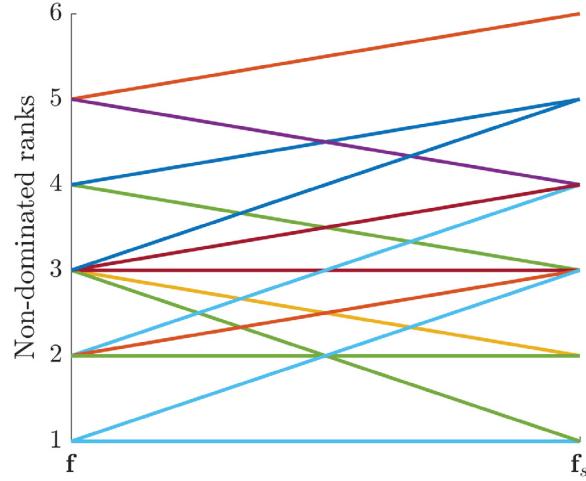
These results corroborate the proposed notion that a performance metric for surrogate models in SBO must be linked with the goal of the optimization process. The error-based metrics which share no correspondence with optimization presented the worst results. When more weight is given to errors among the high-quality solutions, the optimization performance experiences some improvement. Nevertheless, reliance on the sampling method to indicate which are the high-quality solutions can be misleading. Although objective-wise rank-correlation metrics have been shown to be more reliable than the error-based metrics, sometimes they seem to underestimate the effect of the objective-wise discrepancies on the dominance structure. Hence, rank-correlation metrics based on non-dominated ranks presented the best overall results.

Figure 4–3 illustrates a weakness of the objective-wise rank-correlation metrics. Figures 4–3a and 4–3b depict the ranks imposed by two objective functions,  $f_1(\cdot)$  and  $f_2(\cdot)$ , and their corresponding surrogate models,  $f_{s_1}(\cdot)$  and  $f_{s_2}(\cdot)$ . The surrogate models are artificially created such that there is one rank swap for  $f_1$  and three swaps for  $f_2$ . As can be seen, the created surrogate models are fairly good in maintaining the original order. However, as shown in Figure 4–3c, the non-dominated ranks imposed by original and surrogate problems get substantially distorted. This indicates that, for some problems, relatively good objective-wise surrogate models may still generate poor surrogate problems.



(a) Ranks imposed by  $f_1$  and its surrogate,  $f_{s_1}$

(b) Ranks imposed by  $f_2$  and its surrogate,  $f_{s_2}$



(c) Non-dominated ranks imposed by  $\mathbf{f} = [f_1, f_2]$  and  $\mathbf{f}_s = [f_{s_1}, f_{s_2}]$

Figure 4-3: Evolution of rank disruption

As reference, Tables 4–7 and 4–8 show the best combinations of surrogate models selected by NSR and NKT, respectively, and their corresponding IGD. These results indicate, that successful surrogate problems have a NSR of approximately 0.2 or less, or a NKT of approximately 0.25 or less.

Problem	$f_{s_1}$	$f_{s_2}$	$f_{s_3}$	Surrogate List	IGD	NSR
zdt2	2	1	-	$\mathcal{L}_{GRNN}$	0.000000	0.083832
zdt3	1	1	-	$\mathcal{L}_{GRNN}$	0.000025	0.157936
zdt4	2	1	-	$\mathcal{L}_{RBFNN}$	0.000000	0.222490
zdt6	3	1	-	$\mathcal{L}_{RBFNN}$	0.000002	0.219328
dtlz1	4	3	2	$\mathcal{L}_{RBFNN}$	0.025578	0.352105
dtlz2	5	3	5	$\mathcal{L}_{GRNN+RBFNN}$	0.000120	0.202087
dtlz3	1	4	1	$\mathcal{L}_{RBFNN}$	0.300600	0.346668
dtlz4	5	2	3	$\mathcal{L}_{GRNN+RBFNN}$	0.000347	0.192737
dtlz5	5	5	5	$\mathcal{L}_{GRNN+RBFNN}$	0.000002	0.159733

Table 4–7: IGD of the best surrogate models combinations selected through NSR

Problem	$f_{s_1}$	$f_{s_2}$	$f_{s_3}$	Surrogate List	IGD	NKT
zdt2	2	1	-	$\mathcal{L}_{GRNN}$	0.000000	0.102268
zdt3	2	1	-	$\mathcal{L}_{GRNN}$	0.000027	0.230552
zdt4	2	4	-	$\mathcal{L}_{GRNN}$	0.001000	0.281398
zdt6	3	2	-	$\mathcal{L}_{RBFNN}$	0.000005	0.263111
dtlz1	2	4	3	$\mathcal{L}_{RBFNN}$	0.039868	0.409833
dtlz2	7	5	5	$\mathcal{L}_{GRNN+RBFNN}$	0.000094	0.276306
dtlz3	2	4	1	$\mathcal{L}_{RBFNN}$	0.235536	0.370052
dtlz4	5	3	1	$\mathcal{L}_{GRNN+RBFNN}$	0.000345	0.228043
dtlz5	1	1	1	$\mathcal{L}_{RBFNN}$	0.000003	0.202595

Table 4–8: IGD of the best surrogate models combinations selected through NKT

### 4.3 Performance metrics for reduced problem selection

#### 4.3.1 Objective

The goal of this experiment is to evaluate the different performance metrics in the context of objective reduction with a focus on their ability to identify essential objective sets.

#### 4.3.2 Methodology

In this experiment,  $wfg3$ ,  $dtlz5$ ,  $p1$ ,  $p2$  and  $dtlz2$  are used as test problems. See Appendix A for definitions.

The  $wfg3_{(m,2)}$  is an  $m$ -objective problem with 2 essential objectives. Except for  $f_m$ , the other objectives are **not conflicting** with each other. Thus, the essential objective set is given by  $\mathcal{F}_e = \{f_m, f_l\}$  where  $l \in \{1, 2, \dots, m-1\}$ .

The  $dtlz5_{(m,i)}$  is an  $m$ -objective problem with  $i$  essential objectives. The first  $m - i + 1$  objectives are mutually non-conflicting and the essential objective set is given by  $\mathcal{F}_e = \{f_l, f_{m-i+2}, \dots, f_m\}$ , where  $l \in \{1, 2, \dots, m - i + 1\}$ .

$p1_{(3,2)}$  and  $p1_{(6,2)}$  are used to test the ability of the algorithms in detecting redundancy in combinations of objectives. Both problems have 2 essential objectives and the remaining objectives are linear and non-linear combinations of the essential ones. The compound objectives are crafted in a way that makes them always redundant.

Finally,  $dtlz2_{(m,m)}$  is an  $m$ -objective problem in which all the objectives are essential.

Objective-wise metrics are not applicable to objective reduction problems and due to their poor performance in the previous experiment when compared to the other proposed metrics, the NWD and the AKT were also removed from this experiment. Thus, in addition to the  $\kappa$ -discrepancy (DKAP), given by equation 3.22, only DEPI, DETA, NSR and NKT are used. For each problem, 30 Latin-hypercubes of 300 to 500 samples were independently generated. Given a sample set,  $A$ , and the original set of objectives,  $\mathcal{F}$ , the best reduced objective set,  $\mathcal{F}_s$ , is found by solving the optimization problem given in Eq.4.1. Broadly speaking, we are trying to find the reduced problem set of minimum cardinality such that the discrepancy with respect to the full problem is 0. Exhaustive search is used to find the solution of the referred optimization problem.

$$\begin{aligned} & \text{minimize} && |\mathcal{F}_s| \\ & \text{subject to:} && p(\mathcal{F}_s, A) = 0 \\ & && \mathcal{F}_s \subset \mathcal{F} \end{aligned} \tag{4.1}$$

#### 4.3.3 Results and discussion

Table 4–9 shows the median, the minimum and the maximum cardinality of the reduced problem set found by each one of the different performance metrics. It can be seen that the objective-wise rank-correlation based metric, DKAP, was the worst performing metric. It was unable to find reduced objective sets for most of the problems. As expected from the

theoretical analysis, since they are not strictly based on approximations of the true Pareto-front, when compared to other methods, NSR and NKT were able to find more parsimonious sets of objectives with less variation in the majority of the problems. The major advantage of NKT over NSR is the easiness of interpretation. For the identified reduced sets, we know that the probability that a pair-wise dominance relationship is reversed is 0. On the other hand, NSR has shown itself to be able to identify more parsimonious objective sets when compared to NKT. However, the lack of a direct interpretation for NSR makes it hard to determine the reason for why this has happened.

Problem	Metrics				
	DEPI	DETA	NSR	NKT	DKAP
$wfg3_{(15,2)}$	4 <sub>[4,5]</sub>	4 <sub>[4,4]</sub>	4 <sub>[4,5]</sub>	4 <sub>[4,5]</sub>	8 <sub>[8,8]</sub>
$wfg3_{(20,2)}$	4 <sub>[4,4]</sub>	4 <sub>[4,4]</sub>	4 <sub>[4,4]</sub>	4 <sub>[4,4]</sub>	9 <sub>[9,9]</sub>
$wfg3_{(30,2)}$	4 <sub>[4,5]</sub>	4 <sub>[4,5]</sub>	4 <sub>[4,5]</sub>	4 <sub>[4,5]</sub>	9 <sub>[9,9]</sub>
$dtlz5_{(20,7)}$	12 <sub>[8,14]</sub>	10 <sub>[8,12]</sub>	8 <sub>[7,10]</sub>	10 <sub>[8,11]</sub>	20 <sub>[20,20]</sub>
$dtlz5_{(20,5)}$	10 <sub>[8,12]</sub>	9 <sub>[8,10]</sub>	8 <sub>[7,9]</sub>	9 <sub>[7,10]</sub>	20 <sub>[20,20]</sub>
$dtlz5_{(15,7)}$	10 <sub>[8,13]</sub>	9 <sub>[7,11]</sub>	8 <sub>[7,9]</sub>	8 <sub>[7,11]</sub>	15 <sub>[15,15]</sub>
$dtlz5_{(15,5)}$	10 <sub>[7,11]</sub>	9 <sub>[7,12]</sub>	7 <sub>[6,8]</sub>	8 <sub>[6,9]</sub>	15 <sub>[15,15]</sub>
$p1_{(3,2)}$	2 <sub>[2,2]</sub>	2 <sub>[2,2]</sub>	2 <sub>[2,2]</sub>	2 <sub>[2,2]</sub>	3 <sub>[3,3]</sub>
$p2_{(6,2)}$	2 <sub>[2,2]</sub>	2 <sub>[2,2]</sub>	2 <sub>[2,2]</sub>	2 <sub>[2,2]</sub>	6 <sub>[6,6]</sub>
$dtlz2_{(15,15)}$	15 <sub>[15,15]</sub>				

Table 4–9: Median, minimum and maximum cardinality of the reduced problem sets found by each performance metric over 30 independent runs

These results conclude the performance metrics empirical analysis. It could be seen that the proposed metrics, NSR and NKT, not only generalize the evaluation of surrogate problems but also lead to better optimization results and more parsimonious reduced problems when compared to other metrics. In the next chapter, this thesis enters the realm of realistic engineering applications in which some of the investigated performance metrics are used to evaluate surrogate problems related to the design of electrical motors.

## CHAPTER 5

### Computational Experiments with Electrical Motor Design Problems

In the previous chapters, a comprehensive theoretical and empirical analysis of performance metrics for surrogate problems has been presented. The results obtained so far, have allowed us to assess weaknesses and strengths of each one of them in the context of single and, mainly, multi-objective optimization. The main goal of this whole exercise is to provide a methodology for the selection of surrogate problems in challenging engineering problems and this is what this chapter is intended to achieve, at least partially.

More specifically, in this chapter, a number of surrogate problems, based on both objective reduction and surrogate models, is built and evaluated for three, recently published, electrical motor design problems (Salimi, 2017; Ghorbanian et al., 2017; Silva et al., 2017b).

**Three questions are asked for each problem:**

1. Is the presented formulation adequate? In other words, **does the proposed formulation contain essential objectives only?**
2. **Can the most used surrogate models properly represent the given objective functions?**
3. **Can the best surrogate models of each objective together produce an adequate surrogate problem?**

The methodology used to address these questions is described in Section 5.1. In Section 5.2, the IPM motor design problem defined by Salimi (2017) is presented and its respective surrogate problems analyzed. The same is done for the integrated IPM motor-drive design problem defined by Ghorbanian et al. (2017) in Section 5.3, and for the Fractional Slot Concentrated Winding Surface Mounted Permanent Magnet FSCW-SMPM motor design problem defined by Silva et al. (2017b) in Section 5.4.

## 5.1 Methodology

The evaluation of surrogate problems requires data. For the problems investigated in this section, 1000 samples randomly generated within the feasible region of each problem and evaluated using the adequate simulation method were provided by the respective authors. These samples make up the database,  $\mathbf{D} = [\mathbf{X}, \mathbf{Y}]$ .

To address the first question, regarding reduced formulations for defined problems the following procedure is used:

1. Generate all the possible combinations of objectives;
2. Evaluate each formulation with the non-dominated rank correlation metrics, NSR and NKT, and the database  $\mathbf{D}$ ;
3. Extract the Pareto optimal reduced problems;
4. In order to assess the sensitivity of the estimated performance with respect to the sample data<sup>1</sup> :
  - 4.1. Generate a 100 new databases  $\mathbf{D}'_k, k = \{1, \dots, 100\}$  by randomly selecting  $n'$  samples from  $\mathbf{D}$ , where  $n'$  is randomly chosen from [500, 1000];
  - 4.2. Re-evaluate each Pareto optimal formulation with each  $\mathbf{D}'_k$ .

To address the second question, regarding the suitability of commonly used surrogate models for each objective, the following procedure is used.

1. Generate a 100 databases  $\mathbf{D}'_k, k = \{1, \dots, 100\}$  by randomly selecting  $n'$  samples from  $\mathbf{D}$ , where  $n'$  is randomly chosen from [300, 500];
2. For each objective function, evaluate each surrogate model using each one of the databases  $\mathbf{D}'_k$  and 10-fold cross-validation;

Notice that number of samples used for the generation and evaluation of the surrogate models was reduced in order to correspond to the number of samples generally used in the

---

<sup>1</sup> For more details on the sub-sampling technique used to estimate the uncertainty in the reduced problems evaluation, see (Pearson, 2011).

literature in this context. Again, the correlation metrics are transformed into conflict (see Eq. 3.23) which bound them in the interval  $[0, 1]$  where 1 stands for maximum conflict (total inversion) and 0 for complete rank maintenance.

The selected surrogate models are listed in Table 5–1. Moreover, for this part of the experiment, each surrogate model is evaluated by all the presented objective-wise performance metrics.

Surrogate Model	Symbol
1 <sup>st</sup> -order polynomial	P1
2 <sup>nd</sup> -order polynomial	P2
3 <sup>rd</sup> -order polynomial	P3
GRNN ( $\sigma = 0.1$ )	GRNN0.1
GRNN ( $\sigma = 1$ )	GRNN1
GRNN ( $\sigma = 3$ )	GRNN3
GRNN ( $\sigma = 6$ )	GRNN6
Kriging	Kriging
RBFNN ( $\sigma = 0.1$ )	RBFNN0.1
RBFNN ( $\sigma = 1$ )	RBFNN1
RBFNN ( $\sigma = 3$ )	RBFNN3
RBFNN ( $\sigma = 6$ )	RBFNN6

Table 5–1: Selected surrogate models for the machine design problem

To tackle the third question, the best surrogate models of each objective function are combined and evaluated by NKT and NSR. Again, 10-fold cross-validation over all  $\mathbf{D}'_k$  sample sets is used to assess their suitability for the optimization of the essential objective set.

## 5.2 Interior permanent magnet machine

The first motor design problem refers to a 4 pole 24 slots IPM Motor. The version of this problem used here can be found in (Salimi, 2017) and consists of 10 design variables and 5 objectives. The objective functions evaluation through 2D FEA takes on average 10 seconds on a 8-core Intel Xeon E5 1650 CPU (3.5 GHz) with 32 GB RAM. Although this is probably too fast to justify the use of surrogates, this model is well known in the literature (see (Li

et al., 2014, 2015; Salimi and Lowther, 2016; Silva et al., 2016a,b) and (Salimi, 2017)) and allows us to quickly perform experiments on a relatively realistic design problem.

Table 5–2 lists the objective functions.  $T_{avg}$ ,  $T_{str}$ ,  $T_{rip}$  and  $\eta$  are computed with help of the FEM package MagNet®. The  $PM_{vol}$  can be directly computed from the design variables and therefore does not require a surrogate model.

Objectives	Symbol
Average Torque ( $N.m$ )	$T_{avg}$
Starting Torque ( $N.m$ )	$T_{str}$
Torque Ripple (%)	$T_{rip}$
Efficiency (%)	$\eta$
Permanent Magnet Volume ( $m^3$ )	$PM_{vol}$

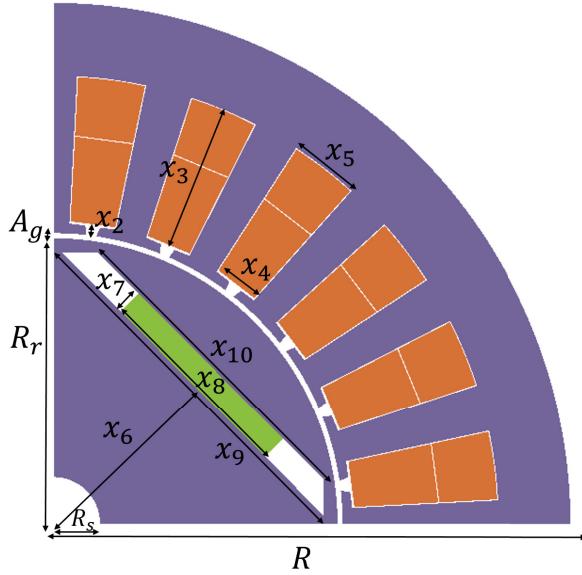
Table 5–2: IPM motor design objectives

Figure 5–1 presents the model cross-section, design variables and fixed parameters. The motor is fed with a balanced three-phase 100Hz current and the current density was kept fixed at  $0.15279A/mm^2$ .

### 5.2.1 Objective reduction

Figure 5–2 presents the Pareto optimal reduced problems and Pareto fronts for the two selected metrics. The small variation observed in the evaluation of the points in the Pareto front indicates that the number of samples was adequate. It can also be seen that the different metrics led to the exact same solution and both point out that all the objectives in this problem are essential.

It is interesting to notice that 2-objective versions of this problem are very popular in the literature (Salimi and Lowther, 2016; Li et al., 2015, 2016; Silva et al., 2017a). In all the cited articles the selected objectives were  $T_{avg}$  and  $T_{rip}$ . The results obtained here, on the other hand, indicate that the most meaningful 2-objective formulation for this problem consists of  $T_{str}$  and  $PM_{vol}$ . Thus, in the future, researchers should reconsider the 2-objective formulation of this problem.



Design Variable	Description	Lower bound	Upper bound
$x_1 = \frac{R_r}{R-A_g}$	Ratio between rotor radius and outer stator radius	0.4	0.7
$x_2$	Winding Slot Depth	0.5 mm	15 mm
$x_3$	Winding Slot Length	5 mm	22 mm
$x_4$	Winding Slot Lower Angle	2°	13°
$x_5$	Winding Slot Upper Angle	2°	13°
$x_6$	PM Depth	10 mm	30 mm
$x_7$	PM Thickness	1 mm	22 mm
$x_8$	PM Width	1 mm	32 mm
$x_9$	PM Slot Lower Width	4 mm	40 mm
$x_{10}$	PM Slot Upper Width	4 mm	40 mm
Fixed Parameter	Description	Value	
$R$	Stator Outer Radius	51 mm	
$R_s$	Shaft Outer Radius	4.5 mm	
$A_g$	Air-gap thickness	0.5 mm	
$L$	Stack Length	203.4 mm	
$S_s$	Stator Steel	M-19 29 Ga	
$R_s$	Rotor Steel	M-19 29 Ga	
$PM$	Permanent Magnet	Samarium Cobalt	
$S_w$	Stator Winding	Copper 100% IACS	
$N$	Number of Turns	10	
$F_f$	Fill Factor	43.07%	

Figure 5–1: IPM motor cross-section, design variables and fixed parameters (Adapted from: (Salimi, 2017))

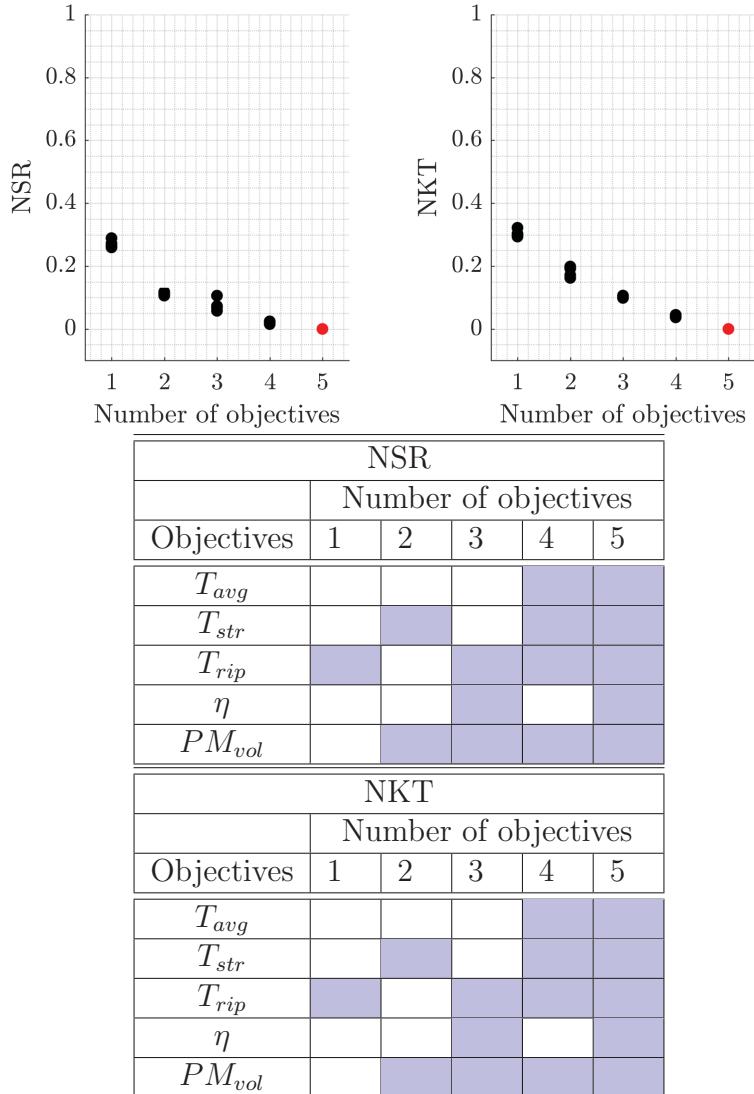


Figure 5–2: Scatter plots of the Pareto optimal IPM reduced problems and the table representation of the Pareto optimal reduced formulations for NSR and NKT. **Highlighted squares indicate the presence and white squares the absence of a given objective function in a given formulation.**

### 5.2.2 Surrogate model selection

In this section the surrogate models listed in Section 5.1 are evaluated for each one of the objectives. As mentioned before, no surrogate model is required for  $PM_{vol}$ .

Tables 5–3 to 5–6 present the median and the interquartile interval of the surrogate models performances for  $T_{avg}$ ,  $T_{str}$ ,  $T_{rip}$  and  $\eta$ , respectively. The figures in boldface indicate the best performances. Among the figures in boldface there is no statistical difference. The figures which are not in boldface can be considered statistically worse than the best performing ones for a confidence level  $\alpha = 95\%$ . As before, Kruskal Wallis and Dunn’s test were used to verify whether the performances were significantly different.

The results show that the second-order polynomial models were the only type of surrogate model among the top-performing for the four objective functions under all performance metrics. Notice, however, that in Table 5–5, all the tested surrogate models had difficulty concerning rank maintenance for the torque ripple. The fact that the second-order polynomials were still the best surrogate model overall may indicate that  $T_{rip}$  is probably multimodal but presents a globally quadratic trend. Possibly, for a better representation of this objective, a higher number of samples would be required. In this scenario, Kriging and the neural networks may outperform the second order polynomials.

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	<b>0.10</b> <sub>[0.10,0.10]</sub>	<b>0.19</b> <sub>[0.19,0.20]</sub>	<b>0.06</b> <sub>[0.06,0.06]</sub>	<b>0.04</b> <sub>[0.04,0.05]</sub>	<b>0.44</b> <sub>[0.43,0.46]</sub>	0.16 <sub>[0.15,0.17]</sub>
P2	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>0.14</b> <sub>[0.13,0.14]</sub>	<b>0.04</b> <sub>[0.04,0.05]</sub>	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.32</b> <sub>[0.28,0.34]</sub>	<b>0.12</b> <sub>[0.10,0.13]</sub>
P3	<b>0.10</b> <sub>[0.08,0.14]</sub>	<b>0.18</b> <sub>[0.17,0.23]</sub>	<b>0.05</b> <sub>[0.05,0.07]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>0.50</b> <sub>[0.35,0.61]</sub>	<b>0.10</b> <sub>[0.08,0.11]</sub>
Kriging	<b>0.09</b> <sub>[0.08,0.10]</sub>	<b>0.18</b> <sub>[0.17,0.19]</sub>	<b>0.06</b> <sub>[0.05,0.06]</sub>	<b>0.04</b> <sub>[0.03,0.04]</sub>	<b>0.43</b> <sub>[0.36,0.52]</sub>	<b>0.13</b> <sub>[0.12,0.15]</sub>
GRNN0.1	0.24 <sub>[0.22,0.26]</sub>	0.31 <sub>[0.30,0.33]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.07 <sub>[0.06,0.07]</sub>	0.62 <sub>[0.51,0.73]</sub>	22.00 <sub>[19.44,24.90]</sub>
GRNN1	0.24 <sub>[0.23,0.25]</sub>	0.32 <sub>[0.31,0.32]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.08 <sub>[0.08,0.08]</sub>	0.84 <sub>[0.82,0.85]</sub>	24.22 <sub>[22.73,25.40]</sub>
GRNN3	0.25 <sub>[0.23,0.27]</sub>	0.32 <sub>[0.32,0.34]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.08 <sub>[0.08,0.08]</sub>	0.87 <sub>[0.87,0.88]</sub>	23.86 <sub>[22.80,24.62]</sub>
GRNN6	0.25 <sub>[0.24,0.27]</sub>	0.32 <sub>[0.32,0.34]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.08 <sub>[0.08,0.08]</sub>	0.89 <sub>[0.88,0.89]</sub>	23.97 <sub>[22.06,24.67]</sub>
RBFNN0.1	0.33 <sub>[0.30,0.36]</sub>	0.41 <sub>[0.40,0.54]</sub>	0.19 <sub>[0.13,0.38]</sub>	0.17 <sub>[0.07,0.33]</sub>	1.07 <sub>[0.61,1.64]</sub>	1.01 <sub>[0.57,4.45]</sub>
RBFNN1	0.15 <sub>[0.10,0.25]</sub>	0.25 <sub>[0.20,0.30]</sub>	0.13 <sub>[0.07,0.22]</sub>	0.08 <sub>[0.05,0.10]</sub>	0.84 <sub>[0.54,1.22]</sub>	<b>0.13</b> <sub>[0.11,0.22]</sub>
RBFNN3	0.15 <sub>[0.11,0.27]</sub>	0.21 <sub>[0.19,0.26]</sub>	0.08 <sub>[0.07,0.18]</sub>	0.06 <sub>[0.05,0.08]</sub>	0.62 <sub>[0.46,1.20]</sub>	<b>0.10</b> <sub>[0.08,0.12]</sub>
RBFNN6	0.16 <sub>[0.12,0.30]</sub>	0.25 <sub>[0.22,0.30]</sub>	0.10 <sub>[0.08,0.15]</sub>	0.09 <sub>[0.07,0.16]</sub>	1.19 <sub>[0.68,2.23]</sub>	<b>0.10</b> <sub>[0.07,0.12]</sub>

Table 5–3: Surrogate model performance for the IPM Average Torque  $T_{avg}$

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	<b>0.10</b> <sub>[0.10,0.10]</sub>	<b>0.20</b> <sub>[0.20,0.20]</sub>	<b>0.08</b> <sub>[0.08,0.08]</sub>	<b>0.06</b> <sub>[0.06,0.06]</sub>	<b>0.48</b> <sub>[0.47,0.50]</sub>	0.19 <sub>[0.18,0.22]</sub>
P2	<b>0.07</b> <sub>[0.06,0.09]</sub>	<b>0.15</b> <sub>[0.15,0.16]</sub>	<b>0.06</b> <sub>[0.06,0.07]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.39</b> <sub>[0.35,0.48]</sub>	<b>0.12</b> <sub>[0.10,0.14]</sub>
P3	<b>0.13</b> <sub>[0.08,0.16]</sub>	<b>0.20</b> <sub>[0.18,0.26]</sub>	0.13 <sub>[0.10,0.20]</sub>	0.09 <sub>[0.08,0.14]</sub>	1.11 <sub>[0.82,2.01]</sub>	<b>0.10</b> <sub>[0.07,0.11]</sub>
Kriging	<b>0.08</b> <sub>[0.07,0.11]</sub>	<b>0.17</b> <sub>[0.15,0.23]</sub>	<b>0.07</b> <sub>[0.06,0.08]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>0.41</b> <sub>[0.37,0.44]</sub>	<b>0.17</b> <sub>[0.14,0.20]</sub>
GRNN0.1	0.22 <sub>[0.21,0.24]</sub>	0.30 <sub>[0.28,0.33]</sub>	0.11 <sub>[0.10,0.11]</sub>	0.09 <sub>[0.08,0.09]</sub>	<b>0.65</b> <sub>[0.60,0.67]</sub>	21.78 <sub>[18.09,24.28]</sub>
GRNN1	0.23 <sub>[0.23,0.25]</sub>	0.32 <sub>[0.31,0.33]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.74 <sub>[0.73,0.75]</sub>	25.05 <sub>[24.53,25.64]</sub>
GRNN3	0.26 <sub>[0.25,0.26]</sub>	0.33 <sub>[0.33,0.34]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.77 <sub>[0.76,0.77]</sub>	26.37 <sub>[25.49,26.88]</sub>
GRNN6	0.27 <sub>[0.25,0.27]</sub>	0.34 <sub>[0.33,0.35]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.10 <sub>[0.10,0.11]</sub>	0.78 <sub>[0.78,0.79]</sub>	26.54 <sub>[25.80,27.53]</sub>
RBFNN0.1	0.37 <sub>[0.34,0.42]</sub>	0.42 <sub>[0.40,0.53]</sub>	0.41 <sub>[0.20,0.81]</sub>	0.33 <sub>[0.11,0.64]</sub>	0.95 <sub>[0.71,1.79]</sub>	3.28 <sub>[0.64,28.46]</sub>
RBFNN1	0.21 <sub>[0.15,0.28]</sub>	0.27 <sub>[0.25,0.33]</sub>	0.20 <sub>[0.12,0.46]</sub>	0.09 <sub>[0.06,0.18]</sub>	0.61 <sub>[0.48,1.15]</sub>	<b>0.15</b> <sub>[0.13,0.25]</sub>
RBFNN3	0.17 <sub>[0.12,0.25]</sub>	0.27 <sub>[0.23,0.34]</sub>	0.12 <sub>[0.11,0.20]</sub>	0.10 <sub>[0.08,0.16]</sub>	1.14 <sub>[0.78,2.29]</sub>	<b>0.11</b> <sub>[0.08,0.15]</sub>
RBFNN6	0.18 <sub>[0.16,0.21]</sub>	0.27 <sub>[0.25,0.31]</sub>	0.18 <sub>[0.14,0.25]</sub>	0.14 <sub>[0.11,0.20]</sub>	1.52 <sub>[1.15,2.04]</sub>	<b>0.12</b> <sub>[0.09,0.15]</sub>

Table 5–4: Surrogate model performance for the IPM Starting Torque  $T_{str}$

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	<b>0.24</b> <sub>[0.24,0.25]</sub>	<b>0.33</b> <sub>[0.32,0.34]</sub>	<b>0.07</b> <sub>[0.07,0.07]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.80</b> <sub>[0.79,0.81]</sub>	0.51 <sub>[0.42,9.39]</sub>
P2	<b>0.26</b> <sub>[0.25,0.29]</sub>	<b>0.34</b> <sub>[0.32,0.34]</sub>	<b>0.07</b> <sub>[0.07,0.08]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>0.77</b> <sub>[0.74,0.80]</sub>	<b>0.33</b> <sub>[0.28,0.36]</sub>
P3	0.38 <sub>[0.34,0.41]</sub>	0.41 <sub>[0.40,0.43]</sub>	0.14 <sub>[0.11,0.16]</sub>	0.09 <sub>[0.08,0.14]</sub>	1.66 <sub>[0.94,2.96]</sub>	<b>0.12</b> <sub>[0.10,0.13]</sub>
Kriging	0.36 <sub>[0.29,0.39]</sub>	0.38 <sub>[0.36,0.40]</sub>	<b>0.07</b> <sub>[0.07,0.07]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.86</b> <sub>[0.77,0.87]</sub>	10.80 <sub>[0.88,15.92]</sub>
GRNN0.1	0.31 <sub>[0.28,0.32]</sub>	<b>0.36</b> <sub>[0.35,0.39]</sub>	<b>0.07</b> <sub>[0.07,0.07]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.85</b> <sub>[0.79,0.87]</sub>	14.50 <sub>[13.02,15.71]</sub>
GRNN1	<b>0.30</b> <sub>[0.29,0.31]</sub>	0.37 <sub>[0.36,0.37]</sub>	0.07 <sub>[0.07,0.07]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	0.86 <sub>[0.86,0.87]</sub>	15.96 <sub>[15.53,16.67]</sub>
GRNN3	0.32 <sub>[0.31,0.33]</sub>	0.37 <sub>[0.37,0.38]</sub>	0.07 <sub>[0.07,0.07]</sub>	0.06 <sub>[0.06,0.06]</sub>	0.86 <sub>[0.86,0.87]</sub>	16.00 <sub>[15.63,16.37]</sub>
GRNN6	0.32 <sub>[0.31,0.33]</sub>	0.38 <sub>[0.37,0.39]</sub>	0.08 <sub>[0.07,0.08]</sub>	0.06 <sub>[0.06,0.06]</sub>	0.86 <sub>[0.86,0.87]</sub>	16.11 <sub>[15.85,16.32]</sub>
RBFNN0.1	0.42 <sub>[0.39,0.54]</sub>	0.48 <sub>[0.44,0.52]</sub>	0.34 <sub>[0.11,1.29]</sub>	0.27 <sub>[0.13,0.50]</sub>	0.95 <sub>[0.76,1.75]</sub>	3.18 <sub>[1.06,11.38]</sub>
RBFNN1	0.39 <sub>[0.34,0.44]</sub>	0.42 <sub>[0.39,0.46]</sub>	0.16 <sub>[0.10,0.26]</sub>	0.14 <sub>[0.09,0.24]</sub>	0.93 <sub>[0.80,1.40]</sub>	<b>0.20</b> <sub>[0.14,0.30]</sub>
RBFNN3	0.39 <sub>[0.37,0.41]</sub>	0.45 <sub>[0.43,0.48]</sub>	0.16 <sub>[0.14,0.25]</sub>	0.11 <sub>[0.10,0.17]</sub>	1.33 <sub>[0.98,2.33]</sub>	<b>0.17</b> <sub>[0.10,0.27]</sub>
RBFNN6	0.42 <sub>[0.40,0.45]</sub>	0.44 <sub>[0.42,0.44]</sub>	0.29 <sub>[0.23,0.35]</sub>	0.14 <sub>[0.12,0.20]</sub>	2.43 <sub>[1.56,3.67]</sub>	<b>0.14</b> <sub>[0.09,0.19]</sub>

Table 5–5: Surrogate model performance for the IPM Torque Ripple  $T_{rip}$

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	<b>0.11</b> <sub>[0.11,0.12]</sub>	<b>0.21</b> <sub>[0.20,0.21]</sub>	<b>0.10</b> <sub>[0.10,0.10]</sub>	<b>0.07</b> <sub>[0.07,0.07]</sub>	<b>0.60</b> <sub>[0.58,0.62]</sub>	66.46 <sub>[63.99,68.33]</sub>
P2	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>0.14</b> <sub>[0.13,0.16]</sub>	<b>0.07</b> <sub>[0.07,0.08]</sub>	<b>0.05</b> <sub>[0.04,0.05]</sub>	<b>0.55</b> <sub>[0.46,0.69]</sub>	<b>0.07</b> <sub>[0.06,27.64]</sub>
P3	<b>0.07</b> <sub>[0.06,0.09]</sub>	<b>0.17</b> <sub>[0.15,0.22]</sub>	0.10 <sub>[0.09,0.16]</sub>	<b>0.06</b> <sub>[0.05,0.08]</sub>	0.94 <sub>[0.76,1.48]</sub>	<b>0.07</b> <sub>[0.05,0.08]</sub>
Kriging	0.11 <sub>[0.10,0.35]</sub>	0.21 <sub>[0.18,0.35]</sub>	<b>0.09</b> <sub>[0.08,0.13]</sub>	0.07 <sub>[0.06,0.10]</sub>	<b>0.71</b> <sub>[0.58,0.78]</sub>	44.75 <sub>[35.92,57.63]</sub>
GRNN0.1	0.30 <sub>[0.29,0.32]</sub>	0.36 <sub>[0.35,0.38]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.80 <sub>[0.78,0.84]</sub>	61.60 <sub>[48.77,79.40]</sub>
GRNN1	0.34 <sub>[0.33,0.35]</sub>	0.39 <sub>[0.38,0.40]</sub>	0.13 <sub>[0.13,0.14]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.74 <sub>[0.74,0.75]</sub>	76.18 <sub>[73.84,77.47]</sub>
GRNN3	0.35 <sub>[0.35,0.36]</sub>	0.40 <sub>[0.39,0.41]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.11 <sub>[0.11,0.11]</sub>	<b>0.72</b> <sub>[0.72,0.73]</sub>	80.58 <sub>[79.44,82.46]</sub>
GRNN6	0.35 <sub>[0.35,0.36]</sub>	0.40 <sub>[0.40,0.41]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.11 <sub>[0.11,0.11]</sub>	<b>0.72</b> <sub>[0.72,0.73]</sub>	81.57 <sub>[80.73,82.15]</sub>
RBFNN0.1	0.41 <sub>[0.38,0.48]</sub>	0.47 <sub>[0.42,0.52]</sub>	0.48 <sub>[0.23,1.18]</sub>	0.35 <sub>[0.14,0.90]</sub>	1.42 <sub>[0.64,1.62]</sub>	44.81 <sub>[0.66,75.71]</sub>
RBFNN1	0.22 <sub>[0.17,0.34]</sub>	0.32 <sub>[0.26,0.40]</sub>	0.26 <sub>[0.15,0.42]</sub>	0.13 <sub>[0.10,0.30]</sub>	0.94 <sub>[0.69,2.07]</sub>	0.15 <sub>[0.12,42.04]</sub>
RBFNN3	0.15 <sub>[0.13,0.28]</sub>	0.25 <sub>[0.22,0.34]</sub>	0.17 <sub>[0.12,0.32]</sub>	0.09 <sub>[0.08,0.13]</sub>	0.98 <sub>[0.70,1.55]</sub>	<b>0.09</b> <sub>[0.07,0.10]</sub>
RBFNN6	0.19 <sub>[0.15,0.28]</sub>	0.28 <sub>[0.23,0.36]</sub>	0.19 <sub>[0.14,0.34]</sub>	0.11 <sub>[0.09,0.15]</sub>	1.07 <sub>[0.88,1.84]</sub>	<b>0.10</b> <sub>[0.06,0.13]</sub>

Table 5–6: Surrogate model performance for the IPM Efficiency  $\eta$

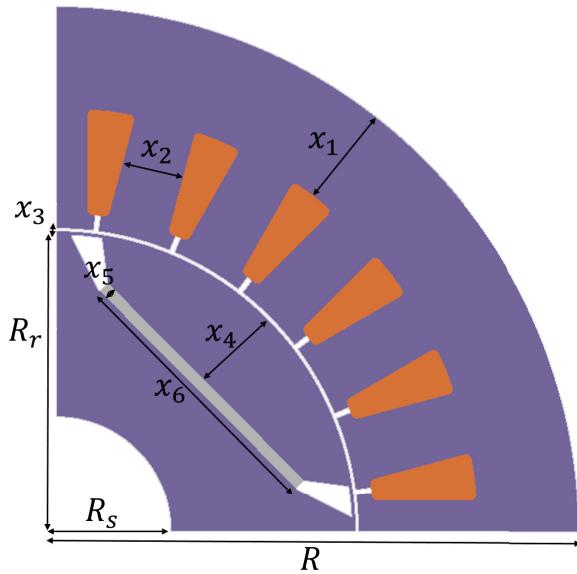
Table 5–7 presents the performance of the surrogate problem generated by the best surrogate models of each objective, according to the correlation metrics based on non-dominated ranking. Despite the difficulty in approximating  $T_{rip}$ , it can be seen that, for these metrics, the full surrogate problem is comparable to the high performing surrogate problems found for the benchmark functions (see Tables 4–7 and 4–8).

Objective Function	Surrogate Model
$T_{avg}$	P2
$T_{str}$	P2
$T_{rip}$	P2
$\eta$	P2
$PM_{vol}$	-
Performance	
NKT	$0.20_{[0.19,0.21]}$
NSR	$0.16_{[0.16,0.17]}$

Table 5–7: Median and interquartile interval of best surrogate models’ performance for the multi-objective IPM motor design problem

### 5.3 Integrated motor-drive design of an Interior Permanent Magnet Motor

In this section, the presented methodologies are used to evaluate surrogate problems related to integrated motor-drive design of an IPM motor. This problem was recently introduced by Ghorbanian et al. (2017) to address issues associated with the combination of motor and inverter performances and consists of 8 variables ( $x_1$  to  $x_6$  are related to the motor geometry,  $x_7$  and  $x_8$  are related to the inverter) and 12 objective functions summarized in Figure 5–3 and Table 5–8, respectively. The simulation model corresponds to a torque-oriented inverter-fed IPM motor which means that torque is fixed for all the candidate designs. Using the methodology described in (Ghorbanian et al., 2017) which uses a combination of Simulink® and MagNet®, each simulation takes on average 10 minutes on a 8-core Intel Xeon E5 1650 CPU (3.5 GHz) with 32 GB RAM.



Design Variable	Description	Lower bound	Upper bound
$x_1$	Stator Yoke Thickness	10 mm	30 mm
$x_2$	Stator Tooth Width	4 mm	12 mm
$x_3$	Air-gap thickness	0.2 mm	0.6 mm
$x_4$	Magnet Inset Depth	9.2 mm	28.5 mm
$x_5$	Magnet Thickness	2 mm	6 mm
$x_6$	Magnet Width	15 mm	45 mm
$x_7$	Switching frequency	4 kHz	16 kHz
$x_8$	DC bus voltage	300 V	500 V
Fixed Parameters	Description	Value	
$R$	Stator Outer Radius	91.45 mm	
$N$	Number of Turns	8	
$F$	Fill Factor	0.45	
$L$	Stack Length	201 mm	
$S_d$	Slot Depth	21.06 mm	
$T_{tt}$	Tooth Tip Thickness	2.77 mm	
$S_{ow}$	Slot Opening Width	0.69 rad	
$F_{tr}$	Top Fillet Radius	1.06 mm	
$F_{br}$	Bottom Fillet Radius	1.06 mm	
$T_{rated}$	Rated torque	40 N.m	

Figure 5–3: IPM motor cross-section, design variables and fixed parameters (Adapted from: (Ghorbanian et al., 2017))

Objectives	Symbol
Cogging Torque ( $N.m$ )	$T_{cog}$
Inverter Cost ( $US\$/kVA$ )	$Iv_{cost} = 20US\$/$ Apparent Power (S)
Inverter Efficiency (%)	$\eta_{Iv}$
Start-up Torque ( $N.m$ )	$T_{str}$
Motor Cost (Material cost) ( $US\$$ )	$M_{cost}$
Current Density ( $A/mm^2$ )	$J$
Motor Efficiency (%)	$\eta_{motor}$
Power Factor	$pf$
Torque Ripple (%)	$T_{rip}$
Speed Overshoot ( $rpm$ )	$n$
Rising Time ( $s$ )	$rt$
System Efficiency (%)	$\eta = \eta_{Iv} \times \eta_{motor}$

Table 5–8: Integrated IPM motor-drive design objectives

### 5.3.1 Objective reduction

Figure 5–4 presents the Pareto fronts and the respective Pareto optimal solution for the objective reduction problem defined over the NKT and NSR. Despite the small variation in the performance estimates, it is not possible to assert with certainty whether the essential objective set consists of 11 or 10 objectives. It is clear however that, within the 12 objectives,  $\eta$  is definitely a redundant objective. Even without any computational analysis tool, this should be clear by the definition of  $\eta$  in terms of  $\eta_{Iv}$  and  $\eta_{motor}$ . The results obtained by NKT and NSR were very similar, in particular, for the reduced problems with 5 objectives or more.

The results also show that this problem has 9-objective versions of very low discrepancy with respect to the original problem. Despite the low discrepancy, it is likely that some essential objectives are not present in those versions and they should only be used after specialist's analysis. For this reason, a more conservative reduction is be adopted which means that the 11-objective formulation will be considered ideal.

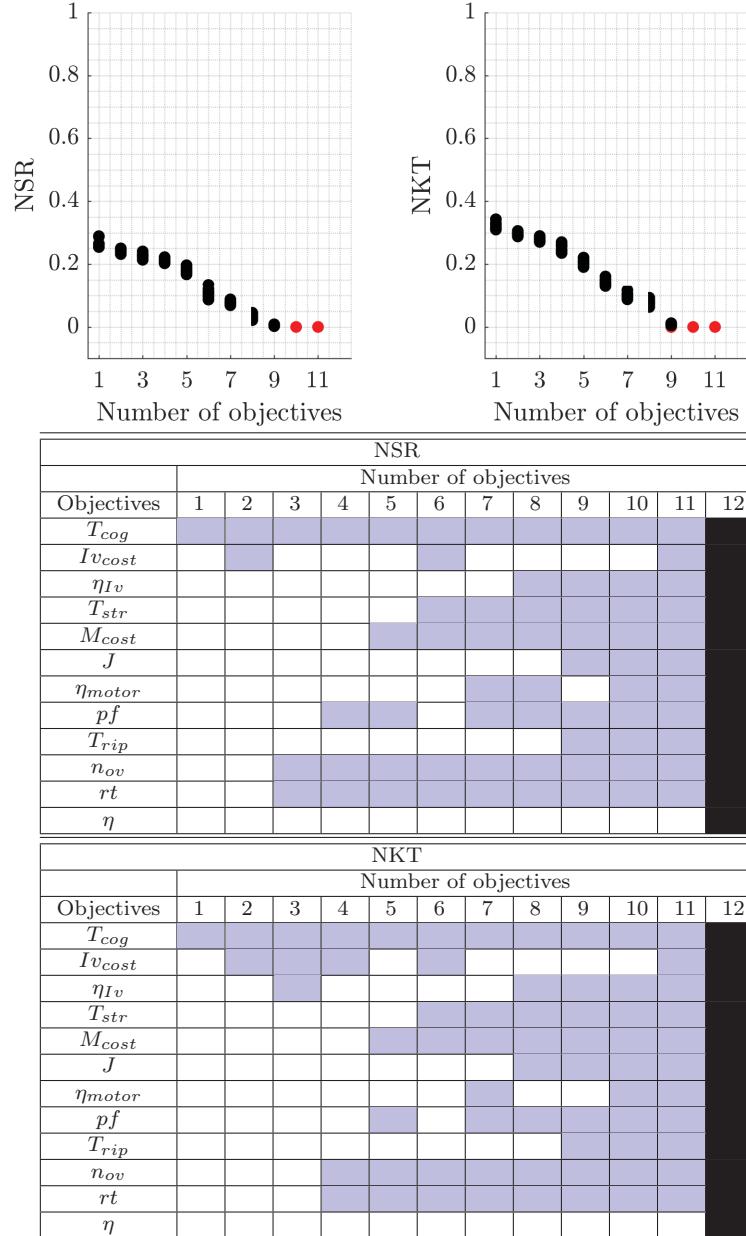


Figure 5–4: Scatter plots of the Pareto optimal Integrated IPM motor-drive reduced problems and the table representation of the Pareto optimal reduced formulations for NSR and NKT. Highlighted squares indicate the presence and white squares the absence of a given objective function in a given formulation. A black column indicates that a formulation with that number of objectives is not Pareto optimal.

### 5.3.2 Surrogate model selection

In this section, the surrogate models' performance for the integrated IPM motor-drive design problem is presented. For obvious reasons,  $M_{cost}$  does not require a surrogate model and hence was kept out of this experiment. Tables 5–9 to 5–18 present the surrogate models' estimated performance for the remaining essential objectives and, as in the previous experiment, the figures in boldface indicate the best performances.

Overall, second and third-order polynomials generated the best surrogate models. For some objectives, such as, inverter cost ( $Iv_{cost}$ ) in Table 5–9, inverter efficiency ( $\eta_{Iv}$ ) in Table 5–10, and starting torque in Table 5–11, the third-order polynomials presented lower (yet not significantly lower) values of mean average percentual error, MAPE, than the second-order ones. However, as any averaged measure, MAPE is very sensitive to outliers. Thus, one bad prediction may have a significant impact on the perceived performance. The fact that the higher MAPE values did not seem to imply higher rank disruptions combined with the Ockham's razor principle allow us to suggest the use of second-order polynomials even for these objectives.

Two exceptions were also observed, the cogging torque in Table 5–17), and the torque ripple in Table 5–18. For these objectives, Kriging was the superior surrogate model. Specifically for the torque ripple, it can be seen that despite Kriging's superiority, second-order polynomials still present a reasonable performance, in particular, concerning the rank maintenance measures given by SR and KT. Similarly to the previous IPM design problem, the torque ripple also seems to present a global quadratic landscape. The same cannot be said about the cogging torque for which the values of KT and SR presented by the second order polynomials were high (around 0.5). Even Kriging had difficulties approximating this objective. Compared with torque ripple where Kriging presented a median SR of 0.01 and a median KT of 0.07, its performance regarding cogging torque was considerably worse ( $SR \approx 0.24$  and  $KT \approx 0.30$ ). Possibly, more sample points are required to get a more faithful representation of this objective.

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.04 <sub>[0.04,0.04]</sub>	<b>0.13</b> <sub>[0.13,0.13]</sub>	0.07 <sub>[0.07,0.07]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.95 <sub>[0.93,0.96]</sub>	0.17 <sub>[0.16,0.18]</sub>
P2	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.09</b> <sub>[0.09,0.10]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>0.04</b> <sub>[0.03,0.04]</sub>	0.87 <sub>[0.85,0.89]</sub>	<b>0.13</b> <sub>[0.12,0.14]</sub>
P3	0.04 <sub>[0.03,0.08]</sub>	0.11 <sub>[0.10,0.16]</sub>	<b>0.06</b> <sub>[0.05,0.08]</sub>	<b>0.04</b> <sub>[0.03,0.04]</sub>	0.92 <sub>[0.88,0.95]</sub>	<b>0.08</b> <sub>[0.08,0.11]</sub>
Kriging	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>0.11</b> <sub>[0.10,0.12]</sub>	<b>0.06</b> <sub>[0.06,0.06]</sub>	<b>0.04</b> <sub>[0.03,0.04]</sub>	0.88 <sub>[0.87,0.90]</sub>	0.17 <sub>[0.15,0.19]</sub>
GRNN0.1	0.10 <sub>[0.08,0.11]</sub>	0.18 <sub>[0.17,0.21]</sub>	0.10 <sub>[0.09,0.10]</sub>	0.07 <sub>[0.06,0.07]</sub>	0.86 <sub>[0.84,0.88]</sub>	4.30 <sub>[3.54,5.34]</sub>
GRNN1	0.05 <sub>[0.04,0.06]</sub>	0.14 <sub>[0.13,0.14]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.09 <sub>[0.09,0.09]</sub>	<b>0.79</b> <sub>[0.78,0.79]</sub>	6.13 <sub>[5.87,6.42]</sub>
GRNN3	0.05 <sub>[0.05,0.07]</sub>	0.15 <sub>[0.14,0.15]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.11 <sub>[0.11,0.11]</sub>	<b>0.76</b> <sub>[0.75,0.77]</sub>	7.34 <sub>[7.08,7.57]</sub>
GRNN6	0.06 <sub>[0.05,0.07]</sub>	0.15 <sub>[0.14,0.17]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.11 <sub>[0.11,0.12]</sub>	<b>0.77</b> <sub>[0.77,0.78]</sub>	7.82 <sub>[7.67,8.00]</sub>
RBFNN0.1	0.20 <sub>[0.15,0.25]</sub>	0.29 <sub>[0.26,0.33]</sub>	0.17 <sub>[0.14,0.22]</sub>	0.13 <sub>[0.12,0.16]</sub>	0.81 <sub>[0.69,0.91]</sub>	4.12 <sub>[2.29,7.01]</sub>
RBFNN1	0.07 <sub>[0.05,0.13]</sub>	0.16 <sub>[0.14,0.18]</sub>	0.08 <sub>[0.07,0.09]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.88 <sub>[0.86,0.91]</sub>	0.22 <sub>[0.18,0.30]</sub>
RBFNN3	0.05 <sub>[0.05,0.07]</sub>	0.14 <sub>[0.13,0.15]</sub>	0.07 <sub>[0.07,0.08]</sub>	<b>0.05</b> <sub>[0.04,0.05]</sub>	0.92 <sub>[0.91,0.98]</sub>	<b>0.13</b> <sub>[0.11,0.15]</sub>
RBFNN6	0.09 <sub>[0.06,0.11]</sub>	0.18 <sub>[0.17,0.22]</sub>	0.10 <sub>[0.09,0.11]</sub>	0.06 <sub>[0.05,0.07]</sub>	1.04 <sub>[0.97,1.21]</sub>	<b>0.09</b> <sub>[0.08,0.11]</sub>

Table 5–9: Surrogate model performance for the IPM motor-drive Inverter Cost ( $Iv_{cost}$ )

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.07</b> <sub>[0.06,0.07]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.49 <sub>[0.49,0.50]</sub>	14.97 <sub>[14.59,15.24]</sub>
P2	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.06</b> <sub>[0.05,0.06]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.36</b> <sub>[0.34,0.37]</sub>	<b>7.38</b> <sub>[7.08,7.93]</sub>
P3	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.04,0.06]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.02</b> <sub>[0.01,0.02]</sub>	<b>0.28</b> <sub>[0.25,0.30]</sub>	<b>3.98</b> <sub>[3.01,4.84]</sub>
Kriging	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.04,0.05]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.32</b> <sub>[0.30,0.38]</sub>	<b>6.36</b> <sub>[5.39,7.39]</sub>
GRNN0.1	0.04 <sub>[0.03,0.05]</sub>	0.13 <sub>[0.12,0.15]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.56 <sub>[0.52,0.59]</sub>	13.71 <sub>[10.41,16.62]</sub>
GRNN1	0.03 <sub>[0.03,0.03]</sub>	0.10 <sub>[0.10,0.11]</sub>	0.09 <sub>[0.08,0.09]</sub>	0.06 <sub>[0.06,0.06]</sub>	0.64 <sub>[0.62,0.66]</sub>	19.85 <sub>[19.61,20.34]</sub>
GRNN3	0.04 <sub>[0.03,0.04]</sub>	0.12 <sub>[0.11,0.13]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.75 <sub>[0.74,0.75]</sub>	24.01 <sub>[23.81,24.19]</sub>
GRNN6	0.04 <sub>[0.03,0.04]</sub>	0.12 <sub>[0.11,0.12]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.78 <sub>[0.78,0.79]</sub>	25.35 <sub>[25.15,25.53]</sub>
RBFNN0.1	0.17 <sub>[0.10,0.30]</sub>	0.23 <sub>[0.20,0.31]</sub>	0.14 <sub>[0.13,0.18]</sub>	0.11 <sub>[0.09,0.13]</sub>	0.82 <sub>[0.75,0.87]</sub>	24.61 <sub>[22.73,27.12]</sub>
RBFNN1	0.02 <sub>[0.01,0.04]</sub>	0.09 <sub>[0.07,0.13]</sub>	0.05 <sub>[0.05,0.06]</sub>	0.03 <sub>[0.03,0.04]</sub>	0.60 <sub>[0.54,0.68]</sub>	16.07 <sub>[13.89,20.43]</sub>
RBFNN3	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.06</b> <sub>[0.05,0.07]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.37</b> <sub>[0.32,0.39]</sub>	<b>5.64</b> <sub>[4.12,6.41]</sub>
RBFNN6	0.02 <sub>[0.01,0.03]</sub>	0.08 <sub>[0.07,0.09]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	0.02 <sub>[0.02,0.03]</sub>	<b>0.34</b> <sub>[0.26,0.36]</sub>	<b>3.30</b> <sub>[0.04,4.97]</sub>

Table 5–10: Surrogate model performance for the IPM motor-drive Inverter Efficiency ( $\eta_{Iv}$ )

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.10 <sub>[0.10,0.10]</sub>	0.20 <sub>[0.20,0.20]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.04 <sub>[0.04,0.04]</sub>	<b>0.48</b> <sub>[0.47,0.49]</sub>	8.57 <sub>[8.31,8.80]</sub>
P2	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.13</b> <sub>[0.13,0.14]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.44</b> <sub>[0.40,0.47]</sub>	<b>6.08</b> <sub>[5.61,6.76]</sub>
P3	<b>0.06</b> <sub>[0.04,0.08]</sub>	<b>0.14</b> <sub>[0.12,0.18]</sub>	<b>0.04</b> <sub>[0.03,0.05]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>0.40</b> <sub>[0.36,0.47]</sub>	<b>0.12</b> <sub>[0.08,5.74]</sub>
Kriging	<b>0.05</b> <sub>[0.04,0.06]</sub>	<b>0.13</b> <sub>[0.12,0.13]</sub>	<b>0.04</b> <sub>[0.03,0.04]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.46</b> <sub>[0.43,0.49]</sub>	<b>6.72</b> <sub>[5.45,7.57]</sub>
GRNN0.1	0.11 <sub>[0.10,0.12]</sub>	0.20 <sub>[0.19,0.22]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.04 <sub>[0.03,0.04]</sub>	<b>0.52</b> <sub>[0.47,0.55]</sub>	7.52 <sub>[6.03,9.55]</sub>
GRNN1	0.09 <sub>[0.08,0.10]</sub>	0.19 <sub>[0.18,0.19]</sub>	0.06 <sub>[0.06,0.06]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.57 <sub>[0.56,0.58]</sub>	10.71 <sub>[10.45,11.13]</sub>
GRNN3	0.12 <sub>[0.11,0.12]</sub>	0.22 <sub>[0.21,0.23]</sub>	0.07 <sub>[0.07,0.07]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.60 <sub>[0.60,0.60]</sub>	12.12 <sub>[12.02,12.19]</sub>
GRNN6	0.12 <sub>[0.12,0.13]</sub>	0.23 <sub>[0.22,0.23]</sub>	0.08 <sub>[0.08,0.08]</sub>	0.06 <sub>[0.06,0.06]</sub>	0.61 <sub>[0.61,0.61]</sub>	12.50 <sub>[12.34,12.59]</sub>
RBFNN0.1	0.20 <sub>[0.14,0.25]</sub>	0.28 <sub>[0.26,0.34]</sub>	0.10 <sub>[0.09,0.12]</sub>	0.07 <sub>[0.06,0.11]</sub>	0.62 <sub>[0.57,0.66]</sub>	11.95 <sub>[10.43,13.09]</sub>
RBFNN1	0.07 <sub>[0.06,0.09]</sub>	0.17 <sub>[0.15,0.20]</sub>	0.04 <sub>[0.04,0.06]</sub>	0.03 <sub>[0.03,0.04]</sub>	0.53 <sub>[0.47,0.58]</sub>	8.21 <sub>[5.96,10.45]</sub>
RBFNN3	0.08 <sub>[0.07,0.10]</sub>	0.18 <sub>[0.16,0.19]</sub>	0.05 <sub>[0.04,0.06]</sub>	0.03 <sub>[0.03,0.03]</sub>	<b>0.45</b> <sub>[0.38,0.50]</sub>	<b>5.91</b> <sub>[0.09,7.96]</sub>
RBFNN6	0.13 <sub>[0.11,0.15]</sub>	0.22 <sub>[0.20,0.24]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.05 <sub>[0.04,0.06]</sub>	0.53 <sub>[0.48,0.67]</sub>	<b>0.10</b> <sub>[0.09,0.12]</sub>

Table 5–11: Surrogate model performance for the IPM motor-drive Start-up Torque ( $T_{str}$ )

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.02 <sub>[0.02,0.02]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.38 <sub>[0.37,0.39]</sub>	0.10 <sub>[0.10,0.11]</sub>
P2	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.28</b> <sub>[0.26,0.29]</sub>	<b>0.09</b> <sub>[0.08,0.10]</sub>
P3	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.04,0.05]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.29</b> <sub>[0.28,0.34]</sub>	<b>0.05</b> <sub>[0.05,0.07]</sub>
Kriging	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.30</b> <sub>[0.27,0.34]</sub>	<b>0.08</b> <sub>[0.07,0.09]</sub>
GRNN0.1	0.07 <sub>[0.06,0.09]</sub>	0.16 <sub>[0.15,0.19]</sub>	0.08 <sub>[0.08,0.09]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.52 <sub>[0.49,0.57]</sub>	2.13 <sub>[1.37,2.43]</sub>
GRNN1	0.04 <sub>[0.03,0.05]</sub>	0.13 <sub>[0.11,0.14]</sub>	0.10 <sub>[0.10,0.11]</sub>	0.08 <sub>[0.08,0.08]</sub>	0.61 <sub>[0.60,0.63]</sub>	5.17 <sub>[4.84,5.38]</sub>
GRNN3	0.06 <sub>[0.04,0.07]</sub>	0.15 <sub>[0.12,0.16]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.70 <sub>[0.70,0.71]</sub>	7.81 <sub>[7.64,8.06]</sub>
GRNN6	0.06 <sub>[0.05,0.07]</sub>	0.15 <sub>[0.14,0.16]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.73 <sub>[0.73,0.74]</sub>	8.82 <sub>[8.66,8.96]</sub>
RBFNN0.1	0.18 <sub>[0.14,0.28]</sub>	0.28 <sub>[0.25,0.36]</sub>	0.16 <sub>[0.14,0.18]</sub>	0.14 <sub>[0.11,0.16]</sub>	0.81 <sub>[0.71,0.92]</sub>	8.19 <sub>[5.81,12.61]</sub>
RBFNN1	0.04 <sub>[0.02,0.06]</sub>	0.12 <sub>[0.10,0.15]</sub>	0.06 <sub>[0.06,0.08]</sub>	0.04 <sub>[0.04,0.05]</sub>	0.48 <sub>[0.41,0.51]</sub>	0.19 <sub>[0.16,0.24]</sub>
RBFNN3	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.07</b> <sub>[0.06,0.09]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.30</b> <sub>[0.27,0.32]</sub>	<b>0.10</b> <sub>[0.08,0.11]</sub>
RBFNN6	0.02 <sub>[0.01,0.02]</sub>	0.08 <sub>[0.07,0.09]</sub>	<b>0.04</b> <sub>[0.03,0.04]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.34</b> <sub>[0.30,0.42]</sub>	<b>0.08</b> <sub>[0.07,0.10]</sub>

Table 5–12: Surrogate model performance for the IPM motor-drive Current Density ( $J$ )

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	0.03 <sub>[0.03,0.03]</sub>	<b>0.28</b> <sub>[0.27,0.28]</sub>	<b>0.07</b> <sub>[0.07,0.07]</sub>
P2	<b>0.01</b> <sub>[0.00,0.01]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.27</b> <sub>[0.26,0.28]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>
P3	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.04,0.07]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.28</b> <sub>[0.26,0.35]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>
Kriging	<b>0.01</b> <sub>[0.00,0.01]</sub>	<b>0.04</b> <sub>[0.04,0.05]</sub>	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.27</b> <sub>[0.25,0.28]</sub>	1.35 <sub>[0.07,2.01]</sub>
GRNN0.1	0.07 <sub>[0.06,0.08]</sub>	0.16 <sub>[0.15,0.18]</sub>	0.10 <sub>[0.10,0.12]</sub>	0.08 <sub>[0.08,0.09]</sub>	0.49 <sub>[0.45,0.53]</sub>	4.49 <sub>[3.35,5.12]</sub>
GRNN1	0.04 <sub>[0.04,0.04]</sub>	0.13 <sub>[0.12,0.13]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.47 <sub>[0.45,0.48]</sub>	8.69 <sub>[8.35,9.27]</sub>
GRNN3	0.05 <sub>[0.05,0.06]</sub>	0.14 <sub>[0.13,0.15]</sub>	0.18 <sub>[0.18,0.19]</sub>	0.16 <sub>[0.16,0.16]</sub>	0.52 <sub>[0.51,0.53]</sub>	13.11 <sub>[12.84,13.39]</sub>
GRNN6	0.05 <sub>[0.05,0.06]</sub>	0.15 <sub>[0.14,0.15]</sub>	0.20 <sub>[0.20,0.20]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.54 <sub>[0.54,0.55]</sub>	14.44 <sub>[14.28,14.74]</sub>
RBFNN0.1	0.15 <sub>[0.12,0.22]</sub>	0.28 <sub>[0.25,0.32]</sub>	0.25 <sub>[0.20,0.31]</sub>	0.24 <sub>[0.19,0.30]</sub>	0.64 <sub>[0.61,0.75]</sub>	17.69 <sub>[10.53,21.74]</sub>
RBFNN1	0.04 <sub>[0.02,0.05]</sub>	0.12 <sub>[0.09,0.15]</sub>	0.08 <sub>[0.07,0.10]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.52 <sub>[0.36,0.58]</sub>	0.21 <sub>[0.16,2.43]</sub>
RBFNN3	0.01 <sub>[0.01,0.02]</sub>	<b>0.06</b> <sub>[0.05,0.07]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	<b>0.03</b> <sub>[0.03,0.03]</sub>	0.36 <sub>[0.29,0.41]</sub>	<b>0.07</b> <sub>[0.06,0.08]</sub>
RBFNN6	0.01 <sub>[0.01,0.02]</sub>	0.08 <sub>[0.07,0.08]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.04 <sub>[0.04,0.05]</sub>	0.44 <sub>[0.36,0.50]</sub>	<b>0.07</b> <sub>[0.06,0.09]</sub>

Table 5–13: Surrogate model performance for the IPM motor-drive Rising Time ( $rt$ )

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.04 <sub>[0.04,0.04]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.07 <sub>[0.07,0.07]</sub>	0.91 <sub>[0.90,0.93]</sub>	7.86 <sub>[7.38,8.15]</sub>
P2	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.08</b> <sub>[0.08,0.09]</sub>	<b>0.06</b> <sub>[0.06,0.07]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	0.89 <sub>[0.87,0.92]</sub>	<b>0.11</b> <sub>[0.10,0.93]</sub>
P3	<b>0.02</b> <sub>[0.02,0.04]</sub>	<b>0.09</b> <sub>[0.08,0.11]</sub>	<b>0.07</b> <sub>[0.06,0.08]</sub>	<b>0.05</b> <sub>[0.04,0.07]</sub>	0.97 <sub>[0.92,1.02]</sub>	<b>0.09</b> <sub>[0.09,0.10]</sub>
Kriging	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.09</b> <sub>[0.09,0.10]</sub>	<b>0.08</b> <sub>[0.07,0.08]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	0.91 <sub>[0.90,0.94]</sub>	7.85 <sub>[0.13,11.24]</sub>
GRNN0.1	0.08 <sub>[0.07,0.09]</sub>	0.18 <sub>[0.17,0.21]</sub>	0.12 <sub>[0.12,0.13]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.85 <sub>[0.82,0.89]</sub>	12.06 <sub>[9.82,14.06]</sub>
GRNN1	0.04 <sub>[0.04,0.05]</sub>	0.12 <sub>[0.12,0.13]</sub>	0.16 <sub>[0.16,0.16]</sub>	0.13 <sub>[0.13,0.14]</sub>	<b>0.67</b> <sub>[0.65,0.68]</sub>	15.68 <sub>[15.16,16.08]</sub>
GRNN3	0.04 <sub>[0.04,0.05]</sub>	0.13 <sub>[0.13,0.14]</sub>	0.20 <sub>[0.20,0.20]</sub>	0.17 <sub>[0.17,0.17]</sub>	<b>0.58</b> <sub>[0.57,0.58]</sub>	17.55 <sub>[17.12,17.66]</sub>
GRNN6	0.05 <sub>[0.04,0.06]</sub>	0.14 <sub>[0.13,0.15]</sub>	0.21 <sub>[0.21,0.21]</sub>	0.18 <sub>[0.18,0.18]</sub>	<b>0.55</b> <sub>[0.54,0.56]</sub>	18.05 <sub>[17.79,18.35]</sub>
RBFNN0.1	0.21 <sub>[0.17,0.28]</sub>	0.33 <sub>[0.27,0.38]</sub>	0.23 <sub>[0.21,0.27]</sub>	0.23 <sub>[0.19,0.27]</sub>	0.73 <sub>[0.62,0.80]</sub>	20.39 <sub>[13.95,24.88]</sub>
RBFNN1	0.05 <sub>[0.04,0.06]</sub>	0.14 <sub>[0.12,0.17]</sub>	0.11 <sub>[0.10,0.15]</sub>	0.07 <sub>[0.06,0.08]</sub>	0.88 <sub>[0.84,0.96]</sub>	11.39 <sub>[1.97,12.60]</sub>
RBFNN3	0.04 <sub>[0.04,0.05]</sub>	0.12 <sub>[0.11,0.13]</sub>	<b>0.09</b> <sub>[0.08,0.09]</sub>	0.06 <sub>[0.06,0.07]</sub>	1.00 <sub>[0.93,1.08]</sub>	<b>0.10</b> <sub>[0.09,0.12]</sub>
RBFNN6	0.07 <sub>[0.05,0.09]</sub>	0.16 <sub>[0.14,0.21]</sub>	0.12 <sub>[0.11,0.15]</sub>	0.09 <sub>[0.08,0.11]</sub>	1.26 <sub>[1.08,1.55]</sub>	<b>0.09</b> <sub>[0.08,0.10]</sub>

Table 5–14: Surrogate model performance for the IPM motor-drive Power Factor ( $pf$ )

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.05 <sub>[0.05,0.05]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.54 <sub>[0.53,0.56]</sub>	<b>0.13</b> <sub>[0.12,0.13]</sub>
P2	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.09</b> <sub>[0.09,0.09]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.44</b> <sub>[0.43,0.46]</sub>	<b>0.13</b> <sub>[0.12,0.14]</sub>
P3	<b>0.03</b> <sub>[0.02,0.05]</sub>	0.12 <sub>[0.09,0.16]</sub>	<b>0.04</b> <sub>[0.04,0.06]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.43</b> <sub>[0.41,0.48]</sub>	<b>0.10</b> <sub>[0.09,0.12]</sub>
Kriging	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.09</b> <sub>[0.09,0.10]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.48</b> <sub>[0.44,0.55]</sub>	<b>0.13</b> <sub>[0.12,0.18]</sub>
GRNN0.1	0.07 <sub>[0.07,0.09]</sub>	0.17 <sub>[0.16,0.17]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.04 <sub>[0.04,0.05]</sub>	0.58 <sub>[0.56,0.61]</sub>	1.73 <sub>[1.14,2.32]</sub>
GRNN1	0.05 <sub>[0.05,0.06]</sub>	0.14 <sub>[0.14,0.15]</sub>	0.08 <sub>[0.08,0.08]</sub>	0.06 <sub>[0.06,0.06]</sub>	0.69 <sub>[0.68,0.70]</sub>	3.78 <sub>[3.41,3.86]</sub>
GRNN3	0.07 <sub>[0.06,0.08]</sub>	0.17 <sub>[0.16,0.17]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.08 <sub>[0.08,0.08]</sub>	0.77 <sub>[0.76,0.78]</sub>	5.95 <sub>[5.87,6.10]</sub>
GRNN6	0.08 <sub>[0.07,0.09]</sub>	0.17 <sub>[0.16,0.18]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.08 <sub>[0.08,0.09]</sub>	0.79 <sub>[0.78,0.79]</sub>	6.95 <sub>[6.81,7.04]</sub>
RBFNN0.1	0.17 <sub>[0.15,0.23]</sub>	0.29 <sub>[0.25,0.34]</sub>	0.13 <sub>[0.11,0.16]</sub>	0.10 <sub>[0.09,0.14]</sub>	0.80 <sub>[0.75,0.89]</sub>	6.68 <sub>[3.55,9.04]</sub>
RBFNN1	0.06 <sub>[0.04,0.08]</sub>	0.13 <sub>[0.12,0.14]</sub>	0.05 <sub>[0.05,0.06]</sub>	0.04 <sub>[0.03,0.04]</sub>	0.57 <sub>[0.48,0.62]</sub>	0.20 <sub>[0.17,0.24]</sub>
RBFNN3	0.04 <sub>[0.04,0.04]</sub>	0.13 <sub>[0.11,0.14]</sub>	0.05 <sub>[0.04,0.06]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>0.44</b> <sub>[0.41,0.46]</sub>	0.15 <sub>[0.13,0.16]</sub>
RBFNN6	0.07 <sub>[0.06,0.09]</sub>	0.16 <sub>[0.15,0.17]</sub>	0.07 <sub>[0.06,0.07]</sub>	0.04 <sub>[0.04,0.05]</sub>	0.54 <sub>[0.48,0.64]</sub>	<b>0.10</b> <sub>[0.08,0.12]</sub>

Table 5–15: Surrogate model performance for the IPM motor-drive Speed Overshoot ( $n_o$ )

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.07 <sub>[0.07,0.07]</sub>	0.16 <sub>[0.16,0.16]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.83 <sub>[0.82,0.85]</sub>	26.45 <sub>[26.21,26.95]</sub>
P2	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.09</b> <sub>[0.09,0.10]</sub>	<b>0.04</b> <sub>[0.03,0.04]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.62</b> <sub>[0.60,0.63]</sub>	<b>19.33</b> <sub>[18.68,20.45]</sub>
P3	<b>0.03</b> <sub>[0.02,0.04]</sub>	<b>0.09</b> <sub>[0.08,0.13]</sub>	<b>0.04</b> <sub>[0.03,0.04]</sub>	0.03 <sub>[0.02,0.04]</sub>	<b>0.55</b> <sub>[0.51,0.59]</sub>	<b>18.49</b> <sub>[15.92,20.45]</sub>
Kriging	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.09</b> <sub>[0.08,0.10]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.59</b> <sub>[0.52,0.64]</sub>	<b>18.41</b> <sub>[15.95,20.05]</sub>
GRNN0.1	0.07 <sub>[0.07,0.09]</sub>	0.17 <sub>[0.16,0.19]</sub>	0.06 <sub>[0.05,0.06]</sub>	0.04 <sub>[0.04,0.04]</sub>	<b>0.66</b> <sub>[0.63,0.72]</sub>	22.77 <sub>[20.65,23.90]</sub>
GRNN1	0.06 <sub>[0.06,0.07]</sub>	0.16 <sub>[0.15,0.17]</sub>	0.07 <sub>[0.07,0.07]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.76 <sub>[0.75,0.77]</sub>	24.60 <sub>[24.14,24.85]</sub>
GRNN3	0.09 <sub>[0.09,0.10]</sub>	0.20 <sub>[0.19,0.20]</sub>	0.09 <sub>[0.08,0.09]</sub>	0.07 <sub>[0.07,0.07]</sub>	0.79 <sub>[0.78,0.79]</sub>	25.14 <sub>[24.98,25.31]</sub>
GRNN6	0.11 <sub>[0.10,0.12]</sub>	0.20 <sub>[0.19,0.21]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.07 <sub>[0.07,0.07]</sub>	0.79 <sub>[0.79,0.80]</sub>	25.26 <sub>[25.18,25.41]</sub>
RBFNN0.1	0.20 <sub>[0.15,0.25]</sub>	0.29 <sub>[0.23,0.35]</sub>	0.11 <sub>[0.09,0.15]</sub>	0.09 <sub>[0.08,0.11]</sub>	0.79 <sub>[0.77,0.86]</sub>	25.41 <sub>[20.79,26.72]</sub>
RBFNN1	0.05 <sub>[0.04,0.08]</sub>	0.13 <sub>[0.11,0.16]</sub>	0.05 <sub>[0.04,0.05]</sub>	0.04 <sub>[0.03,0.04]</sub>	<b>0.62</b> <sub>[0.59,0.70]</sub>	<b>20.11</b> <sub>[18.77,22.26]</sub>
RBFNN3	<b>0.03</b> <sub>[0.03,0.05]</sub>	<b>0.11</b> <sub>[0.10,0.13]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.62</b> <sub>[0.55,0.66]</sub>	<b>19.35</b> <sub>[16.86,20.61]</sub>
RBFNN6	0.06 <sub>[0.04,0.06]</sub>	0.13 <sub>[0.13,0.15]</sub>	0.05 <sub>[0.04,0.05]</sub>	0.03 <sub>[0.03,0.04]</sub>	<b>0.61</b> <sub>[0.53,0.68]</sub>	<b>17.12</b> <sub>[14.85,19.62]</sub>

Table 5–16: Surrogate model performance for the IPM motor-drive Motor Efficiency ( $\eta_{motor}$ )

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.50 <sub>[0.50,0.52]</sub>	0.52 <sub>[0.51,0.54]</sub>	<b>0.09</b> <sub>[0.09,0.09]</sub>	0.05 <sub>[0.05,0.05]</sub>	<b>0.93</b> <sub>[0.92,0.94]</sub>	0.75 <sub>[0.66,0.92]</sub>
P2	0.51 <sub>[0.50,0.52]</sub>	0.51 <sub>[0.50,0.52]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.06 <sub>[0.06,0.07]</sub>	<b>0.96</b> <sub>[0.93,0.98]</sub>	0.32 <sub>[0.28,0.38]</sub>
P3	0.45 <sub>[0.44,0.46]</sub>	0.47 <sub>[0.45,0.48]</sub>	0.13 <sub>[0.11,0.14]</sub>	0.09 <sub>[0.07,0.11]</sub>	1.02 <sub>[0.98,1.07]</sub>	<b>0.14</b> <sub>[0.13,0.16]</sub>
Kriging	<b>0.24</b> <sub>[0.22,0.31]</sub>	<b>0.30</b> <sub>[0.28,0.34]</sub>	<b>0.08</b> <sub>[0.07,0.09]</sub>	<b>0.04</b> <sub>[0.03,0.04]</sub>	<b>0.95</b> <sub>[0.78,0.98]</sub>	0.25 <sub>[0.22,0.37]</sub>
GRNN0.1	<b>0.41</b> <sub>[0.40,0.42]</sub>	<b>0.43</b> <sub>[0.41,0.44]</sub>	0.10 <sub>[0.10,0.10]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	0.99 <sub>[0.98,0.99]</sub>	7.98 <sub>[7.05,9.69]</sub>
GRNN1	0.48 <sub>[0.46,0.50]</sub>	0.51 <sub>[0.50,0.53]</sub>	<b>0.09</b> <sub>[0.09,0.09]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.95</b> <sub>[0.94,0.96]</sub>	10.51 <sub>[9.74,11.30]</sub>
GRNN3	0.51 <sub>[0.49,0.54]</sub>	0.51 <sub>[0.50,0.52]</sub>	<b>0.09</b> <sub>[0.09,0.09]</sub>	0.05 <sub>[0.05,0.06]</sub>	<b>0.94</b> <sub>[0.94,0.95]</sub>	10.12 <sub>[9.56,11.26]</sub>
GRNN6	0.50 <sub>[0.49,0.52]</sub>	0.52 <sub>[0.50,0.53]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.05 <sub>[0.05,0.06]</sub>	<b>0.94</b> <sub>[0.94,0.95]</sub>	10.10 <sub>[9.65,10.86]</sub>
RBFNN0.1	<b>0.42</b> <sub>[0.40,0.45]</sub>	0.46 <sub>[0.44,0.47]</sub>	0.10 <sub>[0.09,0.10]</sub>	<b>0.04</b> <sub>[0.04,0.06]</sub>	0.98 <sub>[0.97,0.99]</sub>	5.88 <sub>[0.63,10.23]</sub>
RBFNN1	0.45 <sub>[0.43,0.47]</sub>	0.46 <sub>[0.45,0.48]</sub>	0.11 <sub>[0.10,0.12]</sub>	0.08 <sub>[0.07,0.10]</sub>	1.06 <sub>[1.02,1.10]</sub>	<b>0.21</b> <sub>[0.19,0.25]</sub>
RBFNN3	0.45 <sub>[0.44,0.46]</sub>	0.47 <sub>[0.46,0.48]</sub>	0.17 <sub>[0.15,0.18]</sub>	0.11 <sub>[0.10,0.15]</sub>	1.25 <sub>[1.09,1.59]</sub>	<b>0.13</b> <sub>[0.12,0.19]</sub>
RBFNN6	0.47 <sub>[0.46,0.48]</sub>	0.48 <sub>[0.47,0.49]</sub>	0.24 <sub>[0.21,0.26]</sub>	0.18 <sub>[0.15,0.22]</sub>	2.47 <sub>[1.56,3.22]</sub>	<b>0.10</b> <sub>[0.08,0.13]</sub>

Table 5–17: Surrogate model performance for the IPM motor-drive Cogging Torque ( $T_{cog}$ )

Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	<b>0.03</b> <sub>[0.03,0.03]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.08 <sub>[0.08,0.08]</sub>	0.06 <sub>[0.06,0.06]</sub>	0.62 <sub>[0.60,0.63]</sub>	0.24 <sub>[0.23,0.25]</sub>
P2	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.10</b> <sub>[0.10,0.11]</sub>	<b>0.04</b> <sub>[0.04,0.05]</sub>	<b>0.03</b> <sub>[0.03,0.03]</sub>	<b>0.46</b> <sub>[0.43,0.47]</sub>	0.17 <sub>[0.17,0.18]</sub>
P3	<b>0.02</b> <sub>[0.02,0.03]</sub>	0.12 <sub>[0.09,0.15]</sub>	<b>0.04</b> <sub>[0.03,0.05]</sub>	<b>0.03</b> <sub>[0.02,0.04]</sub>	<b>0.36</b> <sub>[0.35,0.42]</sub>	<b>0.09</b> <sub>[0.09,0.10]</sub>
Kriging	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.07</b> <sub>[0.07,0.08]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.41</b> <sub>[0.35,0.44]</sub>	<b>0.16</b> <sub>[0.14,0.18]</sub>
GRNN0.1	0.05 <sub>[0.04,0.06]</sub>	0.14 <sub>[0.13,0.16]</sub>	0.08 <sub>[0.07,0.09]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.66 <sub>[0.62,0.69]</sub>	1.04 <sub>[0.82,1.23]</sub>
GRNN1	0.04 <sub>[0.03,0.04]</sub>	0.11 <sub>[0.11,0.12]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.07 <sub>[0.07,0.07]</sub>	0.73 <sub>[0.72,0.74]</sub>	3.11 <sub>[2.91,3.21]</sub>
GRNN3	0.04 <sub>[0.04,0.05]</sub>	0.12 <sub>[0.12,0.13]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.11 <sub>[0.10,0.11]</sub>	0.81 <sub>[0.80,0.82]</sub>	5.77 <sub>[5.62,6.13]</sub>
GRNN6	0.04 <sub>[0.04,0.04]</sub>	0.13 <sub>[0.12,0.13]</sub>	0.15 <sub>[0.15,0.15]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.83 <sub>[0.82,0.84]</sub>	6.93 <sub>[6.56,7.13]</sub>
RBFNN0.1	0.18 <sub>[0.10,0.21]</sub>	0.27 <sub>[0.23,0.33]</sub>	0.17 <sub>[0.15,0.19]</sub>	0.13 <sub>[0.11,0.19]</sub>	0.87 <sub>[0.82,0.93]</sub>	4.99 <sub>[1.66,13.82]</sub>
RBFNN1	0.05 <sub>[0.03,0.08]</sub>	0.13 <sub>[0.12,0.15]</sub>	0.07 <sub>[0.06,0.08]</sub>	0.05 <sub>[0.04,0.07]</sub>	0.59 <sub>[0.53,0.64]</sub>	0.24 <sub>[0.20,0.25]</sub>
RBFNN3	0.04 <sub>[0.03,0.07]</sub>	0.11 <sub>[0.11,0.13]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>0.44</b> <sub>[0.40,0.46]</sub>	<b>0.15</b> <sub>[0.12,0.20]</sub>
RBFNN6	0.06 <sub>[0.05,0.07]</sub>	0.14 <sub>[0.13,0.17]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.05 <sub>[0.04,0.05]</sub>	<b>0.49</b> <sub>[0.42,0.56]</sub>	<b>0.12</b> <sub>[0.10,0.13]</sub>

Table 5–18: Surrogate model performance for the IPM motor-drive Torque Ripple ( $T_{rip}$ )

Table 5–19 presents the average performance of the best surrogate models for the integrated IPM motor-drive design problem. Except for  $T_{cog}$ , all the surrogate models presented very low levels of conflict with FEM, even lower than the ones observed in previous problems. Again, the quality of the full surrogate problem was comparable to the quality of the highest performing surrogate problems found for the benchmark functions (see Tables 4–7 and 4–8).

Objective Function	Surrogate Model	Performance	
		NKT	NSR
$T_{cog}$	Kriging	0.20 <sub>[0.18,0.21]</sub>	0.19 <sub>[0.18,0.21]</sub>
$I_{vcost}$	P2		
$\eta_{Iv}$	P2		
$T_{str}$	P2		
$M_{cost}$	-		
$J$	P2		
$\eta_{motor}$	P2		
$pf$	P2		
$T_{rip}$	Kriging		
$n_{ov}$	P2		
$rt$	P2		

Table 5–19: Median and interquartile interval of best surrogate models’ performance for the integrated IPM motor-drive design problem

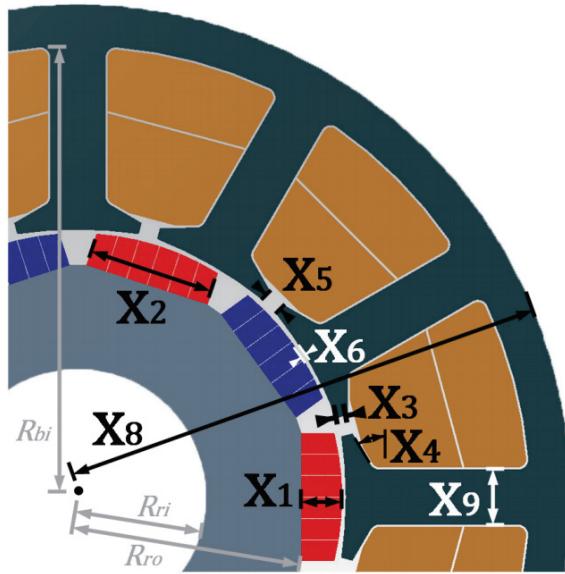
## 5.4 Fractional Slot Concentrated Winding Surface Mounted Permanent Magnet Machine (FSCW-SMPM)

Electric machines are generally optimized with respect to a single operating condition, typically at rated speed. While this is adequate for single-speed applications, optimizing variable-speed drives with respect to only one operating point may lead to sub-optimal performances at others. Since variable-speed EMs are being used by a growing number of applications, in particular, as traction units in transportation systems (e.g. electric cars), optimization of these machines under various operating conditions is a growing necessity.

In this context, Silva et al. (2017b) introduced a multi-objective formulation for the design of FSCW-SMPMs which accounts for multiple operating conditions. In the proposed formulation 18 objectives (6 objectives under 3 operating conditions) are optimized with respect to 9 geometrical variables. Table 5–20 and Figure 5–5 summarize objectives and design variables, respectively. In order to simulate each candidate solution under the 3 different operating conditions, on average, 10 minutes are required, considering an 8-core Intel Xeon E5 1650 CPU (3.5 GHz) with 32 GB RAM using MotorSolve®.

Objectives	Operating Condition	Symbol
Average torque ( $N.m$ )	Maximum speed	$T_{avg}^M$
	Rated speed	$T_{avg}^R$
	Peak current at rated speed	$T_{avg}^P$
Torque ripple(%)	Maximum speed	$T_{rip}^M$
	Rated speed	$T_{rip}^R$
	Peak current at rated speed	$T_{rip}^P$
RMS current density ( $A/mm^2$ )	Maximum speed	$J_{rms}^M$
	Rated speed	$J_{rms}^R$
	Peak current at rated speed	$J_{rms}^P$
Efficiency (%)	Maximum speed	$\eta^M$
	Rated speed	$\eta^R$
	Peak current at rated speed	$\eta^P$
Power factor	Maximum speed	$pf^M$
	Rated speed	$pf^R$
	Peak current at rated speed	$pf^P$
PM eddy current losses ( $kW$ )	Maximum speed	$P_{ePM}^M$
	Rated speed	$P_{ePM}^R$
	Peak current at rated speed	$P_{ePM}^P$

Table 5–20: FSCW-SMPM design objectives



Design Variable	Description	Lower bound	Upper bound
$x_1$	Magnet thickness	11 mm	15 mm
$x_2$	Magnet width	35 mm	48 mm
$x_3$	Tooth tip thickness	4.5 mm	6 mm
$x_4$	Tooth tang angle	17°	23°
$x_5$	Slot opening width	3.5 mm	5 mm
$x_6$	Air gap thickness	0.85 mm	1.15 mm
$x_7$	Stack length	135 mm	181 mm
$x_8$	Stator outer radius	127 mm	173 mm
$x_9$	Tooth width	16 mm	22 mm

Fixed Parameters	Description	Value
$P/S$	Number of poles/slots	10/12
$DC_{bus}$	DC bus voltage	450 V
$\mathcal{R}$	Rated speed	2700 RPM
$\mathcal{M}$	Maximum speed	11000 RPM
$\mathcal{P}$	Peak current	504 A
$R_{ro}$	Rotor outer radius	82.6 mm
$R_{ri}$	Rotor inner radius	55.8 mm
$R_{bi}$	Stator back iron radius	137.4 mm
$I_r$	Rated line current	184 A
$PM$	Magnet material	NdFeB 32/16
$C_m$	Core material	M-19 29 G

Figure 5–5: FSCW-SMPM motor cross-section, design variables and fixed parameters (Adapted from: (Silva et al., 2017b))

### 5.4.1 Objective reduction

Figure 5–6 presents the Pareto fronts obtained after the evaluation of all possible reduced problems. The results indicate that the essential objective set consists of 14 objectives. Independent of the used metric, the solution with 14 objectives presented a discrepancy of 0 in a 100% of the experiments with sub-samples.

Figure 5–7 presents the Pareto optimal reduced problems under both metrics. It can be seen that  $J^R$  and  $J^P$  are considered to be redundant in both cases. Since the current is fixed for each operating condition,  $J$  only depends on the slot area. Thus, an increase or decrease in  $J$  for one operating condition will necessarily imply a corresponding increase or decrease in  $J$  for the others. With respect to  $\eta^R$  and  $\eta^P$ , there seems to be no trivial explanation for their redundancy which is possibly a result of higher order interactions of objectives.

In spite of showing that it is possible to reduce the number of objectives in this problem, the results also show that a given motor performance can be in conflict with itself at different operating conditions. This reinforces the need for considering multiple operating conditions while designing this type of motor.

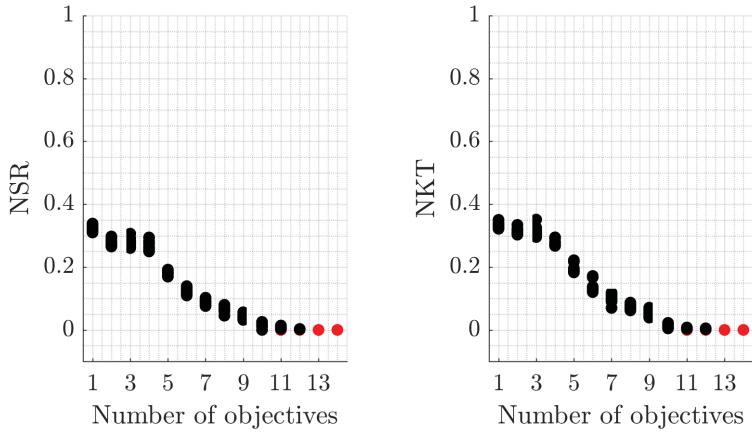


Figure 5–6: Scatter plots of the Pareto optimal FSCW-SMPM (75 kW) reduced problems under NSR and NKT

		NSR																	
		Number of objectives																	
Objectives		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$T_{avg}^M$																			
$T_{avg}^R$																			
$T_{avg}^P$																			
$T_{rip}^M$																			
$T_{rip}^R$																			
$T_{rip}^P$																			
$\eta^M$																			
$\eta^R$																			
$\eta^P$																			
$J^M$																			
$J^R$																			
$J^P$																			
$pf^M$																			
$pf^R$																			
$pf^P$																			
$P_{PM_e}^M$																			
$P_{PM_e}^R$																			
$P_{PM_e}^P$																			

		NKT																	
		Number of objectives																	
Objectives		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$T_{avg}^M$																			
$T_{avg}^R$																			
$T_{avg}^P$																			
$T_{rip}^M$																			
$T_{rip}^R$																			
$T_{rip}^P$																			
$\eta^M$																			
$\eta^R$																			
$\eta^P$																			
$J^M$																			
$J^R$																			
$J^P$																			
$pf^M$																			
$pf^R$																			
$pf^P$																			
$P_{PM_e}^M$																			
$P_{PM_e}^R$																			
$P_{PM_e}^P$																			

Figure 5–7: Table representation of the Pareto optimal reduced formulations for the FSCW-SMPM (75 kW) under NSR and NKT. Highlighted squares indicate the presence and white squares the absence of a given objective function in a given formulation. A black column indicates that a formulation with that number of objectives is not Pareto optimal.

### 5.4.2 Surrogate model selection

In this section, the results regarding the surrogate models' performances for the FSCW-SMPM design objectives are presented. In the tables below the best performances in each metric are shown in boldface. Notice that, in this formulation, the input current is always known. Therefore, no surrogate model is required for the current density calculation. Non-essential objectives were included in this experiment to verify if the operating condition affects the relative surrogate model performance. In other words, we are trying to verify if the best surrogate model for a given objective at a given operating conditions is also the best surrogate model for the same objective at a different operating condition.

Table 5–21 presents the surrogate models performances for torque ripple ( $T_{rip}$ ). For this objective, the relative surrogate model performance varied across the different operating conditions. At maximum speed,  $\mathcal{M}$ , the first-order polynomial was the best surrogate model. For rated speed,  $\mathcal{R}$ , and peak current,  $\mathcal{P}$ , the best surrogate models overall were Kriging and the second-order polynomial, respectively. These results indicate that the operating condition may change the objective function landscape and, hence, change the suitability of the different surrogate models.

The relative success of the surrogate models has also changed for the power factor,  $pf$ , as can be seen in Table 5–22. Although Kriging is among the best surrogates over the three operating conditions, only at maximum speed, is it superior under every metric. For rated speed and peak current, second and third-order polynomials are equally capable.

For the remaining objectives, there was no significant difference among Kriging, second and third-order polynomials. Furthermore, for these objectives, the relative surrogate model performance was stable across the different operating conditions. In this context, again, using the Ockhan's razor principle to break the ties, second-order polynomials were the chosen surrogate models.

$T_{rip}^{\mathcal{M}}$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>0.13</b> <sub>[0.13,0.13]</sub>	<b>0.07</b> <sub>[0.07,0.08]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>0.57</b> <sub>[0.54,0.58]</sub>	<b>67.48</b> <sub>[52.34,77.87]</sub>
P2	0.06 <sub>[0.06,0.08]</sub>	0.15 <sub>[0.14,0.16]</sub>	<b>0.08</b> <sub>[0.07,0.09]</sub>	<b>0.06</b> <sub>[0.05,0.06]</sub>	0.58 <sub>[0.55,0.62]</sub>	92.04 <sub>[74.56,100.81]</sub>
P3	0.13 <sub>[0.11,0.19]</sub>	0.20 <sub>[0.19,0.23]</sub>	0.11 <sub>[0.10,0.14]</sub>	0.07 <sub>[0.07,0.09]</sub>	0.78 <sub>[0.58,0.87]</sub>	<b>0.17</b> <sub>[0.15,0.18]</sub>
Kriging	0.07 <sub>[0.07,0.08]</sub>	0.16 <sub>[0.15,0.17]</sub>	<b>0.08</b> <sub>[0.08,0.09]</sub>	<b>0.06</b> <sub>[0.06,0.06]</sub>	<b>0.55</b> <sub>[0.51,0.59]</sub>	82.17 <sub>[63.04,110.16]</sub>
GRNN0.1	0.10 <sub>[0.10,0.11]</sub>	0.20 <sub>[0.20,0.21]</sub>	0.10 <sub>[0.10,0.11]</sub>	0.07 <sub>[0.07,0.08]</sub>	<b>0.57</b> <sub>[0.53,0.60]</sub>	126.55 <sub>[102.48,143.92]</sub>
GRNN1	<b>0.06</b> <sub>[0.06,0.06]</sub>	<b>0.14</b> <sub>[0.14,0.14]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.08 <sub>[0.08,0.08]</sub>	<b>0.54</b> <sub>[0.53,0.55]</sub>	169.30 <sub>[158.21,187.50]</sub>
GRNN3	<b>0.06</b> <sub>[0.06,0.06]</sub>	<b>0.14</b> <sub>[0.13,0.15]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.11 <sub>[0.11,0.11]</sub>	<b>0.57</b> <sub>[0.56,0.57]</sub>	210.39 <sub>[203.17,225.97]</sub>
GRNN6	<b>0.06</b> <sub>[0.06,0.06]</sub>	<b>0.14</b> <sub>[0.13,0.14]</sub>	0.15 <sub>[0.15,0.15]</sub>	0.12 <sub>[0.11,0.12]</sub>	0.59 <sub>[0.58,0.59]</sub>	233.60 <sub>[221.35,245.02]</sub>
RBFNN0.1	0.24 <sub>[0.17,0.34]</sub>	0.34 <sub>[0.27,0.38]</sub>	0.19 <sub>[0.16,0.25]</sub>	0.15 <sub>[0.13,0.18]</sub>	0.61 <sub>[0.57,0.72]</sub>	227.48 <sub>[199.78,265.69]</sub>
RBFNN1	0.10 <sub>[0.09,0.15]</sub>	0.18 <sub>[0.18,0.21]</sub>	0.10 <sub>[0.09,0.12]</sub>	0.07 <sub>[0.06,0.09]</sub>	<b>0.57</b> <sub>[0.51,0.63]</sub>	96.86 <sub>[0.35,122.25]</sub>
RBFNN3	0.13 <sub>[0.11,0.14]</sub>	0.21 <sub>[0.20,0.22]</sub>	0.11 <sub>[0.10,0.13]</sub>	0.08 <sub>[0.08,0.09]</sub>	0.65 <sub>[0.59,0.70]</sub>	<b>0.17</b> <sub>[0.15,0.30]</sub>
RBFNN6	0.16 <sub>[0.15,0.19]</sub>	0.26 <sub>[0.24,0.28]</sub>	0.15 <sub>[0.14,0.18]</sub>	0.12 <sub>[0.10,0.13]</sub>	0.81 <sub>[0.71,0.91]</sub>	<b>0.16</b> <sub>[0.14,0.18]</sub>
$T_{rip}^{\mathcal{R}}$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.24 <sub>[0.24,0.25]</sub>	0.32 <sub>[0.31,0.32]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.74 <sub>[0.72,0.76]</sub>	141.74 <sub>[127.98,160.80]</sub>
P2	0.14 <sub>[0.13,0.16]</sub>	<b>0.23</b> <sub>[0.22,0.24]</sub>	<b>0.10</b> <sub>[0.10,0.11]</sub>	<b>0.07</b> <sub>[0.07,0.08]</sub>	0.69 <sub>[0.59,0.77]</sub>	<b>34.10</b> <sub>[0.23,54.46]</sub>
P3	0.15 <sub>[0.12,0.19]</sub>	0.24 <sub>[0.21,0.33]</sub>	0.11 <sub>[0.09,0.13]</sub>	0.08 <sub>[0.07,0.09]</sub>	<b>0.56</b> <sub>[0.47,0.64]</sub>	<b>0.16</b> <sub>[0.14,0.17]</sub>
Kriging	<b>0.06</b> <sub>[0.05,0.06]</sub>	<b>0.14</b> <sub>[0.14,0.16]</sub>	<b>0.07</b> <sub>[0.06,0.07]</sub>	<b>0.05</b> <sub>[0.04,0.05]</sub>	<b>0.46</b> <sub>[0.41,0.50]</sub>	<b>44.11</b> <sub>[25.63,57.40]</sub>
GRNN0.1	0.18 <sub>[0.17,0.19]</sub>	0.27 <sub>[0.26,0.29]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.08 <sub>[0.08,0.08]</sub>	0.65 <sub>[0.59,0.75]</sub>	105.94 <sub>[55.26,138.22]</sub>
GRNN1	0.17 <sub>[0.16,0.18]</sub>	0.26 <sub>[0.25,0.27]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.68 <sub>[0.66,0.70]</sub>	169.70 <sub>[162.03,183.92]</sub>
GRNN3	0.22 <sub>[0.21,0.23]</sub>	0.30 <sub>[0.30,0.31]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.66 <sub>[0.65,0.66]</sub>	201.87 <sub>[184.77,218.31]</sub>
GRNN6	0.24 <sub>[0.23,0.25]</sub>	0.31 <sub>[0.31,0.32]</sub>	0.13 <sub>[0.13,0.14]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.65 <sub>[0.64,0.65]</sub>	204.98 <sub>[194.78,219.35]</sub>
RBFNN0.1	0.26 <sub>[0.24,0.30]</sub>	0.34 <sub>[0.31,0.38]</sub>	0.15 <sub>[0.14,0.20]</sub>	0.15 <sub>[0.11,0.20]</sub>	0.66 <sub>[0.59,0.74]</sub>	221.18 <sub>[202.70,267.25]</sub>
RBFNN1	0.13 <sub>[0.11,0.16]</sub>	<b>0.22</b> <sub>[0.21,0.25]</sub>	0.10 <sub>[0.09,0.12]</sub>	0.07 <sub>[0.07,0.09]</sub>	0.61 <sub>[0.53,0.70]</sub>	97.20 <sub>[12.41,121.00]</sub>
RBFNN3	0.12 <sub>[0.11,0.16]</sub>	0.23 <sub>[0.20,0.25]</sub>	0.10 <sub>[0.09,0.11]</sub>	0.07 <sub>[0.07,0.09]</sub>	<b>0.53</b> <sub>[0.47,0.63]</sub>	<b>0.20</b> <sub>[0.16,0.22]</sub>
RBFNN6	0.19 <sub>[0.15,0.22]</sub>	0.26 <sub>[0.24,0.28]</sub>	0.13 <sub>[0.12,0.15]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.77 <sub>[0.72,0.97]</sub>	<b>0.17</b> <sub>[0.15,0.20]</sub>
$T_{rip}^{\mathcal{P}}$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.12 <sub>[0.11,0.12]</sub>	0.21 <sub>[0.21,0.21]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.07 <sub>[0.07,0.07]</sub>	<b>0.34</b> <sub>[0.33,0.35]</sub>	122.98 <sub>[115.91,136.06]</sub>
P2	<b>0.07</b> <sub>[0.06,0.07]</sub>	<b>0.15</b> <sub>[0.15,0.15]</sub>	<b>0.06</b> <sub>[0.06,0.07]</sub>	<b>0.05</b> <sub>[0.04,0.05]</sub>	<b>0.26</b> <sub>[0.24,0.28]</sub>	<b>1.39</b> <sub>[0.14,53.19]</sub>
P3	0.10 <sub>[0.08,0.16]</sub>	0.19 <sub>[0.18,0.20]</sub>	0.08 <sub>[0.07,0.11]</sub>	0.06 <sub>[0.06,0.08]</sub>	0.43 <sub>[0.36,0.77]</sub>	<b>0.15</b> <sub>[0.13,0.16]</sub>
Kriging	<b>0.08</b> <sub>[0.07,0.09]</sub>	<b>0.16</b> <sub>[0.16,0.17]</sub>	<b>0.07</b> <sub>[0.07,0.07]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>0.29</b> <sub>[0.27,0.33]</sub>	108.78 <sub>[77.81,120.67]</sub>
GRNN0.1	0.14 <sub>[0.12,0.15]</sub>	0.23 <sub>[0.21,0.25]</sub>	0.10 <sub>[0.09,0.10]</sub>	0.07 <sub>[0.07,0.08]</sub>	0.42 <sub>[0.40,0.45]</sub>	137.57 <sub>[118.17,154.96]</sub>
GRNN1	<b>0.10</b> <sub>[0.09,0.10]</sub>	<b>0.19</b> <sub>[0.18,0.19]</sub>	0.11 <sub>[0.11,0.12]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.49 <sub>[0.48,0.51]</sub>	179.23 <sub>[167.35,190.02]</sub>
GRNN3	0.11 <sub>[0.11,0.12]</sub>	0.21 <sub>[0.20,0.21]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.58 <sub>[0.57,0.59]</sub>	208.68 <sub>[192.80,223.62]</sub>
GRNN6	0.12 <sub>[0.11,0.12]</sub>	0.21 <sub>[0.21,0.22]</sub>	0.15 <sub>[0.14,0.15]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.61 <sub>[0.60,0.61]</sub>	218.19 <sub>[206.50,227.65]</sub>
RBFNN0.1	0.23 <sub>[0.21,0.32]</sub>	0.36 <sub>[0.29,0.39]</sub>	0.17 <sub>[0.15,0.20]</sub>	0.14 <sub>[0.12,0.17]</sub>	0.64 <sub>[0.55,0.73]</sub>	199.72 <sub>[158.74,290.71]</sub>
RBFNN1	0.12 <sub>[0.10,0.16]</sub>	0.22 <sub>[0.19,0.25]</sub>	0.09 <sub>[0.08,0.10]</sub>	0.07 <sub>[0.06,0.08]</sub>	0.47 <sub>[0.42,0.60]</sub>	107.16 <sub>[85.29,135.31]</sub>
RBFNN3	0.13 <sub>[0.11,0.17]</sub>	0.21 <sub>[0.20,0.24]</sub>	0.10 <sub>[0.09,0.11]</sub>	0.07 <sub>[0.07,0.09]</sub>	0.45 <sub>[0.37,0.54]</sub>	<b>0.19</b> <sub>[0.17,0.23]</sub>
RBFNN6	0.17 <sub>[0.16,0.20]</sub>	0.26 <sub>[0.24,0.27]</sub>	0.14 <sub>[0.13,0.16]</sub>	0.10 <sub>[0.09,0.11]</sub>	0.69 <sub>[0.59,0.87]</sub>	<b>0.14</b> <sub>[0.11,0.16]</sub>

Table 5–21: Surrogate model performance for the FSCW-SMPM motor Torque Ripple ( $T_{rip}$ )

$pf^{\mathcal{M}}$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.06 <sub>[0.06,0.06]</sub>	0.12 <sub>[0.11,0.12]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.09 <sub>[0.08,0.09]</sub>	1.04 <sub>[0.96,1.05]</sub>	56.03 <sub>[39.57,77.30]</sub>
P2	0.05 <sub>[0.05,0.06]</sub>	0.12 <sub>[0.11,0.13]</sub>	0.11 <sub>[0.10,0.12]</sub>	0.08 <sub>[0.08,0.09]</sub>	0.62 <sub>[0.51,0.68]</sub>	<b>0.09</b> <sub>[0.09,0.10]</sub>
P3	<b>0.02</b> <sub>[0.01,0.06]</sub>	<b>0.08</b> <sub>[0.08,0.12]</sub>	<b>0.07</b> <sub>[0.06,0.08]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>	0.39 <sub>[0.36,0.64]</sub>	<b>0.12</b> <sub>[0.10,0.13]</sub>
Kriging	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.09</b> <sub>[0.07,0.11]</sub>	<b>5.08</b> <sub>[0.75,11.60]</sub>
GRNN0.1	0.10 <sub>[0.09,0.13]</sub>	0.21 <sub>[0.20,0.22]</sub>	0.14 <sub>[0.13,0.15]</sub>	0.11 <sub>[0.10,0.12]</sub>	0.62 <sub>[0.58,0.71]</sub>	100.92 <sub>[85.99,121.45]</sub>
GRNN1	0.07 <sub>[0.06,0.07]</sub>	0.14 <sub>[0.13,0.15]</sub>	0.17 <sub>[0.17,0.18]</sub>	0.15 <sub>[0.15,0.15]</sub>	0.64 <sub>[0.62,0.65]</sub>	215.50 <sub>[208.42,224.63]</sub>
GRNN3	0.06 <sub>[0.06,0.07]</sub>	0.13 <sub>[0.12,0.15]</sub>	0.21 <sub>[0.21,0.22]</sub>	0.19 <sub>[0.19,0.19]</sub>	0.56 <sub>[0.54,0.57]</sub>	306.54 <sub>[295.33,319.91]</sub>
GRNN6	0.07 <sub>[0.06,0.07]</sub>	0.14 <sub>[0.13,0.14]</sub>	0.23 <sub>[0.23,0.23]</sub>	0.20 <sub>[0.20,0.20]</sub>	0.54 <sub>[0.53,0.55]</sub>	322.94 <sub>[295.55,341.72]</sub>
RBFNN0.1	0.22 <sub>[0.19,0.31]</sub>	0.31 <sub>[0.27,0.35]</sub>	0.32 <sub>[0.26,0.36]</sub>	0.29 <sub>[0.23,0.37]</sub>	0.69 <sub>[0.60,0.77]</sub>	255.97 <sub>[187.12,386.10]</sub>
RBFNN1	0.05 <sub>[0.05,0.07]</sub>	0.14 <sub>[0.12,0.16]</sub>	0.11 <sub>[0.10,0.13]</sub>	0.08 <sub>[0.08,0.09]</sub>	0.56 <sub>[0.49,0.63]</sub>	87.16 <sub>[0.33,126.13]</sub>
RBFNN3	<b>0.04</b> <sub>[0.03,0.05]</sub>	0.12 <sub>[0.10,0.14]</sub>	0.09 <sub>[0.07,0.11]</sub>	0.06 <sub>[0.05,0.09]</sub>	0.49 <sub>[0.45,0.57]</sub>	<b>0.12</b> <sub>[0.11,8.51]</sub>
RBFNN6	0.06 <sub>[0.03,0.08]</sub>	0.11 <sub>[0.10,0.15]</sub>	0.09 <sub>[0.08,0.11]</sub>	0.07 <sub>[0.06,0.11]</sub>	0.56 <sub>[0.41,0.67]</sub>	<b>0.11</b> <sub>[0.09,0.13]</sub>

$pf^{\mathcal{R}}$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.00 <sub>[0.00,0.00]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.19 <sub>[0.18,0.20]</sub>	105.69 <sub>[100.77,116.65]</sub>
P2	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>	<b>27.28</b> <sub>[23.63,29.64]</sub>
P3	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.04]</sub>	<b>4.34</b> <sub>[1.92,5.36]</sub>
Kriging	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>14.70</b> <sub>[9.64,21.84]</sub>
GRNN0.1	0.05 <sub>[0.04,0.06]</sub>	0.15 <sub>[0.14,0.16]</sub>	0.09 <sub>[0.08,0.10]</sub>	0.07 <sub>[0.06,0.07]</sub>	0.39 <sub>[0.36,0.40]</sub>	156.06 <sub>[131.59,203.59]</sub>
GRNN1	0.01 <sub>[0.01,0.02]</sub>	0.06 <sub>[0.05,0.06]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.46 <sub>[0.44,0.48]</sub>	258.37 <sub>[245.27,294.37]</sub>
GRNN3	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.16 <sub>[0.16,0.16]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.59 <sub>[0.58,0.60]</sub>	336.87 <sub>[320.85,362.06]</sub>
GRNN6	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.63 <sub>[0.62,0.64]</sub>	366.97 <sub>[358.49,385.28]</sub>
RBFNN0.1	0.18 <sub>[0.14,0.25]</sub>	0.27 <sub>[0.25,0.34]</sub>	0.21 <sub>[0.19,0.26]</sub>	0.17 <sub>[0.15,0.21]</sub>	0.66 <sub>[0.58,0.77]</sub>	375.74 <sub>[309.74,465.74]</sub>
RBFNN1	0.03 <sub>[0.02,0.05]</sub>	0.10 <sub>[0.09,0.13]</sub>	0.08 <sub>[0.06,0.11]</sub>	0.06 <sub>[0.04,0.07]</sub>	0.43 <sub>[0.36,0.49]</sub>	207.07 <sub>[161.18,256.01]</sub>
RBFNN3	0.00 <sub>[0.00,0.00]</sub>	0.02 <sub>[0.02,0.04]</sub>	0.02 <sub>[0.02,0.03]</sub>	0.01 <sub>[0.01,0.02]</sub>	0.13 <sub>[0.11,0.17]</sub>	63.23 <sub>[48.93,93.78]</sub>
RBFNN6	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.04</b> <sub>[0.03,0.06]</sub>	<b>10.96</b> <sub>[4.68,13.16]</sub>

$pf^{\mathcal{P}}$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.00 <sub>[0.00,0.00]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.02 <sub>[0.02,0.02]</sub>	0.02 <sub>[0.02,0.02]</sub>	0.10 <sub>[0.09,0.10]</sub>	43.33 <sub>[41.17,46.56]</sub>
P2	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.01,0.02]</sub>	<b>1.03</b> <sub>[0.01,2.44]</sub>
P3	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.28]</sub>
Kriging	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>5.87</b> <sub>[4.00,8.88]</sub>
GRNN0.1	0.05 <sub>[0.04,0.06]</sub>	0.15 <sub>[0.14,0.17]</sub>	0.08 <sub>[0.07,0.09]</sub>	0.07 <sub>[0.06,0.07]</sub>	0.32 <sub>[0.29,0.34]</sub>	107.27 <sub>[91.92,125.53]</sub>
GRNN1	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.12 <sub>[0.11,0.12]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.36 <sub>[0.35,0.37]</sub>	205.65 <sub>[196.90,222.60]</sub>
GRNN3	0.01 <sub>[0.01,0.01]</sub>	0.05 <sub>[0.04,0.07]</sub>	0.16 <sub>[0.16,0.16]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.47 <sub>[0.46,0.48]</sub>	288.83 <sub>[273.81,303.35]</sub>
GRNN6	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.50 <sub>[0.50,0.51]</sub>	309.42 <sub>[289.72,321.66]</sub>
RBFNN0.1	0.19 <sub>[0.13,0.26]</sub>	0.30 <sub>[0.25,0.34]</sub>	0.22 <sub>[0.19,0.26]</sub>	0.20 <sub>[0.17,0.26]</sub>	0.64 <sub>[0.56,0.72]</sub>	353.30 <sub>[247.12,399.91]</sub>
RBFNN1	0.03 <sub>[0.02,0.06]</sub>	0.10 <sub>[0.08,0.13]</sub>	0.07 <sub>[0.06,0.09]</sub>	0.05 <sub>[0.04,0.06]</sub>	0.37 <sub>[0.33,0.50]</sub>	155.55 <sub>[130.84,192.20]</sub>
RBFNN3	0.00 <sub>[0.00,0.00]</sub>	0.02 <sub>[0.01,0.03]</sub>	0.02 <sub>[0.01,0.02]</sub>	0.01 <sub>[0.01,0.02]</sub>	0.09 <sub>[0.07,0.12]</sub>	30.98 <sub>[24.69,37.30]</sub>
RBFNN6	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.32</b> <sub>[0.01,5.09]</sub>

Table 5–22: Surrogate model performance for the FSCW-SMPM motor power factor  $pf$

$T_{avg}^M$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.02 <sub>[0.02,0.02]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.06 <sub>[0.05,0.06]</sub>	0.05 <sub>[0.04,0.05]</sub>	0.22 <sub>[0.21,0.23]</sub>	69.50 <sub>[50.46,75.28]</sub>
P2	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.03<sub>[0.03,0.04]</sub></b>	<b>0.02<sub>[0.02,0.02]</sub></b>	<b>0.02<sub>[0.02,0.02]</sub></b>	<b>0.11<sub>[0.10,0.12]</sub></b>	<b>0.04<sub>[0.04,0.04]</sub></b>
P3	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.03<sub>[0.02,0.03]</sub></b>	<b>0.02<sub>[0.02,0.02]</sub></b>	<b>0.01<sub>[0.01,0.01]</sub></b>	<b>0.10<sub>[0.08,0.13]</sub></b>	<b>0.03<sub>[0.02,0.04]</sub></b>
Kriging	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.01<sub>[0.01,0.02]</sub></b>	<b>0.01<sub>[0.01,0.01]</sub></b>	<b>0.01<sub>[0.01,0.01]</sub></b>	<b>0.06<sub>[0.05,0.07]</sub></b>	[5.92, 23.66]
GRNN0.1	0.07 <sub>[0.06,0.08]</sub>	0.16 <sub>[0.15,0.17]</sub>	0.10 <sub>[0.09,0.11]</sub>	0.08 <sub>[0.07,0.08]</sub>	0.40 <sub>[0.37,0.47]</sub>	215.03 <sub>[191.13,233.99]</sub>
GRNN1	0.02 <sub>[0.02,0.03]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.13 <sub>[0.12,0.13]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.43 <sub>[0.40,0.43]</sub>	252.50 <sub>[240.31,264.57]</sub>
GRNN3	0.03 <sub>[0.02,0.03]</sub>	0.10 <sub>[0.09,0.10]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.52 <sub>[0.52,0.53]</sub>	316.64 <sub>[298.44,337.90]</sub>
GRNN6	0.03 <sub>[0.02,0.03]</sub>	0.10 <sub>[0.10,0.11]</sub>	0.18 <sub>[0.18,0.18]</sub>	0.15 <sub>[0.15,0.15]</sub>	0.55 <sub>[0.55,0.56]</sub>	323.13 <sub>[314.77,342.67]</sub>
RBFNN0.1	0.20 <sub>[0.15,0.26]</sub>	0.30 <sub>[0.25,0.37]</sub>	0.23 <sub>[0.19,0.30]</sub>	0.20 <sub>[0.15,0.25]</sub>	0.72 <sub>[0.56,0.80]</sub>	324.92 <sub>[246.35,438.52]</sub>
RBFNN1	0.04 <sub>[0.02,0.06]</sub>	0.13 <sub>[0.11,0.17]</sub>	0.07 <sub>[0.07,0.08]</sub>	0.07 <sub>[0.05,0.08]</sub>	0.47 <sub>[0.37,0.52]</sub>	176.60 <sub>[129.49,279.70]</sub>
RBFNN3	0.00 <sub>[0.00,0.01]</sub>	0.04 <sub>[0.03,0.05]</sub>	0.04 <sub>[0.02,0.05]</sub>	0.02 <sub>[0.02,0.03]</sub>	0.15 <sub>[0.13,0.20]</sub>	38.68 <sub>[23.00,72.08]</sub>
RBFNN6	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.03<sub>[0.03,0.04]</sub></b>	<b>0.02<sub>[0.02,0.03]</sub></b>	<b>0.02<sub>[0.01,0.02]</sub></b>	<b>0.13<sub>[0.11,0.15]</sub></b>	<b>0.03<sub>[0.03,0.06]</sub></b>

$T_{avg}^R$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.01 <sub>[0.01,0.01]</sub>	0.05 <sub>[0.04,0.05]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.02 <sub>[0.02,0.02]</sub>	0.15 <sub>[0.14,0.16]</sub>	50.79 <sub>[45.20,54.71]</sub>
P2	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.01<sub>[0.01,0.01]</sub></b>	<b>0.01<sub>[0.01,0.01]</sub></b>	<b>0.01<sub>[0.01,0.01]</sub></b>	<b>0.04<sub>[0.04,0.05]</sub></b>	<b>0.01<sub>[0.01,0.02]</sub></b>
P3	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.00<sub>[0.00,0.01]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.02<sub>[0.02,0.03]</sub></b>	<b>0.01<sub>[0.01,1.77]</sub></b>
Kriging	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.01<sub>[0.00,0.01]</sub></b>	<b>0.00<sub>[0.00,0.01]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.02<sub>[0.02,0.03]</sub></b>	<b>2.51<sub>[0.05,4.70]</sub></b>
GRNN0.1	0.05 <sub>[0.04,0.06]</sub>	0.15 <sub>[0.13,0.16]</sub>	0.08 <sub>[0.08,0.09]</sub>	0.07 <sub>[0.06,0.07]</sub>	0.32 <sub>[0.29,0.36]</sub>	157.81 <sub>[121.47,175.69]</sub>
GRNN1	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.38 <sub>[0.37,0.40]</sub>	221.43 <sub>[212.06,234.47]</sub>
GRNN3	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.50 <sub>[0.49,0.51]</sub>	292.95 <sub>[279.04,310.34]</sub>
GRNN6	0.01 <sub>[0.01,0.02]</sub>	0.06 <sub>[0.06,0.06]</sub>	0.18 <sub>[0.18,0.18]</sub>	0.15 <sub>[0.15,0.15]</sub>	0.54 <sub>[0.52,0.54]</sub>	321.47 <sub>[302.70,331.87]</sub>
RBFNN0.1	0.20 <sub>[0.14,0.28]</sub>	0.33 <sub>[0.27,0.39]</sub>	0.21 <sub>[0.19,0.31]</sub>	0.18 <sub>[0.16,0.25]</sub>	0.72 <sub>[0.66,0.80]</sub>	317.01 <sub>[246.68,418.20]</sub>
RBFNN1	0.03 <sub>[0.02,0.04]</sub>	0.10 <sub>[0.09,0.13]</sub>	0.08 <sub>[0.06,0.10]</sub>	0.05 <sub>[0.04,0.07]</sub>	0.43 <sub>[0.37,0.49]</sub>	120.91 <sub>[79.54,155.09]</sub>
RBFNN3	0.00 <sub>[0.00,0.01]</sub>	0.02 <sub>[0.01,0.03]</sub>	0.02 <sub>[0.01,0.03]</sub>	0.01 <sub>[0.01,0.02]</sub>	0.10 <sub>[0.07,0.15]</sub>	22.41 <sub>[11.34,30.19]</sub>
RBFNN6	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.01<sub>[0.01,0.01]</sub></b>	<b>0.01<sub>[0.00,0.01]</sub></b>	<b>0.00<sub>[0.00,0.01]</sub></b>	<b>0.04<sub>[0.03,0.06]</sub></b>	<b>3.14<sub>[0.33,6.80]</sub></b>

$T_{avg}^P$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.00 <sub>[0.00,0.00]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.02 <sub>[0.02,0.02]</sub>	0.02 <sub>[0.02,0.02]</sub>	0.10 <sub>[0.09,0.10]</sub>	<b>0.05<sub>[0.05,0.05]</sub></b>
P2	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.01<sub>[0.01,0.01]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.02<sub>[0.02,0.02]</sub></b>	<b>0.01<sub>[0.01,0.01]</sub></b>
P3	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.01<sub>[0.01,0.02]</sub></b>	<b>0.01<sub>[0.01,0.56]</sub></b>
Kriging	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.01<sub>[0.00,0.01]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.02<sub>[0.02,0.02]</sub></b>	<b>3.58<sub>[2.66,9.27]</sub></b>
GRNN0.1	0.06 <sub>[0.05,0.07]</sub>	0.14 <sub>[0.13,0.17]</sub>	0.09 <sub>[0.08,0.09]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.30 <sub>[0.29,0.33]</sub>	128.93 <sub>[91.11,142.57]</sub>
GRNN1	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.34 <sub>[0.32,0.35]</sub>	188.16 <sub>[179.66,196.72]</sub>
GRNN3	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.16 <sub>[0.16,0.16]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.45 <sub>[0.44,0.46]</sub>	268.73 <sub>[256.58,279.83]</sub>
GRNN6	0.01 <sub>[0.01,0.01]</sub>	0.05 <sub>[0.05,0.06]</sub>	0.18 <sub>[0.18,0.18]</sub>	0.15 <sub>[0.14,0.15]</sub>	0.49 <sub>[0.49,0.50]</sub>	291.91 <sub>[268.15,310.58]</sub>
RBFNN0.1	0.17 <sub>[0.13,0.24]</sub>	0.31 <sub>[0.25,0.35]</sub>	0.26 <sub>[0.21,0.31]</sub>	0.20 <sub>[0.17,0.27]</sub>	0.64 <sub>[0.57,0.74]</sub>	270.81 <sub>[180.84,357.89]</sub>
RBFNN1	0.03 <sub>[0.02,0.04]</sub>	0.09 <sub>[0.08,0.11]</sub>	0.08 <sub>[0.06,0.09]</sub>	0.05 <sub>[0.04,0.06]</sub>	0.39 <sub>[0.34,0.47]</sub>	140.58 <sub>[81.76,176.23]</sub>
RBFNN3	0.00 <sub>[0.00,0.00]</sub>	0.02 <sub>[0.01,0.04]</sub>	0.02 <sub>[0.01,0.03]</sub>	0.01 <sub>[0.01,0.02]</sub>	0.12 <sub>[0.07,0.22]</sub>	32.28 <sub>[13.39,44.19]</sub>
RBFNN6	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.01<sub>[0.00,0.01]</sub></b>	<b>0.00<sub>[0.00,0.01]</sub></b>	<b>0.00<sub>[0.00,0.00]</sub></b>	<b>0.02<sub>[0.02,0.03]</sub></b>	<b>0.97<sub>[0.01,4.52]</sub></b>

Table 5–23: Surrogate model performance for the FSCW-SMPM motor Average Torque ( $T_{avg}$ )

$\eta^{\mathcal{M}}$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.01 <sub>[0.01,0.01]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.18 <sub>[0.17,0.19]</sub>	82.81 <sub>[75.30,87.49]</sub>
P2	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.10</b> <sub>[0.09,0.11]</sub>	<b>27.64</b> <sub>[20.64,29.77]</sub>
P3	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.09</b> <sub>[0.06,0.13]</sub>	<b>8.19</b> <sub>[3.96,10.39]</sub>
Kriging	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.06</b> <sub>[0.05,0.07]</sub>	<b>12.01</b> <sub>[6.65,17.64]</sub>
GRNN0.1	0.05 <sub>[0.05,0.06]</sub>	0.14 <sub>[0.13,0.16]</sub>	0.09 <sub>[0.08,0.09]</sub>	0.07 <sub>[0.06,0.07]</sub>	0.36 <sub>[0.33,0.38]</sub>	147.11 <sub>[120.24,170.24]</sub>
GRNN1	0.01 <sub>[0.01,0.01]</sub>	0.07 <sub>[0.06,0.07]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.41 <sub>[0.39,0.42]</sub>	238.12 <sub>[222.16,258.47]</sub>
GRNN3	0.01 <sub>[0.01,0.02]</sub>	0.07 <sub>[0.06,0.08]</sub>	0.16 <sub>[0.16,0.16]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.53 <sub>[0.52,0.54]</sub>	316.19 <sub>[298.13,331.75]</sub>
GRNN6	0.01 <sub>[0.01,0.01]</sub>	0.07 <sub>[0.06,0.08]</sub>	0.18 <sub>[0.18,0.18]</sub>	0.14 <sub>[0.14,0.15]</sub>	0.56 <sub>[0.55,0.57]</sub>	330.88 <sub>[315.44,345.72]</sub>
RBFNN0.1	0.19 <sub>[0.14,0.26]</sub>	0.29 <sub>[0.26,0.34]</sub>	0.22 <sub>[0.18,0.26]</sub>	0.21 <sub>[0.17,0.28]</sub>	0.65 <sub>[0.55,0.72]</sub>	324.66 <sub>[262.56,412.41]</sub>
RBFNN1	0.03 <sub>[0.02,0.05]</sub>	0.12 <sub>[0.09,0.14]</sub>	0.08 <sub>[0.06,0.09]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.40 <sub>[0.35,0.48]</sub>	158.43 <sub>[128.61,211.91]</sub>
RBFNN3	0.00 <sub>[0.00,0.01]</sub>	0.03 <sub>[0.03,0.05]</sub>	0.02 <sub>[0.02,0.03]</sub>	0.02 <sub>[0.01,0.03]</sub>	0.12 <sub>[0.10,0.22]</sub>	<b>43.46</b> <sub>[30.94,67.32]</sub>
RBFNN6	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.11</b> <sub>[0.10,0.14]</sub>	<b>9.00</b> <sub>[0.03,17.35]</sub>
$\eta^{\mathcal{R}}$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.05 <sub>[0.05,0.05]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.07 <sub>[0.07,0.07]</sub>	0.05 <sub>[0.05,0.06]</sub>	0.42 <sub>[0.41,0.44]</sub>	95.21 <sub>[84.01,110.08]</sub>
P2	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.13</b> <sub>[0.12,0.15]</sub>	<b>46.36</b> <sub>[34.77,53.62]</sub>
P3	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.03</b> <sub>[0.02,0.03]</sub>	<b>0.02</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.10</b> <sub>[0.09,0.12]</sub>	<b>18.29</b> <sub>[0.03,31.80]</sub>
Kriging	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.08</b> <sub>[0.07,0.10]</sub>	<b>24.12</b> <sub>[20.16,32.70]</sub>
GRNN0.1	0.08 <sub>[0.07,0.09]</sub>	0.17 <sub>[0.16,0.18]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.07 <sub>[0.07,0.08]</sub>	0.45 <sub>[0.41,0.48]</sub>	146.32 <sub>[114.12,181.76]</sub>
GRNN1	0.04 <sub>[0.03,0.04]</sub>	0.11 <sub>[0.11,0.12]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.10 <sub>[0.10,0.10]</sub>	0.46 <sub>[0.45,0.47]</sub>	236.41 <sub>[224.68,251.82]</sub>
GRNN3	0.05 <sub>[0.05,0.05]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.16 <sub>[0.15,0.16]</sub>	0.12 <sub>[0.12,0.13]</sub>	0.50 <sub>[0.49,0.51]</sub>	280.15 <sub>[273.84,303.96]</sub>
GRNN6	0.05 <sub>[0.05,0.06]</sub>	0.14 <sub>[0.13,0.14]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.51 <sub>[0.50,0.52]</sub>	310.32 <sub>[288.86,320.80]</sub>
RBFNN0.1	0.19 <sub>[0.17,0.24]</sub>	0.28 <sub>[0.25,0.34]</sub>	0.20 <sub>[0.17,0.26]</sub>	0.20 <sub>[0.15,0.25]</sub>	0.65 <sub>[0.60,0.72]</sub>	291.63 <sub>[243.91,376.72]</sub>
RBFNN1	0.03 <sub>[0.02,0.05]</sub>	0.11 <sub>[0.09,0.15]</sub>	0.08 <sub>[0.07,0.09]</sub>	0.05 <sub>[0.05,0.06]</sub>	0.43 <sub>[0.36,0.51]</sub>	141.93 <sub>[102.86,164.88]</sub>
RBFNN3	<b>0.00</b> <sub>[0.00,0.01]</sub>	0.04 <sub>[0.03,0.06]</sub>	0.03 <sub>[0.03,0.04]</sub>	<b>0.02</b> <sub>[0.01,0.02]</sub>	0.21 <sub>[0.17,0.25]</sub>	<b>66.61</b> <sub>[38.24,84.19]</sub>
RBFNN6	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.03</b> <sub>[0.03,0.05]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.11</b> <sub>[0.10,0.14]</sub>	<b>25.51</b> <sub>[0.05,57.67]</sub>
$\eta^{\mathcal{P}}$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.01 <sub>[0.01,0.01]</sub>	0.05 <sub>[0.05,0.05]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.20 <sub>[0.19,0.21]</sub>	106.66 <sub>[100.95,115.13]</sub>
P2	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>25.97</b> <sub>[19.83,30.10]</sub>
P3	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.00,0.01]</sub>	<b>0.03</b> <sub>[0.03,0.04]</sub>	<b>1.36</b> <sub>[0.01,3.33]</sub>
Kriging	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.04</b> <sub>[0.03,0.05]</sub>	<b>9.00</b> <sub>[1.54,14.67]</sub>
GRNN0.1	0.06 <sub>[0.05,0.07]</sub>	0.14 <sub>[0.13,0.15]</sub>	0.08 <sub>[0.08,0.09]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.40 <sub>[0.38,0.42]</sub>	218.18 <sub>[193.68,236.20]</sub>
GRNN1	0.01 <sub>[0.01,0.02]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.09 <sub>[0.08,0.09]</sub>	0.47 <sub>[0.45,0.48]</sub>	263.97 <sub>[245.84,273.84]</sub>
GRNN3	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.15 <sub>[0.15,0.16]</sub>	0.12 <sub>[0.12,0.12]</sub>	0.59 <sub>[0.57,0.60]</sub>	348.25 <sub>[327.75,366.83]</sub>
GRNN6	0.01 <sub>[0.01,0.01]</sub>	0.07 <sub>[0.06,0.08]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.63 <sub>[0.62,0.63]</sub>	355.42 <sub>[345.97,366.59]</sub>
RBFNN0.1	0.16 <sub>[0.12,0.28]</sub>	0.26 <sub>[0.23,0.34]</sub>	0.20 <sub>[0.18,0.28]</sub>	0.17 <sub>[0.14,0.21]</sub>	0.66 <sub>[0.59,0.75]</sub>	379.97 <sub>[314.87,450.72]</sub>
RBFNN1	0.02 <sub>[0.02,0.06]</sub>	0.10 <sub>[0.09,0.12]</sub>	0.07 <sub>[0.06,0.08]</sub>	0.05 <sub>[0.04,0.06]</sub>	0.45 <sub>[0.40,0.49]</sub>	211.55 <sub>[179.51,256.81]</sub>
RBFNN3	0.00 <sub>[0.00,0.00]</sub>	0.03 <sub>[0.02,0.04]</sub>	0.02 <sub>[0.01,0.03]</sub>	0.01 <sub>[0.01,0.02]</sub>	0.13 <sub>[0.11,0.16]</sub>	46.01 <sub>[40.68,63.58]</sub>
RBFNN6	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.04,0.07]</sub>	<b>6.35</b> <sub>[1.89,15.72]</sub>

Table 5–24: Surrogate model performance for the FSCW-SMPM motor Efficiency  $\eta$

$P_{ePM}^M$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.01 <sub>[0.01,0.01]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.02 <sub>[0.02,0.02]</sub>	0.13 <sub>[0.13,0.14]</sub>	<b>0.05</b> <sub>[0.05,0.05]</sub>
P2	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.07</b> <sub>[0.06,0.08]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>
P3	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.03,0.07]</sub>	<b>5.15</b> <sub>[2.93,8.67]</sub>
Kriging	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.04,0.07]</sub>	<b>3.75</b> <sub>[0.02,8.29]</sub>
GRNN0.1	0.05 <sub>[0.04,0.05]</sub>	0.14 <sub>[0.13,0.15]</sub>	0.08 <sub>[0.07,0.08]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.30 <sub>[0.27,0.33]</sub>	81.18 <sub>[17.90,116.61]</sub>
GRNN1	0.01 <sub>[0.01,0.01]</sub>	0.07 <sub>[0.06,0.07]</sub>	0.11 <sub>[0.11,0.12]</sub>	0.09 <sub>[0.09,0.09]</sub>	0.38 <sub>[0.36,0.39]</sub>	165.44 <sub>[143.92,171.43]</sub>
GRNN3	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.08]</sub>	0.16 <sub>[0.16,0.16]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.50 <sub>[0.50,0.51]</sub>	223.73 <sub>[216.78,239.35]</sub>
GRNN6	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.06,0.08]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.55 <sub>[0.54,0.56]</sub>	256.46 <sub>[242.15,265.40]</sub>
RBFNN0.1	0.14 <sub>[0.11,0.24]</sub>	0.26 <sub>[0.23,0.30]</sub>	0.20 <sub>[0.18,0.23]</sub>	0.18 <sub>[0.15,0.23]</sub>	0.66 <sub>[0.61,0.73]</sub>	243.74 <sub>[174.30,315.54]</sub>
RBFNN1	0.03 <sub>[0.02,0.04]</sub>	0.12 <sub>[0.09,0.15]</sub>	0.08 <sub>[0.06,0.09]</sub>	0.06 <sub>[0.04,0.07]</sub>	0.40 <sub>[0.35,0.47]</sub>	97.92 <sub>[26.95,137.94]</sub>
RBFNN3	0.00 <sub>[0.00,0.01]</sub>	0.03 <sub>[0.02,0.04]</sub>	0.02 <sub>[0.01,0.02]</sub>	0.01 <sub>[0.01,0.02]</sub>	0.15 <sub>[0.09,0.21]</sub>	15.13 <sub>[3.12,29.12]</sub>
RBFNN6	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.08</b> <sub>[0.06,0.09]</sub>	<b>0.03</b> <sub>[0.02,6.65]</sub>
$P_{ePM}^R$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.00 <sub>[0.00,0.00]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.03 <sub>[0.03,0.03]</sub>	0.14 <sub>[0.13,0.14]</sub>	<b>0.06</b> <sub>[0.06,0.07]</sub>
P2	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.05,0.06]</sub>	<b>0.03</b> <sub>[0.03,1.26]</sub>
P3	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.05</b> <sub>[0.04,0.05]</sub>	<b>0.02</b> <sub>[0.02,0.05]</sub>
Kriging	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.06</b> <sub>[0.05,0.07]</sub>	<b>0.02</b> <sub>[0.02,1.44]</sub>
GRNN0.1	0.05 <sub>[0.04,0.06]</sub>	0.14 <sub>[0.13,0.16]</sub>	0.08 <sub>[0.08,0.09]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.36 <sub>[0.33,0.37]</sub>	61.93 <sub>[47.75,76.67]</sub>
GRNN1	0.01 <sub>[0.01,0.01]</sub>	0.07 <sub>[0.06,0.08]</sub>	0.12 <sub>[0.11,0.12]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.44 <sub>[0.42,0.46]</sub>	133.44 <sub>[124.72,143.34]</sub>
GRNN3	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.16 <sub>[0.15,0.16]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.57 <sub>[0.57,0.58]</sub>	191.66 <sub>[183.92,198.63]</sub>
GRNN6	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.61 <sub>[0.60,0.62]</sub>	219.47 <sub>[209.17,226.31]</sub>
RBFNN0.1	0.17 <sub>[0.13,0.26]</sub>	0.28 <sub>[0.25,0.30]</sub>	0.21 <sub>[0.19,0.28]</sub>	0.18 <sub>[0.15,0.25]</sub>	0.67 <sub>[0.60,0.75]</sub>	233.98 <sub>[126.12,307.47]</sub>
RBFNN1	0.03 <sub>[0.02,0.06]</sub>	0.10 <sub>[0.08,0.13]</sub>	0.08 <sub>[0.06,0.10]</sub>	0.05 <sub>[0.04,0.06]</sub>	0.38 <sub>[0.31,0.46]</sub>	32.55 <sub>[0.29,56.70]</sub>
RBFNN3	0.00 <sub>[0.00,0.01]</sub>	0.02 <sub>[0.02,0.04]</sub>	0.02 <sub>[0.01,0.03]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>	0.13 <sub>[0.09,0.17]</sub>	6.73 <sub>[0.18,13.60]</sub>
RBFNN6	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.02]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.06</b> <sub>[0.06,0.08]</sub>	<b>0.04</b> <sub>[0.02,1.99]</sub>
$P_{ePM}^P$						
Surrogate	Performance metrics					
	SR	KT	RMSE	MAE	MAX	MAPE
P1	0.00 <sub>[0.00,0.00]</sub>	0.04 <sub>[0.04,0.04]</sub>	0.02 <sub>[0.02,0.02]</sub>	0.02 <sub>[0.02,0.02]</sub>	0.11 <sub>[0.11,0.12]</sub>	<b>0.06</b> <sub>[0.05,0.06]</sub>
P2	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.04</b> <sub>[0.04,0.04]</sub>	<b>1.56</b> <sub>[0.02,4.34]</sub>
P3	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.02</b> <sub>[0.02,0.03]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>
Kriging	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.03</b> <sub>[0.02,0.04]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>
GRNN0.1	0.05 <sub>[0.04,0.06]</sub>	0.13 <sub>[0.13,0.14]</sub>	0.08 <sub>[0.07,0.09]</sub>	0.06 <sub>[0.06,0.07]</sub>	0.30 <sub>[0.28,0.33]</sub>	74.61 <sub>[49.27,111.55]</sub>
GRNN1	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.08]</sub>	0.11 <sub>[0.11,0.11]</sub>	0.09 <sub>[0.09,0.10]</sub>	0.38 <sub>[0.36,0.39]</sub>	164.79 <sub>[157.69,170.98]</sub>
GRNN3	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.06]</sub>	0.16 <sub>[0.16,0.16]</sub>	0.13 <sub>[0.13,0.13]</sub>	0.51 <sub>[0.50,0.52]</sub>	223.50 <sub>[215.36,239.96]</sub>
GRNN6	0.01 <sub>[0.01,0.01]</sub>	0.06 <sub>[0.05,0.07]</sub>	0.17 <sub>[0.17,0.17]</sub>	0.14 <sub>[0.14,0.14]</sub>	0.55 <sub>[0.54,0.56]</sub>	255.59 <sub>[241.85,261.74]</sub>
RBFNN0.1	0.22 <sub>[0.14,0.31]</sub>	0.27 <sub>[0.25,0.34]</sub>	0.22 <sub>[0.19,0.35]</sub>	0.17 <sub>[0.15,0.22]</sub>	0.65 <sub>[0.57,0.76]</sub>	226.99 <sub>[161.11,307.66]</sub>
RBFNN1	0.03 <sub>[0.02,0.04]</sub>	0.10 <sub>[0.08,0.13]</sub>	0.07 <sub>[0.07,0.10]</sub>	0.05 <sub>[0.04,0.06]</sub>	0.35 <sub>[0.32,0.40]</sub>	74.76 <sub>[27.04,136.33]</sub>
RBFNN3	0.00 <sub>[0.00,0.01]</sub>	0.03 <sub>[0.01,0.05]</sub>	0.01 <sub>[0.01,0.03]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	0.09 <sub>[0.07,0.20]</sub>	10.00 <sub>[0.80,16.27]</sub>
RBFNN6	<b>0.00</b> <sub>[0.00,0.00]</sub>	<b>0.01</b> <sub>[0.01,0.01]</sub>	<b>0.01</b> <sub>[0.00,0.01]</sub>	<b>0.00</b> <sub>[0.00,0.01]</sub>	<b>0.03</b> <sub>[0.03,0.05]</sub>	<b>0.01</b> <sub>[0.01,0.02]</sub>

Table 5–25: Surrogate model performance for the FSCW-SMPM motor Permanent Magnet Eddy Current Losses  $P_{ePM}$

Finally, Table 5–26 presents the average performance of the best surrogate models for the FSCW-SMPM design problem. It can be seen that full surrogate problem performances were extremely high. If we extrapolate the results obtained for the benchmark analytical problems (see Tables 4–7 and 4–8) it is possible that the non-dominated set obtained with this surrogate problem would be very close to an ideal set.

Objective Function	Surrogate Model
$T_{avg}^{\mathcal{M}}$	P2
$T_{avg}^{\mathcal{R}}$	P2
$T_{avg}^{\mathcal{P}}$	P2
$T_{rip}^{\mathcal{M}}$	P1
$T_{rip}^{\mathcal{R}}$	Kriging
$T_{rip}^{\mathcal{P}}$	P2
$J_{rms}^{\mathcal{M}}$	-
$\eta^{\mathcal{M}}$	P2
$pf^{\mathcal{M}}$	Kriging
$pf^{\mathcal{R}}$	P2
$pf^{\mathcal{P}}$	P2
$P_{ePM}^{\mathcal{M}}$	P2
$P_{ePM}^{\mathcal{R}}$	P2
$P_{ePM}^{\mathcal{P}}$	P2
Performance	
NKT	0.12 <sub>[0.10,0.13]</sub>
NSR	0.11 <sub>[0.09,0.12]</sub>

Table 5–26: Median and interquartile interval of best surrogate models’ performance for the FSCW-SMPM motor multi-objective design problem

## 5.5 Summary and discussion

The literature on the use of meta-models for electrical machine design problems is wide. Most of the research, however, has focused on Kriging and a few papers on RBFNNs. This choice of meta-models is understandable given that they are extremely flexible and make no assumption about the shape of the underlying objective functions. However, the results presented in this chapter show that they are rarely the best option.

It is important to remark that most of the objective functions considered here are derived from the variations in energy at steady state operation. Hence, since energy is not a quantity that can vary abruptly (at least, not for the design variables considered here),

these functions are expected to be relatively smooth and therefore easily represented by low order polynomials. The results presented in this chapter give support to this hypothesis and suggest that simpler surrogate models can be used for optimization these types of machine.

If single-objective optimization problems are considered, the presented results even raise doubts on the need for surrogate models. Considering that most of these objectives are easily represented with second order polynomials, it is very likely that, deterministic, local search algorithms, e.g. Sequential Quadratic Programming (SQP), will be extremely efficient in most of them.

Furthermore, the objective reduction methodology, has also proven to be very useful. For the two most challenging problems, in terms of the number of objectives, presented here, it could easily find redundant objectives. In addition, the NSR and NKT-based formulations have been shown to be able to identify both trivial and non-trivial redundancies.

If we consider as a reasonable approximation of the Pareto front, the optimal solution of each objective plus the mid-point of each pair of objectives, an m-objective problem will require  $m + \binom{m}{2}$  solutions. For our 18-objective example, 171 points are required to represent the front, which corresponds to 28.5 hours (not considering the optimization cost). Removing 4 objectives reduces this requirement to 28 points which corresponds to 5 hours. This is a rather simplistic analysis but shows how important it is to eliminate redundant problems, even if only a few.

Overall, the results presented here have the potential to refresh the view of the scientific community on how these problems should be formulated and on which optimization strategies should be used to tackle them.

## CHAPTER 6

### Conclusion

The ability of certain models to explain physical phenomena with a very high degree of accuracy, combined with the evolution of numerical methods, makes the computational simulation of devices and processes possible and precise. With these tools, a significant part of the design process of almost anything is now computer-aided. When pushing a design's performance to the limit, researchers and practitioners often rely on optimization methods. Unfortunately, high-fidelity simulation models tend to be computationally expensive. Thus, optimization methods that, depending on the number of objectives and variables, can require thousands of function evaluations may become impractical.

The literature on this topic usually resorts to surrogate models or objective reduction to alleviate the computational burden of the optimization process. Surrogate models are cheap(er) to compute emulators of the high-fidelity model. They reduce the optimization cost by not requiring the use of the high-fidelity and computationally expensive model for all the function evaluations. Objective reduction is the process of finding redundant objectives in the original problem formulation. By reducing the number of objectives, the computational requirements of the optimization process are also reduced.

In order to select a surrogate model or a reduced set of objectives, one has to evaluate their suitability as substitutes of the original problem. In general, researchers use different definitions of suitability for each approach. In this thesis, we unify both approaches under the name, Surrogate Problems, and propose a definition of quality that is also applicable for both. Although the proposed definition is not useful in practice, it serves as a base for comparison when other quality metrics are taken into consideration.

In the scope of surrogate models, there used to be no strong theory to tie surrogate models and optimization performance. Based on the proposed definitions, this thesis takes a

step in this direction and provides, in the form of theorems and proofs, a formal justification for the use of error and rank based metrics in both single and multi-objective optimization. We also show that when compared to the proposed definitions, the error-based metrics are too restrictive and in numerous scenarios unable to identify high-quality surrogate models. Rank-based metrics, on the other hand, have been shown to be much more adequate for surrogate model-based optimization.

In the scope of reduced problems, we show that the available performance metrics are mostly in accordance with the proposed definition but, since they require faithful representations of the original Pareto front, they become too sensitive to the sample data. To overcome this problem we have proposed an extension of rank-correlation metrics that is based on the non-dominated ranks instead of objective-wise ranks. Thus, instead of concentrating on the Pareto front approximation only, these metrics are able to assess the overall disruption of the Pareto dominance structure.

The empirical results in a number of well-known benchmark analytical optimization problems show that the metrics based on non-dominated ranking not only make the objective reduction methods less sensitive to the sample data but also show their superiority when compared to the objective-wise rank-based metrics. The results indicate that the objective-wise metrics can, in some circumstances, underestimate the overall dominance disruption.

After understanding the performance metrics and some of their implications, we present a comprehensive evaluation of surrogate problems for realistic motor design problems. The results show that for most of the objectives considered, low-order polynomials present the best performance under the majority of the metrics. This knowledge has the potential to influence not only the selection of surrogate models for this kind of problem but also the optimization methodology to tackle them.

## 6.1 Extensions

An interesting road opened by this thesis is the formal analysis of surrogate-based optimization methods. Here we have concentrated only on the conditions for which the

solution of a surrogate problem guarantees optimality of the original problem. Based on metrics that measure the probability of discordance and concordance, this analysis can be extended to situations in which the surrogate models are imperfect and provide lower and upper bounds for the overall optimization method performance.

Another possible extension regards the analysis of performance metrics for surrogate constraints. The optimization problems considered in this thesis are either unconstrained or have constraints that are cheap to compute. Constraints and objective functions, however, play different roles in optimization problems. Therefore, surrogate constraints probably require performance metrics that are able to measure different characteristics. It is also important to remark that for the question asked in this thesis, apart from the used surrogate problem, we have tried to isolate any other factor that would interfere with the optimization performance as much as possible. Thus, at this point, we do not know how the quality of the surrogate problem interacts with other processes that can be incorporated into that SBO methodology. Thus, extensions to this work could also incorporate different sampling methods, different optimization and infill point selection algorithms.

Finally, one of the important lessons of this thesis is that commonly used error-based measures are not of much importance for the optimization performance. In other words, it means that model accuracy is not a requirement for the optimization success. This lesson may have important consequences on how other types of surrogate models, in particular, first principle based models, are seen. Although, under the current standards, some of them may be deemed too inaccurate to be useful, since accuracy is not a requirement anymore, they may even become actual substitutes for the finite element based simulations in the context of optimization.

## **Appendix A - Analytical benchmark problems**

This appendix presents the set of benchmark functions used in this work. The “zdt” and the “dtlz” test sets were introduced in (Zitzler et al., 2000b) and (Deb et al., 2002), respectively. The Matlab® implementations (Tian et al., 2017) of the problems listed below can be found at <https://goo.gl/MVZzkx>.

- $zdt2$

$$\begin{aligned}
 f_1 &= x_1 \\
 f_2 &= g(\mathbf{x}) \cdot [1 - (x_1/g(\mathbf{x}))^2] \\
 g(\mathbf{x}) &= 1 + \frac{9}{n-1} (\sum_{i=2}^n x_i) \\
 0 \leq x_i &\leq 1, i = 1, \dots, n \\
 n &= 20 \text{ (Unless otherwise stated)}
 \end{aligned} \tag{6.1}$$

- $zdt3$

$$\begin{aligned}
 f_1 &= x_1 \\
 f_2 &= g(\mathbf{x}) \cdot \left[ 1 - \sqrt{x_1/g(\mathbf{x})} - x_1/g(\mathbf{x}) \cdot \sin(10\pi x_1) \right] \\
 g(\mathbf{x}) &= 1 + \frac{9}{n-1} (\sum_{i=2}^n x_i) \\
 0 \leq x_i &\leq 1, i = 1, \dots, n \\
 n &= 10 \text{ (Unless otherwise stated)}
 \end{aligned} \tag{6.2}$$

- $zdt4$

$$\begin{aligned}
 f_1 &= x_1 \\
 f_2 &= g(\mathbf{x}) \cdot [1 - (x_1/g(\mathbf{x}))^2] \\
 g(\mathbf{x}) &= 1 + 10(n-1) + \sum_{i=2}^n (x_i^2 - 10 \cos(4\pi x_i)) \\
 0 \leq x_i &\leq 1, i = 1, \dots, n \\
 n &= 8 \text{ (Unless otherwise stated)}
 \end{aligned} \tag{6.3}$$

- $zdt6$

$$\begin{aligned}
 f_1 &= 1 - \exp(-4x_1) \cdot \sin^6(6\pi x_1) \\
 f_2 &= 1 - \left( \frac{f_1(\mathbf{x})}{g(\mathbf{x})} \right)^2 \\
 g(\mathbf{x}) &= 1 + \left( \frac{\sum_{i=2}^n x_i}{n-1} \right)^{0.25} \\
 0 \leq x_i &\leq 1, i = 1, \dots, n \\
 n &= 8 \text{ (Unless otherwise stated)}
 \end{aligned} \tag{6.4}$$

- $dtlz1$

$$\begin{aligned}
f_1(\mathbf{x}) &= \frac{1}{2}x_1x_2(1 + g(\mathbf{x})) \\
f_2(\mathbf{x}) &= \frac{1}{2}x_1x_2(1 - x_2)(1 + g(\mathbf{x})) \\
f_3(\mathbf{x}) &= \frac{1}{2}x_1(1 - x_2)(1 + g(\mathbf{x}))
\end{aligned} \tag{6.5}$$

$$g(\mathbf{x}) = 100 (|\mathbf{x}| + \sum_{i=3}^n (x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5)))$$

$$0 \leq x_i \leq 1, i = 1, \dots, n$$

- $dtlz2$

$$\begin{aligned}
f_1(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos(x_1\pi/2) \cos(x_2\pi/2) \\
f_2(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos(x_1\pi/2) \sin(x_2\pi/2) \\
f_3(\mathbf{x}) &= (1 + g(\mathbf{x})) \sin(x_1\pi/2) \\
g(\mathbf{x}) &= \sum_{i=3}^n (x_i - 0.5)^2
\end{aligned} \tag{6.6}$$

$$0 \leq x_i \leq 1, i = 1, \dots, n$$

- $dtlz2_{(M,M)}$

$$f_j(\mathbf{x}) = \begin{cases} (1 + g(\mathbf{x})) \prod_{i=1}^{M-1} \cos(x_i\pi/2) & , j = 1 \\ (1 + g(\mathbf{x})) \left( \prod_{i=1}^{M-j} \cos(x_i\pi/2) \right) \sin(x_{M-j+1}) & , 1 < j < M \\ (1 + g(\mathbf{x})) \sin(x_1\pi/2) & , j = M \end{cases} \tag{6.7}$$

$$g(\mathbf{x}) = \sum_{i=M}^{M+k-1} (x_i - 0.5)^2 \tag{6.8}$$

$$0 \leq x_i \leq 1, i = 1, \dots, M+k-1, k = 1 \tag{6.9}$$

- $dtlz3$

$$\begin{aligned}
f_1(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos(x_1\pi/2) \cos(x_2\pi/2) \\
f_2(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos(x_1\pi/2) \sin(x_2\pi/2) \\
f_3(\mathbf{x}) &= (1 + g(\mathbf{x})) \sin(x_1\pi/2)
\end{aligned} \tag{6.10}$$

$$g(\mathbf{x}) = 100 (|\mathbf{x}| + \sum_{i=3}^n (x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5)))$$

$$0 \leq x_i \leq 1, i = 1, \dots, n$$

- *dtlz4*

$$\begin{aligned}
f_1(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos(x_1^{100} \frac{\pi}{2}) \\
f_2(\mathbf{x}) &= (1 + g(\mathbf{x})) \sin(x_1^{100} \frac{\pi}{2}) \\
g(\mathbf{x}) &= \sum_{i=2}^n (x_i - 0.5)^2 \\
0 \leq x_i &\leq 1, i = 1, \dots, n
\end{aligned} \tag{6.11}$$

- *dtlz5*

$$\begin{aligned}
f_1(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos\left(\frac{\pi}{4(1+g(\mathbf{x}))}(1 + 2g(\mathbf{x})x_1)\right) \\
f_2(\mathbf{x}) &= (1 + g(\mathbf{x})) \sin\left(\frac{\pi}{4(1+g(\mathbf{x}))}(1 + 2g(\mathbf{x})x_2)\right) \\
g(\mathbf{x}) &= \sum_{i=3}^n x_i^{0.1} \\
0 \leq x_i &\leq 1, i = 1, \dots, n
\end{aligned} \tag{6.12}$$

- *dtlz5<sub>(M,I)</sub>*

$$f_j(\mathbf{x}) = \begin{cases} (1 + g(\mathbf{x})) \prod_{i=1}^{M-1} \cos(\theta_i) & , j = 1 \\ (1 + g(\mathbf{x})) \left( \prod_{i=1}^{M-j} \cos(\theta_i) \right) \sin(\theta_{M-j+1}) & , 1 < j < M \\ (1 + g(\mathbf{x})) \sin(\theta_i) & , j = M \end{cases} \tag{6.13}$$

$$\theta_i(\mathbf{x}) = \begin{cases} (\pi/2)x_i & , i = 1, \dots, I-1 \\ \frac{\pi}{4(1+g(\mathbf{x}))}(1 + 2g(\mathbf{x})x_i) & , i = I, \dots, M-1 \end{cases} \tag{6.14}$$

$$g(\mathbf{x}) = \sum_{i=M}^{M+k-1} (x_i - 0.5)^2 \tag{6.15}$$

$$0 \leq x_i \leq 1, \quad i = 1, \dots, M+k-1, \quad k = 9 \tag{6.16}$$

- *dtlz6*

$$\begin{aligned}
f_1(\mathbf{x}) &= x_1 \\
f_2(\mathbf{x}) &= x_2 \\
f_3(\mathbf{x}) &= (1 + g(\mathbf{x})) \cdot \left( 3 - \sum_{i=1}^2 \frac{f_i}{1+g(\mathbf{x})} (1 + \sin(3\pi f_i)) \right) \\
g(\mathbf{x}) &= 1 + \frac{9}{|\mathbf{x}|} \sum_{i=3}^n x_i \\
0 \leq x_i &\leq 1, i = 1, \dots, n
\end{aligned} \tag{6.17}$$

- $wfg3_{(M)}$  (Huband et al., 2006)

$$f_j(\mathbf{x}) = \begin{cases} \left( \frac{\prod_{i=1}^{M-1} x_i}{g(\mathbf{x})} \right)^{0.02} & , j = 1 \\ \left( A + \min(0, \lfloor g_2(\mathbf{x}) - B \rfloor) \frac{A(B-g_2(\mathbf{x}))}{B} - \min(0, \lfloor C - g_2(\mathbf{x}) \rfloor) \frac{(1-A)(g_2(\mathbf{x})-C)}{1-C} \right)^{0.02} & , 2 < j < 9 \\ \left( A + \min(0, \lfloor g_3(\mathbf{x}) - B \rfloor) \frac{A(B-g_3(\mathbf{x}))}{B} - \min(0, \lfloor C - g_3(\mathbf{x}) \rfloor) \frac{(1-A)(g_3(\mathbf{x})-C)}{1-C} \right)^{0.02} & , 10 < j < M-1 \\ & , j = M \end{cases} \quad (6.18)$$

$$g_1(\mathbf{x}) = \left( \left( \prod_{i=1}^{M-1} x_i \right) \times (1 - x_{M-j+1}) \right) \quad (6.19)$$

$$g_2(\mathbf{x}) = \frac{|g_1(\mathbf{x}) - 0.35|}{|[\lfloor 0.35 - g_1(\mathbf{x}) \rfloor] + 0.35|} \quad (6.20)$$

$$g_2(\mathbf{x}) = \frac{|1 - x_1 - 0.35|}{|[\lfloor 0.35 - 1 - x_1 \rfloor] + 0.35|} \quad (6.21)$$

- $p1$

$$\begin{aligned} f_1(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos(x_1\pi/2) \cos(x_2\pi/2) \\ f_2(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos(x_1\pi/2) \sin(x_2\pi/2) \\ f_3(\mathbf{x}) &= f_1(\mathbf{x})^2 + f_2(\mathbf{x})^2 \\ g(\mathbf{x}) &= \sum_{i=M}^{M+k-1} (x_i - 0.5)^2 \\ 0 \leq x_i &\leq 1, i = 1, \dots, 11 \end{aligned} \quad (6.22)$$

- $p2$

$$\begin{aligned} f_1(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos(x_1\pi/2) \cos(x_2\pi/2) \\ f_2(\mathbf{x}) &= (1 + g(\mathbf{x})) \cos(x_1\pi/2) \sin(x_2\pi/2) \\ f_3(\mathbf{x}) &= f_1(\mathbf{x})^2 + f_2(\mathbf{x})^4 + f_1(\mathbf{x})^2 \times f_2(\mathbf{x}) \\ f_4(\mathbf{x}) &= 6 \times f_1(\mathbf{x})^2 + f_2(\mathbf{x}) \\ f_5(\mathbf{x}) &= f_1(\mathbf{x}) + f_2(\mathbf{x}) + f_1(\mathbf{x}) \times f_2(\mathbf{x}) \\ f_6(\mathbf{x}) &= f_1(\mathbf{x})^2 + f_2(\mathbf{x})^2 \\ g(\mathbf{x}) &= \sum_{i=M}^{M+k-1} (x_i - 0.5)^2 \\ 0 \leq x_i &\leq 1, i = 1, \dots, 11 \end{aligned} \quad (6.23)$$

## **Appendix B - Statistical analysis for the surrogate model selection methods**

- Statistical analysis *zdt2*

- GRNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 189.12, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	1.00000	1.00000	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
KT	1.7e-05	4.4e-07	9.7e-10	5.2e-07	—	—	—	—	—	—	—
SR	1.3e-05	3.2e-07	6.4e-10	3.8e-07	1.00000	—	—	—	—	—	—
NKT	0.00031	1.1e-05	5.0e-08	1.3e-05	1.00000	1.00000	—	—	—	—	—
NSR	9.2e-05	3.0e-06	1.0e-08	3.5e-06	1.00000	1.00000	1.00000	—	—	—	—
AKT	1.9e-05	7.4e-07	3.8e-09	8.6e-07	1.00000	1.00000	1.00000	1.00000	—	—	—
NWD	0.00117	4.7e-05	3.2e-07	5.4e-05	1.00000	1.00000	1.00000	1.00000	1.00000	—	—
DEPI	1.00000	1.00000	1.00000	1.00000	4.2e-08	2.8e-08	1.3e-06	3.4e-07	9.8e-08	6.7e-06	—
DETA	1.9e-05	5.2e-07	1.2e-09	6.1e-07	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	5.0e-08

```
P value adjustment method: holm
```

- RBFNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 1.9676, df = 11, p-value = 0.9986
```

- GRNN+RBFNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 2.6118, df = 11, p-value = 0.9949
```

- Statistical analysis *zdt3*

- GRNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 146.33, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	1.00000	1.00000	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
KT	0.00071	0.00036	1.5e-07	0.00029	—	—	—	—	—	—	—
SR	0.00245	0.00130	8.3e-07	0.00103	1.00000	—	—	—	—	—	—
NKT	0.09717	0.06040	0.00019	0.05114	1.00000	1.00000	—	—	—	—	—
NSR	0.00029	0.00015	4.1e-08	0.00012	1.00000	1.00000	1.00000	—	—	—	—
AKT	0.02860	0.01832	7.5e-05	0.01551	1.00000	1.00000	1.00000	1.00000	—	—	—
NWD	0.02491	0.01530	2.5e-05	0.01260	1.00000	1.00000	1.00000	1.00000	1.00000	—	—
DEPI	0.56191	0.81199	1.00000	0.90792	1.6e-09	1.1e-08	5.6e-06	3.9e-10	2.7e-06	5.1e-07	—
DETA	0.00020	0.00010	2.4e-08	7.8e-05	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	2.1e-10

```
P value adjustment method: holm
```

- RBFNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 13.857, df = 11, p-value = 0.241
```

– GRNN+RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 111.81, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	0.34222	0.94621	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	0.67597	—	—	—	—	—	—	—	—
KT	3.5e-07	5.8e-06	0.05338	1.7e-06	—	—	—	—	—	—	—
SR	5.5e-07	8.7e-06	0.06821	2.7e-06	1.00000	—	—	—	—	—	—
NKT	2.9e-06	4.0e-05	0.17255	1.3e-05	1.00000	1.00000	—	—	—	—	—
NSR	6.0e-05	0.00059	0.74411	0.00022	1.00000	1.00000	1.00000	—	—	—	—
AKT	0.00023	0.00163	0.75637	0.00070	1.00000	1.00000	1.00000	1.00000	—	—	—
NWD	1.1e-05	0.00013	0.34222	4.4e-05	1.00000	1.00000	1.00000	1.00000	1.00000	—	—
DEPI	0.41947	1.00000	1.00000	0.75637	0.04145	0.05338	0.13523	0.65099	0.67597	0.27366	—
DETA	0.00012	0.00108	0.93879	0.00042	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.78622

P value adjustment method: holm

- Statistical analysis *zdt4*

– GRNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 68.689, df = 11, p-value = 2.167e-10
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	1.00000	1.00000	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
KT	1.00000	0.04050	0.10098	0.57612	—	—	—	—	—	—	—
SR	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—	—
NKT	0.23766	0.00281	0.00863	0.07479	1.00000	1.00000	—	—	—	—	—
NSR	0.00080	1.2e-06	5.7e-06	0.00013	0.94382	0.02339	1.00000	—	—	—	—
AKT	0.07572	0.00099	0.00286	0.02411	1.00000	0.63687	1.00000	1.00000	—	—	—
NWD	1.00000	0.11376	0.26483	1.00000	1.00000	1.00000	1.00000	0.43230	1.00000	—	—
DEPI	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.03648	0.84036	1.00000	—
DETA	1.00000	1.00000	1.00000	1.00000	0.73518	1.00000	0.10098	0.00022	0.03363	1.00000	1.00000

P value adjustment method: holm

## – RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 32.733, df = 11, p-value = 0.0005812
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	0.01320	0.01077	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	0.00043	—	—	—	—	—	—	—	—
KT	1.00000	1.00000	0.49906	1.00000	—	—	—	—	—	—	—
SR	1.00000	1.00000	1.00000	0.44117	1.00000	—	—	—	—	—	—
NKT	1.00000	1.00000	1.00000	0.22687	1.00000	1.00000	—	—	—	—	—
NSR	1.00000	1.00000	1.00000	0.46019	1.00000	1.00000	1.00000	—	—	—	—
AKT	1.00000	1.00000	1.00000	0.34058	1.00000	1.00000	1.00000	1.00000	—	—	—
NWD	1.00000	1.00000	1.00000	0.23495	1.00000	1.00000	1.00000	1.00000	1.00000	—	—
DEPI	1.00000	1.00000	0.79922	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—
DETA	1.00000	1.00000	0.76705	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

P value adjustment method: holm

## – GRNN+RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 38.614, df = 11, p-value = 6.16e-05
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.000	—	—	—	—	—	—	—	—	—	—
MAPE	0.023	0.091	—	—	—	—	—	—	—	—	—
MAX	1.000	1.000	0.019	—	—	—	—	—	—	—	—
KT	1.000	1.000	1.000	1.000	—	—	—	—	—	—	—
SR	0.172	0.535	1.000	0.151	1.000	—	—	—	—	—	—
NKT	0.172	0.535	1.000	0.151	1.000	1.000	—	—	—	—	—
NSR	0.092	0.295	1.000	0.080	1.000	1.000	1.000	—	—	—	—
AKT	0.716	1.000	1.000	0.645	1.000	1.000	1.000	1.000	—	—	—
NWD	0.054	0.178	1.000	0.046	1.000	1.000	1.000	1.000	1.000	—	—
DEPI	1.000	1.000	1.000	0.940	1.000	1.000	1.000	1.000	1.000	1.000	—
DETA	0.134	0.416	1.000	0.115	1.000	1.000	1.000	1.000	1.000	1.000	1.000

P value adjustment method: holm

- Statistical analysis *zdt6*

- GRNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 96.673, df = 11, p-value = 8.122e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	1.00000	1.00000	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
KT	0.05323	0.04662	1.4e-05	0.04167	—	—	—	—	—	—	—
SR	0.02471	0.02100	3.1e-06	0.01818	1.00000	—	—	—	—	—	—
NKT	0.01499	0.01286	1.3e-06	0.01103	1.00000	1.00000	—	—	—	—	—
NSR	0.04662	0.04125	1.0e-05	0.03617	1.00000	1.00000	1.00000	—	—	—	—
AKT	0.08220	0.07313	9.0e-05	0.06602	1.00000	1.00000	1.00000	1.00000	—	—	—
NWD	0.03306	0.02826	5.3e-06	0.02471	1.00000	1.00000	1.00000	1.00000	1.00000	—	—
DEPI	1.00000	1.00000	1.00000	1.00000	0.00395	0.00125	0.00062	0.00320	0.01031	0.00190	—
DETA	0.01817	0.01535	1.8e-06	0.01348	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.00081

```
P value adjustment method: holm
```

- RBFNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 42.312, df = 11, p-value = 1.43e-05
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	9.9e-05	0.00056	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	0.00994	—	—	—	—	—	—	—	—
KT	1.00000	1.00000	0.05898	1.00000	—	—	—	—	—	—	—
SR	1.00000	1.00000	0.10411	1.00000	1.00000	—	—	—	—	—	—
NKT	0.32007	0.84508	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—
NSR	1.00000	1.00000	0.00228	1.00000	1.00000	1.00000	1.00000	—	—	—	—
AKT	1.00000	1.00000	0.00072	1.00000	1.00000	1.00000	0.56189	1.00000	—	—	—
NWD	1.00000	1.00000	0.00337	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—
DEPI	0.25851	0.70052	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.47125	1.00000	—
DETA	1.00000	1.00000	0.01655	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

```
P value adjustment method: holm
```

– GRNN+RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 32.431, df = 11, p-value = 0.0006502
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	0.57766	0.00133	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	0.20240	—	—	—	—	—	—	—	—
KT	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—	—	—
SR	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—	—
NKT	1.00000	1.00000	0.01381	1.00000	1.00000	1.00000	—	—	—	—	—
NSR	1.00000	1.00000	0.71060	1.00000	1.00000	1.00000	1.00000	—	—	—	—
AKT	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—
NWD	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—
DEPI	1.00000	1.00000	0.00051	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—
DETA	1.00000	1.00000	0.00391	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

```
P value adjustment method: holm
```

● Statistical analysis *dtlz1*

– GRNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 221.32, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	1.00000	1.00000	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
KT	7.6e-12	9.8e-13	3.6e-10	7.1e-10	—	—	—	—	—	—	—
SR	1.0e-10	1.4e-11	3.8e-09	7.1e-09	1.00000	—	—	—	—	—	—
NKT	0.34568	0.17172	1.00000	1.00000	3.7e-05	0.00021	—	—	—	—	—
NSR	1.00000	0.53316	1.00000	1.00000	3.8e-06	2.5e-05	1.00000	—	—	—	—
AKT	0.00967	0.00375	0.05590	0.07308	0.09163	0.22724	1.00000	1.00000	—	—	—
NWD	6.8e-12	8.6e-13	3.2e-10	6.4e-10	1.00000	1.00000	3.4e-05	3.5e-06	0.08968	—	—
DEPI	1.00000	1.00000	1.00000	1.00000	4.0e-10	4.2e-09	1.00000	1.00000	0.05809	3.6e-10	—
DETA	8.9e-11	1.2e-11	3.4e-09	6.3e-09	1.00000	1.00000	0.00019	2.3e-05	0.22188	1.00000	3.8e-09

```
P value adjustment method: holm
```

## – RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 24.812, df = 11, p-value = 0.009714
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.000	—	—	—	—	—	—	—	—	—	—
MAPE	0.461	1.000	—	—	—	—	—	—	—	—	—
MAX	1.000	1.000	1.000	—	—	—	—	—	—	—	—
KT	1.000	1.000	1.000	1.000	—	—	—	—	—	—	—
SR	1.000	1.000	1.000	1.000	1.000	—	—	—	—	—	—
NKT	0.389	1.000	1.000	1.000	1.000	1.000	—	—	—	—	—
NSR	0.038	1.000	1.000	0.760	1.000	1.000	1.000	—	—	—	—
AKT	0.058	1.000	1.000	0.781	1.000	1.000	1.000	1.000	—	—	—
NWD	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	—	—
DEPI	0.183	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	—
DETA	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

P value adjustment method: holm

## – GRNN+RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 115.66, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.0000	—	—	—	—	—	—	—	—	—	—
MAPE	0.0086	0.0155	—	—	—	—	—	—	—	—	—
MAX	1.0000	1.0000	0.0145	—	—	—	—	—	—	—	—
KT	8.3e-07	1.9e-06	1.0000	1.7e-06	—	—	—	—	—	—	—
SR	1.6e-08	4.2e-08	0.3133	3.7e-08	1.0000	—	—	—	—	—	—
NKT	0.1812	0.2466	1.0000	0.2387	0.1651	0.0217	—	—	—	—	—
NSR	0.0502	0.0797	1.0000	0.0757	0.4558	0.0957	1.0000	—	—	—	—
AKT	0.0452	0.0698	1.0000	0.0667	1.0000	0.5010	1.0000	1.0000	—	—	—
NWD	1.1e-06	2.4e-06	1.0000	2.2e-06	1.0000	1.0000	0.1812	0.5010	1.0000	—	—
DEPI	0.1662	0.2289	1.0000	0.2209	0.1812	0.0252	1.0000	1.0000	1.0000	0.1939	—
DETA	5.8e-07	1.4e-06	1.0000	1.2e-06	1.0000	1.0000	0.1408	0.3944	1.0000	1.0000	0.1592

P value adjustment method: holm

- Statistical analysis *dtlz2*

- GRNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 207.53, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	0.52422	1.00000	—	—	—	—	—	—	—	—	—
MAX	0.29373	0.00666	7.1e-05	—	—	—	—	—	—	—	—
KT	4.7e-06	3.1e-09	1.6e-12	0.13999	—	—	—	—	—	—	—
SR	3.2e-06	1.9e-09	9.1e-13	0.11842	1.00000	—	—	—	—	—	—
NKT	1.00000	0.36696	0.01598	1.00000	0.00153	0.00113	—	—	—	—	—
NSR	1.00000	0.52368	0.02667	1.00000	0.00084	0.00061	1.00000	—	—	—	—
AKT	0.13658	0.00350	5.7e-05	1.00000	0.97153	0.87247	1.00000	1.00000	—	—	—
NWD	1.2e-06	5.7e-10	2.3e-13	0.06854	1.00000	1.00000	0.00055	0.00028	0.64875	—	—
DEPI	0.86126	1.00000	1.00000	0.00021	9.1e-12	5.2e-12	0.03530	0.05694	0.00015	1.4e-12	—
DETA	7.4e-08	2.1e-11	5.1e-15	0.01526	1.00000	1.00000	6.6e-05	3.2e-05	0.25741	1.00000	3.5e-14

```
P value adjustment method: holm
```

- RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 21.873, df = 11, p-value = 0.02538
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00	—	—	—	—	—	—	—	—	—	—
MAPE	1.00	1.00	—	—	—	—	—	—	—	—	—
MAX	1.00	1.00	1.00	—	—	—	—	—	—	—	—
KT	1.00	1.00	1.00	1.00	—	—	—	—	—	—	—
SR	1.00	1.00	1.00	1.00	1.00	—	—	—	—	—	—
NKT	1.00	1.00	1.00	1.00	1.00	1.00	—	—	—	—	—
NSR	1.00	1.00	1.00	1.00	1.00	1.00	—	—	—	—	—
AKT	1.00	1.00	1.00	1.00	0.76	1.00	1.00	1.00	—	—	—
NWD	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	—	—
DEPI	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	—
DETA	0.35	0.40	0.88	1.00	0.22	0.40	1.00	1.00	1.00	1.00	1.00

```
P value adjustment method: holm
```

– GRNN+RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 151.93, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	1.00000	1.00000	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
KT	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—	—	—
SR	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—	—
NKT	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—
NSR	1.00000	0.80412	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—
AKT	1.00000	0.82792	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—
NWD	3.7e-11	7.6e-12	1.2e-09	4.4e-10	7.2e-08	5.5e-08	9.4e-07	2.3e-05	0.00089	—	—
DEPI	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	2.0e-09	—
DETA	2.3e-12	4.2e-13	8.8e-11	3.1e-11	6.6e-09	4.9e-09	9.9e-08	3.0e-06	0.00019	1.00000	1.5e-10

P value adjustment method: holm

● Statistical analysis *dtlz3*

– GRNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 107.4, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	8.2e-08	1.6e-07	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	1.8e-05	—	—	—	—	—	—	—	—
KT	0.00055	0.00087	1.00000	0.02269	—	—	—	—	—	—	—
SR	0.00294	0.00446	1.00000	0.08085	1.00000	—	—	—	—	—	—
NKT	1.00000	1.00000	6.0e-07	1.00000	0.00229	0.01055	—	—	—	—	—
NSR	1.00000	1.00000	7.9e-05	1.00000	0.05853	0.19941	1.00000	—	—	—	—
AKT	0.01727	0.02386	1.00000	0.23244	1.00000	1.00000	0.04690	0.48705	—	—	—
NWD	8.4e-06	1.5e-05	1.00000	0.00081	1.00000	1.00000	4.7e-05	0.00284	1.00000	—	—
DEPI	0.03498	0.04918	0.21963	0.53104	1.00000	1.00000	0.09730	1.00000	1.00000	1.00000	—
DETA	4.7e-05	7.9e-05	1.00000	0.00323	1.00000	1.00000	0.00023	0.01043	1.00000	1.00000	1.00000

P value adjustment method: holm

## – RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 59.771, df = 11, p-value = 1.023e-08
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	0.25590	0.41407	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	0.12016	—	—	—	—	—	—	—	—
KT	1.00000	1.00000	0.16845	1.00000	—	—	—	—	—	—	—
SR	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—	—
NKT	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—
NSR	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—
AKT	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—
NWD	1.00000	1.00000	0.00013	1.00000	1.00000	0.27269	0.30299	0.53151	1.00000	—	—
DEPI	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.01492
DETA	0.01459	0.00740	6.3e-09	0.03516	0.02391	0.00042	0.00049	0.00124	0.00961	1.00000	5.1e-06

P value adjustment method: holm

## – GRNN+RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 20.347, df = 11, p-value = 0.0408
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00	—	—	—	—	—	—	—	—	—	—
MAPE	1.00	1.00	—	—	—	—	—	—	—	—	—
MAX	1.00	1.00	1.00	—	—	—	—	—	—	—	—
KT	1.00	1.00	0.90	1.00	—	—	—	—	—	—	—
SR	1.00	1.00	0.56	1.00	1.00	—	—	—	—	—	—
NKT	1.00	1.00	1.00	1.00	1.00	1.00	—	—	—	—	—
NSR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	—	—	—	—
AKT	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	—	—	—
NWD	1.00	1.00	0.11	1.00	1.00	1.00	1.00	1.00	1.00	—	—
DEPI	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.78	—
DETA	1.00	1.00	0.18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

P value adjustment method: holm

- Statistical analysis *dtlz4*

- GRNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 3.7728, df = 11, p-value = 0.9761
```

- RBFNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 13.878, df = 11, p-value = 0.2398
```

- GRNN+RBFNN

```
Kruskal-Wallis rank sum test
```

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 34.884, df = 11, p-value = 0.000259
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.000	—	—	—	—	—	—	—	—	—	—
MAPE	1.000	1.000	—	—	—	—	—	—	—	—	—
MAX	1.000	1.000	1.000	—	—	—	—	—	—	—	—
KT	1.000	0.019	0.023	1.000	—	—	—	—	—	—	—
SR	1.000	0.049	0.058	1.000	1.000	—	—	—	—	—	—
NKT	1.000	0.041	0.049	1.000	1.000	1.000	—	—	—	—	—
NSR	1.000	0.284	0.325	1.000	1.000	1.000	1.000	—	—	—	—
AKT	1.000	0.427	0.485	1.000	1.000	1.000	1.000	1.000	—	—	—
NWD	1.000	0.814	0.928	1.000	1.000	1.000	1.000	1.000	1.000	—	—
DEPI	1.000	1.000	1.000	1.000	0.315	0.637	0.556	1.000	1.000	1.000	—
DETA	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

```
P value adjustment method: holm
```

- Statistical analysis *dtlz5*

- GRNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 170.5, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	0.04610	1.00000	—	—	—	—	—	—	—	—	—
MAX	1.00000	0.15823	0.00011	—	—	—	—	—	—	—	—
KT	0.00090	1.5e-06	4.6e-12	0.18425	—	—	—	—	—	—	—
SR	0.00066	1.0e-06	2.7e-12	0.15823	1.00000	—	—	—	—	—	—
NKT	1.00000	0.87742	0.00252	1.00000	0.01926	0.01545	—	—	—	—	—
NSR	1.00000	0.13583	8.2e-05	1.00000	0.21219	0.18206	1.00000	—	—	—	—
AKT	1.00000	1.00000	1.00000	0.55065	0.00011	8.2e-05	1.00000	0.49890	—	—	—
NWD	0.01900	9.4e-05	1.3e-09	1.00000	1.00000	1.00000	0.21219	1.00000	0.00241	—	—
DEPI	0.34674	1.00000	1.00000	0.00252	8.5e-10	5.2e-10	0.03622	0.00206	1.00000	1.4e-07	—
DETA	8.7e-06	3.9e-09	1.9e-15	0.00857	1.00000	1.00000	0.00044	0.01049	1.2e-06	1.00000	7.2e-13

```
P value adjustment method: holm
```

- RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 28.535, df = 11, p-value = 0.002679
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00	—	—	—	—	—	—	—	—	—	—
MAPE	1.00	1.00	—	—	—	—	—	—	—	—	—
MAX	1.00	1.00	1.00	—	—	—	—	—	—	—	—
KT	1.00	1.00	1.00	1.00	—	—	—	—	—	—	—
SR	1.00	1.00	1.00	1.00	1.00	—	—	—	—	—	—
NKT	1.00	1.00	1.00	1.00	1.00	1.00	—	—	—	—	—
NSR	1.00	1.00	1.00	1.00	1.00	1.00	—	—	—	—	—
AKT	0.16	0.13	0.95	0.62	0.39	0.23	1.00	1.00	—	—	—
NWD	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	—	—	—
DEPI	0.76	0.63	1.00	1.00	1.00	1.00	1.00	1.00	1.00	—	—
DETA	0.15	0.12	1.00	0.63	0.39	0.22	1.00	1.00	1.00	1.00	—

```
P value adjustment method: holm
```

– GRNN+RBFNN

Kruskal-Wallis rank sum test

```
data: IGD by Metric
Kruskal-Wallis chi-squared = 139.82, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: IGD by Metric
```

	RMSE	MAE	MAPE	MAX	KT	SR	NKT	NSR	AKT	NWD	DEPI
MAE	1.00000	—	—	—	—	—	—	—	—	—	—
MAPE	1.00000	1.00000	—	—	—	—	—	—	—	—	—
MAX	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
KT	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—	—	—
SR	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—	—
NKT	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—
NSR	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	—	—	—	—
AKT	0.05584	0.17659	0.02499	0.02299	0.69951	0.24218	1.00000	0.83572	—	—	—
NWD	1.4e-07	1.4e-06	2.8e-08	2.3e-08	2.7e-05	2.8e-06	0.00018	4.1e-05	1.00000	—	—
DEPI	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.41180	8.5e-06	—
DETA	4.8e-12	8.7e-11	6.4e-13	5.0e-13	3.8e-09	2.1e-10	4.2e-08	6.5e-09	0.02499	1.00000	8.6e-10

```
P value adjustment method: holm
```

**Appendix C - Statistical analysis of surrogate models performances in electric  
motor design problems**

- Surrogate model comparison for the IPM motor design problem of Salimi (2017)

### 1. Average Torque ( $T_{avg}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 252.05, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	6.8e-07	8.2e-08	5.7e-11	5.0e-11	—	—	—	—	—	—	—
P1	9.7e-05	1.6e-05	3.1e-08	2.8e-08	1.00000	—	—	—	—	—	—
P2	2.2e-13	1.2e-14	< 2e-16	< 2e-16	0.51894	0.05029	—	—	—	—	—
P3	0.01209	0.00489	0.00013	0.00012	1.00000	1.00000	0.40490	—	—	—	—
RBFNN0.1	0.00688	0.02456	0.47908	0.48021	< 2e-16	1.5e-15	< 2e-16	9.2e-09	—	—	—
RBFNN1	0.45027	0.19404	0.00600	0.00570	0.02485	0.36765	1.4e-06	1.00000	7.8e-08	—	—
RBFNN3	0.03488	0.01131	0.00012	0.00011	0.36676	1.00000	0.00015	1.00000	1.8e-10	1.00000	—
RBFNN6	0.98126	0.49060	0.02652	0.02554	0.00547	0.11037	1.0e-07	0.52751	1.0e-06	1.00000	1.00000

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 218.4, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	6.3e-06	2.1e-05	1.3e-07	8.6e-09	—	—	—	—	—	—	—
P1	8.1e-05	0.00023	2.2e-06	1.9e-07	1.00000	—	—	—	—	—	—
P2	5.9e-13	3.4e-12	2.1e-15	< 2e-16	0.33225	0.09779	—	—	—	—	—
P3	0.00493	0.01002	0.00041	8.1e-05	1.00000	1.00000	0.22497	—	—	—	—
RBFNN0.1	0.01912	0.00861	0.16433	0.41484	< 2e-16	9.6e-15	< 2e-16	9.2e-10	—	—	—
RBFNN1	0.83732	1.00000	0.18287	0.05804	0.03856	0.16433	5.3e-07	0.63155	2.1e-06	—	—
RBFNN3	0.83732	1.00000	0.18287	0.05804	0.03856	0.16433	5.3e-07	0.63155	2.1e-06	1.00000	—
RBFNN6	1.00000	1.00000	0.89877	0.41454	0.00225	0.01526	4.6e-09	0.15350	0.00011	1.00000	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 243.28, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.40059	—	—	—	—	—	—	—	—	—	—
GRNN3	0.01250	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00284	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00500	2.6e-08	9.9e-12	5.0e-13	—	—	—	—	—	—	—
P1	0.40831	6.8e-05	1.3e-07	1.2e-08	1.00000	—	—	—	—	—	—
P2	1.3e-05	3.1e-12	2.5e-16	< 2e-16	1.00000	0.16847	—	—	—	—	—
P3	0.03033	4.8e-06	1.8e-08	2.2e-09	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	0.00017	0.64495	1.00000	1.00000	2.4e-15	1.5e-10	< 2e-16	5.1e-11	—	—	—
RBFNN1	0.66730	1.00000	1.00000	1.00000	1.1e-07	0.00021	1.8e-11	1.4e-05	0.38256	—	—
RBFNN3	1.00000	1.00000	0.06166	0.01689	0.00070	0.12034	9.8e-07	0.00737	0.00141	1.00000	—
RBFNN6	0.03036	1.00000	1.00000	1.00000	6.7e-11	6.2e-07	2.4e-15	7.0e-08	1.00000	1.00000	0.13606

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 205.15, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.41539	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00331	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	7.4e-05	0.41539	1.00000	—	—	—	—	—	—	—	—
Kriging	0.67026	0.00020	5.5e-08	1.8e-10	—	—	—	—	—	—	—
P1	0.61846	0.00016	3.9e-08	1.2e-10	1.00000	—	—	—	—	—	—
P2	0.00015	9.6e-11	5.3e-16	< 2e-16	0.36149	0.40919	—	—	—	—	—
P3	1.00000	0.05176	0.00027	6.5e-06	1.00000	1.00000	0.07458	—	—	—	—
RBFNN0.1	0.00408	1.00000	1.00000	1.00000	7.6e-08	5.5e-08	8.6e-16	0.00034	—	—	—
RBFNN1	0.16957	1.00000	1.00000	0.89210	3.6e-05	2.8e-05	6.4e-12	0.01707	1.00000	—	—
RBFNN3	1.00000	1.00000	0.24146	0.01639	0.01900	0.01639	1.4e-07	0.67026	0.27585	1.00000	—
RBFNN6	0.00164	1.00000	1.00000	1.00000	1.9e-08	1.4e-08	< 2e-16	0.00014	1.00000	1.00000	0.15358

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 234.7, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.10983	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00211	0.36649	1.00000	—	—	—	—	—	—	—	—
Kriging	0.04752	7.6e-05	5.5e-08	3.4e-11	—	—	—	—	—	—	—
P1	0.13553	0.00039	4.7e-07	4.3e-10	1.00000	—	—	—	—	—	—
P2	1.4e-05	1.1e-09	5.6e-14	< 2e-16	1.00000	0.48022	—	—	—	—	—
P3	0.14598	0.00128	6.3e-06	2.8e-08	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	1.9e-06	0.00321	0.24676	1.00000	2.5e-16	5.3e-15	< 2e-16	4.9e-12	—	—	—
RBFNN1	0.46102	1.00000	1.00000	1.00000	1.8e-06	1.2e-05	5.6e-12	7.7e-05	0.04861	—	—
RBFNN3	1.00000	1.00000	1.00000	0.21252	0.00022	0.00109	5.0e-09	0.00273	0.00131	1.00000	—
RBFNN6	0.23281	1.00000	1.00000	1.00000	3.4e-07	2.5e-06	6.3e-13	2.3e-05	0.11826	1.00000	1.00000

P value adjustment method: holm

## 2. Starting Torque ( $T_{str}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 226.67, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.68484	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.22560	0.68484	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00624	0.00084	1.8e-07	8.9e-09	—	—	—	—	—	—	—
P1	0.00024	2.1e-05	1.3e-09	4.3e-11	1.00000	—	—	—	—	—	—
P2	2.8e-09	9.0e-11	< 2e-16	< 2e-16	0.14664	0.81283	—	—	—	—	—
P3	0.38221	0.15689	0.00255	0.00054	1.00000	1.00000	0.23873	—	—	—	—
RBFNN0.1	5.7e-05	0.00058	0.17809	0.56807	3.9e-16	< 2e-16	< 2e-16	7.4e-08	—	—	—
RBFNN1	1.00000	1.00000	0.08942	0.01875	0.10773	0.00820	5.4e-07	1.00000	5.9e-07	—	—
RBFNN3	1.00000	1.00000	0.11500	0.02610	0.08337	0.00599	3.1e-07	1.00000	1.0e-06	1.00000	—
RBFNN6	1.00000	1.00000	0.03179	0.00599	0.23148	0.02610	3.2e-06	1.00000	9.2e-08	1.00000	1.00000

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 237.36, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.99724	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.28841	0.74724	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00018	3.0e-05	4.3e-09	6.5e-11	—	—	—	—	—	—	—
P1	3.1e-05	4.2e-06	3.2e-10	3.8e-12	1.00000	—	—	—	—	—	—
P2	1.3e-08	1.0e-09	1.0e-14	< 2e-16	1.00000	1.00000	—	—	—	—	—
P3	0.04960	0.01817	0.00011	9.5e-06	1.00000	1.00000	0.83094	—	—	—	—
RBFNN0.1	0.00012	0.00065	0.14778	0.72478	< 2e-16	< 2e-16	< 2e-16	1.8e-10	—	—	—
RBFNN1	1.00000	1.00000	0.80881	0.18340	0.00039	6.9e-05	3.8e-08	0.07603	5.4e-05	—	—
RBFNN3	1.00000	1.00000	0.04095	0.00490	0.03129	0.00840	2.3e-05	0.80881	1.1e-07	1.00000	—
RBFNN6	1.00000	0.83094	0.01268	0.00113	0.09126	0.02915	0.00012	1.00000	1.2e-08	1.00000	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 240.94, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.02527	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00031	0.20136	1.00000	—	—	—	—	—	—	—	—
Kriging	0.01196	3.3e-06	2.0e-10	3.8e-14	—	—	—	—	—	—	—
P1	0.75829	0.00300	1.8e-06	1.7e-09	1.00000	—	—	—	—	—	—
P2	0.00177	2.0e-07	5.5e-12	6.1e-16	1.00000	1.00000	—	—	—	—	—
P3	1.00000	1.00000	1.00000	1.00000	0.00045	0.03277	7.7e-05	—	—	—	—
RBFNN0.1	1.6e-07	0.00154	0.28135	1.00000	< 2e-16	4.0e-14	< 2e-16	0.08482	—	—	—
RBFNN1	1.00000	1.00000	1.00000	0.17265	4.5e-06	0.00379	2.9e-07	1.00000	0.00120	—	—
RBFNN3	0.39894	1.00000	1.00000	0.85795	1.1e-07	0.00023	4.6e-09	1.00000	0.01487	1.00000	—
RBFNN6	4.5e-06	0.01436	1.00000	1.00000	< 2e-16	4.1e-12	< 2e-16	0.28959	1.00000	0.01196	0.10233

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 214.88, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.05692	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00289	0.59336	1.00000	-	-	-	-	-	-	-	-
Kriging	0.12906	0.00024	1.1e-07	6.0e-10	-	-	-	-	-	-	-
P1	0.99808	0.00761	1.3e-05	1.3e-07	1.00000	-	-	-	-	-	-
P2	0.15669	0.00032	1.6e-07	9.5e-10	1.00000	1.00000	-	-	-	-	-
P3	0.00048	0.11076	1.00000	1.00000	4.4e-10	4.9e-08	6.5e-10	-	-	-	-
RBFNN0.1	0.00073	0.26611	1.00000	1.00000	6.5e-11	1.8e-08	1.1e-10	1.00000	-	-	-
RBFNN1	1.00000	1.00000	1.00000	0.47467	0.00035	0.01095	0.00048	0.08647	0.20609	-	-
RBFNN3	7.7e-05	0.06733	1.00000	1.00000	1.4e-12	6.1e-10	2.4e-12	1.00000	1.00000	0.05077	-
RBFNN6	3.1e-08	0.00032	0.07316	0.59336	< 2e-16	1.6e-14	< 2e-16	1.00000	1.00000	0.00021	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 249.99, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.07173	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00082	0.25779	1.00000	-	-	-	-	-	-	-	-
Kriging	0.09220	0.00016	9.4e-08	3.1e-11	-	-	-	-	-	-	-
P1	0.69351	0.00549	1.0e-05	8.9e-09	1.00000	-	-	-	-	-	-
P2	0.02019	1.5e-05	4.8e-09	8.8e-13	1.00000	1.00000	-	-	-	-	-
P3	0.22457	1.00000	1.00000	1.00000	6.7e-06	0.00024	6.8e-07	-	-	-	-
RBFNN0.1	7.1e-10	1.4e-05	0.00704	0.29541	< 2e-16	< 2e-16	< 2e-16	0.03057	-	-	-
RBFNN1	0.00048	0.19278	1.00000	1.00000	1.3e-11	4.1e-09	3.5e-13	1.00000	0.38187	-	-
RBFNN3	0.04951	1.00000	1.00000	1.00000	4.6e-08	5.5e-06	2.3e-09	1.00000	0.01089	1.00000	-
RBFNN6	8.8e-07	0.00278	0.25779	1.00000	5.5e-16	5.1e-13	< 2e-16	0.46613	1.00000	1.00000	0.33167

P value adjustment method: holm

### 3. Torque Ripple ( $T_{rip}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 246.2, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	1.00000	1.00000	1.00000	1.00000	—	—	—	—	—	—	—
P1	0.07080	0.02054	0.00137	5.1e-05	0.00039	—	—	—	—	—	—
P2	0.67239	0.25895	0.03026	0.00218	0.01112	1.00000	—	—	—	—	—
P3	0.00303	0.01047	0.07583	0.42767	0.15855	5.0e-09	4.3e-07	—	—	—	—
RBFNN0.1	7.0e-10	9.0e-09	7.4e-07	3.6e-05	3.7e-06	< 2e-16	< 2e-16	1.00000	—	—	—
RBFNN1	2.5e-05	0.00015	0.00339	0.04436	0.01047	6.6e-14	4.4e-11	1.00000	—	—	—
RBFNN3	1.9e-09	2.3e-08	1.7e-06	7.2e-05	8.2e-06	< 2e-16	< 2e-16	1.00000	1.00000	1.00000	—
RBFNN6	5.6e-08	5.6e-07	2.8e-05	0.00080	0.00011	< 2e-16	8.6e-15	1.00000	1.00000	1.00000	1.00000

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 245.02, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	0.31727	0.24405	1.00000	1.00000	—	—	—	—	—	—	—
P1	0.04095	0.05349	6.7e-05	3.2e-05	7.5e-07	—	—	—	—	—	—
P2	0.92121	1.00000	0.00933	0.00556	0.00030	1.00000	—	—	—	—	—
P3	0.00030	0.00021	0.05349	0.07518	0.38453	3.6e-11	3.3e-08	—	—	—	—
RBFNN0.1	2.2e-08	1.1e-08	0.00015	0.00029	0.00537	< 2e-16	8.6e-15	1.00000	—	—	—
RBFNN1	5.3e-06	3.0e-06	0.00717	0.01188	0.11177	1.1e-15	1.5e-11	1.00000	1.00000	—	—
RBFNN3	1.2e-06	6.6e-07	0.00261	0.00465	0.05253	< 2e-16	1.8e-12	1.00000	1.00000	1.00000	—
RBFNN6	2.3e-10	1.1e-10	4.9e-06	1.0e-05	0.00030	< 2e-16	< 2e-16	1.00000	1.00000	1.00000	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 291.71, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.06444	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00316	0.33451	1.00000	—	—	—	—	—	—	—	—
Kriging	1.00000	0.87873	0.01037	0.00032	—	—	—	—	—	—	—
P1	1.00000	0.06444	0.00016	2.0e-06	1.00000	—	—	—	—	—	—
P2	1.00000	1.00000	0.74059	0.07467	1.00000	0.28911	—	—	—	—	—
P3	3.9e-08	5.4e-05	0.01765	0.19030	1.7e-09	2.5e-12	4.6e-06	—	—	—	—
RBFNN0.1	1.3e-14	7.6e-10	8.8e-06	0.00054	< 2e-16	< 2e-16	1.7e-11	1.00000	—	—	—
RBFNN1	1.6e-12	3.9e-08	0.00018	0.00643	2.1e-14	< 2e-16	1.2e-09	1.00000	1.00000	—	—
RBFNN3	2.1e-11	3.1e-07	0.00080	0.02151	3.4e-13	< 2e-16	1.1e-08	1.00000	1.00000	1.00000	—
RBFNN6	9.4e-14	3.8e-09	3.2e-05	0.00153	1.0e-15	< 2e-16	9.8e-11	1.00000	1.00000	1.00000	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 164.33, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	1.00000	1.00000	1.00000	0.87237	—	—	—	—	—	—	—
P1	0.42614	0.02945	0.00764	0.00224	1.00000	—	—	—	—	—	—
P2	0.05267	0.00173	0.00033	7.4e-05	0.22568	1.00000	—	—	—	—	—
P3	0.01673	0.16551	0.35713	0.63163	0.00390	3.3e-06	1.1e-07	—	—	—	—
RBFNN0.1	1.00000	1.00000	1.00000	1.00000	1.00000	0.00335	0.00012	0.53291	—	—	—
RBFNN1	0.34357	1.00000	1.00000	1.00000	0.08425	3.4e-05	5.7e-07	1.00000	1.00000	—	—
RBFNN3	0.00034	0.01505	0.05293	0.13980	3.2e-05	2.6e-10	9.5e-13	1.00000	0.10433	1.00000	—
RBFNN6	7.1e-09	2.0e-06	1.5e-05	7.4e-05	2.4e-10	< 2e-16	< 2e-16	1.00000	4.6e-05	0.00408	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 276.99, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.14590	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.01566	0.22235	1.00000	—	—	—	—	—	—	—	—
Kriging	1.00000	1.00000	0.30907	0.04266	—	—	—	—	—	—	—
P1	0.24190	0.01791	5.4e-06	7.6e-08	0.10965	—	—	—	—	—	—
P2	1.00000	1.00000	0.38361	0.06079	1.00000	0.07908	—	—	—	—	—
P3	5.4e-06	0.00020	0.06264	0.30907	1.9e-05	1.1e-11	3.1e-05	—	—	—	—
RBFNN0.1	4.1e-11	1.7e-08	0.00035	0.00769	3.6e-10	< 2e-16	7.6e-10	1.00000	—	—	—
RBFNN1	6.5e-10	1.8e-07	0.00190	0.02974	4.8e-09	< 2e-16	1.0e-08	1.00000	1.00000	—	—
RBFNN3	9.7e-11	3.5e-08	0.00059	0.01190	7.8e-10	< 2e-16	1.7e-09	1.00000	1.00000	1.00000	—
RBFNN6	1.7e-14	1.8e-11	2.3e-06	0.00010	2.0e-13	< 2e-16	4.9e-13	1.00000	1.00000	1.00000	1.00000

P value adjustment method: holm

## 4. Efficiency ( $\eta$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 252.46, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.21796	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.09235	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	0.10761	0.00113	2.2e-06	2.6e-07	—	—	—	—	—	—	—
P1	0.00021	2.4e-07	4.7e-11	3.1e-12	1.00000	—	—	—	—	—	—
P2	3.7e-09	3.3e-13	< 2e-16	< 2e-16	0.00873	0.84436	—	—	—	—	—
P3	0.00226	2.2e-05	6.6e-08	1.1e-08	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	3.3e-05	0.00848	0.38575	0.83336	2.9e-13	< 2e-16	< 2e-16	1.1e-13	—	—	—
RBFNN1	1.00000	1.00000	0.13416	0.04852	0.18583	0.00050	1.3e-08	0.00430	1.2e-05	—	—
RBFNN3	0.60385	0.01883	9.1e-05	1.5e-05	1.00000	0.34412	0.00047	0.44153	7.1e-11	0.84436	—
RBFNN6	1.00000	0.09235	0.00078	0.00016	1.00000	0.10446	5.1e-05	0.18583	2.1e-09	1.00000	1.00000

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 241.59, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.13841	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.09171	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	0.80116	0.00589	8.7e-05	4.4e-05	—	—	—	—	—	—	—
P1	0.00173	2.4e-07	4.1e-10	1.5e-10	0.73833	—	—	—	—	—	—
P2	3.8e-09	6.3e-15	< 2e-16	< 2e-16	0.00022	0.31521	—	—	—	—	—
P3	0.00101	1.1e-06	1.0e-08	4.9e-09	0.17926	1.00000	1.00000	—	—	—	—
RBFNN0.1	7.0e-05	0.06901	0.80116	0.91234	7.1e-10	< 2e-16	< 2e-16	1.2e-13	—	—	—
RBFNN1	1.00000	0.80116	0.06513	0.04317	1.00000	0.00503	2.2e-08	0.00235	1.8e-05	—	—
RBFNN3	0.80116	0.00503	7.0e-05	3.4e-05	1.00000	0.80116	0.00028	0.19428	5.0e-10	1.00000	—
RBFNN6	1.00000	0.06479	0.00174	0.00098	1.00000	0.13841	8.1e-06	0.04317	5.7e-08	1.00000	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 218.73, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.06754	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00189	0.31359	1.00000	—	—	—	—	—	—	—	—
Kriging	1.00000	0.06754	0.00022	1.4e-06	—	—	—	—	—	—	—
P1	0.31359	0.00186	1.3e-06	3.0e-09	1.00000	—	—	—	—	—	—
P2	2.1e-05	1.9e-09	2.8e-14	< 2e-16	0.01346	0.27786	—	—	—	—	—
P3	0.87891	0.02296	0.00011	1.2e-06	1.00000	1.00000	0.34546	—	—	—	—
RBFNN0.1	7.8e-07	0.00184	0.25588	1.00000	4.8e-11	2.0e-14	< 2e-16	1.4e-10	—	—	—
RBFNN1	0.00325	0.39091	1.00000	1.00000	2.8e-06	7.1e-09	< 2e-16	2.1e-06	1.00000	—	—
RBFNN3	1.00000	1.00000	1.00000	0.16058	0.16058	0.00581	1.3e-08	0.05484	0.00051	0.22451	—
RBFNN6	0.36400	1.00000	1.00000	1.00000	0.00384	4.7e-05	5.9e-12	0.00159	0.03632	1.00000	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 140.91, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.53880	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.21532	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	0.08433	0.49611	1.00000	1.00000	—	—	—	—	—	—	—
P1	3.8e-07	1.7e-05	0.02299	0.07789	0.20211	—	—	—	—	—	—
P2	9.4e-06	0.00029	0.13520	0.34539	0.79550	1.00000	—	—	—	—	—
P3	1.00000	1.00000	0.21532	0.09139	0.03628	6.0e-07	9.5e-06	—	—	—	—
RBFNN0.1	1.00000	1.00000	0.14250	0.04999	0.01456	1.3e-08	4.5e-07	1.00000	—	—	—
RBFNN1	1.00000	1.00000	0.16594	0.05972	0.01779	1.9e-08	6.2e-07	1.00000	1.00000	—	—
RBFNN3	1.00000	1.00000	0.13520	0.04536	0.01303	1.1e-08	3.8e-07	1.00000	1.00000	1.00000	—
RBFNN6	0.47088	0.07789	0.00011	1.7e-05	2.5e-06	1.4e-14	1.5e-12	1.00000	1.00000	1.00000	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 225, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.25778	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.01505	0.21137	1.00000	—	—	—	—	—	—	—	—
Kriging	1.00000	0.22312	0.00060	5.3e-06	—	—	—	—	—	—	—
P1	0.32746	0.03043	2.1e-05	1.0e-07	1.00000	—	—	—	—	—	—
P2	0.00103	1.7e-05	5.9e-10	4.9e-13	0.32746	1.00000	—	—	—	—	—
P3	1.00000	1.00000	0.63789	0.10037	1.00000	1.00000	0.03765	—	—	—	—
RBFNN0.1	3.2e-09	6.8e-07	0.00289	0.08087	5.4e-15	< 2e-16	< 2e-16	4.8e-06	—	—	—
RBFNN1	0.00032	0.01152	1.00000	1.00000	2.1e-08	2.1e-10	< 2e-16	0.00897	0.78269	—	—
RBFNN3	0.03853	0.38542	1.00000	1.00000	2.4e-05	5.8e-07	4.8e-12	0.20232	0.03308	1.00000	—
RBFNN6	0.00204	0.04539	1.00000	1.00000	2.9e-07	3.7e-09	7.3e-15	0.03008	0.31832	1.00000	1.00000

P value adjustment method: holm

- Surrogate model comparison for the IPM motor-drive design problem of Ghorbanian

and Lowther (2017)

### 1. Cogging Torque ( $T_{cog}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 258.7, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	2.1e-13	-	-	-	-	-	-	-	-	-	-
GRNN3	7.9e-14	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	4.0e-15	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	1.00000	< 2e-16	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	< 2e-16	1.00000	1.00000	1.00000	< 2e-16	-	-	-	-	-	-
P2	2.0e-13	1.00000	1.00000	1.00000	< 2e-16	1.00000	-	-	-	-	-
P3	0.02917	0.00109	0.00071	0.00015	0.00160	5.2e-06	0.00108	-	-	-	-
RBFNN0.1	0.50955	1.1e-06	5.5e-07	6.4e-08	0.04514	5.1e-10	1.1e-06	1.00000	-	-	-
RBFNN1	0.09844	2.4e-05	1.3e-05	1.9e-06	0.00595	2.5e-08	2.3e-05	1.00000	1.00000	-	-
RBFNN3	0.02920	0.00015	8.6e-05	1.5e-05	0.00142	2.7e-07	0.00015	1.00000	1.00000	1.00000	-
RBFNN6	0.00142	0.00446	0.00289	0.00072	3.5e-05	2.5e-05	0.00444	1.00000	1.00000	1.00000	1.00000

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
```

```
Kruskal-Wallis chi-squared = 234.07, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	3.9e-08	-	-	-	-	-	-	-	-	-	-
GRNN3	1.3e-15	0.45580	-	-	-	-	-	-	-	-	-
GRNN6	8.5e-11	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	1.00000	5.2e-11	< 2e-16	4.4e-14	-	-	-	-	-	-	-
P1	2.9e-14	0.95005	1.00000	1.00000	< 2e-16	-	-	-	-	-	-
P2	7.2e-14	1.00000	1.00000	1.00000	< 2e-16	1.00000	-	-	-	-	-
P3	0.22165	0.03610	2.1e-06	0.00141	0.01304	1.4e-05	2.3e-05	-	-	-	-
RBFNN0.1	1.00000	0.00017	2.3e-10	1.7e-06	0.26710	2.8e-09	6.0e-09	1.00000	-	-	-
RBFNN1	0.07089	0.04773	2.1e-06	0.00179	0.00238	1.5e-05	2.5e-05	1.00000	1.00000	-	-
RBFNN3	0.09089	0.03706	1.3e-06	0.00129	0.00335	9.6e-06	1.7e-05	1.00000	1.00000	1.00000	-
RBFNN6	2.9e-05	1.00000	0.01427	0.75689	1.4e-07	0.04773	0.06598	0.75689	0.02131	0.95005	0.82235

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 262.68, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	1.00000	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	1.00000	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	0.65151	0.07351	0.00157	0.00176	-	-	-	-	-	-	-
P1	1.00000	1.00000	1.00000	1.00000	0.00944	-	-	-	-	-	-
P2	0.00213	0.04477	0.69980	0.67670	3.0e-08	0.28010	-	-	-	-	-
P3	9.0e-09	1.1e-06	0.00022	0.00019	9.9e-16	2.5e-05	0.33204	-	-	-	-
RBFNN0.1	1.00000	1.00000	1.00000	1.00000	0.30903	1.00000	0.00811	6.7e-08	-	-	-
RBFNN1	1.3e-08	1.6e-06	0.00034	0.00030	9.9e-16	4.0e-05	0.47947	1.00000	9.9e-08	-	-
RBFNN3	1.5e-12	5.8e-10	4.7e-07	3.9e-07	< 2e-16	3.0e-08	0.00944	1.00000	1.8e-11	1.00000	-
RBFNN6	< 2e-16	8.3e-14	2.5e-10	2.0e-10	< 2e-16	8.7e-12	6.2e-05	0.65151	1.4e-15	0.33204	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 266.76, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.00871	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00087	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00031	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	0.00900	1.00000	1.00000	1.00000	-	-	-	-	-	-	-
P1	1.0e-06	0.96982	1.00000	1.00000	0.96982	-	-	-	-	-	-
P2	0.12045	1.00000	1.00000	1.00000	1.00000	0.13865	-	-	-	-	-
P3	1.00000	8.4e-05	5.0e-06	1.6e-06	8.9e-05	2.2e-09	0.00246	-	-	-	-
RBFNN0.1	1.00000	0.03293	0.00399	0.00166	0.03372	7.8e-06	0.33097	1.00000	-	-	-
RBFNN1	0.22331	2.1e-08	4.3e-10	8.7e-11	2.2e-08	1.4e-14	2.6e-06	1.00000	0.07662	-	-
RBFNN3	0.01155	3.2e-11	3.8e-13	6.1e-14	3.5e-11	< 2e-16	8.4e-09	0.86319	0.00282	1.00000	-
RBFNN6	9.2e-05	3.5e-15	< 2e-16	< 2e-16	3.9e-15	< 2e-16	2.3e-12	0.04688	1.4e-05	0.73880	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 299.04, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	2.9e-07	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00283	0.85558	-	-	-	-	-	-	-	-	-
GRNN6	0.08116	0.10304	1.00000	-	-	-	-	-	-	-	-
Kriging	1.7e-08	1.00000	0.37021	0.02659	-	-	-	-	-	-	-
P1	1.1e-05	1.00000	1.00000	0.49454	1.00000	-	-	-	-	-	-
P2	1.00000	0.00017	0.15622	1.00000	1.9e-05	0.00283	-	-	-	-	-
P3	0.59813	7.8e-13	2.0e-07	3.0e-05	2.1e-14	8.2e-11	0.02958	-	-	-	-
RBFNN0.1	1.00000	2.9e-05	0.05678	0.59813	2.5e-06	0.00055	1.00000	0.08873	-	-	-
RBFNN1	0.85558	6.9e-13	2.7e-07	4.6e-05	1.6e-14	8.5e-11	0.04803	1.00000	0.13612	-	-
RBFNN3	0.01294	< 2e-16	4.9e-12	4.5e-09	< 2e-16	2.2e-16	7.3e-05	1.00000	0.00042	1.00000	-
RBFNN6	0.00021	< 2e-16	1.7e-15	3.9e-12	< 2e-16	< 2e-16	3.8e-07	0.44354	3.3e-06	0.18098	1.00000

P value adjustment method: holm

## 2. Inverter Cost ( $Iv_{cost}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 246.16, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.00013	-	-	-	-	-	-	-	-	-	-
GRNN3	0.02845	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.22741	0.69978	1.00000	-	-	-	-	-	-	-	-
Kriging	3.4e-14	0.02003	7.9e-05	2.2e-06	-	-	-	-	-	-	-
P1	3.5e-08	1.00000	0.12418	0.01307	0.87814	-	-	-	-	-	-
P2	< 2e-16	0.00042	4.5e-07	6.3e-09	1.00000	0.11254	-	-	-	-	-
P3	4.4e-05	1.00000	1.00000	0.33700	0.22741	1.00000	0.01305	-	-	-	-
RBFNN0.1	0.37019	1.1e-10	6.2e-07	2.7e-05	< 2e-16	7.4e-16	< 2e-16	6.8e-11	-	-	-
RBFNN1	0.33700	0.48023	1.00000	1.00000	7.2e-07	0.00637	1.7e-09	0.22741	7.0e-05	-	-
RBFNN3	0.00270	1.00000	1.00000	1.00000	0.00143	0.49089	1.6e-05	1.00000	1.2e-08	1.00000	-
RBFNN6	1.00000	0.00028	0.05060	0.33700	1.4e-13	1.0e-07	< 2e-16	9.3e-05	0.27190	0.48023	0.00537

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 234.95, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	5.4e-05	-	-	-	-	-	-	-	-	-	-
GRNN3	0.02076	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.05386	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	1.2e-12	0.14503	0.00106	0.00030	-	-	-	-	-	-	-
P1	9.3e-08	1.00000	0.24733	0.11837	1.00000	-	-	-	-	-	-
P2	< 2e-16	0.00027	1.4e-07	2.4e-08	1.00000	0.02641	-	-	-	-	-
P3	0.00019	1.00000	1.00000	1.00000	0.28507	1.00000	0.00140	-	-	-	-
RBFNN0.1	0.53491	7.4e-11	7.0e-07	3.6e-06	< 2e-16	9.3e-15	< 2e-16	1.3e-09	-	-	-
RBFNN1	1.00000	0.06377	1.00000	1.00000	2.2e-07	0.00091	1.6e-12	0.09299	0.00233	-	-
RBFNN3	0.05165	1.00000	1.00000	1.00000	0.00032	0.12206	2.7e-08	1.00000	3.2e-06	1.00000	-
RBFNN6	1.00000	0.00170	0.19983	0.38782	3.7e-10	7.5e-06	4.4e-16	0.00376	0.08073	1.00000	0.38309

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 288.69, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.03035	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00022	0.14753	1.00000	-	-	-	-	-	-	-	-
Kriging	2.6e-06	2.7e-11	2.3e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.63840	0.00370	4.2e-06	2.1e-09	0.02419	-	-	-	-	-	-
P2	1.1e-06	8.4e-12	< 2e-16	< 2e-16	1.00000	0.01379	-	-	-	-	-
P3	0.00723	2.4e-06	3.5e-10	3.3e-14	1.00000	1.00000	1.00000	-	-	-	-
RBFNN0.1	1.3e-05	0.02575	0.99734	1.00000	< 2e-16	3.2e-11	< 2e-16	2.4e-16	-	-	-
RBFNN1	1.00000	0.49419	0.00729	2.9e-05	2.3e-05	1.00000	1.1e-05	0.02917	1.4e-06	-	-
RBFNN3	0.12351	0.00015	6.6e-08	1.2e-11	0.19374	1.00000	0.13012	1.00000	1.2e-13	0.34849	-
RBFNN6	1.00000	0.34849	0.00393	1.3e-05	5.1e-05	1.00000	2.4e-05	0.04798	5.3e-07	1.00000	0.49419

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 259.44, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.05028	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00016	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00499	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	1.00000	8.8e-05	2.5e-08	2.9e-06	-	-	-	-	-	-	-
P1	0.00013	3.7e-13	< 2e-16	1.6e-15	0.06270	-	-	-	-	-	-
P2	1.00000	0.00748	1.1e-05	0.00051	1.00000	0.00146	-	-	-	-	-
P3	0.13748	4.1e-07	4.1e-11	8.8e-09	1.00000	1.00000	0.49245	-	-	-	-
RBFNN0.1	1.00000	1.00000	0.02235	0.23642	0.11346	1.9e-07	1.00000	0.00274	-	-	-
RBFNN1	1.00000	0.00014	4.7e-08	5.2e-06	1.00000	0.04856	1.00000	1.00000	0.14582	-	-
RBFNN3	0.00163	3.2e-11	2.2e-16	2.0e-13	0.29331	1.00000	0.01362	1.00000	5.2e-06	0.23642	-
RBFNN6	4.3e-08	< 2e-16	< 2e-16	< 2e-16	0.00024	1.00000	1.1e-06	0.05760	9.6e-12	0.00015	0.73844

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 301.21, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.04410	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00017	0.09754	1.00000	-	-	-	-	-	-	-	-
Kriging	1.7e-05	5.6e-10	1.5e-14	< 2e-16	-	-	-	-	-	-	-
P1	0.10061	0.00019	1.1e-07	5.4e-12	0.47286	-	-	-	-	-	-
P2	4.3e-08	2.4e-13	< 2e-16	< 2e-16	1.00000	0.02834	-	-	-	-	-
P3	0.00926	6.5e-06	2.3e-09	9.8e-14	1.00000	1.00000	0.54514	-	-	-	-
RBFNN0.1	1.9e-05	0.02467	0.87889	1.00000	< 2e-16	1.7e-13	< 2e-16	2.9e-15	-	-	-
RBFNN1	0.91236	0.01044	2.5e-05	5.2e-09	0.03482	1.00000	0.00071	0.91236	2.5e-10	-	-
RBFNN3	0.12805	0.00029	1.8e-07	1.1e-11	0.39239	1.00000	0.02188	1.00000	3.3e-13	1.00000	-
RBFNN6	1.00000	1.00000	0.34341	0.00402	2.7e-07	0.01042	3.3e-10	0.00054	0.00061	0.19078	0.01385

P value adjustment method: holm

### 3. Inverter Efficiency ( $\eta_{Iv}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 288.19, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.23618	-	-	-	-	-	-	-	-	-	-
GRNN3	1.00000	0.97956	-	-	-	-	-	-	-	-	-
GRNN6	1.00000	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	3.5e-09	3.6e-15	1.1e-14	-	-	-	-	-	-	-
P1	1.0e-07	0.01905	5.6e-06	1.1e-05	0.05211	-	-	-	-	-	-
P2	4.6e-12	3.5e-05	7.2e-10	1.8e-09	1.00000	1.00000	-	-	-	-	-
P3	2.7e-13	2.9e-06	4.4e-11	1.1e-10	1.00000	0.59556	1.00000	-	-	-	-
RBFNN0.1	0.45054	5.1e-05	0.07027	0.05046	< 2e-16	8.3e-15	< 2e-16	< 2e-16	-	-	-
RBFNN1	0.05198	1.00000	0.34883	0.45054	1.1e-07	0.09445	0.00045	4.1e-05	3.2e-06	-	-
RBFNN3	2.1e-10	0.00042	2.2e-08	5.0e-08	0.59556	1.00000	1.00000	1.00000	< 2e-16	0.00403	-
RBFNN6	0.00137	1.00000	0.02039	0.03083	2.9e-05	0.97956	0.02381	0.00307	8.1e-09	1.00000	0.10690

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 265.87, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.86329	-	-	-	-	-	-	-	-	-	-
GRNN3	1.00000	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	1.00000	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	6.9e-16	2.2e-09	2.5e-15	4.5e-14	-	-	-	-	-	-	-
P1	1.4e-07	0.00246	3.5e-07	2.5e-06	0.18707	-	-	-	-	-	-
P2	8.1e-10	6.4e-05	2.2e-09	2.1e-08	1.00000	1.00000	-	-	-	-	-
P3	8.6e-09	0.00018	2.1e-08	1.4e-07	1.00000	1.00000	1.00000	-	-	-	-
RBFNN0.1	0.11363	6.6e-05	0.07515	0.02526	< 2e-16	< 2e-16	< 2e-16	< 2e-16	-	-	-
RBFNN1	0.22840	1.00000	0.31702	0.73419	9.1e-08	0.02234	0.00097	0.00204	3.1e-06	-	-
RBFNN3	4.6e-08	0.00110	1.1e-07	8.2e-07	0.30070	1.00000	1.00000	1.00000	< 2e-16	0.01161	-
RBFNN6	0.03624	1.00000	0.05910	0.15372	4.1e-06	0.15900	0.01370	0.02223	6.7e-08	1.00000	0.09675

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 325.35, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.10676	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00056	0.13794	1.00000	—	—	—	—	—	—	—	—
Kriging	6.0e-09	6.7e-14	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.37424	0.00985	1.8e-05	5.7e-07	—	—	—	—	—	—
P2	0.00044	1.4e-07	3.3e-11	2.6e-16	0.60916	0.00858	—	—	—	—	—
P3	1.7e-06	1.1e-10	8.7e-15	< 2e-16	1.00000	6.8e-05	1.00000	—	—	—	—
RBFNN0.1	0.00015	0.05996	1.00000	1.00000	< 2e-16	4.0e-06	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.28924	0.00585	8.3e-06	1.3e-06	1.00000	0.01418	0.00013	1.8e-06	—	—
RBFNN3	2.1e-05	2.3e-09	2.4e-13	< 2e-16	1.00000	0.00063	1.00000	1.00000	< 2e-16	0.00118	—
RBFNN6	0.04353	9.6e-05	1.2e-07	5.6e-12	0.02469	0.32181	1.00000	0.28924	6.1e-13	0.43021	0.89871

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 303.3, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.03231	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00014	0.04134	1.00000	—	—	—	—	—	—	—	—
Kriging	6.9e-05	2.3e-08	5.9e-14	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.05443	5.3e-05	2.7e-08	0.03840	—	—	—	—	—	—
P2	0.00122	9.8e-07	8.3e-12	< 2e-16	1.00000	0.23497	—	—	—	—	—
P3	1.5e-06	3.3e-10	8.2e-16	< 2e-16	1.00000	0.00175	1.00000	—	—	—	—
RBFNN0.1	0.00149	0.20600	1.00000	1.00000	< 2e-16	6.7e-07	1.3e-14	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.23497	0.00272	1.9e-06	0.42419	5.4e-05	3.5e-08	0.02090	—	—
RBFNN3	0.00167	1.5e-06	1.5e-11	2.9e-16	1.00000	0.27935	1.00000	1.00000	2.5e-14	7.6e-05	—
RBFNN6	5.5e-05	1.7e-08	3.9e-14	< 2e-16	1.00000	0.03330	1.00000	1.00000	< 2e-16	1.5e-06	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 329.39, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.15265	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00072	0.14390	1.00000	—	—	—	—	—	—	—	—
Kriging	2.5e-09	3.7e-14	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.15265	0.00314	2.4e-06	3.1e-06	—	—	—	—	—	—
P2	1.1e-05	1.6e-09	2.9e-13	< 2e-16	1.00000	0.00257	—	—	—	—	—
P3	2.4e-06	4.6e-10	1.4e-13	< 2e-16	1.00000	0.00049	1.00000	—	—	—	—
RBFNN0.1	0.00018	0.05698	0.91951	1.00000	< 2e-16	4.1e-07	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.18057	0.00435	3.6e-06	2.0e-06	1.00000	0.00183	0.00034	6.7e-07	—	—
RBFNN3	0.00015	4.2e-08	1.4e-11	< 2e-16	0.81886	0.01588	1.00000	1.00000	< 2e-16	0.01185	—
RBFNN6	0.00838	1.1e-05	1.4e-08	2.1e-13	0.07634	0.27403	1.00000	0.59792	1.6e-14	0.22564	1.00000

P value adjustment method: holm

## 4. Starting Torque ( $T_{str}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 250.21, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.55135	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	0.00284	—	—	—	—	—	—	—	—	—
GRNN6	0.27737	0.00011	1.00000	—	—	—	—	—	—	—	—
Kriging	9.1e-09	0.00108	2.2e-14	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	1.00000	0.47236	0.06038	2.2e-07	—	—	—	—	—	—
P2	6.0e-07	0.01335	4.7e-12	1.1e-14	1.00000	9.9e-06	—	—	—	—	—
P3	0.01272	1.00000	8.0e-06	1.3e-07	0.27737	0.05990	1.00000	—	—	—	—
RBFNN0.1	5.5e-05	1.1e-10	0.05775	0.46195	< 2e-16	3.9e-06	< 2e-16	1.8e-14	—	—	—
RBFNN1	0.55924	1.00000	0.00300	0.00012	0.00098	1.00000	0.01272	1.00000	1.3e-10	—	—
RBFNN3	0.47236	1.00000	0.00189	6.4e-05	0.00169	1.00000	0.01937	1.00000	5.0e-11	1.00000	—
RBFNN6	1.00000	0.00268	1.00000	1.00000	1.9e-14	0.47236	4.1e-12	7.4e-06	0.05979	0.00284	0.00178

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 260.43, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.40882	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	0.02048	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	0.00232	1.00000	—	—	—	—	—	—	—	—
Kriging	1.7e-10	0.00013	4.1e-14	2.7e-16	—	—	—	—	—	—	—
P1	1.00000	1.00000	0.27167	0.05400	1.2e-06	—	—	—	—	—	—
P2	6.4e-09	0.00137	2.9e-12	2.6e-14	1.00000	2.3e-05	—	—	—	—	—
P3	2.5e-05	0.11912	6.5e-08	1.6e-09	1.00000	0.00815	1.00000	—	—	—	—
RBFNN0.1	0.00215	1.3e-08	0.08670	0.40264	< 2e-16	2.8e-06	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.05921	1.00000	0.00127	8.7e-05	0.00311	1.00000	0.02159	0.67057	8.0e-11	—	—
RBFNN3	0.10062	1.00000	0.00261	0.00021	0.00149	1.00000	0.01098	0.43812	3.2e-10	1.00000	—
RBFNN6	1.00000	0.00336	1.00000	1.00000	6.9e-16	0.07209	6.1e-14	3.2e-09	0.31444	0.00014	0.00035

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 288.99, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.27177	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00030	0.78441	—	—	—	—	—	—	—	—	—
GRNN6	2.5e-07	0.02318	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00035	2.2e-10	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.62656	0.00139	1.8e-06	7.1e-05	—	—	—	—	—	—
P2	0.00556	2.1e-08	1.3e-14	< 2e-16	1.00000	0.00152	—	—	—	—	—
P3	1.00000	0.00197	1.3e-07	1.9e-11	0.15600	0.80008	0.75917	—	—	—	—
RBFNN0.1	4.8e-09	0.00239	0.76048	1.00000	< 2e-16	4.3e-08	< 2e-16	1.7e-13	—	—	—
RBFNN1	1.00000	0.01469	2.2e-06	4.7e-10	0.01640	1.00000	0.14358	1.00000	4.8e-12	—	—
RBFNN3	1.00000	0.00444	3.3e-07	4.7e-11	0.04765	1.00000	0.31656	1.00000	3.8e-13	1.00000	—
RBFNN6	0.00472	1.00000	1.00000	0.75917	1.2e-14	0.01640	2.6e-12	5.4e-06	0.16831	7.1e-05	1.3e-05

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 206.2, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.42507	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00195	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	3.2e-06	0.07559	1.00000	—	—	—	—	—	—	—	—
Kriging	0.48918	0.00010	8.7e-09	6.0e-13	—	—	—	—	—	—	—
P1	1.00000	0.01080	6.6e-06	1.9e-09	1.00000	—	—	—	—	—	—
P2	0.04321	1.0e-06	2.1e-11	4.2e-16	1.00000	0.91086	—	—	—	—	—
P3	0.13062	1.3e-05	8.8e-10	5.6e-14	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	0.00012	0.51067	1.00000	1.00000	1.3e-10	1.9e-07	1.8e-13	1.2e-11	—	—	—
RBFNN1	1.00000	1.00000	0.04078	0.00019	0.04933	0.87415	0.00211	0.01080	0.00422	—	—
RBFNN3	0.50875	0.00011	1.0e-08	7.4e-13	1.00000	1.00000	1.00000	1.00000	1.6e-10	0.05291	—
RBFNN6	0.91086	1.00000	0.82578	0.02115	0.00067	0.04454	1.0e-05	0.00010	0.18394	1.00000	0.00074

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 281.85, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.51329	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00240	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	3.6e-06	0.03587	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00028	1.3e-09	1.3e-15	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.03202	2.1e-05	7.0e-09	0.01756	—	—	—	—	—	—
P2	0.00067	4.7e-09	6.9e-15	< 2e-16	1.00000	0.03116	—	—	—	—	—
P3	0.26737	0.00017	1.1e-08	9.5e-13	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	8.8e-09	0.00102	0.31708	1.00000	< 2e-16	6.2e-12	< 2e-16	3.1e-16	—	—	—
RBFNN1	1.00000	0.03587	2.7e-05	9.6e-09	0.01475	1.00000	0.02737	1.00000	9.2e-12	—	—
RBFNN3	1.00000	0.26737	0.00064	5.8e-07	0.00112	1.00000	0.00240	0.51329	1.1e-09	1.00000	—
RBFNN6	0.07661	1.00000	1.00000	0.29792	6.2e-12	0.00240	2.5e-11	4.4e-06	0.01756	0.00282	0.03116

P value adjustment method: holm

## 5. Current Density ( $J$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 311.59, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.12060	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	0.98075	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	5.8e-09	3.2e-13	9.1e-15	—	—	—	—	—	—	—
P1	6.8e-06	0.23578	0.00416	0.00070	0.00240	—	—	—	—	—	—
P2	< 2e-16	8.0e-08	8.2e-12	2.8e-13	1.00000	0.01126	—	—	—	—	—
P3	< 2e-16	1.8e-08	2.4e-12	9.5e-14	1.00000	0.00280	1.00000	—	—	—	—
RBFNN0.1	0.76367	8.8e-05	0.02229	0.08457	< 2e-16	3.4e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.15588	1.00000	1.00000	1.00000	2.5e-09	0.17059	3.5e-08	8.3e-09	0.00016	—	—
RBFNN3	3.5e-09	0.00558	1.3e-05	1.2e-06	0.14040	1.00000	0.36936	0.13453	1.4e-15	0.00346	—
RBFNN6	1.1e-07	0.03583	0.00020	2.3e-05	0.03078	1.00000	0.10719	0.03078	1.4e-13	0.02506	1.00000

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 303.82, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.15434	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	1.8e-09	4.3e-14	2.5e-15	—	—	—	—	—	—	—
P1	4.4e-05	0.54326	0.00922	0.00250	0.00032	—	—	—	—	—	—
P2	1.9e-15	4.3e-07	3.9e-11	3.2e-12	1.00000	0.01055	—	—	—	—	—
P3	2.1e-15	2.8e-07	3.1e-11	2.8e-12	1.00000	0.00599	1.00000	—	—	—	—
RBFNN0.1	0.71151	9.2e-05	0.02812	0.08260	< 2e-16	2.9e-10	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.12727	1.00000	1.00000	0.94385	2.9e-09	0.62166	6.7e-07	4.2e-07	6.4e-05	—	—
RBFNN3	6.8e-11	0.00042	2.7e-07	3.4e-08	0.50285	0.50796	1.00000	1.00000	< 2e-16	0.00058	—
RBFNN6	4.1e-06	0.19206	0.00164	0.00039	0.00209	1.00000	0.04857	0.02812	1.1e-11	0.22929	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 328.61, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.24425	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00386	0.25594	1.00000	—	—	—	—	—	—	—	—
Kriging	3.1e-09	2.1e-13	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.82515	0.03217	0.00030	4.0e-07	0.00014	—	—	—	—	—	—
P2	7.8e-07	1.8e-10	1.8e-14	< 2e-16	1.00000	0.00578	—	—	—	—	—
P3	7.2e-08	1.7e-11	2.0e-15	< 2e-16	1.00000	0.00077	1.00000	—	—	—	—
RBFNN0.1	0.00036	0.05235	0.85057	1.00000	< 2e-16	1.2e-08	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.13648	0.00224	6.0e-06	1.3e-05	1.00000	0.00085	0.00010	2.4e-07	—	—
RBFNN3	0.00069	9.9e-07	5.2e-10	3.3e-14	0.38922	0.36378	1.00000	0.69286	2.5e-16	0.11559	—
RBFNN6	0.00504	1.5e-05	1.5e-08	1.8e-12	0.12953	0.85057	0.85057	0.25594	1.8e-14	0.36378	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 311.27, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.11360	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00185	0.16097	1.00000	—	—	—	—	—	—	—	—
Kriging	9.8e-06	4.4e-09	5.6e-14	< 2e-16	—	—	—	—	—	—	—
P1	0.24445	0.00409	3.4e-06	3.1e-09	0.17216	—	—	—	—	—	—
P2	1.5e-08	1.6e-12	< 2e-16	< 2e-16	1.00000	0.00470	—	—	—	—	—
P3	6.5e-05	5.8e-08	1.9e-12	< 2e-16	1.00000	0.35923	1.00000	—	—	—	—
RBFNN0.1	0.00268	0.18698	1.00000	1.00000	< 2e-16	5.8e-09	< 2e-16	4.1e-16	—	—	—
RBFNN1	1.00000	0.66263	0.00813	6.0e-05	0.00039	1.00000	1.5e-06	0.00185	9.5e-05	—	—
RBFNN3	7.1e-06	3.0e-09	3.3e-14	< 2e-16	1.00000	0.16097	1.00000	1.00000	< 2e-16	0.00030	—
RBFNN6	0.00985	4.0e-05	5.8e-09	1.5e-12	1.00000	1.00000	0.16097	1.00000	3.1e-12	0.13148	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 326.02, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.23375	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00333	0.23364	1.00000	—	—	—	—	—	—	—	—
Kriging	2.6e-08	3.4e-12	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.67698	0.02627	0.00019	1.9e-07	0.00090	—	—	—	—	—	—
P2	1.4e-07	2.6e-11	1.3e-15	< 2e-16	1.00000	0.00271	—	—	—	—	—
P3	8.2e-08	1.6e-11	8.9e-16	< 2e-16	1.00000	0.00166	1.00000	—	—	—	—
RBFNN0.1	0.00037	0.04876	0.83728	1.00000	< 2e-16	7.2e-09	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.34000	0.00896	3.4e-05	1.0e-05	1.00000	4.2e-05	2.5e-05	2.1e-06	—	—
RBFNN3	0.00133	2.8e-06	1.4e-09	8.8e-14	0.60445	0.60445	0.83728	0.67698	1.0e-15	0.05673	—
RBFNN6	0.01162	5.2e-05	5.5e-08	7.5e-12	0.17892	1.00000	0.31461	0.23375	1.2e-13	0.26465	1.00000

P value adjustment method: holm

## 6. Motor Efficiency ( $\eta_{motor}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 274.88, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.85820	0.06129	—	—	—	—	—	—	—	—	—
GRNN6	0.52935	0.02159	1.00000	—	—	—	—	—	—	—	—
Kriging	1.6e-10	1.9e-07	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.00000	0.04514	0.01468	3.8e-07	—	—	—	—	—	—
P2	1.8e-08	1.0e-05	1.4e-14	7.6e-16	1.00000	1.8e-05	—	—	—	—	—
P3	6.6e-05	0.00622	1.7e-09	1.8e-10	0.86887	0.00899	1.00000	—	—	—	—
RBFNN0.1	0.00109	6.1e-06	0.54153	0.85820	< 2e-16	3.3e-06	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.14892	1.00000	0.00012	2.3e-05	0.00081	1.00000	0.01281	0.57388	2.5e-10	—	—
RBFNN3	8.0e-05	0.00878	1.2e-09	1.2e-10	0.54153	0.01281	1.00000	1.00000	< 2e-16	0.78736	—
RBFNN6	0.03861	0.67955	1.3e-05	2.1e-06	0.00544	0.80007	0.05482	1.00000	9.4e-12	1.00000	1.00000

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 271.05, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	0.02716	—	—	—	—	—	—	—	—	—
GRNN6	0.13092	0.00041	1.00000	—	—	—	—	—	—	—	—
Kriging	5.9e-09	5.5e-05	2.3e-14	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	1.00000	0.33762	0.01361	4.7e-07	—	—	—	—	—	—
P2	5.1e-09	4.9e-05	1.9e-14	< 2e-16	1.00000	4.2e-07	—	—	—	—	—
P3	5.7e-06	0.00545	2.9e-10	1.1e-13	1.00000	0.00017	1.00000	—	—	—	—
RBFNN0.1	0.00048	1.2e-07	0.18268	1.00000	< 2e-16	1.7e-05	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.99148	1.00000	0.01041	0.00012	0.00020	1.00000	0.00018	0.01423	2.1e-08	—	—
RBFNN3	0.00238	0.36304	7.2e-07	7.7e-10	0.28742	0.03068	0.27992	1.00000	4.9e-15	0.68263	—
RBFNN6	0.39196	1.00000	0.00209	1.5e-05	0.00124	1.00000	0.00114	0.05310	1.3e-09	1.00000	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 303.57, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.05207	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00032	0.17958	1.00000	—	—	—	—	—	—	—	—
Kriging	5.0e-09	7.0e-15	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.12714	0.00166	2.7e-06	1.8e-06	—	—	—	—	—	—
P2	7.1e-06	9.5e-11	5.1e-15	< 2e-16	1.00000	0.00070	—	—	—	—	—
P3	0.10779	0.00014	2.2e-07	3.8e-11	0.01750	0.89900	0.50110	—	—	—	—
RBFNN0.1	1.3e-05	0.02707	0.72389	1.00000	< 2e-16	5.6e-08	< 2e-16	2.7e-13	—	—	—
RBFNN1	1.00000	0.02209	0.00014	9.7e-08	3.5e-05	1.00000	0.00706	1.00000	1.4e-09	—	—
RBFNN3	0.00567	1.2e-06	4.2e-10	1.4e-14	0.14518	0.12714	1.00000	1.00000	< 2e-16	0.52693	—
RBFNN6	0.70008	0.00341	1.1e-05	3.8e-09	0.00039	1.00000	0.04126	1.00000	3.8e-11	1.00000	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 244.85, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.44978	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00047	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	5.9e-05	0.47340	1.00000	—	—	—	—	—	—	—	—
Kriging	0.36758	3.4e-05	3.5e-10	1.5e-11	—	—	—	—	—	—	—
P1	2.5e-09	0.00111	0.70322	1.00000	< 2e-16	—	—	—	—	—	—
P2	1.00000	0.00071	2.6e-08	1.5e-09	1.00000	2.2e-15	—	—	—	—	—
P3	0.06923	4.0e-06	4.1e-11	1.9e-12	1.00000	< 2e-16	1.00000	—	—	—	—
RBFNN0.1	0.00065	1.00000	1.00000	1.00000	5.8e-10	0.59903	4.1e-08	6.9e-11	—	—	—
RBFNN1	1.00000	0.02244	4.0e-06	3.5e-07	1.00000	2.6e-12	1.00000	0.80108	5.8e-06	—	—
RBFNN3	1.00000	0.00111	5.0e-08	3.1e-09	1.00000	5.9e-15	1.00000	1.00000	7.9e-08	1.00000	—
RBFNN6	1.00000	0.00584	5.4e-07	4.1e-08	1.00000	1.6e-13	1.00000	1.00000	8.3e-07	1.00000	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 308.26, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.85866	—	—	—	—	—	—	—	—	—	—
GRNN3	0.02963	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	7.1e-05	0.08827	1.00000	—	—	—	—	—	—	—	—
Kriging	2.8e-07	6.5e-13	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.38982	0.00789	8.6e-06	2.9e-06	—	—	—	—	—	—
P2	7.3e-05	1.3e-09	6.5e-14	< 2e-16	1.00000	0.00049	—	—	—	—	—
P3	0.00056	3.4e-08	4.4e-12	< 2e-16	1.00000	0.00304	1.00000	—	—	—	—
RBFNN0.1	7.5e-06	0.02409	0.77531	1.00000	< 2e-16	7.7e-07	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.01234	4.5e-05	7.3e-09	0.00096	1.00000	0.04109	0.13433	3.8e-10	—	—
RBFNN3	0.00426	4.8e-07	8.8e-11	4.7e-16	0.77531	0.01918	1.00000	1.00000	< 2e-16	0.51453	—
RBFNN6	1.00000	0.02617	0.00014	3.4e-08	0.00035	1.00000	0.02029	0.06981	1.9e-09	1.00000	0.29941

P value adjustment method: holm

## 7. Power Factor ( $p_f$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 257.85, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.00012	-	-	-	-	-	-	-	-	-	-
GRNN3	0.01914	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.21040	0.74752	1.00000	-	-	-	-	-	-	-	-
Kriging	2.7e-16	0.00247	7.7e-06	7.6e-08	-	-	-	-	-	-	-
P1	9.2e-06	1.00000	1.00000	0.22798	0.01725	-	-	-	-	-	-
P2	< 2e-16	2.9e-05	2.5e-08	1.1e-10	1.00000	0.00032	-	-	-	-	-
P3	3.2e-11	0.15531	0.00293	0.00011	1.00000	0.48258	0.82377	-	-	-	-
RBFNN0.1	0.96297	2.9e-09	5.2e-06	0.00027	< 2e-16	8.4e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	0.17061	0.82377	1.00000	1.00000	1.3e-07	0.27743	2.0e-10	0.00015	0.00019	-	-
RBFNN3	2.2e-07	1.00000	0.31287	0.03508	0.13732	1.00000	0.00521	1.00000	5.7e-13	0.04545	-
RBFNN6	1.00000	0.10693	1.00000	1.00000	3.3e-10	0.02006	2.1e-13	1.8e-06	0.00830	1.00000	0.00172

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 259.86, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	7.3e-05	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00610	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.25470	0.49133	1.00000	-	-	-	-	-	-	-	-
Kriging	7.8e-16	0.00610	7.2e-05	9.0e-08	-	-	-	-	-	-	-
P1	6.3e-06	1.00000	1.00000	0.15319	0.03152	-	-	-	-	-	-
P2	< 2e-16	2.1e-05	6.2e-08	1.6e-11	1.00000	0.00022	-	-	-	-	-
P3	3.5e-11	0.19728	0.00895	7.3e-05	1.00000	0.56083	0.56083	-	-	-	-
RBFNN0.1	0.89605	9.6e-10	6.5e-07	0.00032	< 2e-16	3.1e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	0.00214	1.00000	1.00000	1.00000	0.00024	1.00000	3.1e-07	0.02317	1.3e-07	-	-
RBFNN3	8.0e-05	1.00000	1.00000	0.50284	0.00561	1.00000	1.9e-05	0.19043	1.1e-09	1.00000	-
RBFNN6	1.00000	0.00626	0.18021	1.00000	4.3e-12	0.00093	< 2e-16	3.3e-08	0.07646	0.08238	0.00677

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 308.94, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.80658	—	—	—	—	—	—	—	—	—	—
GRNN3	0.03988	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00032	0.22797	1.00000	—	—	—	—	—	—	—	—
Kriging	1.4e-05	1.4e-10	7.6e-15	< 2e-16	—	—	—	—	—	—	—
P1	0.20979	0.00026	3.6e-07	1.1e-10	0.24497	—	—	—	—	—	—
P2	1.1e-08	1.0e-14	< 2e-16	< 2e-16	1.00000	0.00569	—	—	—	—	—
P3	0.00506	7.2e-07	2.5e-10	2.0e-14	1.00000	1.00000	0.26083	—	—	—	—
RBFNN0.1	4.7e-06	0.01867	0.56125	1.00000	< 2e-16	1.5e-13	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.00866	3.7e-05	3.2e-08	0.02100	1.00000	0.00015	0.58700	1.1e-10	—	—
RBFNN3	0.20768	0.00025	3.3e-07	9.4e-11	0.24533	1.00000	0.00594	1.00000	1.3e-13	1.00000	—
RBFNN6	1.00000	1.00000	0.10376	0.00132	2.6e-06	0.08813	1.3e-09	0.00137	2.5e-05	0.60244	0.08630

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 288.95, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.12300	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00051	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	2.9e-06	0.24511	1.00000	—	—	—	—	—	—	—	—
Kriging	0.26662	3.6e-06	3.1e-10	1.1e-13	—	—	—	—	—	—	—
P1	0.36985	7.9e-06	8.1e-10	3.6e-13	1.00000	—	—	—	—	—	—
P2	1.00000	0.00061	2.7e-07	3.2e-10	1.00000	1.00000	—	—	—	—	—
P3	0.00422	3.4e-09	8.9e-14	< 2e-16	1.00000	1.00000	0.35067	—	—	—	—
RBFNN0.1	0.58428	1.00000	0.58428	0.03744	9.5e-05	0.00019	0.00801	1.5e-07	—	—	—
RBFNN1	1.00000	0.00022	6.7e-08	6.0e-11	1.00000	1.00000	1.00000	0.58428	0.00327	—	—
RBFNN3	0.00019	7.4e-12	< 2e-16	< 2e-16	0.70432	0.58428	0.05310	1.00000	6.8e-10	0.11146	—
RBFNN6	7.7e-08	< 2e-16	< 2e-16	< 2e-16	0.01431	0.00839	0.00020	0.87468	1.2e-14	0.00056	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 318.05, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.03280	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00022	0.09492	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00010	8.4e-09	1.3e-13	< 2e-16	—	—	—	—	—	—	—
P1	0.07738	0.00015	4.0e-08	5.6e-12	1.00000	—	—	—	—	—	—
P2	1.1e-08	5.4e-14	< 2e-16	< 2e-16	1.00000	0.01785	—	—	—	—	—
P3	0.00011	1.4e-08	4.2e-13	< 2e-16	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	7.3e-06	0.01211	0.75597	1.00000	< 2e-16	2.2e-14	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.73580	0.01095	4.6e-05	0.00046	0.19654	9.7e-08	0.00048	1.1e-06	—	—
RBFNN3	0.02341	2.0e-05	2.8e-09	2.2e-13	1.00000	1.00000	0.06426	1.00000	6.2e-16	0.06496	—
RBFNN6	1.00000	1.00000	0.03280	0.00022	0.00010	0.07738	1.1e-08	0.00011	7.4e-06	1.00000	0.02341

P value adjustment method: holm

## 8. Torque Ripple ( $T_{rip}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 231.58, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.00176	—	—	—	—	—	—	—	—	—	—
GRNN3	0.65853	0.89650	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	0.52115	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	7.4e-05	8.1e-10	7.8e-11	—	—	—	—	—	—	—
P1	5.7e-07	1.00000	0.01236	0.00354	0.04747	—	—	—	—	—	—
P2	5.3e-08	0.83863	0.00282	0.00071	0.16217	1.00000	—	—	—	—	—
P3	0.02844	1.00000	1.00000	1.00000	3.5e-06	0.53754	0.22229	—	—	—	—
RBFNN0.1	0.04404	1.7e-11	4.5e-06	2.6e-05	< 2e-16	< 2e-16	< 2e-16	4.4e-09	—	—	—
RBFNN1	1.00000	0.44750	1.00000	1.00000	4.7e-11	0.00275	0.00052	1.00000	3.7e-05	—	—
RBFNN3	0.03094	1.00000	1.00000	1.00000	1.3e-06	0.42832	0.16217	1.00000	3.8e-09	1.00000	—
RBFNN6	1.00000	0.00028	0.28380	0.57402	< 2e-16	4.5e-08	3.7e-09	0.00634	0.16217	0.65853	0.00680

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 254.12, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.00293	—	—	—	—	—	—	—	—	—	—
GRNN3	0.91844	0.64620	—	—	—	—	—	—	—	—	—
GRNN6	0.86291	0.81858	1.00000	—	—	—	—	—	—	—	—
Kriging	3.6e-16	0.00010	3.9e-10	8.6e-10	—	—	—	—	—	—	—
P1	2.6e-07	0.86291	0.00161	0.00258	0.14981	—	—	—	—	—	—
P2	5.6e-08	0.60401	0.00059	0.00095	0.27939	1.00000	—	—	—	—	—
P3	7.2e-08	0.36708	0.00043	0.00067	0.81858	1.00000	1.00000	—	—	—	—
RBFNN0.1	0.03508	4.8e-11	2.5e-05	1.4e-05	< 2e-16	< 2e-16	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.18992	1.00000	1.00000	6.5e-12	0.00015	4.4e-05	3.7e-05	0.00035	—	—
RBFNN3	0.81858	0.86291	1.00000	1.00000	1.8e-09	0.00391	0.00149	0.00102	7.4e-06	1.00000	—
RBFNN6	1.00000	0.00024	0.36708	0.28771	< 2e-16	5.4e-09	1.0e-09	1.8e-09	0.23130	0.87800	0.23130

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 312.51, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.03322	—	—	—	—	—	—	—	—	—	—
GRNN3	5.1e-05	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	2.1e-08	0.07088	1.00000	—	—	—	—	—	—	—	—
Kriging	2.1e-05	7.2e-15	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	1.00000	0.05193	0.00024	7.2e-10	—	—	—	—	—	—
P2	0.04673	8.0e-09	8.0e-14	< 2e-16	0.83810	4.2e-05	—	—	—	—	—
P3	0.12276	1.0e-07	2.6e-12	< 2e-16	0.53369	0.00025	1.00000	—	—	—	—
RBFNN0.1	2.0e-08	0.07088	1.00000	1.00000	< 2e-16	0.00024	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.14914	0.00064	6.1e-07	1.0e-06	1.00000	0.00680	0.02547	5.9e-07	—	—
RBFNN3	0.35038	1.0e-06	3.9e-11	5.7e-16	0.14914	0.00152	1.00000	1.00000	5.3e-16	0.09054	—
RBFNN6	1.00000	0.00833	7.1e-06	1.7e-09	0.00014	0.53369	0.13035	0.31061	1.6e-09	1.00000	0.77701

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 294.05, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.07740	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00173	0.17262	1.00000	—	—	—	—	—	—	—	—
Kriging	2.8e-07	3.3e-11	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.25803	0.00168	1.3e-05	9.1e-05	—	—	—	—	—	—
P2	0.00026	2.2e-07	5.3e-12	9.1e-16	1.00000	0.01993	—	—	—	—	—
P3	3.0e-07	5.7e-11	3.8e-16	< 2e-16	1.00000	7.8e-05	1.00000	—	—	—	—
RBFNN0.1	0.00323	0.24514	1.00000	1.00000	< 2e-16	2.8e-05	3.4e-15	< 2e-16	—	—	—
RBFNN1	1.00000	0.06376	0.00014	5.4e-07	0.00110	1.00000	0.11444	0.00087	1.3e-06	—	—
RBFNN3	0.00014	9.1e-08	1.7e-12	2.6e-16	1.00000	0.01189	1.00000	1.00000	9.6e-16	0.07740	—
RBFNN6	0.02872	0.00012	2.1e-08	1.3e-11	0.23498	0.51160	1.00000	0.17262	4.1e-11	1.00000	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 309.57, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.47506	—	—	—	—	—	—	—	—	—	—
GRNN3	0.01090	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	2.8e-05	0.12572	1.00000	—	—	—	—	—	—	—	—
Kriging	1.8e-06	6.7e-13	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.80706	0.02448	8.5e-05	4.8e-07	—	—	—	—	—	—
P2	0.00012	3.2e-10	1.2e-14	< 2e-16	1.00000	3.9e-05	—	—	—	—	—
P3	0.00035	2.8e-09	2.4e-13	< 2e-16	1.00000	0.00012	1.00000	—	—	—	—
RBFNN0.1	1.4e-06	0.02448	0.80706	1.00000	< 2e-16	5.1e-06	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.07660	0.00070	6.7e-07	6.4e-05	1.00000	0.00242	0.00561	2.2e-08	—	—
RBFNN3	0.04570	4.4e-06	1.4e-09	3.9e-14	0.32474	0.02319	1.00000	1.00000	3.3e-16	0.32474	—
RBFNN6	1.00000	0.00561	1.6e-05	4.4e-09	0.00232	1.00000	0.04207	0.07143	9.5e-11	1.00000	1.00000

P value adjustment method: holm

## 9. Speed Overshoot ( $n_{ov}$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 260.94, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.18225	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	0.22075	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	0.06549	1.00000	—	—	—	—	—	—	—	—
Kriging	1.2e-12	2.3e-05	2.2e-12	6.9e-14	—	—	—	—	—	—	—
P1	0.00023	0.91336	0.00033	4.2e-05	0.04830	—	—	—	—	—	—
P2	6.1e-15	6.1e-07	1.2e-14	2.7e-16	1.00000	0.00589	—	—	—	—	—
P3	0.00078	1.00000	0.00107	0.00018	0.05838	1.00000	0.00848	—	—	—	—
RBFNN0.1	0.02551	1.8e-07	0.01956	0.07036	< 2e-16	2.4e-13	< 2e-16	6.4e-12	—	—	—
RBFNN1	0.00053	1.00000	0.00076	0.00011	0.02611	1.00000	0.00288	1.00000	1.0e-12	—	—
RBFNN3	0.00085	1.00000	0.00117	0.00018	0.01906	1.00000	0.00184	1.00000	2.5e-12	1.00000	—
RBFNN6	1.00000	1.00000	1.00000	1.00000	3.0e-10	0.00490	2.5e-12	0.01156	0.00184	0.00945	0.01389

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 271.26, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.02572	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	0.05694	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	0.00270	1.00000	—	—	—	—	—	—	—	—
Kriging	2.8e-11	0.00345	2.0e-10	2.2e-13	—	—	—	—	—	—	—
P1	0.00158	1.00000	0.00460	8.9e-05	0.04567	—	—	—	—	—	—
P2	5.8e-15	2.8e-05	5.8e-14	< 2e-16	1.00000	0.00098	—	—	—	—	—
P3	1.5e-06	0.32330	6.4e-06	4.9e-08	1.00000	1.00000	0.48187	—	—	—	—
RBFNN0.1	0.02517	1.2e-09	0.00932	0.15189	< 2e-16	5.4e-12	< 2e-16	3.3e-16	—	—	—
RBFNN1	0.32330	1.00000	0.56154	0.05694	6.1e-05	1.00000	1.9e-07	0.03013	4.1e-07	—	—
RBFNN3	1.4e-05	1.00000	5.6e-05	4.4e-07	0.56154	1.00000	0.03906	1.00000	1.6e-15	0.15189	—
RBFNN6	1.00000	0.03906	1.00000	1.00000	7.9e-11	0.00279	1.9e-14	3.3e-06	0.01514	0.44434	2.9e-05

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 303.52, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.97873	-	-	-	-	-	-	-	-	-	-
GRNN3	0.03299	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00023	0.17356	1.00000	-	-	-	-	-	-	-	-
Kriging	2.2e-06	1.0e-11 < 2e-16 < 2e-16	-	-	-	-	-	-	-	-	-
P1	1.00000	0.00637	1.8e-05	1.1e-08	0.00801	-	-	-	-	-	-
P2	3.3e-07	8.4e-13 < 2e-16 < 2e-16	1.00000	0.00237	-	-	-	-	-	-	-
P3	0.00030	1.5e-08	1.6e-12 < 2e-16	1.00000	0.15334	1.00000	-	-	-	-	-
RBFNN0.1	1.9e-05	0.04338	1.00000	1.00000 < 2e-16	3.5e-10 < 2e-16 < 2e-16	-	-	-	-	-	-
RBFNN1	1.00000	0.00767	2.4e-05	1.6e-08	0.00668	1.00000	0.00187	0.14530	5.4e-10	-	-
RBFNN3	0.14530	0.00013	1.1e-07	2.1e-11	0.14530	1.00000	0.05642	1.00000	4.2e-13	1.00000	-
RBFNN6	1.00000	0.68740	0.01835	0.00010	5.6e-06	1.00000	8.9e-07	0.00064	7.8e-06	1.00000	0.19706

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 265.47, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.59024	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00427	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	4.5e-05	0.14814	1.00000	-	-	-	-	-	-	-	-
Kriging	0.08998	1.7e-05	7.9e-10	3.6e-13	-	-	-	-	-	-	-
P1	1.00000	0.01762	1.4e-05	3.7e-08	1.00000	-	-	-	-	-	-
P2	6.6e-05	1.9e-10	3.7e-16 < 2e-16	1.00000	0.01333	-	-	-	-	-	-
P3	0.00427	1.3e-07	1.7e-12	3.7e-16	1.00000	0.21987	1.00000	-	-	-	-
RBFNN0.1	0.00022	0.35852	1.00000	1.00000	4.8e-12	2.9e-07 < 2e-16	6.3e-15	-	-	-	-
RBFNN1	1.00000	0.10499	0.00023	1.2e-06	0.52100	1.00000	0.00147	0.04519	7.6e-06	-	-
RBFNN3	0.00017	7.5e-10	2.0e-15 < 2e-16	1.00000	0.02586	1.00000	1.00000	< 2e-16	0.00332	-	-
RBFNN6	1.00000	0.05520	8.0e-05	3.1e-07	0.83553	1.00000	0.00374	0.08998	2.1e-06	1.00000	0.00757

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 297.06, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.92718	—	—	—	—	—	—	—	—	—	—
GRNN3	0.02064	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	6.1e-05	0.07455	1.00000	—	—	—	—	—	—	—	—
Kriging	9.8e-05	3.0e-09	3.2e-14	< 2e-16	—	—	—	—	—	—	—
P1	0.46028	0.00169	8.6e-07	1.0e-10	0.26650	—	—	—	—	—	—
P2	3.1e-08	6.5e-14	< 2e-16	< 2e-16	1.00000	0.00263	—	—	—	—	—
P3	0.06399	5.5e-05	9.7e-09	5.3e-13	1.00000	1.00000	0.06322	—	—	—	—
RBFNN0.1	2.8e-06	0.01077	0.69474	1.00000	< 2e-16	1.2e-12	< 2e-16	4.2e-15	—	—	—
RBFNN1	0.82791	0.00556	4.8e-06	9.2e-10	0.12628	1.00000	0.00075	1.00000	1.3e-11	—	—
RBFNN3	0.48271	0.00196	1.1e-06	1.4e-10	0.25239	1.00000	0.00228	1.00000	1.6e-12	1.00000	—
RBFNN6	1.00000	1.00000	0.06236	0.00030	1.9e-05	0.23449	3.6e-09	0.02273	1.6e-05	0.46028	0.25239

P value adjustment method: holm

## 10. Rising Time (*rt*)

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 313.47, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.14439	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	2.9e-10	1.5e-13	6.9e-16	—	—	—	—	—	—	—
P1	1.2e-11	0.00017	8.7e-07	1.8e-08	0.43149	—	—	—	—	—	—
P2	< 2e-16	1.1e-09	7.2e-13	3.9e-15	1.00000	0.65234	—	—	—	—	—
P3	2.1e-10	0.00032	3.4e-06	1.2e-07	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	1.00000	0.00019	0.01075	0.09866	< 2e-16	< 2e-16	< 2e-16	5.3e-16	—	—	—
RBFNN1	0.13941	1.00000	1.00000	1.00000	3.3e-10	0.00019	1.3e-09	0.00034	0.00017	—	—
RBFNN3	6.7e-10	0.00182	1.9e-05	6.0e-07	0.10534	1.00000	0.17417	1.00000	5.3e-16	0.00196	—
RBFNN6	4.7e-06	0.23036	0.01000	0.00079	0.00050	0.71247	0.00117	0.64506	5.4e-11	0.23610	1.00000

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 314.55, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.01932	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	4.1e-09	5.2e-14	5.2e-15	—	—	—	—	—	—	—
P1	3.0e-12	0.00130	6.6e-07	1.3e-07	0.34891	—	—	—	—	—	—
P2	< 2e-16	1.2e-08	2.2e-13	2.3e-14	1.00000	0.52679	—	—	—	—	—
P3	1.6e-13	0.00020	6.2e-08	1.1e-08	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	1.00000	5.2e-05	0.02603	0.06140	< 2e-16	< 2e-16	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.02901	1.00000	1.00000	1.00000	1.6e-09	0.00073	5.1e-09	0.00010	0.00010	—	—
RBFNN3	7.9e-10	0.02324	4.3e-05	1.0e-05	0.04006	1.00000	0.06721	1.00000	1.6e-14	0.01471	—
RBFNN6	2.9e-07	0.34891	0.00267	0.00086	0.00135	1.00000	0.00271	0.52679	2.5e-11	0.24487	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 333.69, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.0000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.2721	1.0000	—	—	—	—	—	—	—	—	—
GRNN6	0.0046	0.2624	1.0000	—	—	—	—	—	—	—	—
Kriging	1.3e-10	6.5e-15	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.0037	1.1e-05	1.3e-08	1.2e-12	0.0424	—	—	—	—	—	—
P2	2.0e-08	2.8e-12	< 2e-16	< 2e-16	1.0000	0.3218	—	—	—	—	—
P3	1.7e-05	1.7e-08	8.1e-12	3.8e-16	1.0000	1.0000	1.0000	—	—	—	—
RBFNN0.1	0.0002	0.0315	0.5819	1.0000	< 2e-16	2.9e-15	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.0000	0.3699	0.0139	6.1e-05	9.5e-08	0.1005	7.4e-06	0.0014	1.2e-06	—	—
RBFNN3	0.0028	7.8e-06	8.1e-09	7.1e-13	0.0521	1.0000	0.3699	1.0000	1.6e-15	0.0873	—
RBFNN6	0.3699	0.0104	7.5e-05	5.9e-08	8.6e-05	1.0000	0.0025	0.0873	4.7e-10	1.0000	1.0000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 257.85, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.78150	0.04592	1.00000	—	—	—	—	—	—	—	—
Kriging	4.2e-09	2.2e-06	1.9e-11	4.7e-16	—	—	—	—	—	—	—
P1	1.6e-07	4.2e-05	1.2e-09	6.3e-14	1.00000	—	—	—	—	—	—
P2	1.1e-08	4.9e-06	5.9e-11	1.8e-15	1.00000	1.00000	—	—	—	—	—
P3	0.00040	0.02022	1.2e-05	8.9e-09	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	0.00587	6.4e-05	0.07065	1.00000	< 2e-16	< 2e-16	< 2e-16	1.1e-13	—	—	—
RBFNN1	1.00000	1.00000	1.00000	0.90016	2.2e-09	8.8e-08	6.0e-09	0.00026	0.00840	—	—
RBFNN3	0.03717	0.66554	0.00231	4.8e-06	0.03066	0.16594	0.04592	1.00000	1.6e-10	0.02812	—
RBFNN6	1.00000	1.00000	0.81212	0.02213	8.3e-06	0.00014	1.8e-05	0.04044	1.8e-05	1.00000	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 329.87, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.20038	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00316	0.20038	1.00000	—	—	—	—	—	—	—	—
Kriging	2.8e-09	1.6e-13	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.00264	5.2e-06	2.6e-09	2.6e-13	0.20038	—	—	—	—	—	—
P2	3.3e-09	2.1e-13	< 2e-16	< 2e-16	1.00000	0.20038	—	—	—	—	—
P3	0.00035	7.2e-07	4.7e-10	8.2e-14	1.00000	1.00000	1.00000	—	—	—	—
RBFNN0.1	0.00012	0.02530	0.70501	1.00000	< 2e-16	4.5e-16	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.71862	0.03108	0.00025	1.3e-07	0.02448	1.5e-07	0.00354	5.7e-06	—	—
RBFNN3	0.00289	5.8e-06	3.0e-09	3.3e-13	0.20038	1.00000	0.20038	1.00000	5.7e-16	0.02530	—
RBFNN6	0.71862	0.02802	0.00019	2.7e-07	0.00014	0.70501	0.00016	0.20038	2.3e-09	1.00000	0.70501

P value adjustment method: holm

- Surrogate model comparison for the FSCW-SMPM motor design problem of Silva et al.

(2017b)

### 1. Average Torque at Maximum Speed ( $T_{avg}^{\mathcal{M}}$ ) KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 315.97, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.00237	—	—	—	—	—	—	—	—	—	—
GRNN3	0.11882	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.17602	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	5.7e-10	9.7e-14	2.0e-14	—	—	—	—	—	—	—
P1	9.2e-05	1.00000	0.87202	0.58214	7.6e-08	—	—	—	—	—	—
P2	2.1e-13	0.00287	1.6e-05	6.0e-06	0.09184	0.04328	—	—	—	—	—
P3	1.9e-15	0.00012	3.5e-07	1.2e-07	0.97800	0.00250	1.00000	—	—	—	—
RBFNN0.1	1.00000	2.0e-06	0.00061	0.00136	< 2e-16	2.3e-08	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.12857	1.00000	1.00000	< 2e-16	0.01364	1.7e-09	2.0e-11	0.12857	—	—
RBFNN3	2.5e-10	0.07977	0.00136	0.00061	0.00350	0.51078	1.00000	0.95897	2.2e-15	6.1e-07	—
RBFNN6	7.4e-14	0.00183	8.3e-06	3.0e-06	0.12667	0.02851	1.00000	1.00000	< 2e-16	6.9e-10	1.00000

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 301.79, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.00743	—	—	—	—	—	—	—	—	—	—
GRNN3	0.07678	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.28134	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	7.6e-11	3.3e-13	6.2e-15	—	—	—	—	—	—	—
P1	7.1e-05	1.00000	1.00000	0.37801	1.1e-07	—	—	—	—	—	—
P2	1.6e-14	0.00024	7.2e-06	4.8e-07	0.21472	0.01735	—	—	—	—	—
P3	3.2e-12	0.00099	5.2e-05	5.6e-06	0.42103	0.03647	1.00000	—	—	—	—
RBFNN0.1	1.00000	1.2e-05	0.00037	0.00283	< 2e-16	2.3e-08	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.63450	1.00000	1.00000	1.00000	< 2e-16	0.15239	4.5e-08	7.7e-07	0.01243	—	—
RBFNN3	6.7e-11	0.01705	0.00108	0.00013	0.00778	0.37801	1.00000	1.00000	6.5e-16	1.9e-05	—
RBFNN6	5.1e-14	0.00044	1.5e-05	1.1e-06	0.15239	0.02804	1.00000	1.00000	< 2e-16	1.1e-07	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 332.65, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.21648	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00451	0.27636	1.00000	-	-	-	-	-	-	-	-
Kriging	1.5e-11	3.0e-16	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.72238	0.01997	0.00018	3.0e-07	5.0e-06	-	-	-	-	-	-
P2	0.00056	6.8e-07	3.2e-10	3.1e-14	0.06331	0.38206	-	-	-	-	-
P3	6.0e-06	3.9e-09	1.4e-12	< 2e-16	1.00000	0.01787	1.00000	-	-	-	-
RBFNN0.1	6.4e-05	0.01723	0.45034	1.00000	< 2e-16	5.6e-10	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.88310	0.05068	0.00056	5.5e-10	1.00000	0.00451	6.4e-05	5.1e-06	-	-
RBFNN3	0.00301	6.0e-06	4.6e-09	7.6e-13	0.01844	0.85802	1.00000	1.00000	< 2e-16	0.01844	-
RBFNN6	5.2e-05	2.9e-08	7.2e-12	3.6e-16	0.27252	0.10206	1.00000	1.00000	< 2e-16	0.00053	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 319.64, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.60656	0.60656	-	-	-	-	-	-	-	-	-
GRNN6	0.02508	0.02508	1.00000	-	-	-	-	-	-	-	-
Kriging	2.7e-12	2.8e-12	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.19863	0.19863	7.8e-05	1.4e-07	3.4e-05	-	-	-	-	-	-
P2	1.7e-06	1.7e-06	1.2e-12	< 2e-16	0.71149	0.09337	-	-	-	-	-
P3	4.2e-06	4.2e-06	1.6e-11	3.3e-15	1.00000	0.08707	1.00000	-	-	-	-
RBFNN0.1	0.00162	0.00162	0.81896	1.00000	< 2e-16	1.2e-09	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	1.00000	1.00000	0.35829	1.4e-15	0.01086	5.6e-09	2.8e-08	0.04809	-	-
RBFNN3	0.00539	0.00539	1.7e-07	8.0e-11	0.00508	1.00000	0.86067	3.1e-13	9.6e-05	-	-
RBFNN6	0.00012	0.00012	5.9e-10	9.8e-14	0.09337	0.71149	1.00000	1.00000	< 2e-16	9.9e-07	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 335.07, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.25944	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00685	0.33458	1.00000	-	-	-	-	-	-	-	-
Kriging	4.8e-12	< 2e-16	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.35842	0.00860	5.8e-05	9.3e-08	1.1e-05	-	-	-	-	-	-
P2	2.7e-05	1.6e-08	4.2e-12	2.8e-16	0.26162	0.17680	-	-	-	-	-
P3	5.2e-06	4.0e-09	1.8e-12	2.4e-16	1.00000	0.03880	1.00000	-	-	-	-
RBFNN0.1	7.8e-05	0.01619	0.39961	1.00000	< 2e-16	8.9e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.43368	0.01921	0.00018	2.3e-09	1.00000	0.00137	0.00025	8.4e-07	-	-
RBFNN3	0.01188	5.2e-05	7.7e-08	2.9e-11	0.00273	1.00000	1.00000	0.57277	7.6e-15	0.17680	-
RBFNN6	0.00012	1.1e-07	4.4e-11	4.5e-15	0.12073	0.34525	1.00000	1.00000	< 2e-16	0.00495	1.00000

P value adjustment method: holm

## 2. Torque Ripple at Maximum Speed ( $T_{rip}^{\mathcal{M}}$ ) KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 308.48, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	6.6e-08	-	-	-	-	-	-	-	-	-	-
GRNN3	1.0e-08	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	1.0e-08	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	0.02465	0.15702	0.06597	0.06597	-	-	-	-	-	-	-
P1	2.6e-14	0.72935	1.00000	1.00000	6.8e-05	-	-	-	-	-	-
P2	0.00028	1.00000	1.00000	1.00000	1.00000	0.00862	-	-	-	-	-
P3	1.00000	4.2e-06	1.1e-06	1.1e-06	0.06586	7.5e-11	0.00233	-	-	-	-
RBFNN0.1	0.02962	< 2e-16	< 2e-16	< 2e-16	1.9e-09	< 2e-16	5.4e-13	0.20179	-	-	-
RBFNN1	1.00000	2.3e-05	4.7e-06	4.7e-06	0.45840	8.4e-11	0.01955	1.00000	0.00053	-	-
RBFNN3	1.00000	1.4e-08	2.2e-09	2.2e-09	0.01061	3.5e-15	9.0e-05	1.00000	0.06586	1.00000	-
RBFNN6	0.50442	6.6e-15	5.3e-16	5.3e-16	1.1e-06	< 2e-16	1.0e-09	1.00000	1.00000	0.02905	0.83049

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 296.23, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.8e-05	-	-	-	-	-	-	-	-	-	-
GRNN3	7.4e-06	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	1.1e-05	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	0.18449	0.41126	0.27601	0.33541	-	-	-	-	-	-	-
P1	9.9e-12	0.41126	0.58157	0.49542	0.00011	-	-	-	-	-	-
P2	0.01066	1.00000	1.00000	1.00000	1.00000	0.00485	-	-	-	-	-
P3	1.00000	5.5e-07	2.4e-07	3.6e-07	0.00579	1.3e-12	0.00028	-	-	-	-
RBFNN0.1	0.00252	< 2e-16	< 2e-16	< 2e-16	1.6e-09	< 2e-16	2.9e-12	1.00000	-	-	-
RBFNN1	1.00000	1.2e-06	4.6e-07	7.3e-07	0.03811	2.0e-13	0.00156	1.00000	0.01653	-	-
RBFNN3	1.00000	8.9e-09	3.0e-09	5.1e-09	0.00194	< 2e-16	3.6e-05	1.00000	0.21773	1.00000	-
RBFNN6	0.11481	1.6e-13	3.9e-14	7.7e-14	1.2e-06	< 2e-16	6.4e-09	1.00000	0.41126	1.00000	-

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 294.27, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.65309	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00042	0.40485	-	-	-	-	-	-	-	-	-
GRNN6	3.5e-07	0.00659	1.00000	-	-	-	-	-	-	-	-
Kriging	0.21053	0.00013	1.9e-10	4.1e-15	-	-	-	-	-	-	-
P1	0.00100	1.9e-08	6.1e-16	< 2e-16	1.00000	-	-	-	-	-	-
P2	0.13983	6.3e-05	5.9e-11	1.0e-15	1.00000	1.00000	-	-	-	-	-
P3	1.00000	1.00000	0.03933	0.00034	0.02209	5.8e-05	0.01332	-	-	-	-
RBFNN0.1	1.1e-11	6.8e-06	0.07274	1.00000	< 2e-16	< 2e-16	< 2e-16	1.9e-07	-	-	-
RBFNN1	1.00000	1.00000	0.02209	9.5e-05	0.00860	7.3e-06	0.00479	1.00000	1.7e-08	-	-
RBFNN3	1.00000	1.00000	0.13983	0.00124	0.00083	2.5e-07	0.00042	1.00000	6.5e-07	1.00000	-
RBFNN6	9.9e-07	0.01258	1.00000	1.00000	1.9e-14	< 2e-16	4.8e-15	0.00069	1.00000	0.00022	0.00265

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 142.87, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.39725	-	-	-	-	-	-	-	-	-	-
GRNN3	1.00000	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	1.00000	0.00745	1.00000	-	-	-	-	-	-	-	-
Kriging	1.00000	1.00000	1.00000	0.37296	-	-	-	-	-	-	-
P1	1.00000	1.00000	1.00000	1.00000	1.00000	-	-	-	-	-	-
P2	1.00000	0.03340	1.00000	1.00000	0.90220	1.00000	-	-	-	-	-
P3	0.02036	5.5e-07	0.00207	0.54797	0.00017	0.00184	0.23280	-	-	-	-
RBFNN0.1	0.40515	3.1e-05	0.06123	1.00000	0.00689	0.05547	1.00000	1.00000	-	-	-
RBFNN1	1.00000	0.59240	1.00000	1.00000	1.00000	1.00000	1.00000	0.01126	0.27809	-	-
RBFNN3	0.00481	9.6e-09	0.00026	0.30766	1.2e-05	0.00023	0.09171	1.00000	1.00000	0.00235	-
RBFNN6	8.0e-09	< 2e-16	7.8e-11	1.3e-05	7.3e-13	6.1e-11	1.4e-06	0.70548	0.00404	2.5e-09	0.37649

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 280.59, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00363	0.53669	-	-	-	-	-	-	-	-	-
GRNN6	2.1e-06	0.00407	1.00000	-	-	-	-	-	-	-	-
Kriging	0.17470	0.00066	3.6e-09	2.4e-14	-	-	-	-	-	-	-
P1	0.00084	2.1e-07	2.9e-14	< 2e-16	1.00000	-	-	-	-	-	-
P2	0.05375	0.00011	2.3e-10	8.4e-16	1.00000	1.00000	-	-	-	-	-
P3	1.00000	1.00000	0.82712	0.01421	0.00464	8.5e-06	0.00115	-	-	-	-
RBFNN0.1	1.7e-10	4.6e-06	0.04830	1.00000	< 2e-16	< 2e-16	< 2e-16	6.4e-05	-	-	-
RBFNN1	1.00000	1.00000	0.09340	0.00030	0.00770	7.7e-06	0.00174	1.00000	1.2e-07	-	-
RBFNN3	0.92784	1.00000	1.00000	0.01296	0.00015	2.9e-08	2.2e-05	1.00000	2.5e-05	1.00000	-
RBFNN6	3.4e-06	0.00540	1.00000	1.00000	4.7e-14	< 2e-16	1.7e-15	0.01750	1.00000	0.00043	0.01658

P value adjustment method: holm

### 3. RMS Current Density at Maximum Speed ( $J^M$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 313.85, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.02535	-	-	-	-	-	-	-	-	-	-
GRNN3	0.04955	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.04022	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	1.4e-11	3.0e-12	5.1e-12	-	-	-	-	-	-	-
P1	4.1e-07	0.29549	0.18423	0.21448	4.4e-05	-	-	-	-	-	-
P2	4.4e-15	2.1e-05	7.2e-06	1.0e-05	0.37844	0.21448	-	-	-	-	-
P3	1.2e-14	1.2e-05	4.5e-06	6.4e-06	1.00000	0.10644	1.00000	-	-	-	-
RBFNN0.1	1.00000	7.9e-05	0.00020	0.00015	< 2e-16	3.8e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.37844	0.50796	0.47747	< 2e-16	7.7e-05	1.2e-11	1.7e-11	0.29549	-	-
RBFNN3	9.0e-10	0.01959	0.00938	0.01196	0.00295	1.00000	1.00000	0.64589	2.4e-14	5.4e-07	-
RBFNN6	1.3e-13	0.00014	5.5e-05	7.7e-05	0.16655	0.47747	1.00000	1.00000	< 2e-16	2.3e-10	1.00000

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 322.52, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.06455	-	-	-	-	-	-	-	-	-	-
GRNN3	0.06715	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.02400	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	1.4e-12	1.2e-12	1.4e-11	-	-	-	-	-	-	-
P1	7.4e-07	0.18174	0.17744	0.36640	2.4e-05	-	-	-	-	-	-
P2	1.5e-15	2.3e-06	2.0e-06	1.2e-05	0.47401	0.11647	-	-	-	-	-
P3	1.0e-15	8.1e-07	7.4e-07	4.0e-06	1.00000	0.04761	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00031	0.00034	7.8e-05	< 2e-16	8.5e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.60471	0.60471	0.34842	< 2e-16	0.00011	3.6e-12	1.8e-12	0.34262	-	-
RBFNN3	5.2e-10	0.00487	0.00457	0.01548	0.00385	1.00000	1.00000	0.60471	1.3e-14	2.8e-07	-
RBFNN6	9.0e-14	2.9e-05	2.6e-05	0.00012	0.17344	0.36640	1.00000	1.00000	< 2e-16	1.3e-10	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 331.05, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.18813	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00386	0.29218	1.00000	-	-	-	-	-	-	-	-
Kriging	6.1e-11	1.1e-15	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.39118	0.00620	3.0e-05	3.5e-08	6.3e-05	-	-	-	-	-	-
P2	4.3e-05	1.8e-08	3.1e-12	< 2e-16	0.41168	0.23241	-	-	-	-	-
P3	3.8e-05	5.9e-08	6.1e-11	3.0e-14	1.00000	0.07448	1.00000	-	-	-	-
RBFNN0.1	3.3e-05	0.01114	0.39118	1.00000	< 2e-16	2.2e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.87053	0.05117	0.00058	1.4e-09	0.87053	0.00033	0.00021	3.1e-06	-	-
RBFNN3	0.01722	5.7e-05	6.0e-08	2.2e-11	0.00657	1.00000	1.00000	0.60149	3.8e-15	0.07448	-
RBFNN6	6.3e-05	3.0e-08	5.8e-12	3.8e-16	0.39118	0.28246	1.00000	1.00000	< 2e-16	0.00048	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 315.25, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.06036	0.52929	-	-	-	-	-	-	-	-	-
GRNN6	0.00159	0.03305	1.00000	-	-	-	-	-	-	-	-
Kriging	5.9e-10	1.5e-12	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.45449	0.05020	5.7e-06	1.2e-08	0.00022	-	-	-	-	-	-
P2	8.2e-05	1.1e-06	5.0e-13	< 2e-16	0.71127	0.25873	-	-	-	-	-
P3	0.00014	3.4e-06	1.4e-11	8.6e-15	1.00000	0.20288	1.00000	-	-	-	-
RBFNN0.1	0.00014	0.00470	1.00000	1.00000	< 2e-16	3.1e-10	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	1.00000	0.71127	0.05525	3.7e-13	0.02969	3.9e-07	1.4e-06	0.00911	-	-
RBFNN3	0.04168	0.00215	3.3e-08	2.7e-11	0.00871	1.00000	1.00000	1.00000	4.3e-13	0.00103	-
RBFNN6	0.00067	1.3e-05	1.7e-11	4.4e-15	0.25740	0.71127	1.00000	1.00000	< 2e-16	5.1e-06	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 332.22, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.21562	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00242	0.23189	1.00000	—	—	—	—	—	—	—	—
Kriging	1.0e-10	1.5e-15	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.44309	0.00794	6.3e-05	3.3e-08	5.5e-05	—	—	—	—	—	—
P2	5.8e-05	2.0e-08	7.7e-12	< 2e-16	0.44309	0.21326	—	—	—	—	—
P3	9.2e-07	2.6e-10	1.1e-13	< 2e-16	1.00000	0.00908	1.00000	—	—	—	—
RBFNN0.1	9.5e-05	0.02804	0.51619	1.00000	< 2e-16	2.5e-10	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.14051	0.00120	3.2e-10	0.58316	0.00012	2.2e-06	4.6e-05	—	—
RBFNN3	0.02074	6.3e-05	1.2e-07	1.4e-11	0.00719	1.00000	1.00000	0.22761	4.2e-14	0.03545	—
RBFNN6	0.00022	1.2e-07	6.7e-11	1.5e-15	0.23189	0.38315	1.00000	1.00000	< 2e-16	0.00047	1.00000

P value adjustment method: holm

## 4. Efficiency at Maximum Speed ( $\eta^M$ )

### KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 325.53, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.04719	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00952	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.01988	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	7.9e-08	1.4e-06	3.9e-07	—	—	—	—	—	—	—
P1	7.2e-10	0.00684	0.03571	0.01717	0.35766	—	—	—	—	—	—
P2	3.2e-16	1.2e-06	1.6e-05	4.9e-06	1.00000	0.87819	—	—	—	—	—
P3	< 2e-16	3.6e-10	7.0e-09	1.8e-09	1.00000	0.00952	0.87819	—	—	—	—
RBFNN0.1	1.00000	0.00021	1.9e-05	5.8e-05	< 2e-16	1.3e-14	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.34662	0.08755	0.16834	2.4e-15	6.7e-08	1.1e-13	< 2e-16	0.57538	—	—
RBFNN3	6.5e-05	1.00000	1.00000	1.00000	0.00047	0.74466	0.00325	3.5e-06	2.6e-08	0.00148	—
RBFNN6	< 2e-16	2.7e-07	4.2e-06	1.3e-06	1.00000	0.57538	1.00000	1.00000	< 2e-16	1.3e-14	0.00115

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 329.46, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.13145	—	—	—	—	—	—	—	—	—	—
GRNN3	0.02903	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00490	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	6.1e-09	1.3e-07	2.5e-06	—	—	—	—	—	—	—
P1	4.4e-10	0.00141	0.00965	0.04988	0.36787	—	—	—	—	—	—
P2	< 2e-16	9.1e-08	1.6e-06	2.3e-05	1.00000	0.76603	—	—	—	—	—
P3	< 2e-16	1.1e-11	3.1e-10	7.6e-09	1.00000	0.01064	1.00000	—	—	—	—
RBFNN0.1	1.00000	0.00080	8.8e-05	6.9e-06	< 2e-16	6.3e-15	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.53005	0.15393	0.03362	3.9e-16	1.8e-08	1.5e-14	< 2e-16	0.67475	—	—
RBFNN3	5.5e-05	0.66317	1.00000	1.00000	0.00042	0.67475	0.00249	2.5e-06	1.6e-08	0.00072	—
RBFNN6	7.4e-15	1.5e-06	2.0e-05	0.00023	1.00000	1.00000	1.00000	0.53005	< 2e-16	7.1e-13	0.01414

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 338.28, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.11039	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00209	0.36354	1.00000	—	—	—	—	—	—	—	—
Kriging	9.8e-08	1.4e-12	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.78364	0.00980	7.2e-05	1.4e-07	0.00163	—	—	—	—	—	—
P2	8.4e-05	1.0e-08	2.4e-12	< 2e-16	1.00000	0.11397	—	—	—	—	—
P3	1.2e-08	4.7e-13	< 2e-16	< 2e-16	1.00000	0.00014	0.49181	—	—	—	—
RBFNN0.1	1.2e-05	0.01345	0.38929	1.00000	< 2e-16	8.7e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.90992	0.08401	0.00145	1.7e-07	0.83501	0.00013	2.2e-08	7.5e-06	—	—
RBFNN3	0.11039	0.00028	7.2e-07	5.3e-10	0.04040	1.00000	0.80138	0.00364	1.2e-13	0.13091	—
RBFNN6	1.5e-05	9.7e-10	1.6e-13	< 2e-16	1.00000	0.04334	1.00000	0.83501	< 2e-16	2.5e-05	0.41203

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 329.21, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.39879	0.54941	—	—	—	—	—	—	—	—	—
GRNN6	0.00707	0.01469	1.00000	—	—	—	—	—	—	—	—
Kriging	1.3e-06	4.3e-07	3.5e-13	< 2e-16	—	—	—	—	—	—	—
P1	0.09833	0.05378	1.2e-05	2.9e-09	0.16222	—	—	—	—	—	—
P2	2.2e-06	7.6e-07	7.7e-13	< 2e-16	1.00000	0.19876	—	—	—	—	—
P3	1.6e-10	4.3e-11	< 2e-16	< 2e-16	0.66091	0.00047	0.60251	—	—	—	—
RBFNN0.1	0.00025	0.00058	0.39879	1.00000	< 2e-16	1.1e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	1.00000	0.16222	2.9e-09	0.00353	5.4e-09	1.6e-13	0.01469	—	—
RBFNN3	0.39879	0.24658	0.00022	1.2e-07	0.02951	1.00000	0.03974	3.8e-05	7.0e-10	0.02951	—
RBFNN6	2.4e-06	8.3e-07	8.9e-13	< 2e-16	1.00000	0.20021	1.00000	0.60251	< 2e-16	6.1e-09	0.04097

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 340.36, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.21847	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00529	0.30851	1.00000	—	—	—	—	—	—	—	—
Kriging	1.1e-07	1.4e-11	9.2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.16130	0.00172	6.3e-06	6.4e-09	0.02740	—	—	—	—	—	—
P2	3.2e-05	1.6e-08	3.7e-12	2.6e-16	1.00000	0.39569	—	—	—	—	—
P3	2.4e-10	1.9e-14	< 2e-16	< 2e-16	1.00000	0.00030	0.30851	—	—	—	—
RBFNN0.1	5.5e-05	0.01474	0.39569	1.00000	< 2e-16	3.7e-12	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.66827	0.03949	0.00047	3.4e-06	0.52046	0.00048	9.3e-09	2.7e-06	—	—
RBFNN3	0.30851	0.00522	2.7e-05	3.6e-08	0.01064	1.00000	0.23327	9.0e-05	2.8e-11	0.78013	—
RBFNN6	2.4e-06	5.9e-10	7.4e-14	< 2e-16	1.00000	0.13845	1.00000	0.66827	< 2e-16	5.0e-05	0.05976

P value adjustment method: holm

## 5. Power Factor at Maximum Speed ( $pf^M$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 236.17, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.02125	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00056	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00056	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	3.3e-12	2.2e-09	2.2e-09	-	-	-	-	-	-	-
P1	3.3e-10	0.02125	0.31023	0.31023	0.00185	-	-	-	-	-	-
P2	1.1e-08	0.11481	1.00000	1.00000	0.00018	1.00000	-	-	-	-	-
P3	7.9e-11	0.00300	0.05911	0.05911	0.12959	1.00000	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00011	8.9e-07	8.9e-07	< 2e-16	2.2e-14	1.5e-12	1.1e-14	-	-	-
RBFNN1	0.00780	1.00000	1.00000	1.00000	2.5e-11	0.05431	0.23431	0.00787	2.9e-05	-	-
RBFNN3	4.8e-08	0.21629	1.00000	1.00000	6.0e-05	1.00000	1.00000	1.00000	8.6e-12	0.39327	-
RBFNN6	2.9e-08	0.17360	1.00000	1.00000	8.9e-05	1.00000	1.00000	1.00000	4.6e-12	0.32463	1.00000

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 228.72, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.05372	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00305	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00759	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	2.0e-12	5.7e-10	1.1e-10	-	-	-	-	-	-	-
P1	6.1e-06	0.87573	1.00000	1.00000	2.1e-06	-	-	-	-	-	-
P2	2.1e-07	0.21202	1.00000	0.78186	4.8e-05	1.00000	-	-	-	-	-
P3	6.5e-08	0.02697	0.21202	0.11681	0.07388	1.00000	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00024	5.2e-06	1.7e-05	< 2e-16	1.7e-09	2.5e-11	2.7e-11	-	-	-
RBFNN1	2.0e-06	0.56802	1.00000	1.00000	6.6e-06	1.00000	1.00000	1.00000	4.0e-10	-	-
RBFNN3	7.7e-13	0.00024	0.00668	0.00266	0.07812	0.43816	1.00000	1.00000	< 2e-16	0.71925	-
RBFNN6	2.3e-05	1.00000	1.00000	1.00000	4.9e-07	1.00000	1.00000	1.00000	9.6e-09	1.00000	0.21207

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 307.84, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.16409	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00337	0.32289	1.00000	-	-	-	-	-	-	-	-
Kriging	3.0e-11 < 2e-16 < 2e-16 < 2e-16	-	-	-	-	-	-	-	-	-	-
P1	0.67208	0.01191	8.6e-05	1.4e-07	1.1e-05	-	-	-	-	-	-
P2	0.08543	0.00036	9.0e-07	5.1e-10	0.00066	1.00000	-	-	-	-	-
P3	6.7e-06	2.2e-09	9.7e-13 < 2e-16	1.00000	0.02084	0.20154	-	-	-	-	-
RBFNN0.1	2.8e-05	0.01449	0.39756	1.00000	< 2e-16	1.2e-10	1.6e-13 < 2e-16	-	-	-	-
RBFNN1	0.11513	0.00055	1.6e-06	1.0e-09	0.00043	1.00000	1.00000	0.16409	3.6e-13	-	-
RBFNN3	0.00317	3.3e-06	2.5e-09	4.7e-13	0.02458	1.00000	1.00000	1.00000	< 2e-16	1.00000	-
RBFNN6	0.15987	0.00095	3.3e-06	2.5e-09	0.00025	1.00000	1.00000	0.12595	9.9e-13	1.00000	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 194.86, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.17336	0.27743	-	-	-	-	-	-	-	-	-
GRNN6	0.02043	0.03506	1.00000	-	-	-	-	-	-	-	-
Kriging	9.8e-14	3.9e-13	1.2e-05	0.00034	-	-	-	-	-	-	-
P1	0.00243	0.00120	5.1e-10	3.5e-12 < 2e-16	-	-	-	-	-	-	-
P2	1.00000	1.00000	1.00000	0.42370	2.6e-10	1.9e-05	-	-	-	-	-
P3	0.03433	0.05556	1.00000	1.00000	0.01334	5.7e-10	0.43565	-	-	-	-
RBFNN0.1	1.00000	1.00000	0.00723	0.00043	< 2e-16	0.07368	1.00000	0.00151	-	-	-
RBFNN1	0.41165	0.57233	1.00000	1.00000	1.8e-06	4.8e-09	1.00000	1.00000	0.02347	-	-
RBFNN3	0.00651	0.01249	1.00000	1.00000	0.00137	3.0e-13	0.19144	1.00000	9.6e-05	1.00000	-
RBFNN6	0.37281	0.52509	1.00000	1.00000	2.4e-06	3.6e-09	1.00000	1.00000	0.02043	1.00000	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 306.11, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.07156	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00081	0.24615	1.00000	-	-	-	-	-	-	-	-
Kriging	2.7e-10	4.7e-16	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	1.00000	0.09765	0.00102	2.6e-06	5.0e-07	-	-	-	-	-	-
P2	0.38885	0.00184	4.3e-06	2.5e-09	0.00019	1.00000	-	-	-	-	-
P3	5.9e-05	8.6e-09	2.1e-12	< 2e-16	1.00000	0.00610	0.18317	-	-	-	-
RBFNN0.1	5.7e-06	0.01053	0.41531	1.00000	< 2e-16	5.4e-09	1.6e-12	< 2e-16	-	-	-
RBFNN1	0.38885	0.00184	4.3e-06	2.5e-09	0.00019	1.00000	1.00000	0.18317	1.7e-12	-	-
RBFNN3	0.00845	4.3e-06	1.8e-09	2.1e-13	0.03011	0.30278	1.00000	1.00000	< 2e-16	1.00000	-
RBFNN6	0.06076	7.7e-05	6.8e-08	1.6e-11	0.00383	0.98362	1.00000	0.76852	5.1e-15	1.00000	1.00000

P value adjustment method: holm

## 6. Permanent Magnet Eddy Current Losses ( $P_{ePM}^M$ )

### KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 315.39, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.04588	-	-	-	-	-	-	-	-	-	-
GRNN3	0.02938	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.04409	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	9.3e-11	2.8e-10	1.1e-10	-	-	-	-	-	-	-
P1	2.4e-07	0.13310	0.17782	0.13310	0.00056	-	-	-	-	-	-
P2	< 2e-16	5.0e-07	1.2e-06	5.6e-07	1.00000	0.06742	-	-	-	-	-
P3	8.1e-15	6.4e-07	1.3e-06	7.0e-07	1.00000	0.02165	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00013	6.1e-05	0.00012	< 2e-16	9.3e-12	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.17782	0.13310	0.17782	< 2e-16	5.0e-06	7.6e-15	3.5e-13	0.48259	-	-
RBFNN3	4.2e-09	0.02224	0.03553	0.02339	0.00751	1.00000	0.28629	0.09958	6.3e-14	1.4e-07	-
RBFNN6	1.1e-11	0.00082	0.00156	0.00088	0.10477	1.00000	1.00000	0.45367	< 2e-16	5.8e-10	1.00000

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 324.13, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.04095	—	—	—	—	—	—	—	—	—	—
GRNN3	0.04095	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.04095	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	2.5e-10	2.1e-10	2.5e-10	—	—	—	—	—	—	—
P1	4.0e-07	0.21353	0.20645	0.21353	0.00047	—	—	—	—	—	—
P2	< 2e-16	6.6e-07	5.7e-07	6.6e-07	1.00000	0.04639	—	—	—	—	—
P3	< 2e-16	2.2e-08	1.9e-08	2.2e-08	1.00000	0.00497	1.00000	—	—	—	—
RBFNN0.1	1.00000	0.00024	0.00026	0.00024	< 2e-16	8.0e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.40698	0.40698	0.40698	< 2e-16	5.2e-05	1.4e-13	3.0e-15	0.40698	—	—
RBFNN3	9.1e-09	0.04095	0.03848	0.04095	0.00621	1.00000	0.25985	0.04095	7.7e-13	2.1e-06	—
RBFNN6	3.7e-14	3.8e-05	3.4e-05	3.8e-05	0.57379	0.35851	1.00000	1.00000	< 2e-16	4.6e-11	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 329.1, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.12081	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00184	0.29125	1.00000	—	—	—	—	—	—	—	—
Kriging	5.0e-09	6.2e-14	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.50334	0.00574	2.8e-05	2.8e-08	0.00061	—	—	—	—	—	—
P2	2.5e-05	3.1e-09	4.8e-13	< 2e-16	1.00000	0.11578	—	—	—	—	—
P3	4.8e-06	1.2e-09	4.9e-13	< 2e-16	1.00000	0.01955	1.00000	—	—	—	—
RBFNN0.1	2.4e-05	0.01698	0.48015	1.00000	< 2e-16	3.9e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.08062	0.00097	1.4e-08	0.69549	5.1e-05	9.9e-06	1.1e-05	—	—
RBFNN3	0.02055	3.2e-05	3.6e-08	8.3e-12	0.04958	1.00000	1.00000	0.38267	3.6e-15	0.03445	—
RBFNN6	0.00022	5.5e-08	1.6e-11	8.2e-16	0.78364	0.38267	1.00000	1.00000	< 2e-16	0.00044	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 321.67, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.04057	0.77919	—	—	—	—	—	—	—	—	—
GRNN6	0.00034	0.04112	1.00000	—	—	—	—	—	—	—	—
Kriging	5.0e-07	1.4e-10	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.64979	0.02973	9.9e-06	6.3e-09	0.00630	—	—	—	—	—	—
P2	0.00015	1.6e-07	4.2e-13	< 2e-16	1.00000	0.19249	—	—	—	—	—
P3	5.2e-05	1.3e-07	2.0e-12	2.6e-16	1.00000	0.04543	1.00000	—	—	—	—
RBFNN0.1	2.5e-06	0.00112	0.45941	1.00000	< 2e-16	6.7e-12	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.52860	1.00000	1.00000	0.45941	2.1e-13	0.00111	6.5e-10	1.0e-09	0.02973	—	—
RBFNN3	0.63518	0.02686	8.3e-06	5.0e-09	0.00709	1.00000	0.20557	0.04871	5.1e-12	0.00099	—
RBFNN6	0.00099	2.0e-06	1.1e-11	4.3e-16	1.00000	0.51612	1.00000	1.00000	< 2e-16	1.1e-08	0.52860

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 331.29, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.09986	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00161	0.30939	1.00000	—	—	—	—	—	—	—	—
Kriging	1.2e-09	6.9e-15	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.72107	0.00825	4.2e-05	5.5e-08	0.00012	—	—	—	—	—	—
P2	1.2e-05	7.3e-10	7.7e-14	< 2e-16	1.00000	0.04606	—	—	—	—	—
P3	5.6e-05	2.2e-08	1.4e-11	3.1e-15	1.00000	0.05995	1.00000	—	—	—	—
RBFNN0.1	1.4e-05	0.01438	0.44065	1.00000	< 2e-16	5.5e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.09986	0.00159	1.3e-09	0.72107	1.2e-05	5.7e-05	1.4e-05	—	—
RBFNN3	0.02707	3.8e-05	4.0e-08	1.2e-11	0.02015	1.00000	0.89212	0.83494	3.0e-15	0.02707	—
RBFNN6	0.00045	1.1e-07	3.2e-11	2.5e-15	0.39871	0.39871	1.00000	1.00000	< 2e-16	0.00046	1.00000

P value adjustment method: holm

## 7. Average Torque at Rated Speed ( $T_{avg}^R$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 332.1, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.06650	-	-	-	-	-	-	-	-	-	-
GRNN3	0.02587	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.02579	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	2.3e-11	1.9e-10	2.0e-10	-	-	-	-	-	-	-
P1	7.0e-07	0.16394	0.36503	0.36503	0.00015	-	-	-	-	-	-
P2	1.1e-15	1.5e-06	7.6e-06	7.9e-06	1.00000	0.09942	-	-	-	-	-
P3	< 2e-16	1.2e-09	7.2e-09	7.5e-09	1.00000	0.00057	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00050	0.00013	0.00013	< 2e-16	1.5e-10	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.64513	0.36503	0.36503	< 2e-16	0.00010	2.6e-12	1.3e-15	0.36503	-	-
RBFNN3	4.4e-10	0.00397	0.01244	0.01256	0.01777	1.00000	1.00000	0.02938	1.9e-14	2.2e-07	-
RBFNN6	1.4e-15	1.8e-06	8.6e-06	8.9e-06	1.00000	0.10509	1.00000	1.00000	< 2e-16	3.3e-12	1.00000

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 329.51, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.01218	-	-	-	-	-	-	-	-	-	-
GRNN3	0.03228	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.14363	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	2.2e-09	3.2e-10	7.5e-12	-	-	-	-	-	-	-
P1	3.1e-07	0.36223	0.20590	0.05390	0.00051	-	-	-	-	-	-
P2	3.6e-15	4.8e-05	1.2e-05	7.3e-07	0.97329	0.20249	-	-	-	-	-
P3	< 2e-16	1.3e-07	2.6e-08	1.1e-09	1.00000	0.00413	1.00000	-	-	-	-
RBFNN0.1	1.00000	3.8e-05	0.00015	0.00146	< 2e-16	4.5e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.20249	0.36223	0.97329	< 2e-16	4.6e-05	6.2e-12	4.0e-15	0.36223	-	-
RBFNN3	1.2e-08	0.11812	0.05282	0.00934	0.00469	1.00000	0.58513	0.02458	8.1e-13	2.9e-06	-
RBFNN6	< 2e-16	4.4e-06	9.1e-07	4.4e-08	1.00000	0.05501	1.00000	1.00000	< 2e-16	1.6e-13	0.21617

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 339.36, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.20582	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00455	0.33797	1.00000	-	-	-	-	-	-	-	-
Kriging	2.0e-08	9.1e-13	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.42878	0.00716	5.5e-05	8.5e-08	0.00185	-	-	-	-	-	-
P2	4.8e-05	1.6e-08	5.5e-12	3.5e-16	1.00000	0.19148	-	-	-	-	-
P3	3.3e-09	2.9e-13	< 2e-16	< 2e-16	1.00000	0.00024	0.50946	-	-	-	-
RBFNN0.1	5.3e-05	0.01882	0.42878	1.00000	< 2e-16	9.3e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.63117	0.03831	0.00044	6.3e-07	1.00000	0.00061	8.6e-08	2.7e-06	-	-
RBFNN3	0.07268	0.00040	1.3e-06	8.2e-10	0.02621	1.00000	0.74958	0.00364	4.2e-13	0.33079	-
RBFNN6	2.7e-06	4.1e-10	7.8e-14	< 2e-16	1.00000	0.03721	1.00000	1.00000	< 2e-16	5.2e-05	0.28435

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 332.41, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.06077	0.79260	-	-	-	-	-	-	-	-	-
GRNN6	0.00113	0.05393	1.00000	-	-	-	-	-	-	-	-
Kriging	1.2e-07	1.2e-10	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.94933	0.10681	6.8e-05	1.8e-07	0.00092	-	-	-	-	-	-
P2	0.00065	3.7e-06	1.5e-11	2.3e-15	1.00000	0.24055	-	-	-	-	-
P3	2.9e-07	6.3e-10	5.9e-16	< 2e-16	1.00000	0.00093	0.82864	-	-	-	-
RBFNN0.1	4.6e-06	0.00077	0.42122	1.00000	< 2e-16	8.9e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	0.79260	1.00000	1.00000	0.51739	1.5e-13	0.00598	1.9e-08	1.6e-12	0.02225	-	-
RBFNN3	0.40361	0.01932	3.7e-06	5.2e-09	0.00818	1.00000	0.79260	0.00737	1.4e-12	0.00065	-
RBFNN6	0.00015	6.0e-07	1.3e-12	< 2e-16	1.00000	0.09912	1.00000	1.00000	< 2e-16	2.3e-09	0.42122

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 339.23, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.16990	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00281	0.26864	1.00000	—	—	—	—	—	—	—	—
Kriging	1.2e-07	6.4e-12	4.2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.61793	0.01331	0.00010	1.4e-07	0.00261	—	—	—	—	—	—
P2	0.00021	9.1e-08	3.3e-11	2.0e-15	1.00000	0.23071	—	—	—	—	—
P3	1.6e-09	5.2e-14	< 2e-16	< 2e-16	1.00000	9.0e-05	0.33351	—	—	—	—
RBFNN0.1	2.2e-05	0.01170	0.33351	1.00000	< 2e-16	1.2e-10	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.23071	0.00483	5.3e-08	0.46899	0.00011	6.6e-10	4.5e-05	—	—
RBFNN3	0.22463	0.00183	7.2e-06	5.0e-09	0.01803	1.00000	0.63115	0.00089	2.3e-12	0.16977	—
RBFNN6	6.9e-06	1.0e-09	1.6e-13	< 2e-16	1.00000	0.03221	1.00000	1.00000	< 2e-16	3.3e-06	0.16990

P value adjustment method: holm

## 8. Torque Ripple at Rated Speed ( $T_{rip}^R$ )

### KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 244.42, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	0.10527	—	—	—	—	—	—	—	—	—
GRNN6	0.02504	0.00030	1.00000	—	—	—	—	—	—	—	—
Kriging	1.3e-09	1.4e-06	2.0e-15	< 2e-16	—	—	—	—	—	—	—
P1	0.00658	5.0e-05	0.85670	1.00000	< 2e-16	—	—	—	—	—	—
P2	0.01149	0.32758	4.6e-06	2.3e-10	0.05936	1.5e-11	—	—	—	—	—
P3	1.00000	1.00000	0.30562	0.00308	1.1e-05	0.00070	0.37305	—	—	—	—
RBFNN0.1	0.00429	2.9e-05	0.68966	1.00000	< 2e-16	1.00000	6.6e-12	0.00045	—	—	—
RBFNN1	0.00841	0.28238	2.8e-06	1.2e-10	0.07649	7.3e-12	1.00000	0.32758	3.2e-12	—	—
RBFNN3	0.07004	0.93553	6.7e-05	8.1e-09	0.00943	6.3e-10	1.00000	0.93553	2.9e-10	1.00000	—
RBFNN6	1.00000	1.00000	0.10128	0.00028	1.6e-06	4.5e-05	0.32758	1.00000	2.6e-05	0.29039	0.93553

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 244.11, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.55951	0.06984	—	—	—	—	—	—	—	—	—
GRNN6	0.00239	7.2e-05	1.00000	—	—	—	—	—	—	—	—
Kriging	6.8e-08	6.6e-06	5.7e-15	< 2e-16	—	—	—	—	—	—	—
P1	0.00062	1.5e-05	0.82158	1.00000	< 2e-16	—	—	—	—	—	—
P2	0.28399	1.00000	7.8e-05	4.1e-09	0.01278	4.9e-10	—	—	—	—	—
P3	1.00000	1.00000	0.02488	3.2e-05	0.00065	7.3e-06	1.00000	—	—	—	—
RBFNN0.1	2.2e-05	2.8e-07	0.14576	1.00000	< 2e-16	1.00000	2.6e-12	1.7e-07	—	—	—
RBFNN1	0.23771	1.00000	5.6e-05	2.5e-09	0.01651	2.9e-10	1.00000	1.00000	1.5e-12	—	—
RBFNN3	0.55951	1.00000	0.00027	2.2e-08	0.00478	2.9e-09	1.00000	1.00000	1.9e-11	1.00000	—
RBFNN6	1.00000	1.00000	1.00000	0.00998	6.5e-09	0.00293	0.10741	1.00000	0.00013	0.08708	0.22492

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 270.81, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.11666	—	—	—	—	—	—	—	—	—	—
GRNN3	3.5e-05	0.60425	—	—	—	—	—	—	—	—	—
GRNN6	2.6e-08	0.01981	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00031	1.8e-11	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.63426	1.00000	0.10410	0.00137	2.7e-09	—	—	—	—	—	—
P2	1.00000	0.00057	4.4e-09	5.1e-13	0.08823	0.00951	—	—	—	—	—
P3	1.00000	0.11666	6.7e-05	1.1e-07	0.00249	0.60425	1.00000	—	—	—	—
RBFNN0.1	2.8e-12	9.3e-05	0.19143	1.00000	< 2e-16	2.3e-06	< 2e-16	2.9e-11	—	—	—
RBFNN1	1.00000	0.01981	1.3e-06	4.7e-10	0.00379	0.14740	1.00000	1.00000	2.2e-14	—	—
RBFNN3	1.00000	0.00951	3.9e-07	1.1e-10	0.00853	0.09520	1.00000	1.00000	3.8e-15	1.00000	—
RBFNN6	0.01981	1.00000	1.00000	0.11666	1.9e-13	1.00000	3.5e-05	0.02151	0.00135	0.00224	0.00097

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 131.98, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	1.00000	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	3.2e-07	1.1e-08	6.0e-05	0.00066	—	—	—	—	—	—	—
P1	0.44003	1.00000	0.02688	0.00385	6.0e-15	—	—	—	—	—	—
P2	1.00000	1.00000	1.00000	1.00000	3.1e-08	1.00000	—	—	—	—	—
P3	0.24095	0.05756	1.00000	1.00000	0.50670	5.0e-05	0.09147	—	—	—	—
RBFNN0.1	1.00000	1.00000	1.00000	1.00000	5.0e-07	0.37131	1.00000	0.29030	—	—	—
RBFNN1	1.00000	1.00000	1.00000	1.00000	0.00105	0.00250	1.00000	1.00000	1.00000	—	—
RBFNN3	0.32745	0.06816	1.00000	1.00000	0.05552	2.2e-05	0.11192	1.00000	0.39023	1.00000	—
RBFNN6	0.02688	0.14815	0.00054	4.9e-05	< 2e-16	1.00000	0.09147	5.0e-07	0.02031	2.9e-05	8.8e-08

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 252.54, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.59684	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00015	0.32632	—	—	—	—	—	—	—	—	—
GRNN6	1.9e-07	0.00700	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00077	6.4e-09	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	1.00000	0.11788	0.00151	7.2e-08	—	—	—	—	—	—
P2	1.00000	0.01253	8.7e-08	1.8e-11	0.11364	0.04583	—	—	—	—	—
P3	1.00000	1.00000	0.10114	0.00254	0.00027	1.00000	0.68366	—	—	—	—
RBFNN0.1	8.4e-12	8.3e-06	0.15264	1.00000	< 2e-16	1.0e-06	< 2e-16	8.2e-06	—	—	—
RBFNN1	1.00000	0.56043	0.00012	1.5e-07	0.00091	1.00000	1.00000	1.00000	6.1e-12	—	—
RBFNN3	1.00000	0.15753	9.2e-06	6.1e-09	0.00713	0.42182	1.00000	1.00000	1.1e-13	1.00000	—
RBFNN6	2.1e-05	0.11714	1.00000	1.00000	< 2e-16	0.03737	7.2e-09	0.03649	0.42182	1.7e-05	1.1e-06

P value adjustment method: holm

## 9. RMS Current Density at Rated Speed ( $J^R$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 310.62, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.00027	-	-	-	-	-	-	-	-	-	-
GRNN3	0.05359	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	1.00000	0.11192	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	3.6e-08	2.0e-12	< 2e-16	-	-	-	-	-	-	-
P1	0.05762	1.00000	1.00000	1.00000	1.5e-12	-	-	-	-	-	-
P2	5.7e-13	0.02920	0.00011	7.6e-08	0.07994	9.6e-05	-	-	-	-	-
P3	2.4e-15	0.00021	3.8e-07	2.1e-10	1.00000	3.3e-07	1.00000	-	-	-	-
RBFNN0.1	1.00000	1.6e-07	0.00024	0.04167	< 2e-16	0.00027	< 2e-16	< 2e-16	-	-	-
RBFNN1	0.01024	1.00000	1.00000	1.00000	8.2e-11	1.00000	0.00092	4.5e-06	2.3e-05	-	-
RBFNN3	5.3e-12	0.06536	0.00037	4.2e-07	0.03681	0.00033	1.00000	0.89058	< 2e-16	0.00315	-
RBFNN6	9.1e-15	0.00456	8.7e-06	2.9e-09	0.32627	7.4e-06	1.00000	1.00000	< 2e-16	0.00011	1.00000

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 316.84, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.00035	-	-	-	-	-	-	-	-	-	-
GRNN3	0.11409	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	1.00000	0.13746	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	3.1e-08	3.4e-13	< 2e-16	-	-	-	-	-	-	-
P1	0.13951	1.00000	1.00000	1.00000	1.6e-13	-	-	-	-	-	-
P2	6.7e-13	0.02640	3.3e-05	8.4e-08	0.09063	2.2e-05	-	-	-	-	-
P3	< 2e-16	5.7e-05	2.3e-08	3.3e-11	1.00000	1.4e-08	0.92831	-	-	-	-
RBFNN0.1	1.00000	3.8e-07	0.00113	0.05884	< 2e-16	0.00165	< 2e-16	< 2e-16	-	-	-
RBFNN1	0.00091	1.00000	1.00000	0.23576	8.0e-09	1.00000	0.01272	2.4e-05	1.3e-06	-	-
RBFNN3	9.4e-13	0.02992	4.1e-05	1.1e-07	0.08162	2.6e-05	1.00000	0.89850	< 2e-16	0.01460	-
RBFNN6	4.2e-15	0.00260	1.2e-06	1.6e-09	0.43482	7.3e-07	1.00000	1.00000	< 2e-16	0.00113	1.00000

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 330.38, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.29023	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00797	0.29397	1.00000	—	—	—	—	—	—	—	—
Kriging	6.5e-11	4.0e-15	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.10932	0.00233	8.2e-06	1.3e-06	—	—	—	—	—	—
P2	0.00037	6.7e-07	4.7e-10	4.9e-14	0.14395	0.10405	—	—	—	—	—
P3	2.9e-06	4.8e-09	4.2e-12	8.4e-16	1.00000	0.00179	1.00000	—	—	—	—
RBFNN0.1	8.6e-05	0.01386	0.32869	1.00000	< 2e-16	1.6e-08	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.14395	0.00350	1.4e-05	7.2e-07	1.00000	0.08053	0.00124	3.0e-08	—	—
RBFNN3	0.00038	7.2e-07	5.1e-10	5.5e-14	0.14395	0.10465	1.00000	1.00000	< 2e-16	0.08139	—
RBFNN6	7.9e-06	5.4e-09	1.5e-12	< 2e-16	0.86557	0.00789	1.00000	1.00000	< 2e-16	0.00539	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 310.08, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.72328	0.60976	—	—	—	—	—	—	—	—	—
GRNN6	0.21953	0.17339	1.00000	—	—	—	—	—	—	—	—
Kriging	1.5e-11	2.9e-11	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	1.00000	0.02262	0.00298	9.5e-08	—	—	—	—	—	—
P2	1.1e-05	1.7e-05	3.9e-11	5.4e-13	0.57822	0.00372	—	—	—	—	—
P3	5.2e-07	7.8e-07	3.6e-12	6.8e-14	1.00000	0.00018	1.00000	—	—	—	—
RBFNN0.1	0.00045	0.00030	0.39062	1.00000	< 2e-16	6.6e-07	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.79694	0.25498	9.2e-12	1.00000	7.7e-06	3.7e-07	0.00059	—	—
RBFNN3	0.00441	0.00618	2.9e-07	8.1e-09	0.01182	0.28403	1.00000	0.36750	2.1e-14	0.00360	—
RBFNN6	9.2e-07	1.5e-06	1.4e-12	1.4e-14	1.00000	0.00059	1.00000	1.00000	< 2e-16	6.6e-07	1.00000

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 326.94, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.21256	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00353	0.23365	1.00000	—	—	—	—	—	—	—	—
Kriging	1.5e-10	5.0e-15	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.11375	0.00178	4.4e-06	1.5e-06	—	—	—	—	—	—
P2	0.00019	1.7e-07	5.4e-11	2.3e-15	0.28853	0.04720	—	—	—	—	—
P3	3.2e-05	5.3e-08	4.4e-11	7.4e-15	1.00000	0.00714	1.00000	—	—	—	—
RBFNN0.1	4.0e-05	0.01063	0.37613	1.00000	< 2e-16	1.1e-08	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.04910	0.00045	4.4e-09	1.00000	0.00165	0.00024	3.2e-06	—	—
RBFNN3	0.00804	2.8e-05	2.7e-08	3.9e-12	0.02058	0.51345	1.00000	1.00000	1.0e-15	0.04411	—
RBFNN6	1.1e-05	4.3e-09	6.2e-13	< 2e-16	1.00000	0.00664	1.00000	1.00000	< 2e-16	0.00012	1.00000

P value adjustment method: holm

## 10. Efficiency at Rated Speed ( $\eta^R$ )

### KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 331.59, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.09116	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00953	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.01979	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	3.0e-09	2.5e-07	6.4e-08	—	—	—	—	—	—	—
P1	6.3e-10	0.00305	0.03493	0.01694	0.17350	—	—	—	—	—	—
P2	3.2e-16	3.6e-07	1.6e-05	5.1e-06	1.00000	0.75342	—	—	—	—	—
P3	< 2e-16	1.9e-11	1.6e-09	4.0e-10	1.00000	0.00460	0.58575	—	—	—	—
RBFNN0.1	1.00000	0.00069	2.4e-05	7.8e-05	< 2e-16	2.0e-14	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.58776	0.11264	0.20734	3.7e-16	1.0e-07	2.2e-13	< 2e-16	0.54312	—	—
RBFNN3	6.0e-06	0.33735	1.00000	0.75342	0.00100	1.00000	0.01808	1.0e-05	1.7e-09	0.00028	—
RBFNN6	< 2e-16	2.2e-07	1.0e-05	3.1e-06	1.00000	0.70022	1.00000	0.63150	< 2e-16	1.1e-13	0.01387

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 328.39, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.09960	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00863	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.02313	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	1.3e-08	1.2e-06	2.1e-07	-	-	-	-	-	-	-
P1	2.5e-10	0.00159	0.02382	0.00899	0.44919	-	-	-	-	-	-
P2	< 2e-16	6.8e-08	5.2e-06	9.9e-07	1.00000	0.69806	-	-	-	-	-
P3	< 2e-16	3.2e-11	3.5e-09	5.7e-10	1.00000	0.00956	0.95123	-	-	-	-
RBFNN0.1	1.00000	0.00070	2.0e-05	8.7e-05	< 2e-16	5.5e-15	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.64911	0.10825	0.25552	3.3e-15	4.8e-08	3.1e-14	< 2e-16	0.49760	-	-
RBFNN3	9.9e-06	0.42691	1.00000	0.91567	0.00182	0.95123	0.00540	1.0e-05	2.8e-09	0.00046	-
RBFNN6	6.8e-16	5.2e-07	2.8e-05	6.3e-06	1.00000	1.00000	1.00000	0.60669	< 2e-16	5.0e-13	0.01749

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 338.12, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.20405	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00532	0.37786	1.00000	-	-	-	-	-	-	-	-
Kriging	3.0e-08	1.6e-12	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.28694	0.00316	1.9e-05	2.8e-08	0.00550	-	-	-	-	-	-
P2	6.4e-05	2.5e-08	9.4e-12	9.3e-16	1.00000	0.37786	-	-	-	-	-
P3	5.6e-09	8.6e-13	2.2e-16	< 2e-16	1.00000	0.00052	0.37786	-	-	-	-
RBFNN0.1	3.6e-05	0.01426	0.37786	1.00000	< 2e-16	1.1e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.56423	0.03355	0.00045	1.2e-06	0.81176	0.00104	1.5e-07	1.4e-06	-	-
RBFNN3	0.12906	0.00094	3.7e-06	4.0e-09	0.01648	1.00000	0.56423	0.00154	1.1e-12	0.49258	-
RBFNN6	7.8e-07	8.4e-11	1.2e-14	< 2e-16	1.00000	0.03520	1.00000	1.00000	< 2e-16	2.1e-05	0.09308

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 323.68, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.42878	0.59117	-	-	-	-	-	-	-	-	-
GRNN6	0.01396	0.02863	1.00000	-	-	-	-	-	-	-	-
Kriging	2.1e-05	7.4e-06	2.3e-11	1.1e-15	-	-	-	-	-	-	-
P1	0.12153	0.06841	1.9e-05	1.6e-08	0.42878	-	-	-	-	-	-
P2	1.9e-06	5.9e-07	7.2e-13	< 2e-16	1.00000	0.16285	-	-	-	-	-
P3	5.7e-10	1.6e-10	< 2e-16	< 2e-16	0.37842	0.00060	0.69619	-	-	-	-
RBFNN0.1	0.00051	0.00117	0.53907	1.00000	< 2e-16	5.4e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	1.00000	1.00000	0.59958	3.7e-09	0.00060	1.7e-10	2.5e-14	0.10863	-	-
RBFNN3	0.33735	0.19910	0.00015	2.0e-07	0.17613	1.00000	0.05088	0.00011	9.6e-10	0.00341	-
RBFNN6	9.1e-08	2.6e-08	1.1e-14	< 2e-16	1.00000	0.03677	1.00000	1.00000	< 2e-16	3.8e-12	0.00880

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 340.29, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.13039	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00272	0.30205	1.00000	-	-	-	-	-	-	-	-
Kriging	1.9e-06	1.3e-10	1.1e-14	< 2e-16	-	-	-	-	-	-	-
P1	0.37783	0.00499	2.3e-05	3.5e-08	0.03232	-	-	-	-	-	-
P2	3.7e-05	6.7e-09	1.2e-12	< 2e-16	1.00000	0.16762	-	-	-	-	-
P3	7.6e-10	2.1e-14	< 2e-16	< 2e-16	0.96346	9.4e-05	0.37783	-	-	-	-
RBFNN0.1	2.0e-05	0.01339	0.37783	1.00000	< 2e-16	2.2e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	1.00000	0.09858	0.00172	3.6e-06	0.44628	6.5e-05	1.6e-09	1.1e-05	-	-
RBFNN3	0.22256	0.00156	5.0e-06	5.2e-09	0.08335	1.00000	0.32057	0.00034	2.2e-12	0.29435	-
RBFNN6	3.0e-06	2.3e-10	2.1e-14	< 2e-16	1.00000	0.04124	1.00000	0.89513	< 2e-16	5.3e-06	0.09932

P value adjustment method: holm

## 11. Power Factor at Rated Speed ( $pf^R$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 323.48, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.03351	-	-	-	-	-	-	-	-	-	-
GRNN3	0.03608	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.02503	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	2.2e-10	1.8e-10	4.5e-10	-	-	-	-	-	-	-
P1	2.4e-07	0.19902	0.19013	0.25349	0.00049	-	-	-	-	-	-
P2	7.8e-15	1.9e-05	1.7e-05	3.2e-05	0.69538	0.28362	-	-	-	-	-
P3	< 2e-16	2.5e-08	2.1e-08	4.4e-08	1.00000	0.00514	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00021	0.00024	0.00013	< 2e-16	5.6e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.34452	0.35087	0.29505	< 2e-16	3.7e-05	1.3e-11	3.1e-15	0.36127	-	-
RBFNN3	1.2e-08	0.05098	0.04767	0.06853	0.00394	1.00000	0.67460	0.02766	1.5e-12	3.1e-06	-
RBFNN6	< 2e-16	2.0e-06	1.7e-06	3.4e-06	1.00000	0.09498	1.00000	1.00000	< 2e-16	4.7e-13	0.29592

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 324.16, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.15423	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00972	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.01936	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	1.3e-11	6.0e-09	1.7e-09	-	-	-	-	-	-	-
P1	3.9e-07	0.06021	0.44906	0.32500	0.00066	-	-	-	-	-	-
P2	2.4e-15	4.9e-07	5.2e-05	2.2e-05	1.00000	0.16632	-	-	-	-	-
P3	< 2e-16	2.9e-10	6.4e-08	2.1e-08	1.00000	0.00163	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00124	2.3e-05	5.4e-05	< 2e-16	4.0e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.88543	0.15423	0.22453	< 2e-16	4.0e-05	2.8e-12	5.3e-16	0.35249	-	-
RBFNN3	3.8e-07	0.06021	0.44906	0.32500	0.00066	1.00000	0.16632	0.00163	3.9e-11	4.0e-05	-
RBFNN6	1.1e-15	3.0e-07	3.5e-05	1.4e-05	1.00000	0.14884	1.00000	1.00000	< 2e-16	1.4e-12	0.14884

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 338.64, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.14849	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00237	0.26211	1.00000	-	-	-	-	-	-	-	-
Kriging	1.3e-09	2.0e-14	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.80025	0.01599	0.00012	1.5e-07	9.2e-05	-	-	-	-	-	-
P2	0.00016	6.2e-08	1.9e-11	9.0e-16	0.61865	0.17811	-	-	-	-	-
P3	8.5e-08	1.2e-11	2.2e-15	< 2e-16	1.00000	0.00067	0.80084	-	-	-	-
RBFNN0.1	4.0e-05	0.01826	0.51243	1.00000	< 2e-16	3.8e-10	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.97073	0.06400	0.00068	8.8e-09	1.00000	0.00062	3.9e-07	9.0e-06	-	-
RBFNN3	0.03889	0.00012	2.0e-07	5.7e-11	0.01343	1.00000	1.00000	0.03889	4.5e-14	0.09043	-
RBFNN6	1.4e-05	2.2e-09	3.6e-13	< 2e-16	1.00000	0.04071	1.00000	1.00000	< 2e-16	6.0e-05	0.80084

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 325.89, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.01822	0.43384	-	-	-	-	-	-	-	-	-
GRNN6	0.00044	0.03752	1.00000	-	-	-	-	-	-	-	-
Kriging	1.5e-07	5.7e-11	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.91866	0.04934	6.2e-06	2.5e-08	0.00162	-	-	-	-	-	-
P2	0.00018	3.1e-07	8.6e-14	< 2e-16	1.00000	0.12604	-	-	-	-	-
P3	1.2e-06	1.4e-09	3.3e-16	< 2e-16	1.00000	0.00344	1.00000	-	-	-	-
RBFNN0.1	0.00025	0.02619	1.00000	1.00000	< 2e-16	1.1e-08	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	1.00000	0.20645	0.01381	6.2e-10	0.12686	2.1e-06	1.1e-08	0.00939	-	-
RBFNN3	0.36800	0.01336	5.1e-07	1.4e-09	0.00942	1.00000	0.36800	0.01472	6.0e-10	0.03683	-
RBFNN6	6.2e-06	5.4e-09	3.3e-16	< 2e-16	1.00000	0.01822	1.00000	1.00000	< 2e-16	4.5e-08	0.06858

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 335.33, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.10517	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00188	0.21180	1.00000	-	-	-	-	-	-	-	-
Kriging	2.7e-08	9.1e-13	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.96237	0.02286	9.9e-05	1.8e-07	0.00049	-	-	-	-	-	-
P2	9.9e-05	3.3e-08	3.2e-12	< 2e-16	1.00000	0.10517	-	-	-	-	-
P3	1.8e-07	3.6e-11	3.4e-15	< 2e-16	1.00000	0.00072	1.00000	-	-	-	-
RBFNN0.1	1.5e-05	0.00909	0.40456	1.00000	< 2e-16	1.9e-10	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	1.00000	0.10517	0.00191	2.5e-08	0.96237	9.6e-05	1.7e-07	1.6e-05	-	-
RBFNN3	0.09154	0.00045	5.1e-07	2.7e-10	0.02347	1.00000	0.98838	0.02286	9.6e-14	0.09154	-
RBFNN6	8.5e-06	1.3e-09	6.8e-14	< 2e-16	1.00000	0.02347	1.00000	1.00000	< 2e-16	8.2e-06	0.38018

P value adjustment method: holm

## 12. Permanent Magnet Eddy Current Losses ( $P_{ePM}^P$ )

### KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 317.66, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.07739	-	-	-	-	-	-	-	-	-	-
GRNN3	0.02402	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.01808	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	4.4e-11	5.6e-10	1.1e-09	-	-	-	-	-	-	-
P1	2.7e-07	0.09881	0.25796	0.32039	0.00060	-	-	-	-	-	-
P2	6.0e-16	8.7e-07	6.3e-06	1.0e-05	1.00000	0.13945	-	-	-	-	-
P3	3.2e-16	1.2e-07	8.0e-07	1.3e-06	1.00000	0.02186	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00056	0.00010	6.8e-05	< 2e-16	3.8e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.86239	0.37650	0.32039	< 2e-16	6.8e-05	3.2e-12	8.4e-13	0.32039	-	-
RBFNN3	3.0e-10	0.00279	0.01156	0.01617	0.03214	1.00000	0.32039	1.1e-14	2.7e-07	-	-
RBFNN6	2.4e-15	2.1e-06	1.4e-05	2.3e-05	1.00000	0.21722	1.00000	1.00000	< 2e-16	1.1e-11	1.00000

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 324.03, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.04202	—	—	—	—	—	—	—	—	—	—
GRNN3	0.03138	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.05777	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	1.9e-10	3.7e-10	9.3e-11	—	—	—	—	—	—	—
P1	5.2e-07	0.20505	0.25196	0.16843	0.00033	—	—	—	—	—	—
P2	7.1e-14	5.3e-05	8.0e-05	3.2e-05	0.47643	0.34755	—	—	—	—	—
P3	< 2e-16	4.7e-08	8.1e-08	2.6e-08	1.00000	0.00681	1.00000	—	—	—	—
RBFNN0.1	1.00000	0.00011	7.4e-05	0.00018	< 2e-16	3.4e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.13547	0.10607	0.16843	< 2e-16	5.7e-06	2.1e-12	< 2e-16	0.71366	—	—
RBFNN3	4.4e-08	0.07994	0.10349	0.05900	0.00205	1.00000	0.72532	0.02931	1.5e-12	5.7e-07	—
RBFNN6	3.8e-15	9.0e-06	1.4e-05	5.4e-06	0.85561	0.16843	1.00000	1.00000	< 2e-16	1.3e-13	0.38268

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 336.82, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.21316	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00509	0.32772	1.00000	—	—	—	—	—	—	—	—
Kriging	1.8e-09	7.2e-14	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.61941	0.01453	0.00012	2.1e-07	0.00021	—	—	—	—	—	—
P2	1.2e-05	3.9e-09	9.1e-13	< 2e-16	1.00000	0.05691	—	—	—	—	—
P3	9.4e-08	1.4e-11	2.1e-15	< 2e-16	1.00000	0.00202	1.00000	—	—	—	—
RBFNN0.1	7.7e-05	0.02047	0.50996	1.00000	< 2e-16	4.0e-10	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.94932	0.05575	0.00063	4.6e-08	1.00000	0.00014	1.6e-06	6.1e-06	—	—
RBFNN3	0.01133	3.3e-05	4.8e-08	1.4e-11	0.05575	1.00000	1.00000	0.19216	6.2e-15	0.05691	—
RBFNN6	4.3e-05	2.0e-08	6.2e-12	3.6e-16	1.00000	0.12140	1.00000	1.00000	< 2e-16	0.00043	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 328.13, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.06007	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00089	0.07358	1.00000	—	—	—	—	—	—	—	—
Kriging	1.1e-05	7.5e-09	1.9e-14	< 2e-16	—	—	—	—	—	—	—
P1	0.32084	0.00918	3.1e-06	2.6e-09	0.11216	—	—	—	—	—	—
P2	4.0e-08	7.9e-12	< 2e-16	< 2e-16	1.00000	0.00501	—	—	—	—	—
P3	4.2e-07	3.0e-10	1.3e-15	< 2e-16	1.00000	0.00918	1.00000	—	—	—	—
RBFNN0.1	0.00014	0.02092	1.00000	1.00000	< 2e-16	1.8e-10	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.20421	0.00620	7.8e-07	0.09952	1.8e-09	2.9e-08	0.00128	—	—
RBFNN3	0.20299	0.00449	1.0e-06	6.8e-10	0.19711	1.00000	0.00978	0.01700	4.1e-11	0.05837	—
RBFNN6	0.00011	1.3e-07	8.0e-13	< 2e-16	1.00000	0.32542	1.00000	1.00000	< 2e-16	9.7e-06	0.51619

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 328.31, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.07977	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00127	0.24143	1.00000	—	—	—	—	—	—	—	—
Kriging	1.0e-08	1.0e-13	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.87193	0.01403	5.4e-05	9.4e-08	0.00026	—	—	—	—	—	—
P2	2.6e-05	2.4e-09	1.4e-13	< 2e-16	1.00000	0.05114	—	—	—	—	—
P3	6.5e-05	3.3e-08	1.3e-11	4.5e-15	1.00000	0.04466	1.00000	—	—	—	—
RBFNN0.1	7.4e-06	0.00853	0.40464	1.00000	< 2e-16	6.1e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.09632	0.00190	5.3e-09	0.75854	1.6e-05	4.5e-05	1.2e-05	—	—
RBFNN3	0.09479	0.00028	3.1e-07	1.7e-10	0.01355	1.00000	0.57636	0.40464	3.6e-14	0.07776	—
RBFNN6	7.1e-05	1.0e-08	8.2e-13	< 2e-16	1.00000	0.09479	1.00000	1.00000	< 2e-16	4.7e-05	0.87193

P value adjustment method: holm

### 13. Average Torque at Rated Speed ( $T_{avg}^P$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 329.88, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.06536	-	-	-	-	-	-	-	-	-	-
GRNN3	0.02797	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00363	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	1.4e-08	7.4e-08	1.9e-06	-	-	-	-	-	-	-
P1	1.8e-07	0.09119	0.18103	0.66413	0.01770	-	-	-	-	-	-
P2	< 2e-16	5.9e-07	2.6e-06	5.1e-05	1.00000	0.11387	-	-	-	-	-
P3	< 2e-16	4.2e-12	2.9e-11	1.4e-09	1.00000	0.00011	0.76614	-	-	-	-
RBFNN0.1	1.00000	0.00081	0.00025	1.7e-05	< 2e-16	6.8e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	1.00000	0.76614	0.22512	9.6e-14	0.00020	1.2e-11	< 2e-16	0.20064	-	-
RBFNN3	4.1e-07	0.13167	0.23456	0.76614	0.01052	1.00000	0.07789	5.4e-05	1.9e-10	0.00038	-
RBFNN6	< 2e-16	3.8e-08	1.9e-07	4.5e-06	1.00000	0.02840	1.00000	1.00000	< 2e-16	3.4e-13	0.01798

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 324.11, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.03841	-	-	-	-	-	-	-	-	-	-
GRNN3	0.02670	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.02238	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	1.8e-08	3.5e-08	4.9e-08	-	-	-	-	-	-	-
P1	1.5e-07	0.13378	0.16099	0.18035	0.01208	-	-	-	-	-	-
P2	< 2e-16	5.3e-07	9.8e-07	1.3e-06	1.00000	0.07330	-	-	-	-	-
P3	< 2e-16	1.6e-08	2.9e-08	3.8e-08	1.00000	0.00323	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00017	0.00010	8.0e-05	< 2e-16	1.7e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.25352	0.19858	0.18035	3.3e-16	1.3e-05	4.4e-14	3.0e-15	0.36422	-	-
RBFNN3	9.2e-08	0.10965	0.14113	0.15972	0.01579	1.00000	0.09081	0.00418	9.3e-12	8.8e-06	-
RBFNN6	< 2e-16	2.5e-07	4.9e-07	6.6e-07	1.00000	0.05123	1.00000	1.00000	< 2e-16	1.5e-14	0.06431

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 336.96, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.19747	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00516	0.32659	1.00000	-	-	-	-	-	-	-	-
Kriging	1.1e-06	1.7e-10	2.1e-14	< 2e-16	-	-	-	-	-	-	-
P1	0.33253	0.00557	3.3e-05	5.1e-08	0.03179	-	-	-	-	-	-
P2	9.5e-06	2.7e-09	5.3e-13	< 2e-16	1.00000	0.10741	-	-	-	-	-
P3	4.7e-09	5.7e-13	< 2e-16	< 2e-16	1.00000	0.00045	0.97446	-	-	-	-
RBFNN0.1	4.6e-05	0.01532	0.38027	1.00000	< 2e-16	3.7e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.63308	0.03577	0.00043	2.7e-05	1.00000	0.00018	1.5e-07	2.1e-06	-	-
RBFNN3	0.12931	0.00114	4.3e-06	3.9e-09	0.10741	1.00000	0.29187	0.00213	1.8e-12	0.49621	-
RBFNN6	2.2e-07	2.3e-11	2.0e-15	< 2e-16	1.00000	0.01233	1.00000	1.00000	< 2e-16	6.4e-06	0.04488

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 333.51, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.18639	0.87968	-	-	-	-	-	-	-	-	-
GRNN6	0.00231	0.03645	1.00000	-	-	-	-	-	-	-	-
Kriging	2.9e-06	3.7e-08	6.9e-14	< 2e-16	-	-	-	-	-	-	-
P1	0.56912	0.09067	8.1e-05	5.1e-08	0.03116	-	-	-	-	-	-
P2	0.00010	2.1e-06	1.5e-11	2.9e-16	1.00000	0.19583	-	-	-	-	-
P3	1.5e-08	1.1e-10	< 2e-16	< 2e-16	1.00000	0.00074	0.92819	-	-	-	-
RBFNN0.1	8.9e-06	0.00033	0.19583	1.00000	< 2e-16	1.5e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	0.63643	1.00000	1.00000	0.85069	5.3e-12	0.00092	7.2e-10	7.7e-15	0.04466	-	-
RBFNN3	0.87968	0.18639	0.00027	2.6e-07	0.01222	1.00000	0.10199	0.00025	9.6e-11	0.00270	-
RBFNN6	1.8e-05	2.8e-07	1.0e-12	< 2e-16	1.00000	0.08424	1.00000	1.00000	< 2e-16	6.3e-11	0.03763

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 332.79, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.17243	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00410	0.33241	1.00000	—	—	—	—	—	—	—	—
Kriging	2.0e-08	6.1e-13	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.30302	0.00349	1.8e-05	2.5e-08	0.00362	—	—	—	—	—	—
P2	5.2e-06	6.8e-10	1.3e-13	< 2e-16	1.00000	0.09139	—	—	—	—	—
P3	4.3e-07	1.6e-10	1.1e-13	< 2e-16	1.00000	0.00525	1.00000	—	—	—	—
RBFNN0.1	2.6e-05	0.01413	0.33241	1.00000	< 2e-16	1.0e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.58721	0.03457	0.00042	5.7e-07	0.86396	8.1e-05	5.3e-06	1.5e-06	—	—
RBFNN3	0.11443	0.00064	2.3e-06	1.9e-09	0.01521	1.00000	0.25571	0.01706	4.9e-13	0.37705	—
RBFNN6	6.4e-06	9.2e-10	1.8e-13	< 2e-16	1.00000	0.10186	1.00000	1.00000	< 2e-16	1.0e-04	0.27757

P value adjustment method: holm

## 14. Torque Ripple at Rated Speed ( $T_{rip}^{\mathcal{P}}$ )

### KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 269.16, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.2e-06	—	—	—	—	—	—	—	—	—	—
GRNN3	0.07708	0.24677	—	—	—	—	—	—	—	—	—
GRNN6	1.00000	0.00070	1.00000	—	—	—	—	—	—	—	—
Kriging	1.5e-11	1.00000	0.00063	8.8e-08	—	—	—	—	—	—	—
P1	0.72285	0.01679	1.00000	1.00000	9.1e-06	—	—	—	—	—	—
P2	6.1e-15	0.17120	5.2e-06	1.3e-10	1.00000	3.2e-08	—	—	—	—	—
P3	0.01288	1.00000	1.00000	0.32264	0.19982	1.00000	0.01443	—	—	—	—
RBFNN0.1	0.01296	< 2e-16	3.3e-09	5.6e-05	< 2e-16	7.4e-07	< 2e-16	2.8e-09	—	—	—
RBFNN1	1.00000	0.00063	1.00000	1.00000	7.2e-08	1.00000	1.0e-10	0.30949	6.5e-05	—	—
RBFNN3	1.00000	0.00271	1.00000	1.00000	6.1e-07	1.00000	1.3e-09	0.64340	1.1e-05	1.00000	—
RBFNN6	0.99237	2.9e-12	3.1e-05	0.03413	< 2e-16	0.00168	< 2e-16	8.0e-06	1.00000	0.03735	0.01207

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 253.11, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	2.0e-06	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00861	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.67709	0.02671	1.00000	-	-	-	-	-	-	-	-
Kriging	8.9e-12	1.00000	0.00696	7.8e-06	-	-	-	-	-	-	-
P1	0.33765	0.06741	1.00000	1.00000	3.6e-05	-	-	-	-	-	-
P2	5.2e-15	0.12500	0.00013	3.8e-08	1.00000	2.3e-07	-	-	-	-	-
P3	0.04445	1.00000	1.00000	1.00000	0.00987	1.00000	0.00030	-	-	-	-
RBFNN0.1	0.04712	3.4e-16	5.5e-10	5.3e-06	< 2e-16	1.0e-06	< 2e-16	6.5e-08	-	-	-
RBFNN1	0.73808	0.02274	1.00000	1.00000	6.2e-06	1.00000	2.8e-08	1.00000	6.8e-06	-	-
RBFNN3	1.00000	0.00016	0.13981	1.00000	4.2e-09	1.00000	5.7e-12	0.40583	0.00215	1.00000	-
RBFNN6	1.00000	8.0e-11	6.9e-06	0.00642	< 2e-16	0.00198	< 2e-16	0.00014	1.00000	0.00750	0.33765

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 287.48, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.30831	-	-	-	-	-	-	-	-	-	-
GRNN3	0.00018	0.53765	-	-	-	-	-	-	-	-	-
GRNN6	6.1e-08	0.00771	1.00000	-	-	-	-	-	-	-	-
Kriging	0.00847	7.3e-08	1.3e-14	< 2e-16	-	-	-	-	-	-	-
P1	1.00000	0.50174	0.00043	2.0e-07	0.00419	-	-	-	-	-	-
P2	0.00023	1.8e-10	< 2e-16	< 2e-16	1.00000	9.0e-05	-	-	-	-	-
P3	1.00000	0.06078	2.7e-05	1.2e-08	0.28688	1.00000	0.02646	-	-	-	-
RBFNN0.1	3.7e-11	8.3e-05	0.21805	1.00000	< 2e-16	1.6e-10	< 2e-16	1.2e-11	-	-	-
RBFNN1	1.00000	0.21805	8.6e-05	2.3e-08	0.01502	1.00000	0.00045	1.00000	1.2e-11	-	-
RBFNN3	1.00000	1.00000	0.00291	3.1e-06	0.00063	1.00000	8.7e-06	1.00000	3.8e-09	1.00000	-
RBFNN6	0.00254	1.00000	1.00000	0.58226	2.0e-12	0.00530	1.2e-15	0.00039	0.03118	0.00132	0.02646

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 252.57, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00307	0.44128	—	—	—	—	—	—	—	—	—
GRNN6	2.4e-05	0.01959	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00819	6.4e-06	1.7e-12	2.6e-16	—	—	—	—	—	—	—
P1	0.40545	0.00258	1.5e-08	8.3e-12	1.00000	—	—	—	—	—	—
P2	0.00063	1.5e-07	8.0e-15	< 2e-16	1.00000	0.95917	—	—	—	—	—
P3	1.00000	1.00000	0.78946	0.05536	2.4e-05	0.00452	8.4e-07	—	—	—	—
RBFNN0.1	2.1e-05	0.01788	1.00000	1.00000	< 2e-16	6.5e-12	< 2e-16	0.05175	—	—	—
RBFNN1	1.00000	1.00000	0.77866	0.04275	1.7e-06	0.00096	3.3e-08	1.00000	0.03941	—	—
RBFNN3	1.00000	1.00000	0.14124	0.00452	5.6e-05	0.01144	1.7e-06	1.00000	0.00415	1.00000	—
RBFNN6	5.9e-07	0.00172	1.00000	1.00000	< 2e-16	3.8e-14	< 2e-16	0.00699	1.00000	0.00424	0.00027

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 276.85, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.77415	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00460	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	6.2e-06	0.02779	1.00000	—	—	—	—	—	—	—	—
Kriging	0.00058	1.2e-08	2.8e-14	< 2e-16	—	—	—	—	—	—	—
P1	1.00000	0.11207	0.00018	6.3e-08	0.01195	—	—	—	—	—	—
P2	0.00030	4.3e-09	7.2e-15	< 2e-16	1.00000	0.00673	—	—	—	—	—
P3	1.00000	0.50089	0.00456	1.3e-05	0.01524	1.00000	0.00931	—	—	—	—
RBFNN0.1	1.0e-09	0.00011	0.08523	1.00000	< 2e-16	3.5e-12	< 2e-16	7.4e-09	—	—	—
RBFNN1	1.00000	0.18837	0.00040	1.9e-07	0.00645	1.00000	0.00361	1.00000	1.4e-11	—	—
RBFNN3	1.00000	1.00000	0.01662	4.4e-05	0.00011	1.00000	4.9e-05	1.00000	1.3e-08	1.00000	—
RBFNN6	0.00040	0.35707	1.00000	1.00000	< 2e-16	9.5e-06	< 2e-16	0.00049	0.42122	2.4e-05	0.00201

P value adjustment method: holm

## 15. RMS Current Density at Rated Speed ( $J^P$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 322.26, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.02842	-	-	-	-	-	-	-	-	-	-
GRNN3	0.01466	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.05383	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	3.4e-10	1.2e-09	7.5e-11	-	-	-	-	-	-	-
P1	4.7e-07	0.28999	0.38084	0.17291	0.00031	-	-	-	-	-	-
P2	1.2e-15	7.9e-06	2.1e-05	2.7e-06	1.00000	0.12273	-	-	-	-	-
P3	< 2e-16	1.0e-07	2.8e-07	3.2e-08	1.00000	0.00385	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00012	5.1e-05	0.00032	< 2e-16	8.5e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.32935	0.21532	0.44781	< 2e-16	6.1e-05	2.2e-12	4.4e-14	0.36395	-	-
RBFNN3	1.1e-08	0.05663	0.09669	0.02911	0.00425	1.00000	0.44781	0.02842	8.1e-13	2.7e-06	-
RBFNN6	< 2e-16	2.8e-06	7.6e-06	8.7e-07	1.00000	0.07090	1.00000	1.00000	< 2e-16	4.8e-13	0.32935

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 324.81, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.03891	-	-	-	-	-	-	-	-	-	-
GRNN3	0.03509	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.06102	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	8.2e-11	1.1e-10	2.5e-11	-	-	-	-	-	-	-
P1	7.8e-07	0.28475	0.29400	0.19384	0.00014	-	-	-	-	-	-
P2	1.9e-14	2.6e-05	3.1e-05	1.1e-05	0.50959	0.24411	-	-	-	-	-
P3	< 2e-16	8.8e-09	1.1e-08	3.1e-09	1.00000	0.00192	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00017	0.00015	0.00037	< 2e-16	1.2e-10	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.48368	0.48148	0.63005	< 2e-16	0.00015	5.9e-11	1.9e-15	0.29400	-	-
RBFNN3	1.8e-08	0.05502	0.06048	0.03448	0.00202	1.00000	0.68742	0.01922	1.3e-12	7.5e-06	-
RBFNN6	1.7e-15	6.0e-06	7.3e-06	2.4e-06	0.78912	0.11219	1.00000	1.00000	< 2e-16	7.1e-12	0.48148

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 336.81, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.16786	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00321	0.32289	1.00000	—	—	—	—	—	—	—	—
Kriging	1.8e-10	1.6e-15	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.69906	0.01327	0.00012	1.9e-07	2.8e-05	—	—	—	—	—	—
P2	0.00018	5.9e-08	2.5e-11	1.7e-15	0.32289	0.20114	—	—	—	—	—
P3	6.0e-07	9.7e-11	3.1e-14	< 2e-16	1.00000	0.00328	1.00000	—	—	—	—
RBFNN0.1	3.8e-05	0.01927	0.44746	1.00000	< 2e-16	2.9e-10	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.69906	0.04168	0.00043	4.6e-09	1.00000	0.00139	6.5e-06	3.4e-06	—	—
RBFNN3	0.02230	4.8e-05	8.8e-08	2.8e-11	0.00864	1.00000	1.00000	0.16905	1.1e-14	0.09818	—
RBFNN6	3.4e-05	6.2e-09	1.8e-12	< 2e-16	0.67467	0.08166	1.00000	1.00000	< 2e-16	0.00030	1.00000

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 325.08, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.03728	0.79223	—	—	—	—	—	—	—	—	—
GRNN6	0.00029	0.03254	1.00000	—	—	—	—	—	—	—	—
Kriging	7.1e-08	2.1e-11	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.91774	0.04931	2.2e-05	1.4e-08	0.00094	—	—	—	—	—	—
P2	4.6e-05	5.4e-08	6.0e-14	< 2e-16	1.00000	0.05689	—	—	—	—	—
P3	1.2e-06	2.1e-09	7.3e-15	< 2e-16	1.00000	0.00260	1.00000	—	—	—	—
RBFNN0.1	0.00031	0.03397	1.00000	1.00000	< 2e-16	1.5e-08	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.59685	0.01961	6.8e-11	0.07490	1.5e-07	5.4e-09	0.02062	—	—
RBFNN3	0.12337	0.00258	1.7e-07	3.5e-11	0.02593	1.00000	0.59299	0.03707	4.1e-11	0.00488	—
RBFNN6	5.7e-05	7.1e-08	8.6e-14	< 2e-16	1.00000	0.06344	1.00000	1.00000	< 2e-16	1.8e-07	0.59685

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 340.49, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.21749	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00474	0.28586	1.00000	-	-	-	-	-	-	-	-
Kriging	1.0e-09	5.5e-14	< 2e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.50897	0.01621	0.00012	1.9e-07	0.00015	-	-	-	-	-	-
P2	0.00023	2.3e-07	7.9e-11	6.6e-15	0.45682	0.29807	-	-	-	-	-
P3	2.3e-08	6.0e-12	7.4e-16	< 2e-16	1.00000	0.00051	0.49995	-	-	-	-
RBFNN0.1	6.5e-05	0.01481	0.44819	1.00000	< 2e-16	3.0e-10	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.85304	0.04818	0.00057	2.8e-08	1.00000	0.00199	3.7e-07	4.7e-06	-	-
RBFNN3	0.03651	0.00023	4.1e-07	1.9e-10	0.01214	1.00000	1.00000	0.02074	9.7e-14	0.17302	-
RBFNN6	1.1e-06	2.8e-10	2.6e-14	< 2e-16	1.00000	0.01481	1.00000	1.00000	< 2e-16	1.8e-05	0.30483

P value adjustment method: holm

## 16. Efficiency at Rated Speed ( $\eta^P$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 329.86, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.13098	-	-	-	-	-	-	-	-	-	-
GRNN3	0.01184	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00768	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	1.9e-08	1.9e-06	3.6e-06	-	-	-	-	-	-	-
P1	2.0e-10	0.00086	0.01571	0.02336	0.68289	-	-	-	-	-	-
P2	< 2e-16	3.8e-08	3.4e-06	6.7e-06	1.00000	0.79840	-	-	-	-	-
P3	< 2e-16	2.0e-12	4.0e-10	9.1e-10	0.99092	0.00768	0.90636	-	-	-	-
RBFNN0.1	1.00000	0.00138	4.1e-05	2.2e-05	< 2e-16	6.1e-15	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.77491	0.13098	0.09091	1.0e-14	3.0e-08	2.7e-14	< 2e-16	0.61741	-	-
RBFNN3	1.3e-05	0.36607	1.00000	1.00000	0.00299	0.90636	0.00470	3.4e-06	5.5e-09	0.00050	-
RBFNN6	1.4e-15	4.5e-07	2.7e-05	5.0e-05	1.00000	1.00000	1.00000	0.49099	< 2e-16	7.2e-13	0.01878

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 331.72, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.12336	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00226	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.01414	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	6.0e-09	6.0e-06	3.9e-07	—	—	—	—	—	—	—
P1	3.8e-10	0.00141	0.08857	0.01746	0.36405	—	—	—	—	—	—
P2	< 2e-16	4.9e-08	3.3e-05	2.6e-06	1.00000	0.68598	—	—	—	—	—
P3	< 2e-16	3.1e-12	6.2e-09	3.0e-10	1.00000	0.00403	0.72895	—	—	—	—
RBFNN0.1	1.00000	0.00130	5.0e-06	6.0e-05	< 2e-16	1.6e-14	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.72895	0.04000	0.17209	2.6e-15	7.4e-08	4.2e-14	< 2e-16	0.57125	—	—
RBFNN3	1.6e-05	0.41398	1.00000	1.00000	0.00112	0.89218	0.00403	2.4e-06	7.6e-09	0.00073	—
RBFNN6	2.3e-16	1.5e-07	8.1e-05	6.8e-06	1.00000	0.86757	1.00000	0.57125	< 2e-16	2.0e-13	0.00802

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 337.84, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.21178	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00484	0.32803	1.00000	—	—	—	—	—	—	—	—
Kriging	1.5e-07	1.1e-11	1.2e-15	< 2e-16	—	—	—	—	—	—	—
P1	0.34304	0.00553	4.1e-05	5.6e-08	0.00838	—	—	—	—	—	—
P2	1.2e-05	2.7e-09	7.7e-13	< 2e-16	1.00000	0.10991	—	—	—	—	—
P3	1.6e-09	1.0e-13	< 2e-16	< 2e-16	1.00000	0.00021	0.73360	—	—	—	—
RBFNN0.1	5.3e-05	0.01923	0.37340	1.00000	< 2e-16	5.7e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.87876	0.08051	0.00114	1.2e-06	0.73360	6.7e-05	1.3e-08	9.1e-06	—	—
RBFNN3	0.12690	0.00091	4.0e-06	3.0e-09	0.04170	1.00000	0.32803	0.00131	1.9e-12	0.31119	—
RBFNN6	4.4e-06	7.7e-10	1.7e-13	< 2e-16	1.00000	0.06450	1.00000	0.88714	< 2e-16	2.8e-05	0.21345

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 325.6, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.44893	0.57140	—	—	—	—	—	—	—	—	—
GRNN6	0.00986	0.01654	1.00000	—	—	—	—	—	—	—	—
Kriging	9.7e-06	4.5e-06	8.5e-12	< 2e-16	—	—	—	—	—	—	—
P1	0.10376	0.06758	1.4e-05	4.9e-09	0.41545	—	—	—	—	—	—
P2	1.5e-06	6.8e-07	6.3e-13	< 2e-16	1.00000	0.18444	—	—	—	—	—
P3	5.5e-10	2.3e-10	< 2e-16	< 2e-16	0.56375	0.00094	0.87393	—	—	—	—
RBFNN0.1	0.00062	0.00112	0.57140	1.00000	< 2e-16	4.4e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.87393	1.00000	1.00000	0.86278	2.3e-10	0.00014	2.1e-11	3.0e-15	0.23999	—	—
RBFNN3	0.41545	0.29560	0.00023	1.8e-07	0.10376	1.00000	0.03584	9.3e-05	2.3e-09	0.00161	—
RBFNN6	4.7e-07	2.0e-07	1.2e-13	< 2e-16	1.00000	0.10376	1.00000	1.00000	< 2e-16	4.3e-12	0.01729

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 339.52, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.14469	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00262	0.31336	1.00000	—	—	—	—	—	—	—	—
Kriging	1.2e-06	6.8e-11	7.4e-15	< 2e-16	—	—	—	—	—	—	—
P1	0.31336	0.00265	1.3e-05	1.2e-08	0.04056	—	—	—	—	—	—
P2	2.7e-05	4.1e-09	9.1e-13	< 2e-16	1.00000	0.21217	—	—	—	—	—
P3	5.2e-10	1.2e-14	< 2e-16	< 2e-16	0.91564	0.00014	0.40046	—	—	—	—
RBFNN0.1	3.0e-05	0.01921	0.40519	1.00000	< 2e-16	1.4e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.70182	0.05969	0.00068	6.7e-06	0.54267	0.00012	3.7e-09	6.4e-06	—	—
RBFNN3	0.31336	0.00265	1.3e-05	1.2e-08	0.04056	1.00000	0.21217	0.00014	1.4e-11	0.54267	—
RBFNN6	4.7e-06	3.9e-10	5.5e-14	< 2e-16	1.00000	0.08201	1.00000	0.66715	< 2e-16	2.3e-05	0.08201

P value adjustment method: holm

## 17. Power Factor at Rated Speed ( $pf^P$ )

KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 328.18, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.05053	-	-	-	-	-	-	-	-	-	-
GRNN3	0.01833	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.03071	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	2.2e-08	1.6e-07	6.4e-08	-	-	-	-	-	-	-
P1	1.0e-07	0.08589	0.20694	0.13520	0.02381	-	-	-	-	-	-
P2	< 2e-16	9.0e-09	7.0e-08	2.7e-08	1.00000	0.01510	-	-	-	-	-
P3	< 2e-16	1.1e-10	1.0e-09	3.6e-10	1.00000	0.00063	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00047	9.9e-05	0.00021	< 2e-16	2.5e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.66644	0.41100	0.51010	2.1e-14	6.0e-05	6.7e-15	< 2e-16	0.26864	-	-
RBFNN3	1.8e-08	0.03911	0.09646	0.06281	0.05356	1.00000	0.03715	0.00203	2.9e-12	1.4e-05	-
RBFNN6	1.1e-14	1.1e-05	6.0e-05	2.8e-05	1.00000	0.41100	1.00000	0.50044	< 2e-16	9.4e-11	0.64527

P value adjustment method: holm

SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 324.19, df = 11, p-value < 2.2e-16
```

```
Pairwise comparisons using Dunn's-test for multiple
comparisons of independent samples
```

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.12359	-	-	-	-	-	-	-	-	-	-
GRNN3	0.01519	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.03230	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	4.0e-08	2.1e-06	6.2e-07	-	-	-	-	-	-	-
P1	2.4e-07	0.05747	0.27115	0.18988	0.05291	-	-	-	-	-	-
P2	< 2e-16	1.3e-10	1.3e-08	3.1e-09	1.00000	0.00252	-	-	-	-	-
P3	< 2e-16	2.9e-09	1.2e-07	4.0e-08	1.00000	0.00355	1.00000	-	-	-	-
RBFNN0.1	1.00000	0.00076	3.1e-05	8.9e-05	< 2e-16	1.7e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.62931	0.16490	0.23617	2.9e-14	1.9e-05	< 2e-16	6.5e-15	0.32929	-	-
RBFNN3	1.2e-07	0.04186	0.23617	0.15317	0.07179	1.00000	0.00382	0.00504	7.2e-12	1.1e-05	-
RBFNN6	2.1e-15	7.0e-07	2.6e-05	9.1e-06	1.00000	0.18988	1.00000	1.00000	< 2e-16	1.4e-12	0.23617

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 342.2, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.23455	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00705	0.38392	1.00000	—	—	—	—	—	—	—	—
Kriging	3.3e-07	4.5e-11	5.2e-15	< 2e-16	—	—	—	—	—	—	—
P1	0.30109	0.00437	2.6e-05	4.8e-08	0.02135	—	—	—	—	—	—
P2	1.9e-07	2.4e-11	2.4e-15	< 2e-16	1.00000	0.01629	—	—	—	—	—
P3	4.7e-10	2.7e-14	< 2e-16	< 2e-16	1.00000	0.00024	1.00000	—	—	—	—
RBFNN0.1	5.7e-05	0.01629	0.40037	1.00000	< 2e-16	2.4e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.68922	0.04571	0.00073	7.4e-06	0.74835	4.7e-06	1.7e-08	3.3e-06	—	—
RBFNN3	0.15769	0.00155	6.9e-06	9.8e-09	0.04639	1.00000	0.03656	0.00071	3.7e-12	0.47569	—
RBFNN6	4.9e-05	2.5e-08	8.8e-12	1.0e-15	1.00000	0.30652	1.00000	0.44489	< 2e-16	0.00067	0.47569

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 327.55, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.09438	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.00178	0.09438	1.00000	—	—	—	—	—	—	—	—
Kriging	3.8e-05	5.8e-08	5.2e-13	< 2e-16	—	—	—	—	—	—	—
P1	0.84614	0.05077	7.2e-05	1.2e-07	0.06743	—	—	—	—	—	—
P2	2.0e-07	9.5e-11	< 2e-16	< 2e-16	1.00000	0.00252	—	—	—	—	—
P3	1.0e-06	1.8e-09	3.6e-14	< 2e-16	1.00000	0.00307	1.00000	—	—	—	—
RBFNN0.1	2.9e-06	0.00090	0.22417	1.00000	< 2e-16	1.6e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	0.84614	1.00000	1.00000	0.53598	5.5e-10	0.00416	4.9e-13	2.3e-11	0.01291	—	—
RBFNN3	0.68493	0.03531	4.1e-05	5.8e-08	0.09438	1.00000	0.00373	0.00423	6.7e-12	0.00283	—
RBFNN6	0.00049	1.3e-06	3.0e-11	2.7e-15	1.00000	0.27696	1.00000	1.00000	< 2e-16	2.0e-08	0.36129

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 333.11, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.0000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.1481	1.0000	—	—	—	—	—	—	—	—	—
GRNN6	0.0041	0.2426	1.0000	—	—	—	—	—	—	—	—
Kriging	1.0e-06	1.6e-10	8.1e-15	< 2e-16	—	—	—	—	—	—	—
P1	0.3648	0.0084	2.9e-05	5.5e-08	0.0224	—	—	—	—	—	—
P2	2.8e-07	3.3e-11	1.2e-15	< 2e-16	1.0000	0.0103	—	—	—	—	—
P3	3.1e-07	1.2e-10	2.0e-14	< 2e-16	1.0000	0.0043	1.0000	—	—	—	—
RBFNN0.1	2.3e-05	0.0092	0.3414	1.0000	< 2e-16	2.5e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.0000	1.0000	0.0711	0.0013	4.5e-06	0.6808	1.3e-06	1.3e-06	5.5e-06	—	—
RBFNN3	0.1209	0.0012	2.2e-06	2.1e-09	0.1047	1.0000	0.0569	0.0209	5.3e-13	0.2358	—
RBFNN6	4.7e-06	1.2e-09	8.8e-14	< 2e-16	1.0000	0.0570	1.0000	1.0000	< 2e-16	2.0e-05	0.2169

P value adjustment method: holm

## 18. Permanent Magnet Eddy Current Losses ( $P_{ePM}^P$ )

### KT

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 319.59, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.04686	—	—	—	—	—	—	—	—	—	—
GRNN3	0.00714	1.00000	—	—	—	—	—	—	—	—	—
GRNN6	0.03071	1.00000	1.00000	—	—	—	—	—	—	—	—
Kriging	< 2e-16	1.1e-08	3.3e-07	2.6e-08	—	—	—	—	—	—	—
P1	1.4e-07	0.11625	0.38401	0.16139	0.01223	—	—	—	—	—	—
P2	2.4e-15	4.8e-06	8.2e-05	9.6e-06	1.00000	0.24845	—	—	—	—	—
P3	< 2e-16	2.6e-09	6.3e-08	5.8e-09	1.00000	0.00165	0.80912	—	—	—	—
RBFNN0.1	1.00000	0.00024	1.6e-05	0.00013	< 2e-16	1.4e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	0.47238	0.16139	0.38401	1.5e-15	3.0e-05	6.3e-12	1.5e-15	0.30184	—	—
RBFNN3	4.8e-07	0.18488	0.54059	0.24845	0.00570	1.00000	0.16139	0.00074	6.8e-11	8.2e-05	—
RBFNN6	< 2e-16	5.3e-08	1.4e-06	1.2e-07	1.00000	0.02843	1.00000	1.00000	< 2e-16	1.2e-14	0.01351

P value adjustment method: holm

## SR

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 323.66, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	0.03252	-	-	-	-	-	-	-	-	-	-
GRNN3	0.01701	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.03252	1.00000	1.00000	-	-	-	-	-	-	-	-
Kriging	< 2e-16	1.3e-08	4.3e-08	1.3e-08	-	-	-	-	-	-	-
P1	2.3e-07	0.22129	0.30068	0.22129	0.00623	-	-	-	-	-	-
P2	2.2e-14	3.9e-05	9.2e-05	3.9e-05	1.00000	0.30961	-	-	-	-	-
P3	< 2e-16	5.2e-10	1.6e-09	5.2e-10	1.00000	0.00028	0.30961	-	-	-	-
RBFNN0.1	1.00000	0.00013	5.4e-05	0.00013	< 2e-16	2.6e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.30961	0.22201	0.30961	2.7e-16	3.0e-05	2.4e-11	< 2e-16	0.30961	-	-
RBFNN3	2.6e-07	0.22166	0.30961	0.22166	0.00571	1.00000	0.30961	0.00026	3.2e-11	3.4e-05	-
RBFNN6	< 2e-16	3.0e-07	8.6e-07	3.0e-07	1.00000	0.03506	1.00000	1.00000	< 2e-16	2.1e-14	0.03279

P value adjustment method: holm

## MAE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 331.7, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple  
comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	-	-	-	-	-	-	-	-	-	-
GRNN3	0.24579	1.00000	-	-	-	-	-	-	-	-	-
GRNN6	0.00741	0.33914	1.00000	-	-	-	-	-	-	-	-
Kriging	4.3e-08	4.8e-12	4.3e-16	< 2e-16	-	-	-	-	-	-	-
P1	0.31521	0.00633	4.3e-05	7.4e-08	0.00554	-	-	-	-	-	-
P2	8.6e-06	3.6e-09	9.5e-13	< 2e-16	1.00000	0.11043	-	-	-	-	-
P3	1.8e-07	7.9e-11	3.0e-14	< 2e-16	1.00000	0.00553	1.00000	-	-	-	-
RBFNN0.1	6.1e-05	0.01532	0.34271	1.00000	< 2e-16	4.1e-11	< 2e-16	< 2e-16	-	-	-
RBFNN1	1.00000	0.80432	0.05386	0.00078	1.2e-06	0.92307	0.00014	3.2e-06	3.4e-06	-	-
RBFNN3	0.04449	0.00027	8.1e-07	5.4e-10	0.06481	1.00000	0.61235	0.05252	1.3e-13	0.21611	-
RBFNN6	5.2e-07	1.0e-10	1.5e-14	< 2e-16	1.00000	0.02387	1.00000	1.00000	< 2e-16	1.1e-05	0.21611

P value adjustment method: holm

## MAX

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 332.11, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.00000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.03706	0.93383	—	—	—	—	—	—	—	—	—
GRNN6	0.00035	0.05544	1.00000	—	—	—	—	—	—	—	—
Kriging	7.9e-07	1.0e-10	< 2e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.72775	0.01905	1.0e-05	7.4e-09	0.00813	—	—	—	—	—	—
P2	0.00055	4.2e-07	3.6e-12	< 2e-16	1.00000	0.35950	—	—	—	—	—
P3	2.1e-07	4.4e-11	< 2e-16	< 2e-16	1.00000	0.00179	0.83149	—	—	—	—
RBFNN0.1	1.5e-05	0.00813	0.83149	1.00000	< 2e-16	1.0e-10	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.00000	1.00000	0.54444	0.01905	1.4e-09	0.05544	3.5e-06	4.9e-10	0.00185	—	—
RBFNN3	0.63567	0.01451	6.6e-06	4.4e-09	0.01050	1.00000	0.42506	0.00233	5.7e-11	0.04595	—
RBFNN6	1.4e-05	3.8e-09	7.8e-15	< 2e-16	1.00000	0.04420	1.00000	1.00000	< 2e-16	4.2e-08	0.05364

P value adjustment method: holm

## RMSE

Kruskal-Wallis rank sum test

```
data: mval by model
Kruskal-Wallis chi-squared = 334.18, df = 11, p-value < 2.2e-16
```

Pairwise comparisons using Dunn's-test for multiple comparisons of independent samples

```
data: mval by model
```

	GRNN0.1	GRNN1	GRNN3	GRNN6	Kriging	P1	P2	P3	RBFNN0.1	RBFNN1	RBFNN3
GRNN1	1.0000	—	—	—	—	—	—	—	—	—	—
GRNN3	0.1254	1.0000	—	—	—	—	—	—	—	—	—
GRNN6	0.0025	0.2208	1.00000	—	—	—	—	—	—	—	—
Kriging	2.5e-07	2.0e-11	5.3e-16	< 2e-16	—	—	—	—	—	—	—
P1	0.5264	0.0108	4.1e-05	6.5e-08	0.0061	—	—	—	—	—	—
P2	9.7e-05	4.2e-08	5.0e-12	3.5e-16	1.0000	0.1973	—	—	—	—	—
P3	1.1e-07	1.7e-11	1.1e-15	< 2e-16	1.0000	0.0019	1.0000	—	—	—	—
RBFNN0.1	1.8e-05	0.0094	0.3692	1.0000	< 2e-16	4.6e-11	< 2e-16	< 2e-16	—	—	—
RBFNN1	1.0000	1.0000	0.1973	0.0056	7.8e-08	0.3560	3.8e-05	3.9e-08	4.8e-05	—	—
RBFNN3	0.1471	0.0012	1.9e-06	1.5e-09	0.0447	1.0000	0.6504	0.0137	5.8e-13	0.0900	—
RBFNN6	1.2e-06	1.4e-10	5.2e-15	< 2e-16	1.0000	0.0150	1.0000	1.0000	< 2e-16	3.7e-07	0.0997

P value adjustment method: holm

## References

- Amrhein, M. and Krein, P. T. (2007). Magnetic equivalent circuit modeling of induction machines - Design-oriented approach with extension to 3-D. *Proceedings of IEEE International Electric Machines and Drives Conference, IEMDC 2007*, 2:1557–1563.
- Audet, C., Kokkolaras, M., Le Digabel, S., and Talgorn, B. (2016). Order-based error for managing ensembles of surrogates in derivative-free optimization. *Les Cahiers du Gerad*, G-2016-36.
- Bandyopadhyay, S. and Mukherjee, A. (2015). An algorithm for many-objective optimization with reduced objective computations: A study in differential evolution. *IEEE Transactions on Evolutionary Computation*, 19(3):400–413.
- Barthelemy, J. F. M. and Haftka, R. T. (1993). Approximation concepts for optimum structural design - a review. *Structural Optimization*, 5(3):129–144.
- Bencala, K. E. and Seinfeld, J. H. (1979). An air quality model performance assessment package. *Atmospheric Environment (1967)*, 13(8):1181 – 1185.
- Bischl, B., Trautmann, H., and Weihs, C. (2012). Resampling Methods for Meta-Model Validation with Recommendations for Evolutionary Computation. *Evolutionary Computation*, 20(2):249–275.
- Booker, A. J., Dennis Jr, J., Frank, P. D., Serafini, D. B., Torczon, V., and Trosset, M. W. (1999). A rigorous framework for optimization of expensive functions by surrogates. *Structural optimization*, 17(1):1–13.
- Box, G. E. P. and Wilson, K. B. (1951). On the Experimental Attainment of Optimum Conditions. *Journal of the Royal Statistical Society, Series B*, XIII(1):1–45.
- Brockhoff, D. and Zitzler, E. (2006). Are all objectives necessary? on dimensionality reduction in evolutionary multiobjective optimization. In *Parallel Problem Solving from*

- Nature-PPSN IX*, pages 533–542. Springer.
- Brockhoff, D. and Zitzler, E. (2007). Improving hypervolume-based multiobjective evolutionary algorithms by using objective reduction methods. In *Evolutionary Computation, 2007. CEC 2007. IEEE Congress on*, pages 2086–2093. IEEE.
- Brockhoff, D. and Zitzler, E. (2009a). Objective reduction in evolutionary multiobjective optimization: Theory and applications. *Evol. Comput.*, 17(2):135–166.
- Brockhoff, D. and Zitzler, E. (2009b). Objective reduction in evolutionary multiobjective optimization: Theory and applications. *Evolutionary Computation*, 17(2):135–166.
- Broomhead, D. and Lowe, D. (1988). Multivariable functional interpolation and adaptive networks. *Complex Systems*, 2:321–355.
- Cai, X., Qiu, H., Gao, L., and Shao, X. (2017). Metamodeling for high dimensional design problems by multi-fidelity simulations. *Structural and Multidisciplinary Optimization*, pages 1–16.
- Chai, T. and Draxler, R. R. (2014). Root mean square error (rmse) or mean absolute error (mae)? arguments against avoiding rmse in the literature. *Geoscientific Model Development*, 7(3):1247–1250.
- Clarke, S. M., Griebsch, J. H., and Simpson, T. W. (2005). Analysis of Support Vector Regression for Approximation of Complex Engineering Analyses. *Journal of Mechanical Design*, 127(6):1077–1087.
- Coello, C. A. C., Lamont, G. B., and Veldhuizen, D. A. V. (2006). *Evolutionary Algorithms for Solving Multi-Objective Problems (Genetic and Evolutionary Computation)*. Springer-Verlag New York, Inc., Secaucus, NJ, USA.
- Corne, D. W. and Knowles, J. D. (2003). No free lunch and free leftovers theorems for multiobjective optimisation problems. In *Evolutionary Multi-Criterion Optimization (EMO 2003) Second International Conference*, pages 327–341. Springer LNCS.
- Cox, W. M. and Tikvart, J. A. (1990). A statistical procedure for determining the best performing air quality simulation model. *Atmospheric Environment. Part A. General*

- Topics*, 24(9):2387 – 2395.
- Cui, C., Hu, M., Weir, J. D., and Wu, T. (2016). A recommendation system for meta-modeling: A meta-learning based approach. *Expert Systems with Applications*, 46:33–44.
- Dasgupta, D. and Michalewicz, Z. (2013). *Evolutionary algorithms in engineering applications*. Springer Science & Business Media.
- Deb, K. (2001). *Multi-Objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons, Inc., New York, NY, USA.
- Deb, K. and Saxena, D. (2006). Searching for pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems. In *Proceedings of the World Congress on Computational Intelligence (WCCI-2006)*, pages 3352–3360.
- Deb, K., Thiele, L., Laumanns, M., and Zitzler, E. (2002). Scalable Multi-Objective Optimization Test Problems. In *Congress on Evolutionary Computation (CEC 2002)*, pages 825–830. IEEE Press.
- Di Barba, P. (2010). *Multiobjective Shape Design in Electricity and Magnetism*. Lecture Notes in Electrical Engineering. Springer.
- Díaz-Manríquez, A., Toscano, G., and Coello Coello, C. A. (2016). Comparison of meta-modeling techniques in evolutionary algorithms. *Soft Computing*, pages 1–17.
- Díaz-Manríquez, A., Toscano-Pulido, G., and Gómez-Flores, W. (2011). On the selection of surrogate models in evolutionary optimization algorithms. In *2011 IEEE Congress of Evolutionary Computation (CEC)*, pages 2155–2162.
- Fleming, P. J., Purshouse, R. C., and Lygoe, R. J. (2005). Many-objective optimization: An engineering design perspective. In *International conference on evolutionary multi-criterion optimization*, pages 14–32. Springer.
- Forrester, A. I. and Keane, A. J. (2009). Recent advances in surrogate-based optimization. *Progress in Aerospace Sciences*, 45(13):50 – 79.

- Forrester, A. I. J., Sobester, A., and Keane, A. J. (2008). *Engineering Design via Surrogate Modelling - A Practical Guide*. Wiley.
- Fox, D. G. (1981). Judging Air Quality Model Performance.
- Freitas, A. R., Silva, R. C., and Guimarães, F. G. (2014). On the visualization of trade-offs and reducibility in many-objective optimization. In *Proceedings of the Companion Publication of the 2014 Annual Conference on Genetic and Evolutionary Computation*, GECCO Comp '14, pages 1091–1098, New York, NY, USA. ACM.
- Friedman, J. H. (1991). Multivariate adaptive regression splines. *The Annals of Statistics*, 19(1):1–67.
- Friedman, L. W. (1996). *The simulation metamodel*. Springer US.
- Friedman, L. W. and Friedman, H. H. (1985). Validating the simulation metamodel: Some practical approaches. *Simulation*, 45(3):144–146.
- Friedman, L. W. and Pressman, I. (1988). The metamodel in simulation analysis: Can it be trusted? *Journal of the Operational Research Society*, pages 939–948.
- Frigg, R. (2008). Models in science. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*.
- Frigg, R. and Nguyen, J. (2017). Scientific Representation Is Representation-As. *Philosophy of Science in Practice*, pages 149–179.
- Gal, T. and Leberling, H. (1977). Redundant objective functions in linear vector maximum problems and their determination. *European Journal of Operational Research*, 1(3):176–184.
- Gano, S., Kim, H., and Brown, D. (2006). Comparison of three surrogate modeling techniques: Datascape, kriging, and second order regression. In *11th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, page 7048.
- Gauch, H. G. (2003). *Scientific method in practice*. Cambridge University Press.
- Ghorbanian, V. and Lowther, D. A. (2017). Magnetic and electrical design challenges of inverter-fed permanent magnet synchronous motors. *IEEE Transactions on Magnetics*,

53(6):1–4.

- Ghorbanian, V., Salimi, A., and Lowther, D. (2017). A computer-aided design process for optimizing the size of inverter-fed permanent magnet motors. *IEEE Transactions on Industrial Electronics*, PP(99):1–1.
- Giunta, A. and Watson, L. (1998). A comparison of approximation modeling techniques—polynomial versus interpolating models. In *7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, page 4758.
- Goldstein, G. B. and Dushane, T. (1976). Repro-modeling applied to the simplification of taradcom computer models. Technical report, DTIC Document.
- Gorissen, D., Dhaene, T., and Turck, F. D. (2009). Evolutionary model type selection for global surrogate modeling. *J. Mach. Learn. Res.*, 10:2039–2078.
- Guimarães, F. G. (2008). *Aprendizagem e Busca Local em Algoritmos Meméticos para Projeto Assistido por Computador*. PhD thesis, Universidade Federal de Minas Gerais.
- Guo, X., Wang, Y., and Wang, X. (2016). An objective reduction algorithm using representative pareto solution search for many-objective optimization problems. *Soft Computing*, 20(12):4881–4895.
- Hammond, P. and Sykulski, J. K. (1994). *Engineering electromagnetism: physical processes and computation*. Oxford University Press.
- Hilborn, R. and Mangel, M. (1997). *The ecological detective: confronting models with data*, volume 28. Princeton University Press.
- Huband, S., Hingston, P., Barone, L., and While, L. (2006). A review of multiobjective test problems and a scalable test problem toolkit. *Trans. Evol. Comp*, 10(5):477–506.
- Ishibuchi, H., Masuda, H., and Nojima, Y. (2015). A study on performance evaluation ability of a modified inverted generational distance indicator. In *Proceedings of the 2015 Annual Conference on Genetic and Evolutionary Computation*, pages 695–702. ACM.
- Ishibuchi, H., Tsukamoto, N., and Nojima, Y. (2008). Evolutionary many-objective optimization: A short review. In *Evolutionary Computation, 2008. CEC 2008.(IEEE World Congress on Computational Intelligence)*, pages 1–8. IEEE.

- Congress on Computational Intelligence). IEEE Congress on*, pages 2419–2426. IEEE.
- Iuliano, E. and Quagliarella, D. (2013). Proper Orthogonal Decomposition, surrogate modelling and evolutionary optimization in aerodynamic design. *Computers & Fluids*, 84:327–350.
- Jin, R., Chen, W., and Simpson, T. (2001). Comparative studies of metamodeling techniques under multiple modelling criteria. *Structural and Multidisciplinary Optimization*, 23(1):1–13.
- Jin, R., Du, X., and Chen, W. (2003). The use of metamodeling techniques for optimization under uncertainty. *Structural and Multidisciplinary Optimization*, 25(2):99–116.
- Jin, Y. (2005). A comprehensive survey of fitness approximation in evolutionary computation. *Soft Computing*, 9(1):3–12.
- Jones, D. R. (2001). A taxonomy of global optimization methods based on response surfaces. *J. of Global Optimization*, 21(4):345–383.
- Kendall, M. G. (1938). A new measure of rank correlation. *Biometrika*, 30(1/2):81–93.
- Kern, S., Hansen, N., and Koumoutsakos, P. (2006). Local meta-models for optimization using evolution strategies. In *Proceedings of the 9th International Conference on Parallel Problem Solving from Nature*, PPSN’06, pages 939–948, Berlin, Heidelberg. Springer-Verlag.
- Kleijnen, J. P. (1993). Simulation and optimization in production planning. *Decision Support Systems*, 9(3):269 – 280.
- Kleijnen, J. P. C. (2015). *Design and analysis of simulation experiments*, volume volume 230.
- Knight, W. R. (1966). A computer method for calculating kendall’s tau with ungrouped data. *Journal of the American Statistical Association*, 61(314):436–439.
- Kou, Y. N., Zheng, J. H., Li, Z., and Wu, Q. H. (2017). Many-objective optimization for coordinated operation of integrated electricity and gas network. *Journal of Modern Power Systems and Clean Energy*, pages 1–14.

- Koziel, S. and Leifsson, L. (2013). *Surrogate-Based Modeling and Optimization: Applications in Engineering*. SpringerLink : Bücher. Springer.
- Krige, D. G. (1953). A Statistical Approach to Some Basic Mine Valuation Problems on the Witwatersrand. *OR*, 4(1).
- Landau, L. and Lifshitz, E. (1987). *Fluid Mechanics*. Pergamon, 2nd edition.
- Lebensztajn, L. (2004). Kriging: a useful tool for electromagnetic device optimization. *IEEE Transactions on Magnetics*, 40(2):1196–1199.
- Li, M., Guimares, F., and Lowther, D. A. (2014). A competitive co-evolutionary algorithm for constrained robust design. In *9th IET International Conference on Computation in Electromagnetics (CEM 2014)*, pages 1–2.
- Li, M., Silva, R., aes, F. G., and Lowther, D. (2015). A new robust dominance criterion for multiobjective optimization. *IEEE Transactions on Magnetics*, 51(3):1–4.
- Li, M., Silva, R., and Lowther, D. (2016). Global and local meta-models for the robust design of electrical machines. *International Journal of Applied Electromagnetics and Mechanics*, 51(s1):89–95.
- Li, Y., Ng, S., Xie, M., and Goh, T. (2010). A systematic comparison of metamodeling techniques for simulation optimization in Decision Support Systems. *Applied Soft Computing*, 10(4):1257–1273.
- Lipps, G. F. (1905). *Die Bestimmung der Abhängigkeit zwischen den Merkmalen eines Gegenstandes*.
- López Jaimes, A., Coello, C. A. C., and Urías Barrientos, J. E. (2009). *Online Objective Reduction to Deal with Many-Objective Problems*, pages 423–437. Springer Berlin Heidelberg, Berlin, Heidelberg.
- López Jaimes, A., Coello Coello, C. A., and Chakraborty, D. (2008). Objective reduction using a feature selection technique. In *Proceedings of the 10th Annual Conference on Genetic and Evolutionary Computation*, GECCO '08, pages 673–680, New York, NY, USA. ACM.

- Loshchilov, I., Schoenauer, M., and Sebag, M. (2010). Comparison-based optimizers need comparison-based surrogates. In *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, volume 6238 LNCS, pages 364–373.
- Maxwell, J. (1873). *A Treatise on Electricity and Magnetism*. Clarendon Press.
- Mendes, M. H., Soares, G. L., Coulomb, J.-L., and Vasconcelos, J. A. (2013). A surrogate genetic programming based model to facilitate robust multi-objective optimization: A case study in magnetostatics. *IEEE transactions on magnetics*, 49(5):2065–2068.
- Montgomery, D. C. (2006). *Design and analysis of experiments*. John Wiley & Sons.
- Montgomery, D. C. (2017). *Design and analysis of experiments*. John Wiley & Sons.
- Myers, R. H. and Montgomery, D. C. (1995). *Response Surface Methodology: Process and Product in Optimization Using Designed Experiments*. John Wiley & Sons, Inc., New York, NY, USA, 1st edition.
- Myers, R. H. and Montgomery, D. C. (2002). *Response Surface Methodology: Process and Product in Optimization Using Designed Experiments*. John Wiley & Sons, Inc., New York, NY, USA, 2nd edition.
- Nagelkerke, N. J. (1991). A note on a general definition of the coefficient of determination. *Biometrika*, 78(3):691–692.
- Nappo Jr, C. J. (1975). A method for evaluating the accuracy of air pollution prediction models september 1974. *Environmental Research Laboratories*, page 163.
- Papadrakakis, M., Lagaros, N. D., and Tsompanakis, Y. (1998). Structural optimization using evolution strategies and neural networks. *Computer Methods in Applied Mechanics and Engineering*, 156(14):309 – 333.
- Pearson, K. (1895). Note on regression and inheritance in the case of two parents. *Proceedings of the Royal Society of London*, 58:240–242.
- Pearson, R. (2011). *Exploring Data in Engineering, the Sciences, and Medicine*. Oxford University Press, USA.

- Pilát, M. and Neruda, R. (2013). Aggregate meta-models for evolutionary multiobjective and many-objective optimization. *Neurocomputing*, 116:392–402.
- Pirie, W. (2004). *Spearman Rank Correlation Coefficient*. John Wiley & Sons, Inc.
- Powell, J. D. (1988). Numerical analysis. chapter Radial Basis Function Approximations to Polynomials, pages 223–241. Longman Publishing Group, White Plains, NY, USA.
- Queipo, N. V., Haftka, R. T., Shyy, W., Goel, T., Vaidyanathan, R., and Kevin Tucker, P. (2005). Surrogate-based analysis and optimization. *Progress in Aerospace Sciences*, 41(1):1–28.
- Redhead, M. (1980). Models in Physics. *The British Journal for the Philosophy of Science*, 31(2):145–163.
- Rosen, R. (1991). *Life itself: a comprehensive inquiry into the nature, origin, and fabrication of life*. Columbia University Press.
- Runarsson, T. P. (2004). Constrained evolutionary optimization by approximate ranking and surrogate models. *Parallel Problem Solving From Nature - PPSN VIII, International Conference, Birmingham, Uk, September 18-22, 2004, Proceedings*, Springer, 3242:401–410.
- Runarsson, T. P. (2006). Ordinal Regression in Evolutionary Computation. *Proceedings of Parallel Problem Solving from Nature-PPSN IX*, pages 1048–1057.
- Sacks, J., Welch, W., Mitchell, T., and Wynn, H. (1989). Design and analysis of computer experiments. *Statistical science*, 4(4):409–423.
- Salimi, A. (2017). *Computer-Aided Design of Electrical Machines: The Role of Robustness and Application of Statistical Analysis Techniques in Multi-Objective Design Optimization*. PhD thesis, McGill University.
- Salimi, A. and Lowther, D. A. (2016). On the role of robustness in multi-objective robust optimization: Application to an ipm motor design problem. *IEEE Transactions on Magnetics*, 52(3):1–4.

- Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M., and Tarantola, S. (2008). *Global sensitivity analysis: the primer*. John Wiley & Sons.
- Sarcar, M., Rao, K. M., and Narayan, K. L. (2008). *Computer aided design and manufacturing*. PHI Learning Pvt. Ltd.
- Saxena, D. K., Duro, J. A., Tiwari, A., Deb, K., and Zhang, Q. (2013). Objective reduction in many-objective optimization: Linear and nonlinear algorithms. *IEEE Transactions on Evolutionary Computation*, 17(1):77–99.
- Seada, H. and Deb, K. (2015). U-nsga-iii: A unified evolutionary optimization procedure for single, multiple, and many objectives: Proof-of-principle results. In *International Conference on Evolutionary Multi-Criterion Optimization*, pages 34–49. Springer.
- Silva, R., Li, M., and Lowther, D. A. (2016a). The role of coarse models in space-mapping: A study on an ipm motor optimization. *International Journal of Applied Electromagnetics and Mechanics*, 51(s1):S147–S156.
- Silva, R., Salimi, A., Li, M., Freitas, A. R. R., Guimares, F. G., and Lowther, D. A. (2016b). Visualization and analysis of tradeoffs in many-objective optimization: A case study on the interior permanent magnet motor design. *IEEE Transactions on Magnetics*, 52(3):1–4.
- Silva, R. C. P., Li, M., Rahman, T., and Lowther, D. A. (2017a). Surrogate-based moea/d for electric motor design with scarce function evaluations. *IEEE Transactions on Magnetics*, 53(6):1–4.
- Silva, R. C. P., Rahman, T., Mohammadi, M. H., and Lowther, D. A. (2017b). Multiple operating points based optimization: Application to fractional slot concentrated winding electric motors. *IEEE Transactions on Industrial Electronics*, PP(99):1–1.
- Simpson, T., Lin, D., and Chen, W. (2001). Sampling strategies for computer experiments: design and analysis. *International Journal of Reliability and Safety (IJRS)*, 2(3):209–240.
- Singh, H. K., Isaacs, A., and Ray, T. (2011). A pareto corner search evolutionary algorithm and dimensionality reduction in many-objective optimization problems. *IEEE Transactions on Evolutionary Computation*, 15(4):539–556.

- Sinha, A., Saxena, D. K., Deb, K., and Tiwari, A. (2013). Using objective reduction and interactive procedure to handle many-objective optimization problems. *Appl. Soft Comput.*, 13(1):415–427.
- Sobieszczański-Sobieski, J. and Haftka, R. T. (1997). Multidisciplinary aerospace design optimization: survey of recent developments. *Structural optimization*, 14(1):1–23.
- Spearman, C. (1904). The proof and measurement of association between two things. *The American journal of psychology*, 15(1):72–101.
- Specht, D. F. (1991). A general regression neural network. *IEEE Transactions on Neural Networks*, 2(6):568–576.
- Tian, Y., Cheng, R., Zhang, X., and Jin, Y. (2017). Platemo: a matlab platform for evolutionary multi-objective optimization. *arXiv preprint arXiv:1701.00879*.
- Tichelaar, B. W. and Ruff, L. J. (1989). How good are our best models? jackknifing, bootstrapping, and earthquake depth. *Eos, Transactions American Geophysical Union*, 70(20):593–606.
- Tiegna, H., Amara, Y., and Barakat, G. (2013). Overview of analytical models of permanent magnet electrical machines for analysis and design purposes. *Mathematics and Computers in Simulation*, 90:162 – 177. {ELECTRIMACS} 2011- {PART} I.
- Varadarajan, S., Shen, W., and Pelka, C. J. (2000). Robust concept exploration of propulsion systems with enhanced model approximation capabilities. *Engineering Optimization*, 32(3):309–334.
- Viana, F., Haftka, R., and Watson, L. (2013). Efficient global optimization algorithm assisted by multiple surrogate techniques. *Journal of Global Optimization*, 56(2):669–689.
- Viana, F. A. (2013). Things you wanted to know about the latin hypercube design and were afraid to ask. In *10th World Congress on Structural and Multidisciplinary Optimization, Orlando, Florida, USA (cf. p. 69)*.
- Viana, F. A. C., Haftka, R. T., and Steffen, V. (2009). Multiple surrogates: how cross-validation errors can help us to obtain the best predictor. *Structural and Multidisciplinary*

- Optimization*, 39(4):439–457.
- Voutchkov, I. and Keane, A. (2010). *Multi-Objective Optimization Using Surrogates*, pages 155–175. Springer Berlin Heidelberg, Berlin, Heidelberg.
- Wagner, T., Beume, N., and Naujoks, B. (2007). Pareto-, aggregation-, and indicator-based methods in many-objective optimization. In *Evolutionary multi-criterion optimization*, pages 742–756. Springer.
- Wang, G. G. and Shan, S. (2007). Review of Metamodeling Techniques in Support of Engineering Design Optimization. *Journal of Mechanical Design*, 129(4):370–380.
- Wang, L. and Lowther, D. (2006). Selection of approximation models for electromagnetic device optimization. *IEEE Transactions on Magnetics*, 42(4):1227–1230.
- Wang, Z. and Bovik, A. C. (2009). Mean squared error: Love it or leave it? a new look at signal fidelity measures. *IEEE Signal Processing Magazine*, 26(1):98–117.
- Willmott, C. J. (1981). On the validation of models. *Physical geography*, 2(2):184–194.
- Willmott, C. J. (1982). Some comments on the evaluation of model performance. *Bulletin of the American Meteorological Society*, 63(11):1309–1313.
- Willmott, C. J. (1984). On the evaluation of model performance in physical geography. *Spatial statistics and models*, (1967):443–460.
- Willmott, C. J., Ackleson, S. G., Davis, R. E., Feddema, J. J., Klink, K. M., Legates, D. R., O'Donnell, J., and Rowe, C. M. (1985). Statistics for the evaluation and comparison of models. *Journal of Geophysical Research: Oceans*, 90(C5):8995–9005.
- Willmott, C. J. and Matsuura, K. (2005). Advantages of the mean absolute error (mae) over the root mean square error (rmse) in assessing average model performance. *Climate Research*, 30(1):79–82.
- Wolpert, D. H. and Macready, W. G. (1997). No free lunch theorems for optimization. *IEEE Transactions on Evolutionary Computation*, 1(1):67–82.
- Yao, W., Chen, X., Huang, Y., and van Tooren, M. (2014). A surrogate-based optimization method with rbf neural network enhanced by linear interpolation and hybrid infill strategy.

- Optimization Methods and Software*, 29(2):406–429.
- Yu, B. and Popplewell, K. (1994). Metamodels in manufacturing: a review. *International Journal of Production Research*, 32(4):787–796.
- Yuan, Y., Ong, Y. S., Gupta, A., and Xu, H. (2017). Objective reduction in many-objective optimization: Evolutionary multiobjective approaches and comprehensive analysis. *IEEE Transactions on Evolutionary Computation*, PP(99):1–1.
- Zhan, D., Cheng, Y., and Liu, J. (2017). Expected improvement matrix-based infill criteria for expensive multiobjective optimization. *IEEE Transactions on Evolutionary Computation*.
- Zhang, Q. and Li, H. (2007). Moea/d: A multiobjective evolutionary algorithm based on decomposition. *IEEE Transactions on Evolutionary Computation*, 11(6):712–731.
- Zitzler, E., Deb, K., and Thiele, L. (2000a). Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. *Evolutionary Computation*, 8(2):173–195.
- Zitzler, E., Deb, K., and Thiele, L. (2000b). Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. *Evolutionary Computation*, 8(2):173–195.
- Zitzler, E. and Thiele, L. (1998). Multiobjective optimization using evolutionary algorithms - a comparative case study. In *Proceedings of the 5th International Conference on Parallel Problem Solving from Nature*, PPSN V, pages 292–304, London, UK, UK. Springer-Verlag.

## List of Abbreviations

**AKT** Adapted Kendall's Tau. 77

**AMS** American Meteorological Society. 15

**CAD** Computer-Aided Design. 3, 4, 13

**CCD** Central Composite Design. 33

**CFD** Computational Fluid Dynamics. 4

**DOE** Design of Experiment. 33, 34

**FEA** Finite Element Analysis. 13

**FEM** Finite Element Method. 4, 13, 101

**FSCW-SMPM** Fractional Slot Concentrated Winding Surface Mounted Permanent Magnet Machine. vii, 98, 114, 115, 117, 125, 185

**IGD** Inverse Generational Distance. 29

**IPM** Interior Permanent Magnet. 17, 98, 100, 106, 150, 159

**LHS** Latin-hypercube Sampling. 34, 35

**MAE** Mean Absolute Error. 15, 56, 57

**MAPE** Mean Absolute Percentual Error. 15, 56, 57, 59

**MARS** Multivariate Adaptive Regression Splines. 13

**MAX** Maximum Absolute Error. 15, 56, 57

**MEC** Magnetic Equivalent Circuit. 10

**MOEA/D** Multi-objective Evolutionary Algorithm Based on Decomposition. 84, 90

**MOOP** Multi-objective Optimization Problems. 25, 28

**MSE** Mean Squared Error. 56, 57, 60

**NWD** Normalized Weighted Disagreement. 79, 80

**PCA** Principal Component Analysis. 20

**RBF** Radial basis functions. 12

**RMSE** Root Mean Squared Error. 15, 56, 57

**RMSPE** Root Mean Squared Percentual Error. 15

**ROM** Reduced Order Model. 13

**SBO** Surrogate-based Optimization. 7, 8, 17, 21, 52, 53, 55, 57, 59, 62, 69

**SBOM** Surrogate-based Optimization Methods. 25

**SOOP** Single Objective Optimization Problems. 25

**SQP** Sequential Quadratic Programming. 126

**SVR** Support Vector Regression. 13